Extraction of TMD distributions from data

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(Regensburg University)

Workshop:
TMD Studies: from JLab to EIC
May 7, 2021

Extraction of TMD with `artemide`

- Main features
  - Position space
  - ζ-prescription
  - (maximum) Perturbative matching
  - Strict data cuts
- Code
  - `artemide`
    - Fortran 95
    - https://github.com/VladimirovAlexey/artemide-public
  - `artemide-DataProcessor`
    - Python
    - https://github.com/VladimirovAlexey/artemide-DataProcessor

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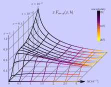
Low-energy
E228, E605,...
PHENIX

Drell-Yan fit

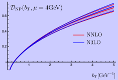
High-energy
ATLAS, LHCb,...
CDF, D0

Unpolarized DY

uTMDPDF (5 params)



RAD (1 param)



[V.Bertone,I.Scimemi,AV;1902.08474]

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→ **Drell-Yan fit** ←

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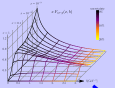

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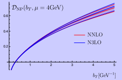


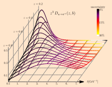
[V.Bertone,I.Scimemi,AV;1902.08474]

Unpolarized SIDIS

HERMES
COMPASS

→ **SIDIS fit** →

TMDFF (4 params)



[I.Scimemi,AV;1912.06532]

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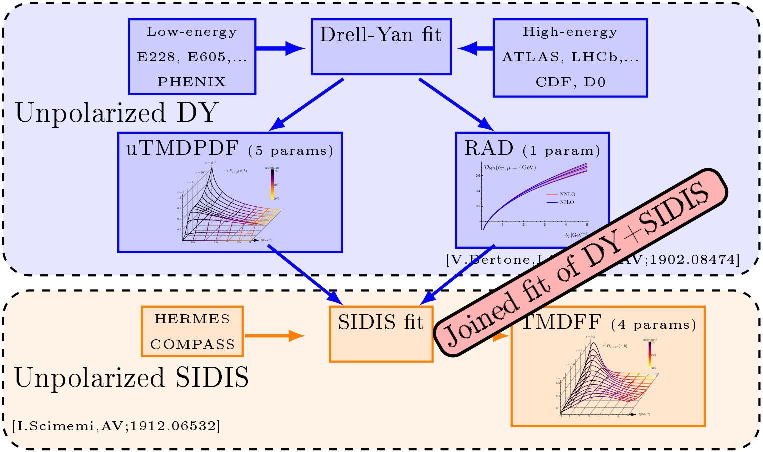
Low-energy
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NNLO
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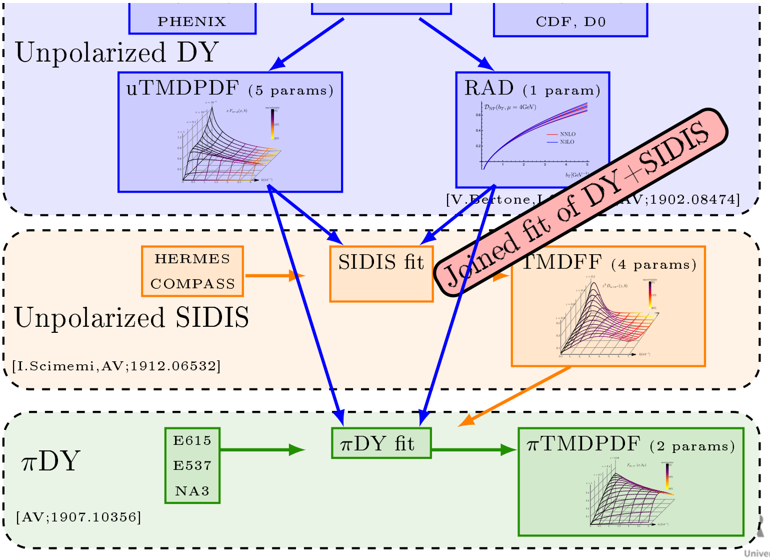
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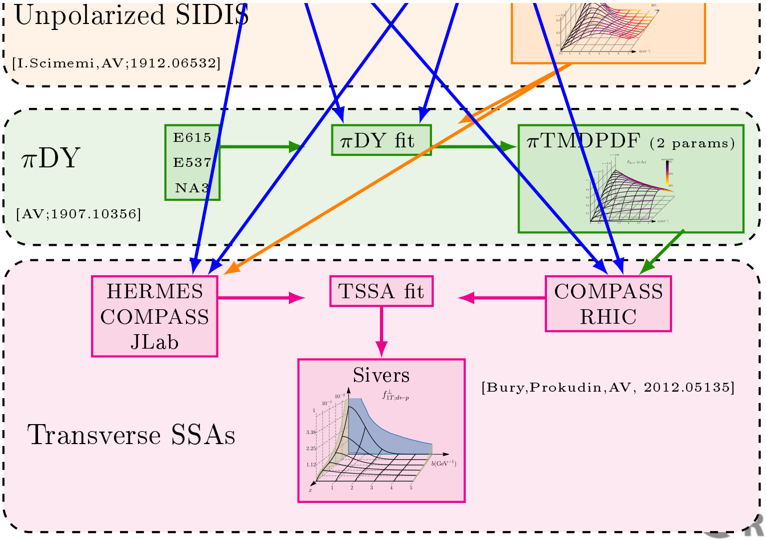
πDY
E615
E537
NA3
πDY fit
πTMDPDF (2 params)
[AV;1907.10356]



