

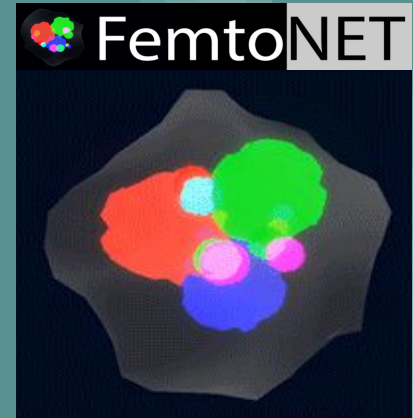
# Gluon Spatial Distributions in the Nucleon

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# Femtography of the nucleon

**Femtography** - is **data driven visualizations** of the phase space (momentum and spatial) distribution of the quarks and gluons inside of the proton using a variety of **deeply virtual scattering processes**.

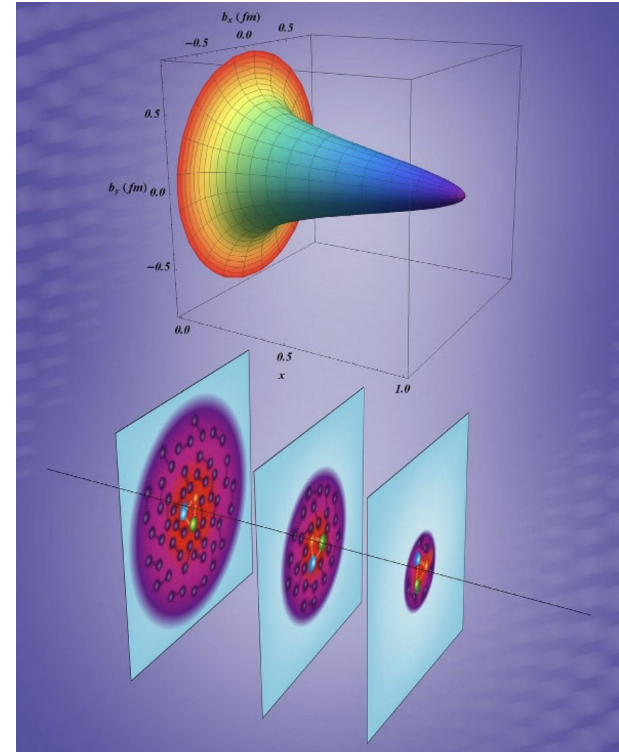
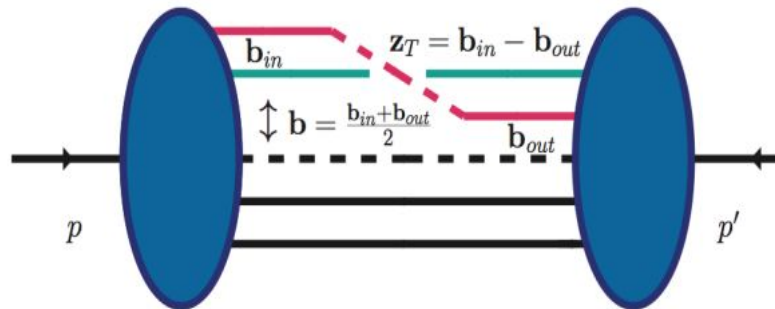


Image credit: **Rafael Dupre**

# Generalized parton distributions

$$F_{\Lambda, \Lambda'}^{[\Gamma]}(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' | \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{W}\left(-\frac{z}{2}, \frac{z}{2} | n\right) \psi\left(\frac{z}{2}\right) | p, \Lambda \rangle \Big|_{z^+ = z_T = 0}$$



$b_T$  Relative average transverse position from the center of momentum of the system  $\longleftrightarrow \Delta_T$

$k_T$  Relative average transverse momentum (integrated)  $\longleftrightarrow z_T$

