QED Radiations in Semi-inclusive Deep Inelastic Scatterings

TMD Studies: from JLab to EIC, May 6th-7th, 2021

Tianbo Liu (刘天博)

Key Laboratory of Particle Physics and Particle Irradiation (MOE)
Institute of Frontier and Interdisciplinary Science, Shandong University

In collaboration with: W. Melnitchouk, J.W. Qiu, N. Sato

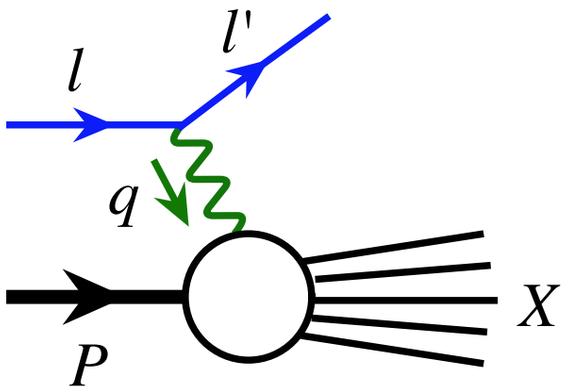
山东大学(青岛)
SHANDONG UNIVERSITY, QINGDAO

粒子物理与粒子辐照教育部重点实验室
Key Laboratory of Particle Physics and Particle Irradiation (MOE)
粒子科学技术研究中心
Research Center for Particle Science and Technology

Lepton-Hadron Deep Inelastic Scattering

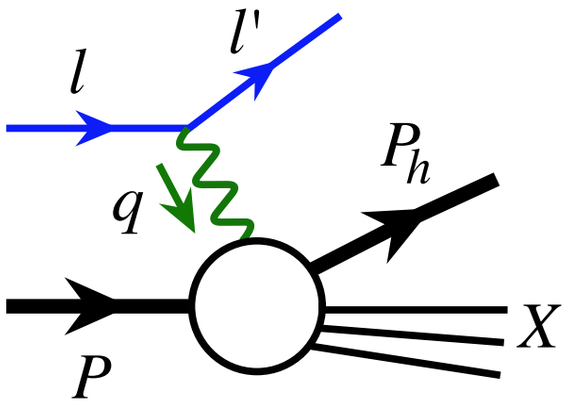
Inclusive DIS

- Large momentum transfer, Q $\gg \Lambda_{\rm QCD}$, provides a probe to "see" quarks/gluons indirectly.
- Collinear factorization: $\sigma \propto H(Q) \otimes \phi_{a/P}(x, \mu^2)$
  overall corrections suppressed by $1/Q^n$
- Not sensitive to confined motions at a hadronic scale.



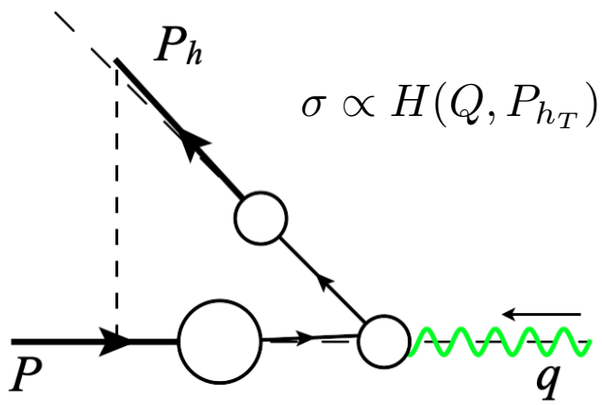
Semi-inclusive DIS

- An additional and adjustable momentum scale, P_{hT}.
- Flavor dependence by selecting different type of observerd hadron: *e.g. pions, kaons, ...*
- Enable us to explore the emergence of color neutral hadrons from colored quarks/gluons.



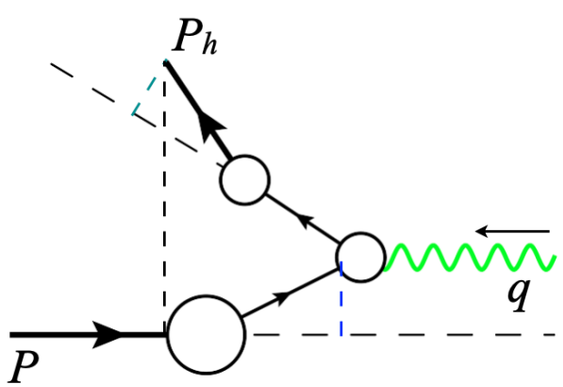
SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



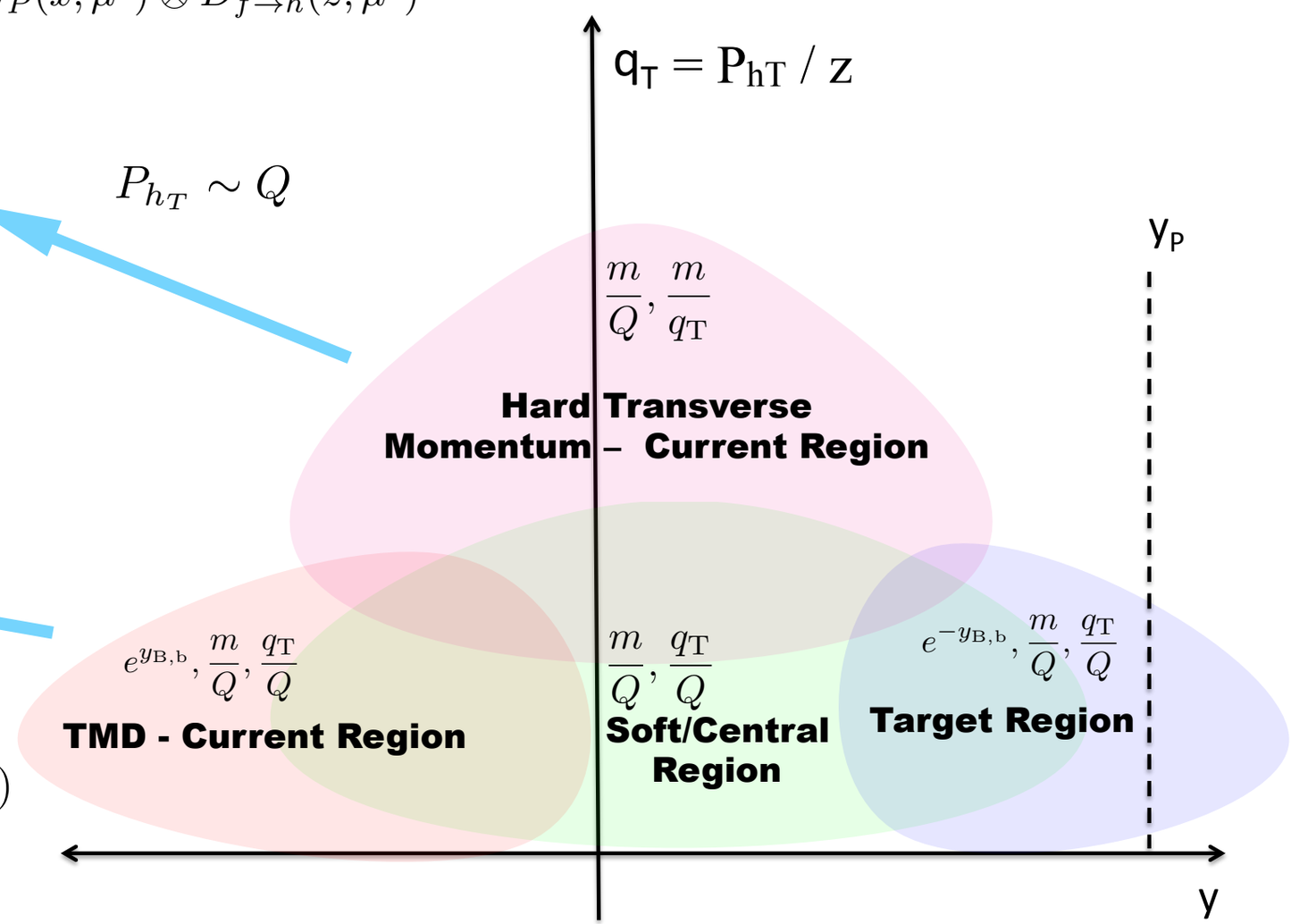
$$\sigma \propto H(Q, P_{h_T}) \otimes \phi_{a/P}(x, \mu^2) \otimes D_{f\to h}(z, \mu^2)$$