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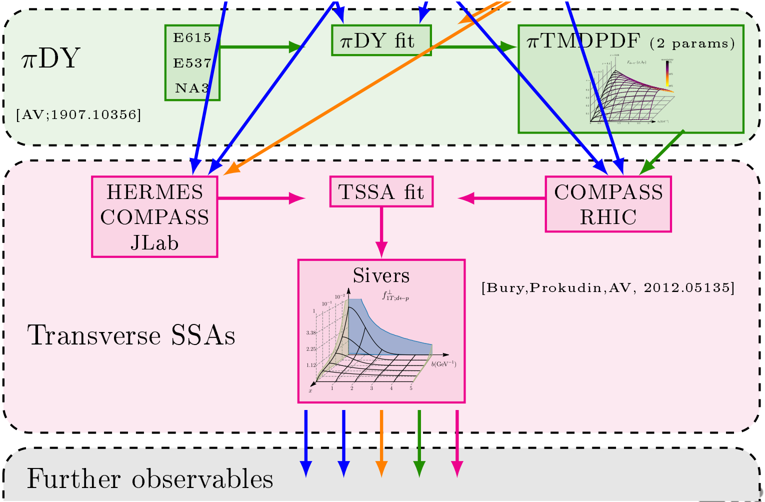
πDY
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πTMDPDF (2 params)
[AV;1907.10356]
HERMES
COMPASS
JLab
TSSA fit
COMPASS
RHIC
Sivers
[Bury,Prokudin,AV, 2012.05135]
Transverse SSAs
Further observables

In my talk I am going to focus on particular aspects of TMD phenomenology

Data selection for TMD factorization

- Theoretical justification
- Experimental evidence
- Impact on present and future data-sets

PDF-bias and problem of TMD modeling

- Structure of (modern) TMD models
- PDF-bias
- Flavor dependence **(work in progress)**

Problem with determination of CS-kernel

- Correlation between TMDs and CS-kernel
- ζ-prescription

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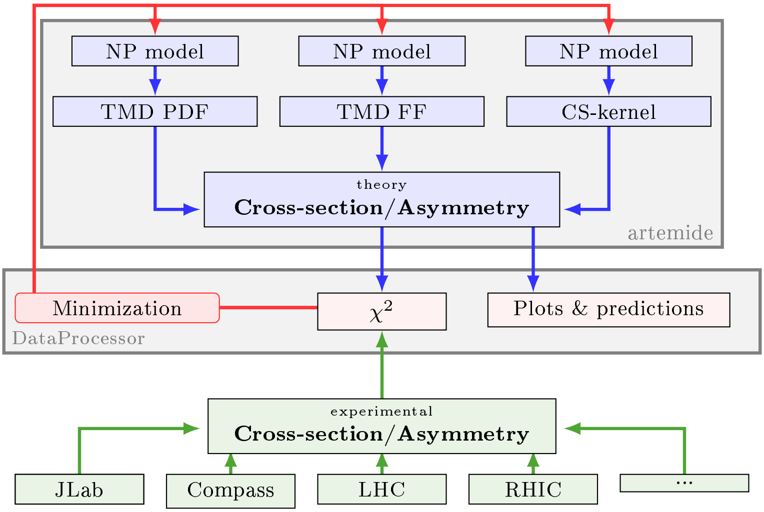
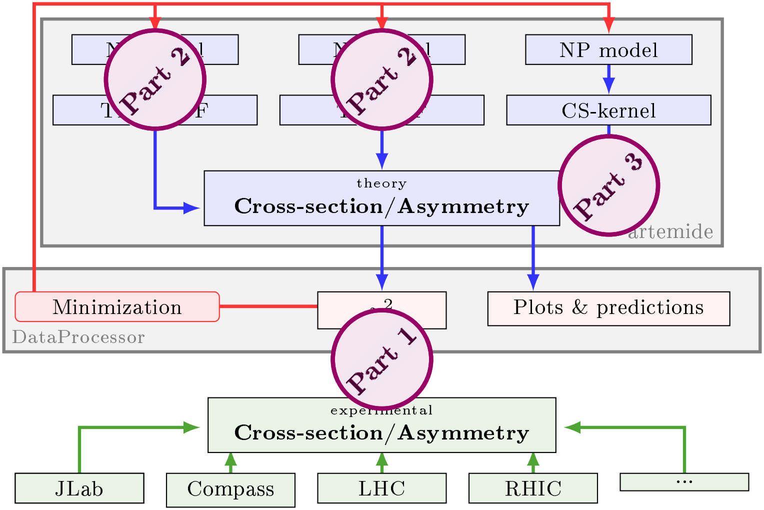

NP model
NP model
NP model
TMD PDF
TMD FF
CS-kernel
theory
Cross-section/Asymmetry
artemide
Minimization
χ^2
Plots & predictions
DataProcessor
experimental
Cross-section/Asymmetry
JLab
Compass
LHC
RHIC
...


Part 2
Part 2
NP model
CS-kernel
theory
Cross-section/Asymmetry
Part 3
artemide
Minimization
Plots & predictions
DataProcessor
Part 1
Cross-section/Asymmetry
JLab
Compass
LHC
RHIC
...

Part I
Data cuts

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TMD factorization is a systematic expansion of **hadronic tensor** in power of $\frac{q_T}{Q}$

$$W^{\mu\nu} = \underbrace{W_0^{\mu\nu}}_{F\otimes D} + \overbrace{\underbrace{\frac{q_T}{Q}W_1^{\mu\nu}}_{\substack{\partial F\otimes D\\ F\otimes\partial D\\ \Upsilon\otimes D\\ F\otimes\Delta}}}^{\text{not present in } F_{UU},\, F_{UT}^{\sin}} + \underbrace{\frac{q_T^2}{Q^2}W_2^{\mu\nu}}_{\substack{\partial^2 F\otimes D\\ \partial F\otimes\partial D\\ \Upsilon\otimes\Delta\\ \partial F\otimes\Delta\\ \dots\\ \text{twist-4}\\ \dots}} + \dots$$

- Leading-power term is well-investigated
- Something is known about NLP W_1 (factorization is not proven)
- Nothing is known about NNLP W_2

Important: TMD factorization is $Q\to\infty$, q_T=fixed.
Could be corrections $\sim\frac{\Lambda}{Q}$ even at $q_T\to 0$.

