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Transverse momentum imaging
Lecture 1

Hampton University Graduate School (e-HUGS) 2021

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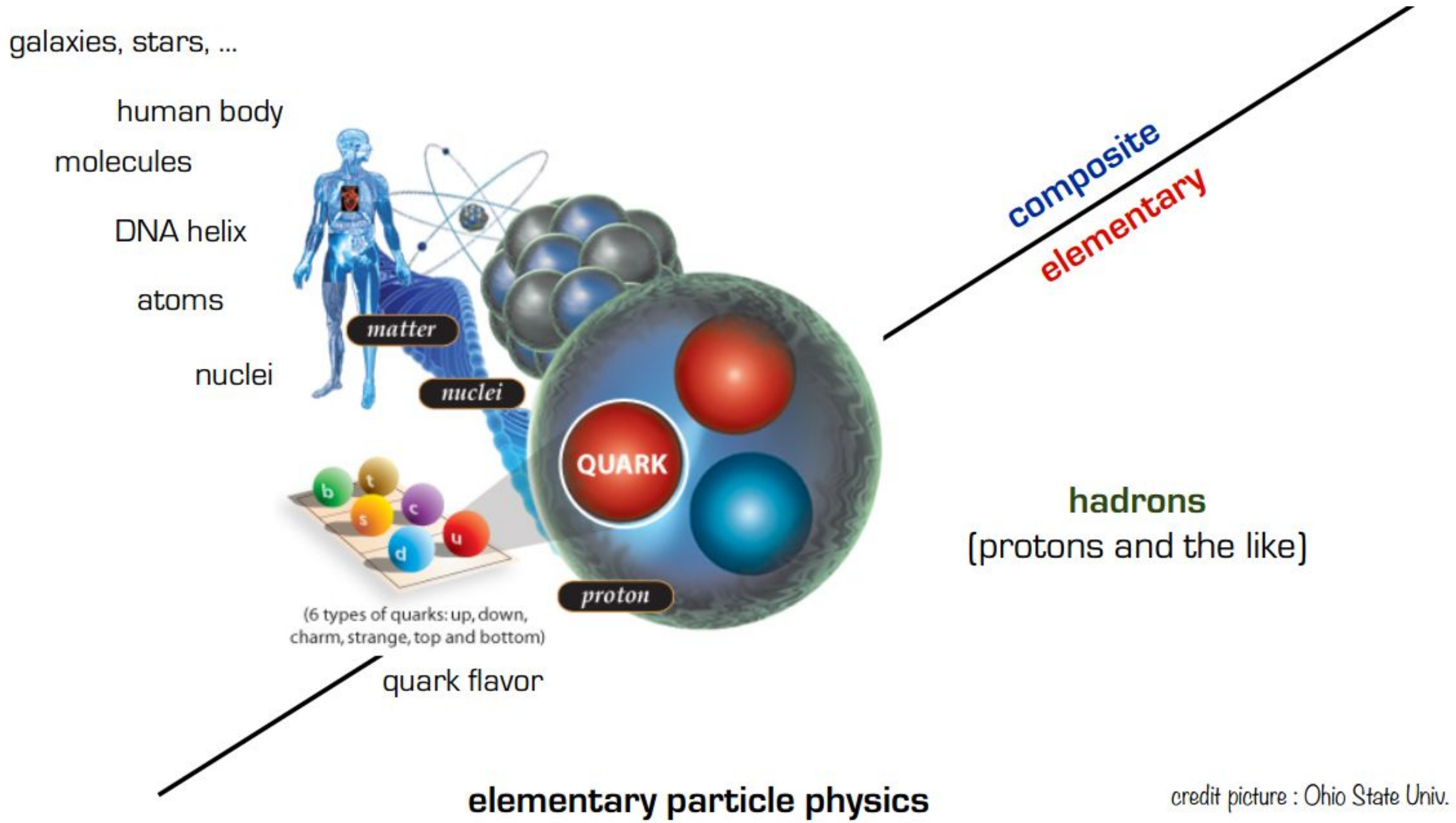
Introduction

The core of matter



Protons and neutrons: **> 99 % of the visible mass** in our world

Quantum Chromodynamics (QCD)

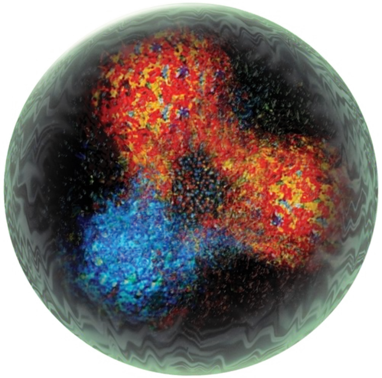
quarks and gluons (partons) are the elementary degrees of freedom in QCD, but they manifest only in bound states (**hadrons**)

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a,\mu\nu}F_{a,\mu\nu} + \overline{\psi}(i\not{D} - m)\psi$$

quark field

$$D_\mu = \partial_\mu - igT^a A^a_\mu$$

gluon field



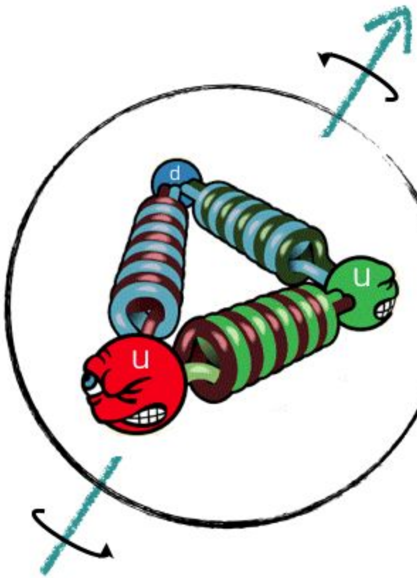
Can we understand the properties of hadrons in terms of quarks and gluons?

Global properties

Can we understand the

mass, spin, size
of hadrons

in terms of
quarks and gluons?



Confinement

Can we understand

hadron formation
and
confinement

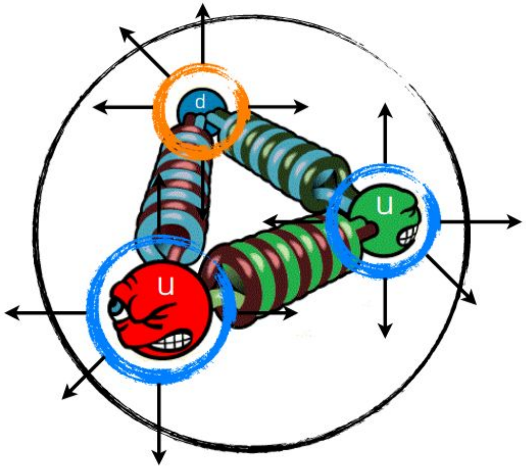
in terms of
quarks and gluons?

Internal structure

Can we understand the

structure of hadrons

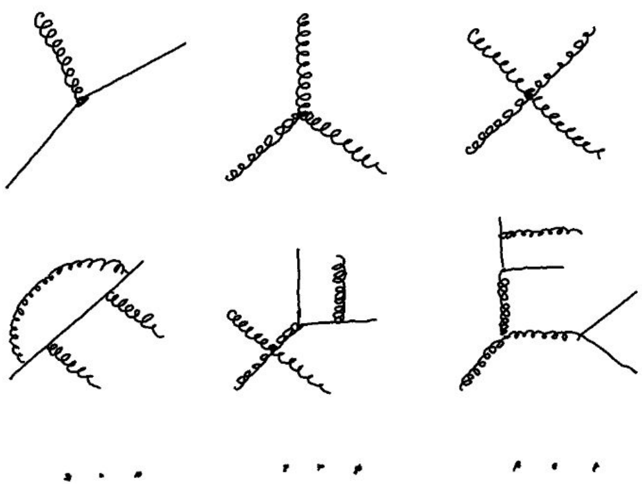
in terms of
quarks and gluons?



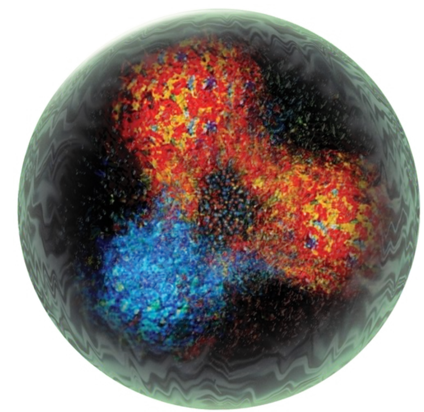
How should we “use” QCD ?

