HUGS: Introduction to QCD

- **Lec. 1: Fundamentals of QCD**
- **Lec. 2: Matching observed hadrons to quarks and gluons**
- **Lec. 3: QCD for cross sections with identified hadrons**
- **Lec. 4: QCD for cross sections with polarized beam(s)**

HUGS 2020 was canceled due to COVID-19
HUGS 2021 will be the first virtual school

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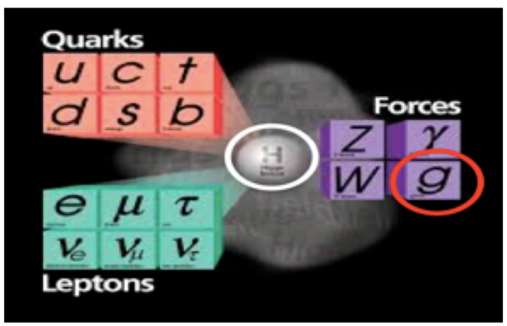
QCD is everywhere in our universe

- ❑ **Understanding where did we come from?** **Global Time:** →

  - ○ **QCD at high temperature, high densities, phase transition, …**
  - ○ ***Facilities – Relativistic heavy ion collisions: SPS. RHIC. the LHC. …***

- ❑ **Understanding the visible world at 3°K – what are we made of?**



  - ○ **How to understand the emergence and properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
  - ○ **How does the glue bind us all?**
  - ○ ***Facilities – CEBAF, EIC, …***

Nuclear Femtography

Search for answers to these questions at a Fermi scale!

Physical observables

Cross sections with identified hadron(s)
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not a perturbative scale

Follow a two-step approach:

1) Purely infrared safe quantities

2) Observables with identified hadron(s)

Fully infrared safe observables – I

Fully inclusive, without any identified hadron!

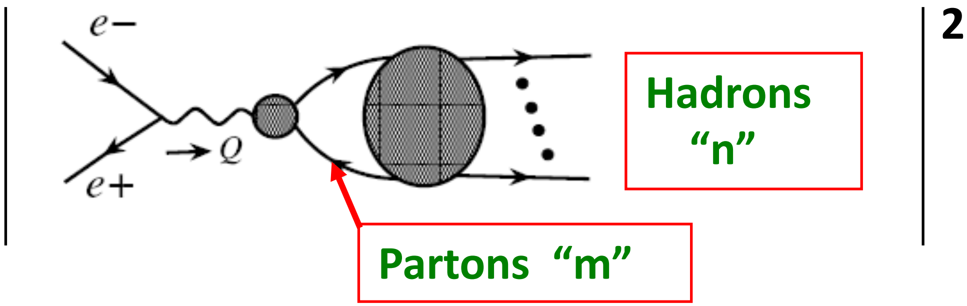
$$\sigma^{\rm total}_{e^+e^-\to{\rm hadrons}} = \sigma^{\rm total}_{e^+e^-\to{\rm partons}}$$

The simplest observable in QCD

e^+e^- ➔ hadrons inclusive cross sections

- e^+e^- ➔ hadron **total** cross section – not a specific hadron!

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto$$



If there is no quantum interference between partons and hadrons,

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto \sum_n \mathrm{P}_{e^+e^-\to n} = \sum_n \boxed{\sum_m \mathrm{P}_{e^+e^-\to m}\mathrm{P}_{m\to n}} = \sum_m \mathrm{P}_{e^+e^-\to m} \boxed{\sum_n \mathrm{P}_{m\to n}} \quad \mathbf{=1}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} \propto \sum_m \mathrm{P}_{e^+e^-\to m}$$

$$\Rightarrow \boxed{\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}}$$

Finite in perturbation theory – KLN theorem

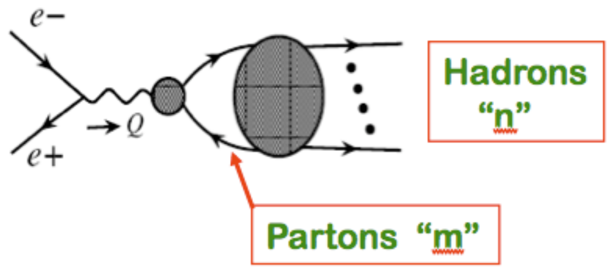
- e^+e^- ➔ parton total cross section:

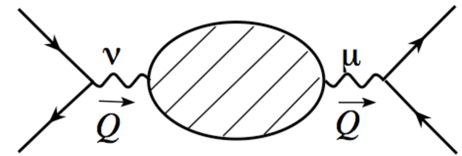
$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

Calculable in pQCD

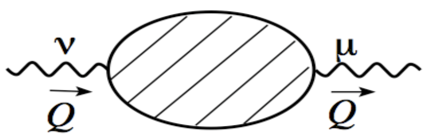
Infrared safety of e^+e^- total cross sections

❑ Optical theorem:

$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\Big|$ 

$\Big|^2 \propto \mathrm{Im}$ 

❑ Time-like vacuum polarization:



$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$

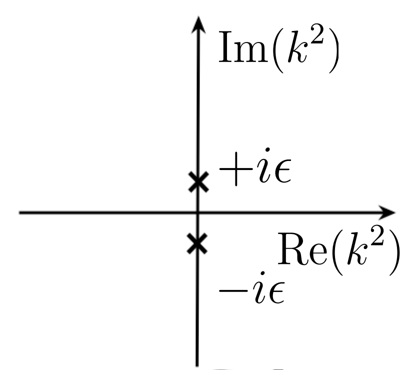
IR safety of $\sigma^{\rm tot}_{e^+e^-\to\rm partons}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

❑ IR safety of $\Pi(Q^2)$:

If there were pinched poles in Π(Q²),

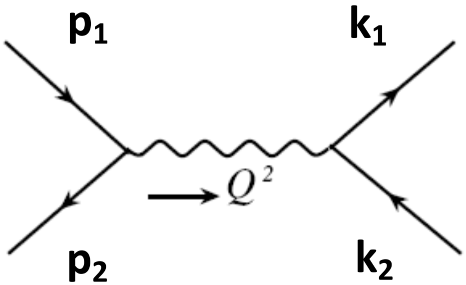
- ✧ **real partons moving away from each other**
- ✧ **cannot be back to form the virtual photon again!**

Rest frame of the virtual photon



Lowest order (LO) perturbative calculation

- Lowest order Feynman diagram:



- Invariant amplitude square:

$$
\begin{aligned}
|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 &= e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right] \\
&\times \mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right] \\
&= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]
\end{aligned}
$$

$$
\boxed{\begin{aligned} s &= (p_1+p_2)^2 \\ t &= (p_1-k_1)^2 \\ u &= (p_2-k_1)^2 \end{aligned}}
$$

- Lowest order cross section:

$$
\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s = Q^2
$$

Threshold constraint

$$
\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}
$$

One of the best tests for the number of colors

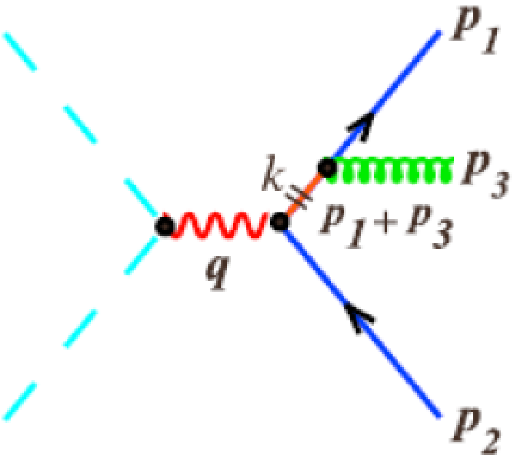
Next-to-leading order (NLO) contribution

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

IR as $x3 \to 0$
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

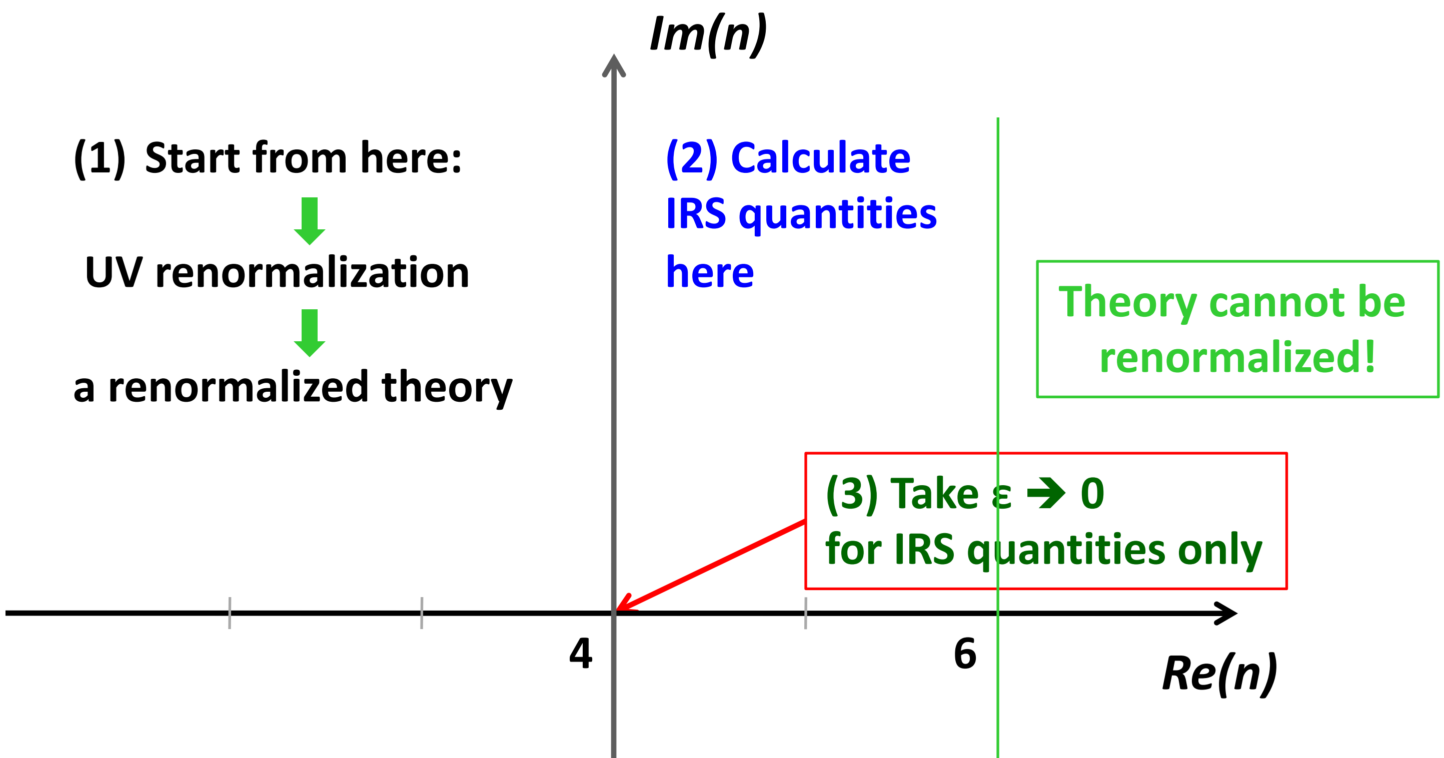
Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

- **Complex *n*-dimensional space:**

$$\int d^n k \, F(k, Q)$$



$G^{(\mathrm{ren})}$

$G^{(\mathrm{un-ren})} \longrightarrow G^{(\mathrm{ren})}$

UV-finite, IR-finite

Dimensional regularization for both IR and CO

- **NLO with a dimensional regulator:**

  - **Real:** $$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

  - **Virtual:** $$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

  - **NLO:** $$\boxed{\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_{2}^{(0)}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]}$$

    No ε dependence!

  - **Total:** $$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O\left(\alpha_s^2\right) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Go beyond the inclusive total cross section?