Image credit: A. Rajan, M. Engelhardt, S. Liuti **PRD 98 (2018)**

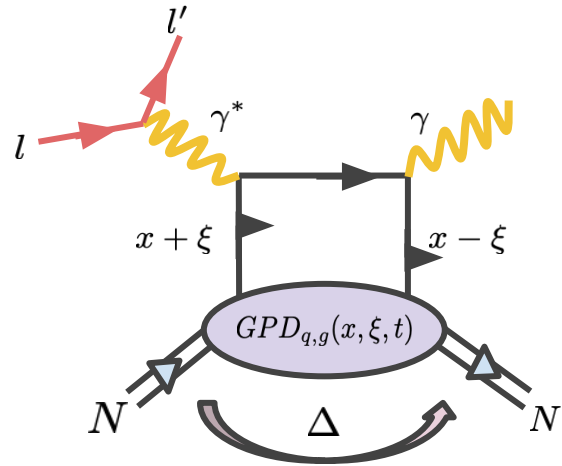
X. Ji **PRL. 78 (1997)**

A. Radyushkin **PRD. 56 (1997)**

D. Muller, et. al. **(1994)**

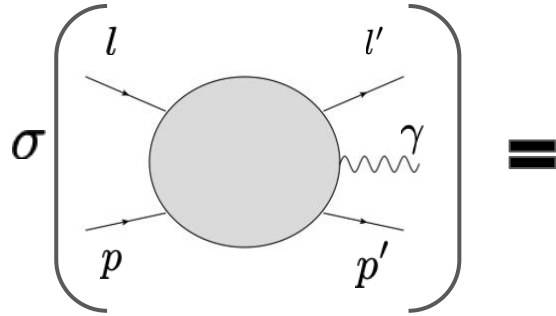
M. Diehl **Phys.Rep. (2003)**

## How do we image the gluon distribution in the proton?



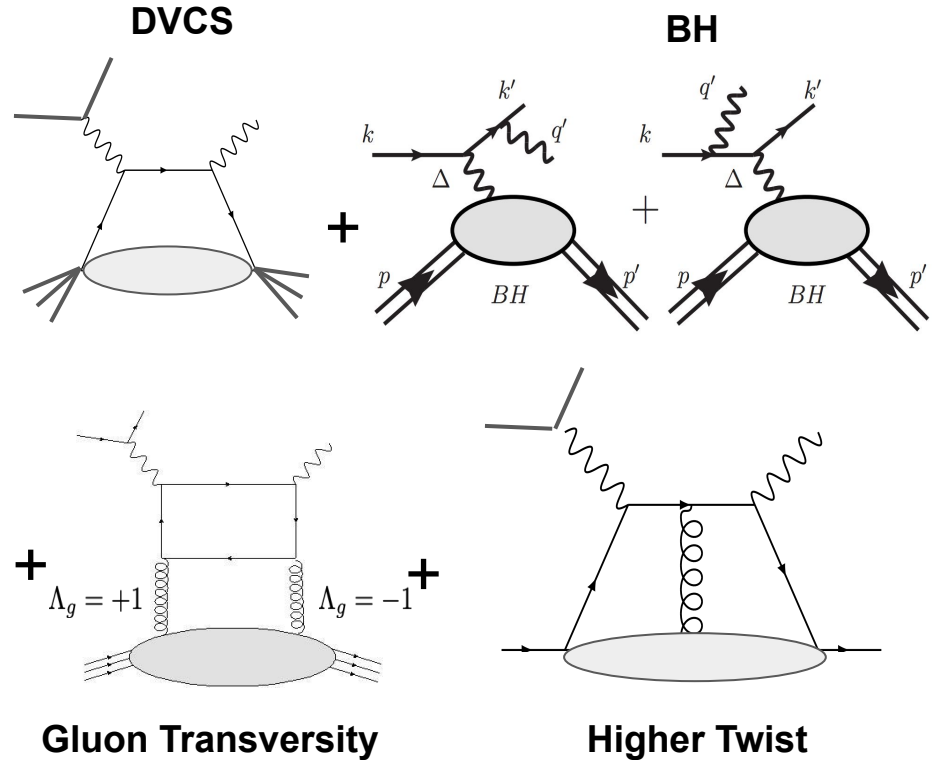
Deeply virtual exclusive processes represent a **new paradigm** for measuring the fundamental properties of nuclei by probing the quantum mechanical **phase space distributions** of the quarks and gluons.

