EXTRACTION OF TMD DISTRIBUTIONS FROM DATA

Alessandro Bacchetta

3DSPIN
MAPPING THE PROTON IN 3D

European Research Council

INFN Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

RESULTS OBTAINED WITH CONTRIBUTIONS FROM

Valerio Bertone

Chiara Bissolotti

Giuseppe Bozzi

Filippo Delcarro

Fulvio Piacenza

Cristian Pisano

Marco Radici

Andrea Signori

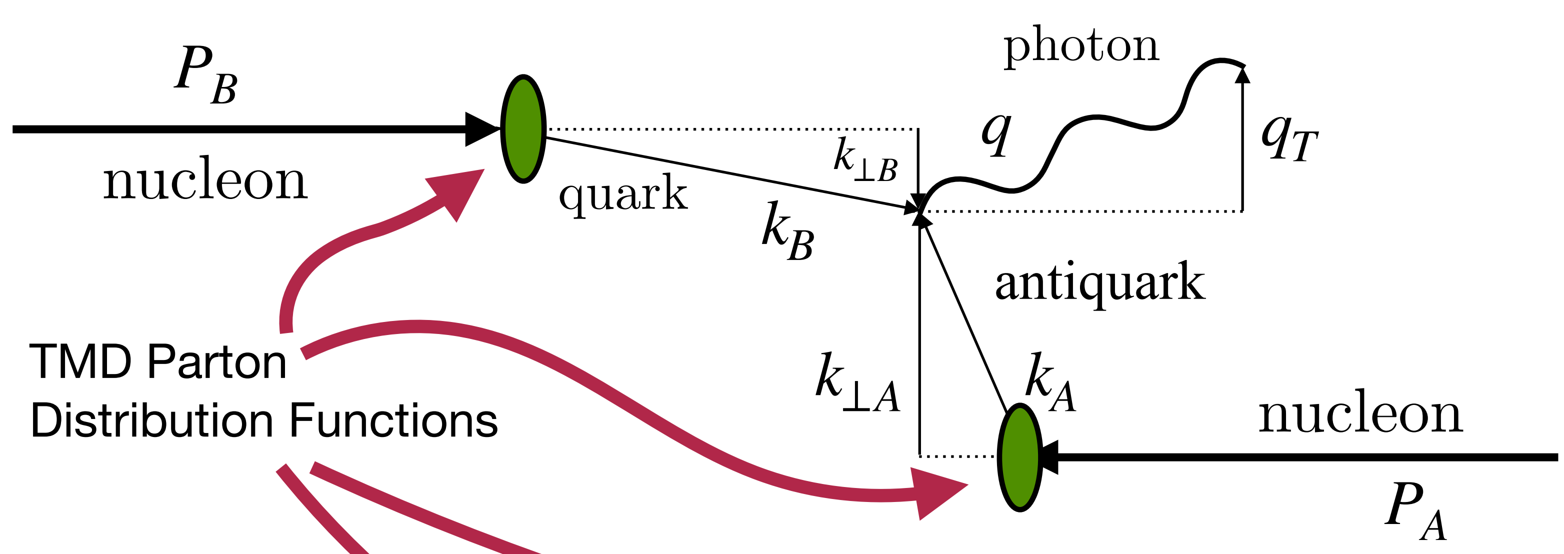
Matteo Cerutti

SOME IMPORTANT POINTS

- Perturbative accuracy
- Choice of data points
- Consistency polarized/unpolarized
- Positivity bounds

UNPOLARISED QUARK TMDS

TMDS IN DRELL-YAN PROCESSES



W term

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

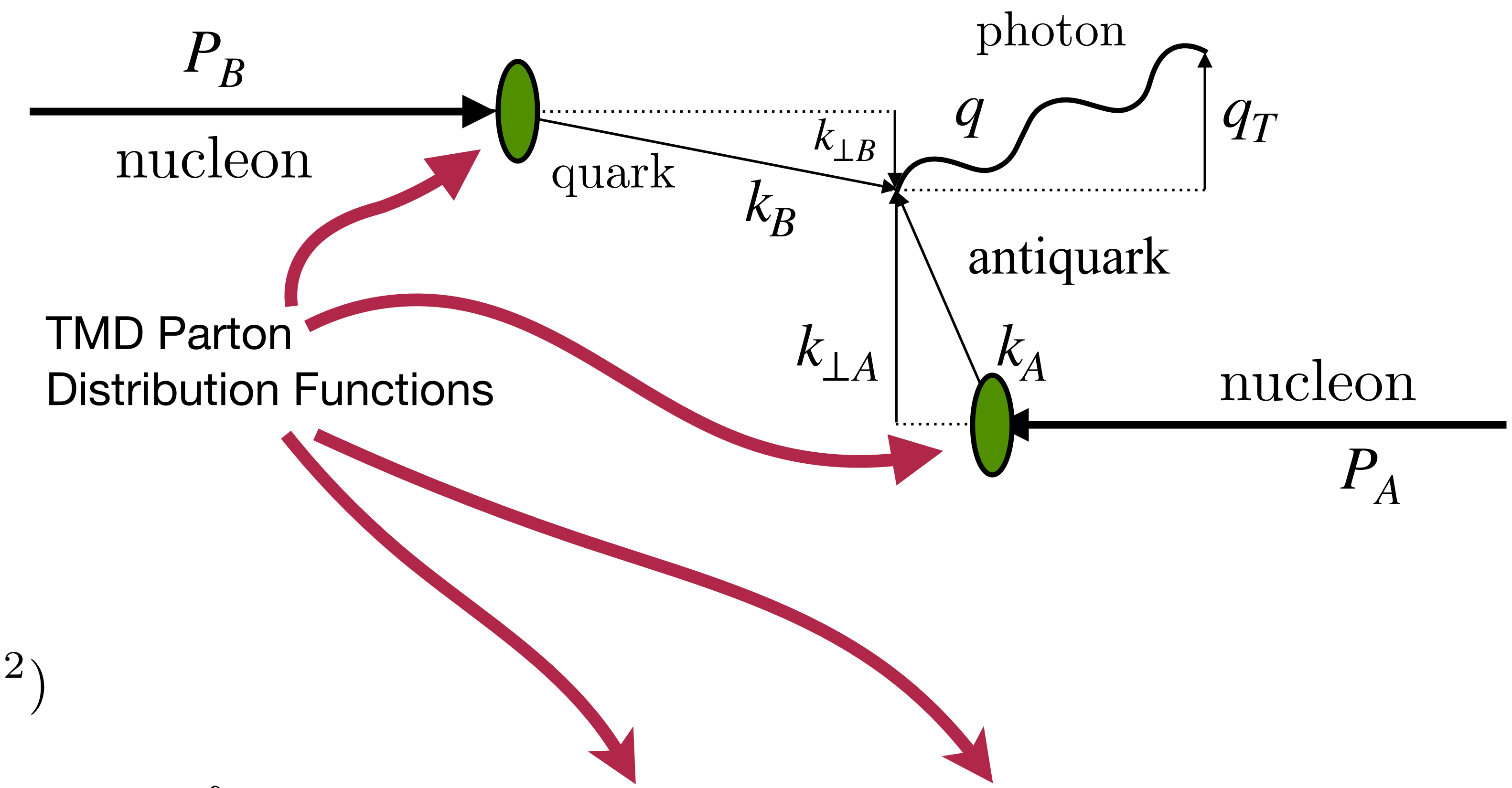
$$= \boxed{\sum_a \mathcal{H}^{1a}_{UU}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a\left(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2\right) f_1^{\bar{a}}\left(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2\right) \delta^{(2)}\left(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\right)}$$

$$+ Y^1_{UU}\left(Q^2, \boldsymbol{q}_T^2\right) + \mathcal{O}\left(M^2/Q^2\right)$$

The W term, dominates at low transverse momentum ($q_T \ll Q$)

So far, the Y term has been excluded in the Pavia analyses

TMDS IN DRELL-YAN PROCESSES



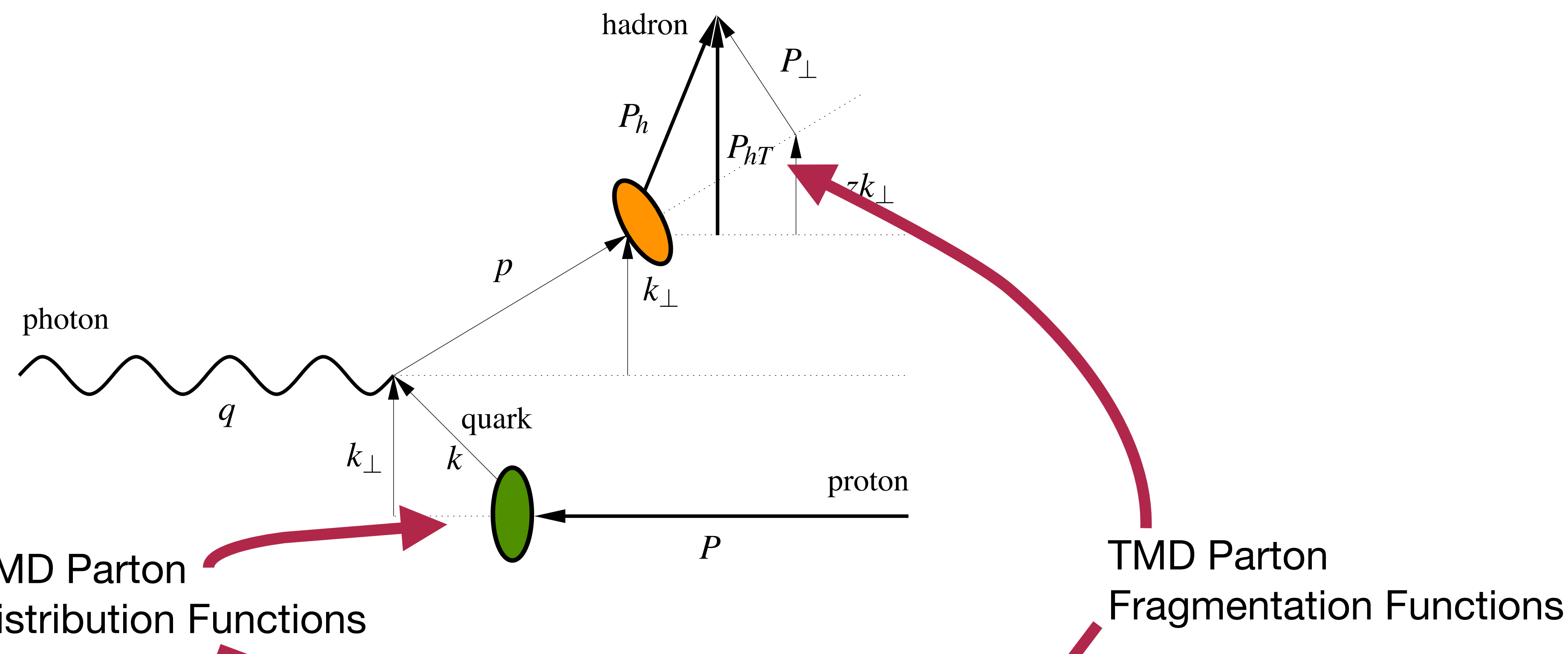
$$
\begin{aligned}
&F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2) \\
&\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2)\, f_1^{\bar{q}}(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}) \\
&= \sum_q \mathcal{H}_{UU}^{1q}(Q^2,\mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2)\, \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)
\end{aligned}
$$

At small transverse momentum, the dominant part is given by TMDs.

The analysis is usually done in Fourier-transformed space

TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS



$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)$$

$$= x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a\big(x,\boldsymbol{k}_\perp^2;\mu^2\big)\, D_1^{a\to h}\big(z,\boldsymbol{P}_\perp^2;\mu^2\big)\,\delta\big(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp\big)$$

$$= x\sum_a \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q\big(x,z^2 b_\perp^2;\mu^2\big)\,\hat{D}_1^{a\to h}\big(z,b_\perp^2;\mu^2\big)$$

hard factor

TMD STRUCTURE

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2\boldsymbol{k}_\perp e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} f_1^q(x, \boldsymbol{k}_\perp^2; \mu^2)$$

perturbative Sudakov form factor

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \big(C_{qi} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^q(x, b_T)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

collinear PDF

matching coefficients (perturbative)

nonperturbative part of evolution

nonperturbative part of TMD

see, e.g.,
Collins, "Foundations of Perturbative QCD" (11)

PERTURBATIVE ORDER OF EACH INGREDIENT

Order in powers of α_s

ingredients in perturbative Sudakov form factor

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evol.
LL		-	1	
NLL		1	2	
NNLL		2	3	
N^{3}LL		3	4	

LOGARITHMIC ACCURACY

	Sudakov form factor	matching coeff.
LL	$\alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right)$	C^0
NLL	$\alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$	C^0
NLL′	$\alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$	$\left(C^0 + \alpha_S C^1\right)$

the difference between the two is NNLL $\alpha_S^n \ln^{2n-2}\left(\frac{Q^2}{\mu_b^2}\right)$