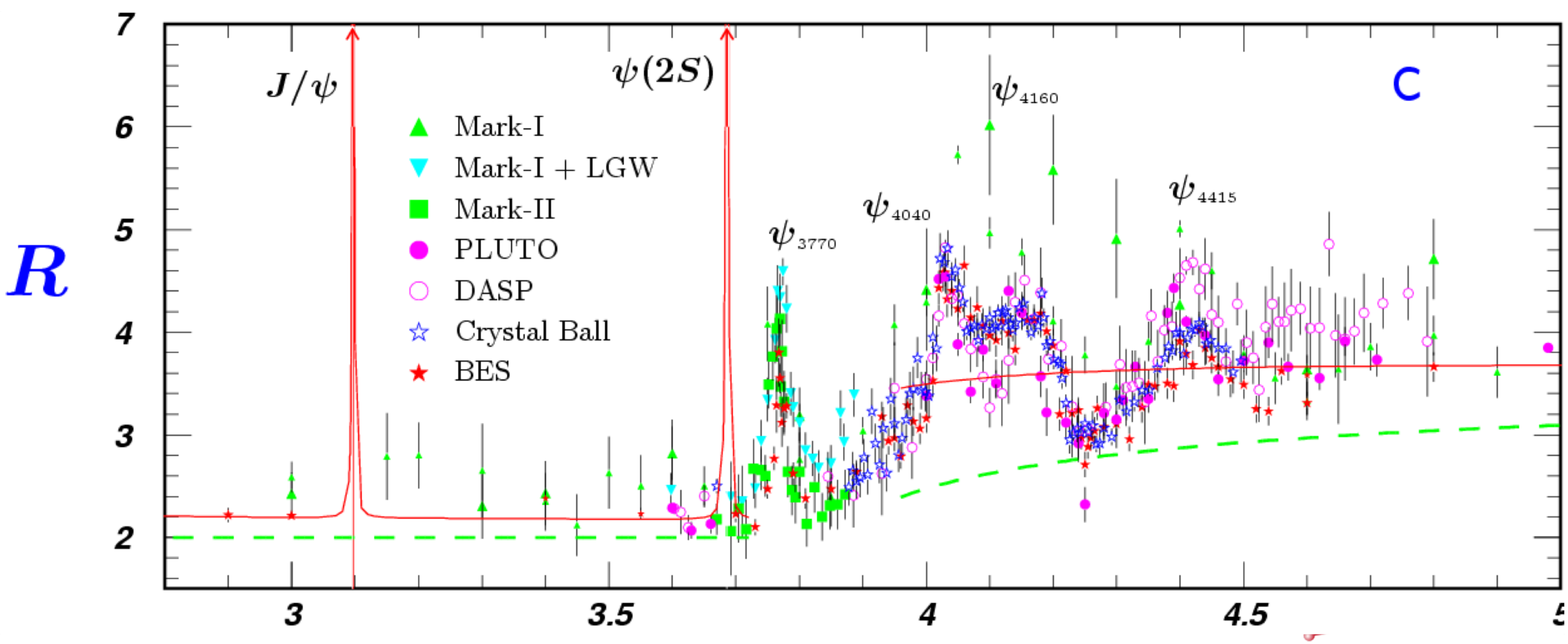
Hadronic cross section in e+e- collision

❑ Normalized hadronic cross section:

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Fully infrared safe observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\mathrm{Jets}}_{e^+e^-\to\mathrm{hadrons}} = \sigma^{\mathrm{Jets}}_{e^+e^-\to\mathrm{partons}}$$

Jets – "trace" or "footprint" of partons

Thrust distribution in e^+e^- collisions

etc.

Jets – trace of partons

- ❑ Jets – "total" cross-section with a limited phase-space

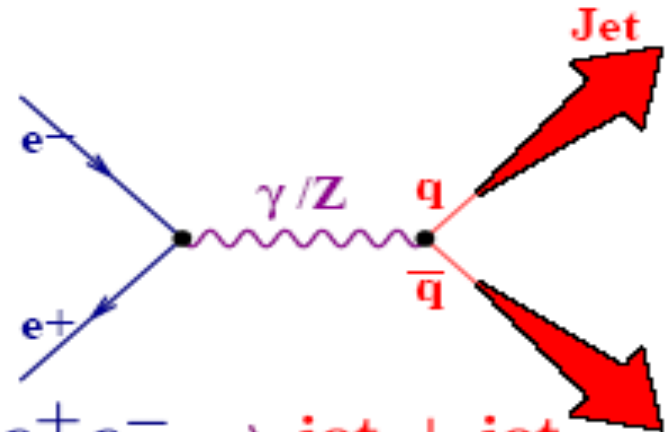
  Not any specific hadron!

- ❑ Q: will IR cancellation be completed?

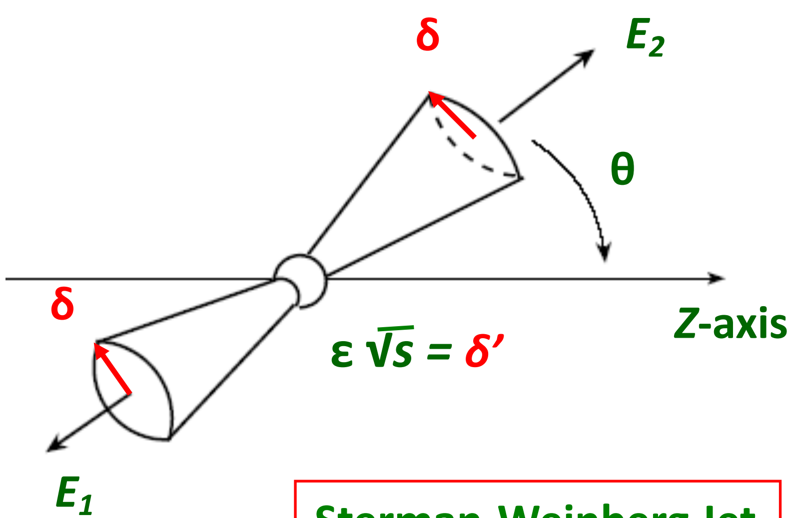
  - ✧ Leading partons are moving away from each other

  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton

- ❑ Many Jet algorithms



$$e^+e^- \rightarrow \text{jet} + \text{jet}$$



Sterman-Weinberg Jet

Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \quad 0 \le \lambda \le 1$$

Physical meaning:

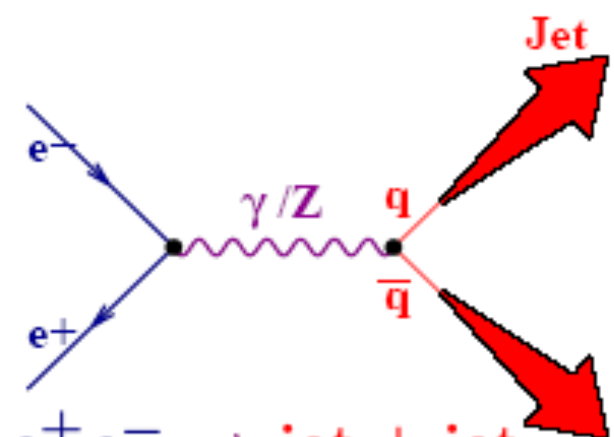
Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without this parton – inclusiveness!