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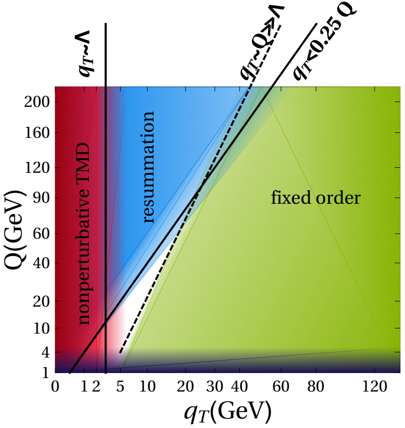
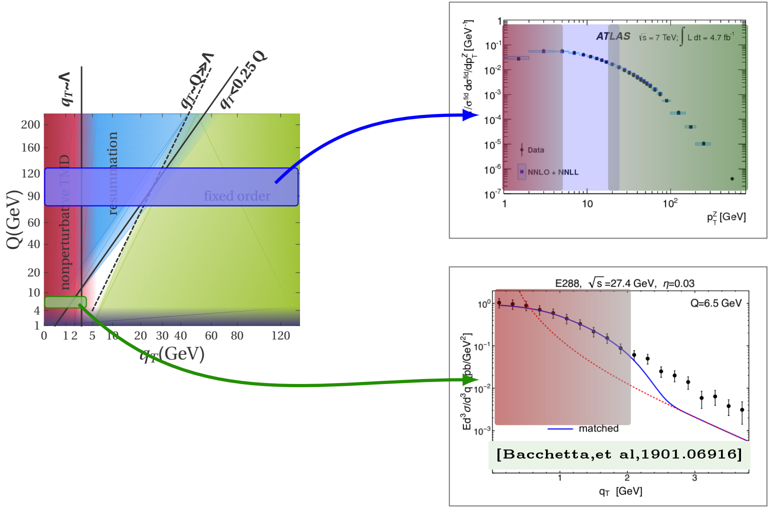
Factorization regions

$q_T \lesssim \delta Q$	TMD factorization	$= \begin{cases} q_T \lesssim \Lambda & \text{nonpertrubative regime} \\ q_T \gg \Lambda & \text{"resummation" regime} \end{cases}$
$q_T \sim Q \gg \Lambda$	collinear factorization	



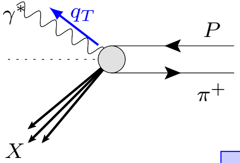


Here I draw $\delta = 0.25$, how to justify this choice?

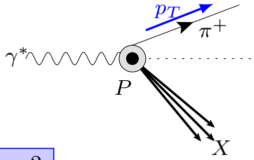
$$\delta^2 = \frac{q_T^2}{Q^2} \sim 0.06 \ll 1$$

The factorization for SIDIS is done in the Breit frame

Breit frame



Lab frame



$$q_T^2 = \frac{p_T^2}{z^2}\frac{1+\gamma^2}{1-\varsigma^2}$$

$$\gamma = \frac{2Mx}{Q}$$

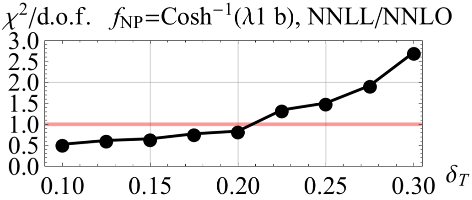
$$\varsigma = \gamma\frac{m}{zQ}$$

$$x_1 = -x\frac{2}{\gamma^2}\left(1 - \sqrt{1+\gamma^2\left(1-\frac{\mathbf{q}_T^2}{Q^2}\right)}\right), \qquad z_1 = z\frac{x_1}{x}\frac{1+\sqrt{1-\varsigma^2}}{2\left(1-\frac{\mathbf{q}_T^2}{Q^2}\right)}$$

- **Only in the Breit frame one can justify the parton model**

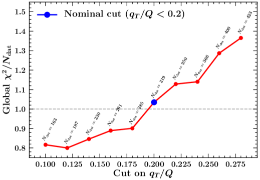
Value of δ from the data

1. Fit data at some small δ ($\chi/N_{pt} \sim 1$)
2. Increase δ and fit again, starting from the previous minimum (repeat)
3. At some moment the χ^2/N_{pt} blows up



[Scimemi,AV, 1706.01473]

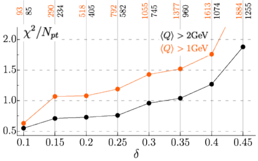
DY



$\delta \sim 0.2$

[Bacchetta,Bertone,et al,1912.07550]

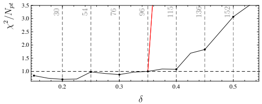
SIDIS



$\delta \sim 0.25$

[Scimemi,AV,1912.06532]

Sivers asymmetry



$\delta \sim 0.3$

[Bury,Prokudin,AV,2103.03270]

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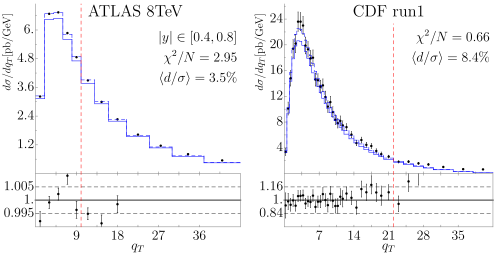
Data indicates that $\delta \sim 0.2 - 0.25$

But...

