MC simulations with dedicated events

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TMD Studies: from JLab to EIC
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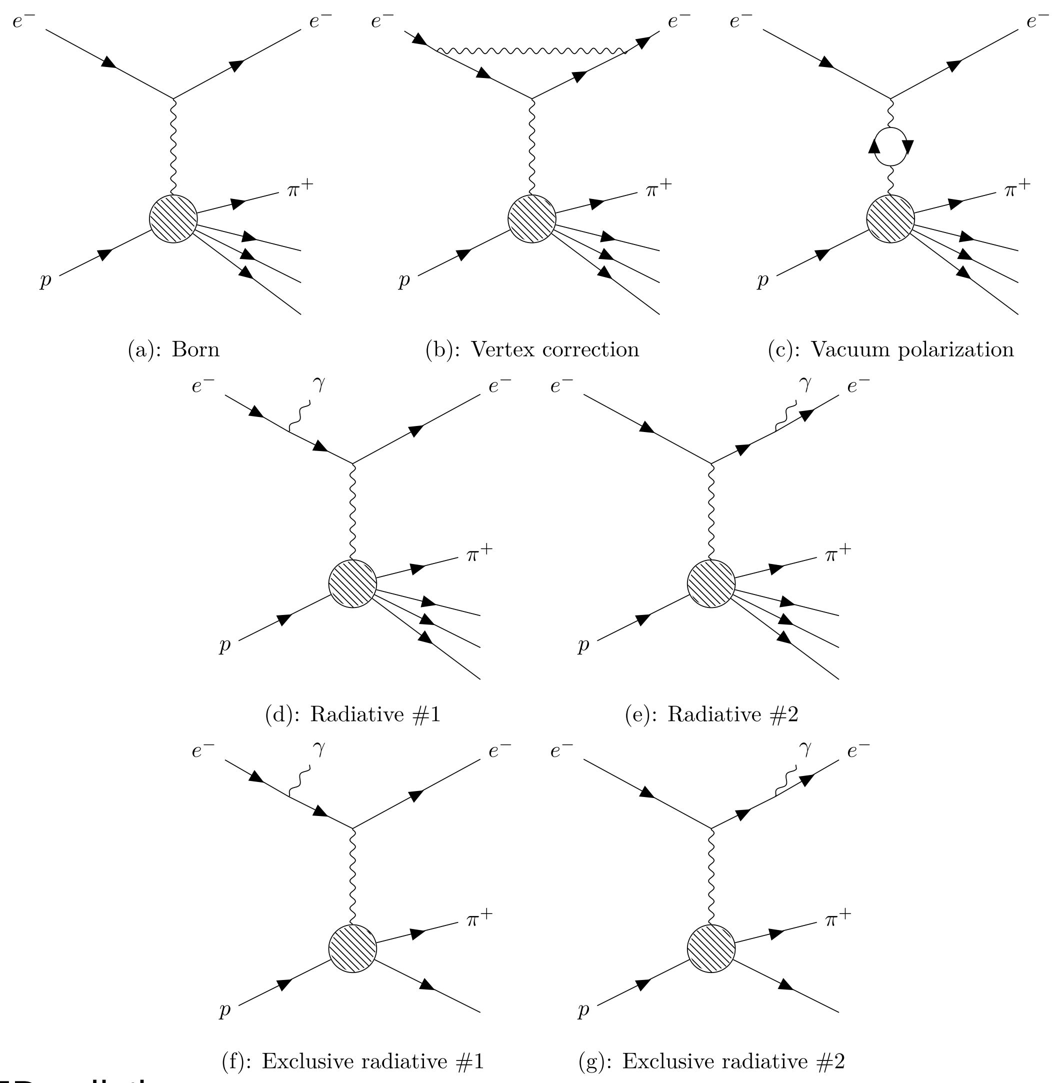
QED radiative effects in SIDIS

- Leading order QED radiative corrections to SIDIS were calculated in ***PRD 100, 033005 (2019)***, including
  - Subleading twist structure functions
  - Non-zero lepton mass
  - Exclusive tail
  - Ultra-relativistic approximation
- Bardin-Shumeiko method used to cancel infrared divergence
- Using these results, we made a Monte-Carlo event generator



Akushevich, Igor, and Ilyichev, Alexander. "Lowest order QED radiative effects in polarized SIDIS." *Physical Review D* 100.3 (2019): 033005.

