Jet fragmentation functions at the EIC

(arXiv:2106.15624)

Fanyi Zhao

University of California, Los Angeles

In collaboration with Zhong-Bo Kang, Kyle Lee and Ding Yu Shao

Jul 30, 2021

Introduction

- Studies of jets have been used as an important probe to test the fundamental properties of hadrons. Study 3D structure of hadrons (encoded in **TMDPDFs**) is one of the most important goals at the future electron ion collider (EIC)

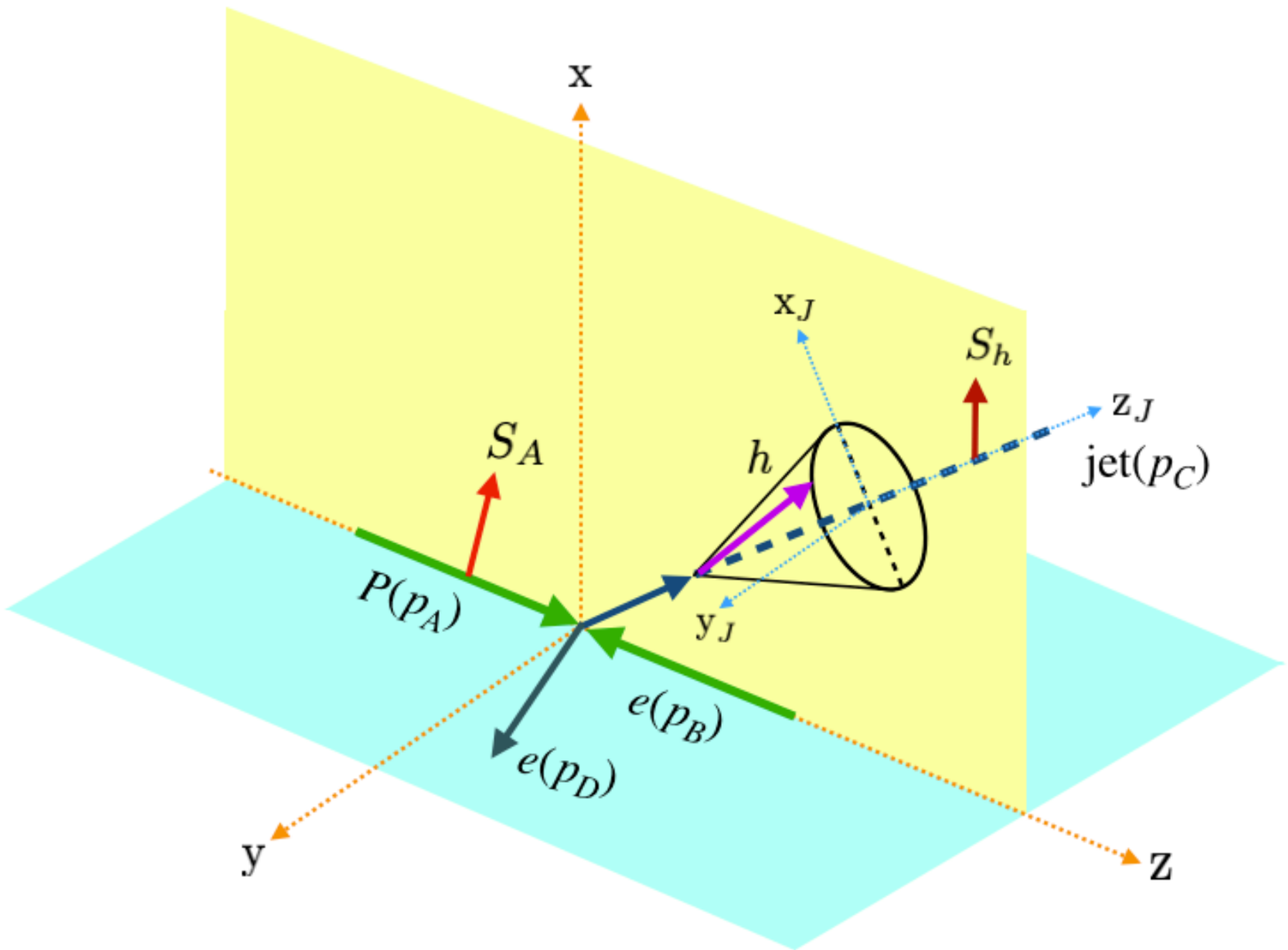
- In the process of $ep \to e + \text{jet}(h) + X$, the hadron's distribution inside jet (**TMDJFFs**) correlate with the parton distribution functions (**TMDPDFs**) of the initial proton

$$\Downarrow$$

TMDJFFs are extremely useful for probing the 3D structure of protons at EIC

Theoretical framework

- All the possible spin asymmetries in back-to-back electron-jet production with jet fragmentation process, $ep \to e + \text{jet}(h) + X$, at the future electron ion collider (EIC)



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$$\boldsymbol{q}_T = \boldsymbol{p}_{C,T} + \boldsymbol{p}_{D,T}$$

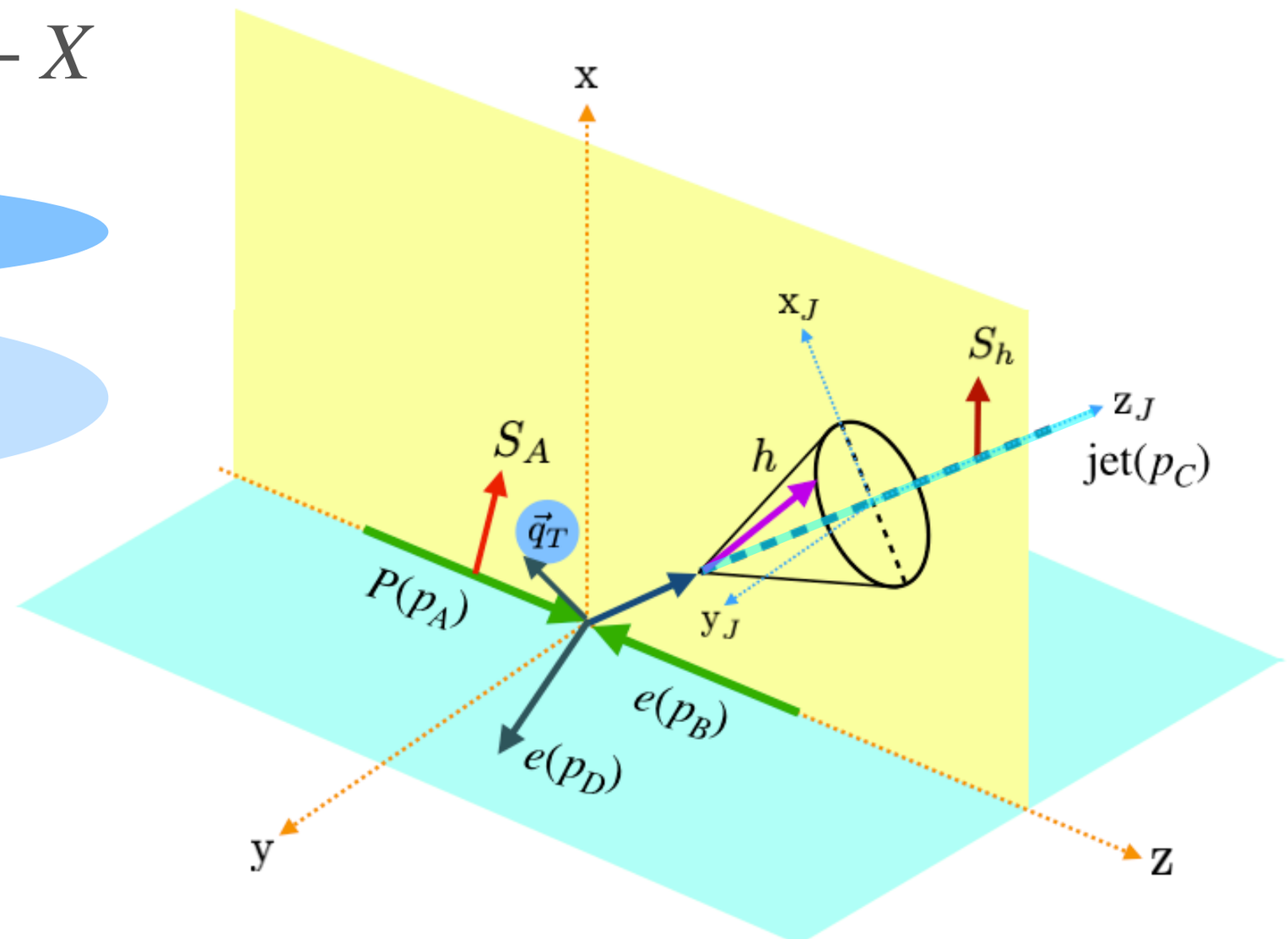
$$\boldsymbol{p}_T = \frac{\boldsymbol{p}_{C,T} - \boldsymbol{p}_{D,T}}{2}$$