Expansion of observable in powers of the coupling constant α:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} \ldots = ??$$



? $\Longrightarrow$

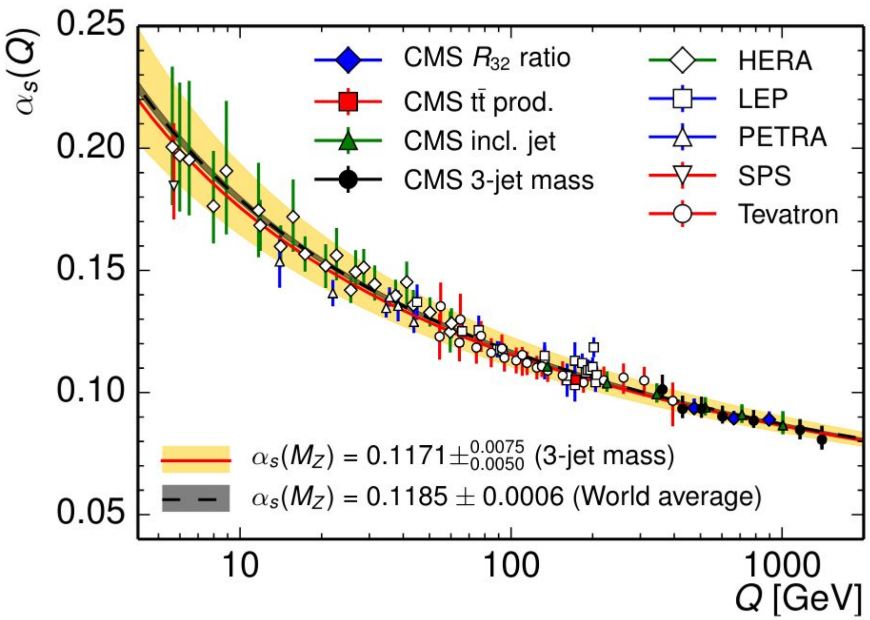


$$Q \sim M_N \sim 1 \text{ GeV}$$

How should we "use" QCD ?

Expansion of observable in powers of the coupling constant α:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} \ldots = ??$$



High energy → convergence
→ perturbative QCD

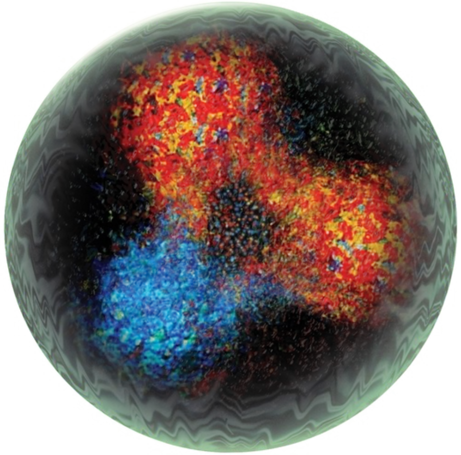
Low energy (hadronic scales)
→ non-perturbative QCD

need alternative techniques

Hadronic physics

Two macro areas to investigate:

1. **Hadron *structure*** : "hadrons → partons" transition



Hadronic physics

Two macro areas to investigate:

1. **Hadron *structure*** : "hadrons → partons" transition
2. **Hadron *formation*** : "partons → hadrons" transition
   (hadronization)

Credit picture: R. Seidl

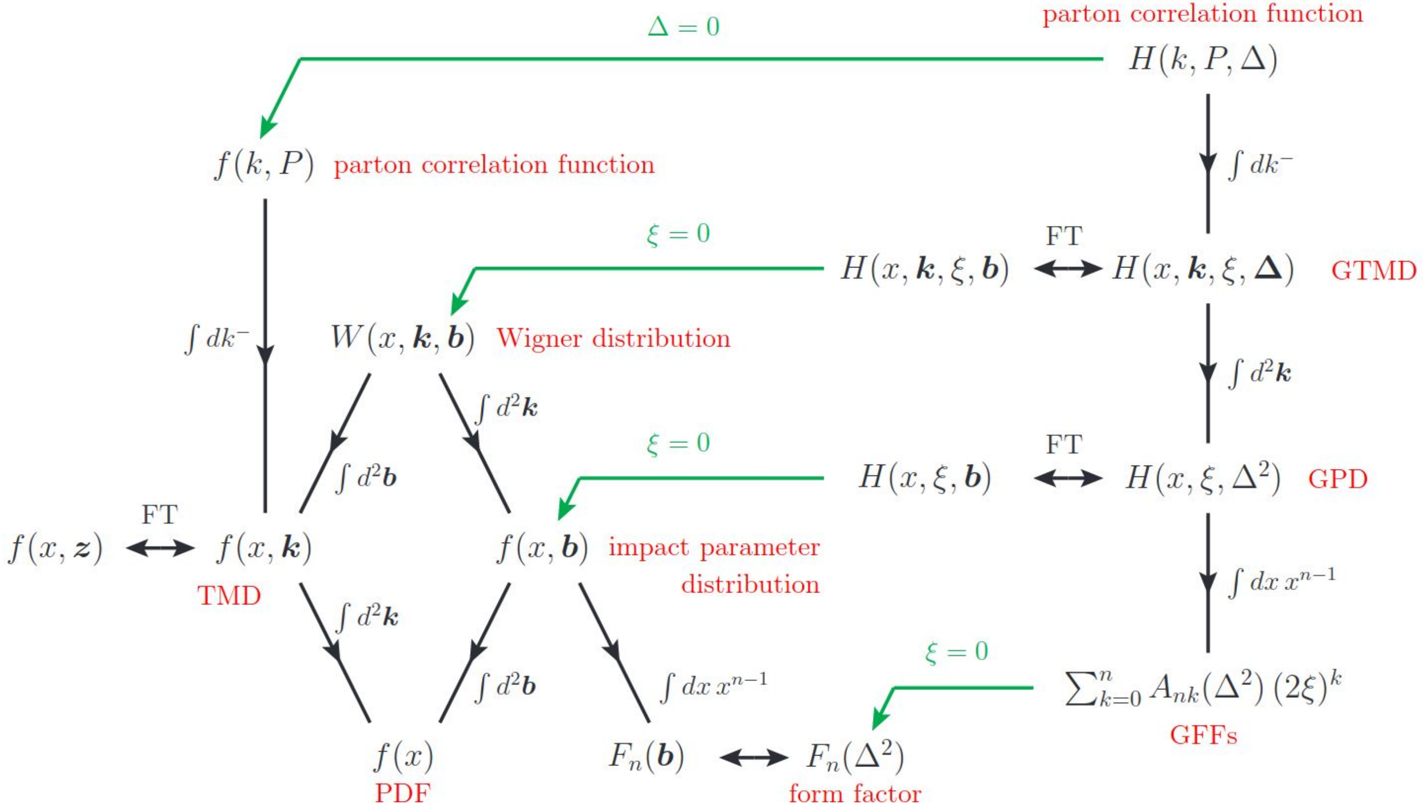
Hadronic physics

The motivations are:

1. **Conceptual** : understand confinement, dynamical breaking of chiral symmetry …
   make sense of the world!

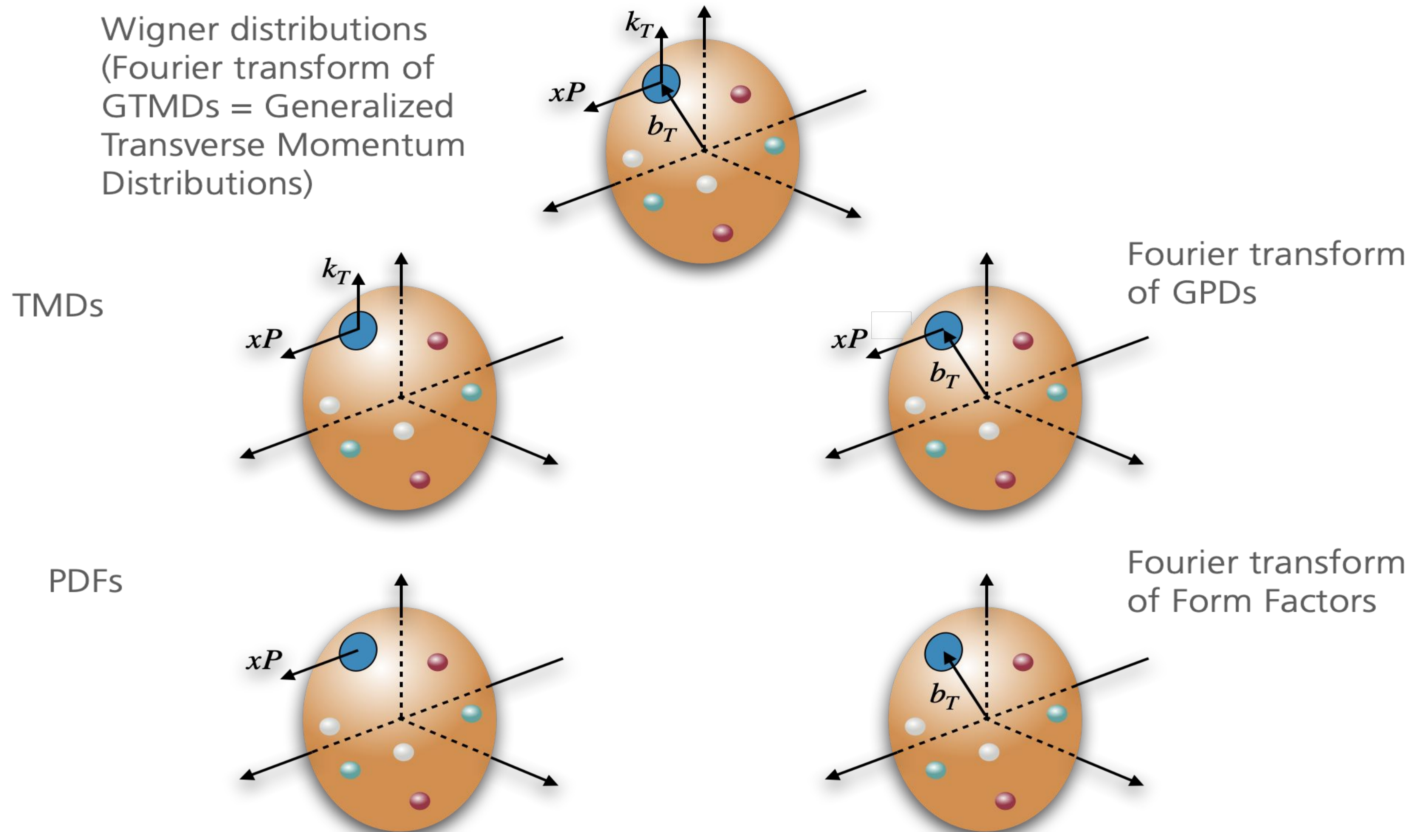
2. **Practical** : improve our understanding of scattering experiments …
   and *unlock >99% of the energy stored in matter!*

