Spin-orbit correlations in Monte Carlo simulations

Focus on recent work on the polarized quark fragmentation process

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in collaboration with X. Artru and A. Martin
The collinear nucleon structure at leading twist is described by unpolarised $f_1$, helicity $g_1$, transversity $h_1$.

<table>
<thead>
<tr>
<th>quark</th>
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<th>L</th>
<th>T</th>
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Anselmino et al., PRD 92, 114023 (2015)

Transversity is the less known.
nucleon structure at leading twist

The collinear nucleon structure at leading twist is described by unpolarised $f_1$, helicity $g_1$, transversity $h_1$

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<td>T</td>
<td>$h_1^T(x, k_T^2)$</td>
<td>$h_{1L}^T(x, k_T^2)$</td>
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<td>(pretzelosity)</td>
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+ 5 other TMD PDFs when the intrinsic $k_T$ is taken into account → correlations between the nucleon spin and the spin and $k_T$ of partons

~ basically unknown, but Sivers

In SIDIS some TMDs are coupled to the unpolarized FF $D_1$

- $A_{UU}^{\cos \phi_h} \sim f_1 \otimes D_1$
- $A_{UT}^{\sin (\phi_h - \phi_S)} \sim f_{1T} \otimes D_1$
- $A_{LT}^{\cos (\phi_h - \phi_S)} \sim g_{1T} \otimes D_1$

~ Cahn effect

Sivers asymmetry

Kotzinian-Mulders

what about MC simulations of these effects?

- $A_{UT}^{\sin (\phi_h + \phi_S - \pi)} \sim h_1 \otimes H_1^\perp$
- $A_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$
- $A_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$
- $A_{UT}^{\sin (3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$

Collins asymmetry

~ Boer-Mulders

Kotzinian-Mulders

pretzelosity
MC implementation of spin-orbit effects

Monte Carlo event generators (MCEGs) are important tools
event generation in the full phase space, access to correlations,
multi-dimensional studies, phenomenology

A systematic implementation of all leading order effects related to the intrinsic transverse momentum
of quarks and to their spin degree of freedom in complete MCEGs is still missing

Different effects considered separately
- TMD PDFs
  $k_T$ dependence in $f_1 \rightarrow «$primordial kT» in PYTHIA? (but PDFs in PYTHIA do not depend on $k_T$)
  Cahn and Sivers effects implemented in LEPTO Kotzinian ’05

- TMD FFs
  unpolarized quarks, $D_1 \rightarrow$ Lund String Model, Cluster Model .. (is relation with theory OK?)
  polarized quarks $\rightarrow$ (T) Collins effect $H_1^T$, (L) jet handedness
  **recursive string+3P0 model $\rightarrow$ this talk**
  stand alone MC AK et al., PRD 100 (2019) 1, 014003, PRD 97 (2018) 7, 074010
  inclusion in PYTHIA 8.2 AK and L. Lönnblad, PoS DIS2019 (2019) 179

  extended NJL-jet model
  stand alone MC Matevosyan et al., PRD 95 (2017) 1, 014021
The string+3P0 model with pseudoscalar (PS) meson production


The string+3P0 model with pseudoscalar (PS) meson production

Assuming e.g. \( q_A = u \)

\( \pi^+ \) and \( \pi^- \) are emitted on opposite sides
- qualitative agreement with data
- predicts also a di-hadron asymmetry

A quantum mechanical formulation of the string+\( ^3P_0 \) model is used for simulations
elementary splitting: emission of a PS

string decay = recursive repetition of the elementary splitting \( q \rightarrow h + q' \)

\[
\text{hadron} = \{ h, Z = p^+/k^+, \quad p_T = k_T - k'_T \}
\]

\[
\text{quark'} = \{ q', k', \rho(S_{q'}) \}
\]

\[
\text{quark} = \{ q, k, \rho(S_q) \}
\]


Splitting in flavour \( \otimes \) momentum \( \otimes \) spin space
\rightarrow Transition Amplitude
\[
T_{q',h,q}(Z,p_T|k_T)
\]

Splitting Probability
\rightarrow Splitting Function
\[
F_{q',h,q}(Z,p_T|k_T,S_q) = tr T_{q',h,q} \rho(S_q) T_{q',h,q}^\dagger
\]

Spin transfer to \( q' \)
\rightarrow spin density matrix of \( q' \)
\[
\rho(S_{q'}) \propto T_{q',h,q} \rho(S_q) T_{q',h,q}^\dagger
\]

the very basic ingredients for the MC simulations
**elementary splitting: emission of a PS**