$$z_h = \frac{p_{\bar{h}}}{p_{\bar{J}}}$$

$$\boldsymbol{j}_\perp$$

$$\boldsymbol{S}_{h\perp}$$

$$\boldsymbol{S}_T$$



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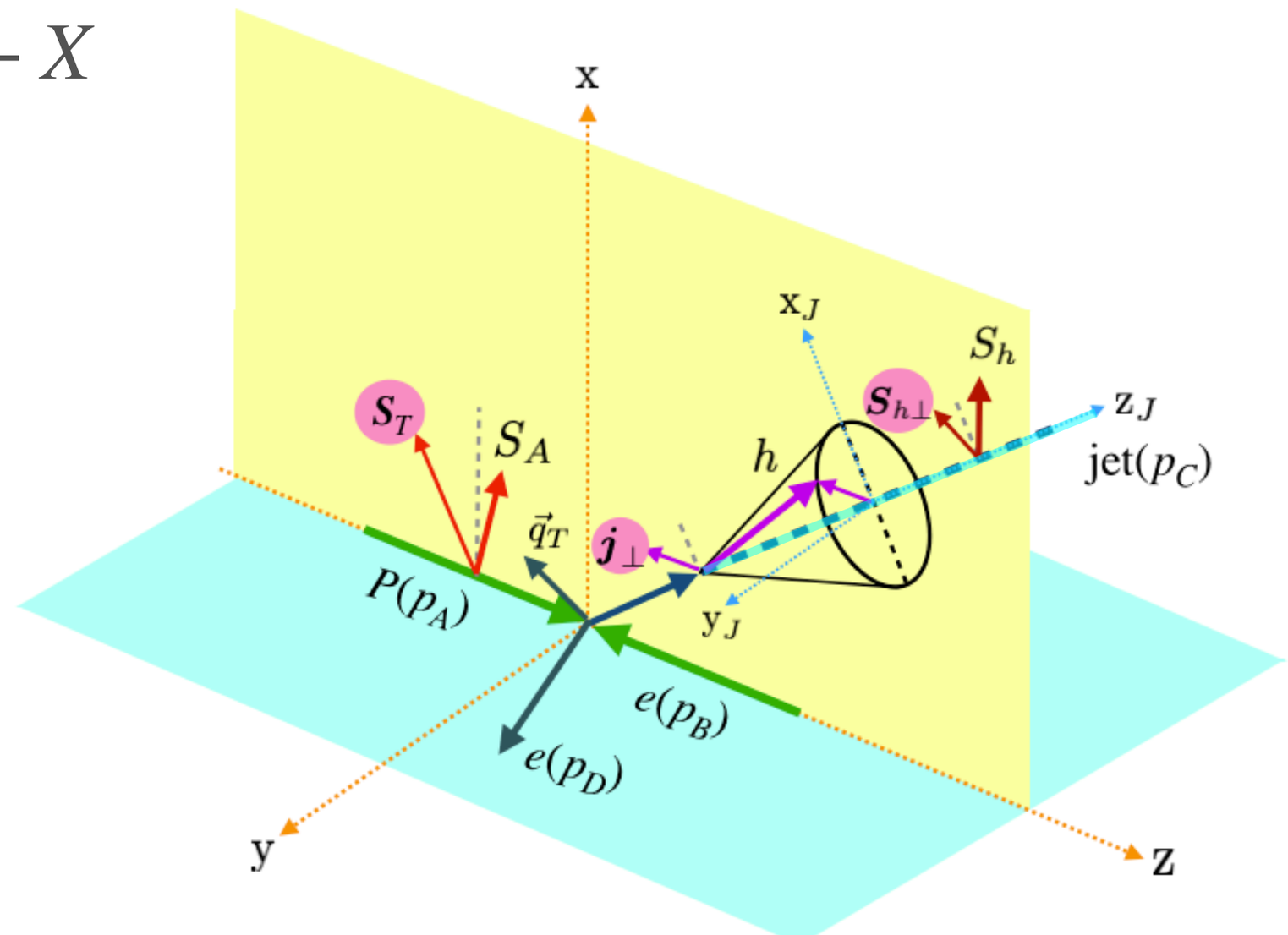

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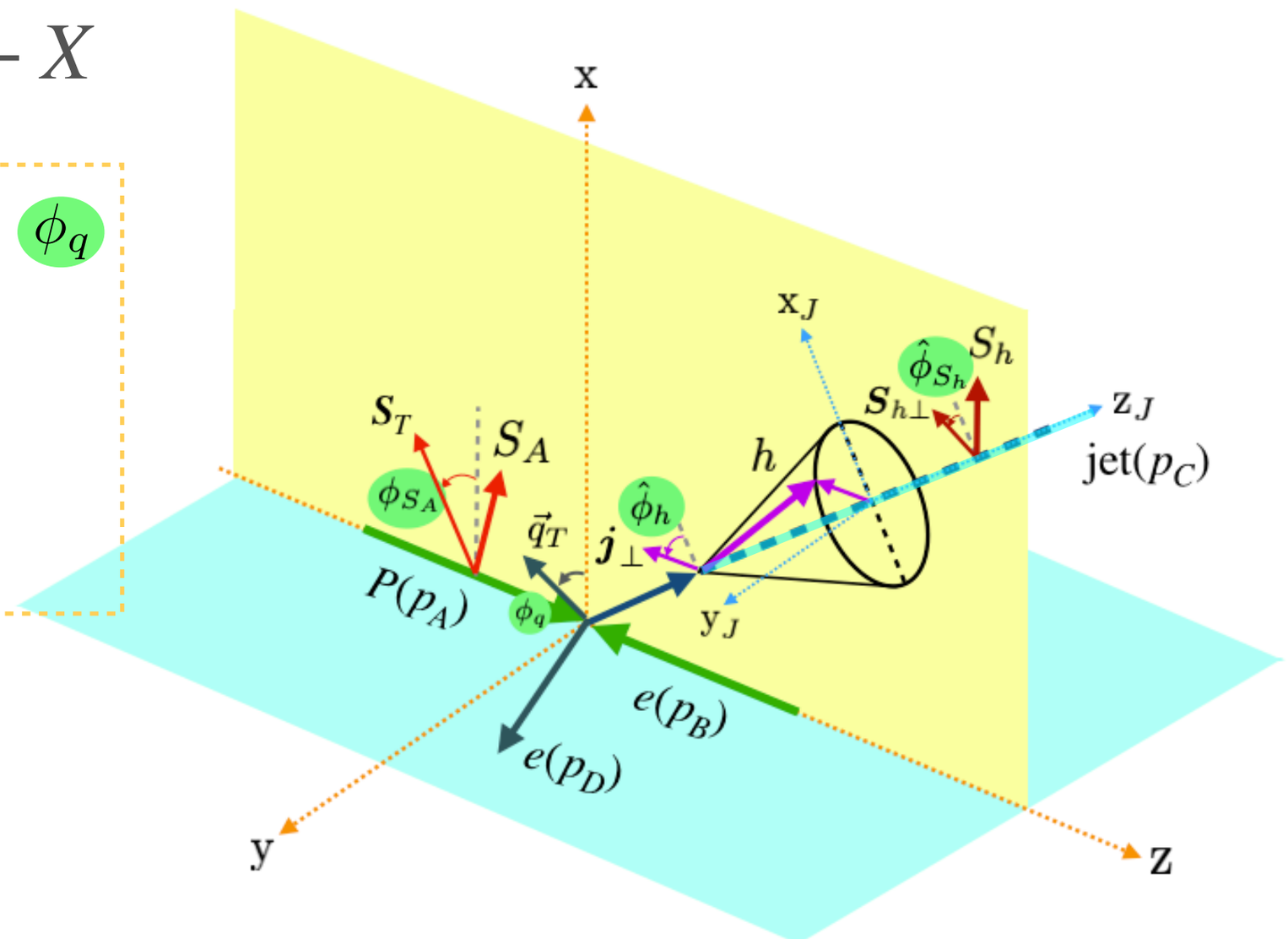
$$\boldsymbol{S}_T \quad \phi_{S_A}$$