# Deeply Virtual Compton Scattering



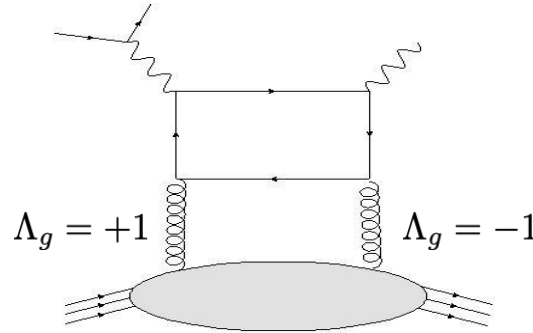
DVCS is known to probe **generalized parton distributions** and is accompanied by various background processes.

X. Ji, **PRD. 55 (1997)**



B.Kriesten, S.Liuti, et. al. **PRD. 101 (2020)**

# Gluon Transversity Observables in DVCS

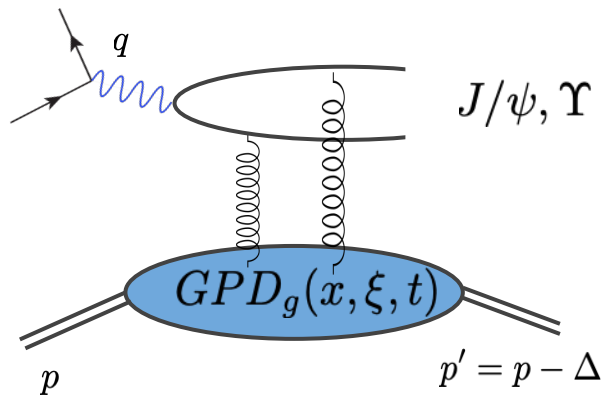


Accessing the  
**chiral-odd** gluon  
structure of the nucleon

$$\begin{aligned}
 F_{UU}^{\cos 2\phi} = & -2 \frac{\alpha_S}{2\pi} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \Re \left[ \sqrt{1-\xi^2} \left( \tilde{\mathcal{H}}_T^g + (1-\xi) \frac{\mathcal{E}_T^g + \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{H} + \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2} (\mathcal{E} + \tilde{\mathcal{E}})^* \right) \right. \\
 & + \sqrt{1-\xi^2} \left( \tilde{\mathcal{H}}_T^g + (1+\xi) \frac{\mathcal{E}_T^g - \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{H} - \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2} (\mathcal{E} + \tilde{\mathcal{E}})^* \right) \\
 & + \frac{\sqrt{t_0-t}}{2M} \left( \tilde{\mathcal{H}}_T^g + (1+\xi) \frac{\mathcal{E}_T^g - \tilde{\mathcal{E}}_T^g}{2} \right) (\mathcal{E} + \xi \tilde{\mathcal{E}})^* \\
 & \left. - \sqrt{1-\xi^2} \left( \mathcal{H}_T^g + \frac{t_0-t}{M^2} \tilde{\mathcal{H}}_T^g - \frac{\xi^2}{1-\xi^2} \mathcal{E}_T^g + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T^g \right) (\mathcal{E} - \xi \tilde{\mathcal{E}})^* \right]
 \end{aligned}$$

# Exclusive Measurements of Gluon Distributions

Exclusive electroproduction of vector mesons (such as the  $J/\psi$ ) probe the gluon content of nuclei.



EIC White Paper [arXiv: 1212.1701](#)

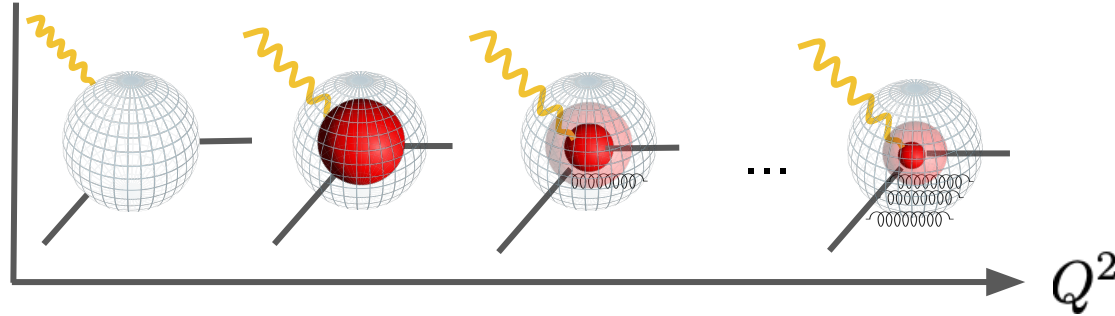
Gluon GPDs enter the vector meson production cross section.