$P_{h_T} \sim Q$



$P_{h_T} \ll Q$

$$\sigma \propto H(Q) \otimes \phi_{a/P}(x, k_T, \mu^2) \otimes D_{f\to h}(z, p_T, \mu^2)$$



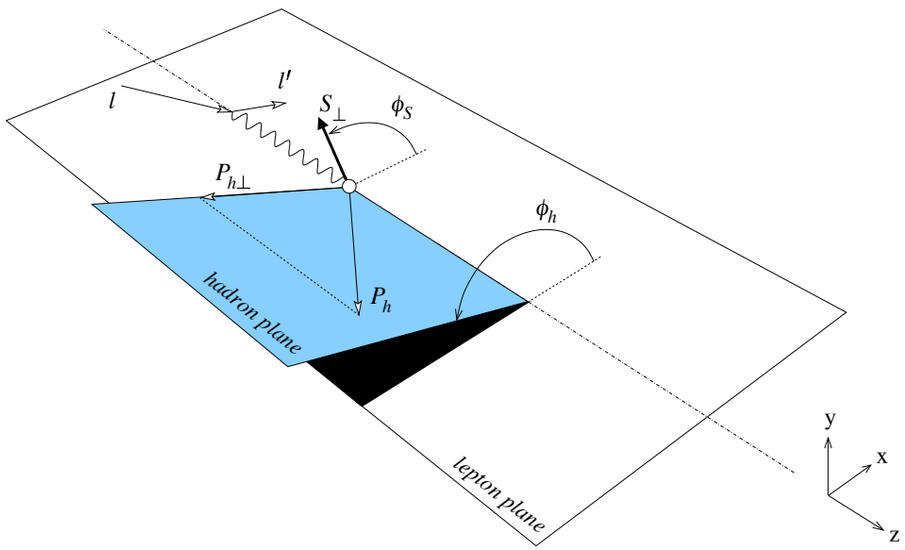
P_{hT} is defined in the photon-hadron frame

[Figure from JHEP10(2019)122]

Structure Functions

SIDIS differential cross section

18 structure functions $F(x, z, Q^2, P_{hT})$,
(one photon exchange approximation)



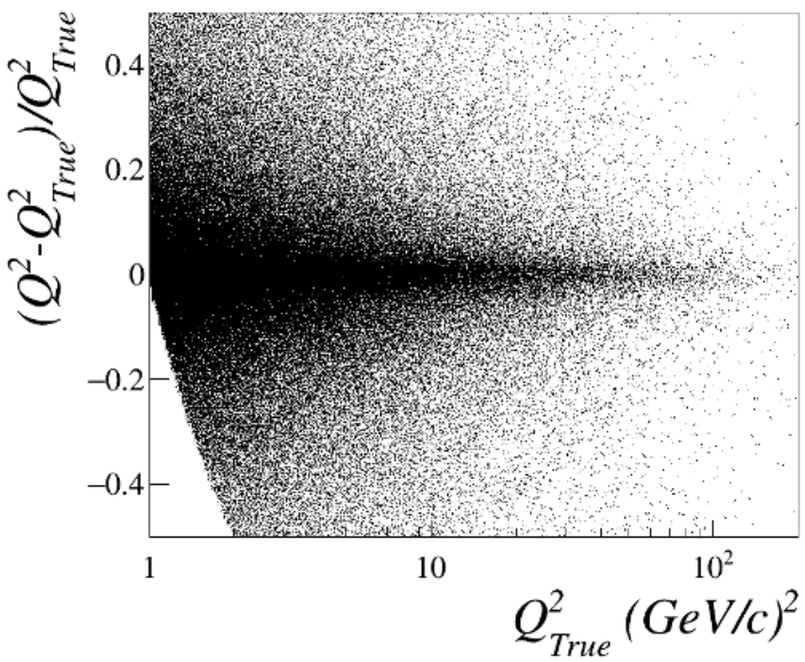
[Trento conventions, PRD70,117504 (2004)]

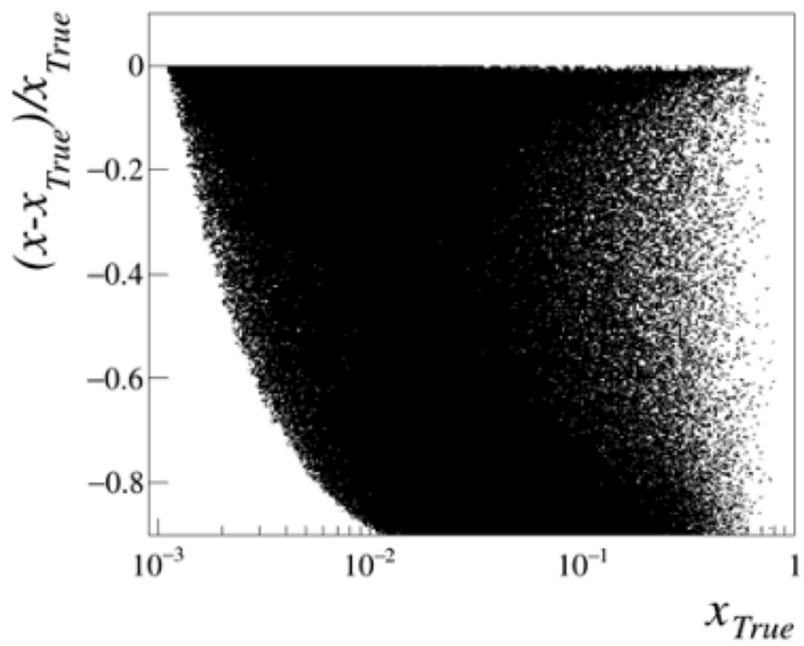
$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,dz\,dP_T^2\,d\phi_h\,d\psi} \\
&= \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right) \\
&\times \Big\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h + \lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h \\
&+ S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h + \epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right] + \lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL} + \sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right] \\
&+ S_T\Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin(\phi_h-\phi_S) + \epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S) + \epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S) \\
&\quad + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big] \\
&+ \lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \\
&\quad + \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S + \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}
\end{aligned}
$$

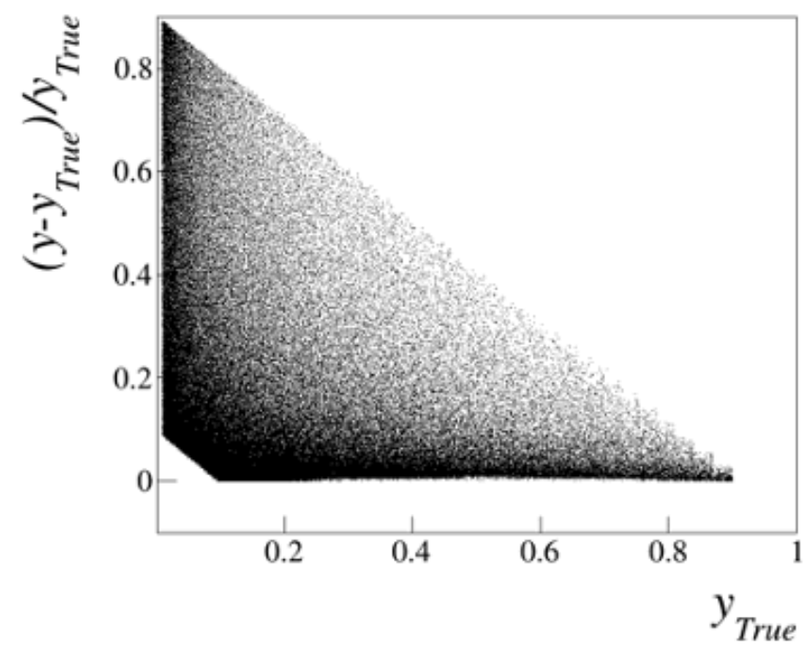
Need to know the photon-hadron frame

Kinematics is Smeared by QED Radiations

[Figures from A. Bressan's talk @ CFNS Ad-Hoc Meeting: Radiative Corrections 2020]







