

HUGS: Introduction to QCD

- **Lec. 1: Fundamentals of QCD**
- **Lec. 2: Matching observed hadrons to quarks and gluons**
- **Lec. 3: QCD for cross sections with identified hadrons**
- **Lec. 4: QCD for cross sections with polarized beam(s)**

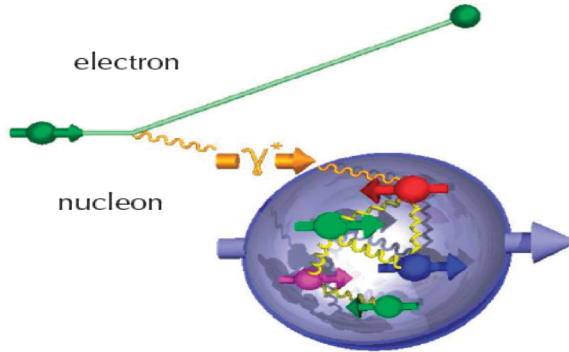
*HUGS 2020 was canceled due to COVID-19
HUGS 2021 will be the first virtual school*

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Lepton-hadron facility is special

□ The new generation of “Rutherford” experiment:



- ✧ A controlled “probe” – virtual photon
- ✧ Can either break or not break the hadron

One facility covers all!
(JLab, COMPASS, EIC, ...)

✧ Inclusive events: $e+p/A \rightarrow e'+X$

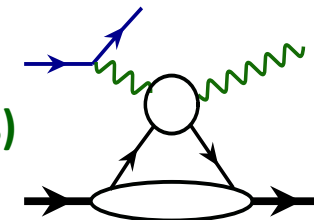
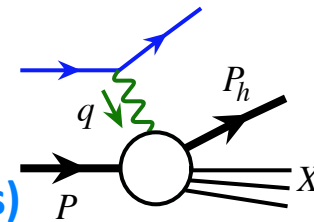
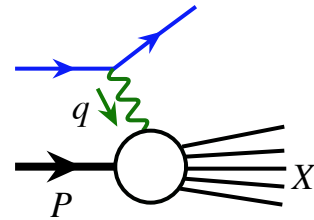
Detect only the scattered lepton in the detector
(Modern Rutherford experiment!)

✧ Semi-Inclusive events: $e+p/A \rightarrow e'+h(p,K,p,jet)+X$

Detect the scattered lepton in coincidence with identified hadrons/jets
(Initial hadron is broken – confined motion! – cleaner than h-h collisions)

✧ Exclusive events: $e+p/A \rightarrow e'+p'/A'+h(p,K,p,jet)$

Detect every things including scattered proton/nucleus (or its fragments)
(Initial hadron is NOT broken – tomography!
– almost impossible for h-h collisions)



Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

□ Cross section:

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q, \vec{s}) \approx \sigma_{AB}^{(2)}(Q, \vec{s}) + \frac{Q_s}{Q} \sigma_{AB}^{(3)}(Q, \vec{s}) + \frac{Q_s^2}{Q^2} \sigma_{AB}^{(4)}(Q, \vec{s}) + \dots$$

□ Spin-averaged cross section:

$$\sigma = \frac{1}{2} [\sigma(\vec{s}) + \sigma(-\vec{s})] \quad \text{– Positive definite}$$

□ Asymmetries or difference of cross sections:

▪ **both beams polarized**

$$A_{LL}, A_{TT}, A_{LT}$$

– Not necessary positive!

$$A_{LL} = \frac{[\sigma(+, +) - \sigma(+, -)] - [\sigma(-, +) - \sigma(-, -)]}{[\sigma(+, +) + \sigma(+, -)] + [\sigma(-, +) + \sigma(-, -)]} \quad \text{for } \sigma(s_1, s_2)$$

▪ **one beam polarized**

$$A_L, A_N$$

$$A_L = \frac{[\sigma(+)] - \sigma(-)]}{[\sigma(+)] + \sigma(-)]} \quad \text{for } \sigma(s) \quad A_N = \frac{\sigma(Q, \vec{s}_T) - \sigma(Q, -\vec{s}_T)}{\sigma(Q, \vec{s}_T) + \sigma(Q, -\vec{s}_T)}$$

Two roles of the proton spin program

□ Proton is a composite particle:

Spin is a consequence of internal dynamics of the bound state

For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

- ➔ Decomposition of proton spin in terms of quark and gluon d.o.f.
helps understand the dynamics of a fundamental QCD bound state
 - Nucleon is a building block all hadronic matter
(> 95% mass of all visible matter)

□ Use the spin as a tool – asymmetries:

Cross section is a probability – classically measured

Spin asymmetry – the difference of two cross sections
involving two different spin states

Asymmetry could be a pure quantum effect!

Spin of a composite particle

□ Spin:

- ✧ Pauli (1924): two-valued quantum degree of freedom of electron
- ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant \hbar)
- ✧ Composite particle = Total angular momentum when it is at rest

□ Spin of a nucleus:

- ✧ Nuclear binding: 8 MeV/nucleon \ll mass of nucleon
- ✧ Nucleon number is fixed inside a given nucleus
- ✧ Spin of a nucleus = sum of the valence nucleon spin

□ Spin of a nucleon – Naïve Quark Model:

- ✧ If the probing energy \ll mass of constituent quark
- ✧ Nucleon is made of three constituent (valence) quark
- ✧ Spin of a nucleon = sum of the constituent quark spin



State:
$$|p \uparrow\rangle = \sqrt{\frac{1}{18}} [u \uparrow u \downarrow d \uparrow + u \downarrow u \uparrow d \uparrow - 2u \uparrow u \uparrow d \downarrow + \text{perm.}]$$

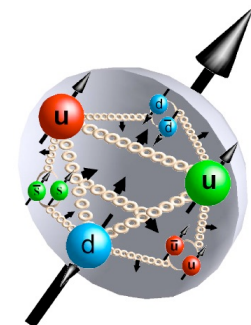
Spin:
$$S_p \equiv \langle p \uparrow | S | p \uparrow \rangle = \frac{1}{2}, \quad S = \sum_i S_i$$

Carried by valence quarks

Spin of a composite particle

□ Spin of a nucleon – QCD:

- ✧ Current quark mass \ll energy exchange of the collision
- ✧ Number of quarks and gluons depends on the probing energy



□ Angular momentum of a proton at rest:

$$S = \sum_f \langle P, S_z = 1/2 | \hat{J}_f^z | P, S_z = 1/2 \rangle = \frac{1}{2}$$

□ QCD Angular momentum operator:

$$J_{\text{QCD}}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M_{\text{QCD}}^{0jk} \quad \leftarrow \quad M_{\text{QCD}}^{\alpha\mu\nu} = T_{\text{QCD}}^{\alpha\nu} x^\mu - T_{\text{QCD}}^{\alpha\mu} x^\nu$$

Energy-momentum tensor

✧ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma} \gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D})) \psi_q \right]$$

Angular momentum density

✧ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]$$

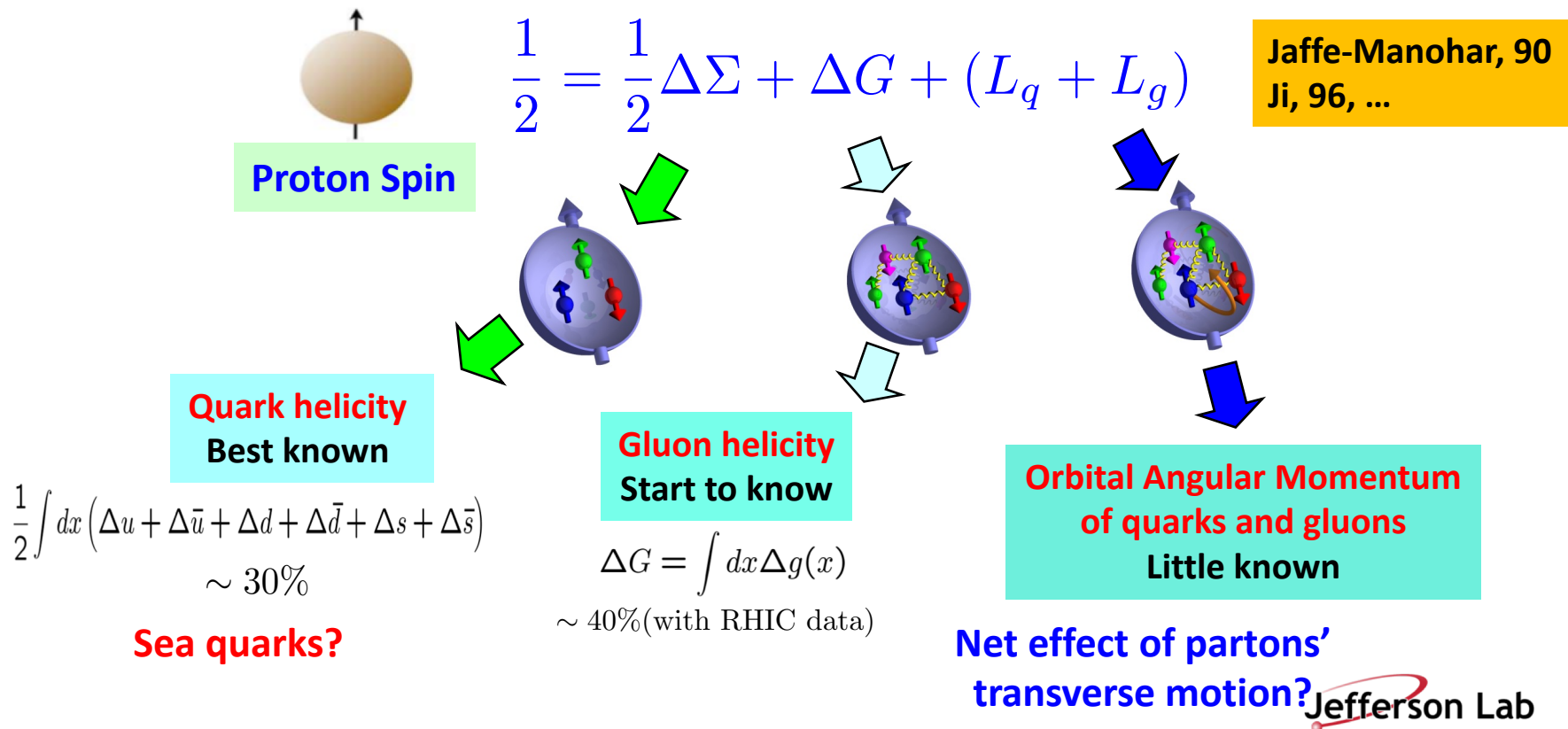
Need to have the matrix elements of these partonic operators measured independently

Current understanding for Proton Spin

□ **The sum rule:**
$$S(\mu) = \sum_f \langle P, S | \hat{J}_f^z(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$

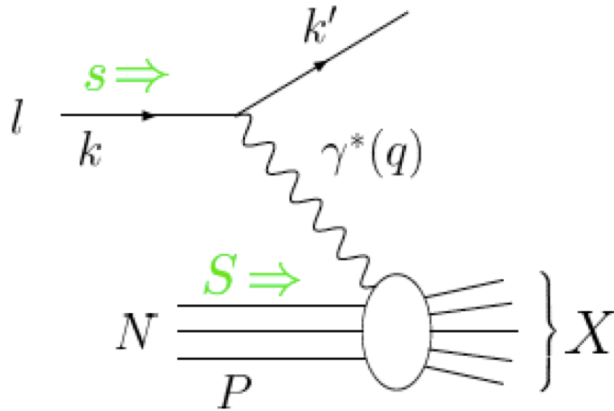
- Infinite possibilities of decompositions – connection to observables?
- Intrinsic properties + dynamical motion and interactions

□ **An incomplete story:**



Polarized deep inelastic scattering

□ DIS with polarized beam(s):



“Resolution”

$$Q \equiv \sqrt{-q^2}$$

$$\frac{\hbar}{Q} = \frac{2 \times 10^{-16} \text{m}}{Q/\text{GeV}} \lesssim 10^{-16} \text{m} = 1/10 \text{fm}$$

“Inelasticity” – known as Bjorken variable

$$x_B = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ Recall – from lecture 2:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x_B, Q^2) + \frac{1}{p \cdot q} \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) F_2(x_B, Q^2) \\ + iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p \cdot q} g_1(x_B, Q^2) + \frac{(p \cdot q) S_\sigma - (S \cdot q) p_\sigma}{(p \cdot q)^2} g_2(x_B, Q^2) \right]$$

✧ Polarized structure functions:

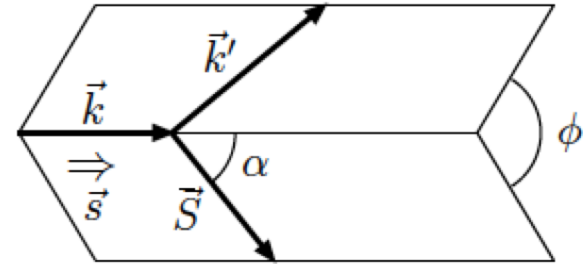
$$g_1(x_B, Q^2), g_2(x_B, Q^2)$$

Polarized deep inelastic scattering

□ Extract the polarized structure functions:

$$\mathcal{W}^{\mu\nu}(P, q, \mathbf{S}) - \mathcal{W}^{\mu\nu}(P, q, -\mathbf{S})$$

✧ Define: $\angle(\hat{k}, \hat{S}) = \alpha$,
and lepton helicity λ



✧ Difference in cross sections with hadron spin flipped

$$\frac{d\sigma^{(\alpha)}}{dx dy d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx dy d\phi} = \frac{\lambda e^4}{4\pi^2 Q^2} \times$$

$$\times \left\{ \cos \alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2 x^2 y^2}{Q^2} \right] g_1(x, Q^2) - \frac{2m^2 x^2 y}{Q^2} g_2(x, Q^2) \right\} \right.$$

$$\left. - \sin \alpha \cos \phi \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2 x^2 y^2}{Q^2} \right)} \left(\frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right) \right\}$$