Y. Guo, X. Ji, Y. Liu [arXiv:2103.11506](#)

$$\frac{d\sigma}{dt} \propto \left(1 - \frac{t}{4M^2}\right) E_2^2 - 2E_2(H_2 + E_2) + (1 - \xi^2)(H_2 + E_2)^2$$

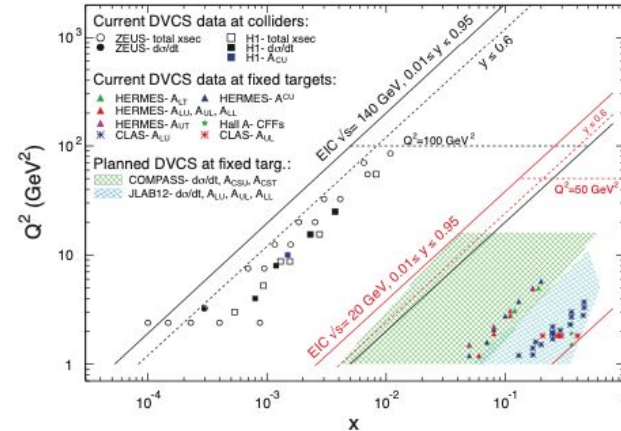
$$E_2 = \int_0^1 dx E_g(x, \xi, t) \quad H_2 = \int_0^1 dx H_g(x, \xi, t)$$

# Gluon distributions through scaling violations



$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \left[ P_{QQ} \otimes F_2 + 2e^2 P_{QG} \otimes xg(x) \right]$$

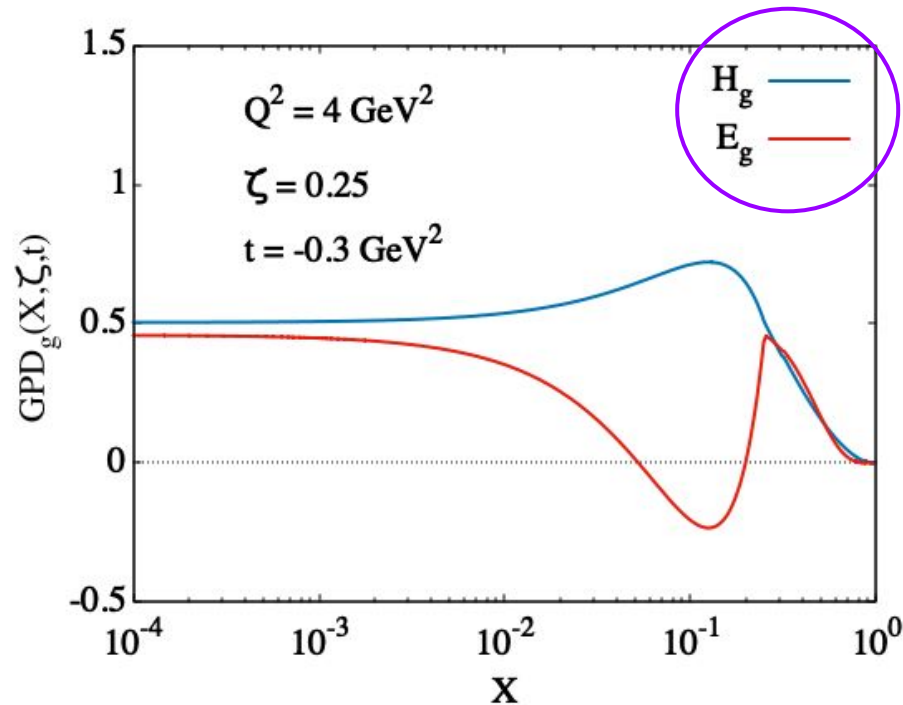
A **lever arm in  $Q^2$**  hopefully allows us to use perturbative evolution to extract the gluon distribution through **scaling violations**.