- For asymmetries (ratios of cross-section) δ could be larger
- The method outcome strongly dependents on precision of the data
  - Precise data → more sensitivity to small effects, e.g. power corrections
  - E.g. for ATLAS ($\sim 0.5\%$ accuracy) at $\delta \sim 0.2$ deviation is $\sim 2 - 3\%$:(
  - E.g. for CDF ($\sim 5\%$ accuracy) at $\delta \sim 0.2$ deviation is $\sim 2 - 3\%$:)
- There could be models which **incorporates** power corrections to factorization into NP-behavior of TMDs
  - The result of extraction is not a TMD distribution (although it could perfectly describe the data), e.g. it violates universality
  - Anyway, at some moment TMD factorization fails (→ next slide)



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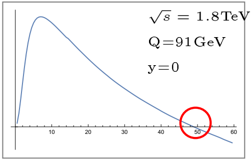
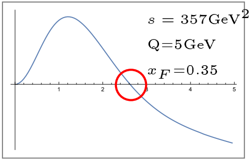
The cross-section with LP TMD factorization eventually became negative.
It happens at large q_T

Tevatron Z-boson

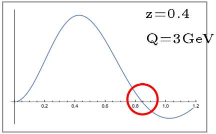


COMPASS πDY

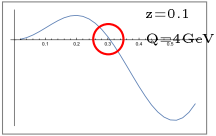


$$\frac{q_T}{Q} \sim 0.5 - 0.7$$

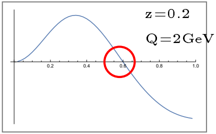
HERMES SIDIS



COMPASS SIDIS



JLab SIDIS



- The position of node depends also on process and x
- For large-Q bins the node can go down to $q_T/Q \sim 0.35$ (πDY at COMPASS!)

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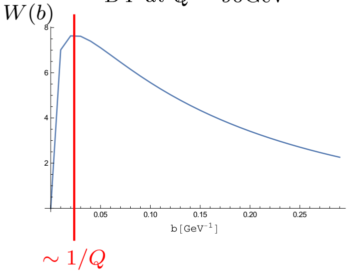
$$d\sigma \simeq \int d^2 b e^{-ibq_T} W(b) \simeq \int dbb\, J_0(bq_T) W(b)$$

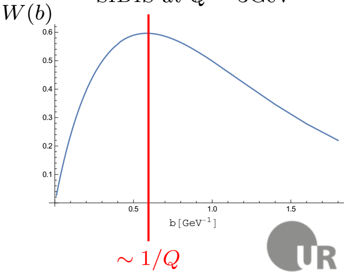
When 2D Fourier is positive definite?

- 1D cos-transformation → Bochner's theorem → "non-growing function"
- Generally, it is a complicated question see e.g.[Giraud,Peschanski,1405.3155]
- The first requirement(but not sufficient): $W(b)$ has maximum at $b = 0$.

DY at $Q \sim 90$GeV



SIDIS at $Q \sim 3$GeV



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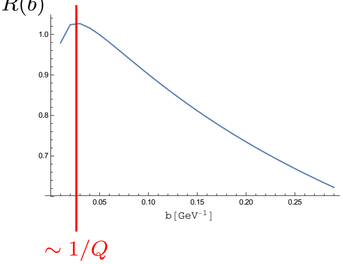
This fall down is (mainly) due to TMD perturbative evolution !

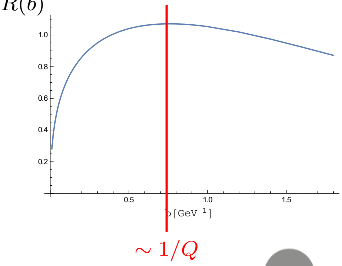
$$W(b,Q;x,z) \simeq R(b,Q)^2 F(x,b)D(z,b)$$

- In ζ-prescription F and D are (almost) monotonous functions
- $R(b,Q) = \exp(-\mathcal{D}(b,Q)\ln(Q^2/\zeta_Q(b))$

DY at $Q \sim 90$GeV



SIDIS at $Q \sim 3$GeV



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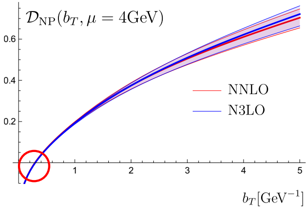
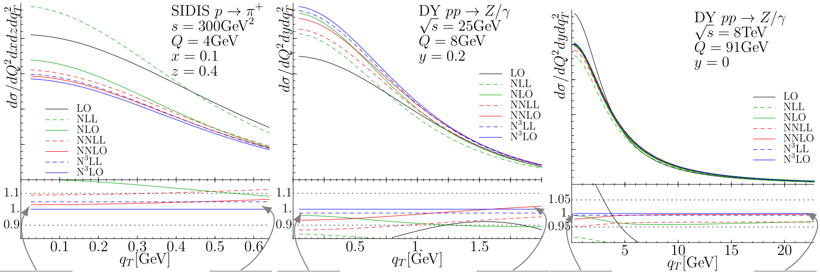


Perturbation theory = predictive power



~ 3%	~ 5%	~ 7%	~ 2%	~ 2%	< 1%
~ 16%	~ 8%	~ 3%	~ 10%	~ 2%	~ 3%

Difference between NNLO and N^3LO is not that important

Difference between NLO and NNLO is important (**especially at low-energy!**)

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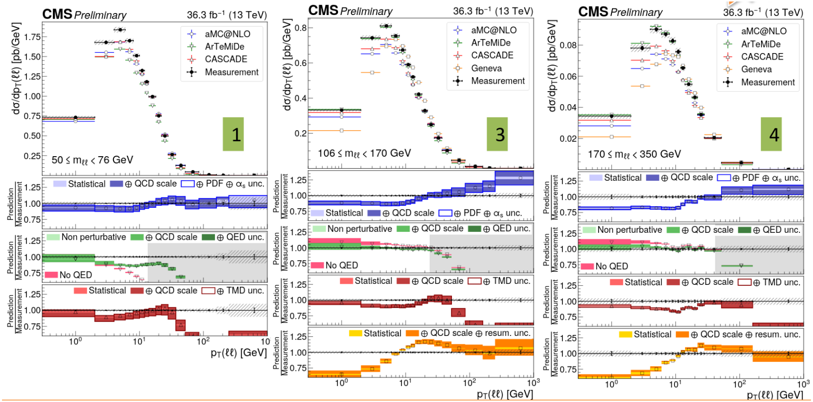
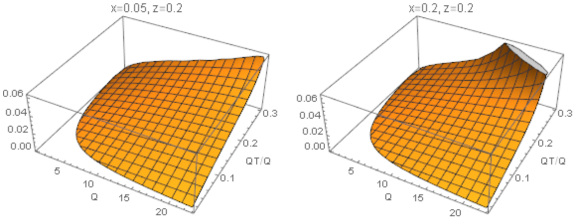
Perturbation theory = predictive power