String decay = recursive repetition of the elementary splitting \( q \rightarrow h + q' \)

\[
\text{hadron} = \{h, \quad Z = p^+/k^+, \quad \rho(S_{q'})\}
\]

hadron type with transverse mass \( \varepsilon_h = (m_h^2 + p_T^2)^{1/2} \)

\[
\text{longitudinal momentum fraction} \quad \rho(S)
\]

\[
\text{transverse momentum w.r.t string axis}
\]

\[
\text{quark}' = \{q', k', \rho(S_{q'})\}
\]

\[
\text{quark} = \{q, k, \text{flavour}\}
\]

\[
\text{spin density matrix} (2x2, Pauli spinors)
\]

**X. Artru, Z. Belghobsi DSPIN-2011, 2013**

- **Expression for the Transition Amplitude**

\[
T_{q',h,q} = C_{q',h,q} \times \left(1 - \frac{Z}{\varepsilon_h^2}\right)^{a/2} \times e^{-\frac{b_L}{2} \varepsilon_h^2} \times e^{-\frac{b_T}{2} k_T^2} \times \tilde{g} \left(\varepsilon_h^2\right) \times [\mu + \sigma_z \sigma \cdot k'_T] \times \sigma_z \times \hat{\mu}^{-1/2} (k_T)
\]

\[
T_{q',h,q} = \text{Lund String Fragmentation Model} \times \text{3P}_0\text{ operator} \times \text{PS coupling} \times ..
\]

- **Few free parameters**

  \(a, b_L, b_T\) \rightarrow string fragmentation dynamics (Lund Model, e.g. PYTHIA, LEPTO)

  \(\mu\) **complex mass** from \(3\text{P}_0\) mechanism \rightarrow responsible for spin effects (\(\text{Im}(\mu)\) \rightarrow transverse)

- **Input function** \(\tilde{g}\) \rightarrow governs spin-independent \(k_T - k'_T\) correlations

  \[
  \text{correlations} \rightarrow \text{Model M18} \quad \text{PRD 97 (2018) 7, 074010}
  \]

  \[
  \text{NO correlations} \rightarrow \text{Model M19 (much simpler)} \quad \text{PRD100 (2019) no.1, 014003}
  \]
M18 and M19 have been implemented in stand alone MC programs
give very similar results, in spite for M19 being much simpler

Next slide, simulated Collins analysing power $a_{u\uparrow\rightarrow h+X}$
with initial conditions

$u$ quarks fully transversely polarized along $\hat{y}$
Energy calculated from a $\{x_B, Q^2\}$ sample of SIDIS events
no primordial KT

Values of the free parameters

$a = 0.9$
$b_L = 0.5 \text{ (GeV/c}^2\text{)}^{-2}$
$b_T = 5.17 \text{ (GeV/c)}^{-2}$
$\mu = (0.42 + i0.76) \text{ GeV/c}^2$

Azimuthal spectrum of hadrons

$\frac{dN_h}{d\phi} \propto 1 + a_{u\uparrow\rightarrow h+X} \sin(\phi_h - \phi_{s_u})$

Collins analysing power

$a_{u\uparrow\rightarrow h+X} = 2 \langle \sin(\phi_h - \phi_{s_u}) \rangle$

see AK, X. Artru, Z. Belghobsi, F. Bradamante, A. Martin PRD 100 (2019) 1, 014003
Stand alone simulations: comparison with SIDIS data (M19)

Collins asymmetry on protons $A_{Coll}^p \simeq \frac{h_1^u}{f_1^u} a_{u^{\uparrow \rightarrow \pi + X}}$

MC $\rightarrow$ Collins analysing power $a_{u^{\uparrow \rightarrow \pi + X}}$

scaled by $\lambda \sim \langle h_1^u / f_1^u \rangle = 0.055 \pm 0.010$

only $u$ quarks fully transversely polarized

$\lambda$ is estimated by comparison with $A_{Coll}^p$ for $\pi^-$

the model describes the main properties of data

Di-hadron asymmetry for $h^+ h^-$ pairs

MC $\rightarrow$ same scale factor $\lambda$

same mechanism as for Collins
Improving the model

The model gives already a good description of the main properties of data with few parameters, same mechanism for Collins and dihadron asymmetries, jet handedness (not shown here).