QED radiative effects in SIDIS

- Total SIDIS cross-section with RC is

$$\sigma = \sigma^B + \frac{\alpha}{\pi}(\delta_{\text{vert}} + \delta_{\text{vac}})\sigma^B + \sigma^{AMM} + \int \sigma^R \, d^3\boldsymbol{k}$$

- Radiative part σ^R is divided into soft and hard parts by cutoff in photon energy $\bar{k}_0$.

$$\sigma = \underbrace{\sigma^B + \frac{\alpha}{\pi}(\delta_{\text{vert}} + \delta_{\text{vac}})\sigma^B + \sigma^{AMM} + \overbrace{\int_0^{\bar{k}_0} \sigma^R \, d^3\boldsymbol{k}}^{\text{difficult}}}_{\text{soft part}} + \underbrace{\int_{\bar{k}_0}^{\infty} \sigma^R \, d^3\boldsymbol{k}}_{\text{hard part}}$$

- Events are randomly chosen to be soft or hard, based on total soft/hard cross-sections.
- Integral over soft part of σ^R is computationally expensive.

QED radiative effects in SIDIS

- To handle soft part of σ^R, split into components σ^R_{IR} (σ^R_F), with (without) infrared divergence, in such a way that

$$\sigma^R = \sigma^R_{IR} + \sigma^R_F, \quad \int_0^{\bar{k}_0} \sigma^R_{IR}\, d^3\boldsymbol{k} = \frac{\alpha}{\pi}\delta_S \sigma^B$$

- Difficult term becomes

$$\int_0^{\bar{k}_0} \sigma^R\, d^3\boldsymbol{k} = \frac{\alpha}{\pi}\delta_S \sigma^B + \int_0^{\bar{k}_0} \sigma^R_F\, d^3\boldsymbol{k} \quad (\text{second term} \approx 0)$$

- Infrared divergence in δ_S adds with δ_{vert} to form δ_{VS} (without infrared divergence).

$$\sigma \approx \underbrace{\sigma^B + \frac{\alpha}{\pi}(\delta_{VS} + \delta_{\text{vac}})\sigma^B + \sigma^{AMM}}_{\text{soft part}} + \underbrace{\int_{\bar{k}_0}^{\infty} \sigma^R\, d^3\boldsymbol{k}}_{\text{hard part}}, \quad \delta_{VS} = \delta_S + \delta_{\text{vert}}$$

Event generator technical details

- Generator hosted at https://github.com/duanebyer/sidis
- Divided into two components
  - C++ library for calculating SIDIS cross-sections
  - Monte-Carlo generator for producing events
- Use FOAM library from ROOT for event generation
  - Spatial indexing tree ("foam") is initialized before events can be generated
  - The foam allows events to be generated with a weight close to 1 using Markov chain Monte-Carlo method
- Features
  - Radiative corrections (of course), except for exclusive tail
  - Leading and subleading structure functions
  - Efficient kinematic cuts (without event rejection)
  - Custom structure functions can be provided, either as ROOT-compiled shared libraries, or on grid

Event generator technical details

- Parameter file is written by user
- Produce the foam

```
sidisgen --initialize <params>
```

- Generate events

```
sidisgen --generate <params>
```

- Produces a ROOT file containing the events
- Check what parameters a previous set of events were generated with

```
sidisgen --inspect <ROOT file>
```

```
num_events    100000
seed          1
# Initial conditions.
beam_energy   11.0
beam          muon
target        proton
hadron        pi+
# Use Prokudin structure functions.
sf_set        prokudin
# RC methods.
rc_method     approx
soft_threshold 0.01
# Cuts.
x_cut         0.01    0.1
Q_sq_cut      1.0     2.0
theta_h_cut   5       18
```

Structure functions

- By default, use structure functions from ***JHEP 06 (2019) 007***
- Mathematica implementation can be found at https://github.com/prokudin/WW-SIDIS
- Combines Wandzura-Wilczek-type approximation with Gaussian approximation for TMDs and FFs.
  - Gaussian approximation: Simplifies $k_\perp$ and $p_\perp$ dependence of TMDs and FFs, allowing for analytic evaluation of convolution integrals
  - Wandzura-Wilczek-type approximation: use $\left|\frac{\langle \bar{q} g q \rangle}{\langle \bar{q} q \rangle}\right| \ll 1$ to express some TMDs and FFs in terms of others. Reduce down to eight basis functions
- WW-type and Gaussian approximations are also available for user-defined TMDs and FFs

Bastami, Saman, et al. "Semi-inclusive deep-inelastic scattering in Wandzura-Wilczek-type approximation." *Journal of High Energy Physics* 2019.6 (2019): 1-73.

