The Lattice (G)TMD Project

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What do we compute – and why

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, \Delta_T, S, \ldots) \equiv \frac{1}{2}\langle P + \Delta_T/2, S|\ \bar{q}(-b/2)\ \Gamma\ \mathcal{U}[-b/2, b/2]\ q(b/2)\ |P - \Delta_T/2, S\rangle$$

- Operator separation b Fourier conjugate to quark momentum k.
  - Transverse component $b_T \longleftrightarrow k_T$
  - Longitudinal component $b \cdot P \longleftrightarrow x$ (momentum fraction)
- (Transverse) momentum transfer Δ_T Fourier conjugate to impact parameter $r_T \longrightarrow$ GTMDs
- Dirac structures $\Gamma \longrightarrow$ spin physics
- Now, what about the gauge link $\mathcal{U}$ . . .

What do we compute – and why

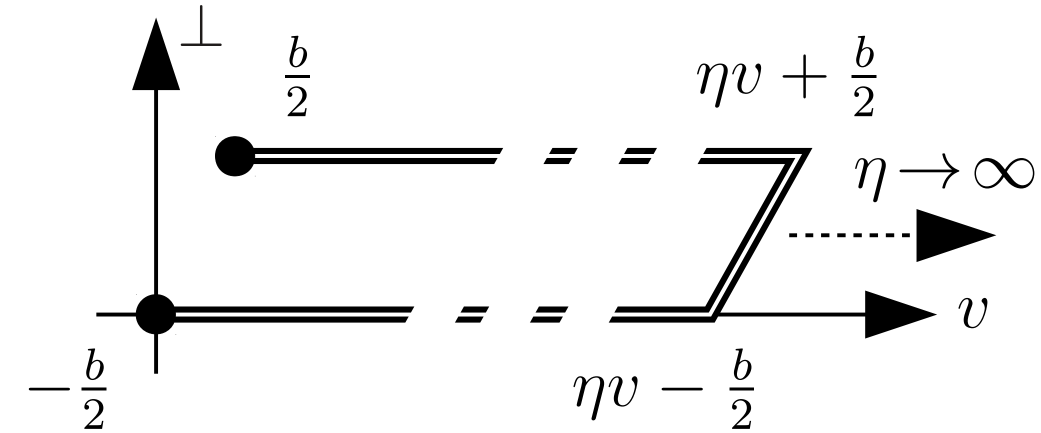
$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, \Delta_T, S, \ldots) \equiv \frac{1}{2}\langle P + \Delta_T/2, S|\ \bar{q}(-b/2)\ \Gamma\ \mathcal{U}[-b/2, b/2]\ q(b/2)\ |P - \Delta_T/2, S\rangle$$

Role of the gauge link $\mathcal{U}$:

Staple-shaped $\mathcal{U}[-b/2, b/2] \longrightarrow$ final state interactions

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \dfrac{P \cdot v}{|P||v|}$



Are interested in $\hat{\zeta} \longrightarrow \infty$

Systematic decomposition of $\widetilde{\Phi}$ into invariant amplitudes (TMDs)

$$\frac{1}{2}\widetilde{\Phi}^{[\gamma^\mu]}_{\text{unsubtr.}} = P^\mu \widetilde{A}_2 - i m_N^2 b^\mu \widetilde{A}_3 - i m_N \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha S_\beta \widetilde{A}_{12} + \frac{m_N^2}{v\cdot P} v^\mu \widetilde{B}_1 + \frac{m_N}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} P_\nu v_\alpha S_\beta \widetilde{B}_7$$
$$-\frac{i m_N^3}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} b_\nu v_\alpha S_\beta \widetilde{B}_8 - \frac{m_N^3}{v\cdot P}(b\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \widetilde{B}_9 - \frac{i m_N^3}{(v\cdot P)^2}(v\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \widetilde{B}_{10}$$

and similarly for other Γ structures (32 amplitudes altogether for TMDs)

- Essential use of Lorentz covariance in this calculational scheme
- Frames in which TMDs are defined phenomenologically and in which lattice calculations are performed differ
- Use of invariant amplitudes permits direct translation of results
- For (G)TMDs, all separations (b, v) are spacelike $\Rightarrow$ can find a frame in which they are purely spatial

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}}{\tilde{f}_1^{[1](0)}} = m_H \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse ("T") direction in an unpolarized ("U") hadron; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Analogously, Sivers shift (in a polarized hadron):

$$\langle k_y \rangle_{TU} = -m_N \frac{\widetilde{A}_{12B}}{\widetilde{A}_{2B}}$$

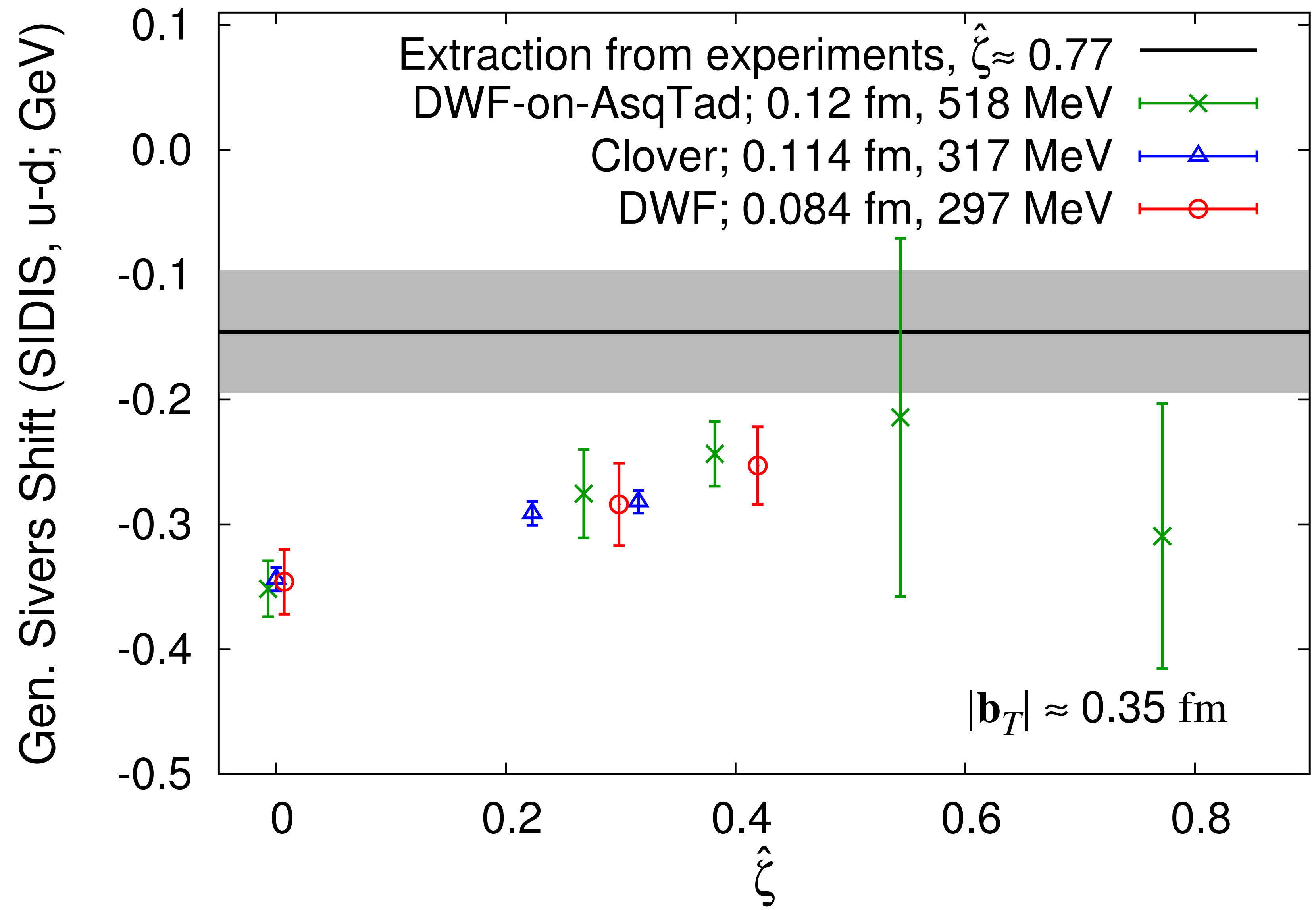
Generalized tensor charge (no k-weighting) :

$$\frac{\tilde{h}_1^{[1](0)}}{\tilde{f}_1^{[1](0)}} = -\frac{\widetilde{A}_{9B} - (m_N^2 b^2/2)\widetilde{A}_{11B}}{\widetilde{A}_{2B}}$$

... and more.