✧ Spin orientation:

$$\alpha = 0 : \Rightarrow g_1$$

$$\alpha = \pi/2 : \Rightarrow y g_1 + 2 g_2, \text{ suppressed } m/Q$$

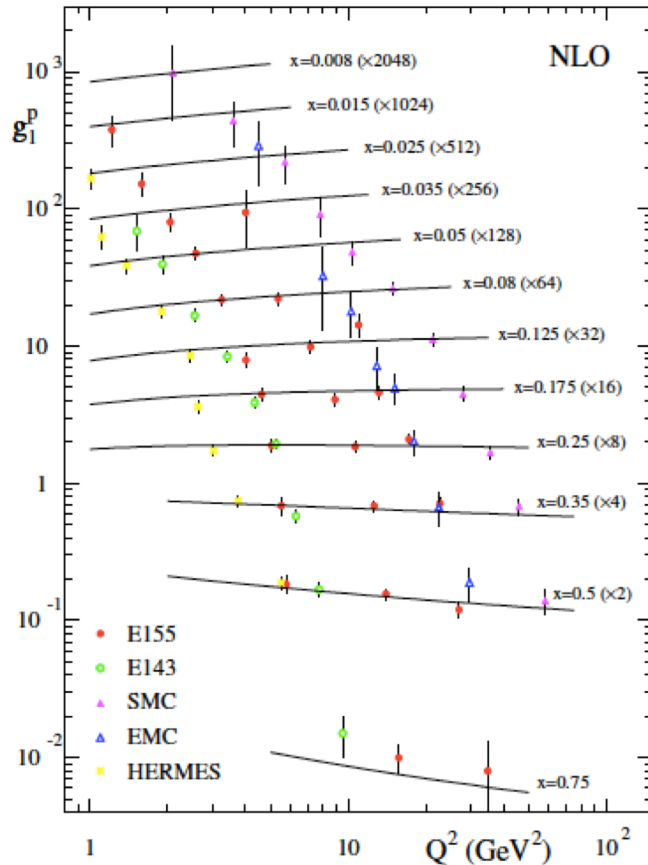
Polarized deep inelastic scattering

Spin asymmetries – measured experimentally:

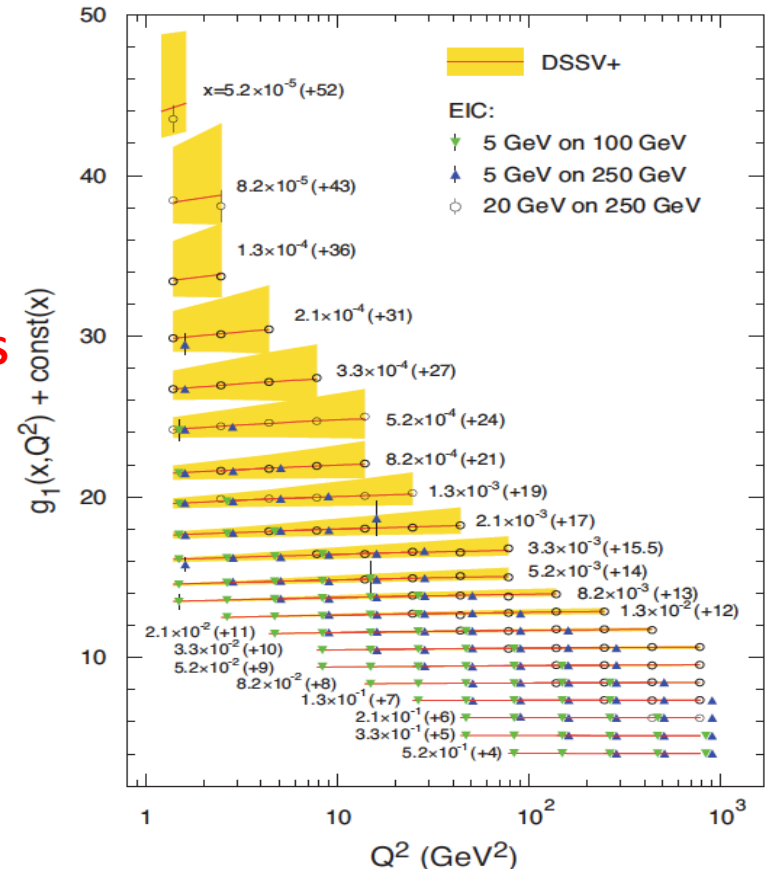
Longitudinal polarization – $\alpha = 0$

$$A_{\parallel} = \frac{d\sigma(\rightarrow\leftarrow) - d\sigma(\rightarrow\rightarrow)}{d\sigma(\rightarrow\leftarrow) + d\sigma(\rightarrow\rightarrow)} = D(y) \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \equiv D(y) A_1(x, Q^2)$$

Known function

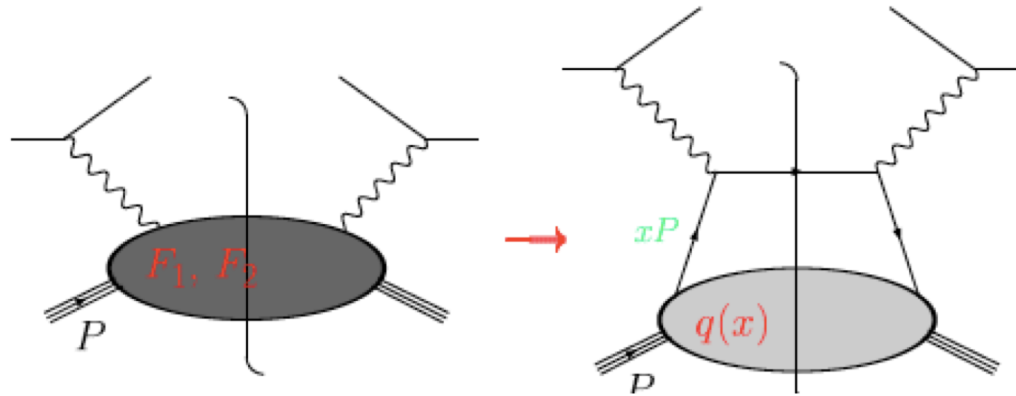


Polarized DIS
at EIC



Polarized deep inelastic scattering

□ Parton model results – LO QCD:



✧ Structure functions:

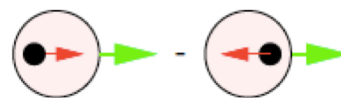
$$F_1(x) = \frac{1}{2} \sum_q e_q^2 [q(x) + \bar{q}(x)]$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)]$$

$$g_1 = \frac{1}{2} \left[\frac{4}{9} (\Delta u + \Delta \bar{u}) + \frac{1}{9} (\Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}) \right]$$

✧ Polarized quark distribution:

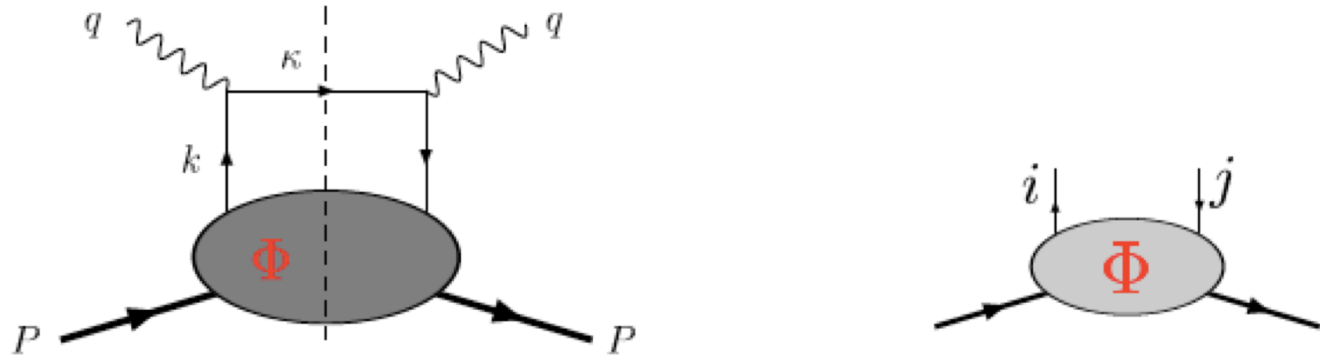
$$\Delta f(\xi) \equiv f^+(\xi) - f^-(\xi)$$



**Information on nucleon's
spin structure**

Polarized deep inelastic scattering

□ Systematics polarized PDFs – LO QCD:



✧ Two-quark correlator:

$$\begin{aligned}\Phi_{ij}(k, P, S) &= \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X} (2\pi)^4 \delta^4(P - k - P_X) \langle PS | \bar{\psi}_j(0) | X \rangle \langle X | \psi_i(0) | PS \rangle \\ &= \int d^4z e^{ik \cdot z} \langle PS | \bar{\psi}_j(0) \psi_i(z) | PS \rangle\end{aligned}$$

✧ Hadronic tensor (one-flavor):

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4} \delta((k+q)^2) \text{Tr}[\Phi \gamma^\mu (\not{k} + \not{q}) \gamma^\nu]$$

Polarized deep inelastic scattering

✧ General expansion of $\phi(x)$:

must have general expansion in terms of P , \not{n} , \not{s} etc.

$$\phi(x) = \frac{1}{2} [q(x)\gamma \cdot P + s_{\parallel}\Delta q(x)\gamma_5\gamma \cdot P + \delta q(x)\gamma \cdot P\gamma_5\gamma \cdot S_{\perp}]$$

✧ 3-leading power quark parton distribution:

$$q(x) = \frac{1}{4\pi} \int dz^- e^{iz^-xP^+} \langle P, S | \bar{\psi}(0) \gamma^+ \psi(0, z^-, \mathbf{0}_{\perp}) | P, S \rangle$$

$$\Delta q(x) = \frac{1}{4\pi} \int dz^- e^{iz^-xP^+} \langle P, S | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(0, z^-, \mathbf{0}_{\perp}) | P, S \rangle$$

$$\delta q(x) = \frac{1}{4\pi} \int dz^- e^{iz^-xP^+} \langle P, S | \bar{\psi}(0) \gamma^+ \gamma_{\perp} \gamma_5 \psi(0, z^-, \mathbf{0}_{\perp}) | P, S \rangle$$

“unpolarized” – “longitudinally polarized” – “transversity”

Polarized deep inelastic scattering

□ Physical interpretation:

$$q(x) = \frac{1}{2} \sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 + \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 \right]$$

$$\Delta q(x) = \frac{1}{2} \sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 - \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 \right]$$

$$\delta q(x) = \frac{1}{2} \sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left| \langle X | \mathcal{P}^\uparrow \psi_+(0) | P, S_\perp = \frac{1}{2} \rangle \right|^2 - \left| \langle X | \mathcal{P}^\downarrow \psi_+(0) | P, S_\perp = \frac{1}{2} \rangle \right|^2 \right]$$

Spin projection:

$$\mathcal{P}^\pm \equiv \frac{1 \pm \gamma_5}{2} \quad \text{and} \quad \mathcal{P}^{\uparrow\downarrow} \equiv \frac{1 \pm \gamma_\perp \gamma_5}{2}$$

Basics for spin observables

□ Factorized cross section:

$$\sigma_{h(p)}(Q, s) \propto \langle p, \vec{s} | \mathcal{O}(\psi, A^\mu) | p, \vec{s} \rangle$$

$$\text{e.g. } \mathcal{O}(\psi, A^\mu) = \bar{\psi}(0) \hat{\Gamma} \psi(y^-) \quad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5 \gamma^\mu, \sigma^{\mu\nu}$$

□ Parity and Time-reversal invariance:

$$\langle p, \vec{s} | \mathcal{O}(\psi, A^\mu) | p, \vec{s} \rangle = \langle p, -\vec{s} | \mathcal{PT} \mathcal{O}^\dagger(\psi, A^\mu) \mathcal{T}^{-1} \mathcal{P}^{-1} | p, -\vec{s} \rangle$$

$$\square \text{ IF: } \langle p, -\vec{s} | \mathcal{PT} \mathcal{O}^\dagger(\psi, A^\mu) \mathcal{T}^{-1} \mathcal{P}^{-1} | p, -\vec{s} \rangle = \pm \langle p, -\vec{s} | \mathcal{O}(\psi, A^\mu) | p, -\vec{s} \rangle$$

$$\text{or } \langle p, \vec{s} | \mathcal{O}(\psi, A^\mu) | p, \vec{s} \rangle = \pm \langle p, -\vec{s} | \mathcal{O}(\psi, A^\mu) | p, -\vec{s} \rangle$$

Operators lead to the “+” sign  spin-averaged cross sections

Operators lead to the “-” sign  spin asymmetries

□ Example:

$$\mathcal{O}(\psi, A^\mu) = \bar{\psi}(0) \gamma^+ \psi(y^-) \Rightarrow q(x)$$

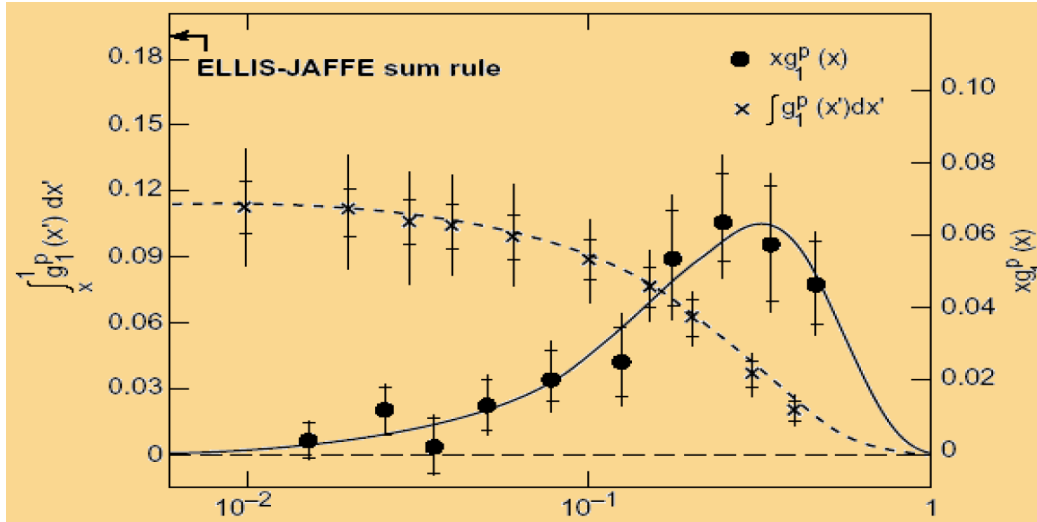
$$\mathcal{O}(\psi, A^\mu) = \bar{\psi}(0) \gamma^+ \gamma_5 \psi(y^-) \Rightarrow \Delta q(x)$$

$$\mathcal{O}(\psi, A^\mu) = \bar{\psi}(0) \gamma^+ \gamma^\perp \gamma_5 \psi(y^-) \Rightarrow \delta q(x) \rightarrow h(x)$$

$$\mathcal{O}(\psi, A^\mu) = \frac{1}{xp^+} F^{+\alpha}(0) [-i\varepsilon_{\alpha\beta}] F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Proton “spin crisis” – excited the field