PERTURBATIVE ORDER OF EACH INGREDIENT

Order in powers of α_S

hard factor and matching coefficients

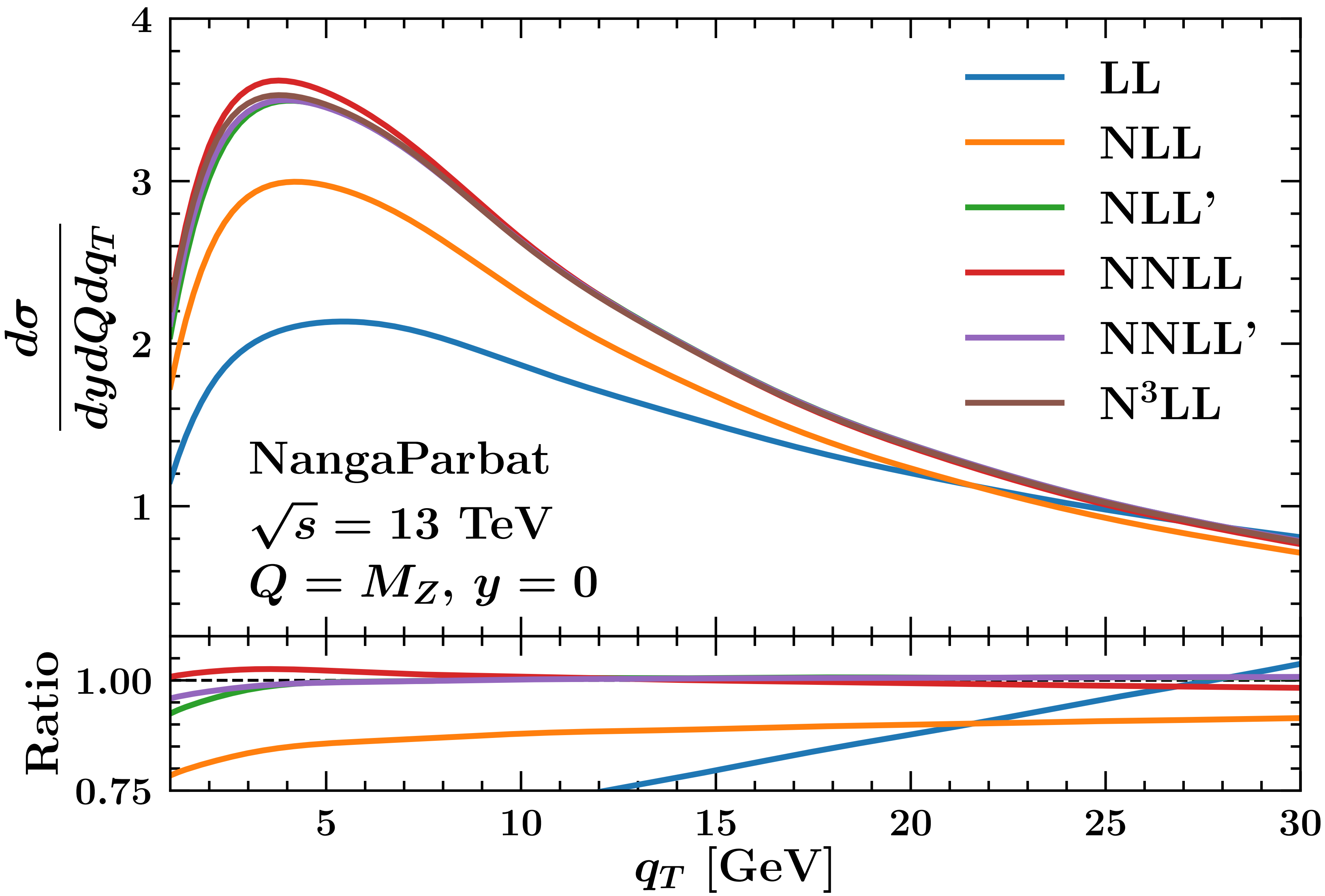
ingredients in perturbative Sudakov form factor

Accuracy	H and C	K and γ_F	γ_K	PDF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL’	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL’	2	2	3	NNLO
N^3LL	2	3	4	NNLO
N^3LL’	3	3	4	N^3LO

order α^3 matching coeff. only available since last year
Lou, Yang, Zhu, Zhu, arXiv:2012.03256

COMPARISON OF DIFFERENT ORDERS

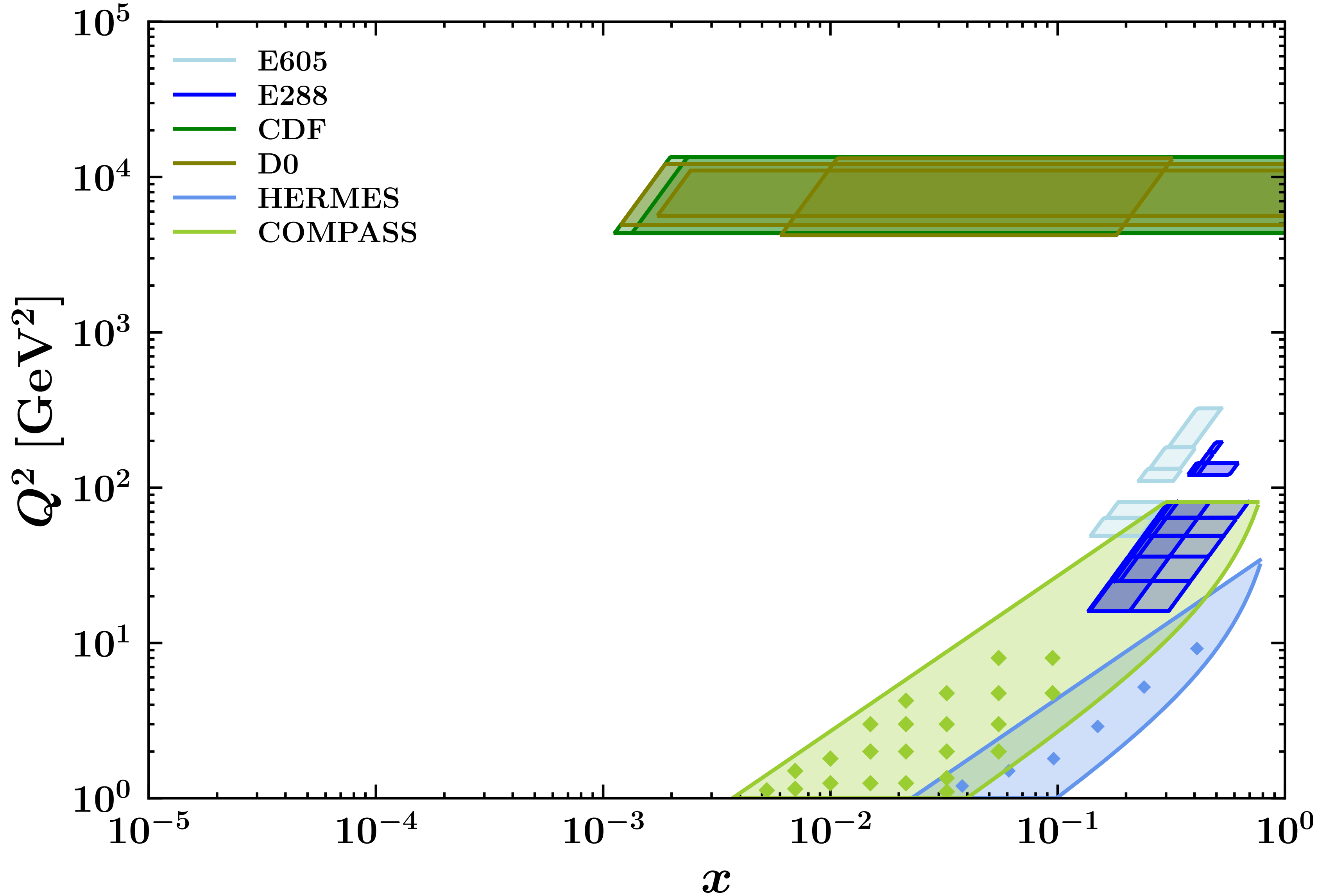
V. Bertone's talk at LHC EW WG General Meeting, Dec 2019
https://indico.cern.ch/event/849342/



RECENT TMD FITS OF UNPOLARIZED DATA

	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

x–Q² COVERAGE PV17



Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

DATA SELECTION IN PAVIA 2017

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5 \text{ GeV}$$

Total number of data points: 8059

Total χ^2/dof = 1.55

The TMD “eight-thousander” fit

Nanga Parbat, Kashmir, 8126 m

DATA SELECTION IN PAVIA 2017

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5 \text{ GeV}$$

Total number of data points: 8059

Total χ^2/dof = 1.55

We checked also

$$P_{hT} < \text{Min}[0.2\,Q, 0.5\,Qz] + 0.3\,\text{GeV}$$

Total number of data points: 3380

Total χ^2/dof = 0.96

$$P_{hT} < 0.2\,Qz$$

Total number of data points: 477

Total χ^2/dof = 1.02

PV17 – RESULTING TMDS

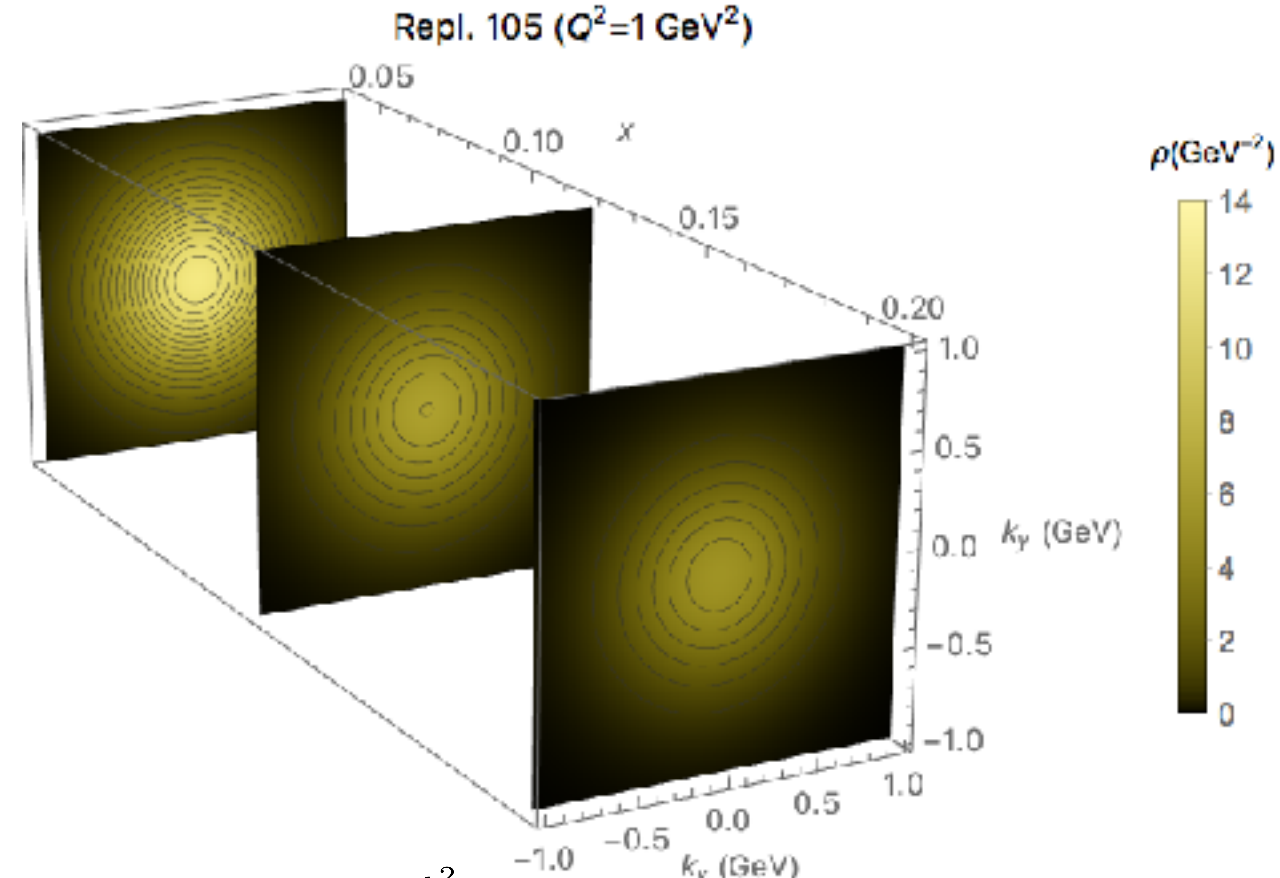
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in b_T space

plot in $k_\perp$ space

$$\hat{f}_{\rm NP}(x, b_T) = e^{-g_1(x)\frac{b_T^2}{4}} \left(1 - \frac{\lambda g_1^2(x)}{1 + \lambda g_1(x)} \frac{b_T^2}{4}\right)$$

- Guassian + weighted Gaussian
- nontrivial x dependence
- no flavor dependence



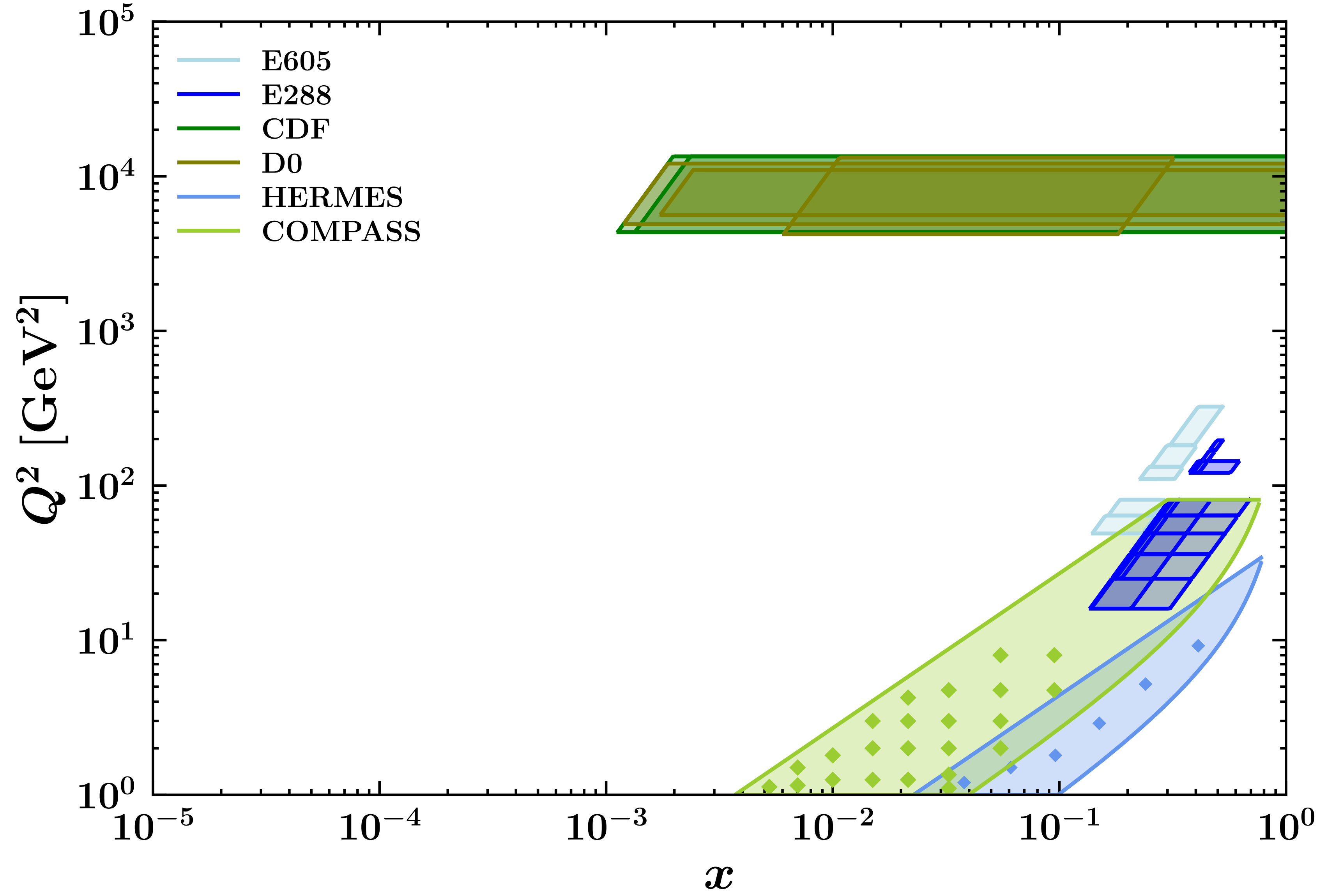
$$g_K(b_T) = -\frac{g_2}{2} b_T^2$$ Gaussian

$$\hat{D}_{\rm NP}(z, b_T) = \frac{g_3(z)\ e^{-g_3(z)\frac{b_T^2}{4z^2}} + (\lambda_F/z^2) g_4^2(z) \left(1 - g_4(z) \frac{b_T^2}{4z^2}\right)\ e^{-g_4^2(z)\frac{b_T^2}{4z^2}}}{z^2 \left(g_3(z) + (\lambda_F/z^2) g_4^2(z)\right)}$$

TMD Frag. Func.