Kinematic variables are smeared so much due to QED radiations

Kinematics experienced by the parton could be very different from those reconstructed from observed momenta: l, l', P, P_h

Virtual photon direction is also smeared → *trouble with "photon-hadron frame"*

How to Handle QED Radiation in SIDIS

Radiative correction (RC) to Born kinematics

$$\sigma_{\text{measured}} = \sigma_{\text{No QED radiation}} \otimes \eta_{\text{RC}}$$

Radiative correction factor

Problems or challenges

The determination of RC factor relies on Monte Carlo simulation.

Usually depends on the physics we want to extract, hence introducing bias.

Almost impossible to determine the virtual photon event by event, and thus the *true photon-hadron frame*, which is essential for SIDIS/TMD extractions.

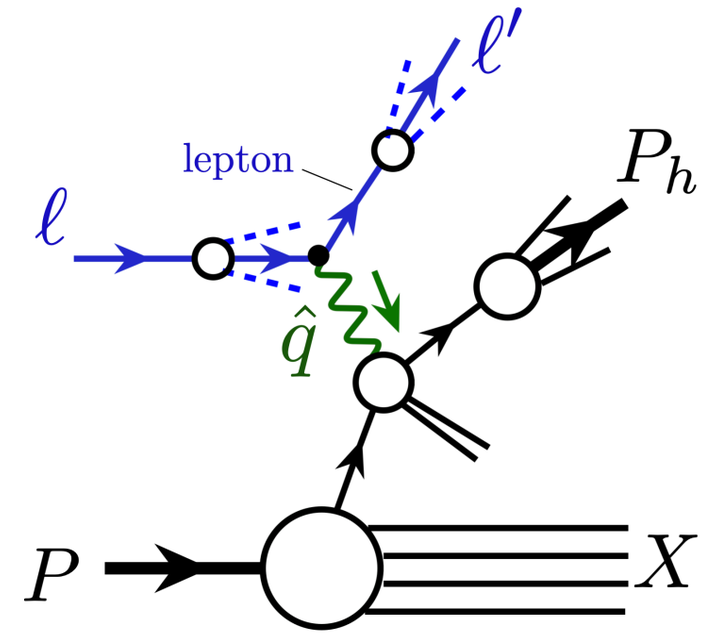
Basic ideas of our approach

- Treat QED as part of the reaction, not trying to match Born kinematics. — No RC!
- Generalize the QCD factorization to include Electroweak theory, resum the logarithmic enhanced QED contributions.
- Same systematic and RG improved treatment of QED radiation in DIS and SIDIS.

SIDIS Cross Section with QED Radiations

Differential cross section

$$d\sigma_{\ell P\to\ell' P_h X} = \frac{1}{2S}\left|\overline{M}_{\ell P\to\ell' P_h X}\right|^2 d\mathrm{PS}$$



one photon exchange approximation:

$$E_{\ell'}E_{P_h}\frac{d^6\sigma_{\ell P\to\ell' P_h X}}{d^3\ell'\,d^3P_h} = \frac{\alpha^2}{2S}\int d^4\hat{q}\left(\frac{1}{\hat{q}^2}\right)^2 \widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q})\,\widetilde{W}_{\mu\nu}(\hat{q},P,P_h)$$

Hadronic tensor:

$$\widetilde{W}_{\mu\nu}(P,P_h,\hat{q}) = \sum_{X_h}\langle P|J_\mu(0)|P_hX_h\rangle\langle P_hX_h|J_\mu(0)|P\rangle\prod_x^{X_h}\frac{d^3p_x}{(2\pi)^3 2E_x}\,\delta^{(4)}\Big(\hat{q}+P-P_h-\sum_x^{X_h}p_x\Big)$$

Leptonic tensor:

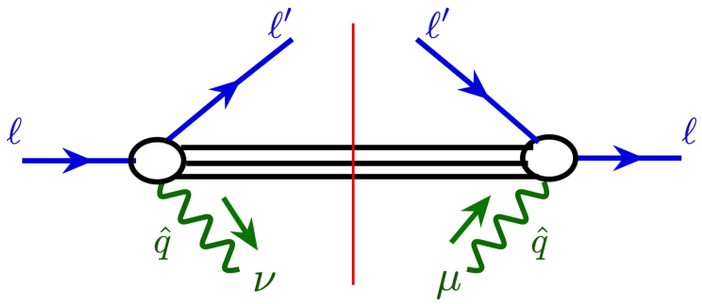
$$\widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q}) = \sum_{X_L}\langle\ell|j^\mu(0)|\ell' X_L\rangle\langle\ell' X_L|j^\mu(0)|\ell\rangle\prod_x^{X_L}\frac{d^3k_x}{(2\pi)^3 2E_x}\,\delta^4\Big(\ell-\ell'-\hat{q}-\sum_x^{X_L}k_x\Big)$$

The lowest order recovers no QED radiation expression:

$$\widetilde{L}^{\mu\nu(0)}(\ell,\ell',\hat{q}) = 2\big[\ell^\mu\ell'^\nu+\ell'^\mu\ell^\nu-\ell\cdot\ell' g^{\mu\nu}\big]\delta^{(4)}(\ell-\ell'-\hat{q})$$

Lepton Structure Functions

Current conserved decomposition of leptonic tensor



$$\widetilde{L}^{\mu\nu}(\ell,\ell',\hat{q}) = -\widetilde{g}^{\mu\nu}L_1 + \frac{\widetilde{\ell}^{\mu}\widetilde{\ell}^{\nu}}{\ell\cdot\ell'}L_2 + \frac{\widetilde{\ell}'^{\mu}\widetilde{\ell}'^{\nu}}{\ell\cdot\ell'}L_3 + \frac{\widetilde{\ell}^{\mu}\widetilde{\ell}'^{\nu}+\widetilde{\ell}'^{\mu}\widetilde{\ell}^{\nu}}{2\ell\cdot\ell'}L_4$$

$$\widetilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{\hat{q}^{\mu}\hat{q}^{\nu}}{\hat{q}^2}, \quad \widetilde{\ell}^{\mu} = \widetilde{g}^{\mu\nu}\ell_{\nu} = \ell^{\mu} - \frac{\ell\cdot\hat{q}}{\hat{q}^2}\hat{q}^{\mu}, \quad \widetilde{\ell}'^{\mu} = \widetilde{g}^{\mu\nu}\ell'_{\nu} = \ell'^{\mu} - \frac{\ell'\cdot\hat{q}}{\hat{q}^2}\hat{q}^{\mu}$$

Lepton structure functions: $L_i(\xi_B,\zeta_B,\hat{\mathbf{q}}_T^2,Q^2), \quad i=1,2,3,4$

$$\xi_B = \frac{\hat{q}\cdot\ell'}{\ell\cdot\ell'}, \quad \frac{1}{\zeta_B} = -\frac{\hat{q}\cdot\ell}{\ell\cdot\ell'} \quad \hat{\boldsymbol{q}}_T^2 = \widehat{Q}^2 - \frac{\xi_B}{\zeta_B}Q^2$$