□ EMC (European Muon Collaboration '87) – “the Plot”:



$$g_1(x) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

✧ Combined with earlier SLAC data:

$$\int_0^1 g_1^p(x) dx = 0.126 \pm 0.018$$

✧ Combined with:

$$g_A^3 = \Delta u - \Delta d \quad \text{and} \quad g_A^8 = \Delta u + \Delta d - 2\Delta s$$

from low energy neutron & hyperon β decay



$$\Delta\Sigma = \sum_q [\Delta q + \Delta \bar{q}] = 0.12 \pm 0.17$$

□ “Spin crisis” or puzzle:

✧ Strange sea polarization is sizable & negative

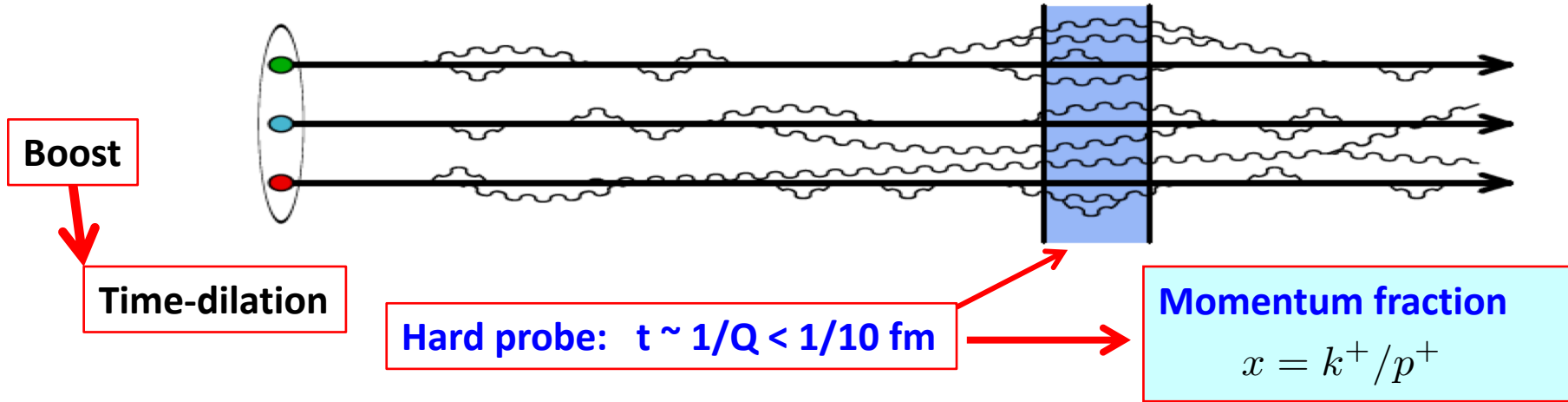
✧ Very little of the proton spin is carried by quarks



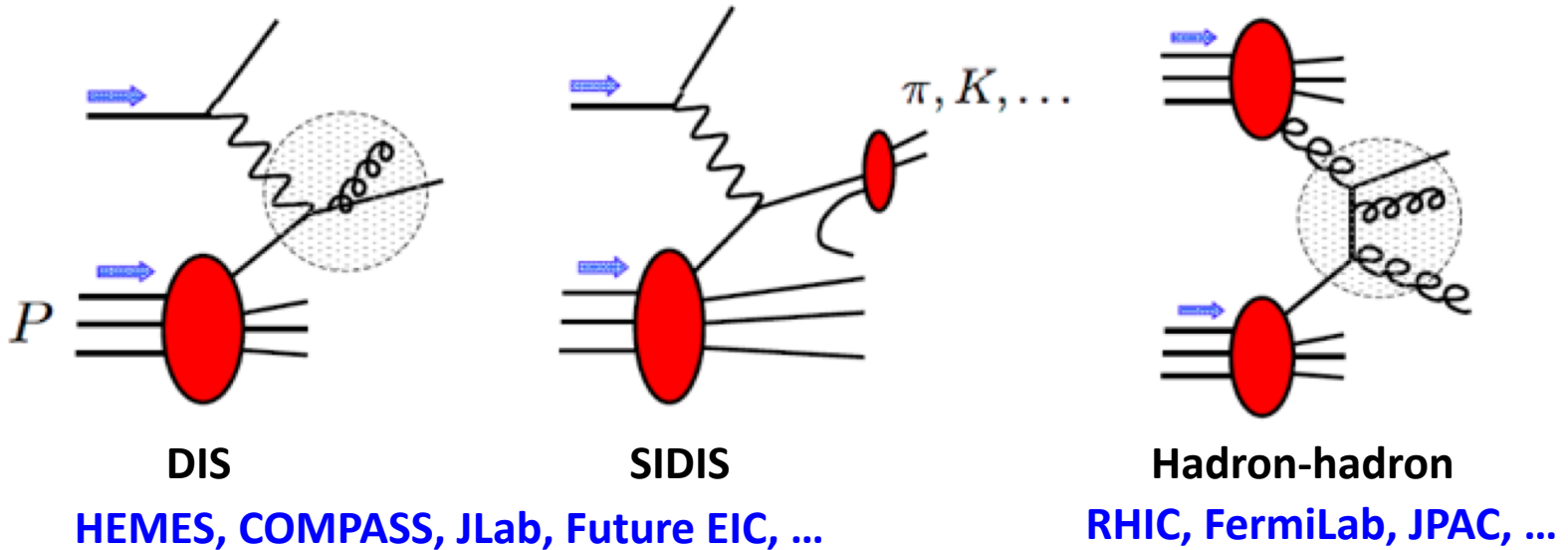
*New era of
spin physics*

Probes and facilities

High energy scattering – to see quarks and gluons:

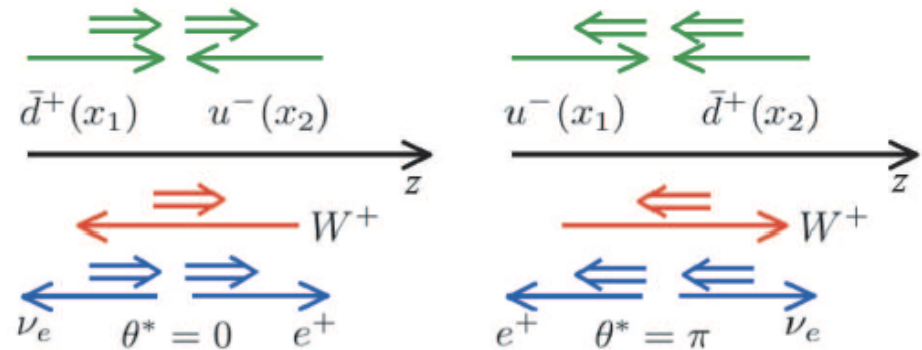
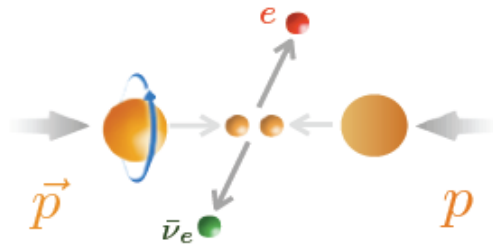


Spin Probes:



Determination of Δq and $\Delta \bar{q}$

□ W's are left-handed:



□ Flavor separation:

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

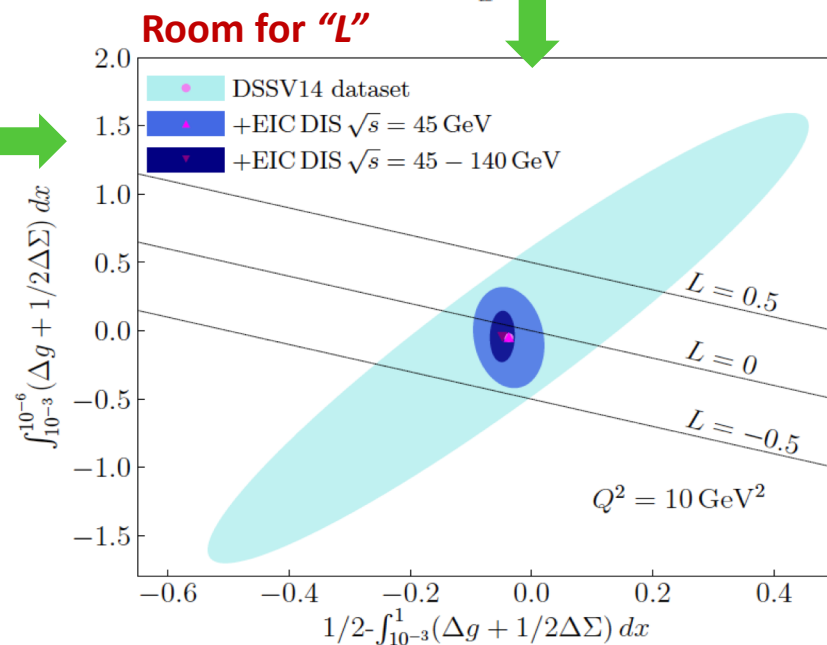
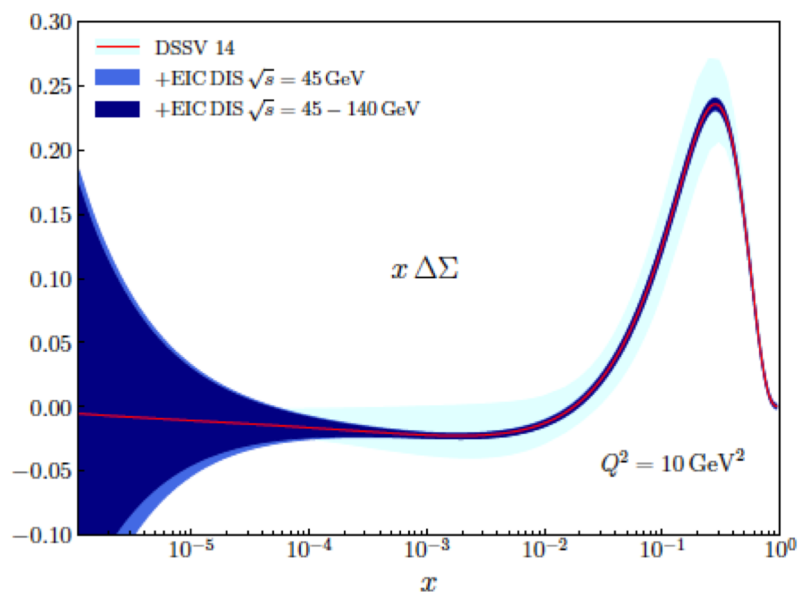
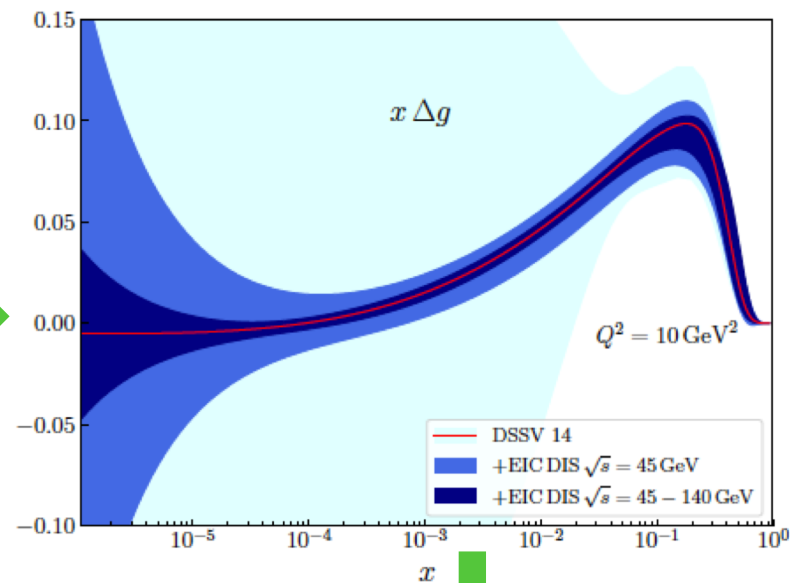
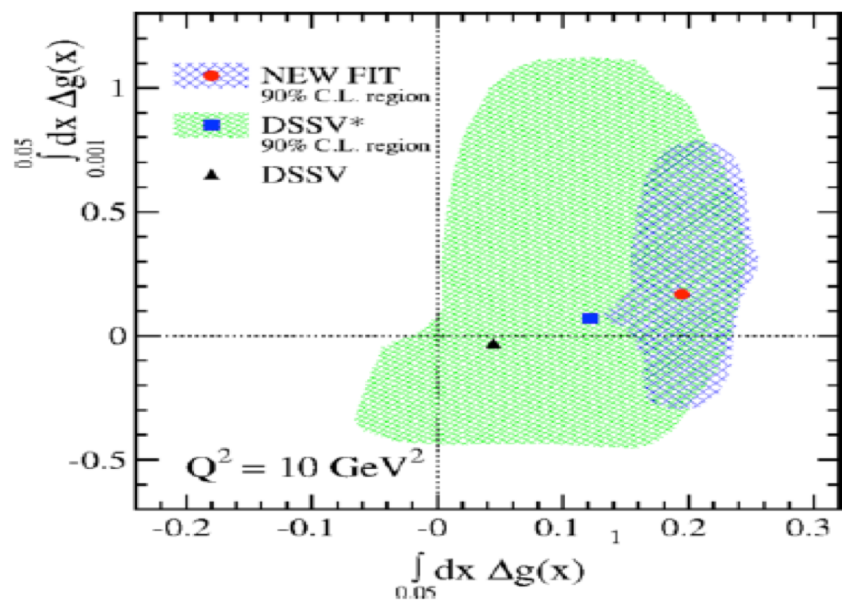
Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

□ Complications:

High order, W's p_T -distribution at low p_T

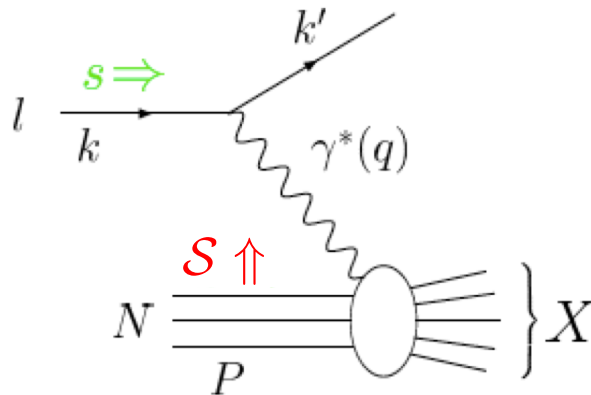
What the EIC can do/help – EIC Yellow Report?