A first comparison with phenomenology

Sivers shift; dependence of SIDIS limit on $\hat{\zeta}$, with $|b_T| = 0.35$ fm

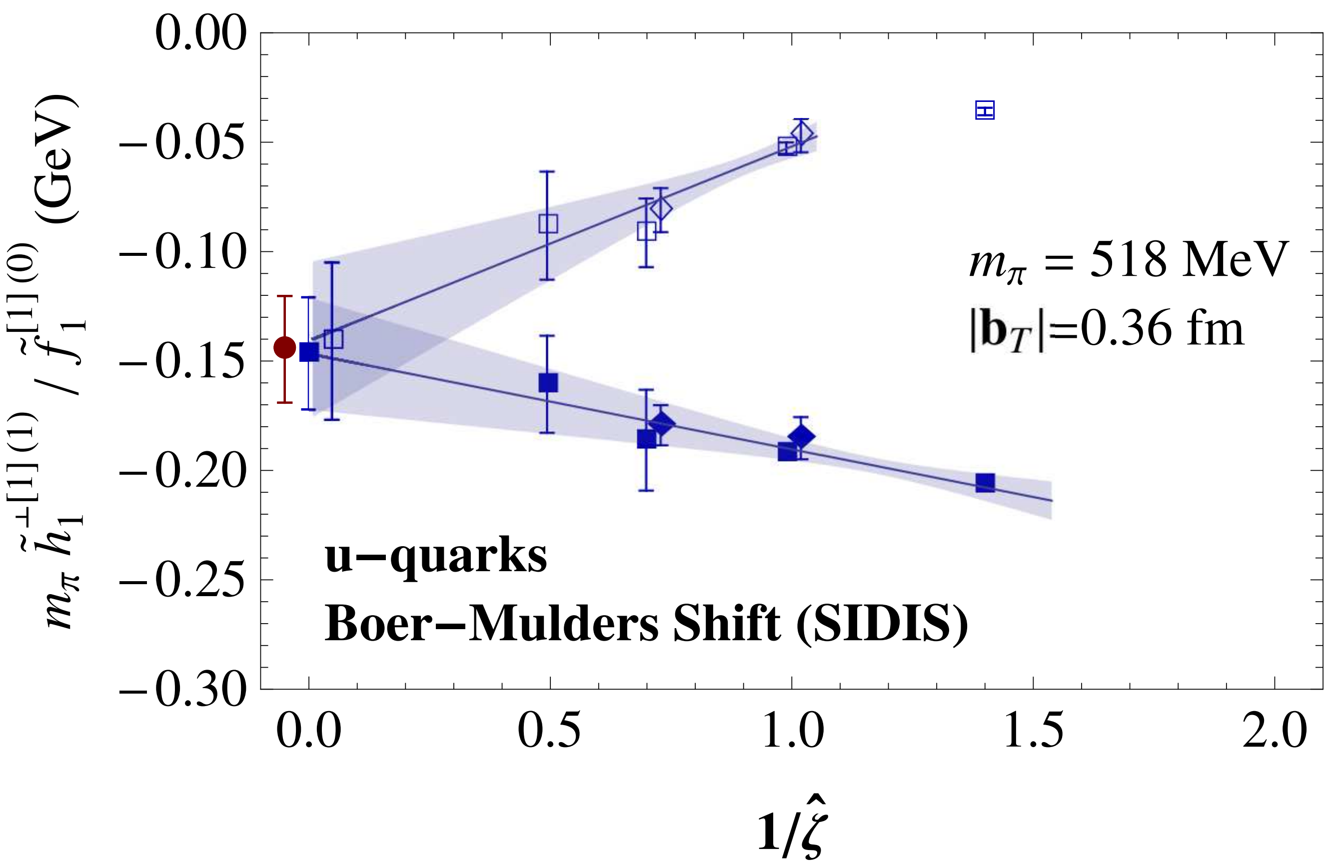


Challenges

- Reaching the large $\hat{\zeta}$ regime
- Reaching the physical pion mass
- Renormalization – operator mixing