$$z_h = \frac{p_{\bar{h}}}{p_{\bar{J}}}$$

$$\boldsymbol{j}_\perp \quad \hat{\phi}_h$$

$$\boldsymbol{S}_{h\perp} \quad \hat{\phi}_{S_h}$$



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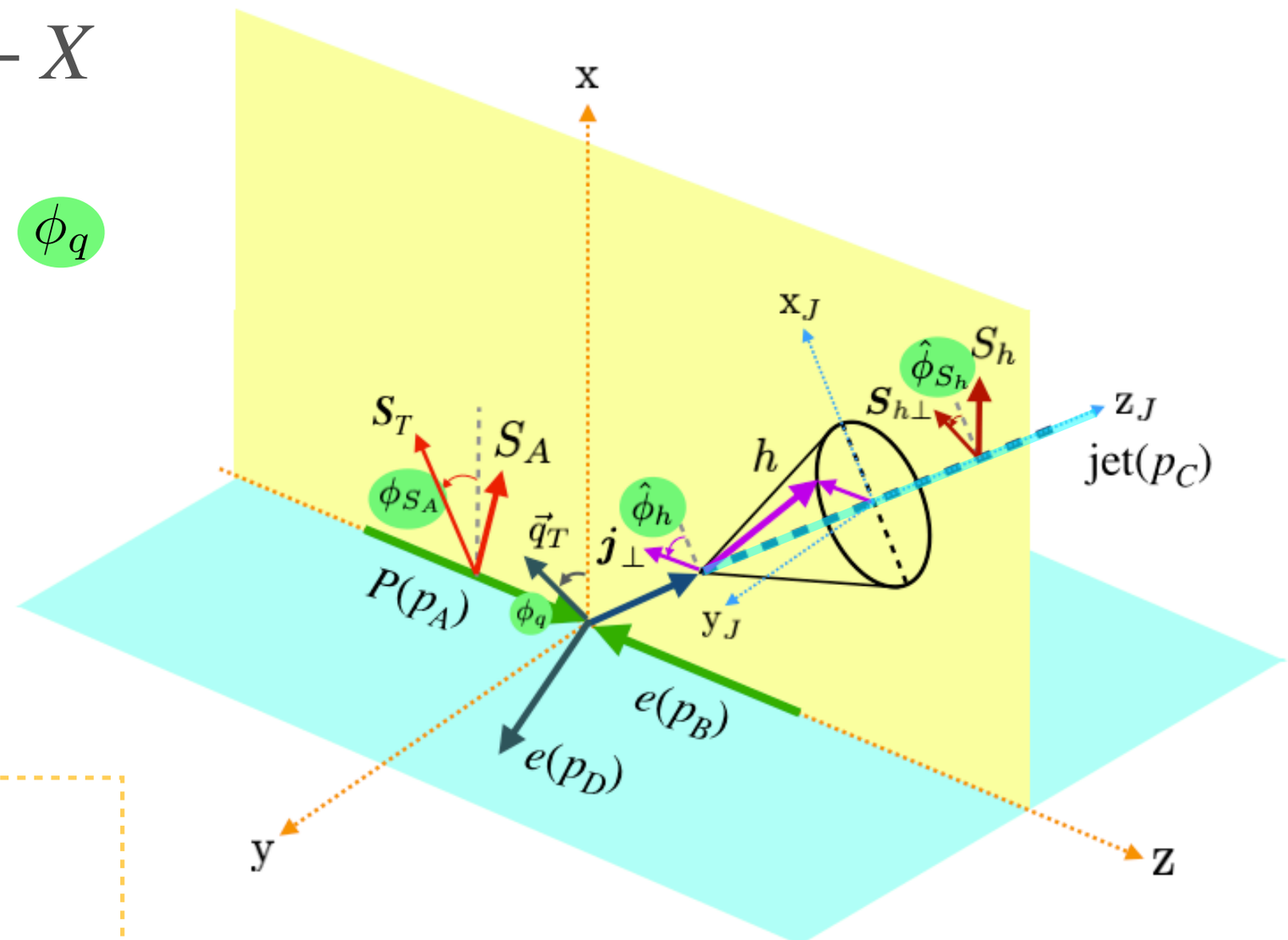

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TMDPDFs

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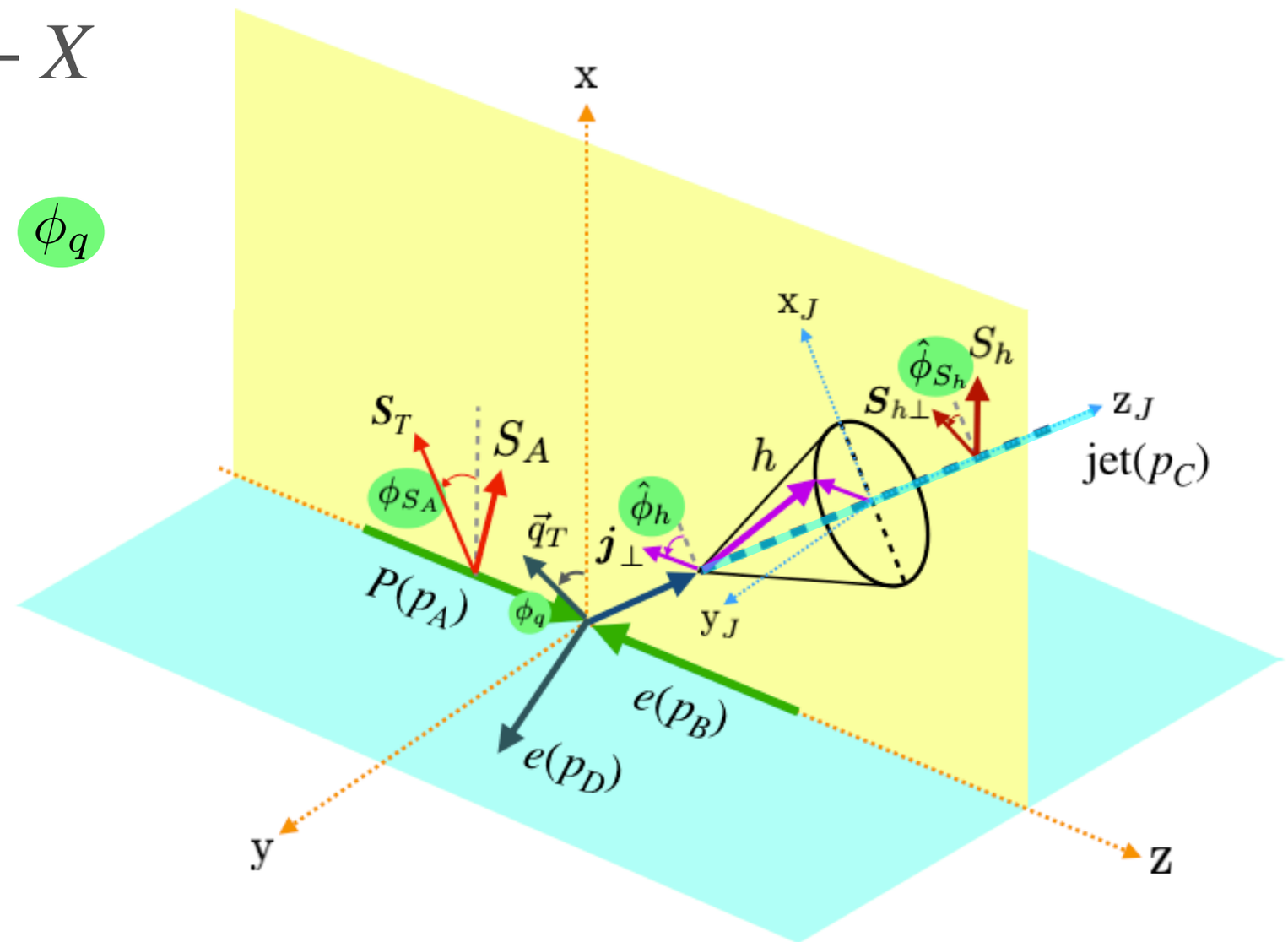
TMDJFFs $\boldsymbol{j}_\perp$ $\hat{\phi}_h$