Transverse spin phenomena in QCD

- 40 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



Single Transverse-Spin Asymmetry (SSA)

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS **vanishes** if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

□ DIS cross section: $\sigma(\vec{s}_\perp) \propto L^{\mu\nu} W_{\mu\nu}(\vec{s}_\perp)$

□ Leptonic tensor is symmetric: $L^{\mu\nu} = L^{\nu\mu}$

□ Hadronic tensor: $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp | j_\mu^\dagger(0) j_\nu(y) | P, \vec{s}_\perp \rangle$

□ Polarized cross section:

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} [W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)]$$

□ Vanishing single spin asymmetry:

$$A_N = 0 \iff \langle P, \vec{s}_\perp | j_\mu^\dagger(0) j_\nu(y) | P, \vec{s}_\perp \rangle \\ \stackrel{?}{=} \langle P, -\vec{s}_\perp | j_\nu^\dagger(0) j_\mu(y) | P, -\vec{s}_\perp \rangle$$

A_N for inclusive DIS

□ Define two quantum states:

$$\langle \beta | \equiv \langle P, \vec{s}_\perp | j_\mu^\dagger(0) j_\nu(y) \quad | \alpha \rangle \equiv | P, \vec{s}_\perp \rangle$$

□ Time-reversed states:

$$|\alpha_T\rangle = V_T |P, \vec{s}_\perp\rangle = | -P, -\vec{s}_\perp \rangle$$

$$\begin{aligned} |\beta_T\rangle &= V_T [j_\mu^\dagger(0) j_\nu(y)]^\dagger |P, \vec{s}_\perp\rangle \\ &= (V_T j_\nu^\dagger(y) V_T^{-1}) (V_T j_\mu(0) V_T^{-1}) | -P, -\vec{s}_\perp \rangle \end{aligned}$$

□ Time-reversal invariance:

$$\langle \alpha_T | \beta_T \rangle = \langle \alpha | V_T^\dagger V_T | \beta \rangle = \langle \alpha | \beta \rangle^* = \langle \beta | \alpha \rangle$$

$$\begin{aligned} \longrightarrow \langle -P, -\vec{s}_\perp | (V_T j_\nu^\dagger(y) V_T^{-1}) (V_T j_\mu(0) V_T^{-1}) | -P, -\vec{s}_\perp \rangle \\ = \langle P, \vec{s}_\perp | j_\mu^\dagger(0) j_\nu(y) | P, \vec{s}_\perp \rangle \end{aligned}$$

A_N for inclusive DIS

□ Parity invariance:

$$1 = U_P^{-1} U_P = U_P^\dagger U_P$$

$$\langle -P, -\vec{s}_\perp | (V_T j_\nu^\dagger(\mathbf{y}) V_T^{-1}) (V_T j_\mu(0) V_T^{-1}) | -P, -\vec{s}_\perp \rangle$$

$$\downarrow$$

$$\langle P, -\vec{s}_\perp | (U_P V_T j_\nu^\dagger(\mathbf{y}) V_T^{-1} U_P^{-1}) (U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}) | P, -\vec{s}_\perp \rangle$$

$$\downarrow$$

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(-\mathbf{y}) j_\mu(0) | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(0) j_\mu(\mathbf{y}) | P, -\vec{s}_\perp \rangle$$

$$= \langle P, \vec{s}_\perp | j_\mu^\dagger(0) j_\nu(\mathbf{y}) | P, \vec{s}_\perp \rangle$$

□ Polarized cross section:

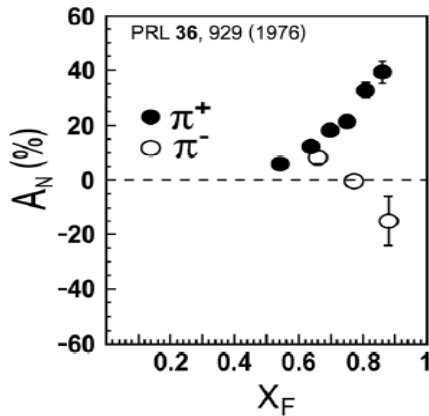
$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} [W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)]$$

$$= L^{\mu\nu} [W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp)] = 0$$

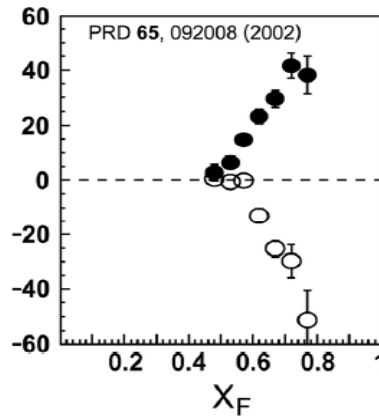
A_N in hadronic collisions

A_N - consistently observed for over 35 years!

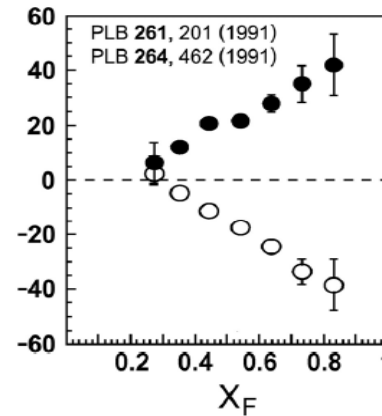
ANL – 4.9 GeV



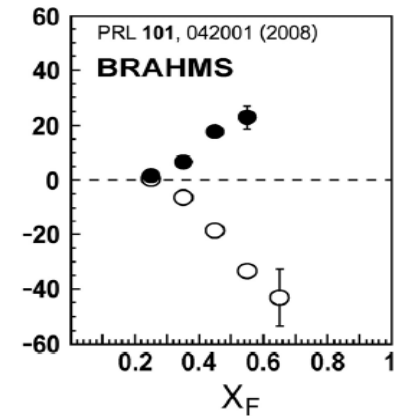
BNL – 6.6 GeV



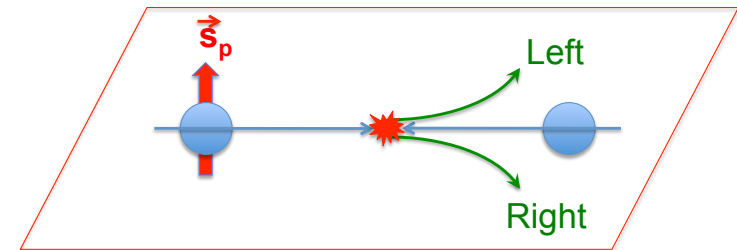
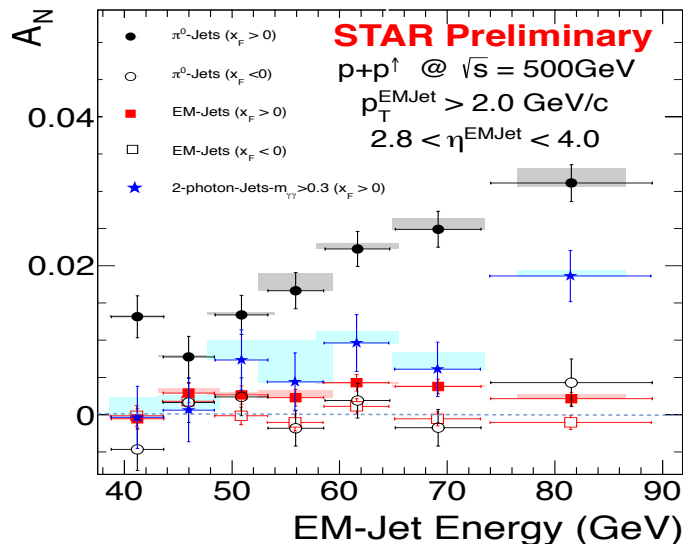
FNAL – 20 GeV



BNL – 62.4 GeV



Survived the highest RHIC energy:



$$A_N \equiv \frac{\Delta\sigma(l, \vec{s})}{\sigma(l)} = \frac{\sigma(l, \vec{s}) - \sigma(l, -\vec{s})}{\sigma(l, \vec{s}) + \sigma(l, -\vec{s})}$$

Do we understand this? Jefferson Lab

A_N in hadronic collisions

Kane, Pumplin, Repko, PRL, 1978

□ Early attempt:

Cross section:

$$\sigma_{AB}(p_T, \vec{s}) \propto \left[\text{diagram 1} + \text{diagram 2} + \dots \right]^2$$

Asymmetry:

$$\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) = \text{diagram} \propto \alpha_s \frac{m_q}{p_T}$$

Too small to explain available data!

□ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

□ Vanish without parton's transverse motion:

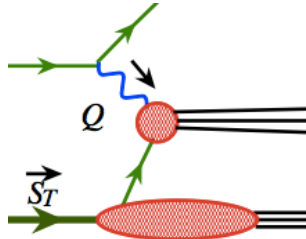


A direct probe for parton's transverse motion,

Spin-orbital correlation, QCD quantum interference

Current understanding of TSSAs

□ Symmetry plays important role:

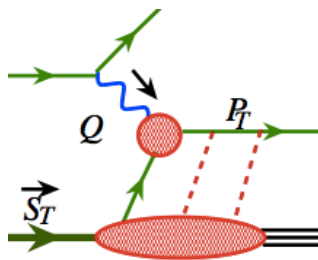


Inclusive DIS
Single scale
 Q

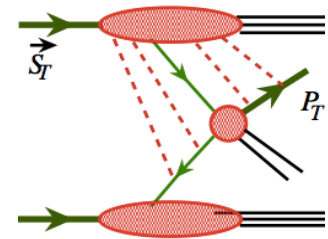
Parity
Time-reversal

→ $A_N = 0$

□ One scale observables – $Q \gg \Lambda_{\text{QCD}}$:



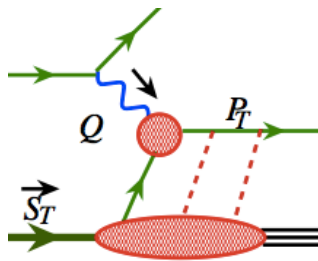
SIDIS: $Q \sim P_T$



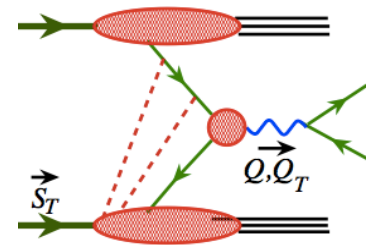
DY: $Q \sim P_T$; Jet, Particle: P_T

Collinear factorization
Twist-3 distributions

□ Two scales observables – $Q_1 \gg Q_2 \sim \Lambda_{\text{QCD}}$:



SIDIS: $Q \gg P_T$



DY: $Q \gg P_T$ or $Q \ll P_T$

TMD factorization
TMD distributions

How collinear factorization generates TSSA?

□ Collinear factorization beyond leading power:

$$\sigma(Q, \vec{s}) \propto \left| \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ + \\ \text{Diagram 3} \\ + \dots \end{array} \right|^2 \left(\frac{\langle k_{\perp} \rangle}{Q} \right)^n - \text{Expansion}$$

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q) H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

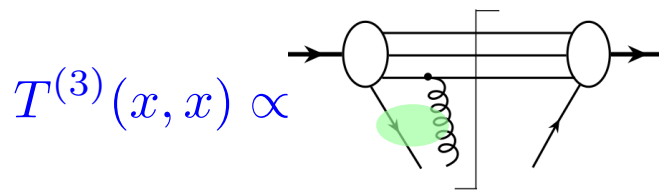
Too large to compete!

Three-parton correlation

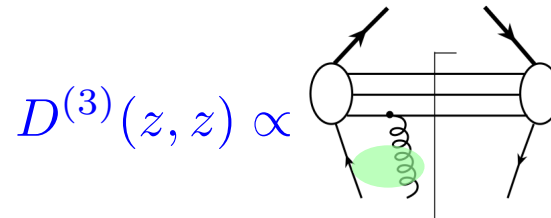
□ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \dots$$



Qiu, Sterman, 1991, ...



Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Twist-3 distributions relevant to A_N

□ Twist-2 distributions:

▪ Unpolarized PDFs:

$$q(x) \propto \langle P | \bar{\psi}_q(0) \frac{\gamma^+}{2} \psi_q(y) | P \rangle$$

$$G(x) \propto \langle P | F^{+\mu}(0) F^{+\nu}(y) | P \rangle (-g_{\mu\nu})$$

▪ Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_{\parallel} | \bar{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} \psi_q(y) | P, S_{\parallel} \rangle$$

$$\Delta G(x) \propto \langle P, S_{\parallel} | F^{+\mu}(0) F^{+\nu}(y) | P, S_{\parallel} \rangle (i\epsilon_{\perp\mu\nu})$$

□ Two-sets Twist-3 correlation functions:

No probability interpretation!

$$\tilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+}{2} [\epsilon^{s_T \sigma n \bar{n}} F_{\sigma}^+(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle$$

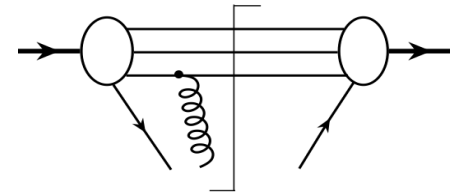
Kang, Qiu, 2009

$$\tilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [\epsilon^{s_T \sigma n \bar{n}} F_{\sigma}^+(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (-g_{\rho\lambda})$$

$$\tilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} [i s_T^{\sigma} F_{\sigma}^+(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle$$

$$\tilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [i s_T^{\sigma} F_{\sigma}^+(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!