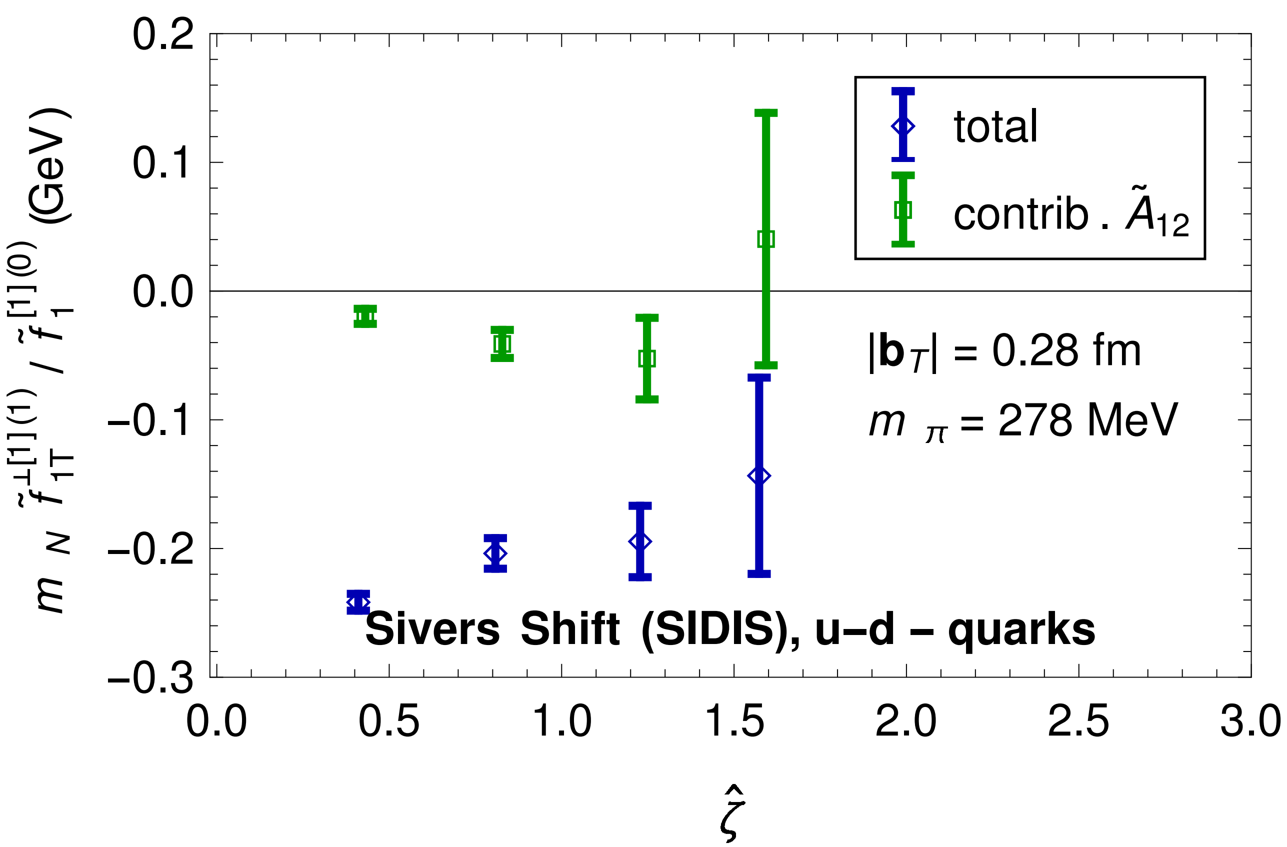
Approaching large $\hat{\zeta}$

Open/green symbols: contribution from $\widetilde{A}$ amplitudes only



Boer-Mulders shift in a pion

Fit function $a + b/\hat{\zeta}$

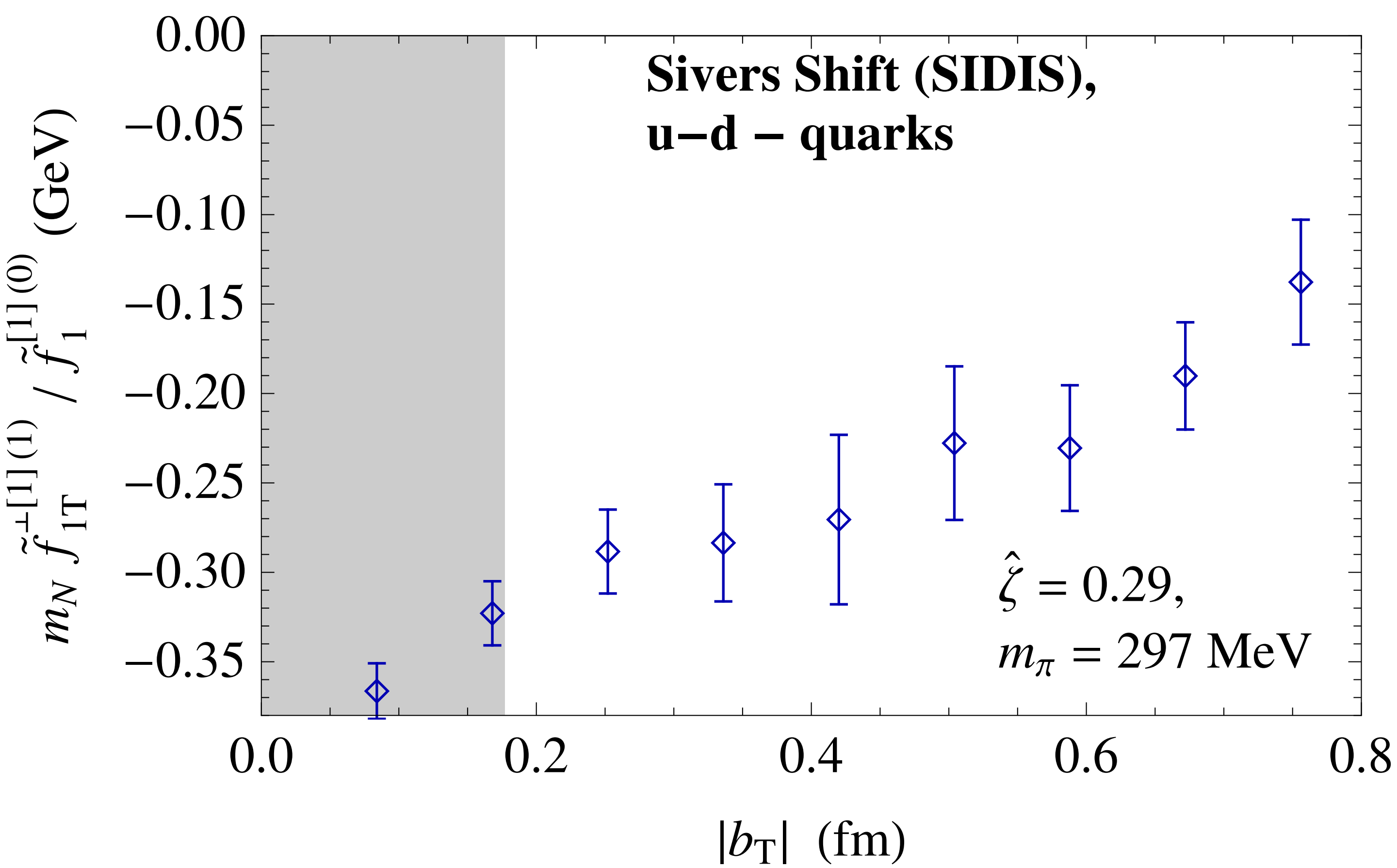