EIC Yellow Report [arXiv: 2103.05419](https://arxiv.org/abs/2103.05419)  
 EIC White Paper [arXiv: 1212.1701](https://arxiv.org/abs/1212.1701)

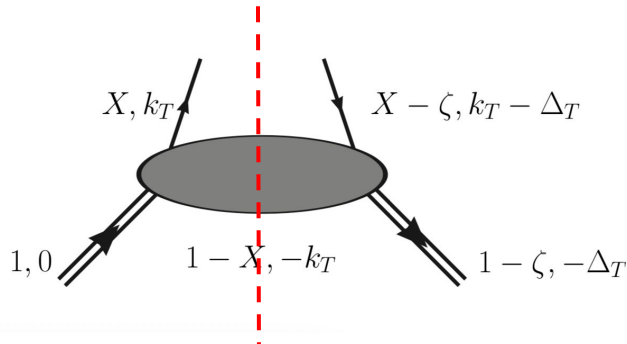


## Flexible Gluon GPD Model



Allows us to calculate  
**gluon angular  
momentum**  
observables as they  
appear in experiment.

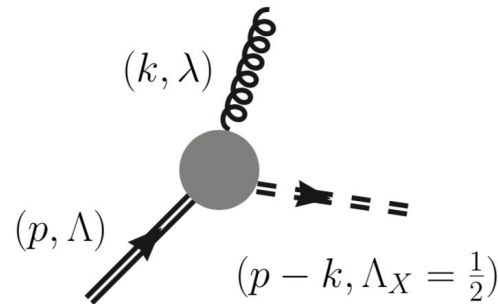
# Reggeized Spectator Gluon GPD Model



$$A_{\Lambda' \lambda'_g, \Lambda \lambda_g} = \int \frac{d^2 k_\perp}{1-X} \sum_{\Lambda_X} \phi_{\lambda'_g \Lambda'}^{* \Lambda_X}(k', p') \phi_{\lambda_g \Lambda}^{\Lambda_X}(k, p)$$

- 7 Fitted parameters and the initial scale for perturbative evolution.

$$H^g(X, \zeta, t) = \begin{cases} H_{M_X}^{M_\Lambda}(X, \zeta, t) R_p^{\alpha, \alpha'}(X, t), & \zeta \leq X \leq 1 \\ a_g X^2 - a_g \zeta X + H(\zeta, t), & 0 \leq X < \zeta \\ 0 & -1 + \zeta \leq X < 0 \end{cases}$$


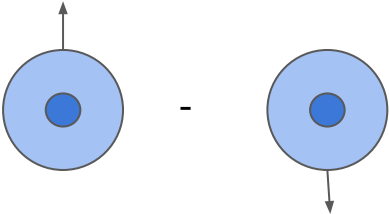
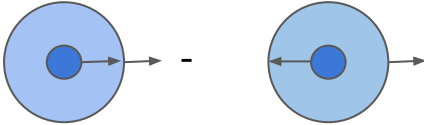
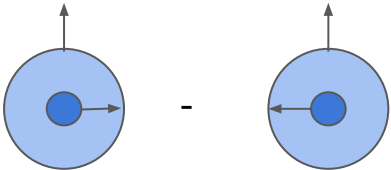


$$\phi_{\lambda_g \Lambda}^{\Lambda_X}(k, p) = \Gamma(k) \frac{\bar{U}_{\Lambda_X}(p - k) U_\Lambda(p)}{k^2 - m_g^2} \not{\epsilon}_{\lambda_g}^*(k)$$