The hadron structure landscape



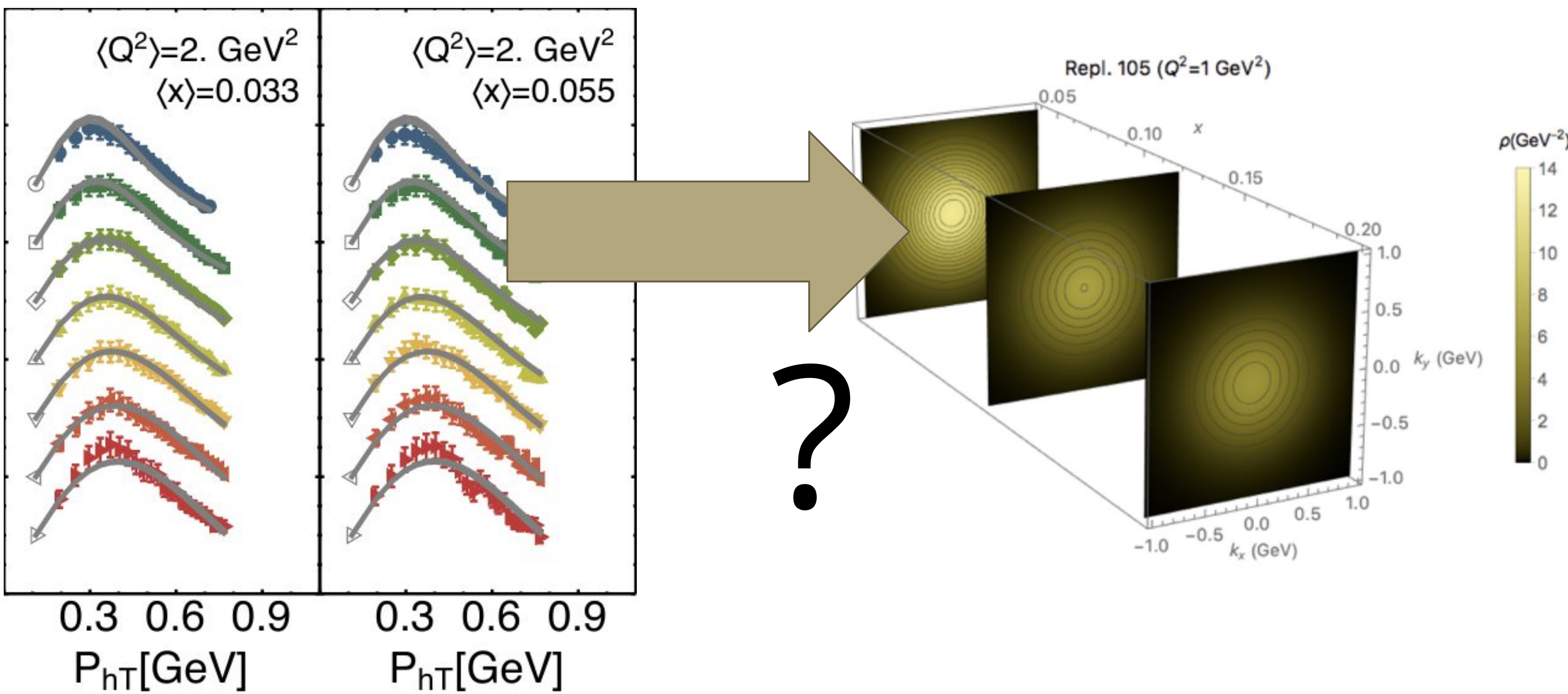
Credit picture: M. Diehl - [arXiv 1512.01328]

The hadron structure landscape



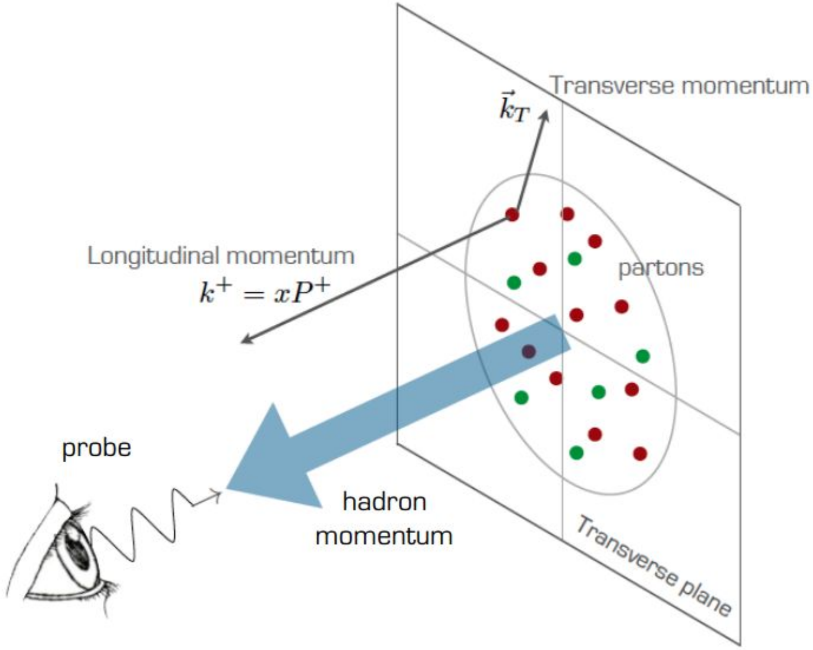
see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

Transverse momentum imaging



Parton distribution functions (PDFs)

“Maps” of hadron *structure* in momentum space



$f_1(x)$

1D structure
in momentum space

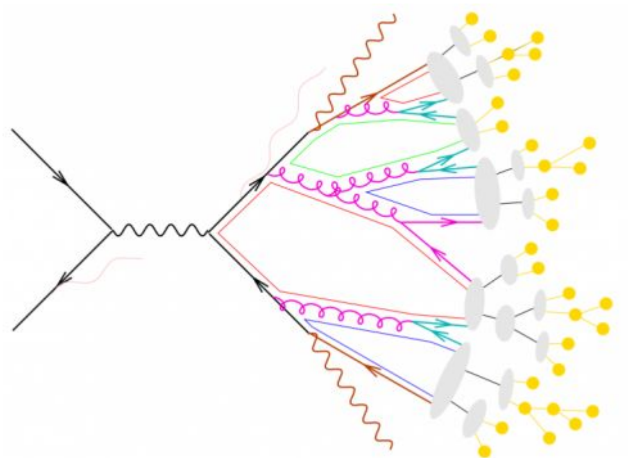
$f_1(x, k_T^2)$

3D structure
in momentum space

Credit picture: A. Bacchetta

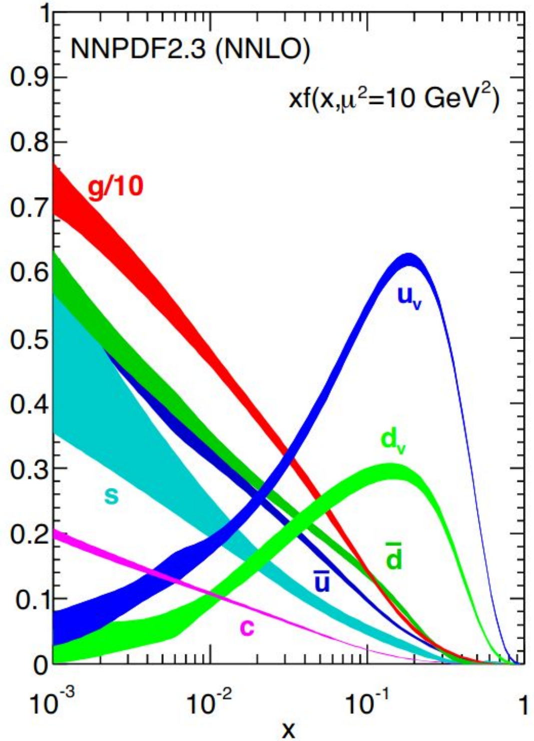
Fragmentation functions (FFs)