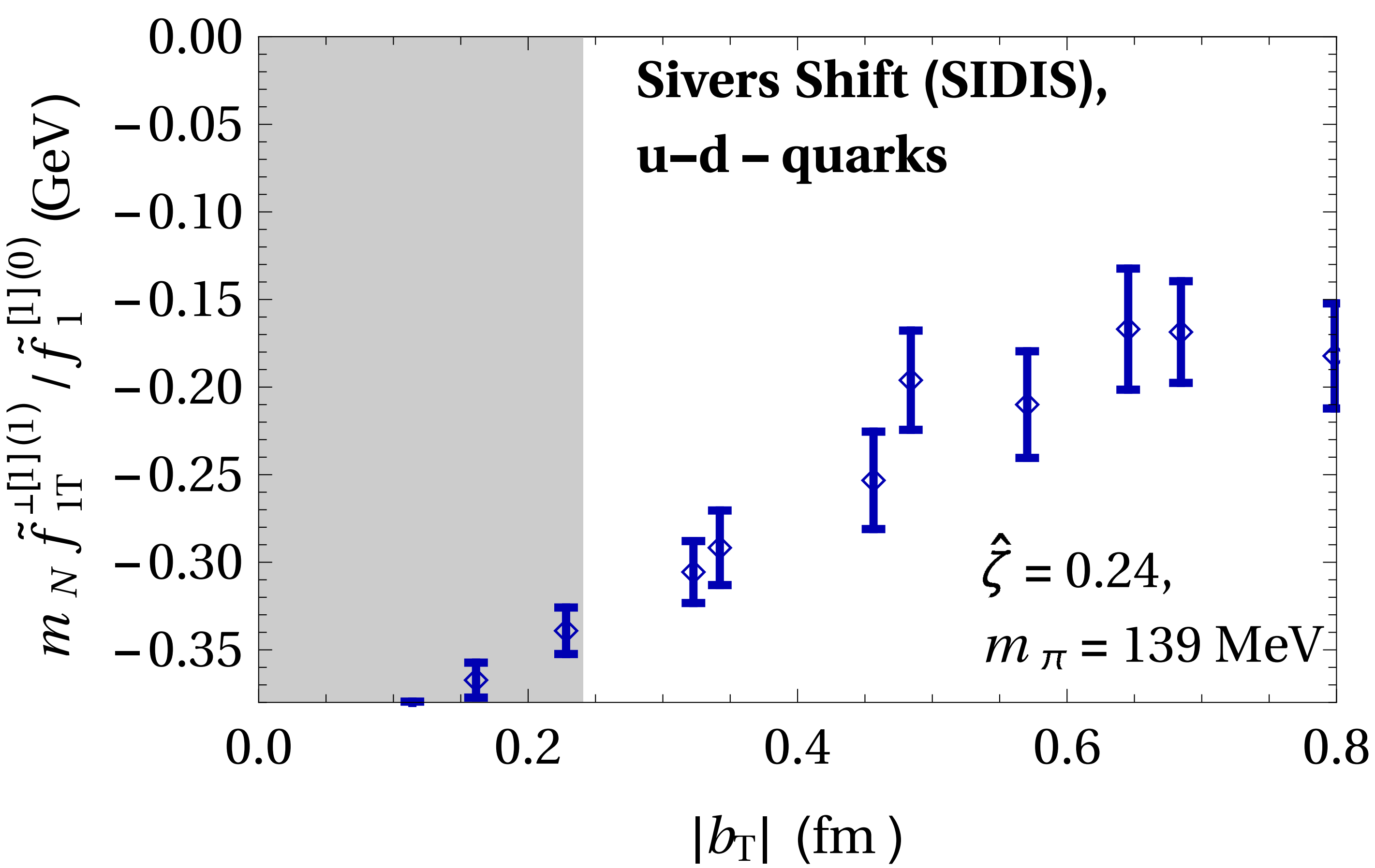
Sivers shift in a nucleon – preliminary

Using momentum smearing

Reaching the physical pion mass

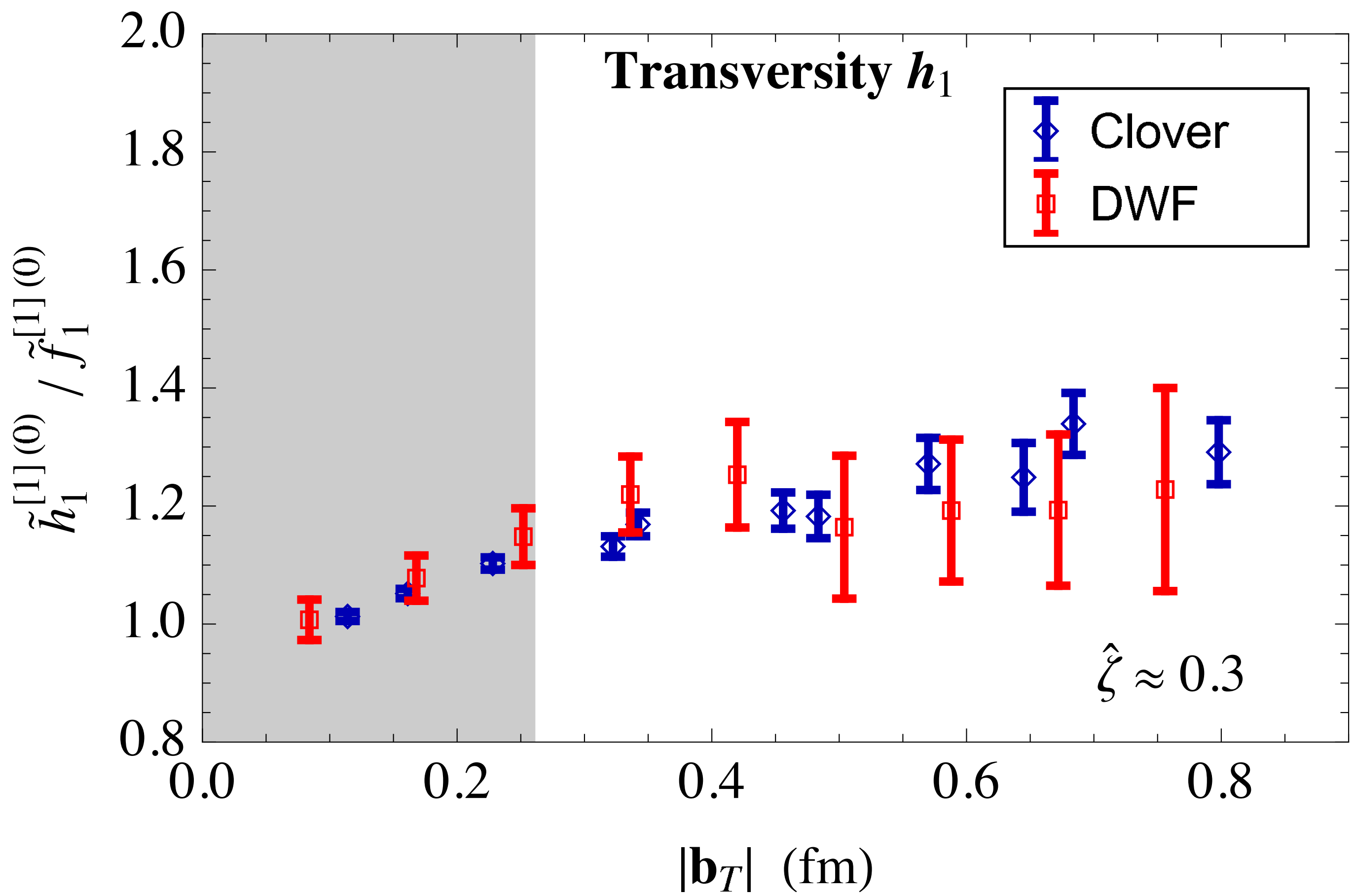
Dependence of SIDIS limit on $|b_T|$