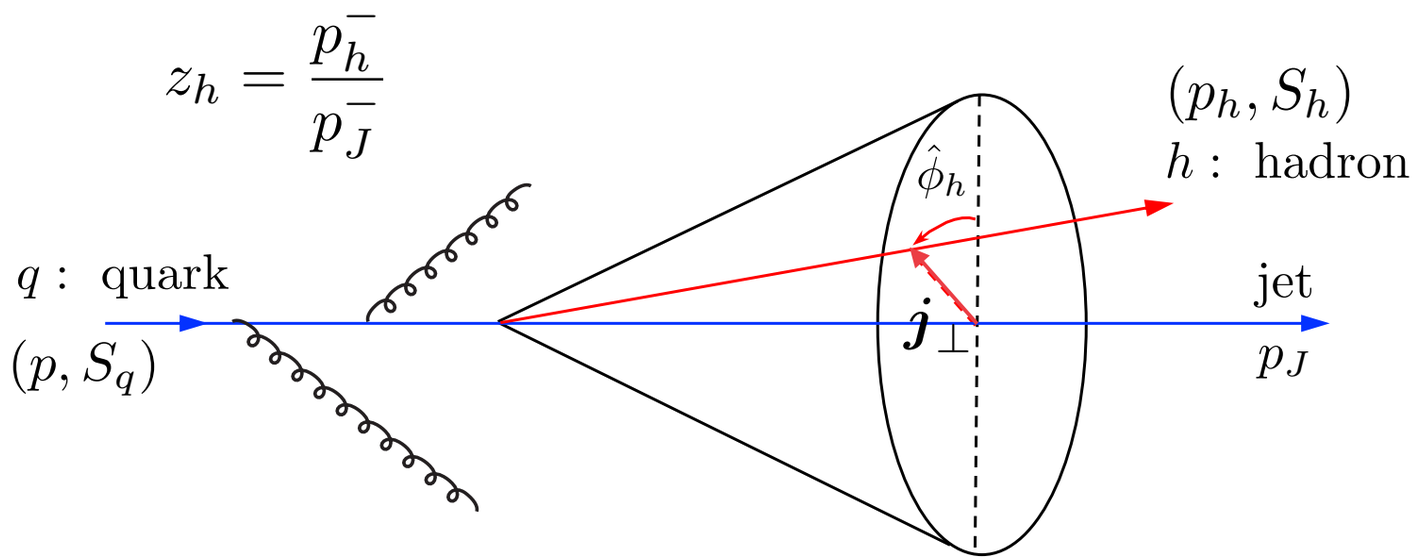
$$\boldsymbol{S}_{h\perp} \quad \hat{\phi}_{S_h}$$



Theoretical framework

TMDJFFs

$$p(p_A, S_A) + e(p_B) \to (\text{jet}(\eta_J, p_{JT}, R)\ h(z_h, j_\perp, S_h)) + e(p_D) + X$$



Leading TMDFFs

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1\mathrm{L}}^\perp$
T	$D_{1\mathrm{T}}^\perp$	$G_{1\mathrm{T}}$	$H_1, H_{1\mathrm{T}}^\perp$

Leading TMDJFFs

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\ h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{h/q}$
T	$\mathcal{D}_{1T}^{\perp\ h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}, \mathcal{H}_{1T}^{\perp\ h/q}$

Transverse momentum dependent FFs/JFFs for quarks. Here U, L, and T represent unpolarized, longitudinally, and transversely polarized state

Example

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)} + \cdots$$

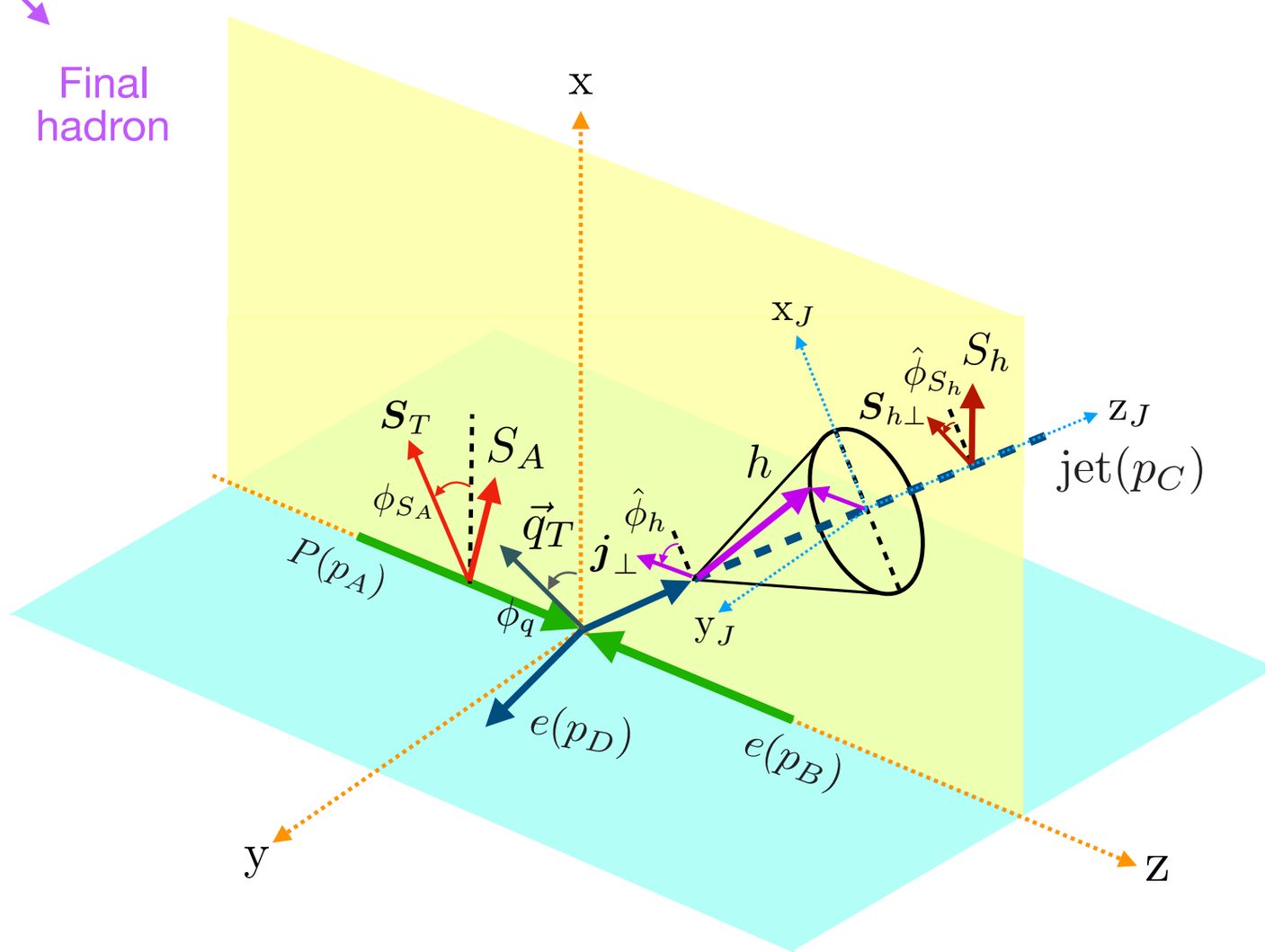
$$A_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)} = \frac{F_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)}}{F_{UU,U}}$$

$$ep \to e + \mathrm{jet}(h) + X$$

incoming proton

incoming electron

Final hadron



Example

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$$A_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)} = \frac{F_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)}}{F_{UU,U}}$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

$$F_{UU,U}^{\cos(\phi_q - \hat{\phi}_h)} \sim h_1^\perp \otimes \mathcal{H}_1^{\perp\ h/q}$$