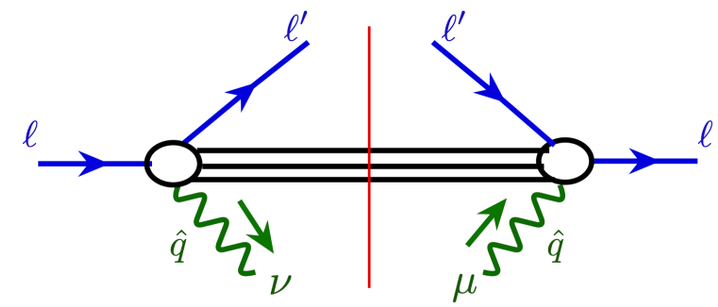
In lepton back-to-back frame:

$$\ell^{\mu} = (\ell^+, 0, \mathbf{0}_T), \quad \ell'^{\mu} = (0, \ell'^-, \mathbf{0}_T) \qquad \ell^+ = \ell'^- = Q/\sqrt{2}$$

$$\hat{q}^{\mu} = \left(\hat{q}^+, \hat{q}^-, \hat{\boldsymbol{q}}_T\right) = \left(\xi_B\ell^+, -\frac{1}{\zeta_B}\ell'^-, \hat{\boldsymbol{q}}_T\right)$$

Lepton SFs in Helicity Basis

Basis vectors and polarization vectors:



$$T^\mu = \frac{\sqrt{\xi_B\zeta_B}}{Q}\widetilde{\ell}^\mu + \frac{1}{\sqrt{\xi_B\zeta_B}Q}\widetilde{\ell}'^\mu,$$

$$X^\mu = -\frac{\widehat{Q}\sqrt{\xi_B\zeta_B}}{Q\sqrt{\hat{\boldsymbol{q}}_T^2}}\widetilde{\ell}^\mu + \frac{\widehat{Q}}{Q\sqrt{\xi_B\zeta_B}\sqrt{\hat{\boldsymbol{q}}_T^2}}\widetilde{\ell}'^\mu,$$

$$Y^\mu = \varepsilon^{\mu\nu\rho\sigma}Z_\nu T_\rho X_\sigma,$$

$$Z^\mu = \frac{\hat{q}^\mu}{Q}$$

$$\epsilon_0^\mu(\hat{q}) = T^\mu,$$

$$\epsilon_+^\mu(\hat{q}) = -\frac{1}{\sqrt{2}}X^\mu - \frac{i}{\sqrt{2}}Y^\mu,$$

$$\epsilon_-^\mu(\hat{q}) = \frac{1}{\sqrt{2}}X^\mu - \frac{i}{\sqrt{2}}Y^\mu,$$

Helicity basis lepton structure functions:

$$\begin{aligned}\widetilde{L}^{\mu\nu} &= \epsilon_0^{*\mu}\epsilon_0^\nu L_{00} + (\epsilon_+^{*\mu}\epsilon_+^\nu + \epsilon_-^{*\mu}\epsilon_-^\nu)L_{++} + (\epsilon_+^{*\mu}\epsilon_-^\nu + \epsilon_-^{*\mu}\epsilon_+^\nu)L_{+-} \\ &\quad - \epsilon_0^{*\mu}(\epsilon_+^\nu - \epsilon_-^\nu)L_{0+} - (\epsilon_+^\mu - \epsilon_-^\mu)^*\epsilon_0^\nu L_{+0} \\ &= T^\mu T^\nu L_{00} + (X^\mu X^\nu + Y^\mu Y^\nu)L_{TT} \\ &\quad + (T^\mu X^\nu + T^\nu X^\mu)L_\Delta + (Y^\mu Y^\nu - X^\mu X^\nu)L_{\Delta\Delta},\end{aligned}$$

Expansion in α:

$$L_{\rho\sigma} = e^2\sum_{N=0}^{\infty}\left(\frac{\alpha}{\pi}\right)^N L_{\rho\sigma}^{(N)}$$

Leading order: $L_{TT}^{(0)} = 2\,\delta(\xi-1)\delta(\frac{1}{\zeta}-1)\delta^{(2)}(\hat{\boldsymbol{q}}_T)$ *the other three vanish.*

Factorization of Lepton Structure Function

CSS factorization

"W+Y" formalism:

$$L_{TT}(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2)=\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}e^{i\hat{\boldsymbol{q}}_T\cdot\boldsymbol{b}}\widetilde{W}_{TT}(\xi_B,\zeta_B,Q^2,b)+Y_{TT}(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2)$$

b-space resummed form:

lepton distribution function (LDF)

lepton fragmentation function (LFF)

$$\widetilde{W}_{TT}(\xi_B,\zeta_B,Q^2,b)=2\int_{\xi_B}^1\frac{d\xi}{\xi}\int_{\zeta_B}^1\frac{d\zeta}{\zeta^2}\boxed{f(\xi)}\boxed{D(\zeta)}C_f(\lambda)C_D(\eta)$$
$$\times\exp\left\{-\int_{\mu_b^2}^{\mu_Q^2}\frac{d\mu'^2}{\mu'^2}\left[A\big(\alpha(\mu')\big)\ln\frac{\mu_Q^2}{\mu'^2}+B\big(\alpha(\mu')\big)\right]\right\}$$

Expansion in α:

$$A=\sum_{N=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^N A^{(N)}$$

$$B=\sum_{N=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^N B^{(N)}$$

$$C_{f,D}=\sum_{N=0}^{\infty}\left(\frac{\alpha}{\pi}\right)^N C_{f,D}^{(N)}$$

$$A^{(1)}=1,\qquad C_f^{(0)}(\lambda)=\delta(1-\lambda)$$

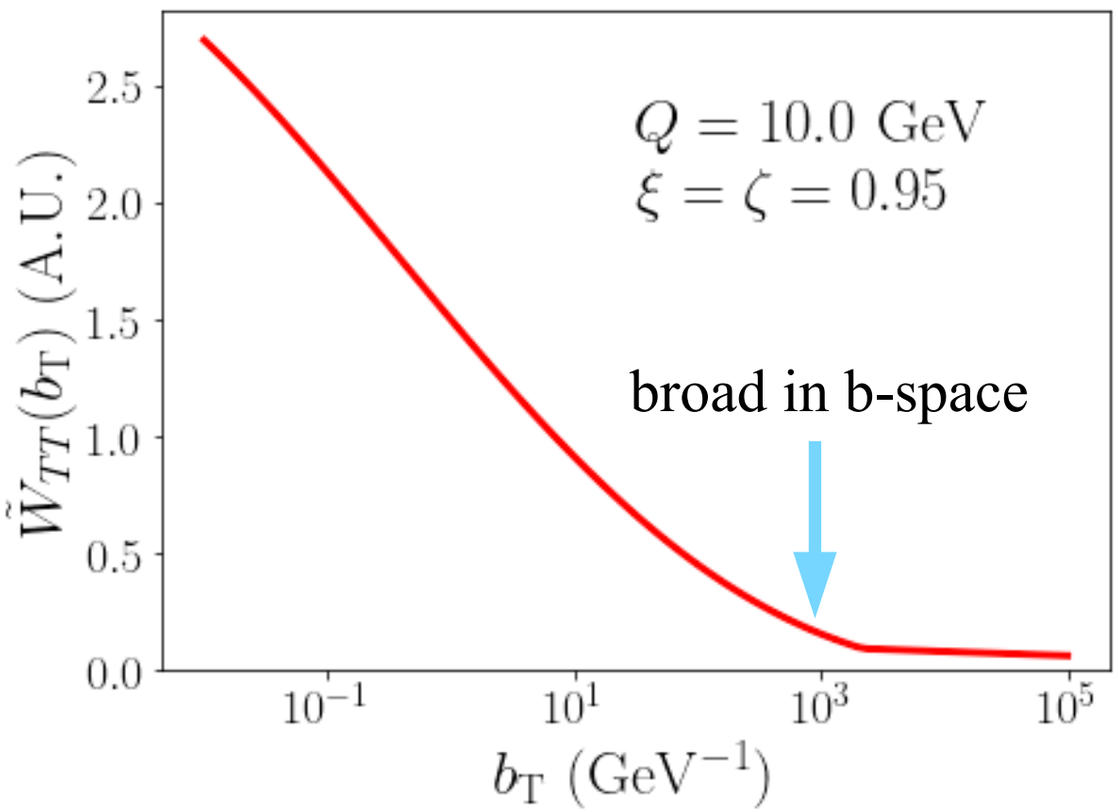
$$B^{(1)}=-\frac{3}{2}\qquad C_D^{(0)}(\eta)=\delta(1-\eta)$$