Quark Parametrization **PRD. 84 (2010), PRC 88 (2013)**

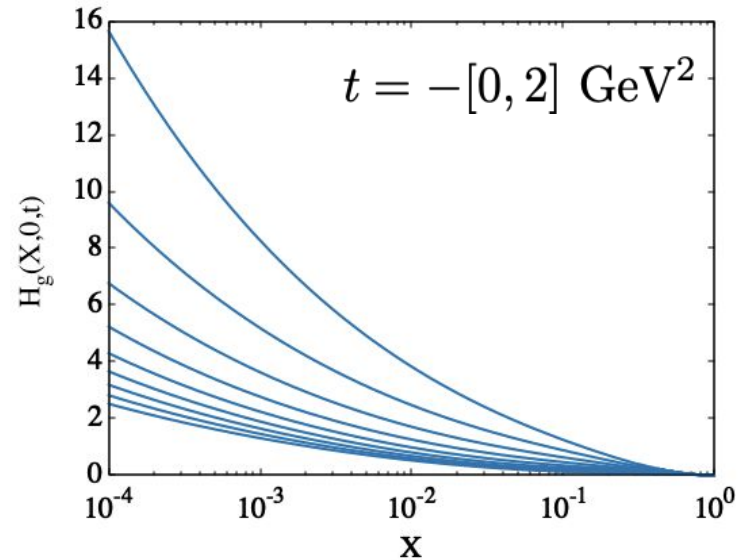
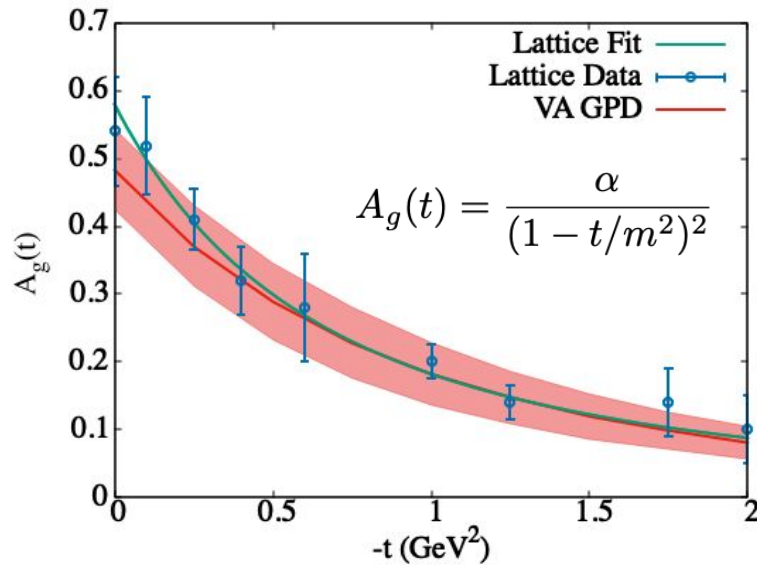
$$R_p^{\alpha, \alpha'} = X^{-[\alpha + \alpha'(1-X)^p t]}$$

## GPDs from Helicity Amplitudes

GPD	Phase	Helicity Composition
$H$	1	
$\Delta_T E$	$e^{i\phi}$	
$\tilde{H}$	1	
$\xi \Delta_T \tilde{E}$	$e^{i\phi}$	

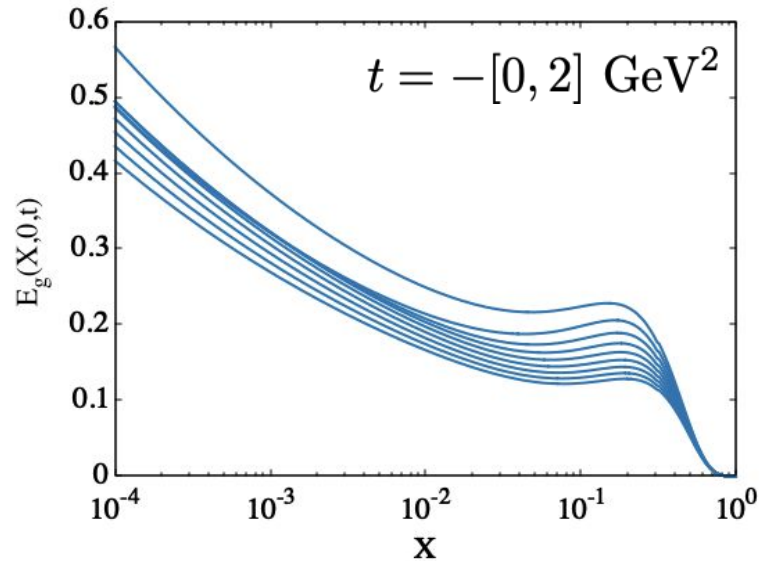
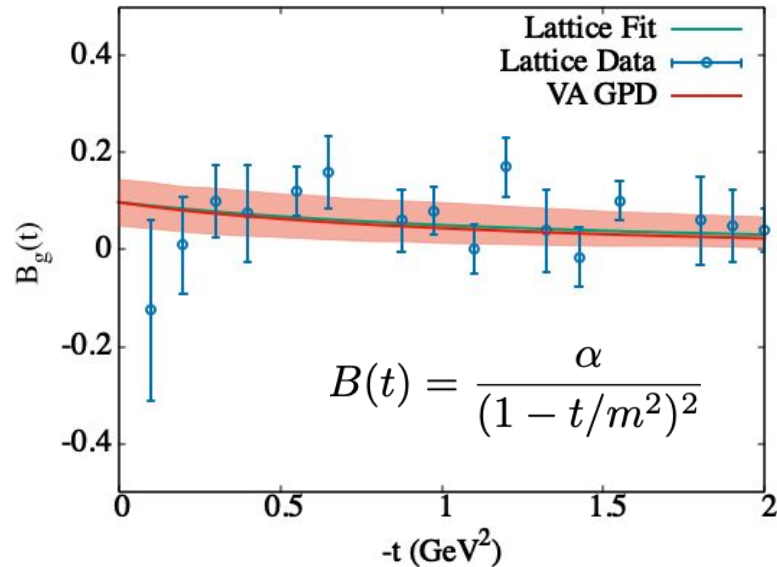
## Gluon GPD $H(X,0,t)$