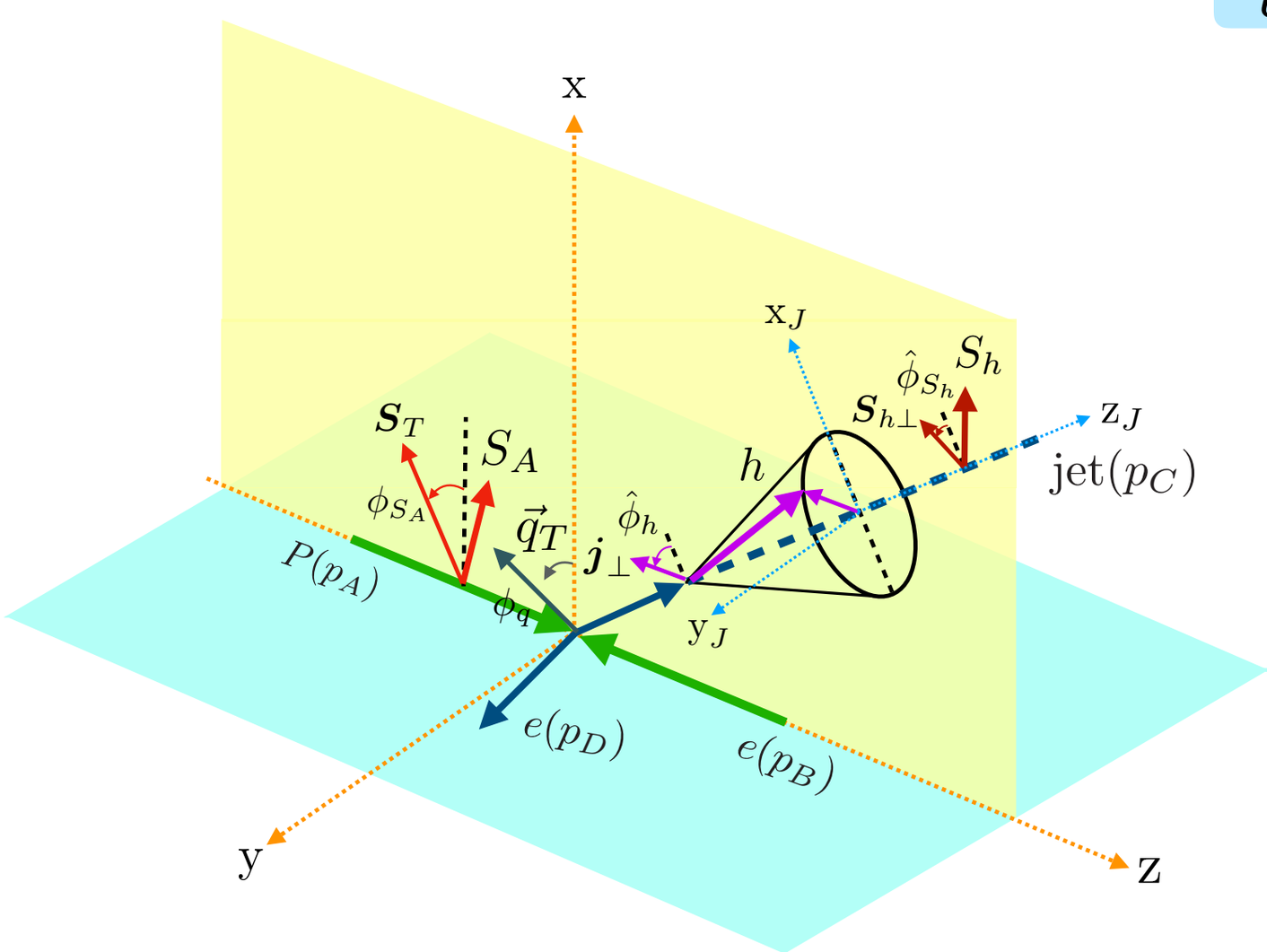
Boer-Mulders function

arXiv: [0912.5194]

match to

Collins function

arXiv: [1505.05589]



$$ep \to e + \text{jet}(h) + X$$

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\ h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{h/q}$
T	$\mathcal{D}_{1T}^{\perp\ h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}$, $\mathcal{H}_{1T}^{\perp\ h/q}$

TMDJFFs for quarks.

Example

$$A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} = \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}}{F_{UU,U}}$$

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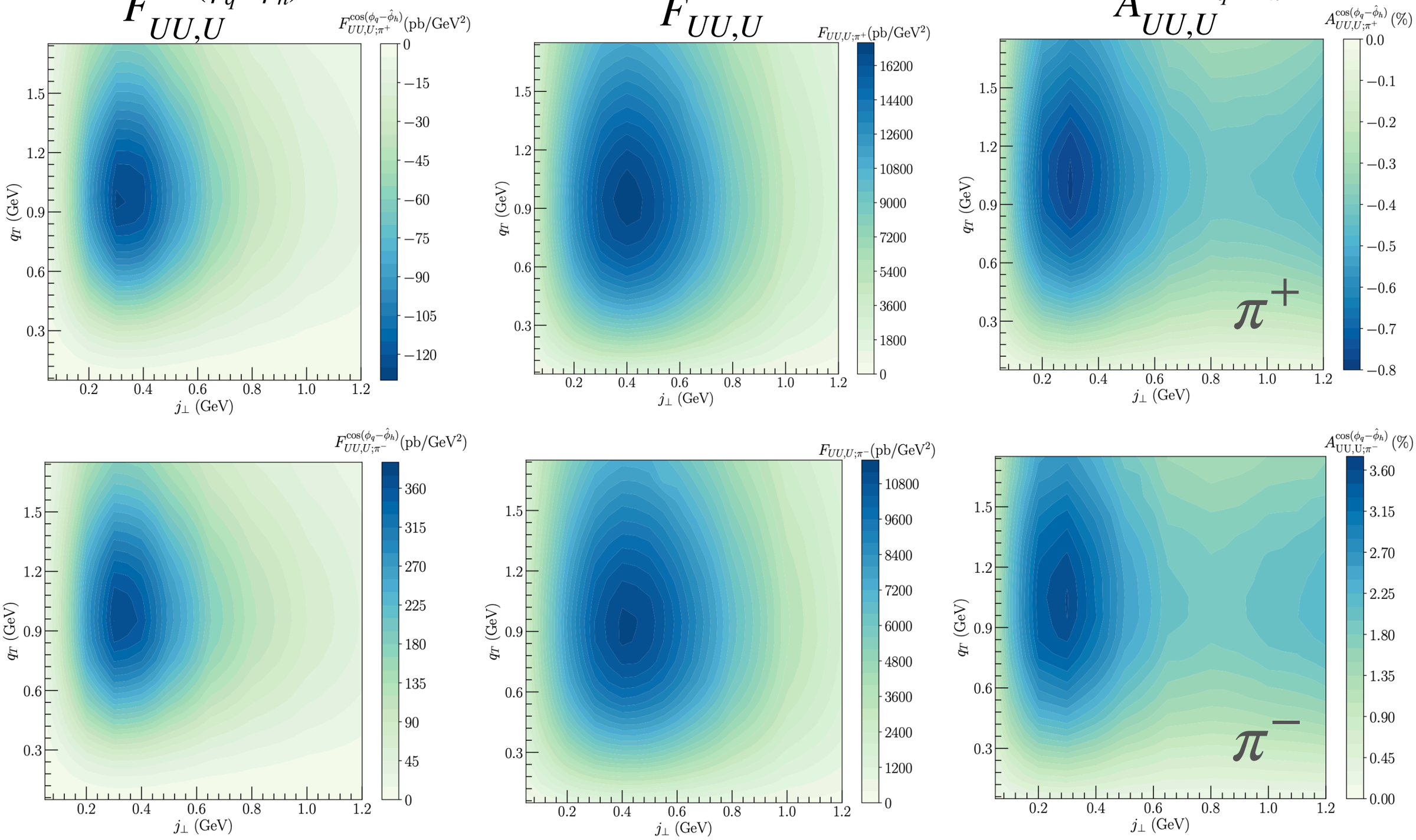