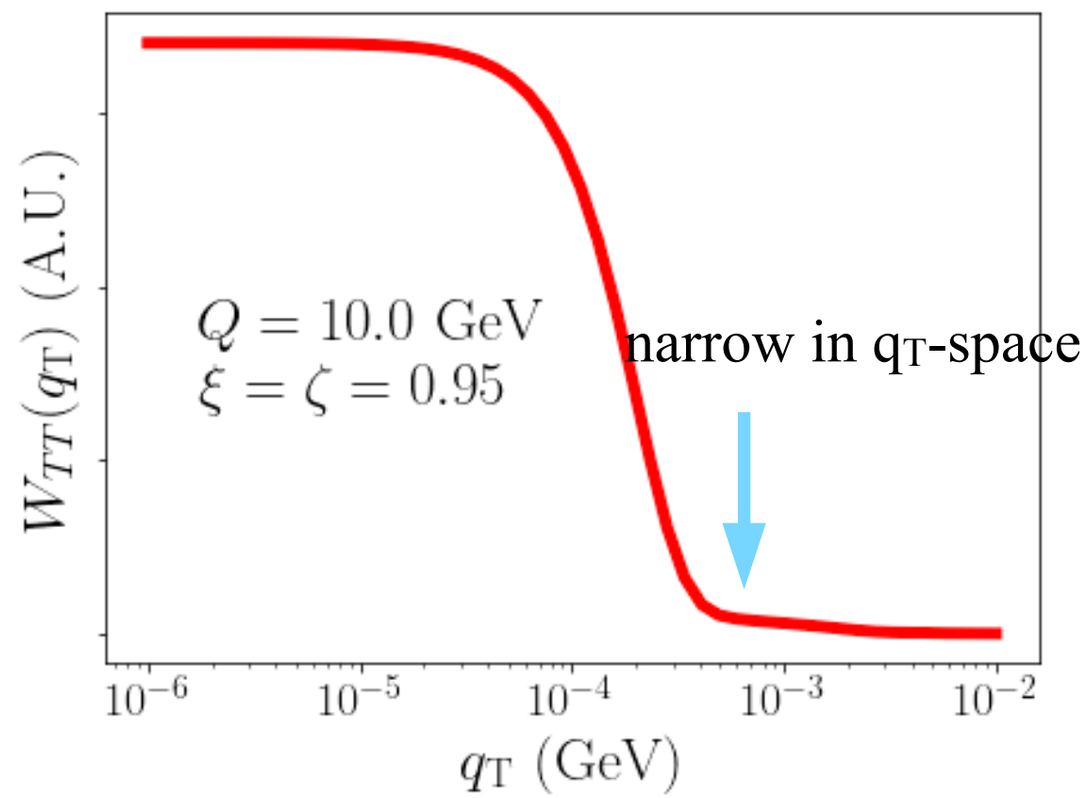
$$C_f^{(1)}(\lambda)=\frac{1}{2}(1-\lambda)-\left(\frac{1+\lambda^2}{1-\lambda}\right)_+\ln\frac{\mu_{\overline{\rm MS}}}{\mu_b}-2\delta(1-\lambda),$$

$$C_D^{(1)}(\eta)=\frac{1}{2\eta}(1-\eta)-\frac{1}{\eta}\left(\frac{1+\eta^2}{1-\eta}\right)_+\ln\frac{\mu_{\overline{\rm MS}}}{\mu_b}-2\delta(1-\eta)$$

Lepton TMD

QED shower generates very small transverse momentum





Collinear LDF and LFF are good approximation of lepton TMDs.

Impact on hadron P_{hT} in "photon-hadron frame" is mainly caused by logarithmic enhanced collinear radiation.

LDF and LFF

Lepton distribution function:

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi}\, e^{i\xi\ell^+ z^-} \langle e|\, \overline{\psi}_i(0)\gamma^+ \Phi_{[0,z^-]}\, \psi_i(z^-)|e\rangle$$

LO: $f^{(0)}_{i/e}(\xi) = \delta_{ie}\delta(1-\xi)$

NLO($\overline{\text{MS}}$): $f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$

Lepton distribution function:

$$D_{e/j}(\zeta) = \frac{\zeta}{2}\sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^-/\zeta}\, \mathrm{Tr}\left[\gamma^+ \langle 0|\, \overline{\psi}_j(0)\Phi_{[0,\infty]}\, |e,X\rangle\langle e,X|\psi_j(z^-)\Phi_{[z^-,\infty]}|0\rangle\right]$$

LO: $D^{(0)}_{e/j}(\zeta) = \delta_{ej}\delta(1-\zeta)$

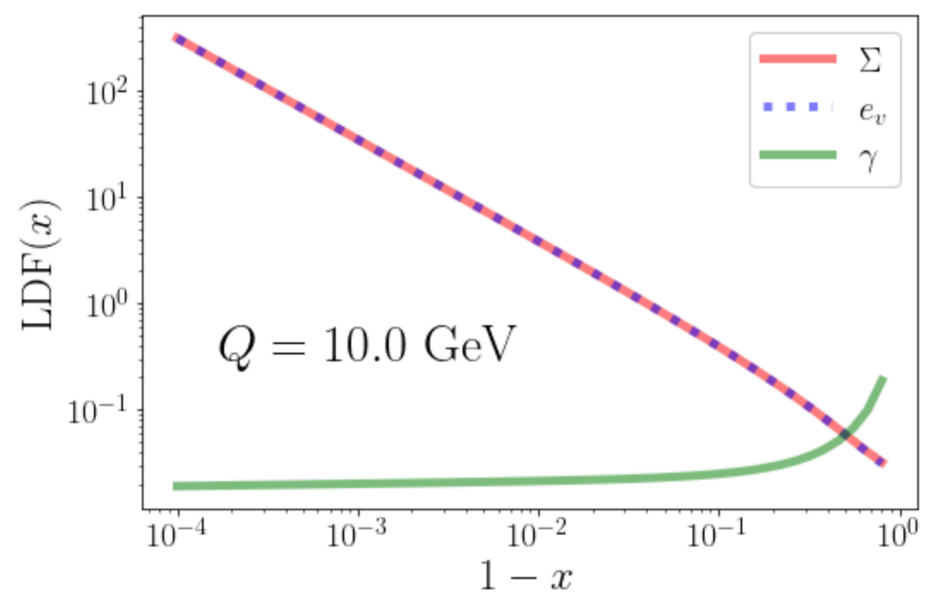
NLO($\overline{\text{MS}}$): $D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$

Resum:

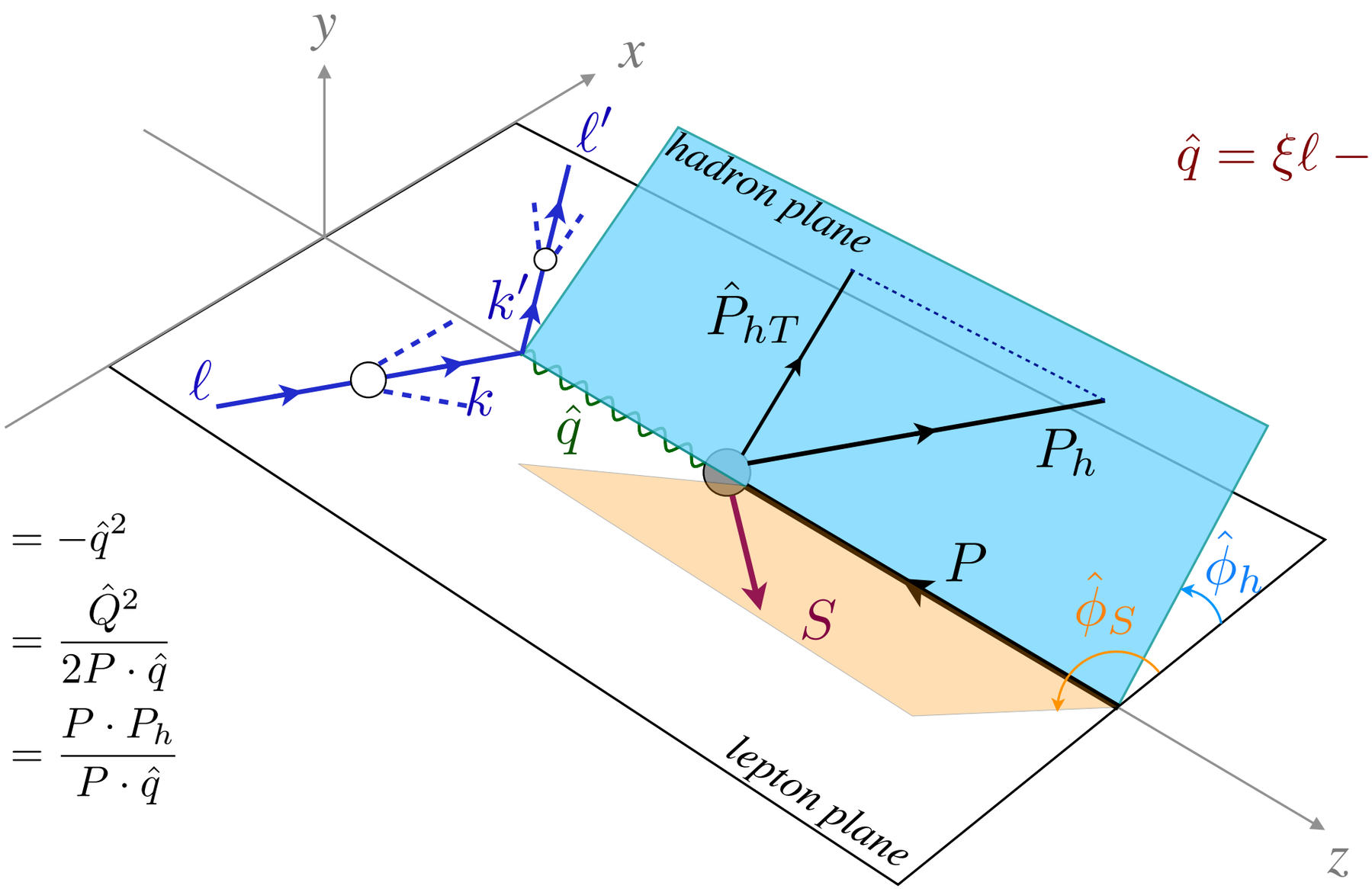
$$\frac{\partial}{\partial \ln\mu^2}\begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\gamma} \\ P_{\gamma e} & P_{\gamma\gamma} \end{pmatrix} \otimes \begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix}$$