Some results on single spin asymmetries

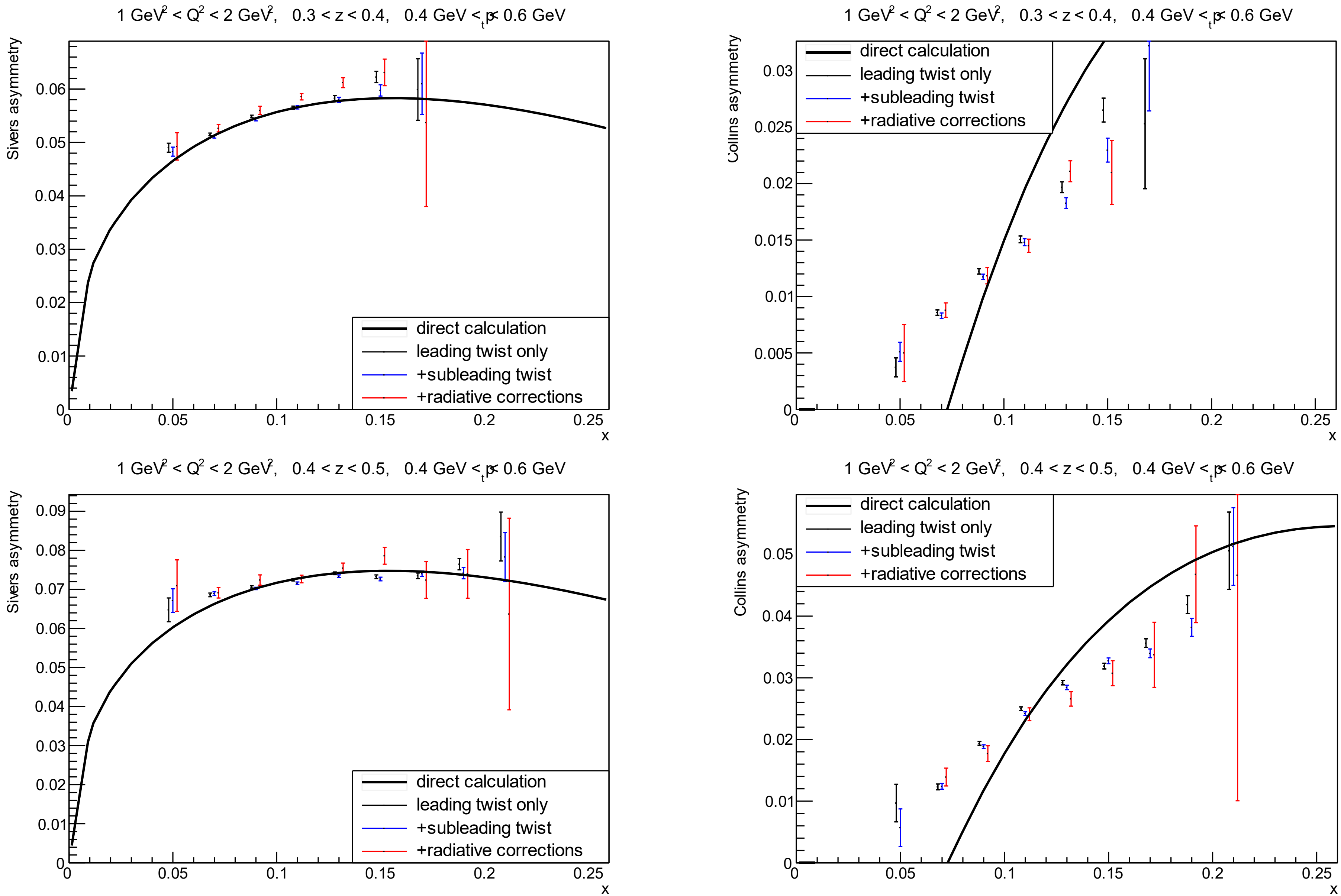
- Simulated 60 days of events
- Pol. lumi.: $1 \times 10^{35}\ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1}$
- # of events: 8×10^{8}
- Electron beam at $11\ \mathrm{GeV}$, unpolarized
- Proton target, transversely polarized
- Leading hadron π^+ (π^- in the future).
- Three cases
  - Leading twist SF only, no RC
  - Leading + subleading twist, no RC
  - Leading + subleading twist, with RC

- $1\ \mathrm{GeV} < |\boldsymbol{p}_e| < 7\ \mathrm{GeV}$
- $8° < \theta_e < 24°$
- $2.5\ \mathrm{GeV} < |\boldsymbol{p}_h| < 7.5\ \mathrm{GeV}$
- $8° < \theta_h < 24°$
- $W > 2.3\ \mathrm{GeV}$
- $W' > 1.6\ \mathrm{GeV}$
- $0.3 < z < 0.7$

Case	Cross-section (GeV^{-2})	Relative change
leading twist	3.9740×10^{-6}	0%
+subleading	3.9728×10^{-6}	-0.03%
+RC	4.025×10^{-6}	$+1.3\%$

Some results on single spin asymmetries

Sivers and Collins asymmetries as a function of x



Summary and next steps

- Event generator is already useful.
- Future improvements to be made:
  - Exclusive tail contribution
  - Collinear factorization support
  - Versatility, especially with user-provided structure functions
- Some studies we plan:
  - Effect of higher twist and radiative corrections on the extraction of leading twist structure functions
  - Compare against previous radiative corrections estimations on 6 GeV data
  - Investigate the intermediate region between TMD and collinear factorization
  - …

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