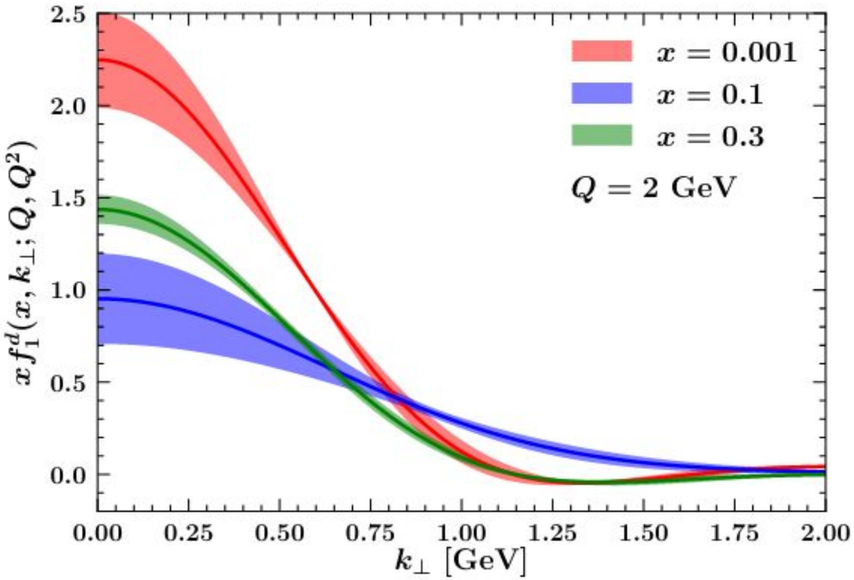
"Maps" of hadron *formation* in momentum space



$D_1^h(z)$	single-hadron collinear FF
$D_1^h(z, P_T^2)$	single-hadron TMD FF
$D_1^{h_1\,h_2}(z, \zeta)$	di-hadron FF
$J(s)$	inclusive jet FF
$\mathcal{G}^h(s, z)$	in-jet FF

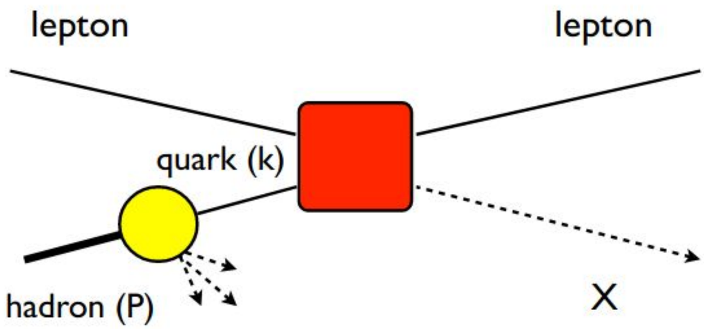
collinear & TMD PDFs





arXiv 1912.07550 (PV19 extraction)

Operator definition (PDFs)

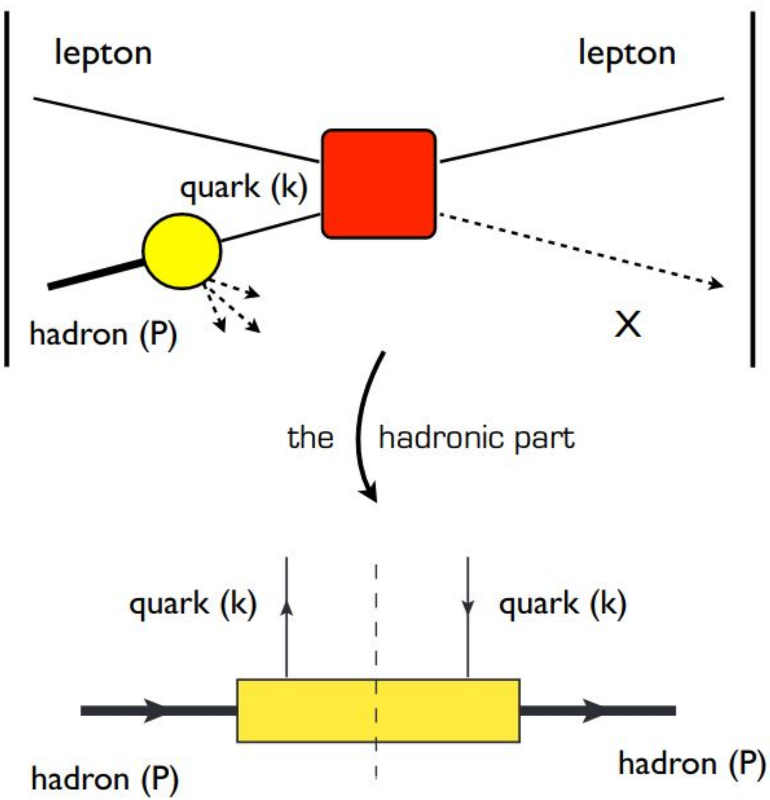


Scattering process with hadron in initial state :
(e.g. Deep Inelastic Scattering - DIS)

need a “hadron → parton” transition

(Parton Distribution Function)

Operator definition (PDFs)



PDFs defined as traces of Φ :

$$F^{[U]}(x, k_T^2) \sim \mathrm{Tr}\,[\Phi\,\Gamma]\;,\;\; \Gamma = \gamma^+\,,\ldots$$

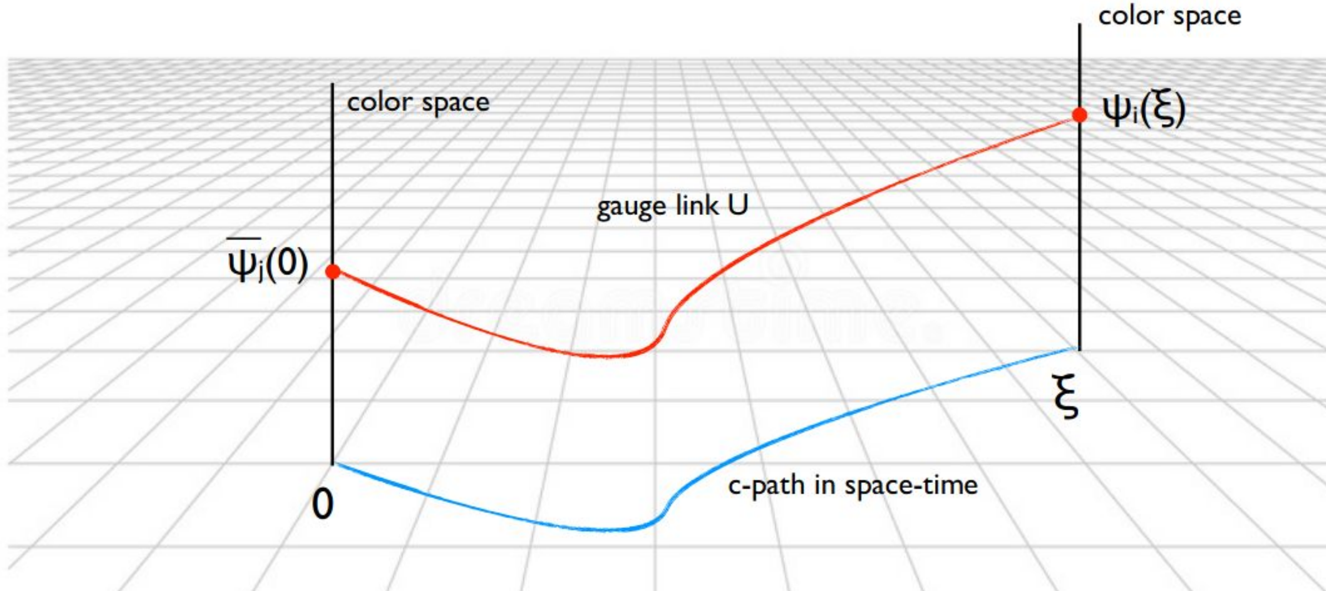
(***8 functions*** that depend on parton kinematics and gauge link U)

Hadronic part described as a ***universal*** "quark-quark correlation function" in space-time

$$\Phi_{ij}(k,P) = \mathrm{F.T.}\,\langle P|\,\overline{\psi}_j(0)\,U\,\psi_i(\xi)\,|P\rangle$$

Geometric structure

$$\Phi(k,P) = \text{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle \longrightarrow f_1^{a\ [U]}(x,k_T^2)\ \not{P} + \cdots$$



$$U(0,\xi) = \mathcal{P}\ \exp\left\{ -ig_s \int_{c(0,\xi)} ds_\mu A^{\mu a}(s) t^a \right\}$$

A selection of useful references

The HUGS pedagogical page

This is a list of references in preparation for and in support of the HUGS program. Further specific references will be suggested by the speakers. You are also welcome to browse the similarly aimed CTEQ pedagogical page, and to send us your comments and suggestions (hugs@jlab.org).