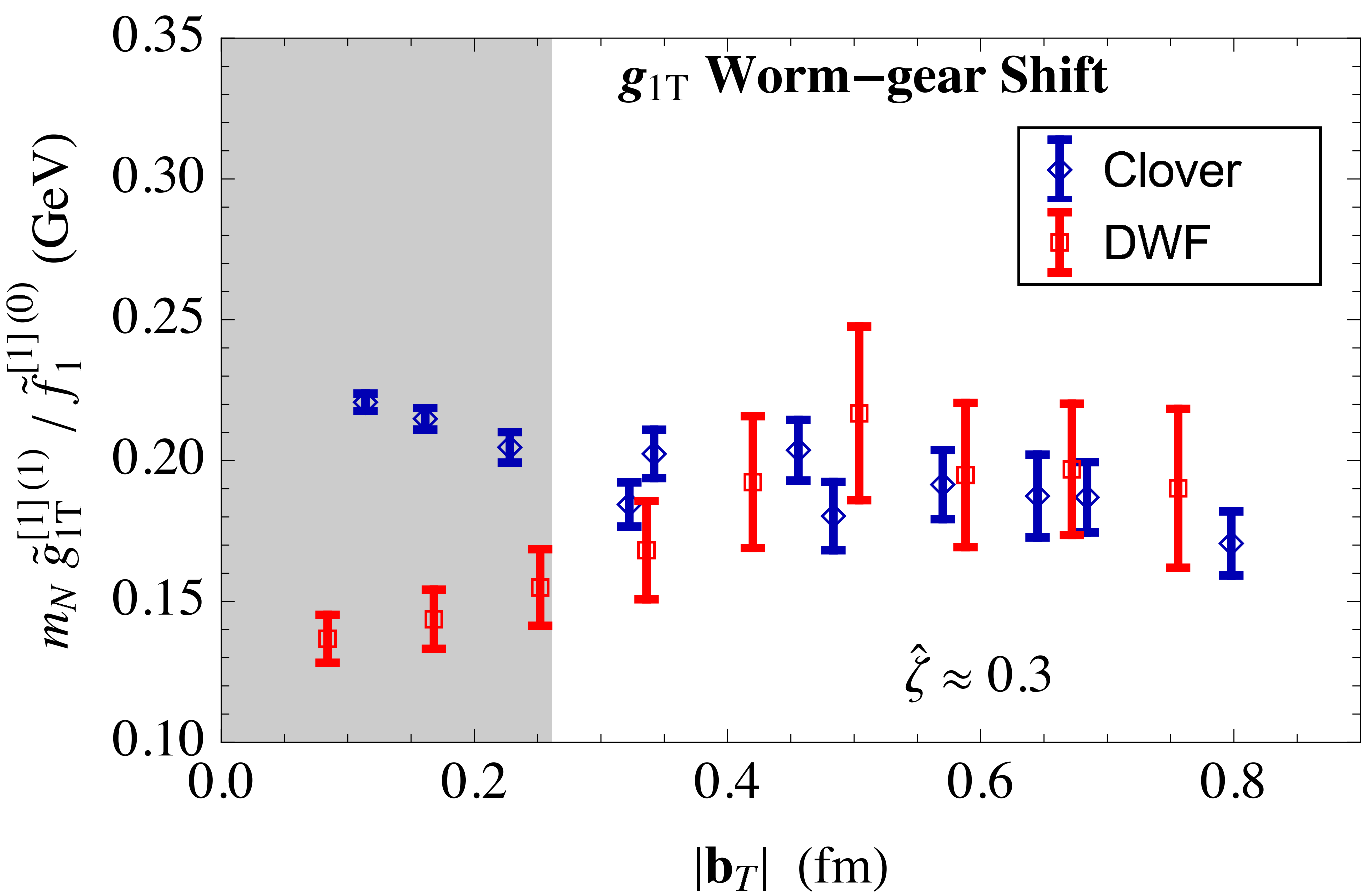




Operator mixing effects revealed using different quark discretizations

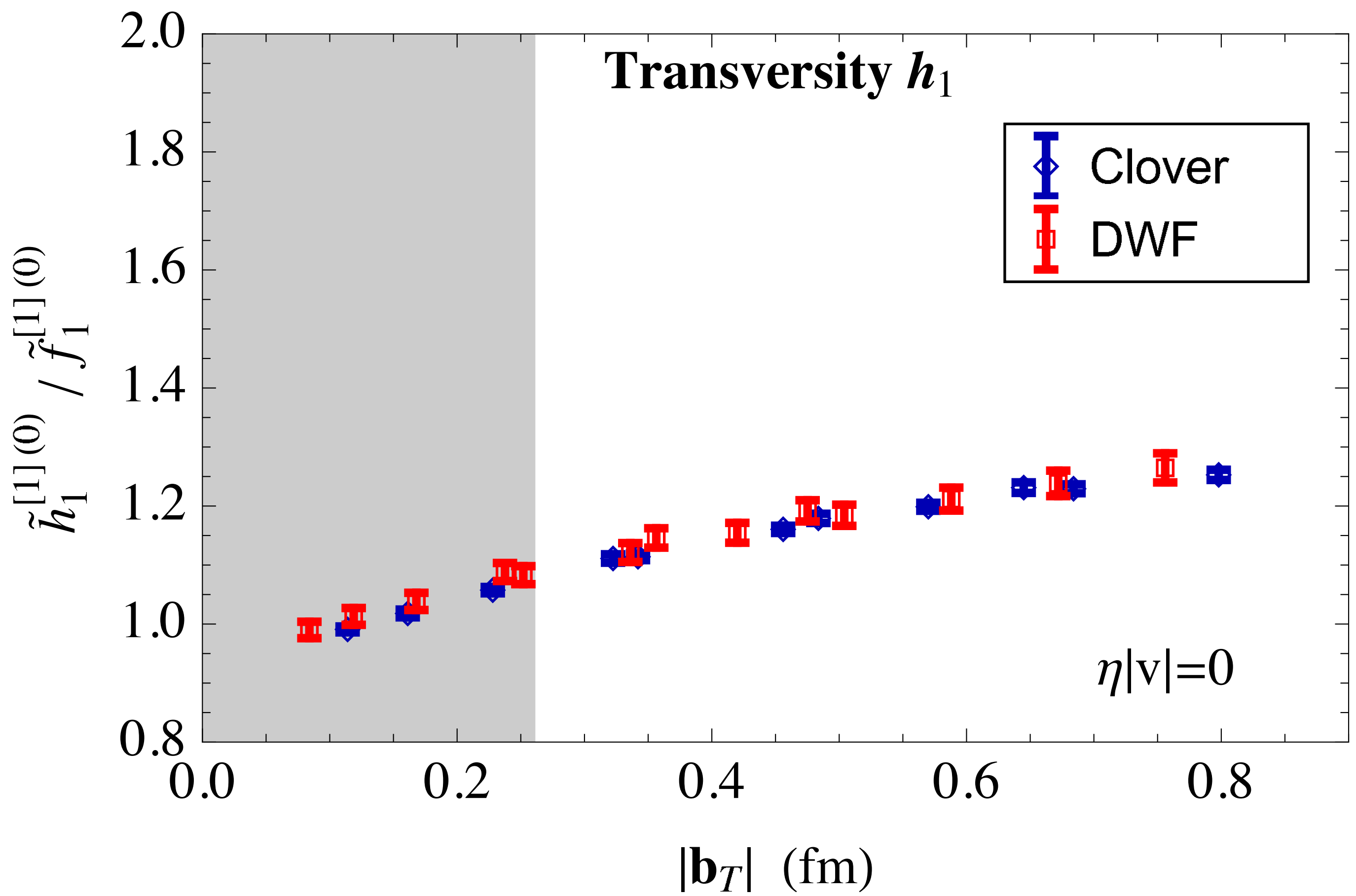
Dependence of SIDIS limit on $|b_T|$, $m_\pi \approx 300\,\mathrm{MeV}$