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1 \ \text{for all } n \quad \Rightarrow \quad \sigma^{(\text{tot})}$

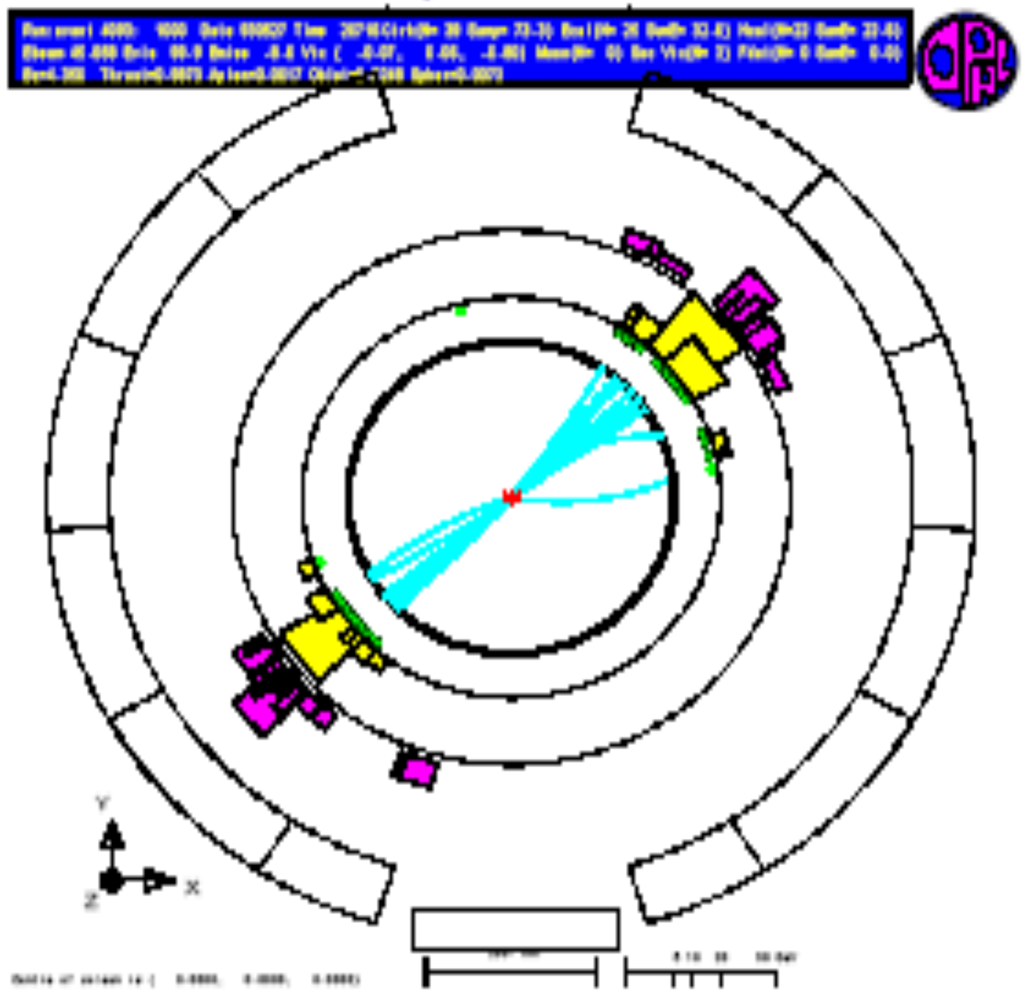
An early clean two-jet event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):

LEP ($\sqrt{s} = 90 - 205$ GeV)



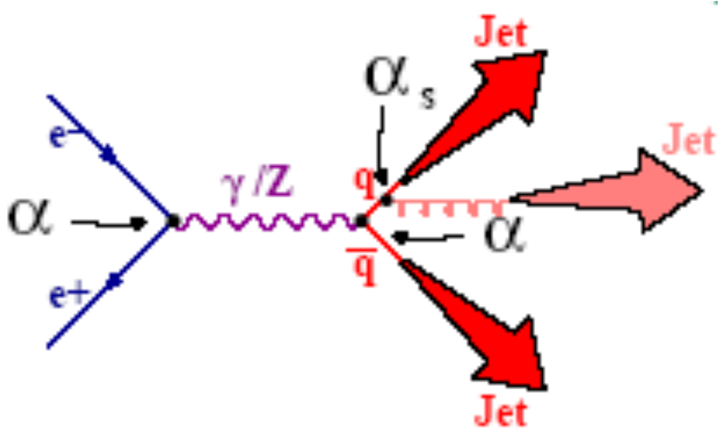
$e^+e^- \rightarrow \text{jet} + \text{jet}$