General texbooks

- Donnelly, Formaggio, Holstein, Milner, Surrow - Foundations of Nuclear and Particle Physics (2017)
  - Short, focused chapters covering practically all past, present, and near future HUGS topics!
- Povh, Rith, Scholz, Zetsche, Rodejohann - Particles and Nuclei (2015)
  - Good introductory level text
- Griffiths - Introduction to Elementary Particles (2008)
  - Another good introductory level text, more focused on the elementary particle aspects
- Halzen, Martin - Quarks and leptons (2008)
  - More advanced, treats QCD in some detail

(Perturbative) QCD

- W. K. Tung, Perturbative QCD and the parton structure of the nucleon
- K. Kovarik, P. M. Nadolski, D. E. Soper, Hadron structure in high-energy collisions
- B. Poetter, Calculational Techniques in Perturbative QCD: The Drell-Yan Process
- Textbooks:
  - J. Collins - Foundations of Perturbative QCD (2011)
  - Kovchegov, Levin - Quantum Chromodynamics at High Energy (2012)
- Check also:
  - The "Suggested QCD literature" list of references by T. Rogers

3D Structure of Nucleons

- Introductory:
  - A. Bacchetta, Transverse Momentum DIstributions (a.k.a."Trento lectures", 2012)
  - P. Mulders, Transverse-momentum distributions and beyond: setting up the nucleon tomography, lectures at the Galileo Galilei Institute (2015)
  - M. Diehl, Introduction to GPDs and TMDs, Eur.Phys.J. A52 (2016) 149
- P. Mulders, Transverse momentum dependence in structure functions in hard scattering processes
- M. Diehl, Lectures on GPDs, Varenna (ITA), 2011
- M. Diehl, Generalized Parton Distributions, Phys.Rept. 388 (2003) 41

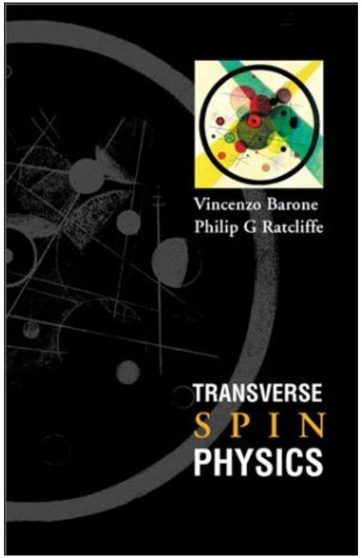
https://www.jlab.org/education/hugs/references

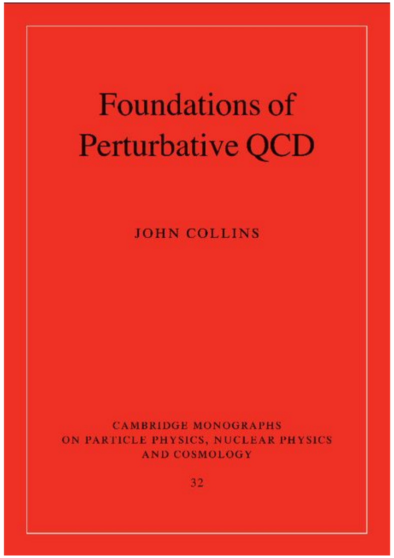
Lecture notes

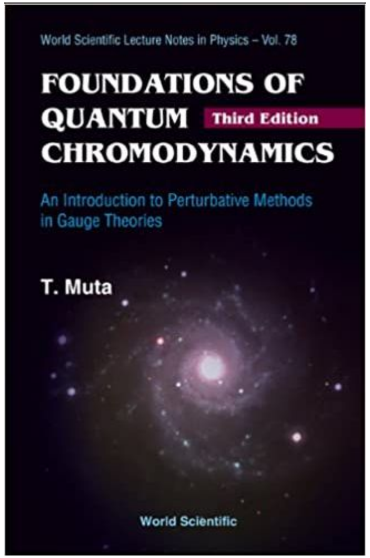
- Barone - Cabeo lecture notes:
 https://www.fe.infn.it/cabeo_school/2010/cabeo_school_2010.pdf
- Bacchetta - Trento lecture notes:
 https://www2.pv.infn.it/~bacchett/teaching/Bacchetta_Trento2012.pdf
- Jaffe - Erice lecture notes: https://arxiv.org/pdf/hep-ph/9602236.pdf
- Mulders - GGI lecture notes: http://www.nat.vu.nl/~mulders/tmdreview-vs3.pdf
- ...

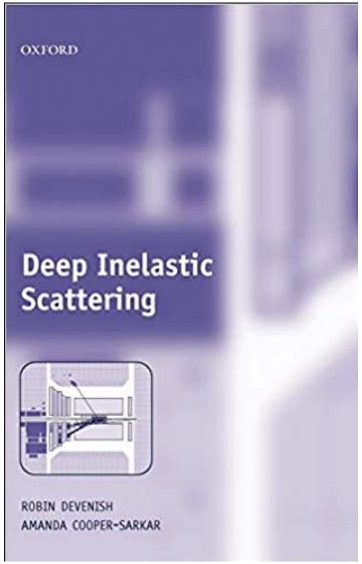
Books

- Barone, Ratcliffe: *Transverse Spin Physics*
- Collins: *Foundations of perturbative QCD*
- Devenish, Cooper-Sarkar: *Deep Inelastic Scattering*
- Muta: *Foundations of Quantum Chromodynamics*
- ...









Papers

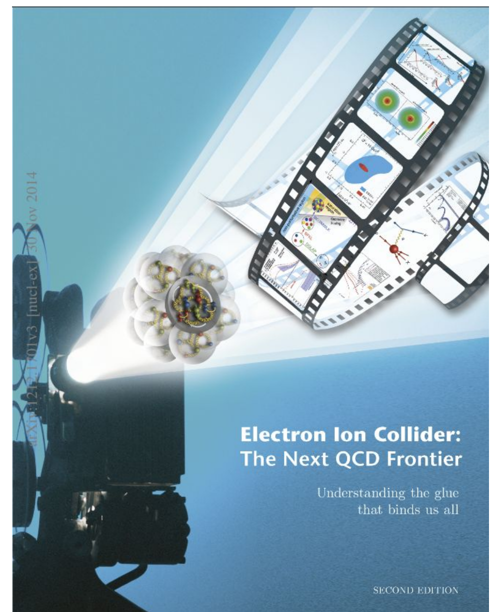
- EPJ-A topical issue: *The 3D structure of the nucleon*
  https://link.springer.com/journal/10050/topicalCollection/AC_628286e999d9a60c9a780398df15f93d
- Diehl: *Introduction to GPDs and TMDs*
  https://inspirehep.net/literature/1408303
- Bacchetta et al.: *Single spin asymmetries: the Trento conventions*
  https://inspirehep.net/literature/660999
- Collins: *Light cone variables, rapidity and all that*
  https://inspirehep.net/literature/443368
- Metz-Vossen*: Parton fragmentation functions*
  https://inspirehep.net/literature/1475000
- *Scimemi: A short review on recent developments in TMD factorization and implementation*
  https://inspirehep.net/literature/1716549
- ...

Experimental overviews

- Dudek et al.: *Physics opportunities with the 12 GeV upgrade at Jefferson Lab*
  https://inspirehep.net/literature/1125972

- Accardi et al.: *Electron Ion Collider: The next QCD Frontier - understanding the glue that binds us all*
  https://inspirehep.net/literature/1206324

- ...

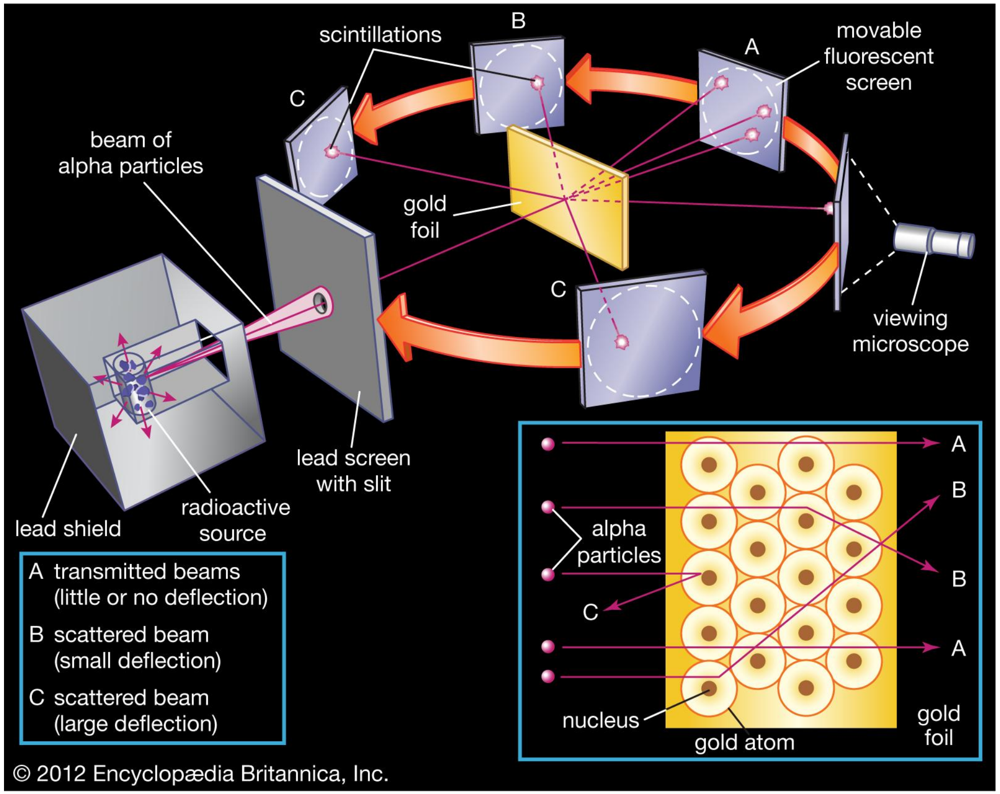


Plan of these lectures

1. **DIS and partons**
2. **From DIS to SIDIS**
3. **Symmetries and universality**
4. **Factorization, evolution, matching**
5. **Phenomenology**