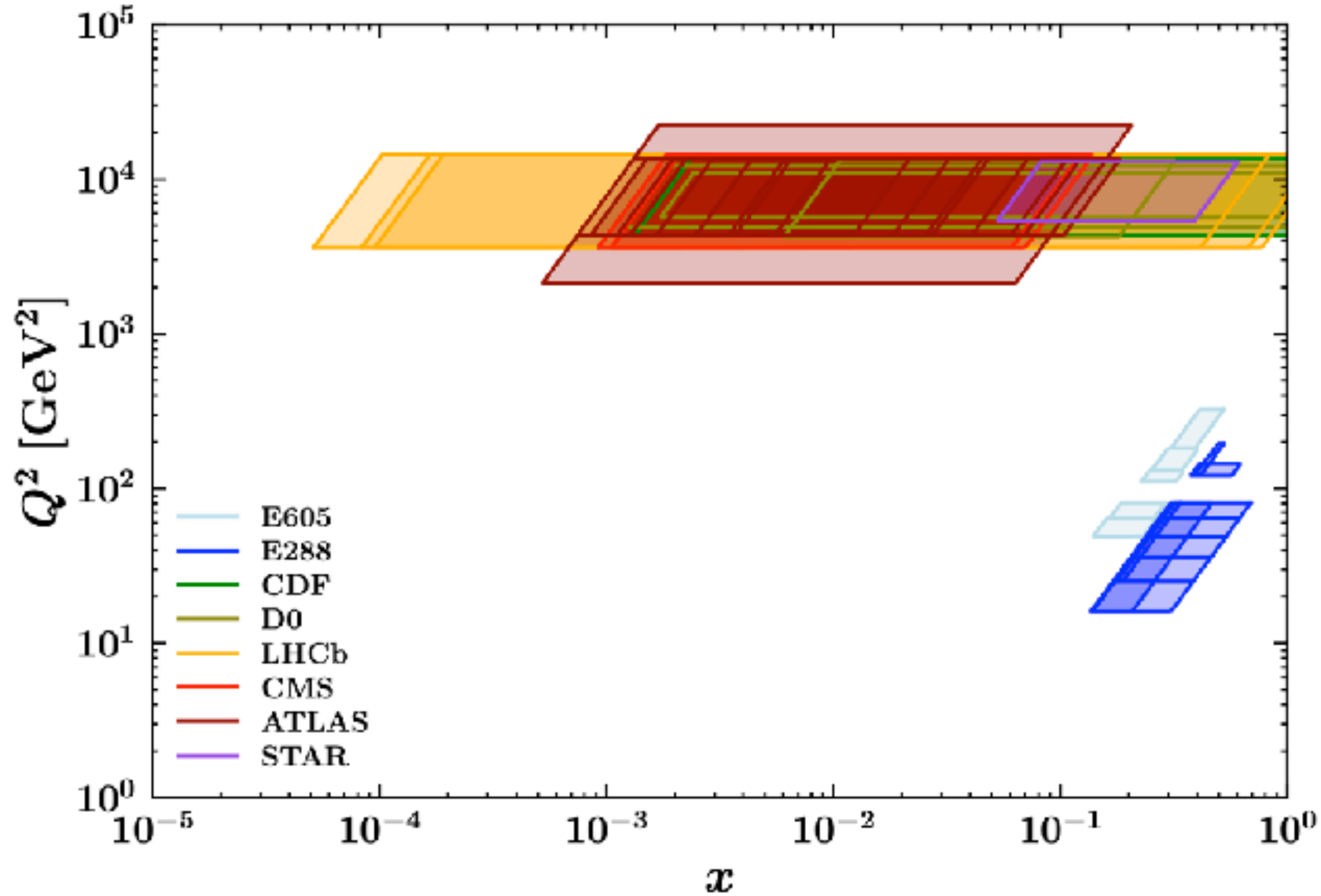
11 free parameters

x–Q² COVERAGE PV17



Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

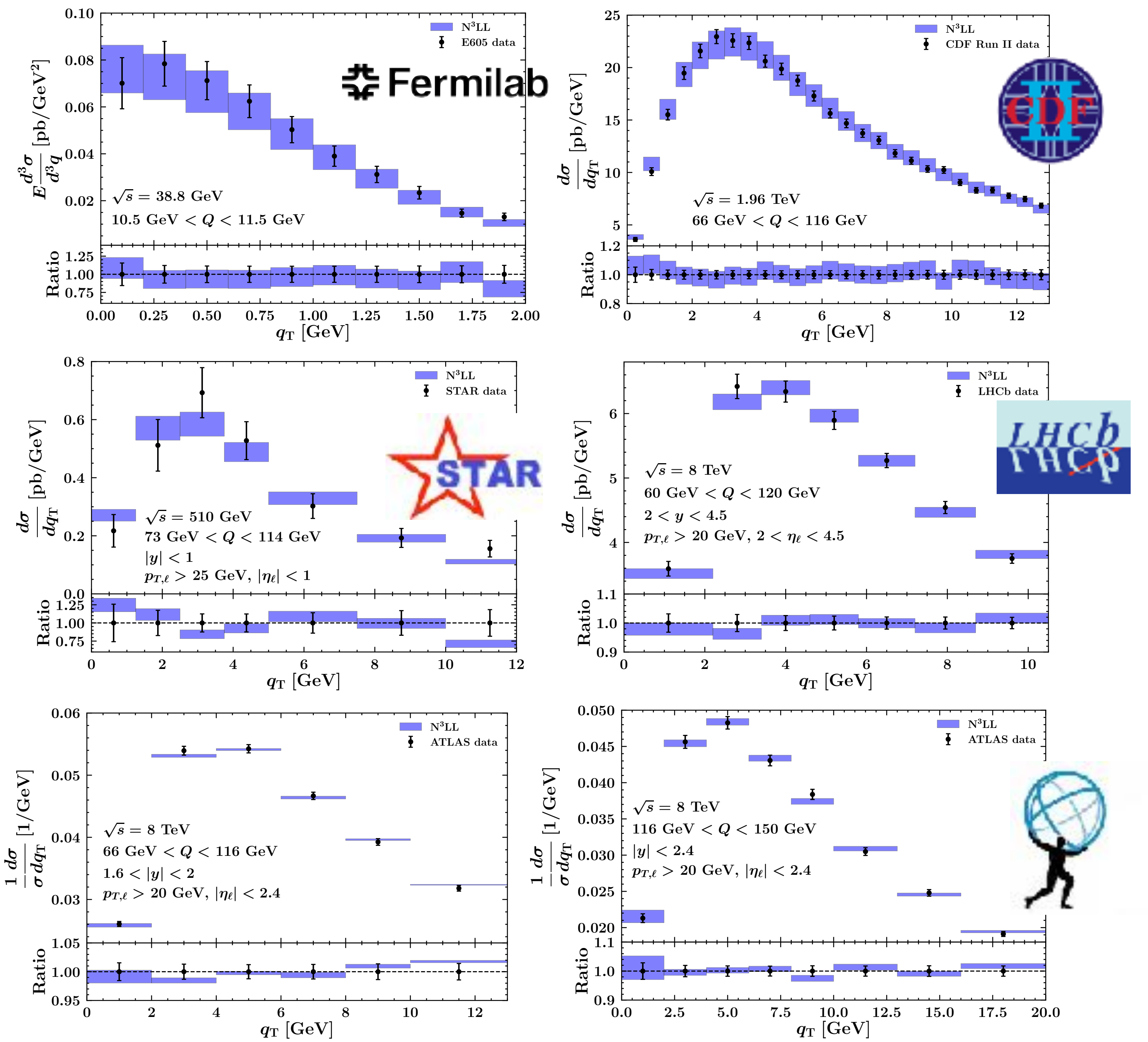
x–Q^2 COVERAGE PV19



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

THE PAVIA19 EXTRACTION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

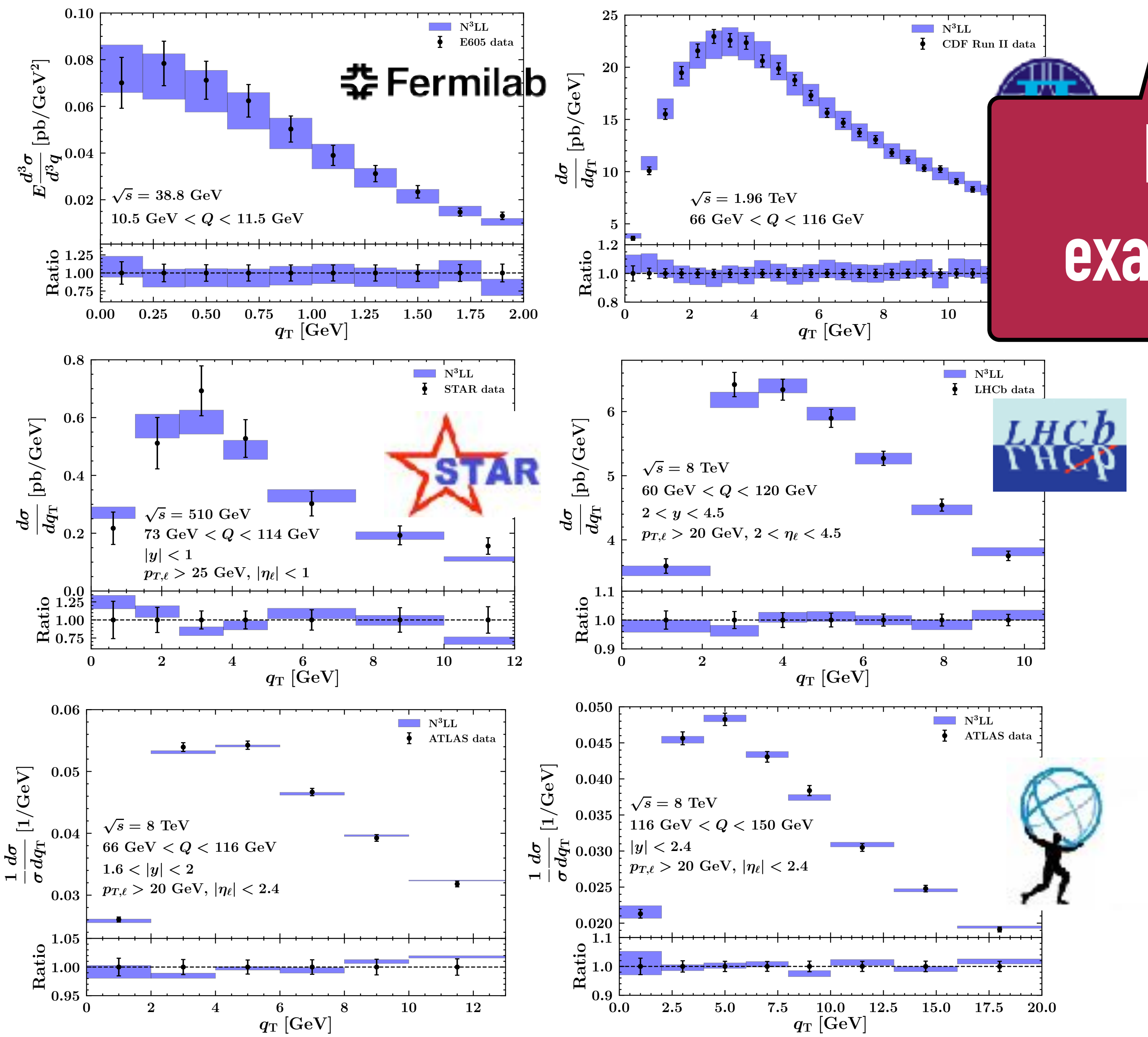


Data selection: $q_T/Q < 0.2$

Number of data points: 353

PV19 – DATA COMPARISION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



Pavia19: first DY fit at N³LL, exactly reproduces normalization

Data selection: $q_T/Q < 0.2$

Number of data points: 353

Global χ^2/dof = 1.02

The TMD "Sugarloaf" fit

Pão de Açucar (Sugarloaf Mountain), Brasil, 396 m

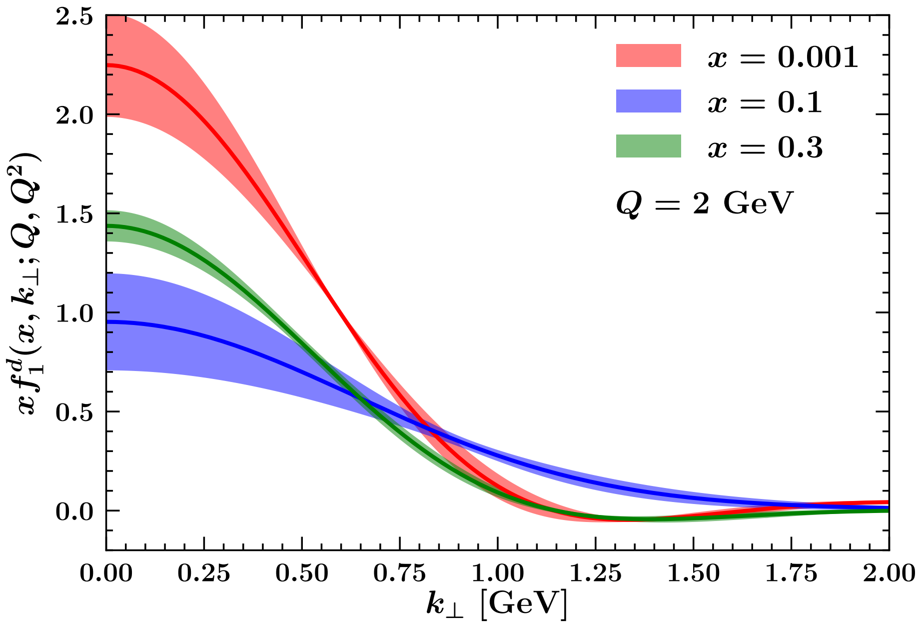
PV19 – RESULTING TMDS

expression in b_T space

$$f_{\rm NP}(x, b_T, \zeta) = \left[\frac{1-\lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x) \frac{b_T^2}{4} \right) \right]$$

$$\times \exp\left[-\left(g_2 + g_{2B} b_T^2\right) \ln\left(\frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right] ,$$

plot in $k_\perp$ space



- q-Guassian + Gaussian
- nontrivial x dependence
- no flavor dependence
- non-Gaussian nonperturbative TMD evolution