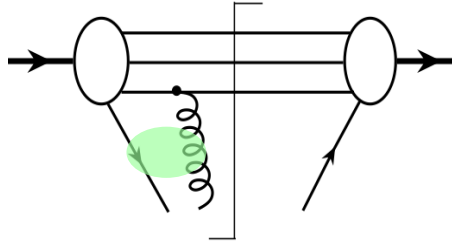
□ Twist-3 fragmentation functions:

“Interpretation” of twist-3 correlation functions

Qiu, Sterman, 1991, ...

Measurement of direct QCD quantum interference:

$$T^{(3)}(x, x, S_{\perp}) \propto$$



Interference between a single active parton state and an active two-parton composite state

“Expectation value” of QCD operators:

$$\langle P, s | \bar{\psi}(0) \gamma^+ \psi(y^-) | P, s \rangle \rightarrow \langle P, s | \bar{\psi}(0) \gamma^+ \left[\epsilon_{\perp}^{\alpha\beta} s_{T\alpha} \int dy_2^- F_{\beta}^+(y_2^-) \right] \psi(y^-) | P, s \rangle$$

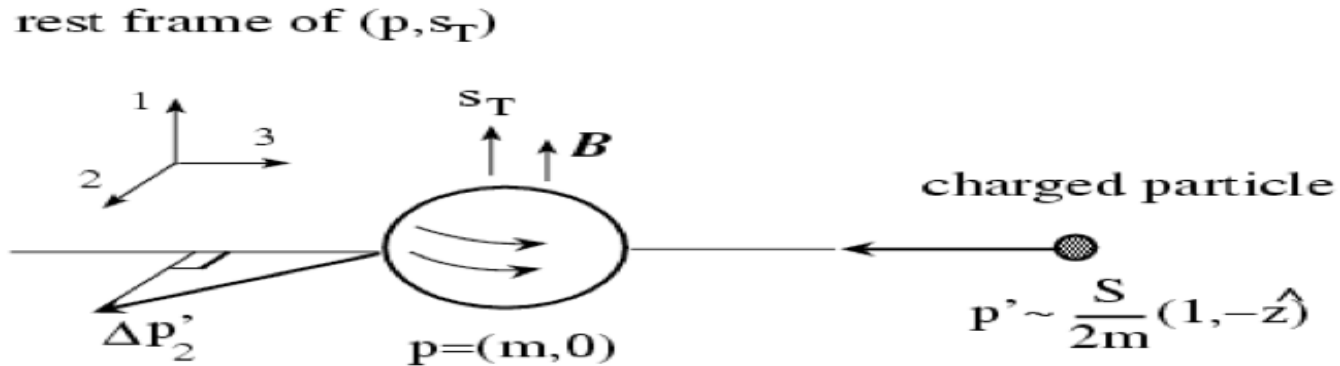
$$\langle P, s | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(y^-) | P, s \rangle \rightarrow \langle P, s | \bar{\psi}(0) \gamma^+ \left[i g_{\perp}^{\alpha\beta} s_{T\alpha} \int dy_2^- F_{\beta}^+(y_2^-) \right] \psi(y^-) | P, s \rangle$$

How to interpret the “expectation value” of the operators in RED?

A simple example

□ The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



□ Change of transverse momentum:

$$\frac{d}{dt} p'_2 = e(\vec{v}' \times \vec{B})_2 = -e v_3 B_1 = e v_3 F_{23}$$

□ In the c.m. frame:

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

$$\implies \frac{d}{dt} p'_2 = e \epsilon^{s_T \sigma n \bar{n}} F_{\sigma}^+$$

□ The total change:

$$\Delta p'_2 = e \int dy^- \epsilon^{s_T \sigma n \bar{n}} F_{\sigma}^+(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Test QCD at twist-3 level

Kang, Qiu, 2009

Scaling violation – “DGLAP” evolution:

$$\underbrace{\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \tilde{T}_{q,F} \\ \tilde{T}_{\Delta q,F} \\ \tilde{T}_{G,F}^{(f)} \\ \tilde{T}_{G,F}^{(d)} \\ \tilde{T}_{\Delta G,F}^{(f)} \\ \tilde{T}_{\Delta G,F}^{(d)} \end{pmatrix}}_{(x, x + x_2, \mu, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K_{qG}^{(f)} & K_{qG}^{(d)} & K_{q\Delta G}^{(f)} & K_{q\Delta G}^{(d)} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K_{\Delta qG}^{(f)} & K_{\Delta qG}^{(d)} & K_{\Delta q\Delta G}^{(f)} & K_{\Delta q\Delta G}^{(d)} \\ K_{Gq}^{(f)} & K_{G\Delta q}^{(f)} & K_{GG}^{(ff)} & K_{GG}^{(fd)} & K_{G\Delta G}^{(ff)} & K_{G\Delta G}^{(fd)} \\ K_{Gq}^{(d)} & K_{G\Delta q}^{(d)} & K_{GG}^{(df)} & K_{GG}^{(dd)} & K_{G\Delta G}^{(df)} & K_{G\Delta G}^{(dd)} \\ K_{\Delta Gq}^{(f)} & K_{\Delta G\Delta q}^{(f)} & K_{\Delta GG}^{(ff)} & K_{\Delta GG}^{(fd)} & K_{\Delta G\Delta G}^{(ff)} & K_{\Delta G\Delta G}^{(fd)} \\ K_{\Delta Gq}^{(d)} & K_{\Delta G\Delta q}^{(d)} & K_{\Delta GG}^{(df)} & K_{\Delta GG}^{(dd)} & K_{\Delta G\Delta G}^{(df)} & K_{\Delta G\Delta G}^{(dd)} \end{pmatrix}}_{(\xi, \xi + \xi_2; x, x + x_2, \alpha_s)} \otimes \underbrace{\begin{pmatrix} \tilde{T}_{q,F} \\ \tilde{T}_{\Delta q,F} \\ \tilde{T}_{G,F}^{(f)} \\ \tilde{T}_{G,F}^{(d)} \\ \tilde{T}_{\Delta G,F}^{(f)} \\ \tilde{T}_{\Delta G,F}^{(d)} \end{pmatrix}}_{\int d\xi \int d\xi_2}$$

Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q)H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

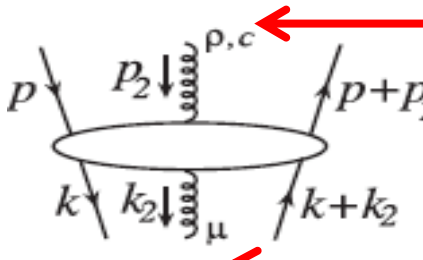
DGLAP for f_2 : $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3 : $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

Evolution kernels – an example

Kang, Qiu, 2009

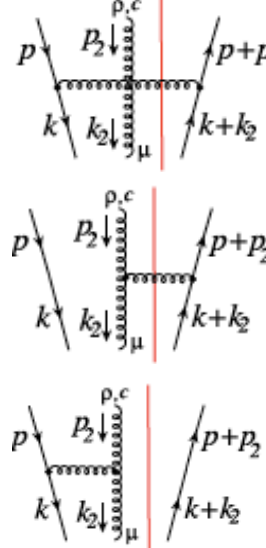
□ Quark to quark:



$$\mathcal{P}_{q,F}^{(LC)} = \frac{1}{2} \gamma \cdot P \left(\frac{-1}{\xi_2} \right) (i\epsilon^{sr\rho n\bar{n}}) \tilde{C}_q,$$

$$\mathcal{V}_{q,F}^{LC} = \frac{\gamma^+}{2P^+} \delta\left(x - \frac{k^+}{P^+}\right) x_2 \delta\left(x_2 - \frac{k_2^+}{P^+}\right) (i\epsilon^{sr\sigma n\bar{n}} [-g_{\sigma\mu}]) C_q.$$

□ Feynman diagram calculation:



$$\int d\xi \int d\xi_2 \mathcal{T}_{q,F}(\xi, \xi + \xi_2) \delta(\xi_2) \frac{1}{\xi} \int^{\mu_F^2} \frac{dk_T^2}{k_T^2} \left[C_F - \frac{C_A}{2} \right] \frac{\alpha_s}{2\pi} \left(\frac{1+z^2}{1-z} \right)$$

$$\int d\xi \int d\xi_2 \mathcal{T}_{q,F}(\xi, \xi + \xi_2) \delta(\xi - x) \frac{1}{\xi_2} \int^{\mu_F^2} \frac{dk_T^2}{k_T^2} \left[\frac{C_A}{2} \right] \frac{\alpha_s}{2\pi} \left(\frac{1}{2} \frac{2x + \xi_2}{x + \xi_2} \right) - \int^{\mu_F^2} \frac{dk_T^2}{k_T^2} \left[\frac{C_A}{2} \right] \frac{\alpha_s}{2\pi} \mathcal{T}_{q,F}(x, x)$$

$$\int d\xi \int d\xi_2 \mathcal{T}_{q,F}(\xi, \xi + \xi_2) \delta(\xi + \xi_2 - x) \frac{1}{\xi} \int^{\mu_F^2} \frac{dk_T^2}{k_T^2} \left[\frac{C_A}{2} \right] \frac{\alpha_s}{2\pi} \left(\frac{1}{2} \frac{1+z}{1-z} \right) - \int^{\mu_F^2} \frac{dk_T^2}{k_T^2} \left[\frac{C_A}{2} \right] \frac{\alpha_s}{2\pi} \mathcal{T}_{q,F}(x, x)$$

Semi-inclusive DIS (SIDIS)

□ Process:

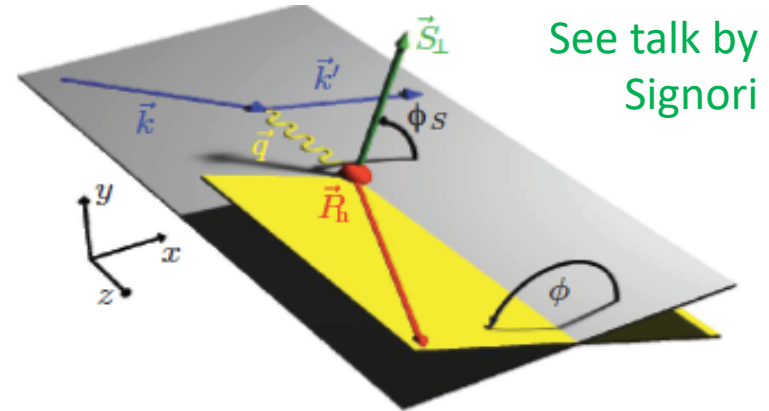
$$e(k) + N(p) \longrightarrow e'(k') + h(P_h) + X$$

□ Natural event structure:

In the photon-hadron frame: $P_{h_T} \approx 0$

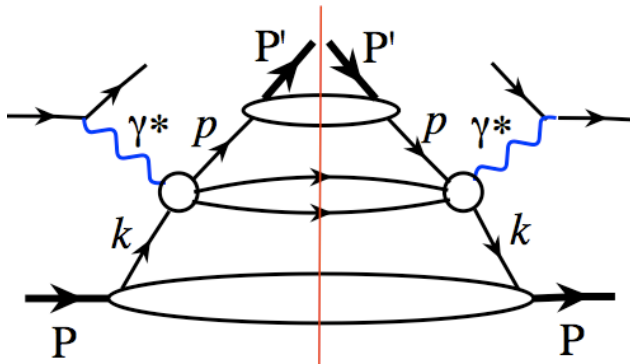
Semi-Inclusive DIS is a natural observable with TWO very different scales

$Q \gg P_{h_T} \gtrsim \Lambda_{\text{QCD}}$ Localized probe sensitive to parton's transverse motion



See talk by Signori

□ Collinear QCD factorization holds if P_{h_T} integrated:



$$d\sigma_{\gamma^* h \rightarrow h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f \rightarrow f'} \otimes D_{f' \rightarrow h'}$$

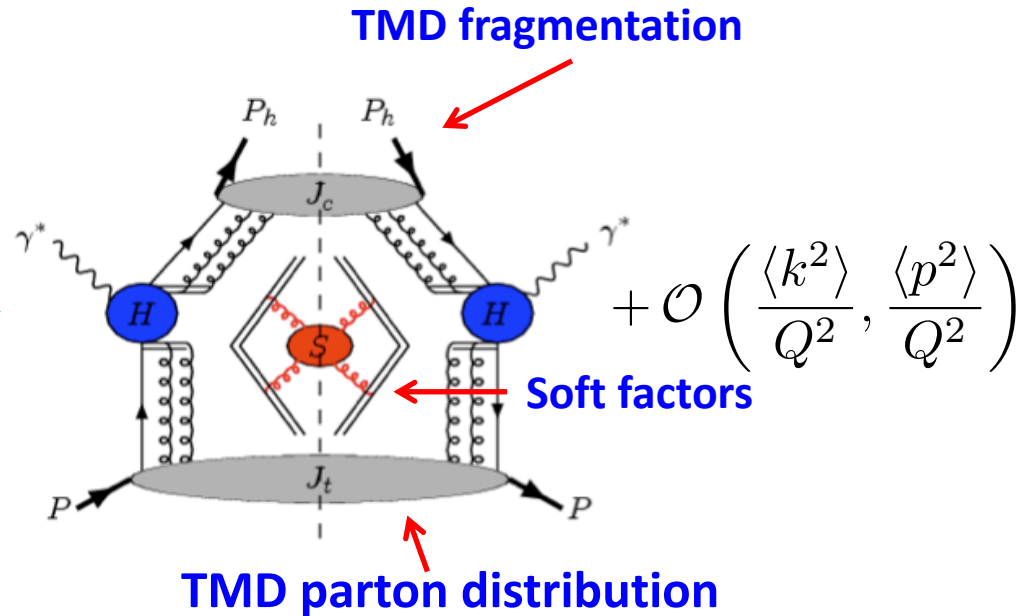
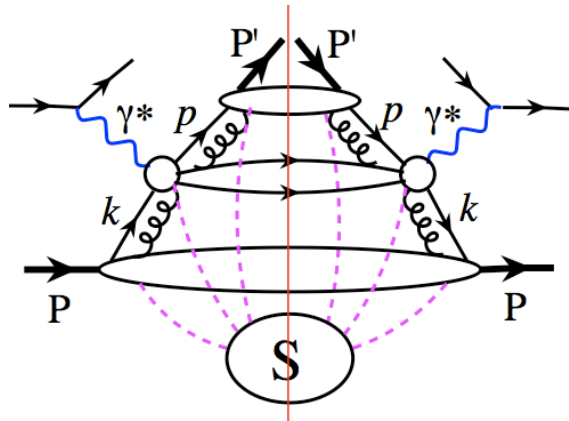
$$z = \frac{P_h \cdot p}{q \cdot p} \quad y = \frac{q \cdot p}{k \cdot p}$$