1. DIS & partons

Geiger / Marsden / Rutherford experiment



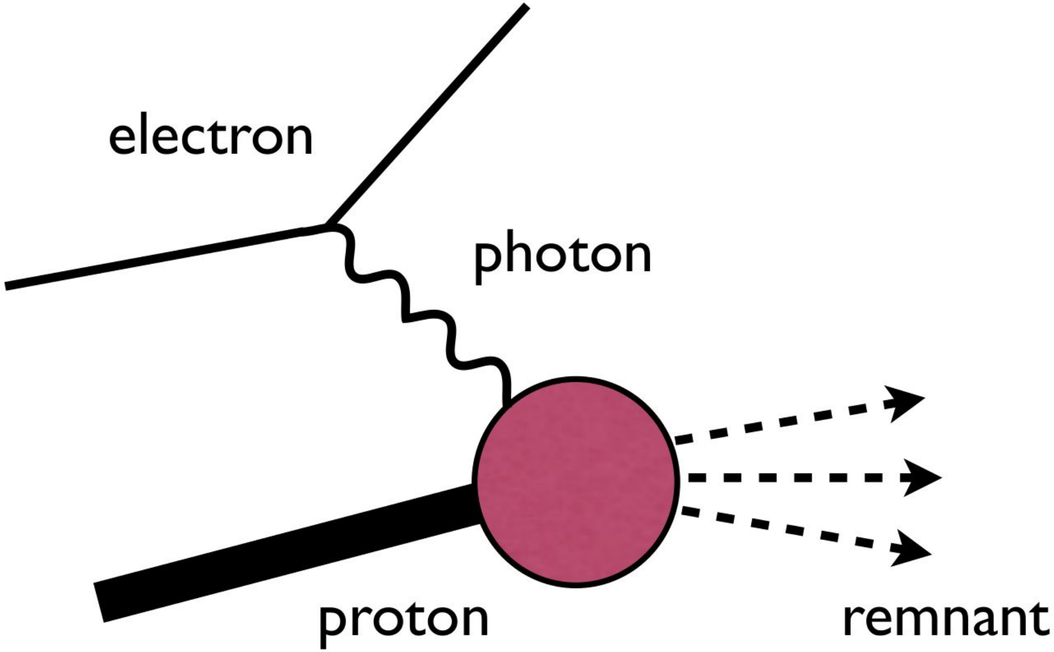
~1910

Scattering of alpha particles on gold:

discovery of the atomic nucleus

Deep-inelastic scattering

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)$$



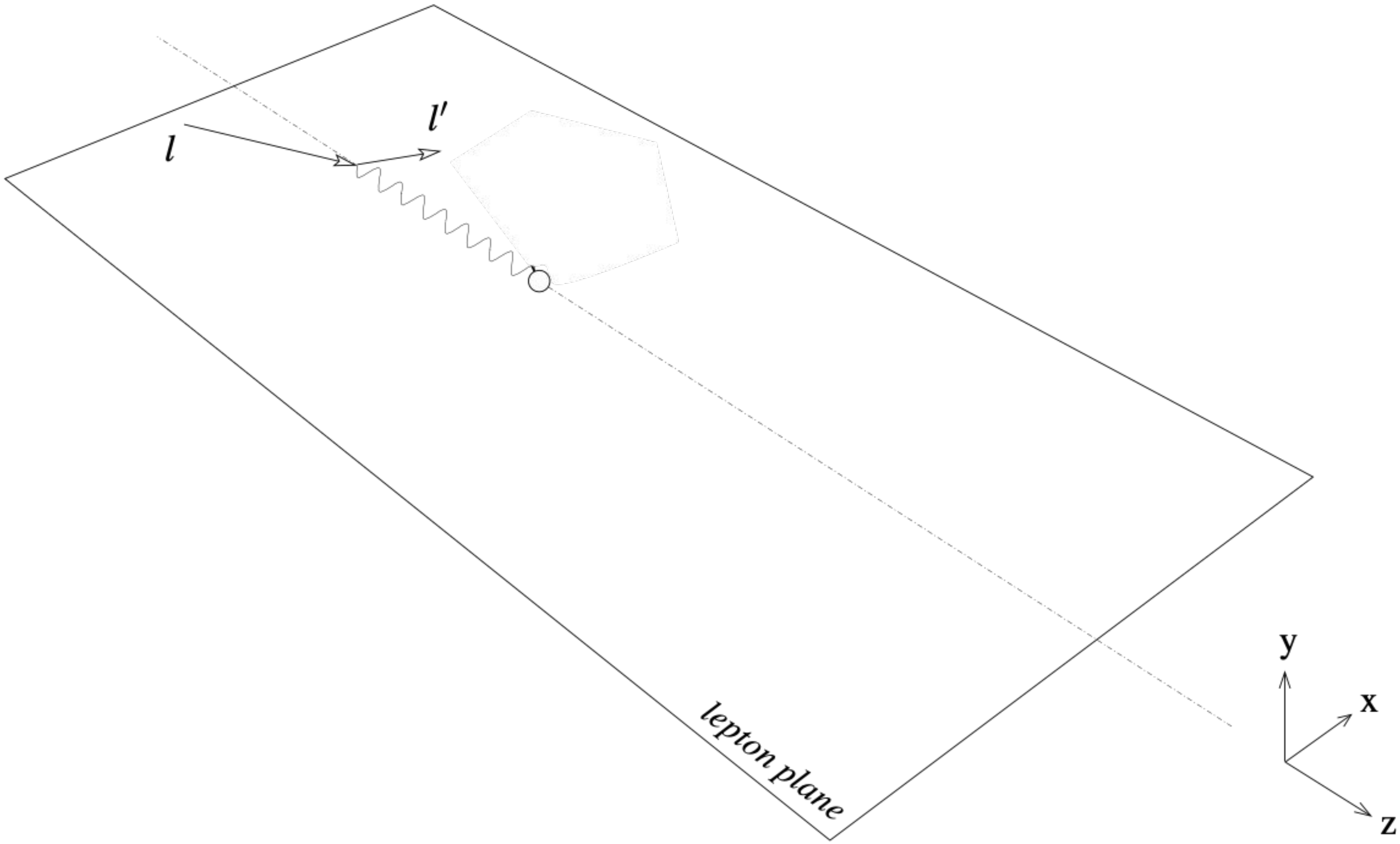
MIT-SLAC experiments ('60-'70)

Scattering of electrons off protons to test hadrons' substructure:

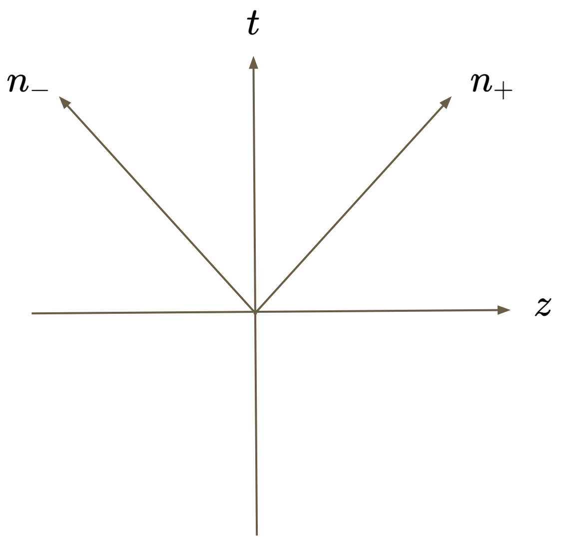
First evidence of free point-like spin-½ constituents (partons) inside the proton

Deep-inelastic scattering

$$l(\ell) + N(P) \rightarrow l'(\ell') + X(P_X)$$



Light cone variables



Choice of a basis :

$$\{n_+,\, n_-\}$$

$$n_+^2 \;=\; n_-^2 \;=\; 0$$

$$n_+ \cdot n_- \;=\; 1$$

$$V^\mu \;=\; \left(V^0,\, V^1,\, V^2,\, V^3\right) \;=\; \left[V^+,\, V^-,\, \mathbf{V}_T\right]$$

$$V^\pm \;=\; \frac{1}{\sqrt{2}}\left(V^0 \pm V^3\right)\,, \quad \mathbf{V}_T \;=\; \left(V^1, V^2\right)$$

$$V^2 \;=\; 2V^+V^- \;-\; |\mathbf{V}_T|^2$$

Projectors on the transverse space:

$$g_T^{\mu\nu} \;=\; g^{\mu\nu} \;-\; n_+^\mu n_-^\nu \;-\; n_-^\mu n_+^\nu$$

$$\epsilon_T^{\mu\nu} \;=\; \epsilon^{\mu\nu\rho\sigma}\, n_-^\rho\, n_+^\sigma$$

Kinematics

With a nucleon target, we have four “external” vectors at our disposal: “spin”, $P,\ \ell,\ \ell'$

We can build the following invariants

$$s = (P + \ell)^2$$

$$W^2 = (P + q)^2$$

$$Q^2 = -q^2 = -(\ell - \ell')^2$$

… and variables :

$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot \ell}$$

Deep-inelastic regime (*Bjorken limit*):

$$Q^2,\ P \cdot q \to +\infty \qquad \left(\gg M^2 \ \text{ in practice}\right)$$

$$x_B \quad \text{fixed}$$

Spin 1/2

The spin is described by means of a density operator (matrix, standard Quantum Mechanics)

Spin J → (up to) rank 2J tensor (e.g.: spin ½ → rank 1 tensor = spin vector)

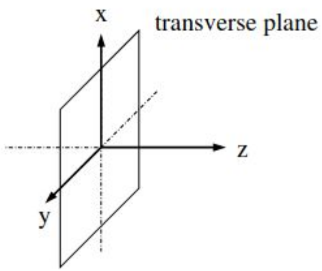
Spin 1/2

$$\rho = \frac{1}{2}\left(1 + S^i\,\sigma^i\right)$$

→ identity operator 1 and Pauli matrices

→ spin 3-vector S $S^i = \left(S_T^x,\, S_T^y,\, S_L\right)$

(z chosen as longitudinal direction)