9 free parameters

X DEPENDENCE IN TMDS

PV17

$$g_1(x) = N_1 \frac{(1-x)^{\alpha}\ x^{\sigma}}{(1-\hat{x})^{\alpha}\ \hat{x}^{\sigma}}$$

PV19

$$g_1(x) \ = \ \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{x}{\alpha}\right)\right] \ ,$$

$$g_{1B}(x) \ = \ \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2} \ln^2\left(\frac{x}{\alpha_B}\right)\right] \ .$$

NANGA PARBAT: A PUBLIC PLATFORM FOR TMD STUDIES

https://github.com/MapCollaboration/NangaParbat

Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/vbertone/NangaParbat/releases

For the last development branch you can clone the master code:

```
git clone git@github.com:vbertone/NangaParbat.git
```

PROBLEMS WITH SIDIS NORMALIZATION

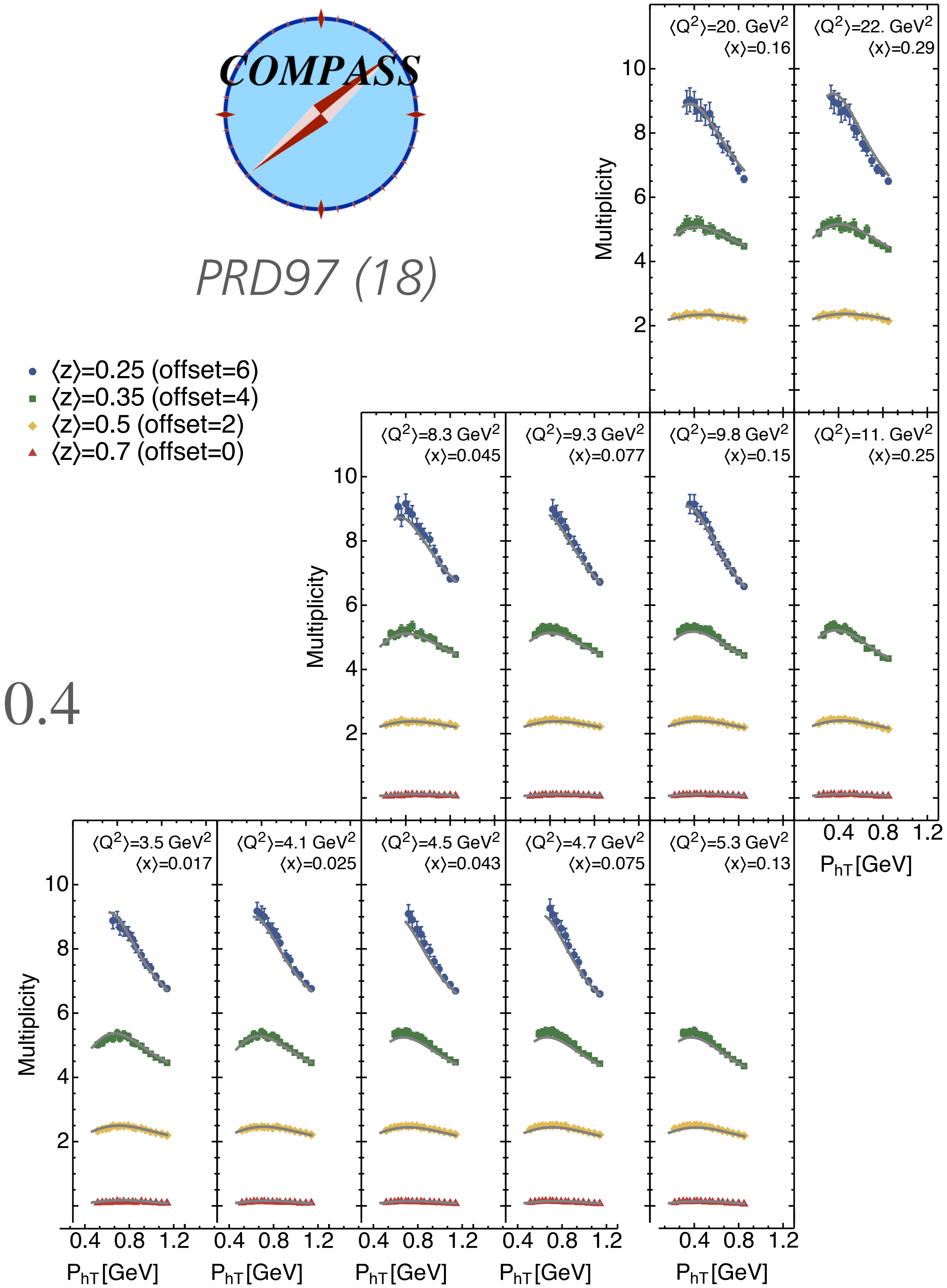
Comparing the PV17 extraction with the new COMPASS data, without normalization factors, at NLL the agreement is very good

PRD97 (18)

NLL $\qquad H = 1$

NLL’ $\qquad H = 1 + \frac{C_F}{\pi}\left(-4 + \frac{\pi^2}{12}\right)\alpha_S \approx 1 - 0.4$

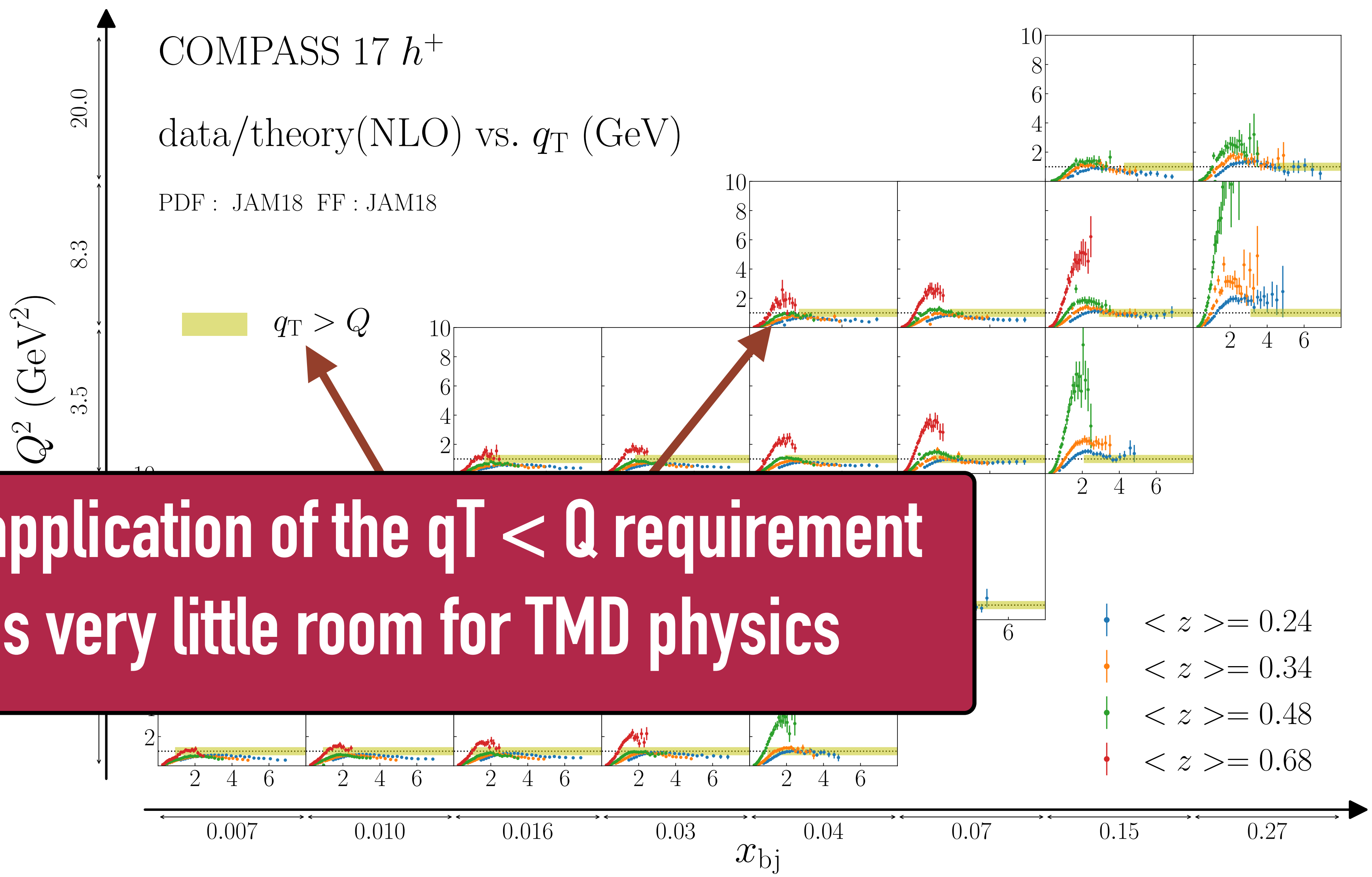
Going to NLL’ or NNLL the situation worsens!



from F. Piacenza’s PhD thesis

PROBLEMS WITH HIGH TRANSVERSE MOMENTUM

Gonzalez-Hernandez, Rogers, Sato, Wang arXiv:1808.04396



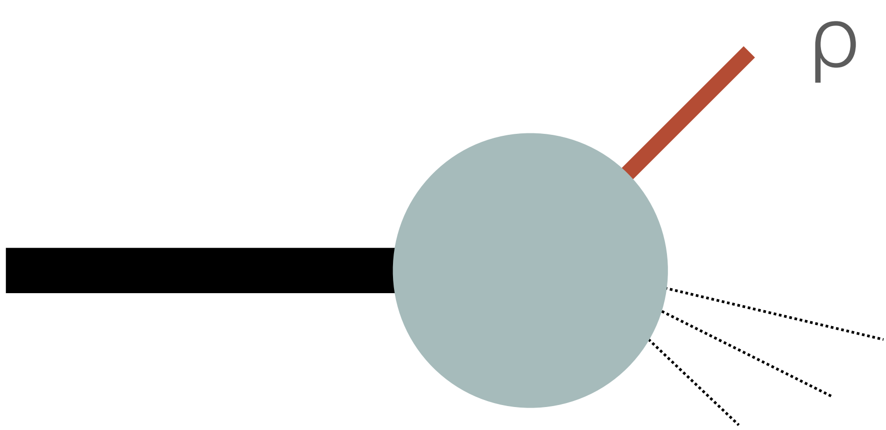
a strict application of the qT < Q requirement leaves very little room for TMD physics

My personal opinion is that we should be less strict

TMD REGIONS AND DECAYS

Harut's observation

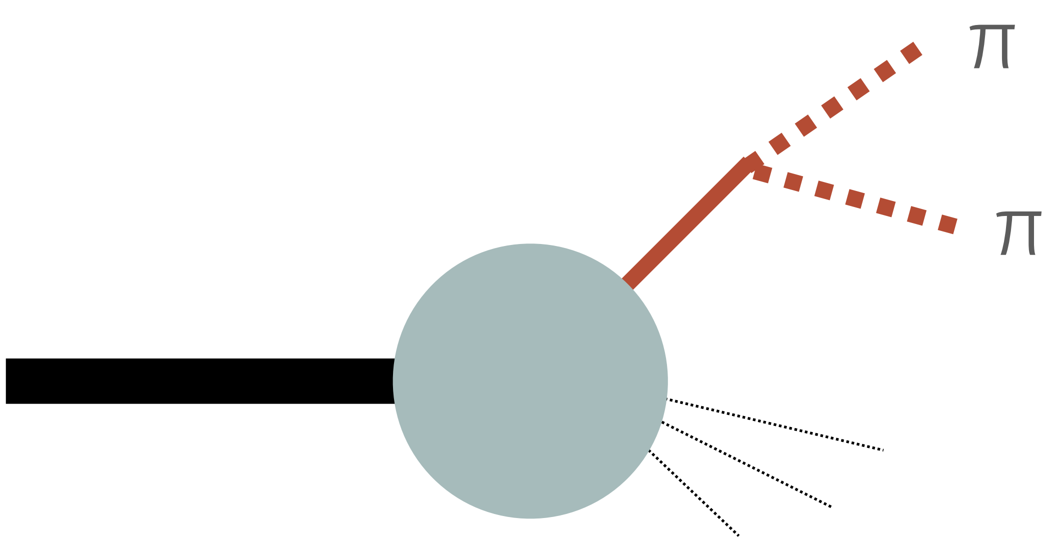
$q_T \leq 0.2\, Q$ $\quad$ $q_T = \dfrac{P_{h\perp}}{z}$ $\quad$ $Q = 10\ \text{GeV}$



$P_{\rho\perp} = 1\ \text{GeV}$

$z_\rho = 0.5$

ρ in TMD region



$P_{\pi\perp} = 0.7\ \text{GeV}$

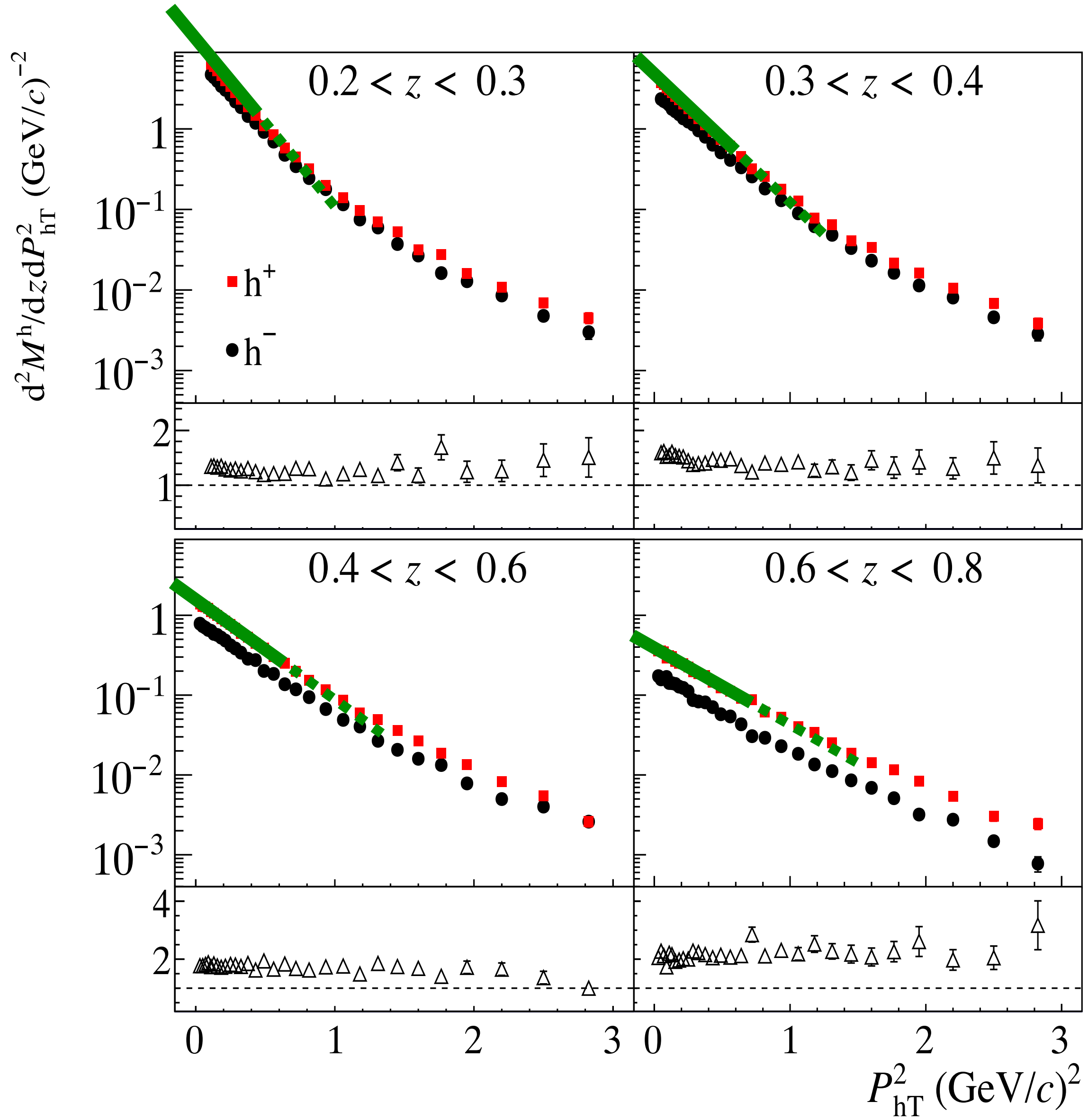
$z_\pi = 0.1$

decay π NOT in TMD region?