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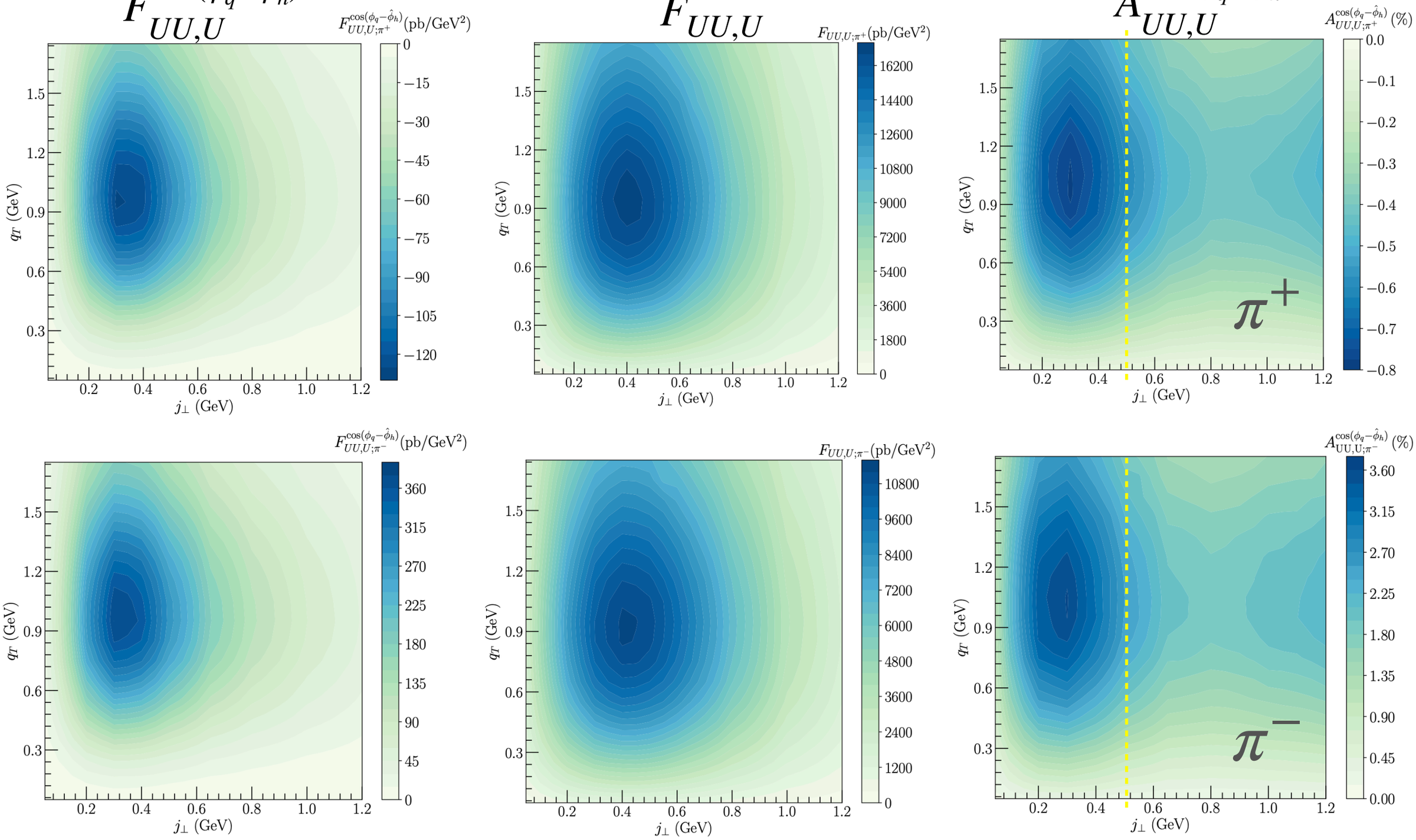

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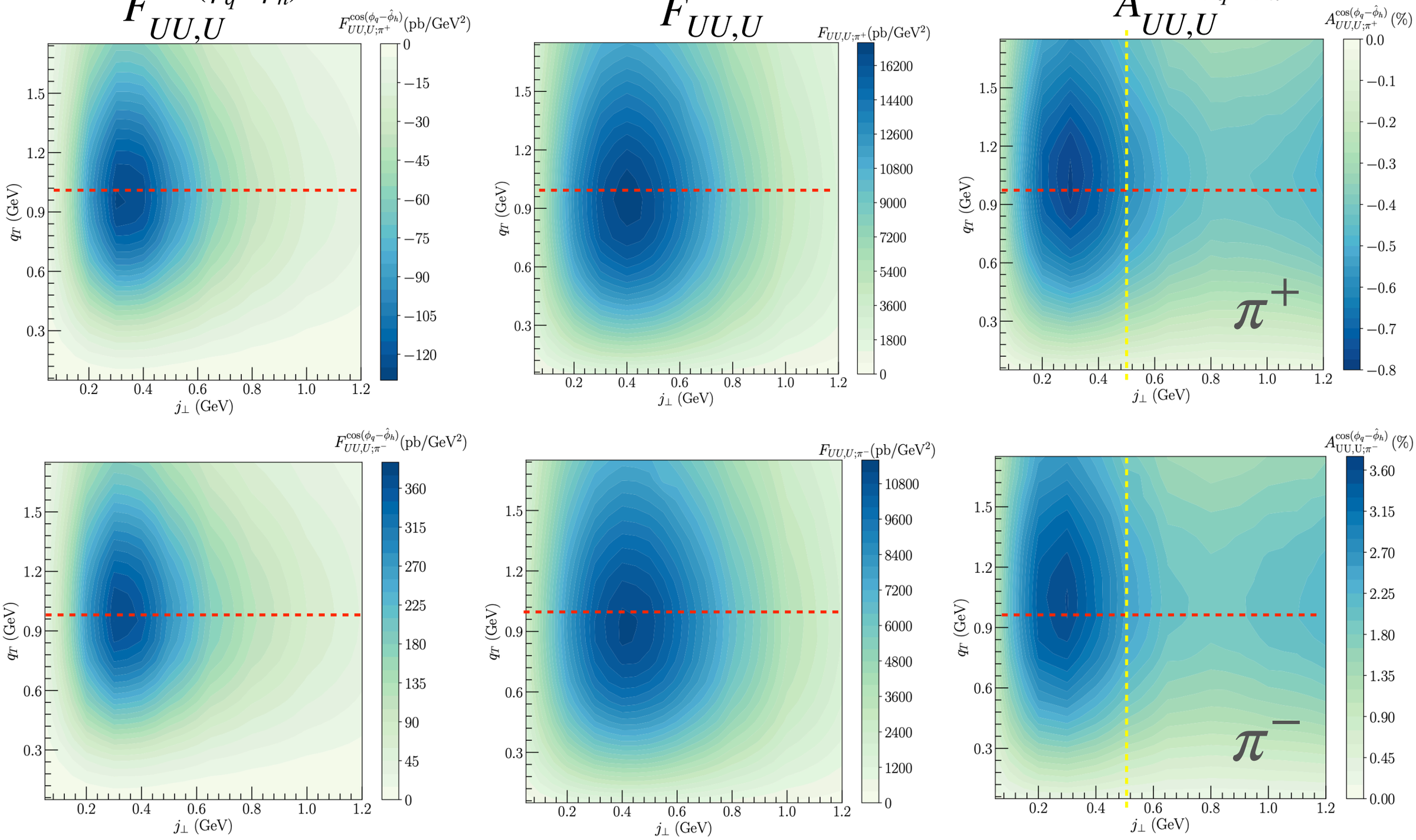