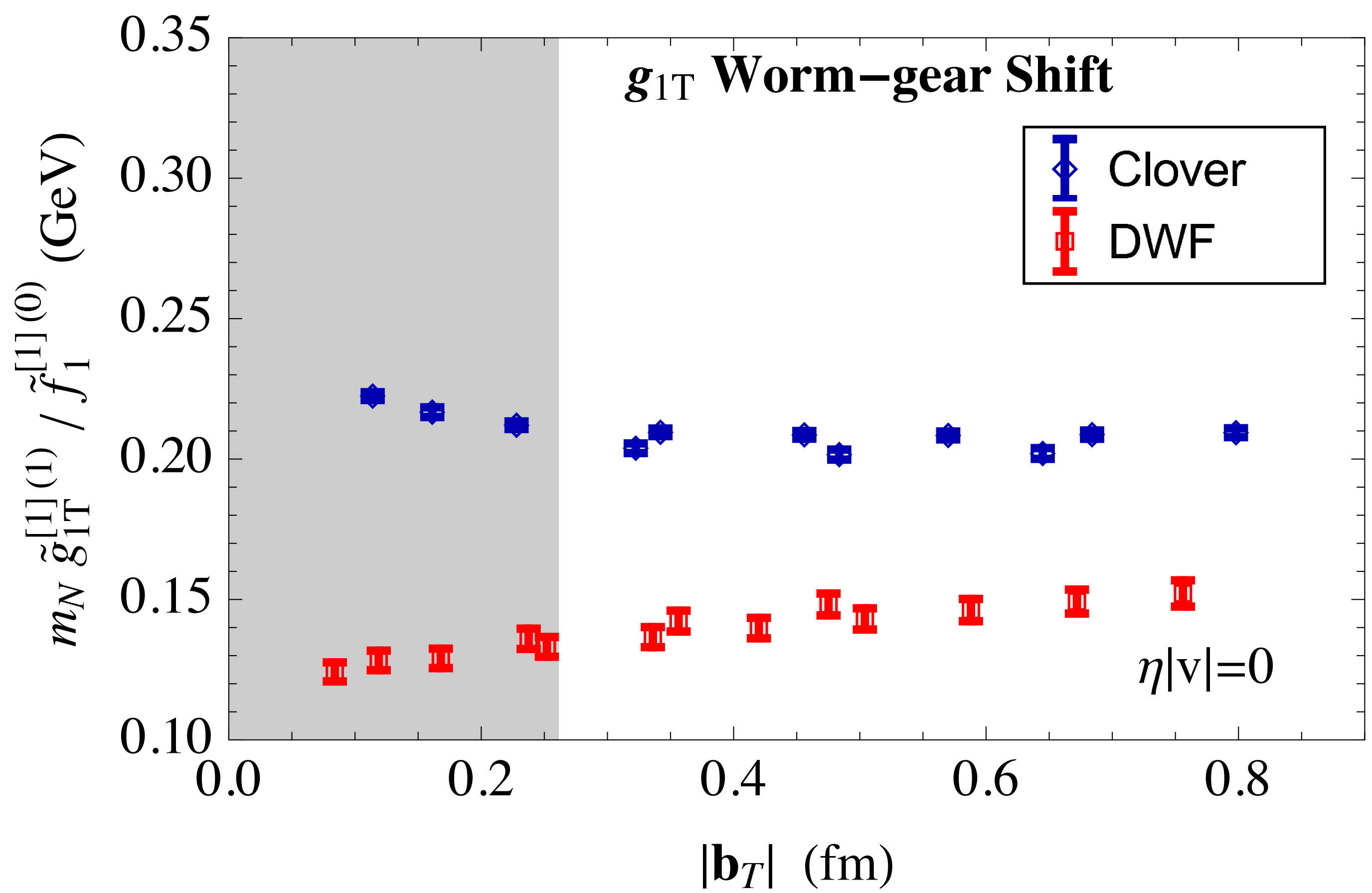




Operator mixing effects revealed using different quark discretizations

Dependence on $|b_T|$, straight link, $m_\pi \approx 300\,\mathrm{MeV}$





$\longrightarrow$ Lattice perturbation theory M. Constantinou et al.

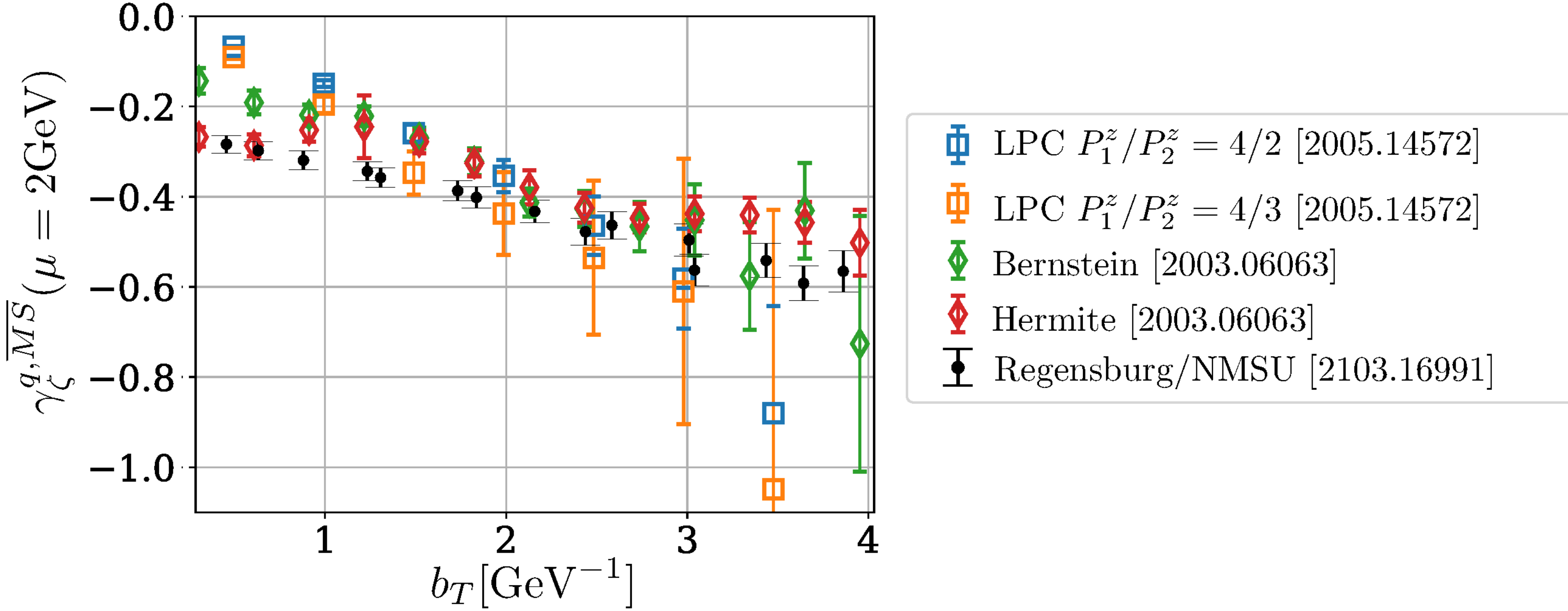
$\longrightarrow$ Nonperturbative mixing patterns P. Shanahan et al.

Further information to be gained from varying lattice spacing

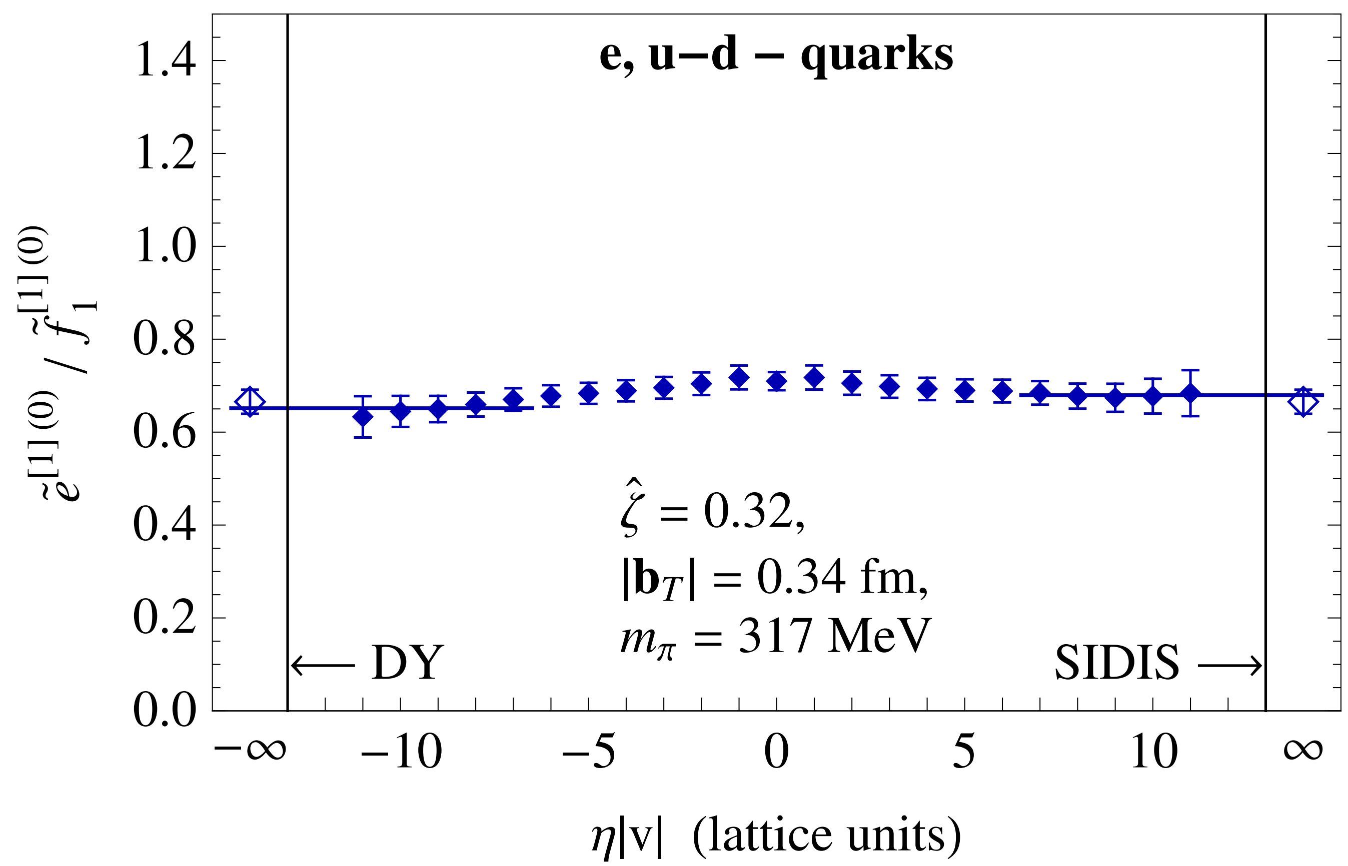
A wealth of extended topics ...