TMD REGIONS AND DATA

COMPASS Collab., arXiv:1709.07374

$\langle Q \rangle = 3.1$ GeV



It seems that the same "physics" is dominating at least for $0 \leq P_{hT} \leq 0.7$ GeV,

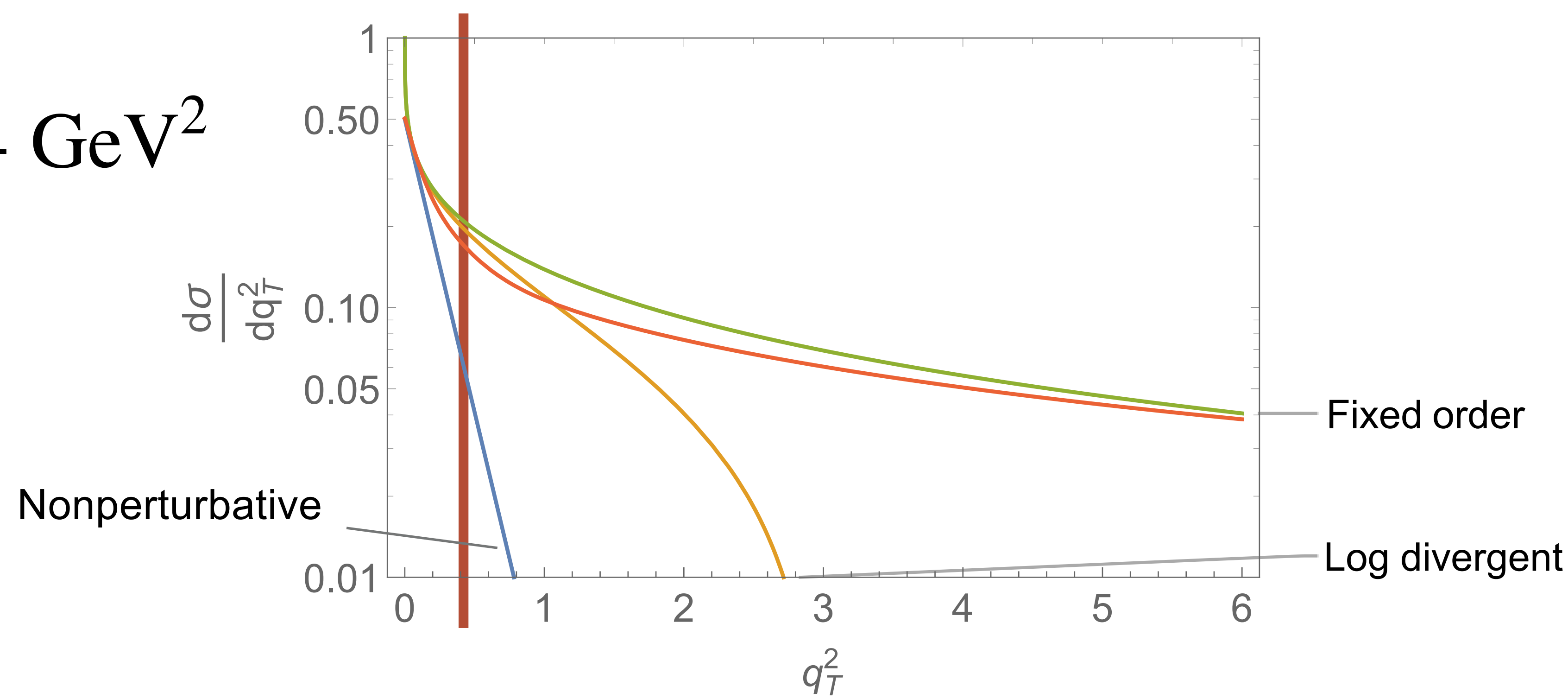
which means $0 \leq q_T \leq 2.8$ GeV in the lowest-z bin

TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

Perturbative approach:
TMD region = where the log divergence of the fixed-order calculation dominates (resummation is required)

Nonperturbative approach:
TMD region = where either the log divergence OR the nonperturbative contributions dominate

$$\langle Q^2 \rangle = 4\ \mathrm{GeV}^2$$



TMD region (ideal situation)

TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

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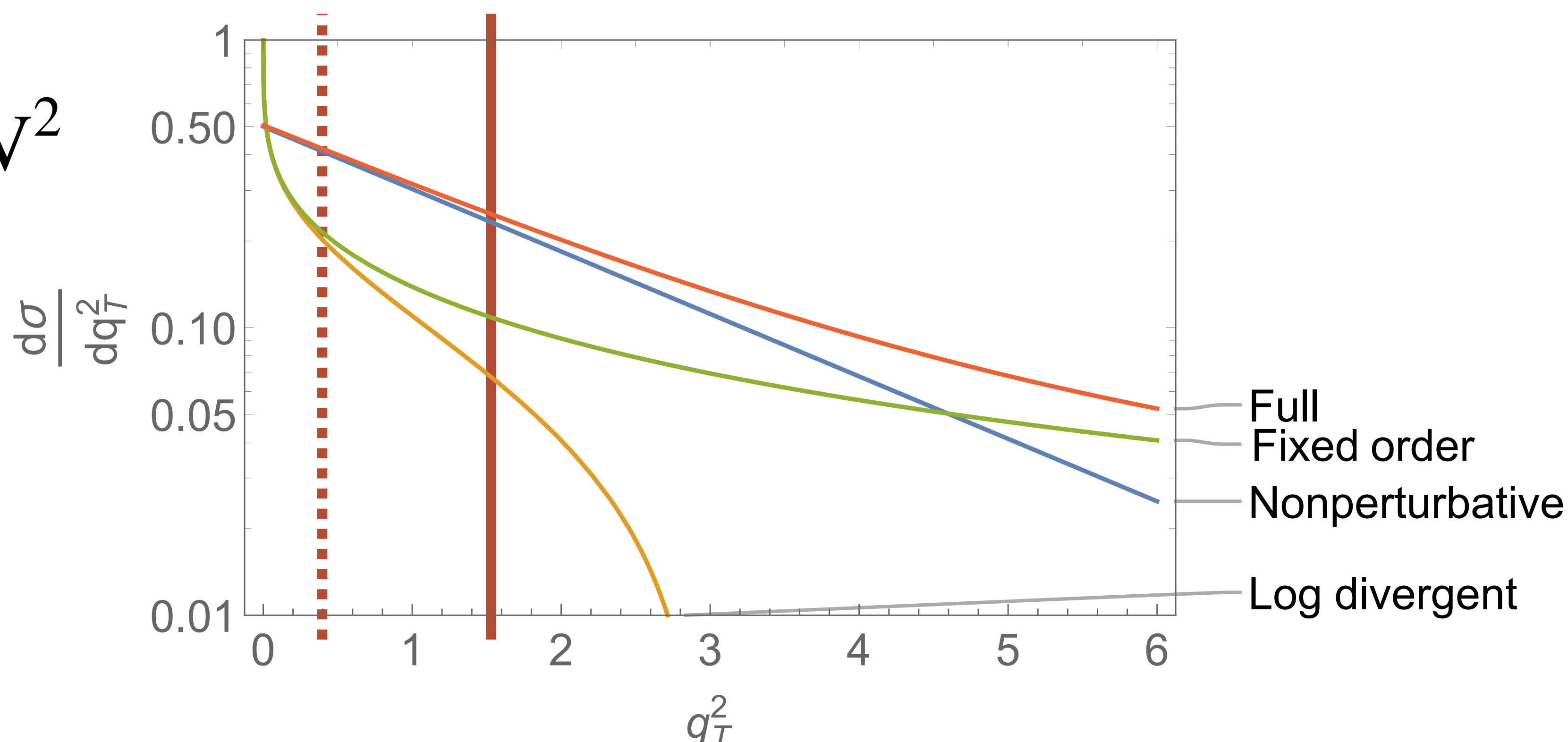


TMD region?

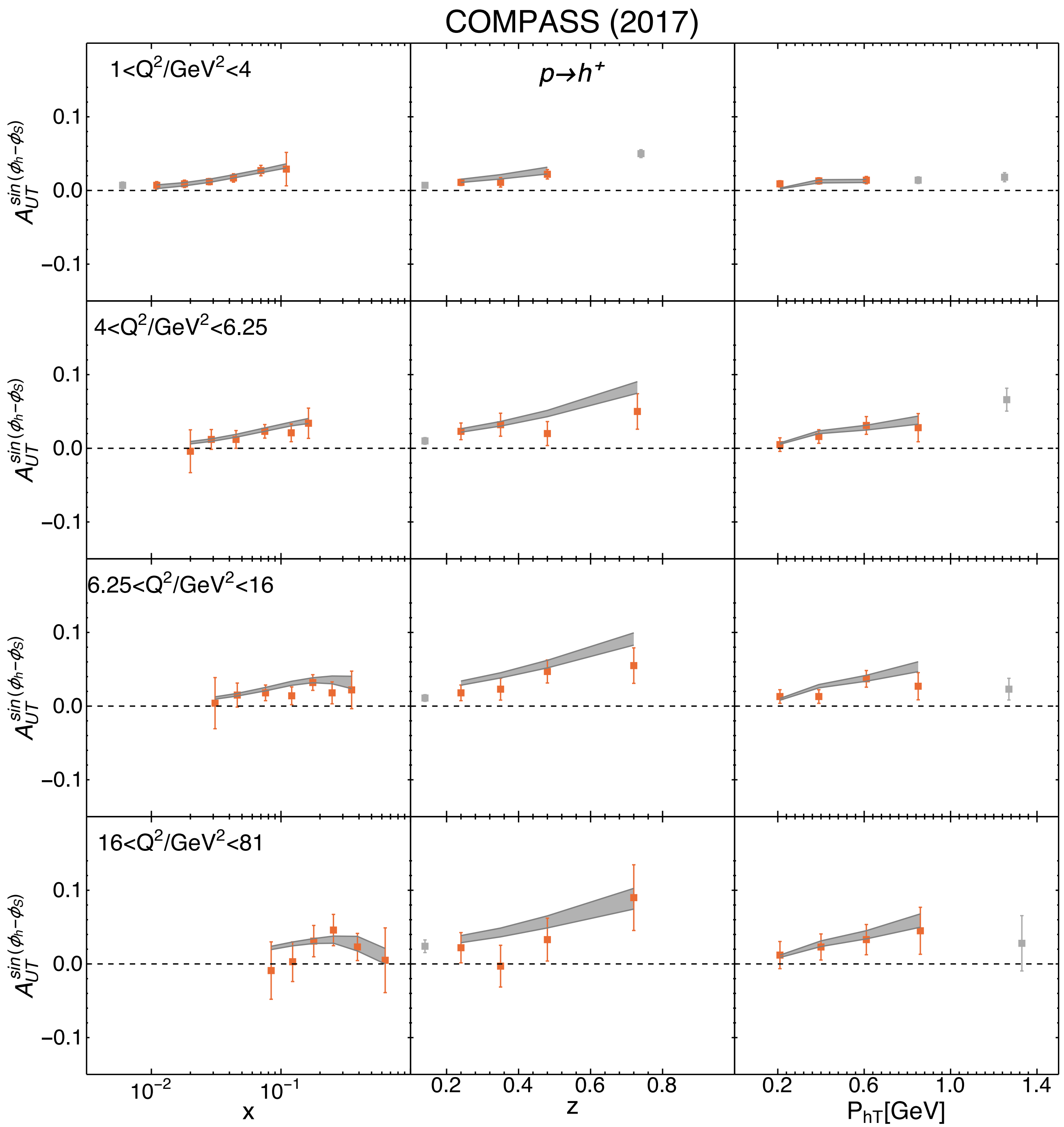
GENERAL CONSIDERATIONS

- Simple Gaussians are not sufficient
- Nontrivial x-dependence is required
- At least NLL should be used
- A study with unpolarized TMDs without the above characteristics is an exploration or toy model, not an extraction
- No flavor dependence is needed for the moment (note however that some flavor dependence is already generated by the collinear PDFs)
- The identification of the region of applicability of the TMD formalism is still an open issue