A clean trace of two partons – a pair of quark and antiquark

Discovery of a gluon jet

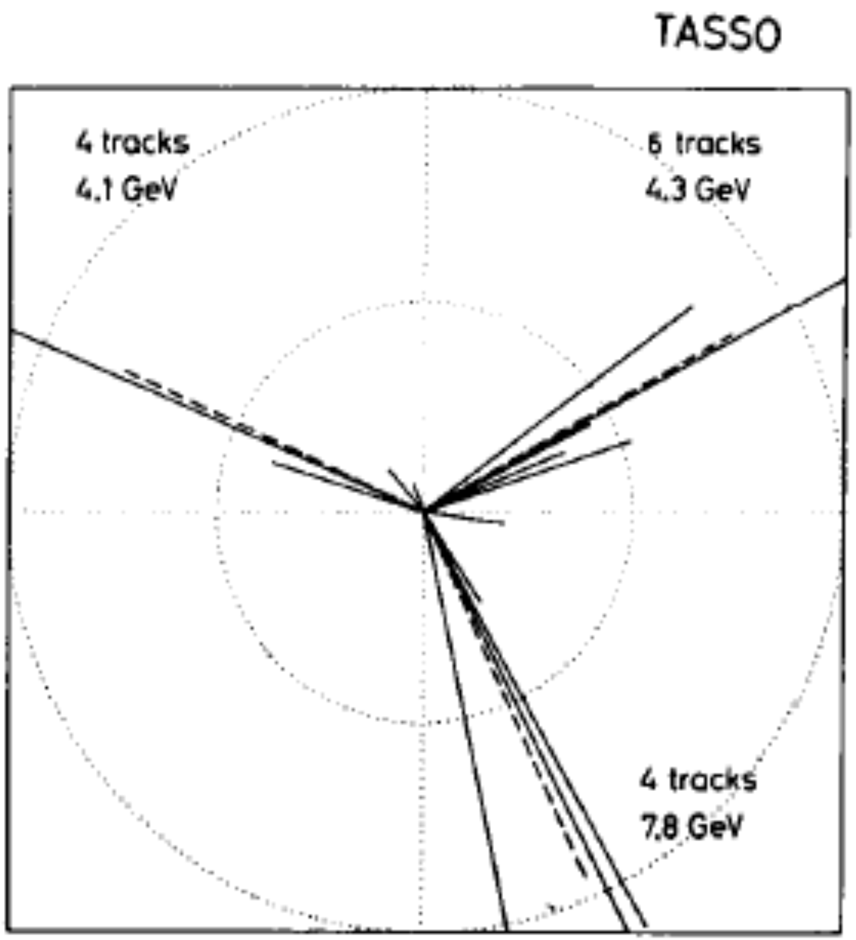
First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



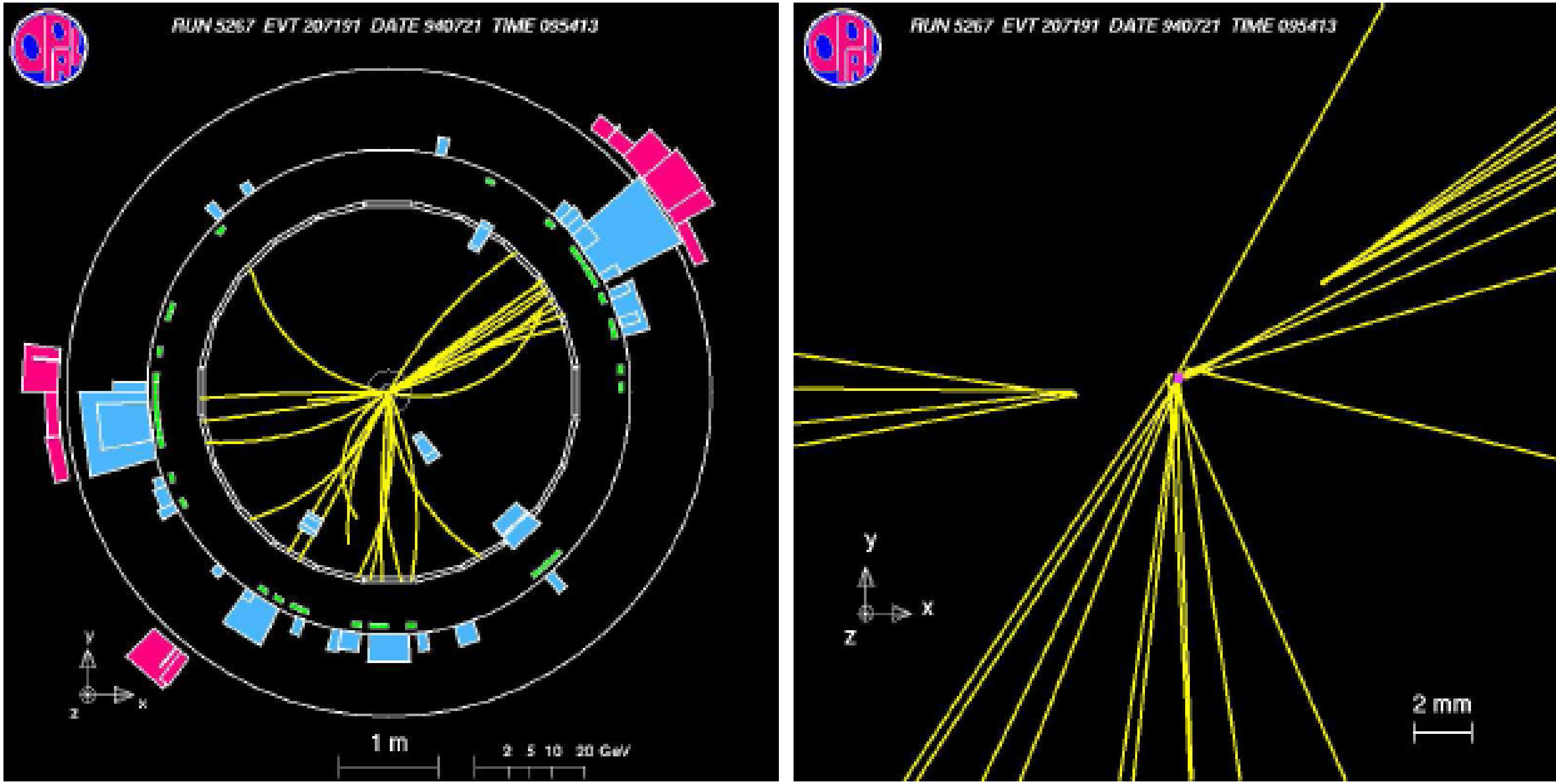
TASSO Collab., Phys. Lett. B86 (1979) 243

MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged three-jet event from LEP

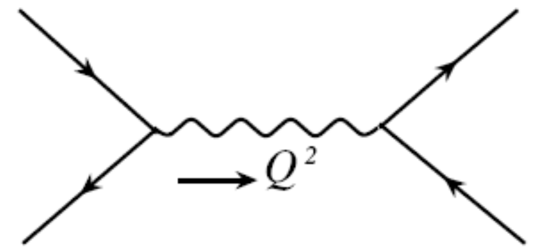


Gluon Jet

Two-jet cross section in e+e- collisions

- **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



- **Two-jet in pQCD:**

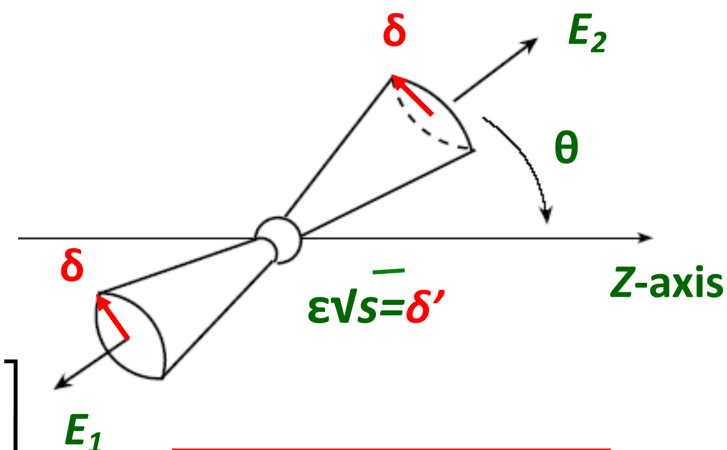
$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $$C_n = C_n\left(\delta\right)$$