B.Bilin,DIS2021

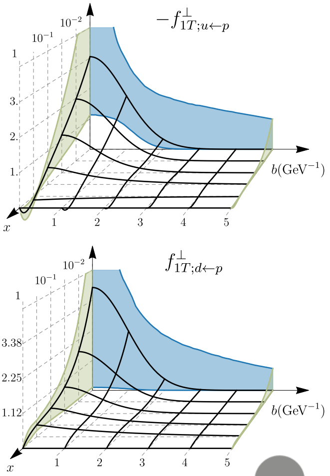


$$\text{asymmetry} \simeq \frac{\text{something}}{F_{UU}}$$

- Eventually $F_{UU} = 0$ (in TMD factorization)



Dataset name	Ref.	Reaction	# Points
Compass08	[36]	$d^{\uparrow}+\gamma^{*}\to\pi^{+}$	1 / 9
		$d^{\uparrow}+\gamma^{*}\to\pi^{-}$	1 / 9
		$d^{\uparrow}+\gamma^{*}\to K^{+}$	1 / 9
		$d^{\uparrow}+\gamma^{*}\to K^{-}$	1 / 9
Compass16	[39]	$p^{\uparrow}+\gamma^{*}\to h^{+}$	5 / 40
		$p^{\uparrow}+\gamma^{*}\to h^{-}$	5 / 40
Hermes	[35]	$p^{\uparrow}+\gamma^{*}\to\pi^{+}$	11 / 64
		$p^{\uparrow}+\gamma^{*}\to\pi^{-}$	11 / 64
		$p^{\uparrow}+\gamma^{*}\to K^{+}$	12 / 64
		$p^{\uparrow}+\gamma^{*}\to K^{-}$	12 / 64
JLab	[41, 42]	$p^{\uparrow}+\gamma^{*}\to\pi^{+}$	1 / 4
		$p^{\uparrow}+\gamma^{*}\to\pi^{-}$	1 / 4
		$p^{\uparrow}+\gamma^{*}\to K^{+}$	1 / 4
		$p^{\uparrow}+\gamma^{*}\to K^{-}$	0 / 4
SIDIS total			63
CompassDY	[40]	$\pi^{-}+d^{\uparrow}\to\gamma^{*}$	2 / 3
Star.W+	[43]	$p^{\uparrow}+p\to W^{+}$	5 / 5
Star.W-		$p^{\uparrow}+p\to W^{-}$	5 / 5
Star.Z		$p^{\uparrow}+p\to\gamma^{*}/Z$	1 / 1
DY total			13
Total			76



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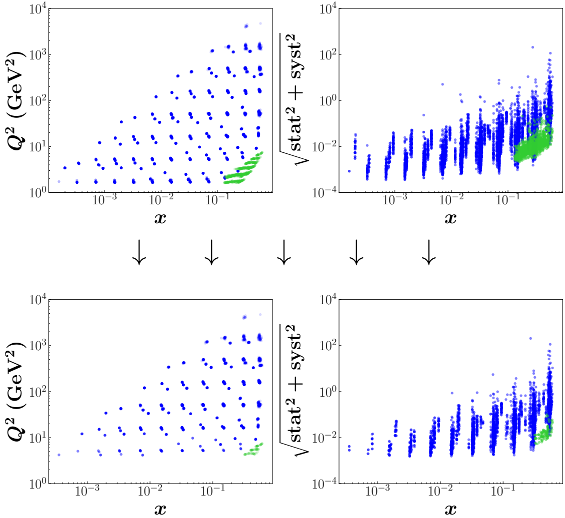
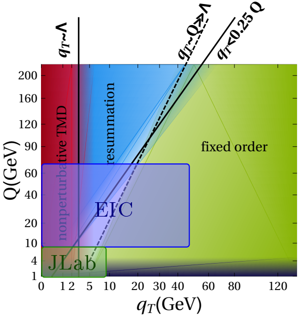
plots by A.Prokudin



Conclusion for part 1

- **There is a natural limit of TMD factorization** $q_T < (0.2 - 0.3)Q$
  - This limit is required from by theory
  - This limit is also seen in the data
- Pushing this limit higher does not help practically
  - At $q_T \sim 0.5Q$ cross-section become negative
  - It is pure perturbative effect
- Ways out:
  - Interpolate to fix order (works only at large Q)
  - Introduce $b_{\min}$
  - Go to power corrections
  - ...
- **A lot of stuff to explore** especially at lower energy JLab → EIC



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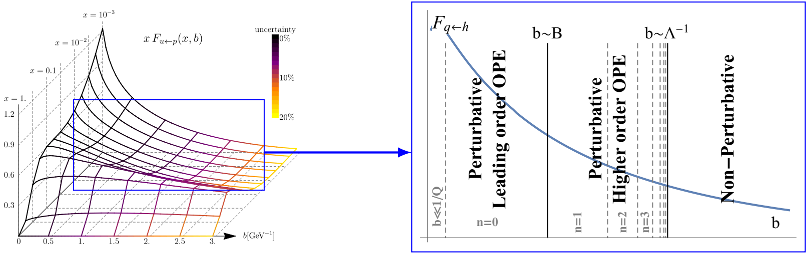
Part II
PDF-bias

TMD distributions are independent 3D function for each flavor

TMDPDF $= F(x,b)$, TMDFF$= D(z,b)$

Too much freedom!



$$F(x,b) = \left[q(x) + \alpha_s \left(p(x)\ln(b^2\mu^2) + ...\right) + \alpha_s^2 ...\right] + b^2 ... + ...$$

Lead.power OPE
up N^{3}LO

Higher power OPE
e.g. [V.Moos,AV,2008.01744]

$$F(x,b) = C(x,b) \otimes q(x) f_{NP}(x,b)$$

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refs. are defined in [V.Moos,AV,2008.01744]