SIVERS
QUARK TMDS

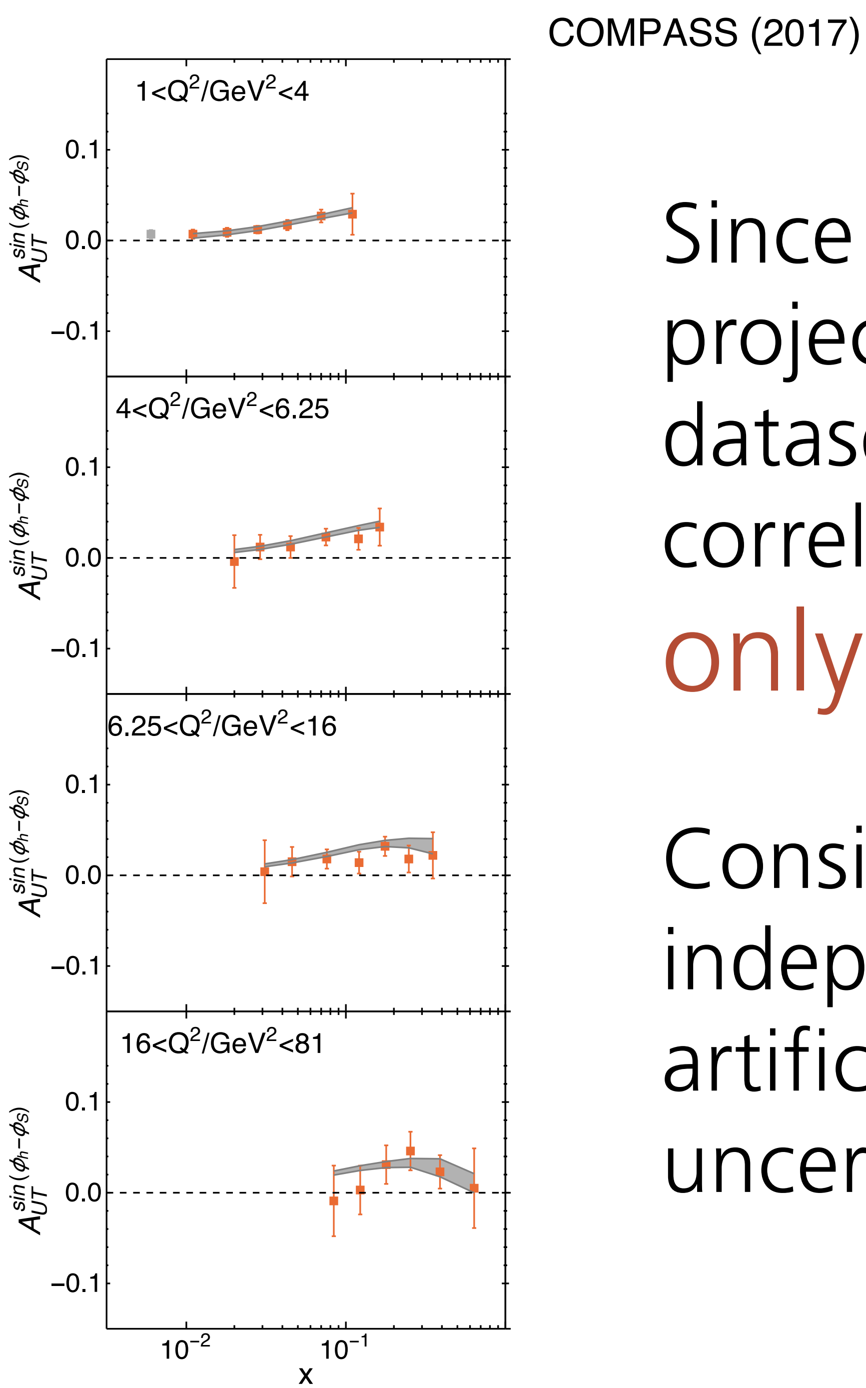
1ST IMPORTANT POINT: CHOICE OF DATA

Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



1^{ST} IMPORTANT POINT: CHOICE OF DATA

Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



Since the x, z, and P_{hT} projections come from the same dataset and are strongly correlated, we consider **only the x projection**

Considering all three projections independently, leads to an artificial underestimate of the uncertainties

2ND IMPORTANT POINT: CONSISTENCY WITH UNPOLARIZED TMDS

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z,\boldsymbol{P}_{hT}^2,Q^2) \approx \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}}$$

NLL analysis

$$\hat{f}_{1T}^{\perp(1)a}(x,b_T^2;Q^2) = e^{S(\mu_b^2,Q^2)}\ e^{g_K(b_T)\ln(Q^2/Q_0^2)} f_{1T}^{\perp(1)a}(x;\mu_b^2)\ \hat{f}_{1T\mathrm{NP}}^{\perp(1)a}(x,b_T^2)$$

this ingredient is the same and must be fixed by unpolarized TMD studies

$$\hat{f}_1^a(x,b_T^2;Q^2) = e^{S(\mu_b^2,Q^2)}\ e^{g_K(b_T)\ln(Q^2/Q_0^2)}\ f_1^a(x;\mu_b^2)\ \hat{f}_{1\mathrm{NP}}^a(x,b_T^2)$$

CAVEAT: EVOLUTION OF COLLINEAR PART

NLL analysis

$$\hat{f}_{1T}^{\perp(1)a}(x, b_T^2; Q^2) = e^{S(\mu_b^2, Q^2)}\ e^{g_K(b_T)\ln(Q^2/Q_0^2)} f_{1T}^{\perp(1)a}(x; \mu_b^2)\ \hat{f}_{1T\mathrm{NP}}^{\perp(1)a}(x, b_T^2)$$

The evolution of $f_{1T}^{\perp(1)}(x)$ is nontrivial and no exact solutions are available.
We applied the same evolution as f_1. This is an approximation that does not affect much the results if the range of Q is small.

3RD IMPORTANT POINT: CHOICE OF FUNCTIONAL FORM

$$f_{1T\,\mathrm{NP}}^{\perp}(x, k_T^2) = \frac{(1 + \lambda_S\, k_T^2)\, e^{-k_T^2/M_1^2}}{K\pi\, (M_1^2 + \lambda_S M_1^4)}\, f_{1\mathrm{NP}}(x, k_T^2)$$

$$f_{1T}^{\perp(1)a}(x; Q_0^2) = \frac{N_{\mathrm{Siv}}^a}{G_{\max}^a}\, x^{\alpha_a}(1 - x)^{\beta_a}\big[1 + A_a\, T_1(x) + B_a\, T_2(x)\big]\, f_1^a(x; Q_0^2)$$

5 parameters for up, down, sea +2
= 17 free parameters

Why did we choose such a form?

DETOUR: POSITIVITY BOUNDS

$$\frac{k_T}{M}\left|f_{1T}^{\perp}(x,k_T^2)\right| \leq f_1(x,k_T^2)$$

for any value of x and k_T

Analogous to

$$f_1(x) \geq 0 \qquad \left|g_1(x)\right| \leq f_1(x) \qquad \left|h_1(x)\right| \leq \frac{1}{2}\Big(f_1(x) + g1(x)\Big)$$

These bounds are essential
- to interpret the PDFs as probability densities
- to guarantee that cross sections are never negative

THE SIMPLEST EXAMPLE

Proton-antiproton Drell-Yan at fixed rapidity

$$y = 0 \quad \Rightarrow \quad x_A = x_B = \frac{Q}{\sqrt{s}}$$

$$d\sigma \propto \sum_q e_q^2 \left[\left(f_1^q(x) \right)^2 - \left(g_1^q(x) \right)^2 \right]$$

If I have a region where

$|g_1^{u+\bar{u}}| > f_1^{u+\bar{u}}$ and $|g_1^{d+\bar{d}}| > f_1^{d+\bar{d}}$

the cross section will become negative

Be like a cross section

stay

always

positive

RESULTS

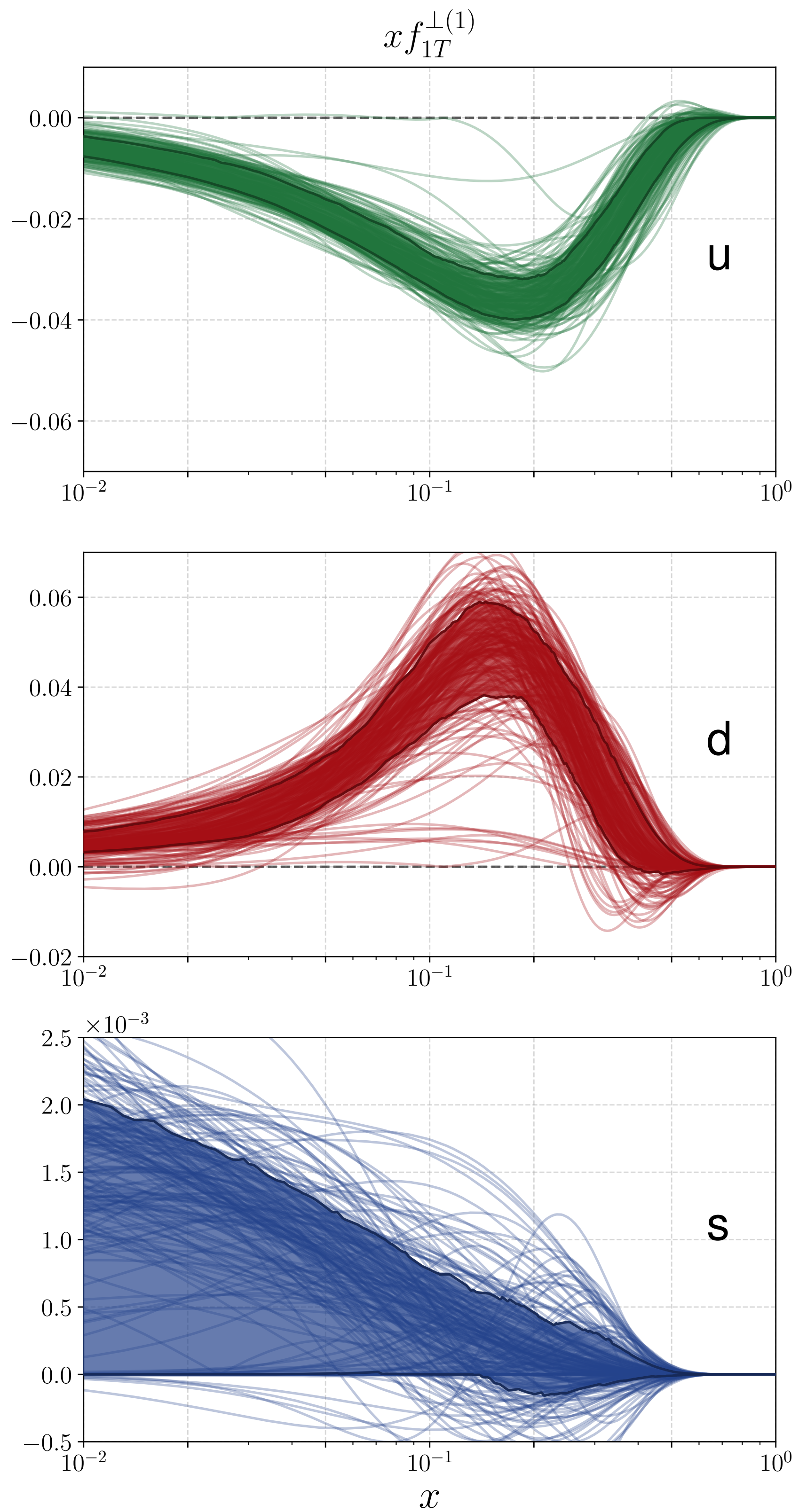
Slightly updated results w.r.t. arXiv (due to the inclusion of correlated errors)

Total number of data points: 118
Total χ^2/dof = 1.08

	M_1	λ_S	α_d	α_u	α_s	
All replicas	0.81 ± 0.35	-0.50 ± 0.73	1.13 ± 0.98	0.16 ± 0.16	1.61 ± 1.52	
Replica 105	0.78	−0.42	0.42	0.12	0.32	
	β_d	β_u	β_s	A_d	A_u	A_s
All replicas	5.64 ± 4.32	1.47 ± 1.41	4.64 ± 4.55	0.79 ± 8.63	-0.98 ± 3.10	-0.68 ± 6.83
Replica 105	9.70	0.86	0.20	−0.88	−0.08	−1.52
	B_d	B_u	B_s	N^d_{Siv}	N^u_{Siv}	N^s_{Siv}
All replicas	2.05 ± 5.01	2.35 ± 4.57	0.29 ± 3.32	$-4.89 \times 10^{-6} \pm 1.00$	-0.07 ± 0.50	0.02 ± 0.64
Replica 105	0.98	1.49	0.89	−1.00	0.29	0.44