We have improved it further following two directions...

a) Exploit the true predictive power of the model via a more complete simulation of the event: interface M19 with PYTHIA 8.2 for SIDIS and introduction of transversity PDF

*in collaboration with L. Lönnblad*

*the first step towards a systematic implementation of spin effects in PYTHIA!*

b) Improve the description of the polarized fragmentation process: extend M19 by introducing vector mesons → *the NEW model M20*
PYTHIA+3P0

M19 is interfaced with PYTHIA 8.2 for the simulation of polarized SIDIS
→ spin effects introduced for the first time in the hadronization part of a complete MCEG
→ parameterizations for $u^p$ and $d^p$ transversity PDFs implemented
→ PYTHIA+3P0 allows to simulate the Collins and dihadron asymmetries

Simulation of SIDIS off protons @ COMPASS kinematics
ISR/FSR switched OFF, no intrinsic $k_{\perp}$
complex mass retuned to $\mu = (0.78 + i0.38)$ GeV/c$^2$

Collins asymmetry PYTHIA+3P0

Collins asymmetry M19

Nice description of data!
trend vs $z_h$ is modified in PYTHIA
also good description of di-hadron asymmetries

PYTHIA+3P0 will be available soon!!
AK and L. Lönnblad, in preparation
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→ interface M19 with PYTHIA 8.2 for SIDIS
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the first step towards a systematic implementation of spin effects in PYTHIA!

b) Improve the description of the polarized fragmentation process
→ extend M19 by introducing vector mesons → the NEW model M20

spin effects with PS and VM production are treated systematically, for the first time
elementary splitting: emission of a VM


\[ T_{q',h,q}^{sh}(M,Z,p_T,s_h\mid k_T) \]

\[ F_{q'hq}(M,Z,p_T\mid k_T,s_q) = \text{tr} \ T_{q',h,q}^{sh} \rho(s_q) T_{q',h,q}^{sh\dagger} \]

\[ \hat{\rho}_{shsh'}(h) \propto \text{tr} \ T_{q',h,q}^{sh} \rho(s_q) T_{q',h,q}^{sh\dagger} \]

\[ dN/d\Omega \propto \mathcal{M}_{sh} \hat{\rho}_{shsh'} \mathcal{M}_{sh'}^{\dagger} \]

\[ \bar{D}_{shsh'} = \mathcal{M}_{sh'}^{\dagger} \mathcal{M}_{sh} \]

\[ \rho(s_{q'}) = \bar{D}_{shsh'} T_{q',h,q}^{sh} \rho(s_q) T_{q',h,q}^{sh\dagger} \]
Complicated recipe!

respects entanglement $q' \leftrightarrow$ momenta of decay hadrons  
(Collins ‘88, Knowles ‘88)

Form of the NEW splitting amplitude

$$T_{q', h, q}^{s_h} = \text{relativistic BW} \times \text{Lund String Fragmentation Model} \times \xi_0 \text{ operator} \times \text{VM coupling} \times \ldots$$

VM coupling $\rightarrow$ complex free parameters

$G_L \rightarrow$ coupling of $q$ to VM with linear $L$ polarisation along the string axis

$G_T \rightarrow$ coupling of $q$ to VM with linear $T$ polarisation w.r.t the string axis

Only two new parameters in the model

$$|G_L|/|G_T| \rightarrow \text{global Collins effect of the VM (depends on VM polarisation!)}$$

$$\theta_{LT} = \arg(G_L/G_T) \rightarrow \text{oblique polarisation (LT)}$$


paper in preparation
For each event define initial quark $q_A \equiv q_1$, i.e. flavour $(u, d, s)$, momentum, density matrix $\rho(q_A)$

1. Generate a $q_2\bar{q}_2$ pair and form the hadron $h_1(q_A\bar{q}_2)$, VM with prob. $\frac{f_{VM}}{f_{VM}+f_{PS}}$

2. Generate $M_{h_1}$ (if VM), $k_{2T}, Z_1$, using $F_{q_2h_1q_A} \rightarrow$ construct $p_1$

3. If $h_1 = PS$ go to 4.
   If $h_1 = VM$ $\rightarrow$ calculate $\hat{\rho}(h)$
   a) generate decay hadrons in VM rest frame and boost to string frame
   b) construct the decay matrix $\tilde{D}$

4. Calculate the spin density matrix of $q_2$

- Iterate points 1-4 until the exit condition (enough renamining mass to produce at least one baryonic resonance)
Stand alone simulations with M20

Values of the free parameters used in simulations

all mesons

\[ a = 0.9 \]
\[ b_L = 0.5 \text{(GeV/c}^2\text{)}^{-2} \]
\[ b_T = 8.43 \text{(GeV/c}^2\text{)}^{-2} \]
\[ \mu = (0.42 + i0.76) \text{GeV/c}^2 \]