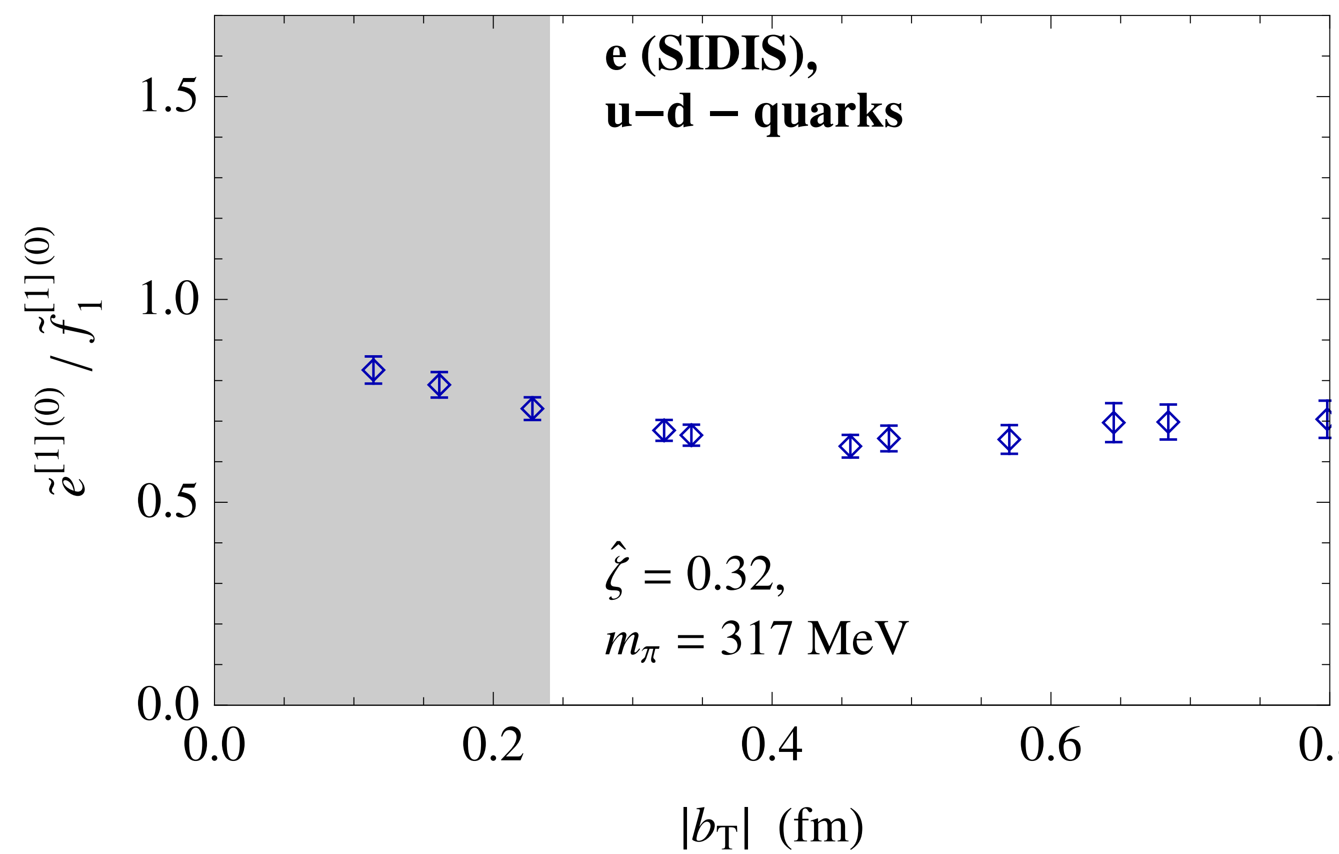
- Collins-Soper kernel
- Twist 3 TMD observables
- TMD flavor ratios
- x-dependence of TMD observables
- Quark spin-orbit coupling