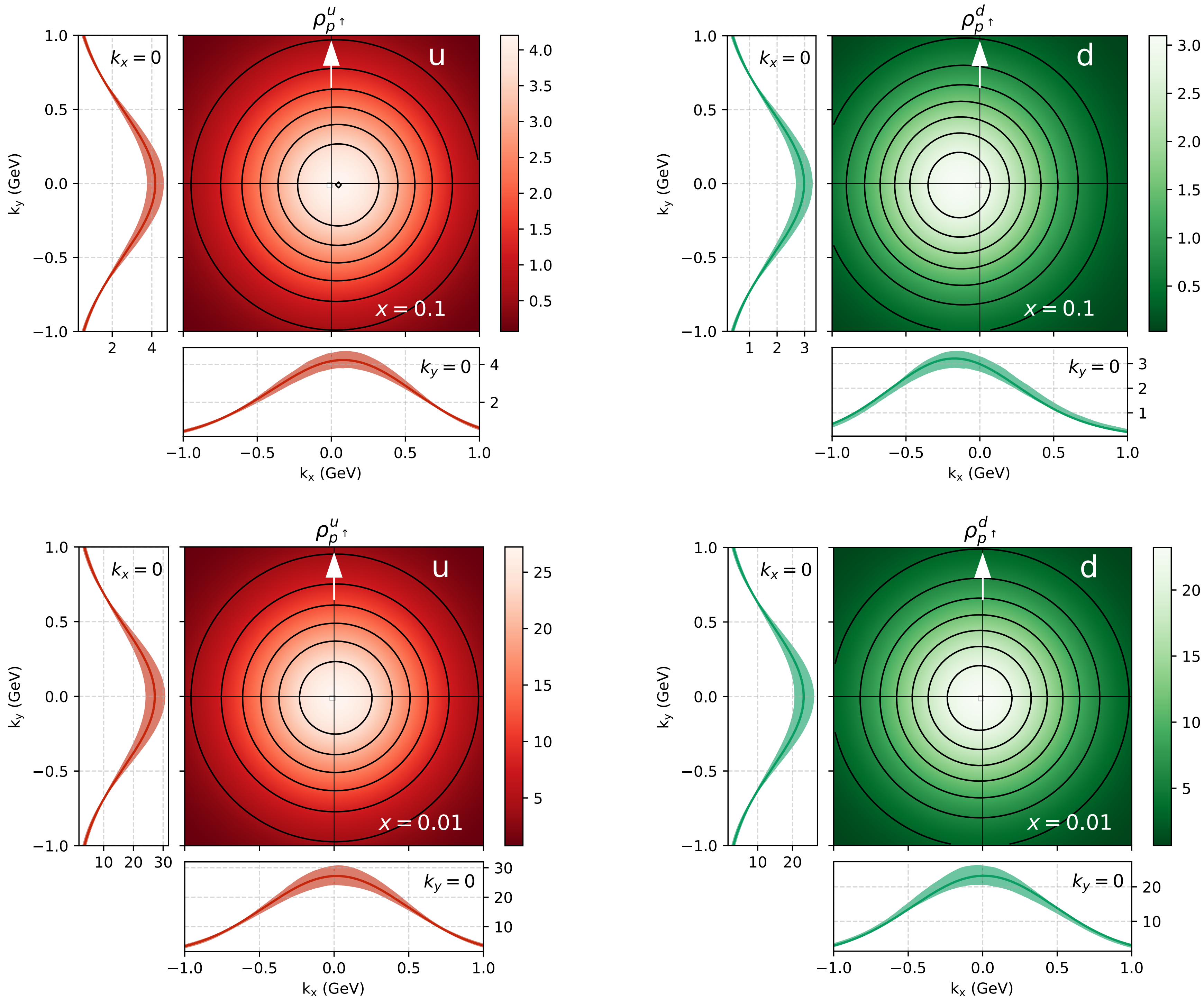
Grids available, please ask

SIVERS FUNCTION



Q= 2GeV

3D STRUCTURE IN MOMENTUM SPACE

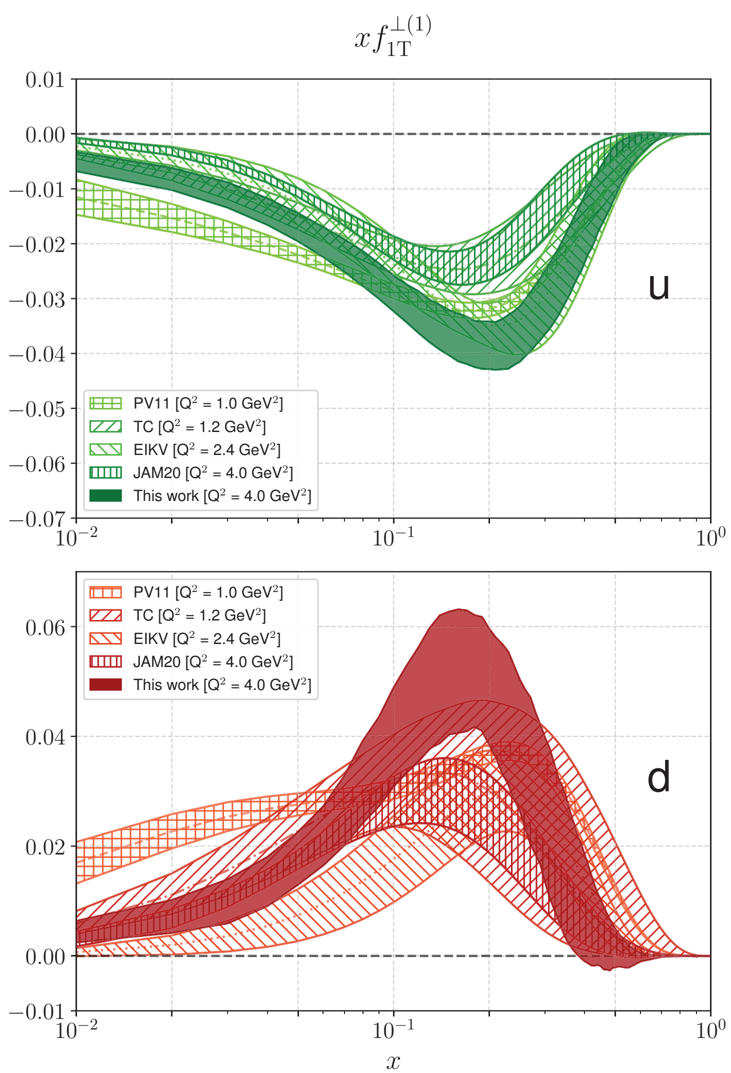


Q= 2GeV

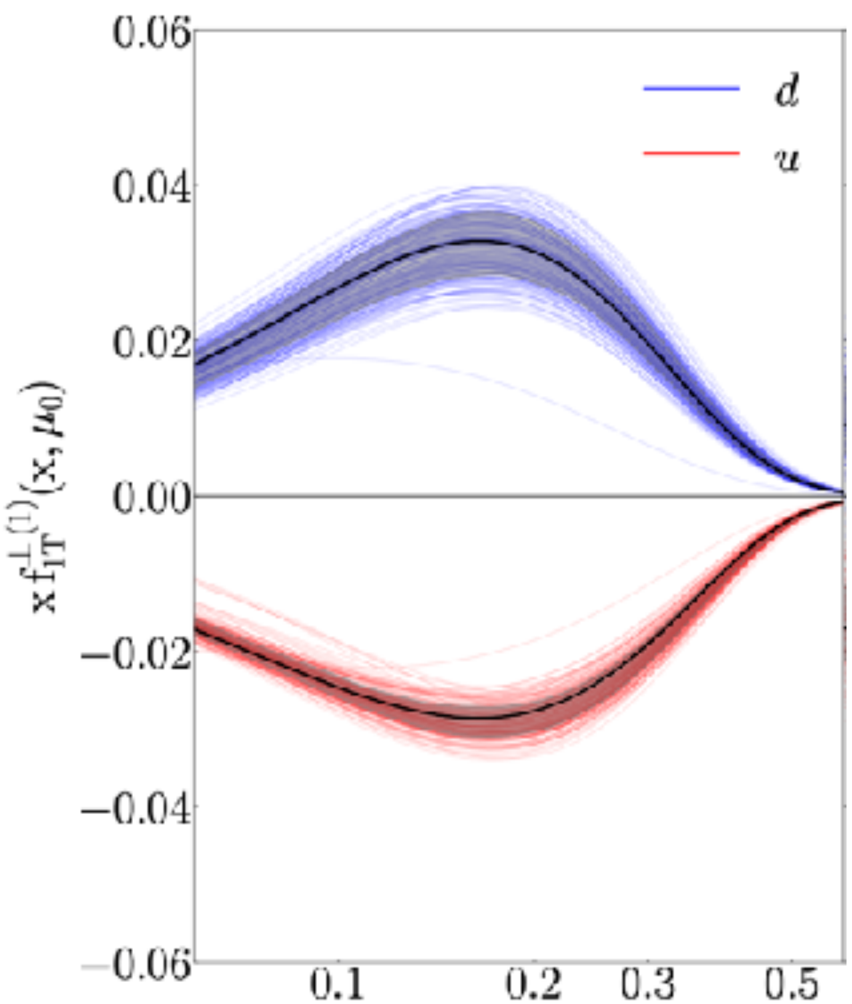
Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278

SIVERS FUNCTION: COMPARISON WITH ECHEVARRIA, KANG, TERRY

Without RHIC W & Z data



Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



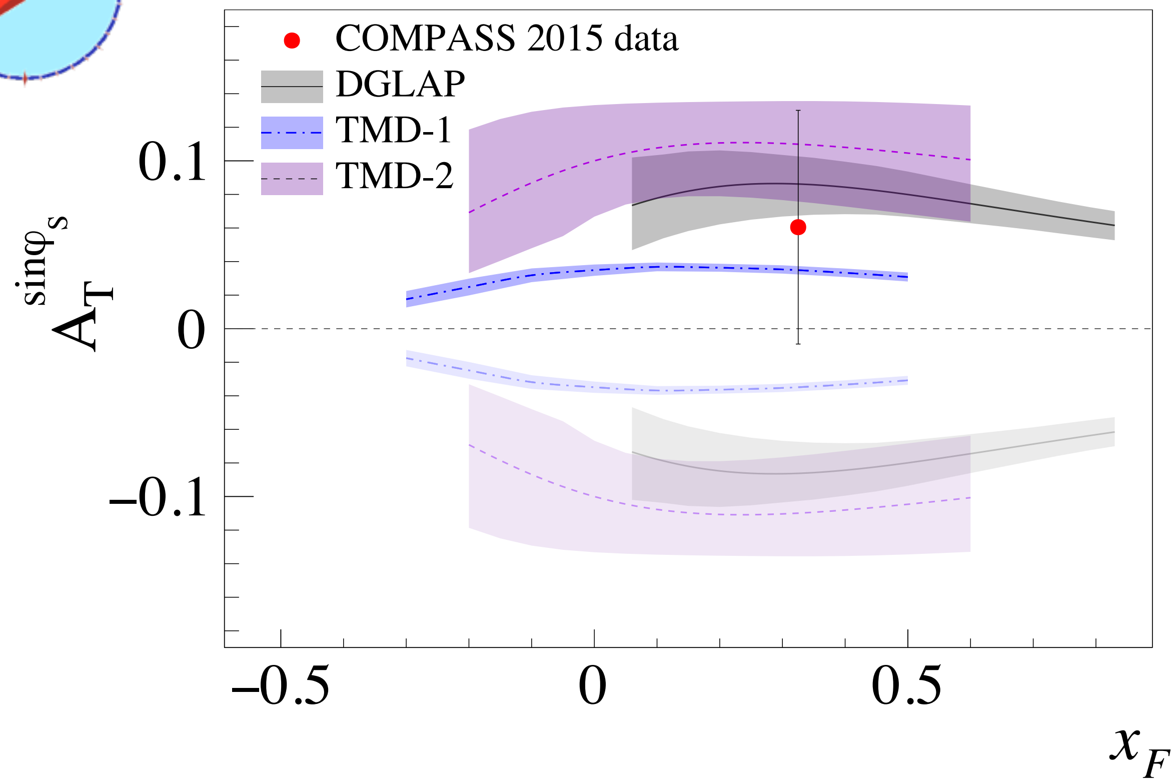
Echevarria, Kang, Terry, arXiv:2009.10710

WHAT ABOUT THE COMPARISON WITH DRELL-YAN?

Sivers function SIDIS = – Sivers function Drell-Yan

Collins, PLB 536 (02)

arXiv:1704.00488

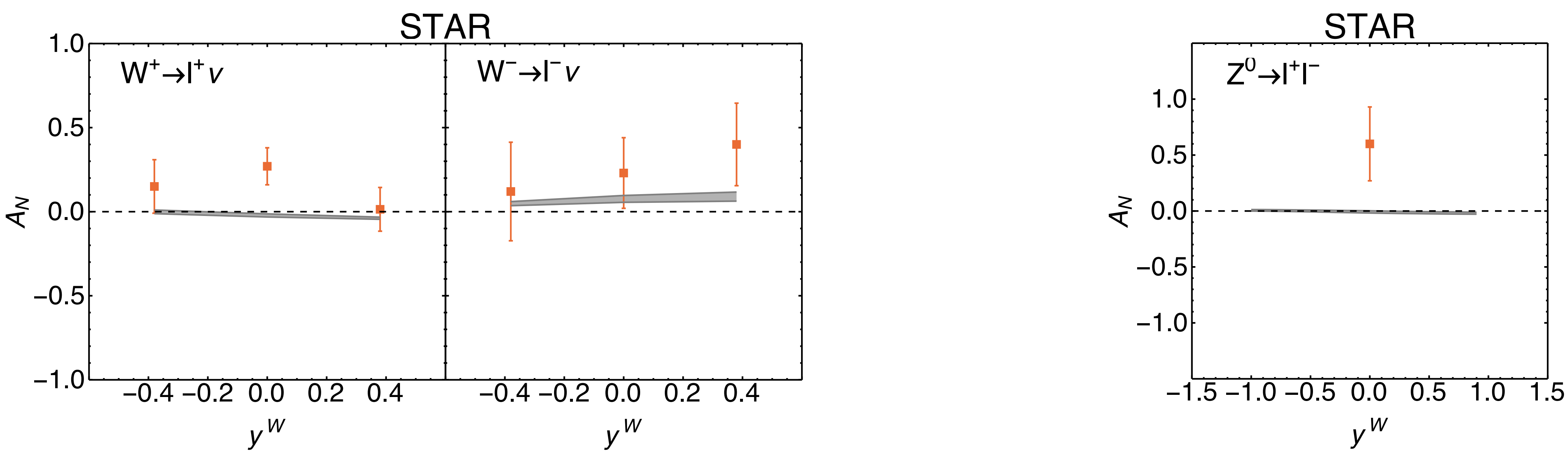


sign change

no sign change

COMPARISON WITH DRELL-YAN DATA

F. Delcarro's talk at POETIC 2019

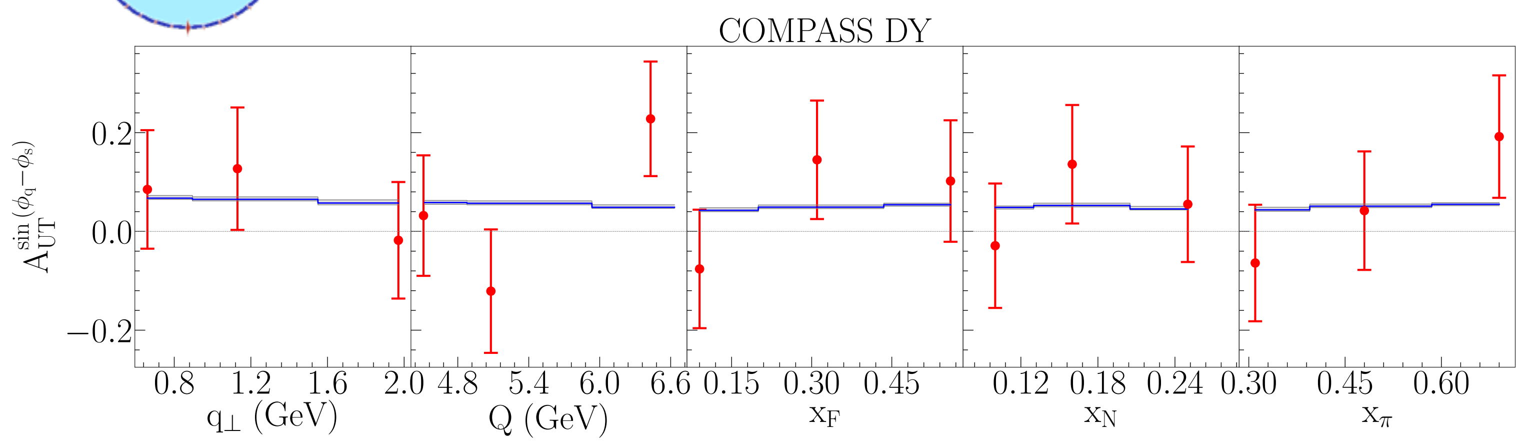


Roughly speaking, the contribution to χ^2 of these data is about 14 for 7 data points.