Dipole fit of Lattice QCD calculated moments (P.E. Shanahan, W. Detmold **PRD 99, (2019)**) allows us to fit the the gluon GPD  $t$ -dependence.



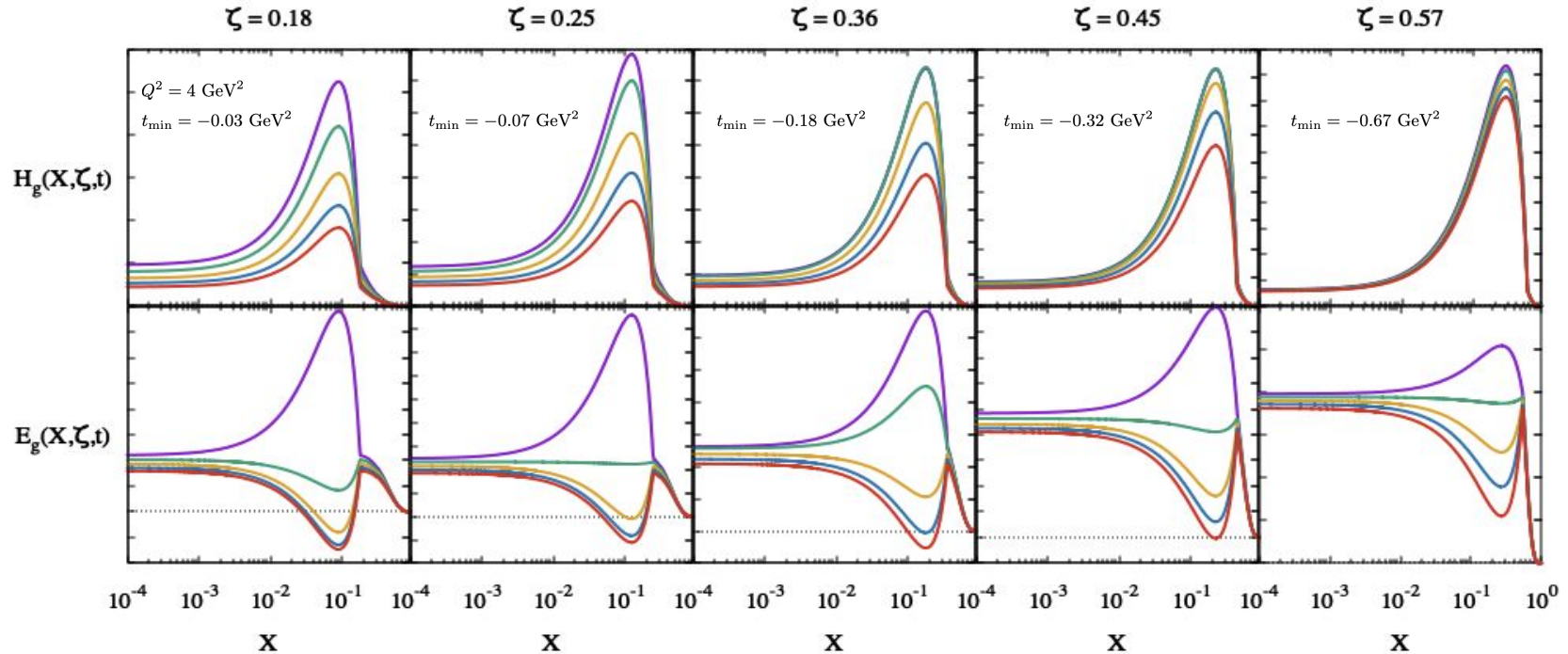
## Gluon GPD $E(X,0,t)$

Similarly, we use a dipole fit of Lattice QCD calculated moments (P.E. Shanahan, W. Detmold **PRD 99, (2019)**) to fit the gluon GPD  $t$ -dependence, but there is **much flexibility in this fit**.



# Full Gluon GPD

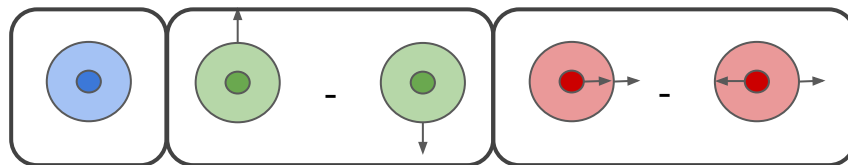
Allows us to calculate **gluon observables** as they appear in experimental cross sections.



## Momentum Space to Transverse Position Space

Probability density of finding a parton at transverse position  $\mathbf{b}$  from the center of momentum as a function of the parton and proton polarization.

$$\rho_{\Lambda\lambda}^q(\mathbf{b}) = H_q(\mathbf{b}^2) + \frac{b^i}{M} \epsilon_{ij} S_T^j \frac{\partial}{\partial b} E_q(\mathbf{b}^2) + \Lambda\lambda \tilde{H}_q(\mathbf{b}^2)$$