□ “Total c.m. energy”: $s_{\gamma^* p} = (p + q)^2 \approx Q^2 \left[\frac{1 - x_B}{x_B} \right] \approx \frac{Q^2}{x_B}$

Semi-inclusive DIS (SIDIS)

□ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



□ Low P_{hT} – TMD factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f \rightarrow h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

□ High P_{hT} – Collinear factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

□ P_{hT} Integrated - Collinear factorization:

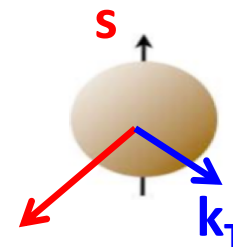
$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

The Present: TMDs

□ Power of spin – many more correlations:

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \odot$		$h_1^\perp = \odot \uparrow - \odot \downarrow$ Boer-Mulders
	L		$g_{1L} = \odot \rightarrow - \odot \leftarrow$ Helicity	$h_{1L}^\perp = \odot \rightarrow \uparrow - \odot \leftarrow \downarrow$
	T	$f_{1T}^\perp = \odot \uparrow - \odot \downarrow$ Sivers	$g_{1T}^\perp = \odot \rightarrow \uparrow - \odot \leftarrow \downarrow$	$h_1 = \odot \uparrow - \odot \downarrow$ Transversity $h_{1T}^\perp = \odot \rightarrow \uparrow - \odot \leftarrow \downarrow$

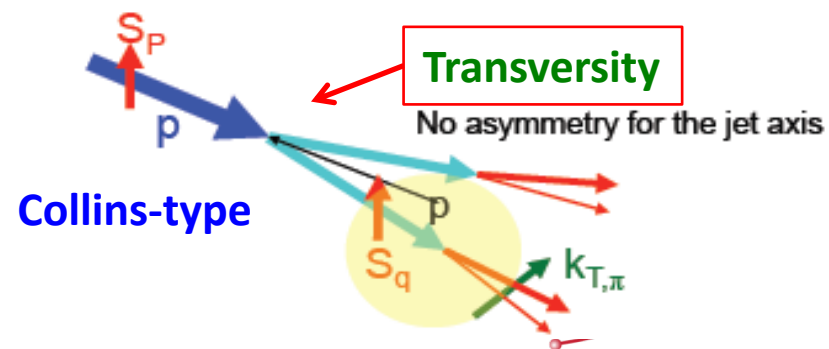
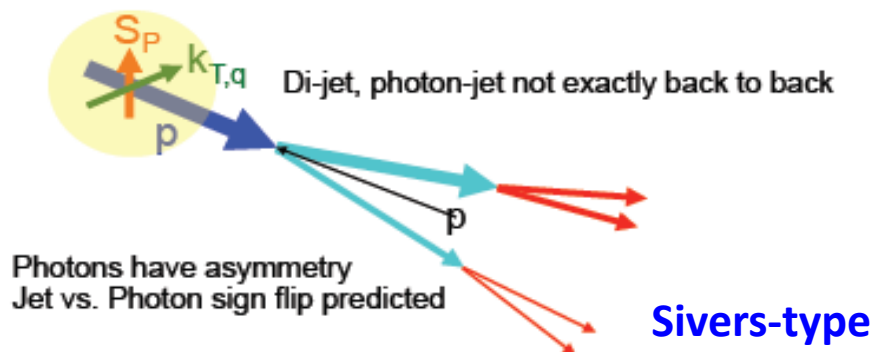
Nucleon Spin
 Quark Spin
 Similar for gluons



Require **two** Physical scales

More than one TMD contribute to the same observable!

□ A_N – single hadron production:

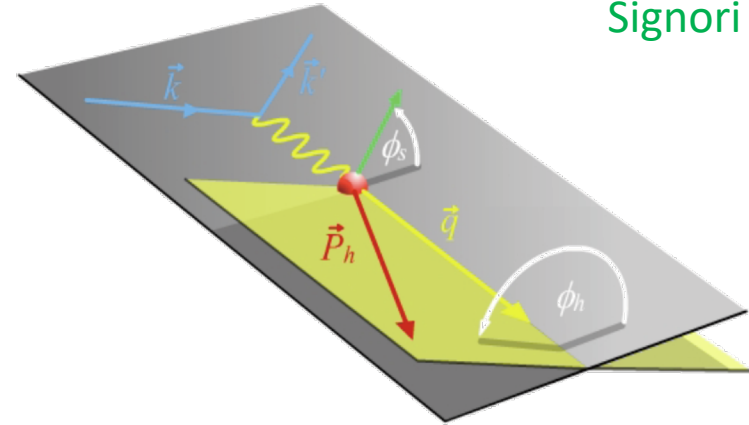


SIDIS is the best for probing TMDs

□ Naturally, two scales & two planes:

See talk by Signori

$$\begin{aligned}
 A_{UT}(\phi_h^l, \phi_S^l) &= \frac{1}{P} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \\
 &= A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_S) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_S) \\
 &+ A_{UT}^{\text{Pretzelosity}} \sin(3\phi_h - \phi_S)
 \end{aligned}$$



□ Separation of TMDs:

$$A_{UT}^{\text{Collins}} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{\text{Sivers}} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{\text{Pretzelosity}} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

← Collins frag. Func. from e⁺e⁻ collisions



Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable):
jet, identified hadron, photon, ...

Boosted 3D nucleon structure

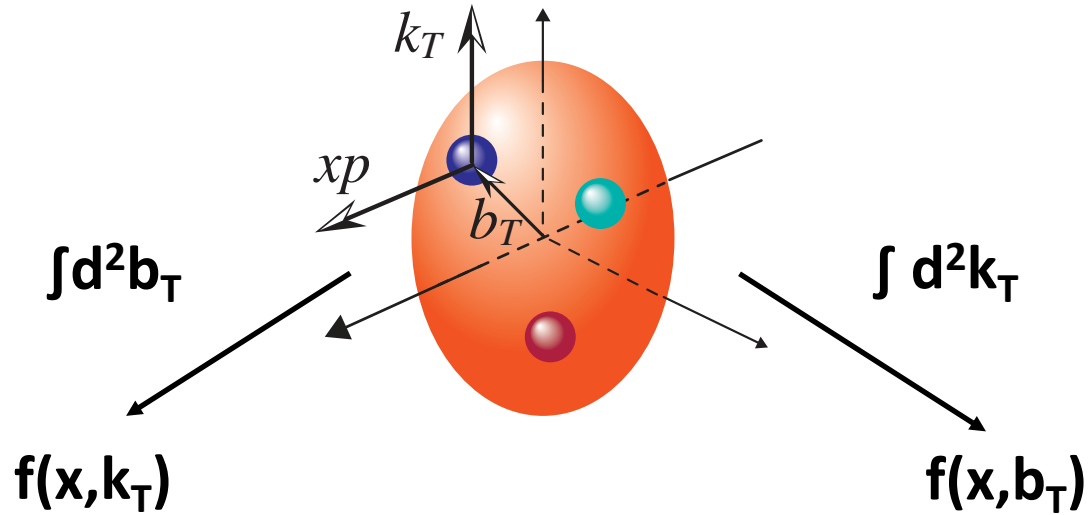
High energy probes “see” the boosted partonic structure:

Momentum Space

Coordinate Space

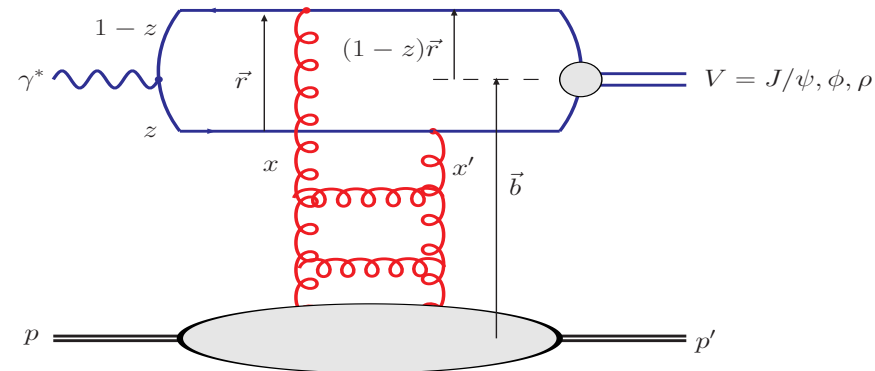
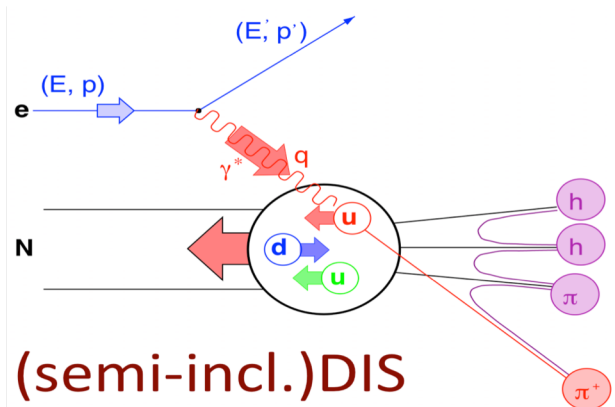
TMDs

GPDs



3D momentum space images

2+1D coordinate space images



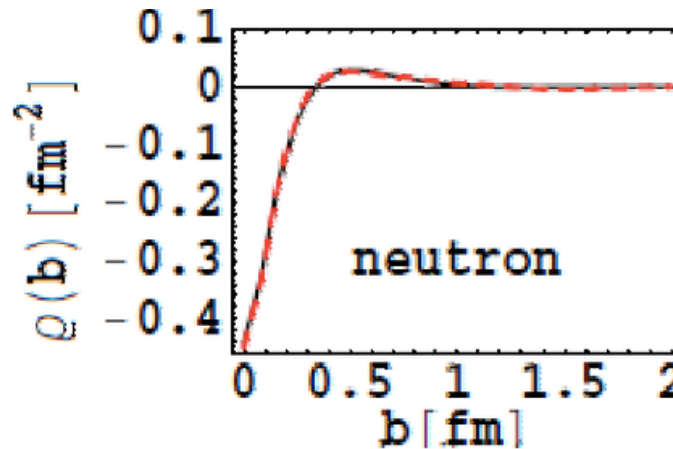
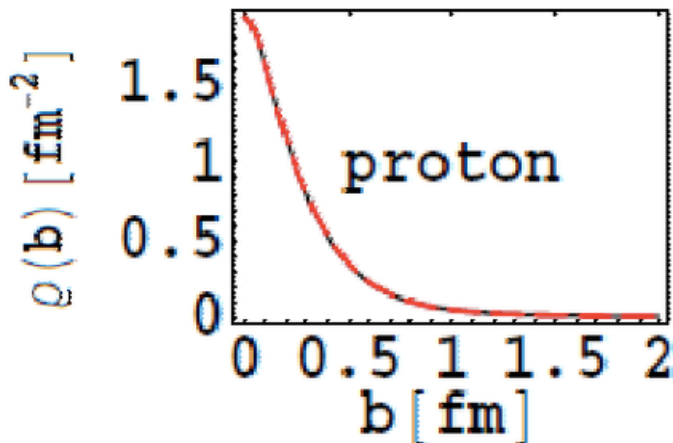
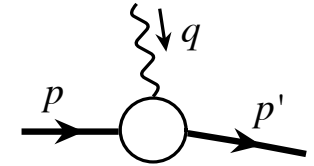
Proton's radius in color distribution?

□ The “big” question:

How color is distributed inside a hadron? (clue for color confinement?)

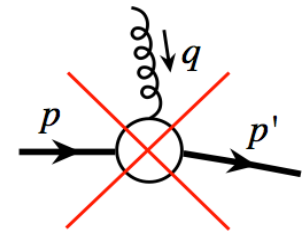
□ Electric charge distribution:

Elastic electric form factor \longrightarrow Charge distributions



□ But, NO color elastic nucleon form factor!