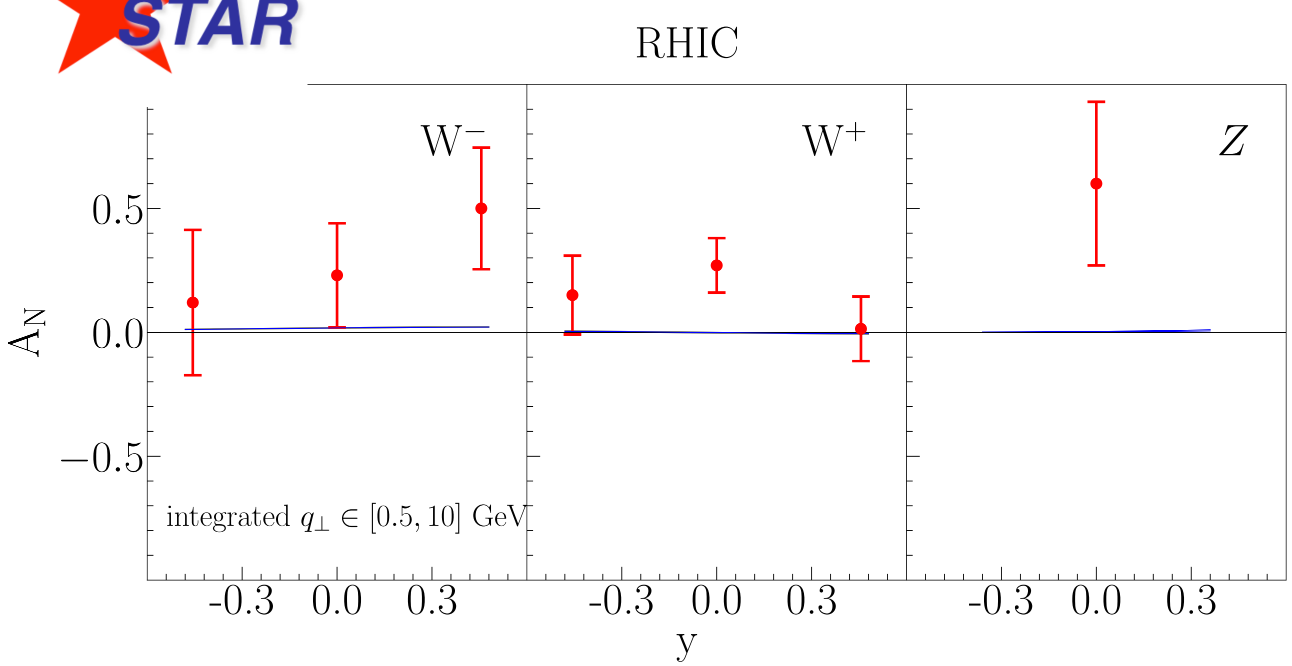
This would change the total χ^2/dof to something like 1.15.

PROBLEMS WITH DY DATA?

Echevarria, Kang, Terry, arXiv:2009.10710



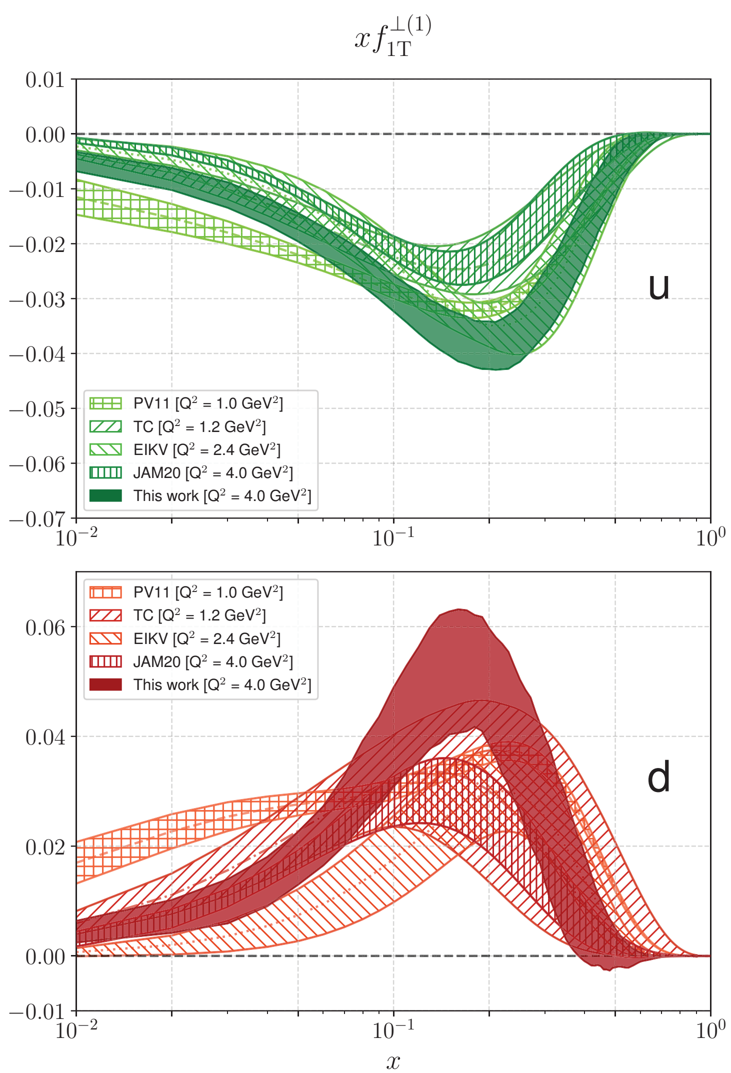
Collab	Ref	Process	Q_{avg}	N_{data}	χ^2/N_{data}
COMPASS	[46]	$\pi^- p \to \gamma^* X$	5.34	15	0.658
RHIC	[47]	$pp \to W^+ X$	M_W	8	2.189
		$pp \to W^- X$	M_W	8	1.684
		$pp \to Z^0 X$	M_Z	1	3.270



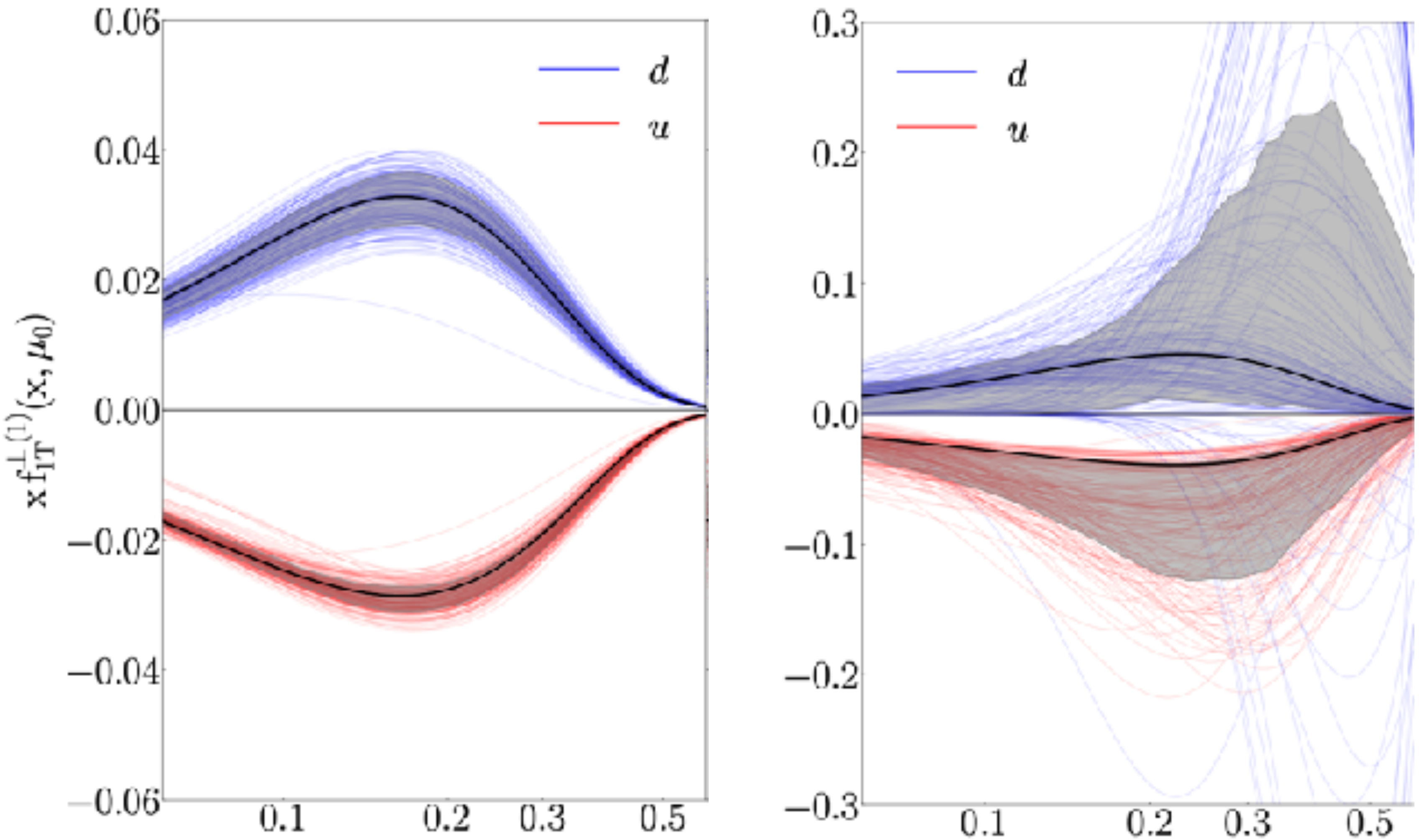
SIVERS FUNCTION

Without RHIC W & Z data

With RHIC W & Z data



Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



Echevarria, Kang, Terry, arXiv:2009.10710

CONCLUSIONS

- We performed a state-of-the-art extraction of the Sivers function from SIDIS data
- Our extraction of the Sivers function is not well compatible with STAR Drell-Yan data. We are including the data in an updated fit, but we don't think they'll have a significant effect.
- If data require a large violation of positivity bounds, it means that there is something wrong with the theoretical analysis

BACKUP SLIDES

LOW-b_T MODIFICATIONS

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

- The justification is to recover the integrated result ("unitarity constraint")
- Modification at low b_T is allowed because resummed calculation is anyway unreliable there

PAVIA 2019 b_* PRESCRIPTION

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \big(C_{qi} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^q(x, b_T)$$

$$\mu_0 = 1\,\text{GeV}$$

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\rm max}\left(\frac{1 - e^{-b_T^4/b_{\rm max}^4}}{1 - e^{-b_T^4/b_{\rm min}^4}}\right)^{1/4} \qquad b_{\rm max} = 2e^{-\gamma_E}$$

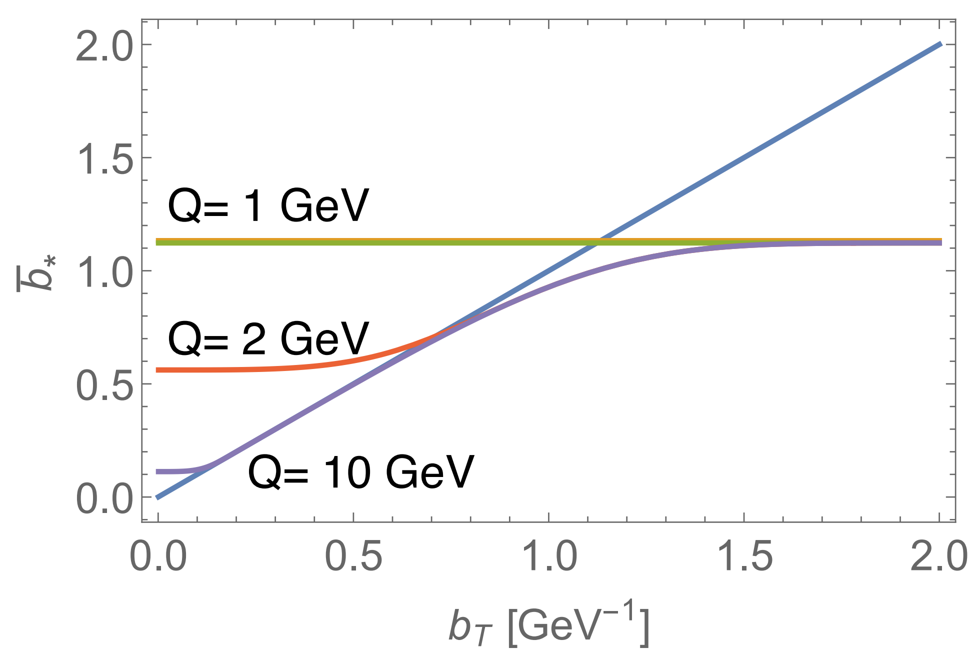
$$b_{\rm min} = \frac{2e^{-\gamma_E}}{Q}$$

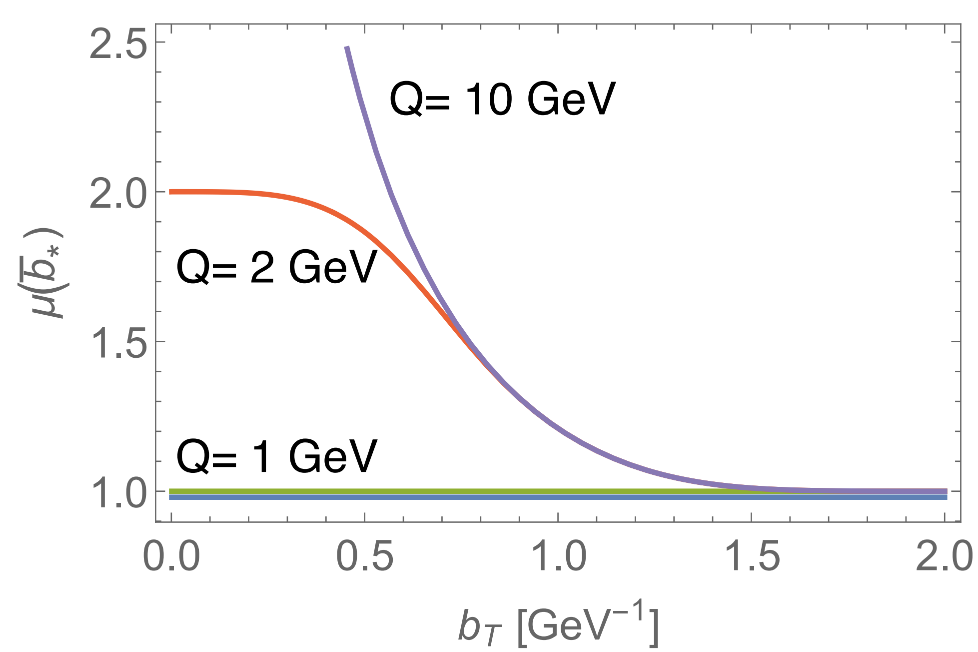
These are all choices that should be at some point checked/challenged

EFFECTS OF b$_*$ PRESCRIPTION

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \qquad b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$





No significant effect at high Q, but large effect at low Q (inhibits perturbative contribution)