Covariant spin vector

$$S^\mu = \left(0,\, S^i\right)$$

Spin 1

see https://inspirehep.net/literature/530045 for more details

The spin is described by means of a density operator (matrix, standard Quantum Mechanics)

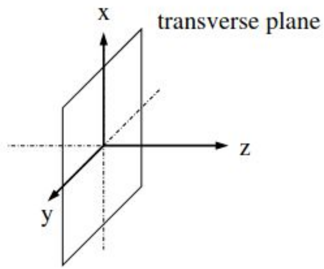
Spin J → (up to) rank 2J tensor (e.g.: spin ½ → rank 1 tensor = spin vector)

Spin 1

$$\rho = \frac{1}{3}\left(1 + \frac{3}{2} S^i \Sigma^i + 3\, T^{ij}\, \Sigma^{ij}\right)$$

→ spin 3-vector "S" and spin tensor "T"

→ identity 1, 3D Pauli matrices Σ and their generalization to rank-2



$$\boldsymbol{S} = (S_T^x, S_T^y, S_L)\,,$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$

Spin - take home message

Spin 1/2

$$\rho = \frac{1}{2}\left(1 + S^i \sigma^i\right)$$

→ spin 3-vector “S”

Spin 1

$$\rho = \frac{1}{3}\left(1 + \frac{3}{2} S^i \Sigma^i + 3\, T^{ij} \Sigma^{ij}\right)$$

→ spin 3-vector “S” and spin tensor “T”

A polarized deuteron (spin 1) has more “degrees of freedom” compared to a polarized nucleon (spin ½) . This leads to a richer spin structure:

Additional structure functions and partonic distributions (see also E. Long’s lectures)

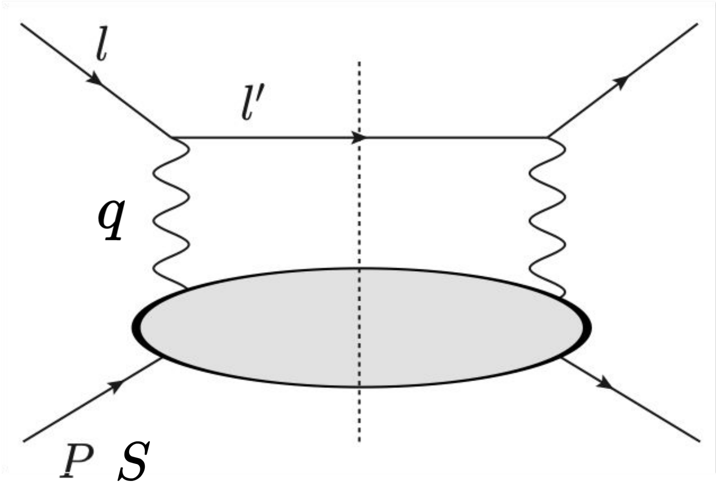
Cross section

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

CUT DIAGRAM notation for one-photon exchange approximation.

Represents the product of two Feynman amplitudes (→ cross section), one at the left and one at the right of the “cut”

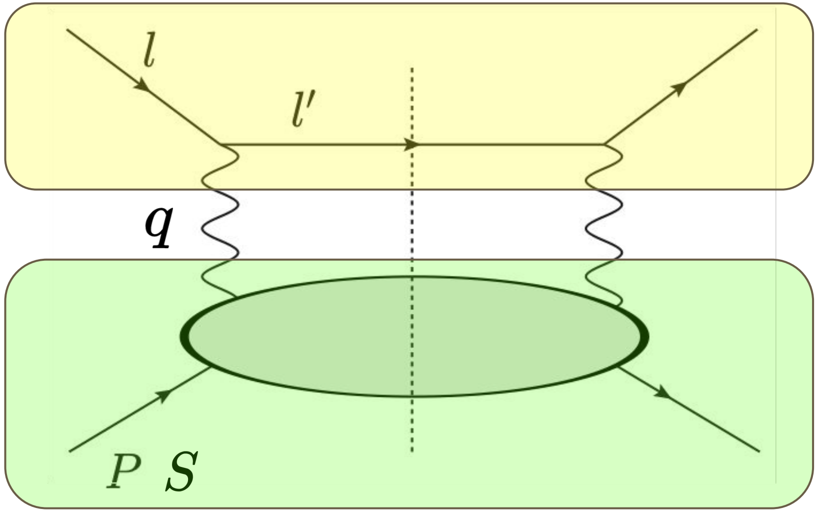
The dashed line represents the “cut”: particles that go to the final state (on-shell)



Cross section

$$\frac{d^3\sigma}{dx_B\,dy\,d\phi_S} = \frac{\alpha^2\, y}{2\,Q^4} L_{\mu\nu}(l, l', \lambda_e)\, 2MW^{\mu\nu}(q, P, S)$$

Leptonic tensor - QED
(completely
calculable)



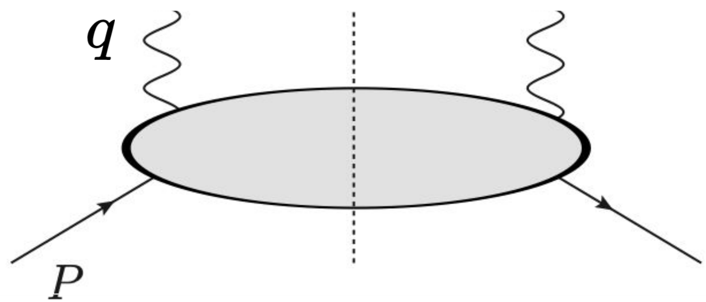
Hadronic tensor - QCD
(NOT completely
calculable)

The hadronic tensor (unpolarized)

$$2\,M\,W_{\mu\nu}(q,P) \;=\; \sum_X \int \frac{d^3P_X}{2E_X}\,\delta^4(P+q\,-\,P_X)\,\langle P|\,J_\mu^\dagger(0)\,|P_X\rangle\,\langle P_X|\,J_\nu(0)|P\rangle$$

The scattering electron “feels” the electromagnetic current J in the target

$J_\mu(\xi) \;=\; :\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi):$ (in case of weak interaction (W,Z) the current is different)

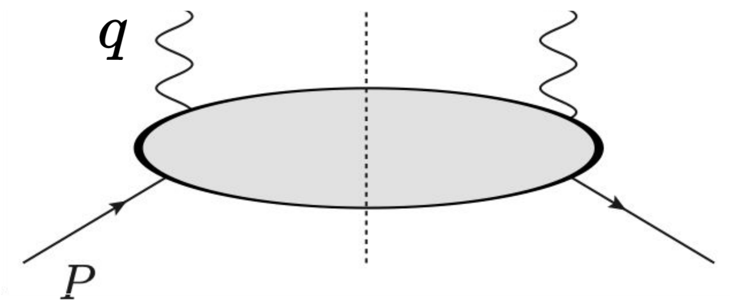


The hadronic tensor (unpolarized)

We can parametrize W using the available vectors and the symmetries of the theory:

$$2\,M\,W_{\mu\nu}(q,P) = \left[A\,g_{\mu\nu} + B\,\frac{q_\mu q_\nu}{q^2} + C\,\frac{P_\mu\,P_\nu}{M^2} + D\,\frac{P_\mu\,q_\nu + P_\nu\,q_\mu}{M^2}\right]$$

Conditions: **parity** invariance, **time-reversal** invariance, **gauge** invariance and *hermiticity*



Weak interaction: no parity → additional terms!
Spin → additional terms!

Structure functions

$$M\,W^{\mu\nu}(q,P) = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) F_1\left(x_B,\, Q^2\right) + \frac{1}{P\cdot q}\left(P^\mu - \frac{P\cdot q}{q^2} q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2} q^\nu\right) F_2\left(x_B,\, Q^2\right) =$$

$$= -g_\perp^{\mu\nu} F_{UU,T}\left(x_B, Q^2\right) + \hat{t}^{\,\mu}\,\hat{t}^{\,\nu} F_{UU,L}\left(x_B,\, Q^2\right)$$

F1, F2: “standard” unpolarized DIS structure functions

FT, FL: structure functions in the {t, z} basis (direct connection with the photon polarization)

Orthogonal and normalized basis

$$z^\mu = -\,\hat{q}^{\,\mu} \equiv q^\mu \,/\, Q$$

$$\hat{t}^{\,\mu} = \frac{2\,x_B}{Q\sqrt{1+\gamma^2}}\left(P^\mu - \frac{P\cdot q}{q^2} q^\mu\right)$$

Transverse projectors

$$g_\perp^{\mu\nu} = g^{\mu\nu} + \hat{q}^{\,\mu}\,\hat{q}^{\,\nu} - \hat{t}^{\,\mu}\,\hat{t}^{\,\nu}$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\,\hat{t}_{\,\rho}\,\hat{q}_{\,\sigma}$$

Polarized case - spin 1/2

$$W^{\mu\nu}(q,P,S) \sim -g_\perp^{\mu\nu} F_{UU,T} + \hat{t}^{\mu}\hat{t}^{\nu} F_{UU,L}$$

$$+ i\, S_L \epsilon_\perp^{\mu\nu} \boxed{F_{LL}} + i\left(\hat{t}^{\mu}\epsilon_\perp^{\nu\rho} - \hat{t}^{\nu}\epsilon_\perp^{\mu\rho}\right) S_\rho \boxed{F_{LT}^{\cos\phi}}$$

Two additional structure functions for the nucleon:
longitudinal and **transverse** target polarization → related to "standard" g1 and g2 functions

Transverse beam polarization is proportional to electron mass and thus suppressed

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$$+\, i\, S_L \epsilon_\perp^{\mu\nu}\, F_{LL} + i\Big(\hat{t}^{\,\mu}\epsilon_\perp^{\nu\rho} - \hat{t}^{\,\nu}\epsilon_\perp^{\mu\rho}\Big) S_\rho\, F_{LT}^{\cos\phi}$$

$$+ \boxed{\{b_1\,,\, b_2\,,\, b_3,\, b_4\}} \longrightarrow T^{\mu\nu} \text{ tensor polarized terms}$$

Two additional structure functions for the nucleon:
longitudinal and **transverse** target polarization → related to “standard” g1 and g2 functions

Transverse beam polarization is proportional to electron mass and thus suppressed

For a deuteron there are four additional structures associated with the **tensor polarization**!