Name	Function	Twist of leading matching	Twist-2 distributions in matching	Twist-3 distributions in matching	Order of leading power coef.function	Ref.
unpolarized	$f_1(x,b)$	tw-2	$f_1(x)$	–	N^{3}LO (α_s^3)	[21, 22]
Sivers	$f_{1T}^{\perp}(x,b)$	tw-3	–	$T(-x,0,x)$	NLO (α_s^1)	[23]
helicity	$g_{1L}(x,b)$	tw-2	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s^1)	[16, 17]
worm-gear T	$g_{1T}(x,b)$	tw-2/3	$g_1(x)$	$\mathcal{T}_g(x)$	LO (α_s^0)	[13, 14]
transversity	$h_1(x,b)$	tw-2	$h_1(x)$	$\mathcal{T}_h(x)$	NNLO (α_s^2)	[19]
Boer-Mulders	$h_1^{\perp}(x,b)$	tw-3	–	$\delta T_\epsilon(-x,0,x)$	LO (α_s^0)	[14]
worm-gear L	$h_{1L}^{\perp}(x,b)$	tw-2/3	$h_1(x)$	$\mathcal{T}_h(x)$	LO (α_s^0)	[13, 14]
pretzelosity	$h_{1T}^{\perp}$	tw-3/4	–	$\mathcal{T}_h(x)$	LO (α_s^0)	eq.(4.8)

- Twist-2 and twist-3 contributions at all powers of b^2 (tree)
- Typical expression (here for Sivers function):

$$f_{1T}^{\perp}(x,b) = \pm\pi\Big\{T_q(x) + \sum_{n=1}^{\infty}\left(\frac{x^2b^2M^2}{4}\right)^n \int_0^1 du \int dy \frac{\delta(x-uy)}{(n+1)!(n-1)!}\left(\frac{\bar{u}}{u}\right)^n \frac{1+(n-1)u+u^2}{1-u} T_q(y)\Big\}$$

  - $T(x) = T(-x,0,x)$ is Qiu-Sterman function
  - $\text{TMDs}_{proton} \sim \text{TMDs}_{nuclei}$
- Non-trivial matching for pretzelosity
  - Leading term: $h_{1T}^{\perp}(x,b) = -x^2 \int_x^1 \frac{du}{u}\frac{1-u^2}{u}\mathcal{T}_h\left(\frac{x}{u}\right) + \text{tw-4}$

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Matching is essential part on nowadays TMD phenomenology

- The region $5\text{GeV} \lesssim q_T < 0.25Q$ is accurately described by $f_{NP} \sim 1$
  - LHC, Tevatron, RHIC, ($\to$ EIC)
- It is observed that $q(x)$ carries the most part of x-dependence. I.e. $f_{NP}(x,b) \sim f_{NP}(b)$
  - Greatly reduces the parametric freedom
- It is observed that $q_f(x)$ carries the most part of the flavor dependence, i.e. $f_{NP}(x,b) \sim$ flavor-independent
- In fact, the simplest model $F_f(x,b) \sim C \otimes q_f(x) f_{NP}(b)$ capable to describe the most part of the data rather accurately

Matching to PDF leads to high predictive power, but is also a pitfall $\to$

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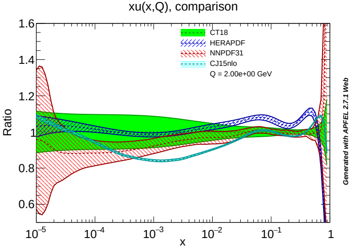

Result of a TMD fit is 100% dependent on PDF in use!

- Different PDF set are different
  - Especially in a "TMD-important" region $x \sim 0.1 - 0.5$
  - Different flavor decomposition
- As the result:

PDF & FF sets	χ^2/N_{pt}
HERA20 & DSS	0.76
HERA20 & JAM19	0.93
NNPDF31 & DSS	1.00
NNPDF31 & JAM19	1.65
HERA20 & DSS (N^3LO)	0.88
NNPDF31 & DSS (N^3LO)	1.31

SIDIS+DY fit [SV19]

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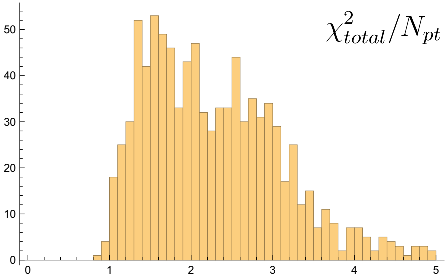
Obviously, one must include PDF uncertainty into the fit

Including PDF uncertainty "straightforwardly"

- PDF uncertainty is **larger** than the experimental precision
  - LHC 5-7% vs. 1%
  - Low energy DY 10-50% vs. 10%
  - SIDIS 10-50% vs.
- TMD physics (in comparison to DIS) is sensitive to different x-domain
- **Strongly depends on the set**



$\leftarrow$ SV19 fit
NNPDF+DSS
SIDIS+DY
$N_{pt} = 1039$
fit made for central replica
$\leftarrow$ distribution of χ^2 for
1000 replicas of NNPDF

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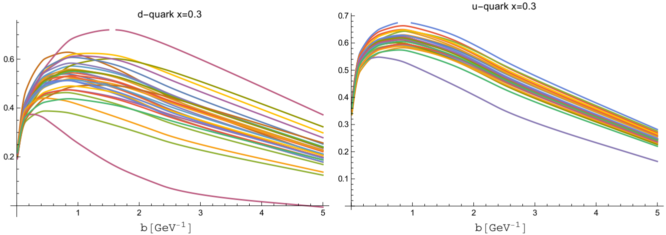
PDF essentially changes behavior of TMD = **PDF-bias**



SV19 fit, 40 random replicas of NNPDF3.1

In fact, each PDF replica must be equipped by its own f_{NP}
It will partially compensate PDF-bias
So, together they form a TMD distribution

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Inclusion of PDF uncertainty into TMD fit (work in progress)

Computationally intensive work

- Represent PDF uncertainty as MC replicas (1000 replicas)
- Make a fit of TMD distribution, based on each replica

One could expect that result would be less dependent on PDF

Inclusion of PDF uncertainty into TMD fit (work in progress)