QED DGLAP evolution

Similar for LFF



True Photon-Hadron Frame



$$\hat{q} = \xi\ell - \frac{1}{\zeta}\ell' \neq \ell - \ell'$$

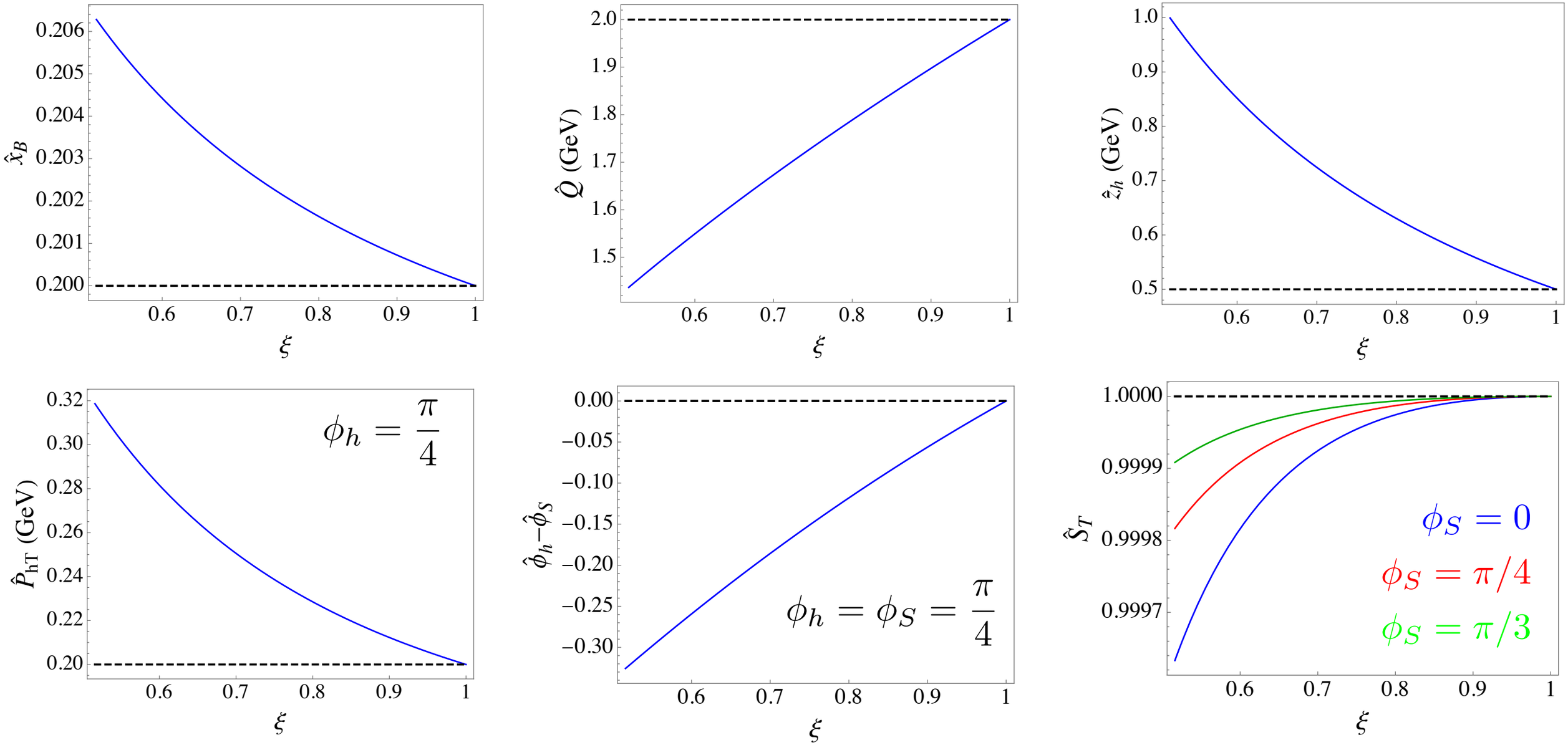
$$\hat{Q}^2 = -\hat{q}^2$$

$$\hat{x}_B = \frac{\hat{Q}^2}{2P\cdot\hat{q}}$$

$$\hat{z}_h = \frac{P\cdot P_h}{P\cdot\hat{q}}$$

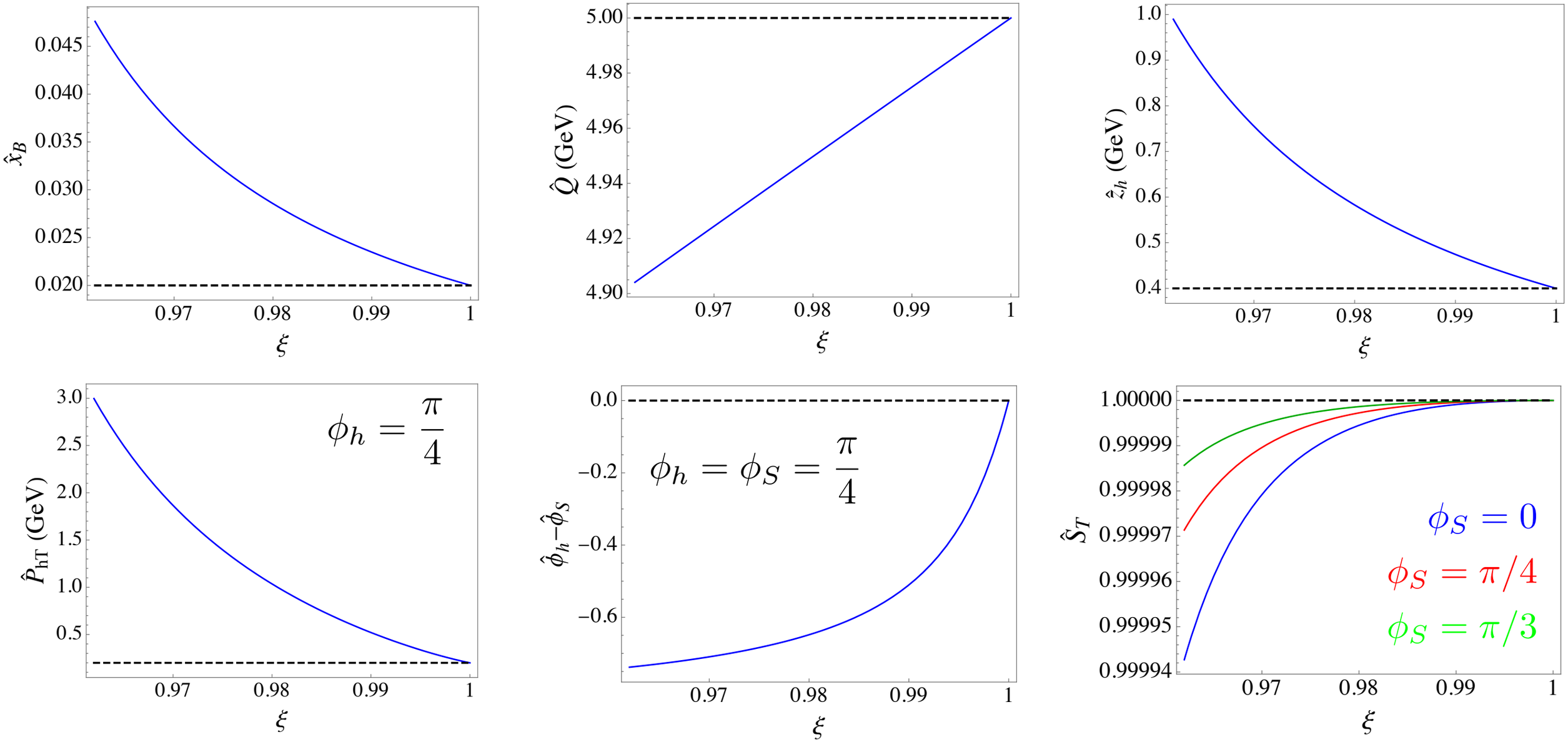
$\hat{Q}^2, \hat{x}_B, \hat{z}_h, \hat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \hat{S}_T, \hat{S}_L$ are functions of $\xi, \zeta, Q^2, x_B, z_h, P_{hT}, \phi_h, \phi_S, S_T, S_L$

Shift of Kinematics



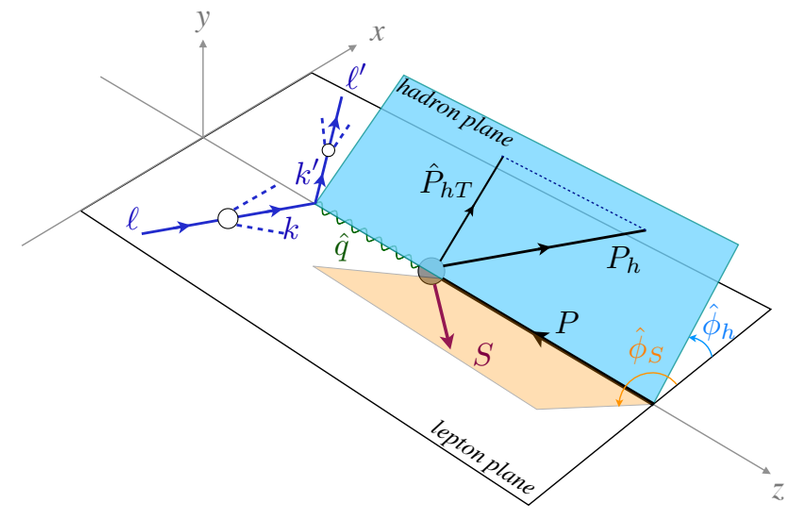
$$\sqrt{s} = 4.64\,\text{GeV}, \quad x_B = 0.2, \quad Q = 2\,\text{GeV},$$
$$z_h = 0.5, \quad P_{hT} = 0.2\,\text{GeV} \quad \zeta = 1$$

Shift of Kinematics



$$\sqrt{s} = 140\,\text{GeV}, \quad x_B = 0.02, \quad Q = 5\,\text{GeV},$$
$$z_h = 0.4, \quad P_{hT} = 0.2\,\text{GeV} \quad \zeta = 1$$