VM production

\[ f_{VM}/f_{PS} = 0.62 \ (0.725) \text{ for } u,d \ (s) \]

\[ |G_L|/|G_T| = 1 \quad \theta_{LT} = 0 \quad \text{as in PYTHIA 8} \]

following the NR quark model \textit{Czyzewski '96}

sensitivity to parameters values also explored

Initial conditions

\( u \) quarks fully transversely polarized along \( \hat{y} \)

Energy calculated from a \( \{x_B, Q^2\} \) sample of SIDIS events

no primordial KT
Collins analysing power as function of rank

- classical picture reproduced
  \( \rho^+ \) have opposite effect w.r.t \( \pi^+ \)
- quark spin information decays along the chain
  faster decay in M20
Collins analysing power for $\rho$ and decay $\pi$

$\rho^+$ analysing power
- opposite to $\pi^+$
- $\sim 3$ times smaller than $\pi^+$
- as expected from the M20 prediction

$\rho^-$, $\rho^0 \sim$ to $\rho^+$ for $|G_L|/|G_T| = 1$

decay $\pi^+ \rightarrow$ larger analysing power at large $z_h$ and large $p_T$ w.r.t $\rho^+$
- large $z_h \rightarrow \pi^+$ emitted along $\hat{z}$ from longitudinally polarized $\rho^+$
- large $p_T \rightarrow \pi^+$ emitted along $\hat{p}_T^\rho$ from rank 1 $\rho^+$
  - emitted along $\hat{z} \times \hat{p}_T^\rho$ from rank 2 $\rho^+$
- small $p_T \rightarrow \pi^+$ emitted along $-\hat{p}_T^\rho$ from rank 1 $\rho^+$

$\frac{a^{u^+\rightarrow\rho+X}}{a^{u^+\rightarrow\pi+X}} \bigg|_{rank=1} = -\frac{|G_L|^2}{2|G_T|^2 + |G_L|^2}$

$\rho^+$ from $\rho^+ \rightarrow \pi^+\pi^0$

AK et al., PRD 100 (2019) 1, 014003,
model M19 $\rightarrow$ only PS

TMD Studies: from JLab to EIC
Albi KERBIZI (INFN Trieste)
Effect of VM decays on transverse spin asymmetries

Large effect on Collins analysing power w.r.t M19
- different trends
- average analysing power diluted by 50%

Di-hadron analysing power calculated using the relative transverse momentum

\[ R_T = z_2 p_{1T} / z - z_1 p_{2T} / z \]

\[ a^{u \rightarrow h^+h^-+X} = 2 \langle \sin(\phi_R - \phi_{S_A}) \rangle \]

50% dilution w.r.t M19
- effect at \( \rho^0 \) peak due to

\( \rho^0 \rightarrow \pi^+\pi^- \) symmetric w.r.t \( R_T \leftrightarrow -R_T \)
Comparison with SIDIS data

MC scaled by a factor $\lambda$ depending on $|G_L|/|G_T|$ and $\theta_{LT}$

Large variations for $\pi^+$ due to different values of $|G_L|/|G_T|$ and $\theta_{LT}$

somewhat smaller for $\pi^-$

$\rightarrow$ both parameters are important

hint for $|G_L| > 1, \theta_{LT} < 0$?

more precise data would help to fix the free parameters

TMD Studies: from JLab to EIC  Albi KERBIZI (INFN Trieste)
Sensitivity to free parameters: Collins effect for $\rho$ mesons

from AK talk at DIS-2021

COMPASS results: NEW! Measurements feasible

they could be used to fix the parameters

Hint for $\frac{|G_L|}{|G_T|} > 1$ (in particular from $p_T$)

Strong dependence on $|G_L|/|G_T|$
both size and shapes change

TMD Studies: from JLab to EIC

Albi KERBIZI (INFN Trieste)
The string+3P0 model with PS meson emission (M18, M19) implemented in a stand alone MC → describes the main features of Collins and di-hadron asymmetries!

M19 has been interfaced to PYTHIA 8
→ parameterisations for the transversity PDF implemented
→ more complete description of TSA
   the code will available very soon (AK and L. Lönnblad, in preparation)

For the first time implementation of the string+3P0 model with PS and VM production (M20) in a stand alone MC (paper in preparation)

→ detailed study of Collins effect for VM
→ only 4 free parameters for spin effects, to be fixed from comparison with data
   \( \mu \) OK, hints for \(|G_L|/|G_T| > 1\) and \(\theta_{LT} < 0\)

more precise data would help (COMPASS 2021-2022 d run, JLab12 ..)

ongoing work, promising results ...