Cross section (polarized nucleon)

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4} L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \Bigg\{ \left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1-y)\, F_{UU,L} + S_L \lambda_e\, y \left(1 - \frac{y}{2}\right) F_{LL} + |\boldsymbol{S}_T| \lambda_e\, y \sqrt{1-y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s} \Bigg\}$$

F...: functions of x, Q

Up to now no partons ...

How do quarks and gluons emerge in this description?

For a summary see e.g. https://inspirehep.net/literature/732275

Light cone dominance

$$2\,M\,W_{\mu\nu}(q,P,S) \;=\; \sum_X \int \frac{d^3 P_X}{2E_X}\,\delta^4(P+q\,-\,P_X)\,\langle PS|\,J^\dagger_\mu(0)\,|P_X\rangle\,\langle P_X|\,J_\nu(0)|PS\rangle$$

By using :

- properties of delta
- completeness on the intermediate states and
- translating the argument of the current

$$2MW_{\mu\nu}(q,P,S) \;=\; \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\,\cdot\xi}\left\langle PS\right|\left[J^\dagger_\mu(\xi),\,J_\nu(0)\right]\left|PS\right\rangle$$

- Causality implies: $\xi^2 > 0$
- Riemann-Lebesgue lemma implies W is zero unless: $\xi^+ \longrightarrow 0$

Thus , the W is dominated by what happens at $\xi^2 \simeq 0$ → We can use the OPE!

Operator Product Expansion (OPE)

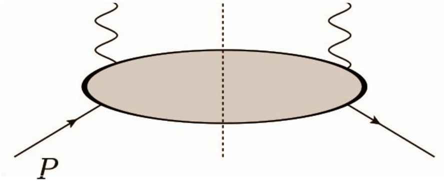
See Muta's book for details

$$A(x)\,B(y) \;=\; \sum_n C_n(x-y)\,O_n(x)\,, \qquad \text{when } |x-y| \text{ is small}$$

operators

Coefficients (lower n : more dominant)

Thanks to the light cone dominance, we can apply this to the hadronic tensor:

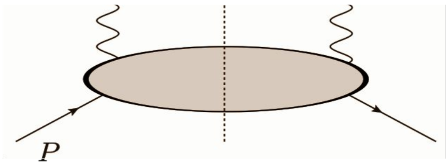


$$2MW_{\mu\nu}(q,P,S) \;=\; \tfrac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J^{\dagger}_{\mu}(\xi),\,J_{\nu}(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) \;=\; :\,\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi)\,:$$

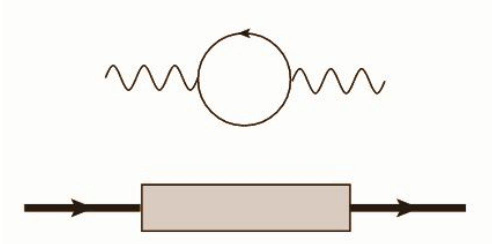
Operator Product Expansion (OPE)

See Muta's book for details



$$2MW_{\mu\nu}(q,P,S) \;=\; \frac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J^\dagger_\mu(x),\,J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) \;=\; :\,\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi)\,:$$



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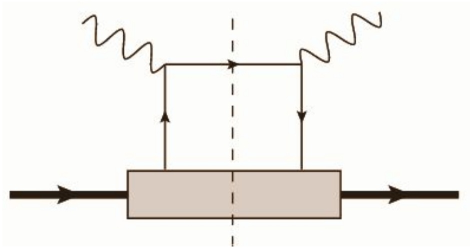

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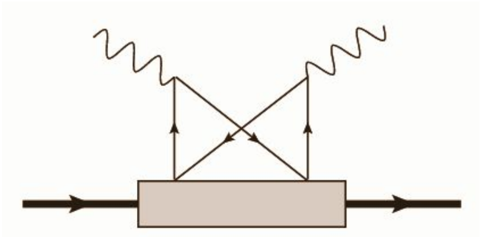
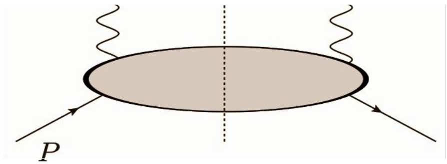


Disconnected — irrelevant

quark-antiquark — dominant

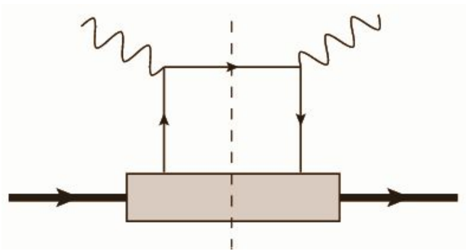
higher-twist — suppressed

Partonic interpretation



$$2MW_{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4\xi\, e^{i\,q\cdot\xi}\left\langle PS\right|\left[J_\mu^\dagger(x),\, J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) = \,:\overline{\psi}(\xi)\, Q\,\gamma_\mu\,\psi(\xi):$$



quark-antiquark

$$2MW^{\mu\nu}(q,P,S) = \sum_q e_q^2\,\frac{1}{2}\,\mathrm{Tr}\left[\Phi(x,S)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right]$$

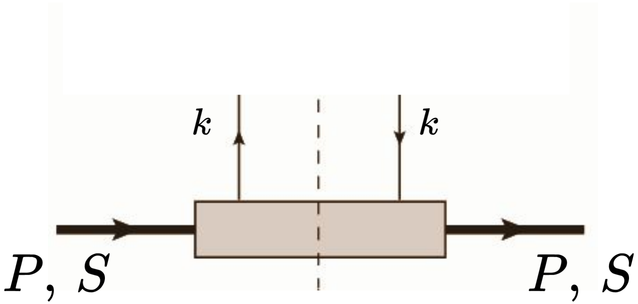
ϕ(x,S) : "collinear" quark correlator

$x_B \simeq x \equiv k^+/P^+$ → measure collinear parton dynamics

The quark transverse momentum is integrated out in DIS

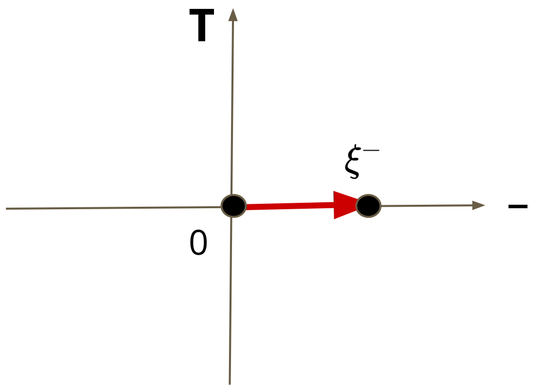
Quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\,\psi_i(\xi) \big| PS\rangle$$



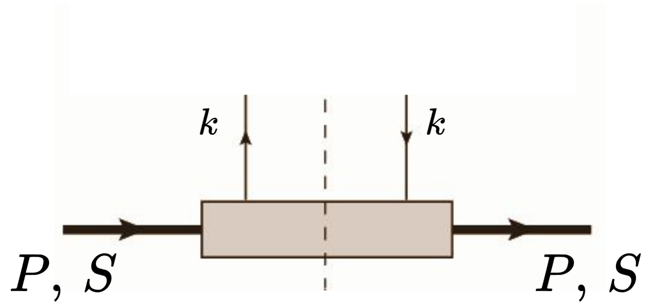
$$\Phi_{ij}(x,S) = \int dk^+\, dk^-\, d^2\mathbf{k}_T\, \delta\big(k^+ - xP^+\big)\,\Phi(k,P,S) =$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{i\,k\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\,\psi_i(\xi) \big| PS\rangle_{\,\xi^+ = \xi_T = 0}$$



Parton physics on the light cone

Collinear parton distribution functions



$\Phi_{ij}(k, P, S)$: non-perturbative hadron structure matrix

$$\Phi(x, S, T) = \frac{1}{2} f_1(x) \not{n}_+ +$$ → unpolarized PDF

$$\frac{1}{2} g_1(x) S_L \gamma_5 \not{n}_+ +$$ → longitudinally polarized PDF (helicity)

$$\frac{1}{2} h_1(x) i\sigma_{\mu\nu} \gamma_5 n_+^\mu S_T^\nu +$$ → transversely polarized PDF (transversity)

$$\frac{1}{2} f_{1\,LL}(x) S_{LL} \not{n}_+$$ → Tensor polarized PDF

"Leading twist" approximation

Ok, but …

what about transverse momentum?