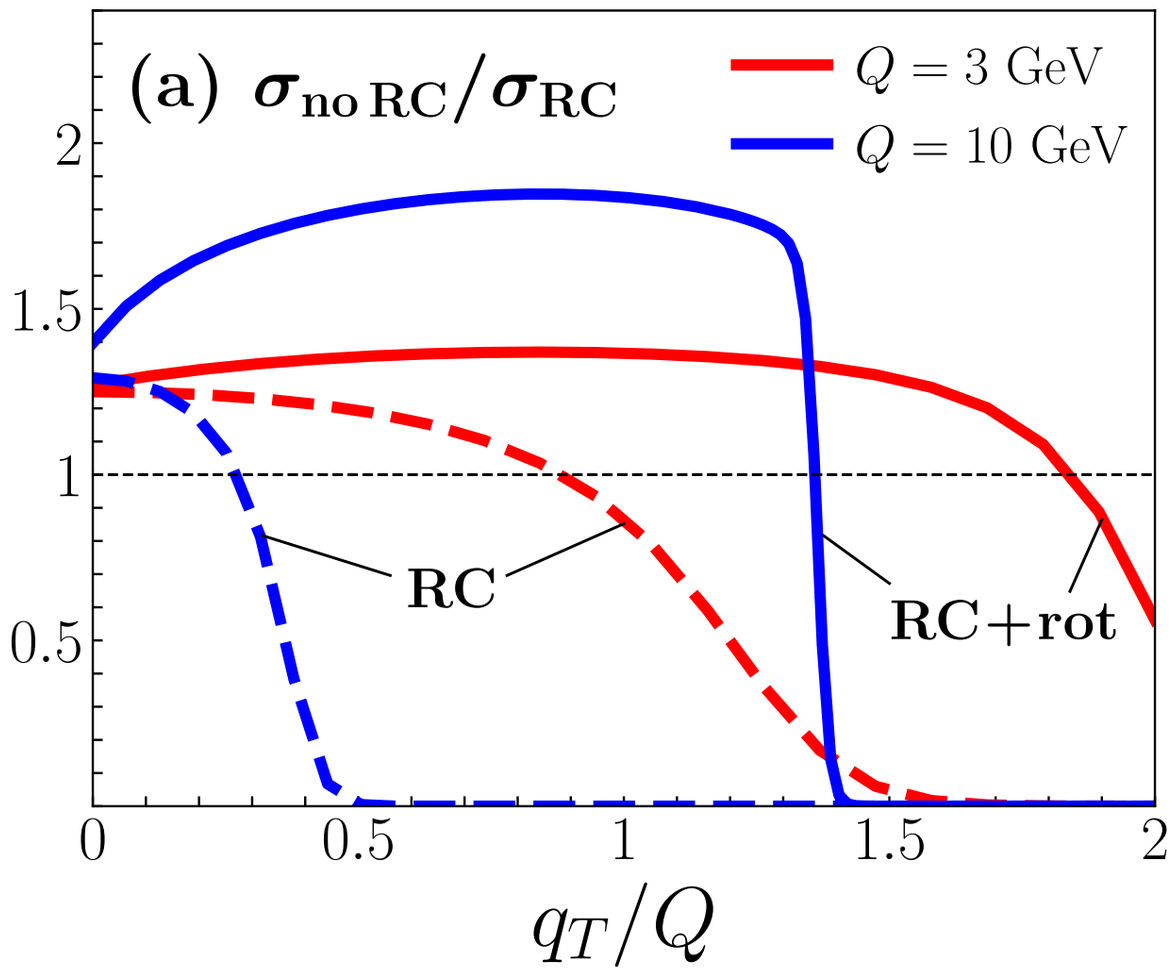
SIDIS with Collinear QED Factorization



$$
\begin{aligned}
&\frac{d\sigma}{dx dy dz dP_T^2 d\phi_h d\psi} \\
&\Big[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right) \\
&\Big\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h + \lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h \\
&+ S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h + \epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right] + \lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL} + \sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right] \\
&+ S_T\Big[(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)})\sin(\phi_h-\phi_S) + \epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S) + \epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S) \\
&\quad + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S + \sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big] \\
&+ \lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \\
&\quad + \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S + \sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\} \\
&\{Q^2, x, y, \gamma, \epsilon, z, P_T, \phi_h, \phi_S, S_T, S_L\} \to \{\hat{Q}^2, \hat{x}_B, \hat{y}, \hat{\gamma}, \hat{\epsilon}, \hat{z}_h, \hat{P}_{hT}, \hat{\phi}_h, \hat{\phi}_S, \hat{S}_T, \hat{S}_L\} \Big] \\
&\otimes f_{e/e}(\xi) \otimes D_{e/e}(\zeta)\left(\frac{\hat{x}_B}{x_B\xi\zeta}\right)
\end{aligned}
$$

Jacobian between the two frames

Impact of QED Effects: Example I

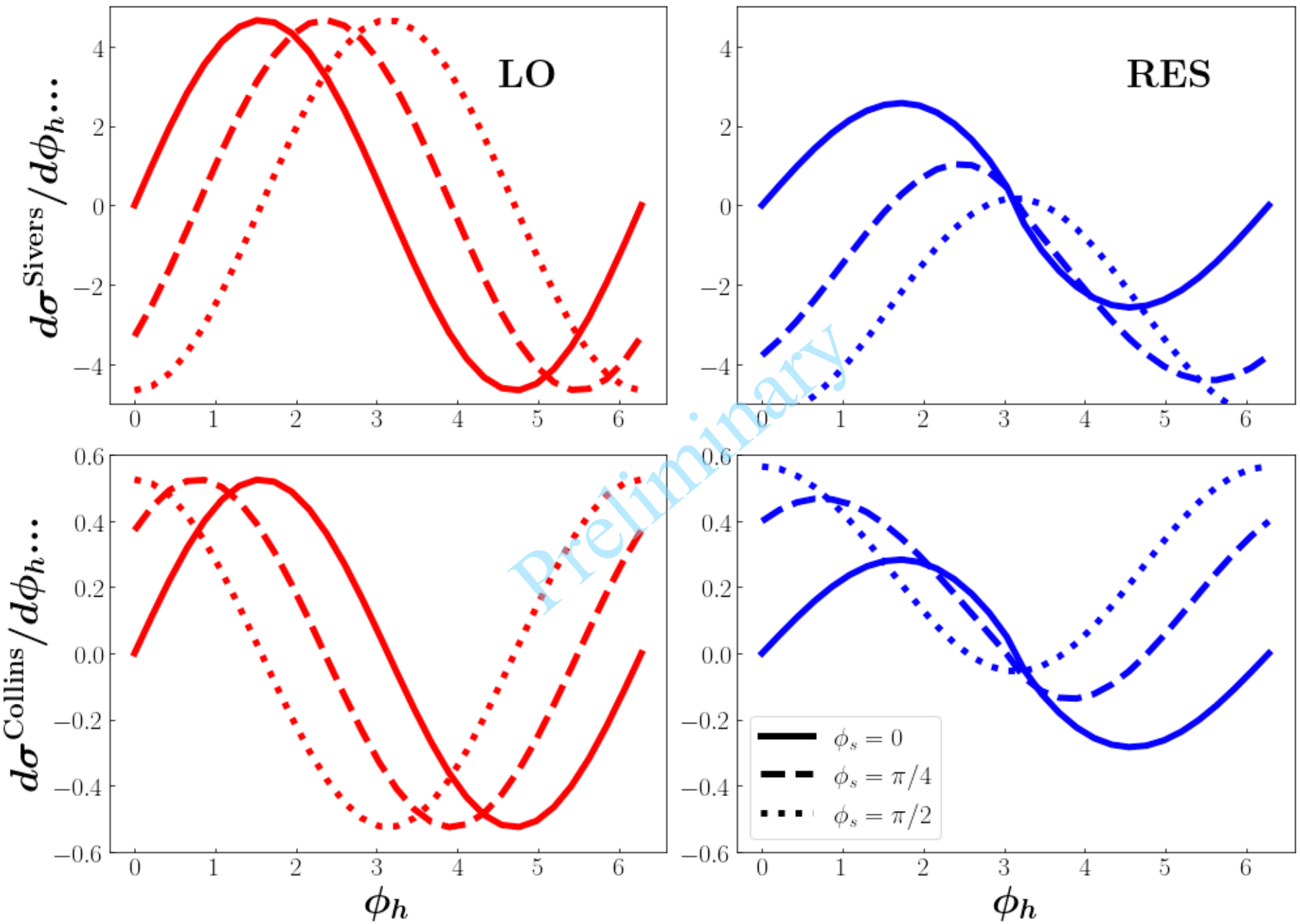


$$\sqrt{s} = 140\,\text{GeV}$$
$$y = 0.4$$
$$z_h = 0.5$$

Lorentz transform from experimental "photon-hadron frame" to the true photon-hadron frame (It is a rotation in target rest frame)

The rotation effect is huge.

Impact of QED Effects: Example II



Nontrivial effects on azimuthal modulations.

Summary

- QED radiation effects are important in SIDIS, and hence precise extractions of TMDs.
  - Experimental "photon-hadron frame" does not coincide with the *true photon hadron frame*, where the factorization works.
  - Almost impossible to determine/reconstruct the *true photon hadron frame* event by event.
  - Challenge to match to Born kinematics without introducing model/theory bias.
- We propose a factorized approach to treat QED radiations.
  - Treat QED radiation as a part of the production cross section.
  - Generalize QCD factorization to include QED. All perturbatively calculable hard parts are IR safe.
  - Transverse momentum generated by QED shower is small, and one can apply collinear factorization for the leptonic tensor.
  - Huge and nontrivial effects on P_T dependence and azimuthal modulations.

Thank you!