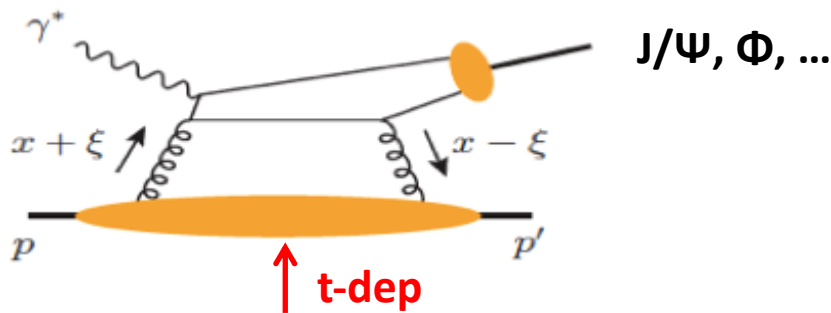
Hadron is colorless and gluon carries color



\longrightarrow Parton density's spatial distributions – a function of x as well (more “proton”-like than “neutron”-like?) – GPDs

Spatial distribution of gluons

Exclusive vector meson production:



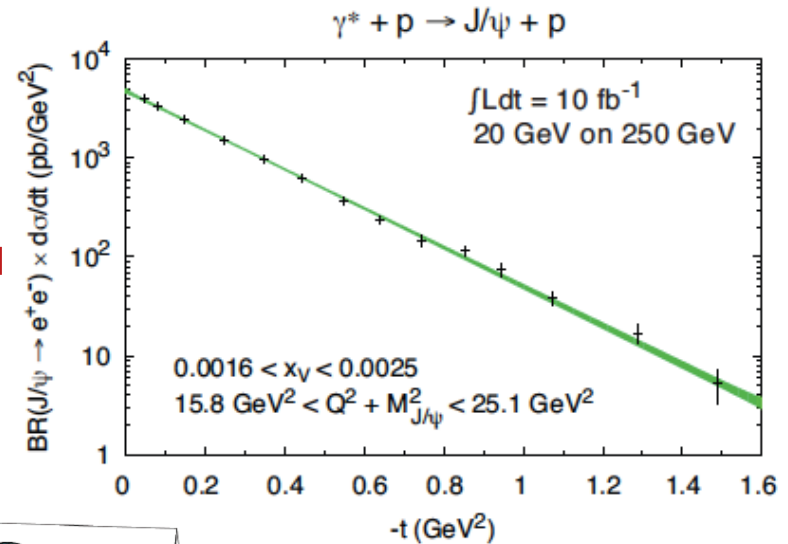
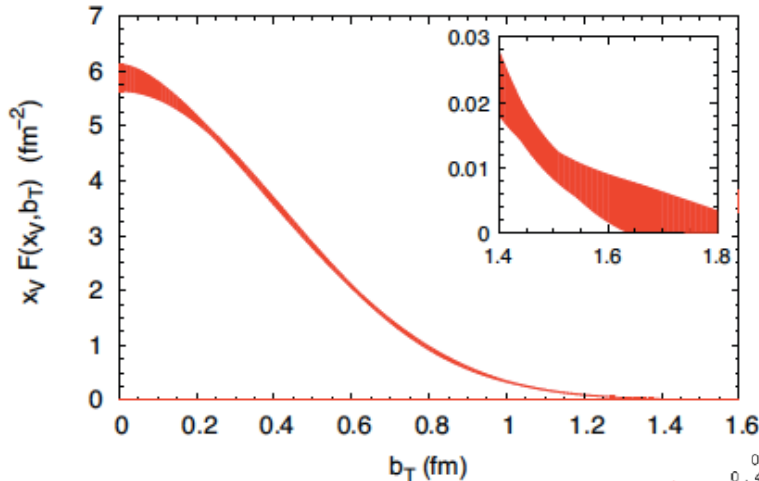
$$\frac{d\sigma}{dx_B dQ^2 dt} \quad \text{EIC-WhitePaper}$$

Fourier transform of the t-dep

Spatial imaging of glue density

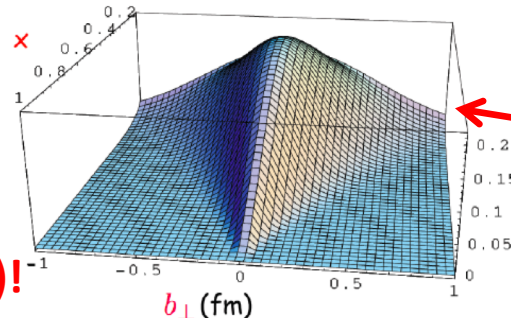
Resolution $\sim 1/Q$ or $1/M_Q$

Gluon imaging from simulation:



Only possible at the EIC

Proton radius?



How spread at small-x?
 Color confinement

Orbital angular momentum

OAM: Correlation between parton's position and its motion
 – in an averaged (or probability) sense

□ **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial}) \right]^3 \psi_q$$

□ **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D}) \right]^3 \psi_q$$

□ **Difference between them:**

Hatta, Yoshida, Burkardt,
 Meissner, Metz, Schlegel,
 ...

✧ **generated by a “torque” of color Lorentz force**

$$\begin{aligned} \mathcal{L}_q^3 - L_q^3 \propto & \int \frac{dy^- d^2y_T}{(2\pi)^3} \langle P' | \bar{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-) \\ & \times \underbrace{\sum_{i,j=1,2} [\epsilon^{3ij} y_T^i F^{+j}(z^-)]}_{\text{“Chromodynamic torque”}} \Phi(z^-, y) \psi(y) | P \rangle_{y^+=0} \end{aligned}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Partonic motion seen by a hard probe

Meissner, Metz, Schiegel, 2009

□ Fully unintegrated distribution:

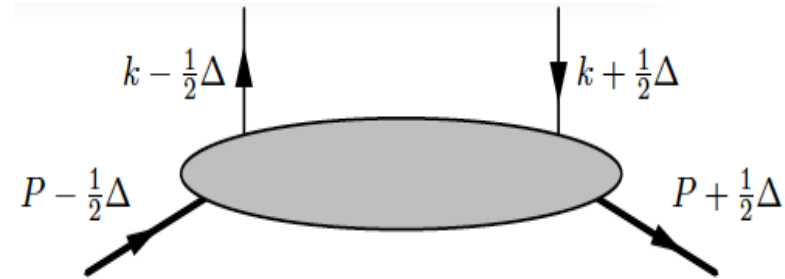
$$W_{\lambda\lambda'}^{[\Gamma]}(P, k, \Delta, N; \eta) = \frac{1}{2} \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \Gamma \mathcal{W}(-\frac{1}{2}z, \frac{1}{2}z | n) \psi(\frac{1}{2}z) | p, \lambda \rangle$$

– not factorizable in general

□ Generalized TMDs – hard probe:

$$\mathcal{W}(x, k_T, \Delta)_\Gamma = \int dk^2 W(P, k, \Delta)_\Gamma$$

– could be factorized assuming **on-shell parton** for the hard probe



□ Wigner function:

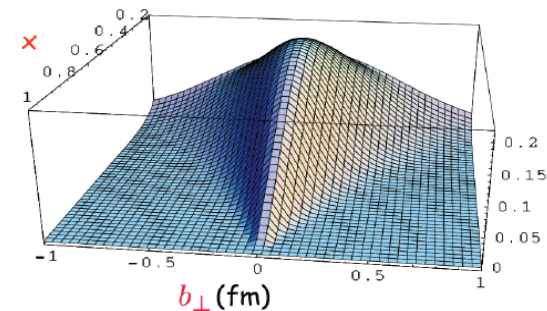
$$W(x, k_T, b) \propto \int d^3 \Delta e^{i\vec{b} \cdot \vec{\Delta}} \mathcal{W}(x, k_T, \Delta)_{\Gamma=\gamma^+}$$

Belitsky, Ji, Yuan

□ Connection to all other known distributions:

$W(x, k_T, b) \Rightarrow$ **Tomographic image of nucleon**

$$q(x, b_\perp) = \int d^2 k_T db^- W(x, k_T, b)_{\gamma^+}$$



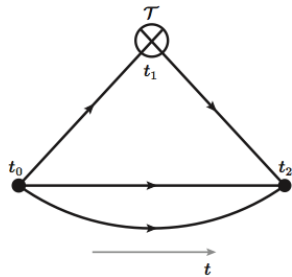
Burkardt, 2002

$\mathcal{W}(x, k_T, \Delta)_\Gamma \Rightarrow$ **TMDs** ($\Delta = 0$), **GPDs** ($\int d^2 k_T$), **PDFs** ($\Delta = 0, \int d^2 k_T$)

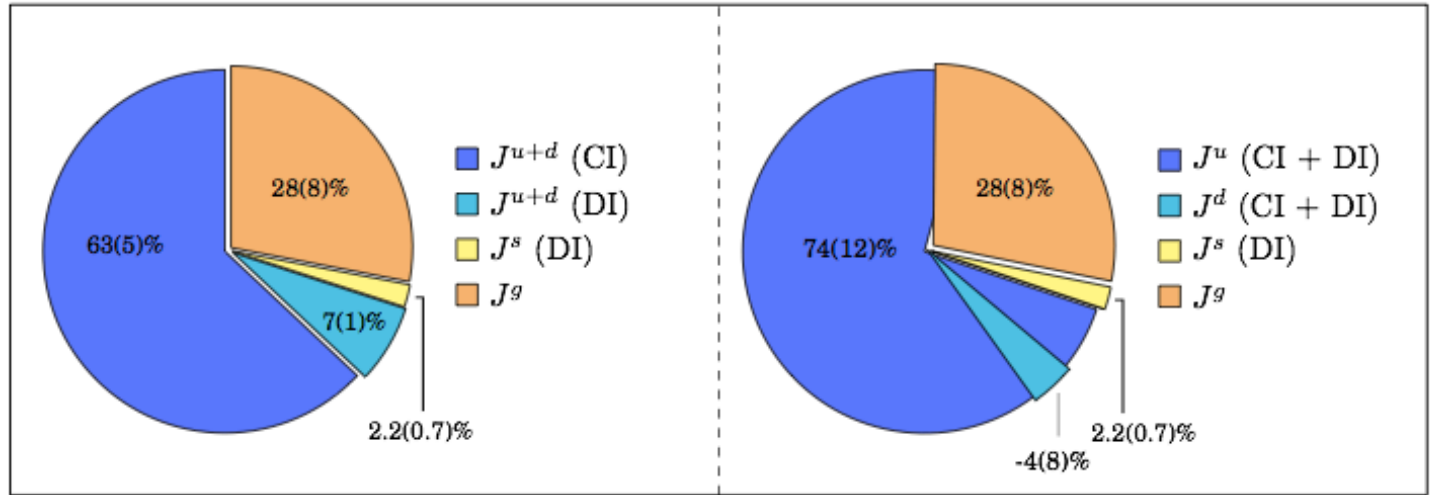
Nucleon spin and OAM from lattice QCD

QCD Collaboration:

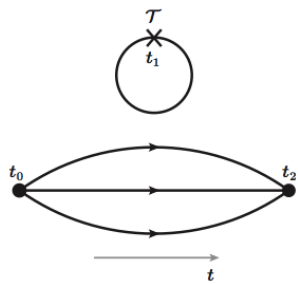
[Deka *et al.* arXiv:1312.4816]



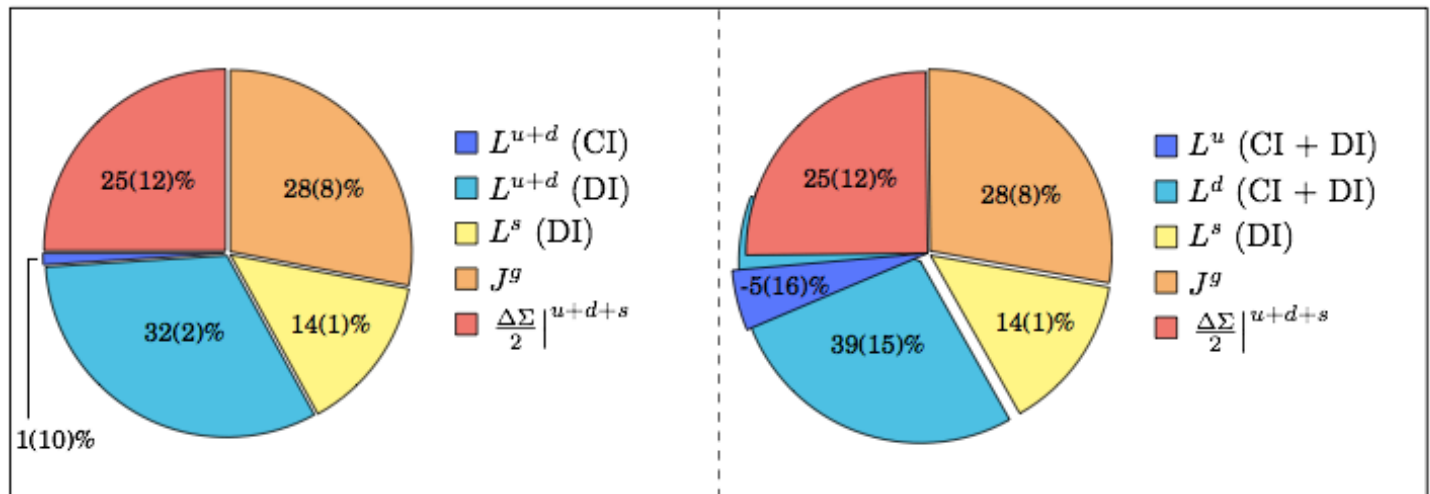
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)

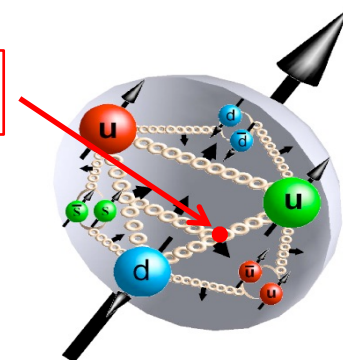


(c)

Summary and outlook

- ❑ QCD has been extremely successful in interpreting and predicting high energy experimental data!
- ❑ But, we still do not know much about hadron structure – The emerging phenomena of QCD!
- ❑ Nuclear Femtography – QCD at a Fermi scale requires two-scale probes. Major advance in both measurement and factorization of two-scale observables!
- ❑ Lepton-Hadron facility is ideal for two-scale observables
- ❑ TMDs and GPDs, accessible by high energy scattering with polarized beams, encode important information on hadron's 3D structure – distributions as well as motions of quarks and gluons

$< 1/10 \text{ fm}$



Thank you!