Collins-Soper kernel

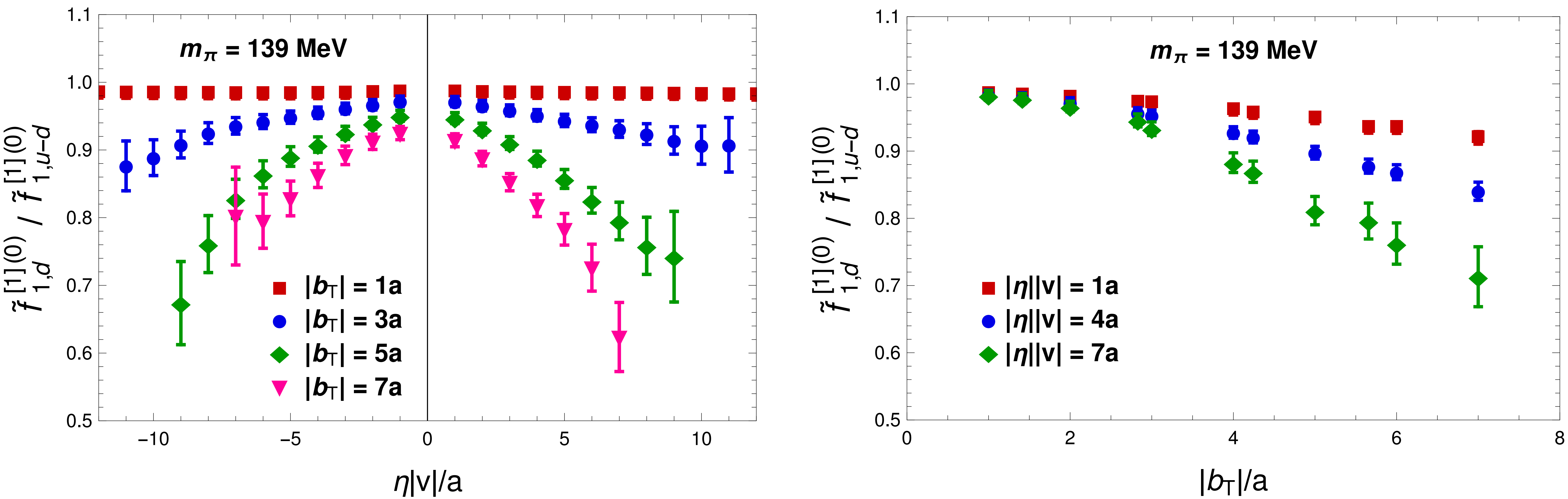


Twist 3 TMD observables – generalized scalar charge

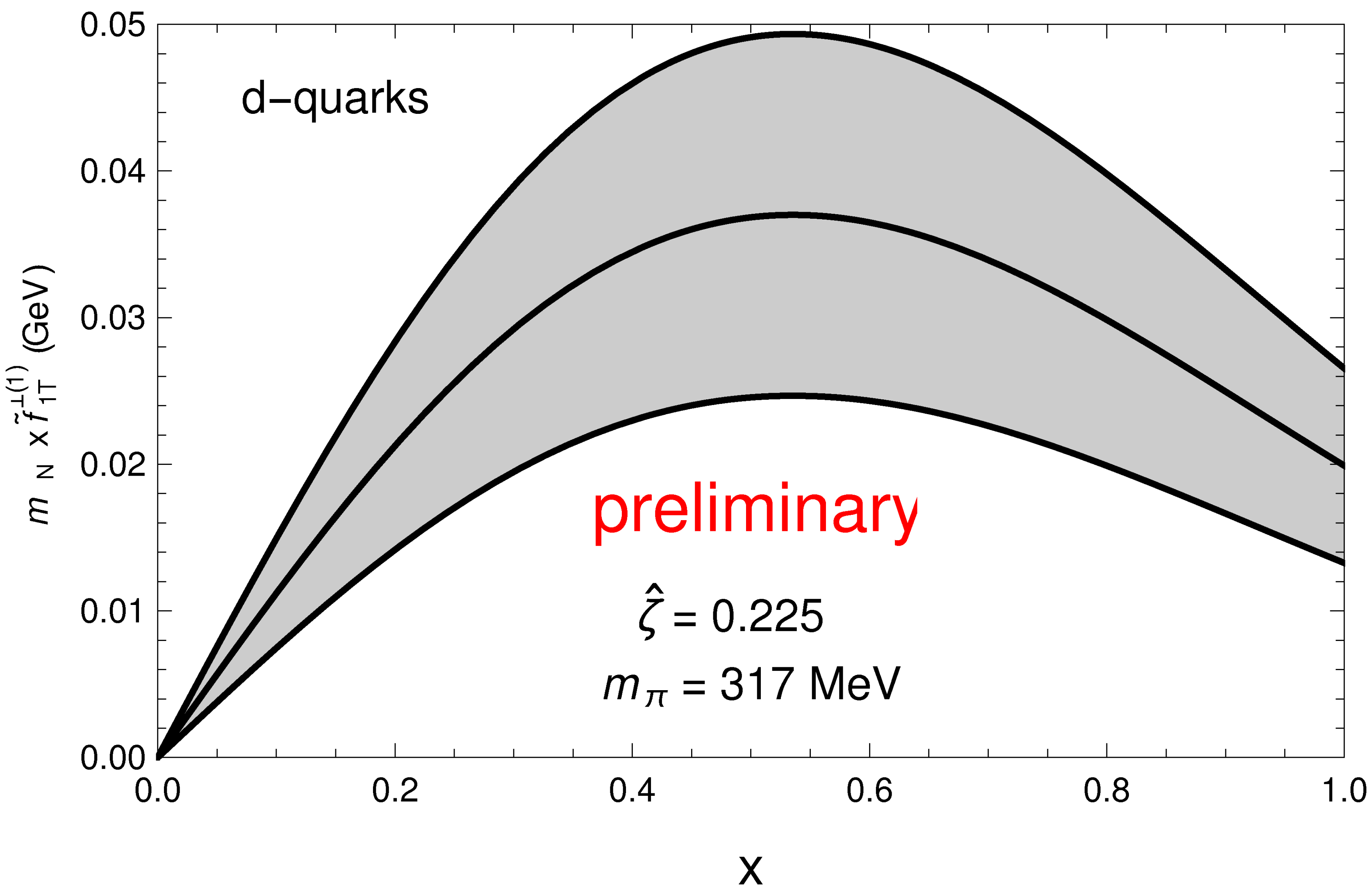




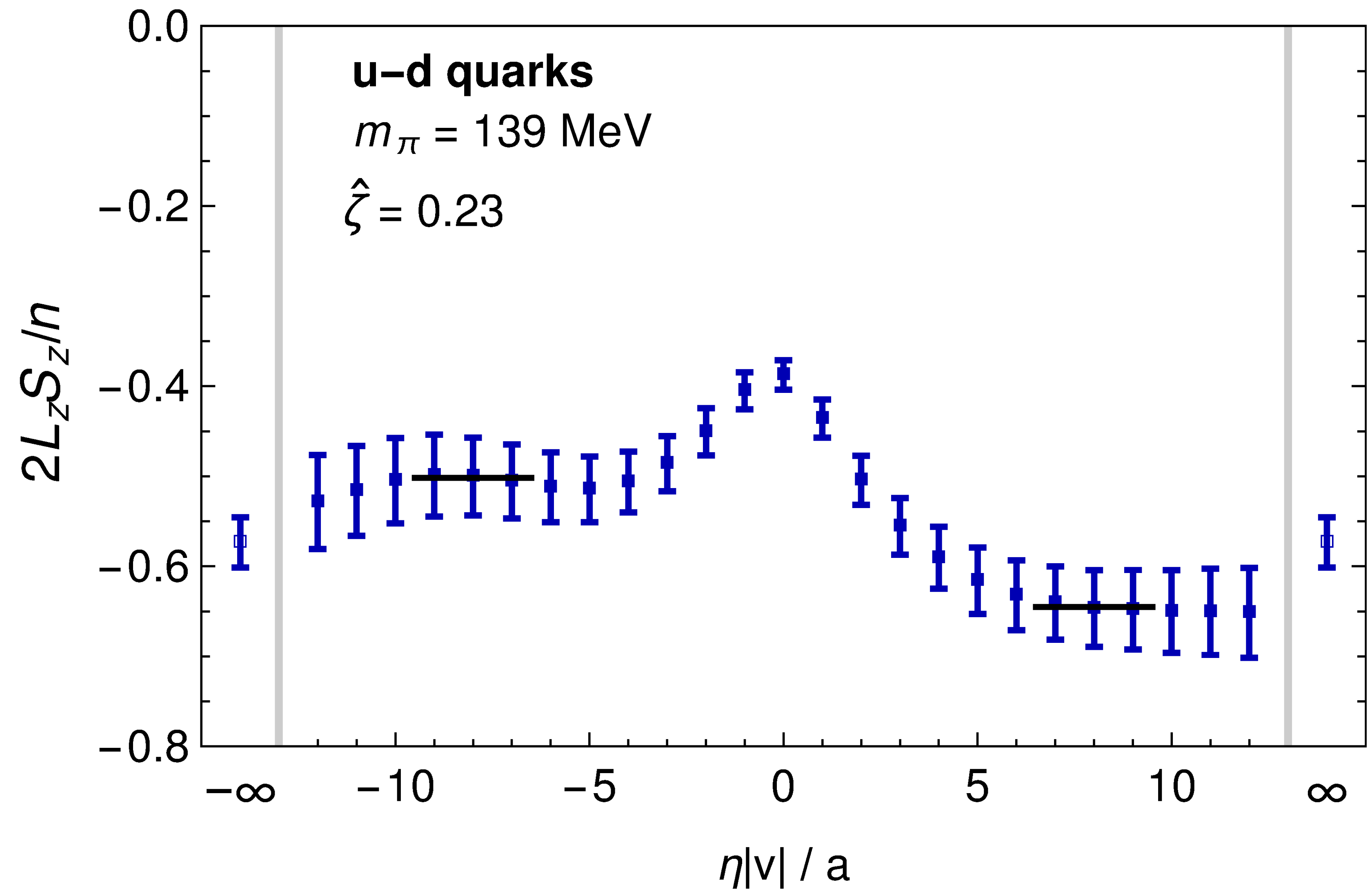
TMD flavor ratios



Dependence of Sivers shift on momentum fraction x



Quark spin-orbit coupling



For comparison, in a polarized proton (from C. Alexandrou et al., PRD 101 (2020) 094513; 2003.08486):

$\langle L^u \rangle = -0.22(3)$, $\langle 2S^u \rangle = 0.86(2) \Rightarrow \langle L^u \rangle \langle 2S^u \rangle = -0.2$

$\langle L^d \rangle = 0.26(2)$, $\langle 2S^d \rangle = -0.42(2) \Rightarrow \langle L^d \rangle \langle 2S^d \rangle = -0.1$

$\Rightarrow \langle L^u \rangle \langle 2S^u \rangle - \langle L^d \rangle \langle 2S^d \rangle = -0.1$

Comments and questions?