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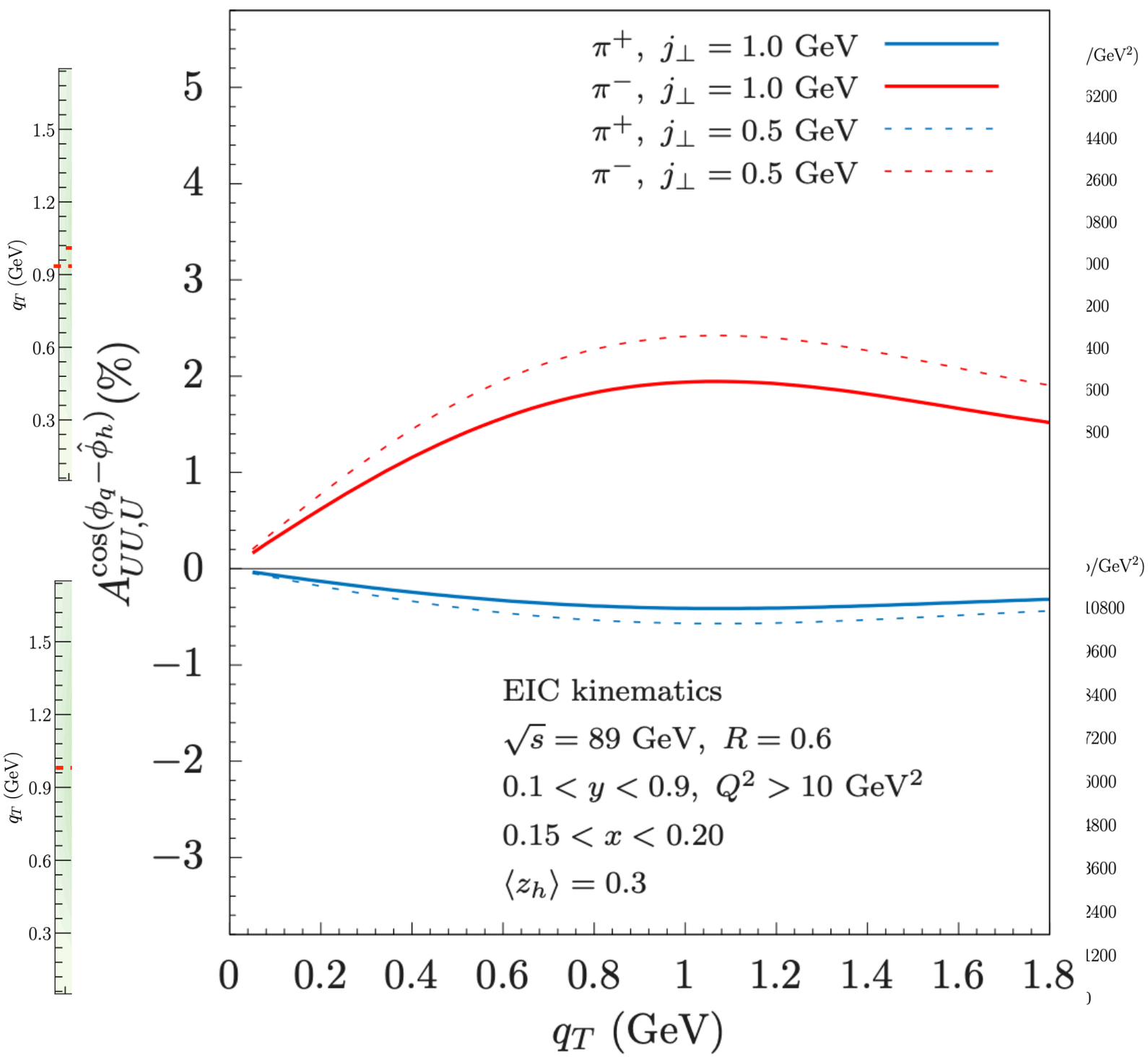