- **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln\left(\delta\right)\ln\left(\delta'\right)+3\ln\left(\delta\right)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$



Sterman-Weinberg Jet

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as}\quad Q\to\infty$$

Basics of jet finding algorithms

- **Recombination jet algorithms (almost all e+e- colliders):**

Recombination metric: $y_{ij} = \dfrac{M_{ij}^2}{E_{c.m.}^2}$ $\quad M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

for Durham k_T

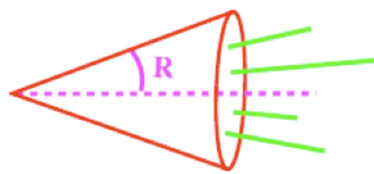
✧ **different algorithm = different choice of M_{ij}^2:**

✧ **Combine the particle pair (i,j) with the smallest : y_{ij}** $\quad (i,j) \to k$

$$e.g.\ \mathrm{E\ scheme:}\quad p_k = p_i + p_j$$

✧ **iterate until all remaining pairs satisfy:** $\quad y_{ij} > y_{cut}$

- **Cone jet algorithms (CDF, …, colliders):**



✧ **Cluster all particles into a cone of half angle R to form a jet:**

✧ **Require a minimum visible jet energy:** $\quad E_{jet} > \epsilon$

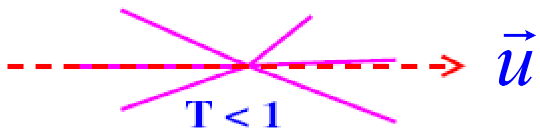
Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right) \dfrac{\Delta_{ij}^2}{R^2}$

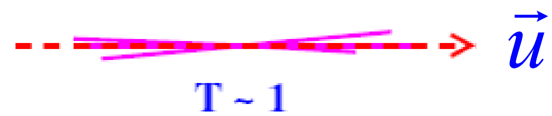
✧ **Classical choices:** ***p=1* – "k_T algorithm", *p= -1* – "anti-k_T", …**

Thrust distribution

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|} \right)$$





- **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to\mathrm{hadrons}}}{dT} \quad \textbf{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

  - Contribution from p=0 particles drops out the sum

  - Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

The harder question

- ❑ **Question:**

  How to test QCD in a reaction with identified hadron(s)?
  – to probe the quark-gluon structure of the hadron

- ❑ **Facts:**

  Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

  Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

- ❑ **Solution – Factorization:**

  ✧ **Isolate the calculable dynamics of quarks and gluons**

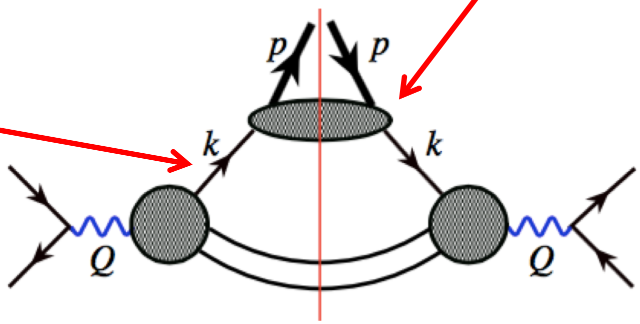
  ✧ **Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions**
  – provide information on the partonic structure of the hadron

Observables with ONE identified hadron

- Creation of an identified hadron:

Not necessary to be dominated by one parton, which is always virtual!

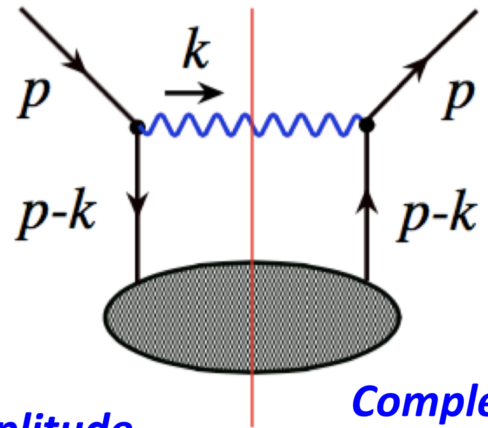
Non-perturbative!



– in a "cut-diagram" notation

- "Square" of the diagram with a *"unobserved gluon"*:

"Cut-line" – final-state



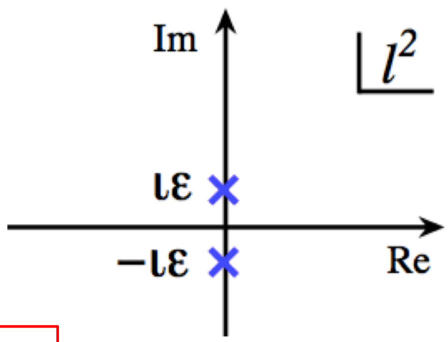
Amplitude

Complex conjugate of the Amplitude

$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Pinch singularity & pinch surface

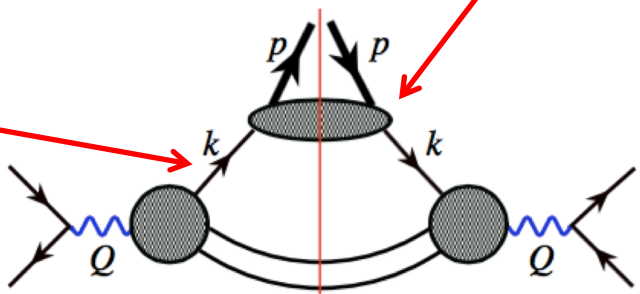
Two parts connected by a "classical" parton

Observables with ONE identified hadron

❑ Creation of an identified hadron:

Not necessary to be dominated by one parton, which is always virtual!