Computationally intensive work

- Represent PDF uncertainty as MC replicas (1000 replicas)
- Make a fit of TMD distribution, based on each replica

One could expect that result would be less dependent on PDF

NO

We have observed that “simple” (5 params!) model from SV19 does not fit all PDF sets equally well

Reason: absence of flavor dependence

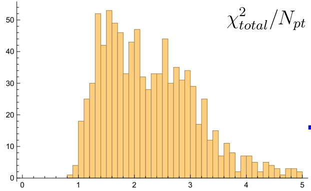
Solution: add flavor dependence $f_{NP} \to f_{NP}^{u,d,\bar{u},\bar{d},rest}$

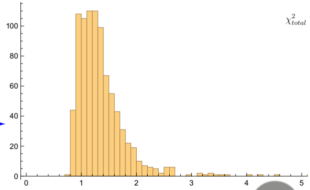
DY only (457 points)
SV19 model

input PDF .	χ^2/N_{pt}
HERA20	0.97
NNPDF31	1.14
CT18	1.26
MSHT20	1.39

DY only (457 points)
flavor-dependent model

input PDF .	χ^2/N_{pt}
HERA20	0.90
NNPDF31	0.97
CT18	0.98
MSHT20	0.89





Plot for NNPDF3.1, but similar picture for other PDFs

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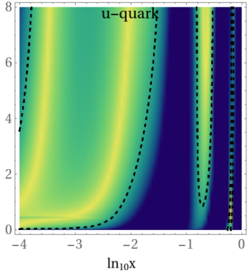
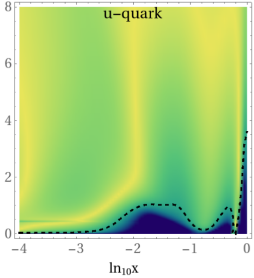
Now, TMDs based on different PDFs are in agreement

Intersection of TMDs based on HERA and CT18 within uncertanties

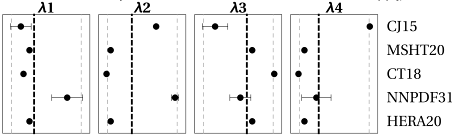
SV19 model



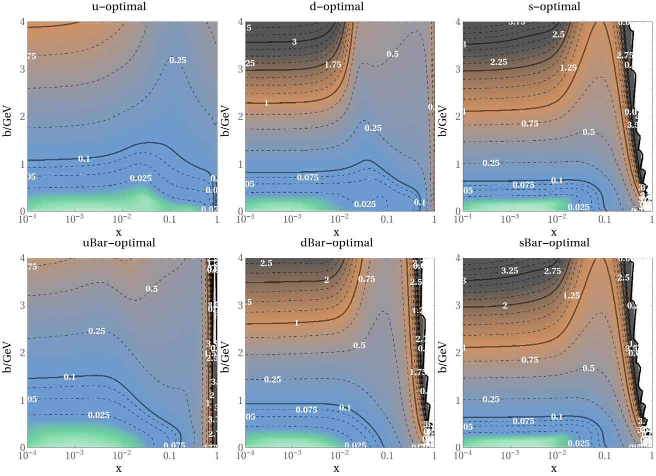
flavor-dependent model



Meanwhile the NP parameters of model strongly distributed



HERA20 model4.0 uncertanty in TMD



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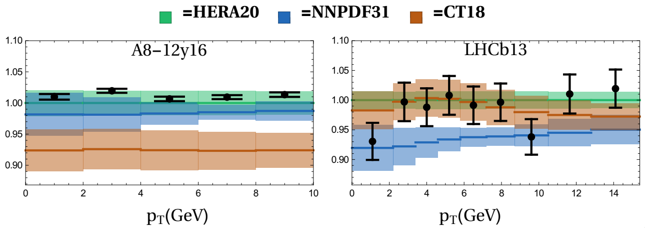
Conclusion for part 2

- **Matching of TMD to PDF is important!**
- TMD totally dependent on the PDF in use
  - There is no agreement between PDF sets
  - Uncertainty in PDF lead to crazy TMDs → fit each PDF replica
- To compensate PDF-bias one needs flavor dependence
  - Results for different PDFs are in agreement
  - TMDs are in agreement
  - Uncertainty on TMD is much larger
- **Work in progress**
- Future: one needs joint fit of PDF + TMD



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Part III
Decorrelation of TMD evolution

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TMD evolution depends on non-perturbative CS-kernel

$$\mathcal{D} = -\frac{1}{2}K = \frac{1}{2}F_{q\bar{q}} = -\frac{1}{2}\gamma_\nu^{f_\perp}$$

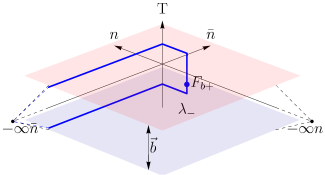
$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(b,\mu) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

CS is fundamental QCD function

$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\mathrm{Tr}\int_0^1 d\beta \langle 0|F_{b+}(-\lambda_- n + b\beta) W_{C'}|0\rangle}{\mathrm{Tr}\langle 0|W_{C'}|0\rangle} + Z_{\mathcal{D}}(\mu)$$

- Independent observable
- Measures QCD-vacuum



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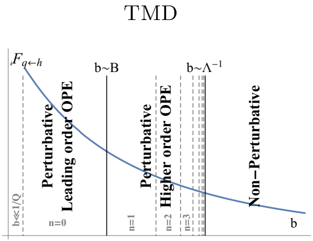
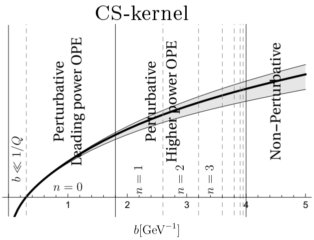
Thus, TMD distributions are functionals of CS-kernel

$$f_1[\mathcal{D}](x, b; \mu, \zeta)$$

- Is it a problem? YES, because we extract simultaneously $\mathcal{D}$ and TMDs
  - Extraction is not universal!
    - E.g. one cannot use $\mathcal{D}$ from lattice, together with TMDs from pheno.
  - In principle, very large/broad pull of data will reduce correlation
  - Problem of comparison/interpretation of result
- One makes situation worse by splitting to perturbative and NP parts
  - Keep CS-kernel a whole function