Basic fundamentals about spin

Some fundamentals about spin

□ Spin in non-relativistic quantum mechanics:

✧ Spin as an intrinsic angular momentum of the particle

– three spin vector:

$$\vec{S} = (S_x, S_y, S_z)$$

– angular momentum algebra:

$$\begin{aligned} [S_i, S_j] &= i\epsilon_{ijk} S_k & \epsilon_{123} &= +1 \\ \longrightarrow [S^2, S_j] &= 0 \end{aligned}$$

✧ S^2, S_z have set of simultaneous eigenvectors: $|S, m\rangle$

$$\begin{aligned} S^2 |S, m\rangle &= S(S+1)\hbar^2 |S, m\rangle & S &= 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \\ S_z |S, m\rangle &= m\hbar |S, m\rangle & -S &\leq m \leq S \end{aligned}$$

✧ Spin d.o.f. are decoupled from kinematic d.o.f.

$$\Psi_{\text{Schr}}(\vec{r}) \longrightarrow \Psi_{\text{Schr}}(\vec{r}) \times \chi_m$$

where χ_m s a $(2S+1)$ – component “spinor”

Some fundamentals about spin

□ Spin-1/2:

✧ Two component spinors:

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

✧ Operators could be represented by Pauli-matrices:

$$\mathcal{S}_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathcal{S}_y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \mathcal{S}_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

✧ Eigenstates to $\vec{\mathcal{S}}^2$ and \mathcal{S}_z :

$$\chi_z^\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_z^\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

✧ Eigenvalues:

$$\mathcal{S}_z \chi_z^\uparrow = +\frac{1}{2} \chi_z^\uparrow \quad \mathcal{S}_z \chi_z^\downarrow = -\frac{1}{2} \chi_z^\downarrow$$

Particles in these states are “polarized in z-direction”

Some fundamentals about spin

□ General superposition:

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_z^\uparrow + b\chi_z^\downarrow \quad (\chi^\dagger\chi = 1)$$

$$\langle S_z \rangle = \chi^\dagger S_z \chi = \left(+\frac{1}{2} \right) |a|^2 + \left(-\frac{1}{2} \right) |b|^2 = \frac{1}{2} [|a|^2 - |b|^2]$$

✧ **Example:** $a = b = 1/\sqrt{2}$ $\chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \langle S_z \rangle = 0$

✧ **Notice:** $\langle S_x \rangle = \chi^\dagger S_x \chi = +\frac{1}{2}$

✧ **Eigenstate to S_x** $\chi_x^\uparrow = \frac{1}{\sqrt{2}} [\chi_z^\uparrow + \chi_z^\downarrow]$

✧ **Arbitrary direction \vec{n} with $|\vec{n}| = 1$**

$$S_n = \vec{n} \cdot \vec{S} = n_x S_x + n_y S_y + n_z S_z = \frac{1}{2} \begin{pmatrix} n_z & n_x - in_y \\ n_x + in_y & -n_z \end{pmatrix}$$

A state that is an eigenstate to this operator: "polarized in \vec{n} - direction"

\vec{n} = Polarization vector

Eigenvalues = $\pm 1/2$

Some fundamentals about spin

□ Spin in the relativistic theory:

Physics is invariant under Lorentz transformation:
boost, rotations, and translations in space and time

✧ Poincare group – 10 generators: $\mathcal{P}^\mu, \mathcal{M}^{\mu\nu}$

✧ Pure rotations: $J_i = -\frac{1}{2} \epsilon_{ijk} \mathcal{M}^{jk}$ re boosts: $\mathcal{K}_i = \mathcal{M}^{i0}$

Total angular momentum: $[J_i, J_j] = i \epsilon_{ijk} J_k$

✧ Two group invariants (fundamental observables):

$$\mathcal{P}_\mu \mathcal{P}^\mu = \mathcal{P}^2 = m^2$$

$$\mathcal{W}_\mu \mathcal{W}^\mu \quad \text{where} \quad \mathcal{W}_\mu = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \mathcal{M}^{\nu\rho} \mathcal{P}^\sigma \quad \text{Pauli-Lubanski}$$

✧ Fact: $[\mathcal{W}_\mu, \mathcal{W}_\nu] = i \epsilon_{\mu\nu\rho\sigma} \mathcal{W}^\rho \mathcal{P}^\sigma \quad \longrightarrow \quad [W^i, W^j] = i m \epsilon_{ijk} W^k$
If acting on states at the rest

✧ Spin: $\mathcal{S}_i \equiv \frac{1}{m} \mathcal{W}^i = J_i$

Note: $\mathcal{W}_\mu \mathcal{W}^\mu$ has eigenvalues $m^2 S(S+1)$

Some fundamentals about spin

✧ Recall: constructed eigenstates to \vec{S}^2 and $\vec{n} \cdot \vec{S}$:

$$\begin{aligned} \mathcal{W}_\mu \mathcal{W}^\mu |p, S\rangle &= m^2 S(S+1) |p, S\rangle & S &= \frac{1}{2} \\ -\frac{W \cdot n}{m} |p, S\rangle &= \pm \frac{1}{2} |p, S\rangle & W^\mu &= \mathcal{W}^\mu|_{\text{at rest}} \end{aligned}$$

✧ “Polarization operator”:

$$\mathcal{P} \equiv -\frac{W \cdot n}{m}$$

✧ “Covariant polarization vector”:

$$n^\mu \text{ with } n^2 = -1, \quad n \cdot p = 0$$

✧ For Dirac particles:

$$\mathcal{P} = \frac{1}{2} \gamma_5 \gamma_\mu n^\mu$$

➡ Projection operators to project out the eigenstates of \mathcal{P} :

$$\frac{1}{2} (\mathbb{1} \pm \gamma_5 \not{n})$$

✧ Longitudinal polarization:

$$\vec{n} = \vec{p}/|\vec{p}|, \quad n^0 = 0$$

➡ $\mathcal{P} = \frac{1}{2} \gamma_5 \gamma_\mu n^\mu = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|}$ with eigenvalues $\pm \frac{1}{2}$

$$\frac{\vec{J} \cdot \vec{p}}{|\vec{p}|} u_\pm(p) = \pm \frac{1}{2} u_\pm(p) \equiv \frac{\lambda}{2} u_\pm(p) \quad \rightsquigarrow \lambda \text{ “helicity”}$$

Massless particle:

$$\frac{\vec{J} \cdot \vec{p}}{|\vec{p}|} \rightarrow \gamma_5$$

helicity = chirality _{1 Lab}

Some fundamentals about spin

✧ **Transverse polarization:** $n^\mu = (0, \vec{n}_\perp, 0)$ (for \vec{p} in z direction)

➡ $\mathcal{P} = \gamma_0 \vec{J} \cdot \vec{n} = \gamma_0 J_\perp \neq J_\perp$

✧ **Transversity, not “transverse spin”, has the eigenvalue:** $\pm \frac{1}{2}$

$$\gamma_0 J_\perp u_{\uparrow\downarrow}(p) = \pm \frac{1}{2} u_{\uparrow\downarrow}(p)$$

with spinors: $u_\uparrow^{(x)} = \frac{1}{\sqrt{2}} [u_+ + u_-]$

Same as in non-relativistic theory

➡ **Transverse polarization, or transversity, not “transverse spin”, is invariant under the “boosts along \vec{p} ”**

✧ **Projection operator with both longitudinal and transverse components:**

$$\frac{1}{2} \not{p} \left[\mathbb{1} - s_\parallel \gamma_5 + \gamma_5 \not{s}_\perp \right] \quad \text{at high energy}$$

with $s_\parallel \sim \lambda, s_\perp \sim n_\perp$

Some fundamentals about spin

□ Back to Spin-1/2:

✧ A free spin-1/2 particle obeys Dirac equation

$$(\not{p} - m) u(p) = 0 \quad \text{where } \not{p} = \gamma_\mu p^\mu$$

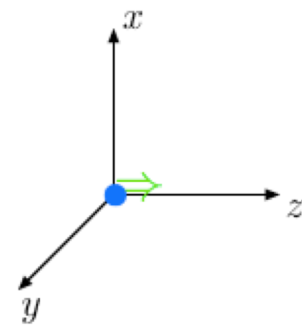
with 4-component solutions:

$$\Psi(x) = \begin{cases} e^{-i p \cdot x} u(p) & \text{positive energy} \rightarrow \text{particle} \\ e^{+i p \cdot x} v(p) & \text{negative energy} \rightarrow \text{antiparticle} \end{cases}$$

Each with “two” solutions: “spin up/down”

✧ If it is at rest,

$$u^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u^- = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



They are eigenstates to the spin operator : S_z

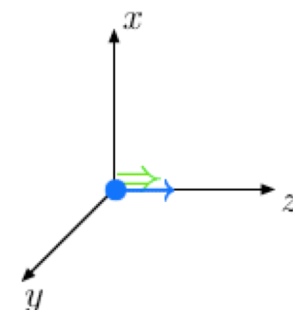
$$S_z u^\pm = \pm \frac{1}{2} u^\pm$$

“polarized in z-direction”

Some fundamentals about spin

- ✧ Boost the particle to momentum $p = (E, 0, 0, p_z)$

$$\rightarrow u^+ = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix} \quad u^- = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix}$$



- ✧ Eigenstates of the helicity operator:

$$\frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} u^\pm = \pm \frac{1}{2} u^\pm$$

- ✧ Also eigenstates of the Pauli-Lubanski (polarization) operator:

$$\frac{1}{2} \gamma_5 \not{n} u^\pm = \pm \frac{1}{2} u^\pm$$

where the polarization vector $n = (p_z, 0, 0, E)/m$

- ✧ At high energy, $E \approx p_z$ also become eigenstates to chirality γ_5 :

$$\gamma_5 u^\pm = \pm \frac{1}{2} u^\pm$$

Some fundamentals about spin

□ Back to rest frame:

- ✧ Construct eigenstates to the spin operator \mathcal{S}_x :

$$\mathcal{S}_x u^\uparrow = +\frac{1}{2} u^\uparrow \quad \mathcal{S}_x u^\downarrow = -\frac{1}{2} u^\downarrow$$

with $u^\uparrow = \frac{1}{\sqrt{2}} [u^+ + u^-]$ $u^\downarrow = \frac{1}{\sqrt{2}} [u^+ - u^-]$

“polarized along x-direction”

- ✧ Boost the particle to momentum $p = (E, 0, 0, p_z)$

→ $u^\uparrow = \frac{N}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ \frac{p_z}{E+m} \\ -\frac{p_z}{E+m} \end{pmatrix}$ $u^\downarrow = \frac{N}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ \frac{p_z}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$

Still has

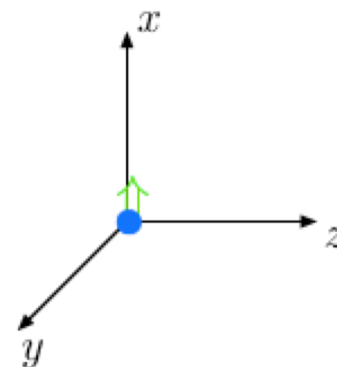
$$u^\uparrow = (u^+ + u^-)/\sqrt{2}$$

- ✧ Still the eigenstates of the Pauli-Lubanski (polarization) operator:

$$\frac{1}{2} \gamma_5 \not{n} u^{\uparrow\downarrow} = \pm \frac{1}{2} u^{\uparrow\downarrow} \quad \text{where } n = (0, 1, 0, 0)$$

- ✧ But, no longer eigenstates of the transverse-spin operator:

$$\mathcal{S}_x u^\uparrow \neq +\frac{1}{2} u^\uparrow$$



Parity and Time-reversal invariance

□ In quantum field theory, **physical observables** are given by **matrix elements** of quantum field operators

□ Consider two quantum states: $|\alpha\rangle$ $|\beta\rangle$

□ Parity transformation:

$$|\alpha_P\rangle \equiv U_P |\alpha\rangle \quad |\beta_P\rangle \equiv U_P |\beta\rangle$$

$$\langle\alpha_P|\beta_P\rangle = \langle\alpha|U_P^\dagger U_P|\beta\rangle = \langle\alpha|\beta\rangle$$

□ Time-reversal transformation:

$$|\alpha_T\rangle \equiv V_T |\alpha\rangle \quad |\beta_T\rangle \equiv V_T |\beta\rangle$$

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|V_T^\dagger V_T|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

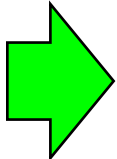
Parity and Time-reversal invariance

□ Parton fields under P and T transformation:

$$U_P \psi(y_0, \vec{y}) U_P^{-1} = \gamma^0 \psi(y_0, -\vec{y})$$

$$V_T \psi(y_0, \vec{y}) V_T^{-1} = (i\gamma^1 \gamma^3) \psi(-y_0, \vec{y})$$

$$\mathcal{J} = i\gamma^1 \gamma^3$$

 $\langle P, \vec{s}_\perp | \bar{\psi}(0) \Gamma_i \psi(y^-) | P, \vec{s}_\perp \rangle$
 $= \langle P, -\vec{s}_\perp | \bar{\psi}(0) \left[\mathcal{J} \left(\Gamma_i^\dagger \right)^* \mathcal{J}^\dagger \right] \psi(y^-) | P, -\vec{s}_\perp \rangle$

□ Quark correlations contribute to polarized X-sections:

$$T_i(x; \vec{s}_\perp) = -T_i(x; -\vec{s}_\perp) \quad \langle \text{Green arrow} \rangle \quad \mathcal{J} \left(\Gamma_i^\dagger \right)^* \mathcal{J}^\dagger = -\Gamma_i$$

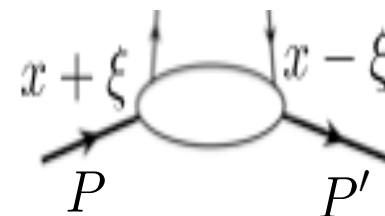
$$\Gamma_i = \gamma^\mu \gamma_5, \quad \sigma^{\mu\nu} \quad \text{or} \quad \sigma^{\mu\nu} (i\gamma_5)$$

$$\Gamma_i = I, \quad i\gamma_5, \quad \gamma^\mu \quad \text{contribute to spin-avg X-sections:}$$

GPDs – role in solving the spin puzzle

□ Quark “form factor”:

$$\begin{aligned}
 F_q(x, \xi, t, \mu^2) &= \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' | \bar{\psi}_q(\lambda/2) \frac{\gamma \cdot n}{2P \cdot n} \psi_q(-\lambda/2) | P \rangle \\
 &\equiv H_q(x, \xi, t, \mu^2) [\bar{U}(P') \gamma^\mu U(P)] \frac{n_\mu}{2P \cdot n} \\
 &+ E_q(x, \xi, t, \mu^2) \left[\bar{U}(P') \frac{i\sigma^{\mu\nu} (P' - P)_\nu}{2M} U(P) \right] \frac{n_\mu}{2P \cdot n}
 \end{aligned}$$



with $\xi = (P' - P) \cdot n/2$ and $t = (P' - P)^2 \Rightarrow -\Delta_\perp^2$ if $\xi \rightarrow 0$

$$\tilde{E}_q(x, \xi, t, Q)$$

Different quark spin projection

□ Total quark’s orbital contribution to proton’s spin:

Ji, PRL78, 1997

$$\begin{aligned}
 J_q &= \frac{1}{2} \lim_{t \rightarrow 0} \int dx x [H_q(x, \xi, t) + E_q(x, \xi, t)] \\
 &= \frac{1}{2} \Delta q + L_q
 \end{aligned}$$

□ Connection to normal quark distribution:

$$H_q(x, 0, 0, \mu^2) = q(x, \mu^2)$$

The limit when

$\xi \rightarrow 0$