Non-perturbative!



– in a "cut-diagram" notation

❑ On-shell approximation:

$$\sigma_{e^+e^-\to h(p)X} \approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,k;\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$$\boxed{\hat{k}^2=0} \qquad \approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,\hat{k};\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right) + \ldots$$

$$\approx \sum_f \int dz\, \mathcal{H}_{e^+e^-\to f(k)}(Q,\frac{p}{z};\sqrt{S}) \int \frac{d^4k}{(2\pi)^4} \delta(z-\frac{p\cdot n}{k\cdot n}) \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$$\approx \sum_f \int dz\; \hat{\sigma}_{e^+e^-\to f(k)}(Q,z;\sqrt{S})\; D_{f(k)\to h(p)X}(z,p;\Lambda_{\rm QCD}) + \ldots$$

Hard collision to produce an on-shell parton
– Perturbatively calculable!

FF: Probability for the parton to become the observed hadron
– Non-perturbative, universal!

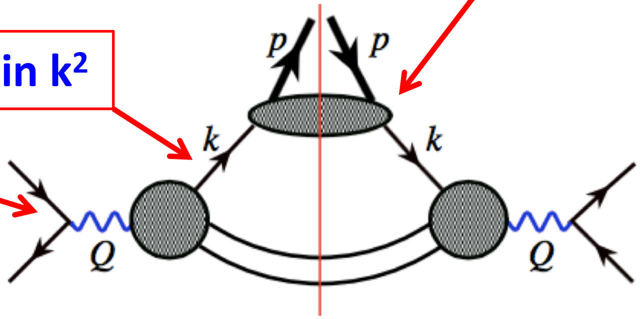
Observables with ONE identified hadron

- ❑ Need more observables to fix the "FFs":

Non-perturbative!

Pinch in k^2

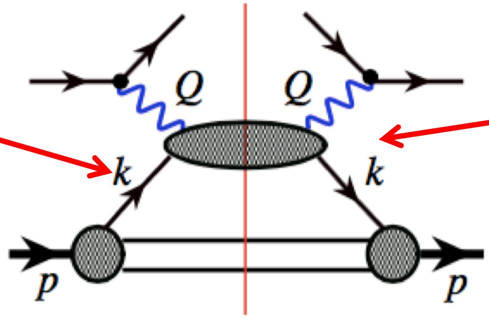
Perturbative!



– in a "cut-diagram" notation

- ❑ Identified initial hadron:

Pinch in k^2



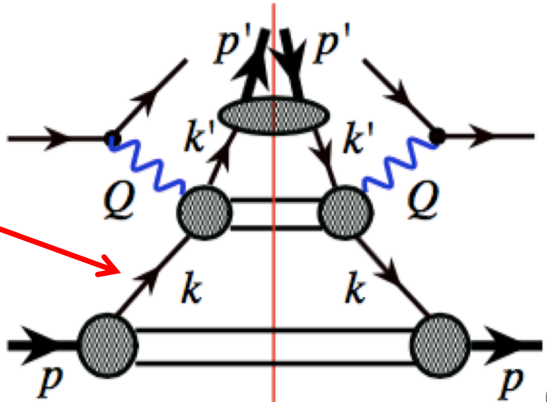
Perturbative!

Non-perturbative!

- ❑ Initial + created identified hadron(s):

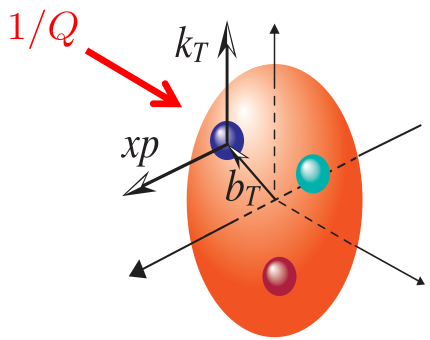
Pinch in both k^2 and k'^2

Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!



Probes for 3D hadron structure

- Single scale hard probe is too "localized":



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> $1/Q$

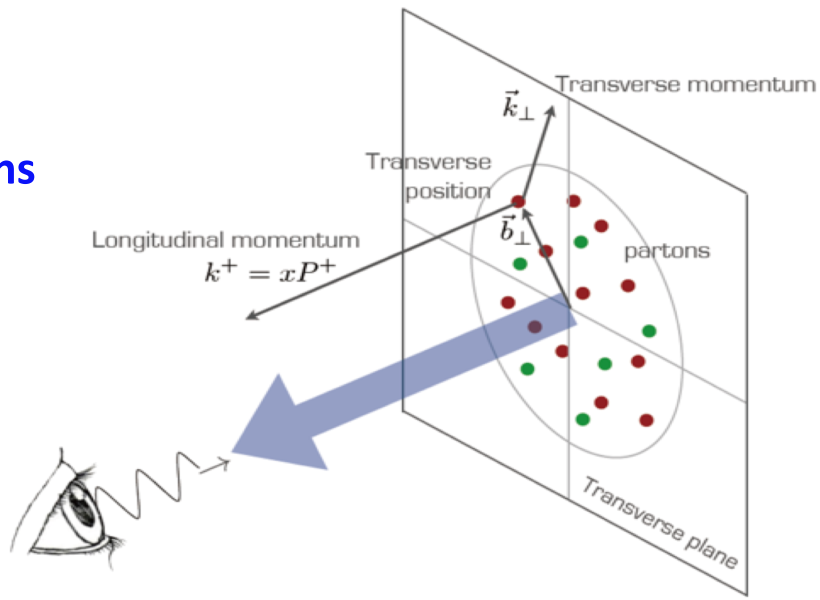
- Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\mathrm{QCD}}$$

Hard scale: Q_1 To localize the probe particle nature of quarks/gluons

"Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm

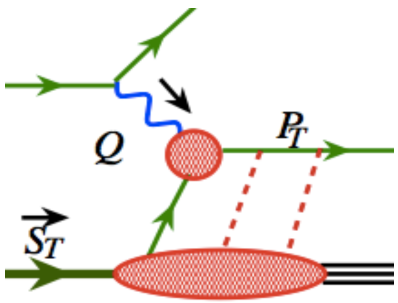
Hit the hadron "very hard" without breaking it, clean information on the structure!



“See” hadron’s 3D partonic structure?