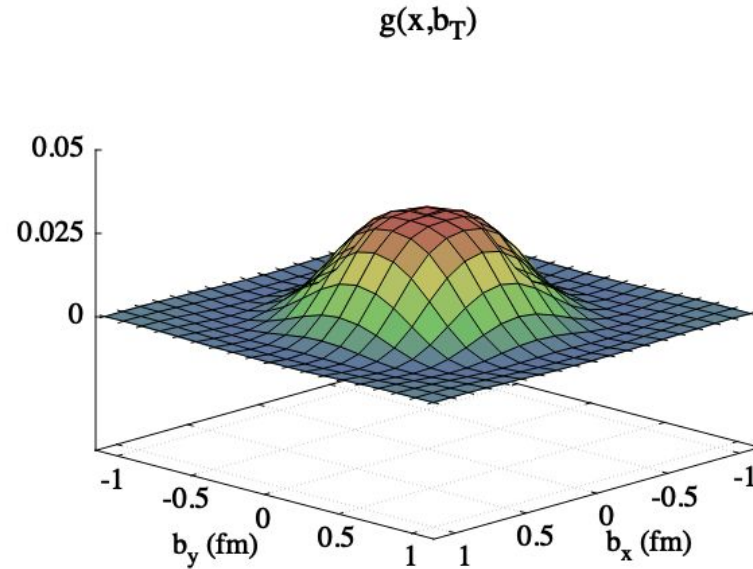


M. Burkardt **PRD. 62 (2000)**

M. Burkardt **Int.J.Mod.Phys.A 18 (2003)**

# Gluon Transverse Position Space Images

Unpolarized gluon distribution in an unpolarized proton.





## Conclusions



- Generalized Parton Distributions give us direct access to information of the fundamental properties of the nucleon contained in the EMT of QCD.
- Theoretical Models of GPDs are necessary to make estimates of the size of experimental observables and calculate sum rules such as those for OAM.
- Gluon GPD models will allow us to calculate cross sections for gluon pieces of DVCS cross sections at EIC kinematics.
- Using a flexible parametrization of gluon GPDs we can construct pictures of what we would expect gluon transverse spatial images to look like.