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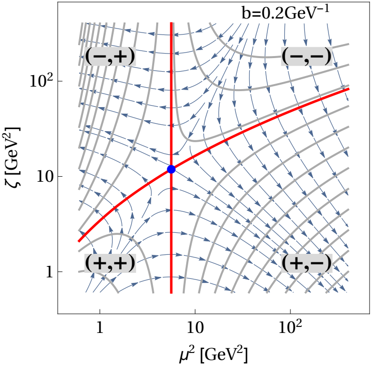
There are several solutions for this problem
my-preferred is ζ-prescription



- In a nutshell: define TMD at $\zeta(b, \mu)$
  - $\zeta(b, \mu)$ = equi-evolution line (NP!)
- Equivalent to fixed-point definition
  - TMDs on the same equi-evolution line are the same (by definition!)
- Generally: does not matter which line use as reference
- But there is one very special line = which passes though the saddle point
  - TMD on this line = optimal TMD

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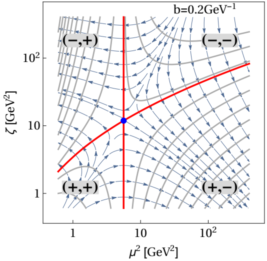
Why optimal TMD is optimal?

1. At saddle point $\mathcal{D} = 0$

$$(\text{optimal})f_1(x,b) = f_1[\mathcal{D}=0](x,b;(\mu,\zeta)_{\text{saddle}})$$

2. Optimal equi-potential line is continuous (important for small-b matching)
3. Greatly simplifies all equations



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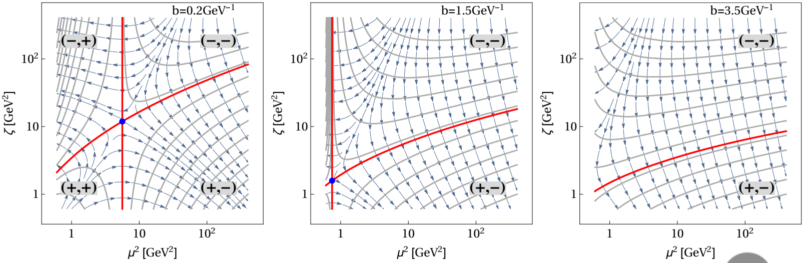
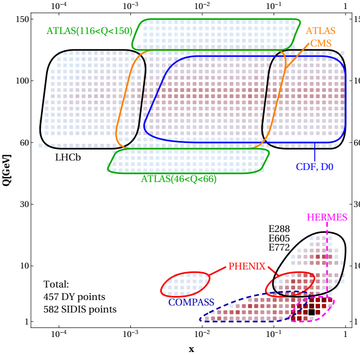
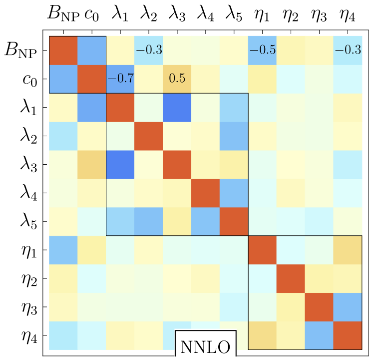
Everything is nonperturbative!
Position of saddle-point, ζ-line,....
Solve all equations in terms of $\mathcal{D}$!

- At large-b saddle-point goes below Λ_{QCD}.
  - Not possible to build perturbative-like solution
  - But there is an **exact** solution! (see [Scimemi,AV,1912.06532,app.C])

$$2\mathcal{D} + 2\beta(a_s)\frac{\partial g(a_s,\mathcal{D})}{\partial a_s} - \Gamma_{\text{cusp}}(a_s)\frac{\partial g(a_s,\mathcal{D})}{\partial \mathcal{D}} + \gamma_V(a_s) = 0. \quad , \quad g(a_s,0)=0$$

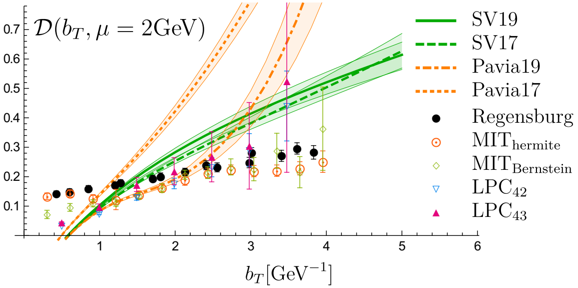



ATLAS(116<Q<150)
ATLAS
CMS
LHCb
CDF, D0
ATLAS(46<Q<66)
HERMES
E288
E605
E772
PHENIX
COMPASS
Total:
457 DY points
582 SIDIS points
Q[GeV]
x


B_{NP} c_0 λ_1 λ_2 λ_3 λ_4 λ_5 η_1 η_2 η_3 η_4
−0.3
−0.5
−0.3
−0.7
0.5
NNLO

Conclusion for part 3

- Independent extraction of evolution and TMDs is cumbersome task
- **Keep CS-kernel as a whole function!**
- Fixed-scale schemes are preferable
  - ζ-prescription



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Conclusion

- Extraction of TMDs is a very peculiar task
  - Involves several NP functions
  - Requires strict data-cuts
  - Perturbative input is important
  - **Many open theoretical questions**
- JLab
  - There will be not much **pure** TMD-factorizable data
    - Large-x
  - Paradise to study power corrections
    - Higher-twist TMDs
    - Mass/kinematic corrections
    - Interesting and weakly studied field
    - current/next frontier of QCD
  - Going to be the challenge for theoreticians
- EIC
  - Will happen in 10+ years (I doubt that our understanding will remain the same)

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Always take into account uncertainties!