❑ **Two-scale observables are natural in lepton-hadron collisions:**

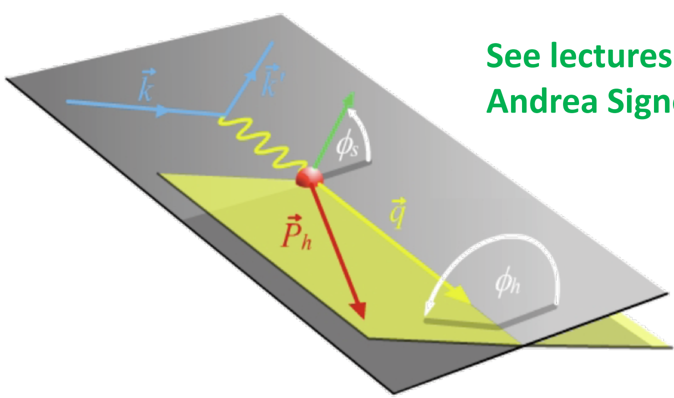
✧ **Semi-inclusive DIS:**



+ …

SIDIS: $Q \gg P_T$

Parton’s confined motion encoded into TMDs

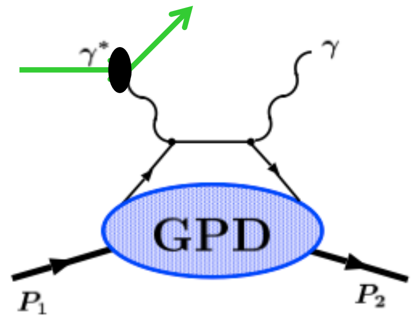


See lectures by
Andrea Signori

Two scales, two planes, Angular modulation, …

✧ **Exclusive DIS:**

See lectures by
Carlos Munoz Camacho

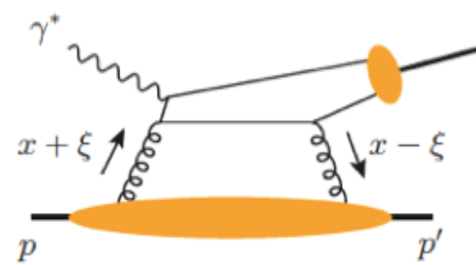


+ …

Imaging quarks

DVCS: $Q^2 \gg |t|$

Parton’s spatial imaging from Fourier transform of GPDs’ t-dependence



J/Ψ, Φ, …

Imaging gluons

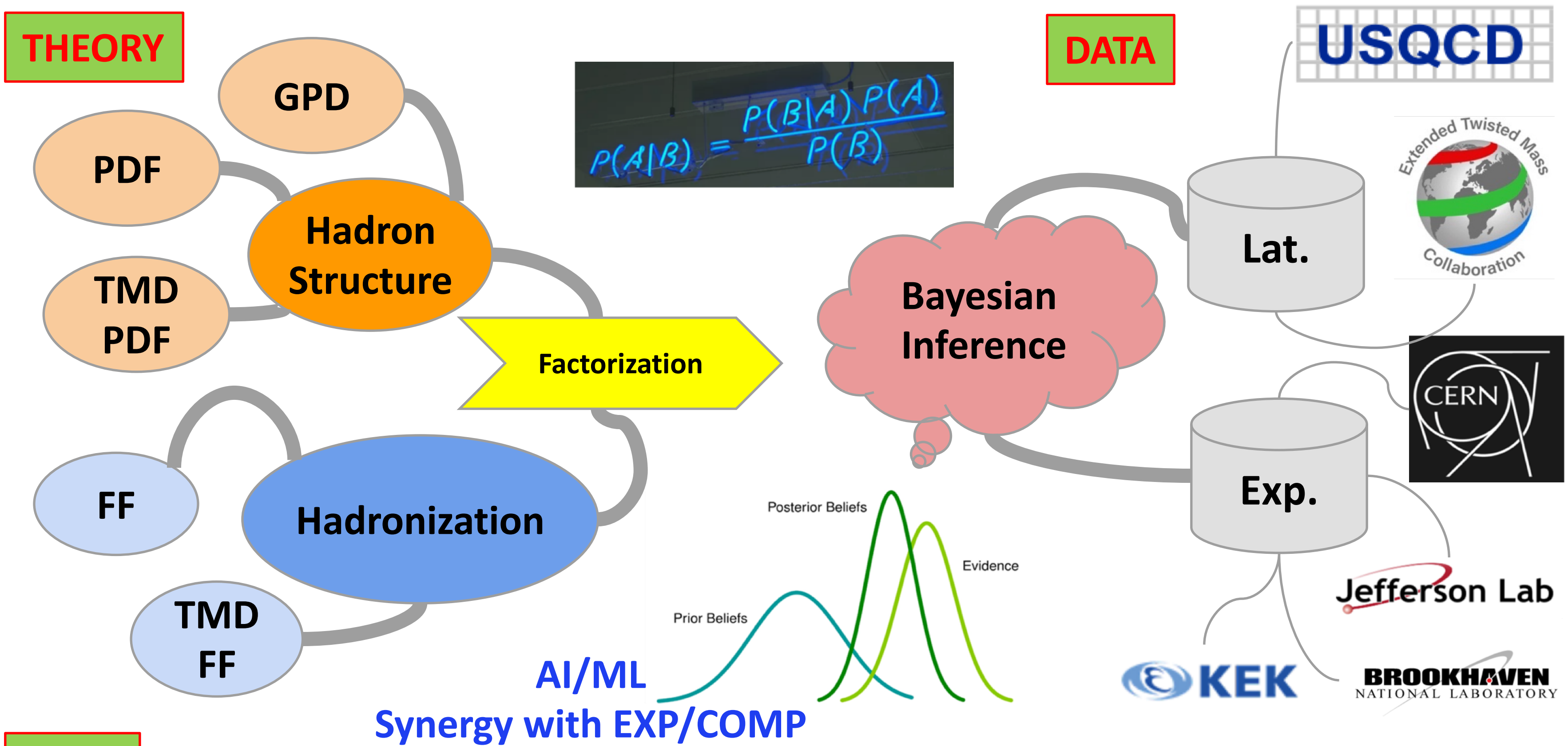
Heavy quarkonium: $Q^2+M^2 \gg |t|$

Imaging the glue only at EIC

Jefferson Lab

Observables with identified hadrons – Phenomenology

❑ **Need QCD global analyses of all data on factorizable cross sections!**



GOAL