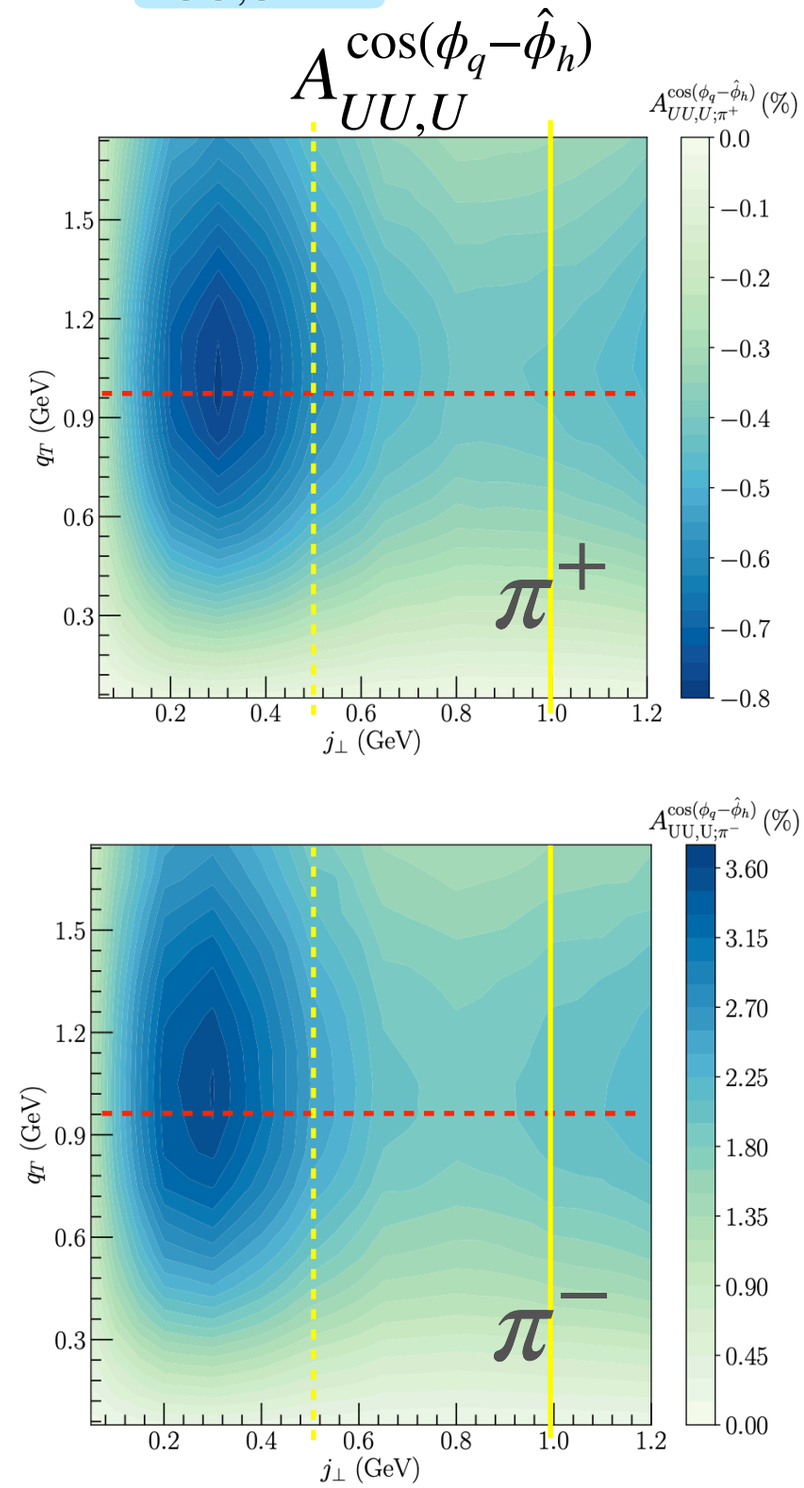

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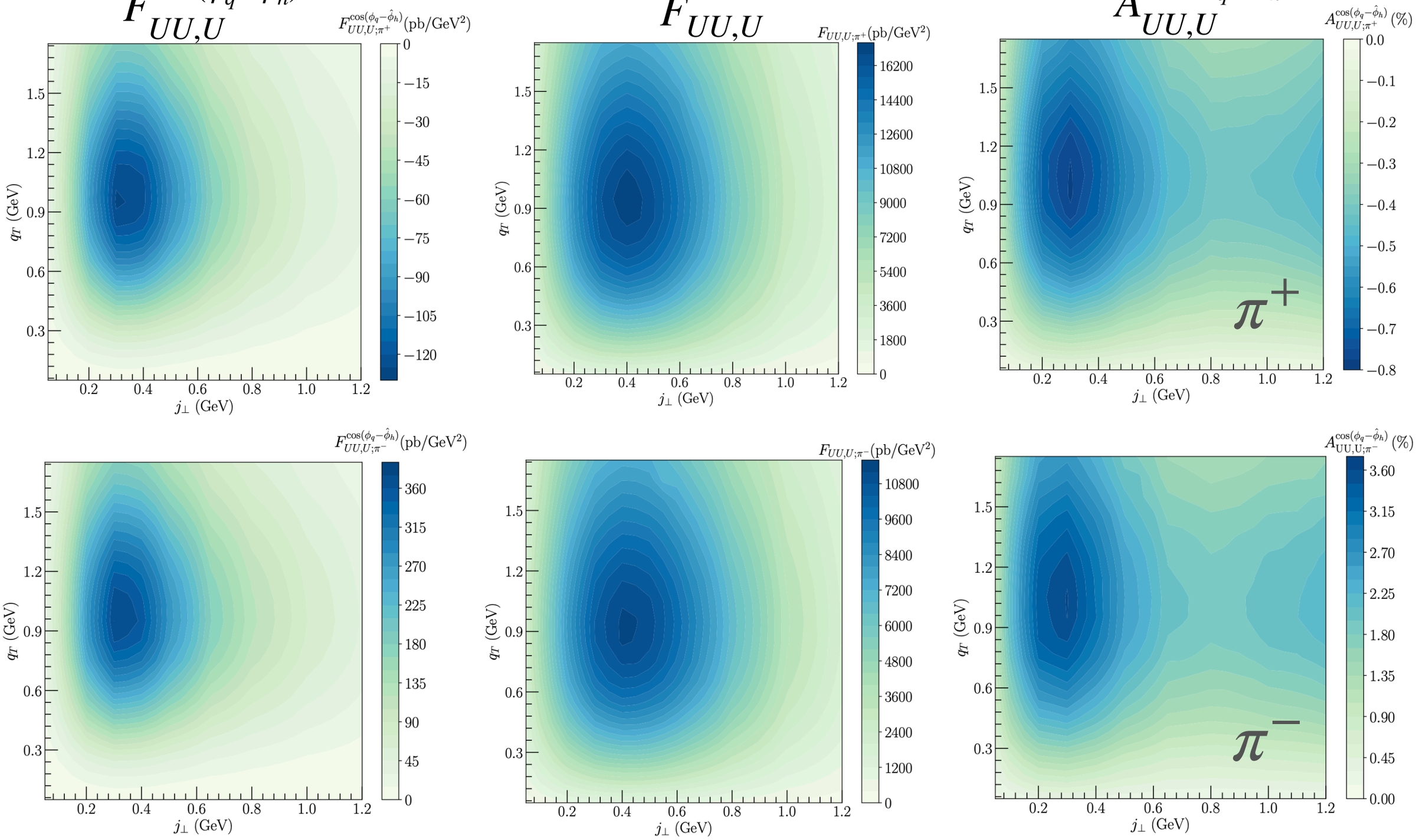

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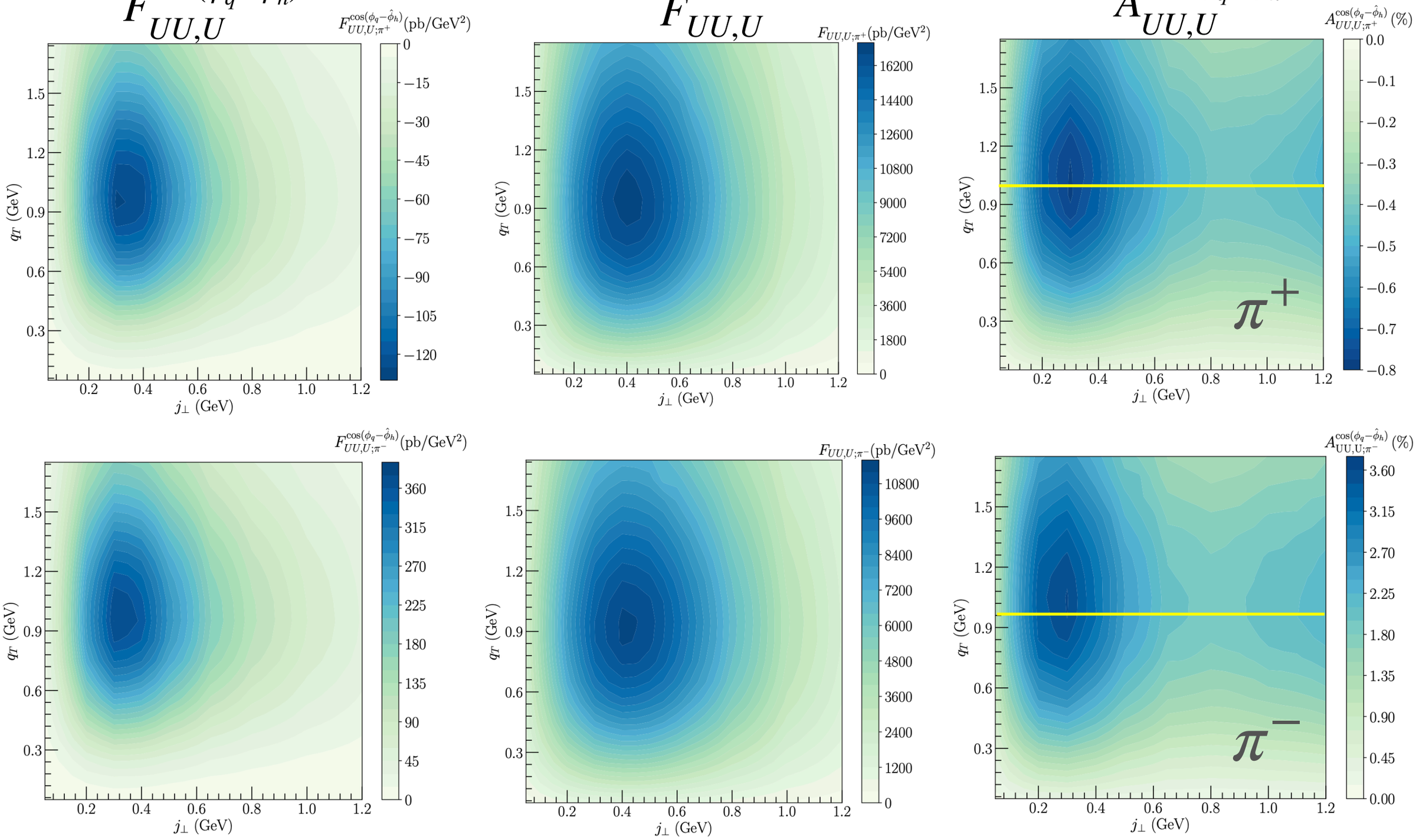

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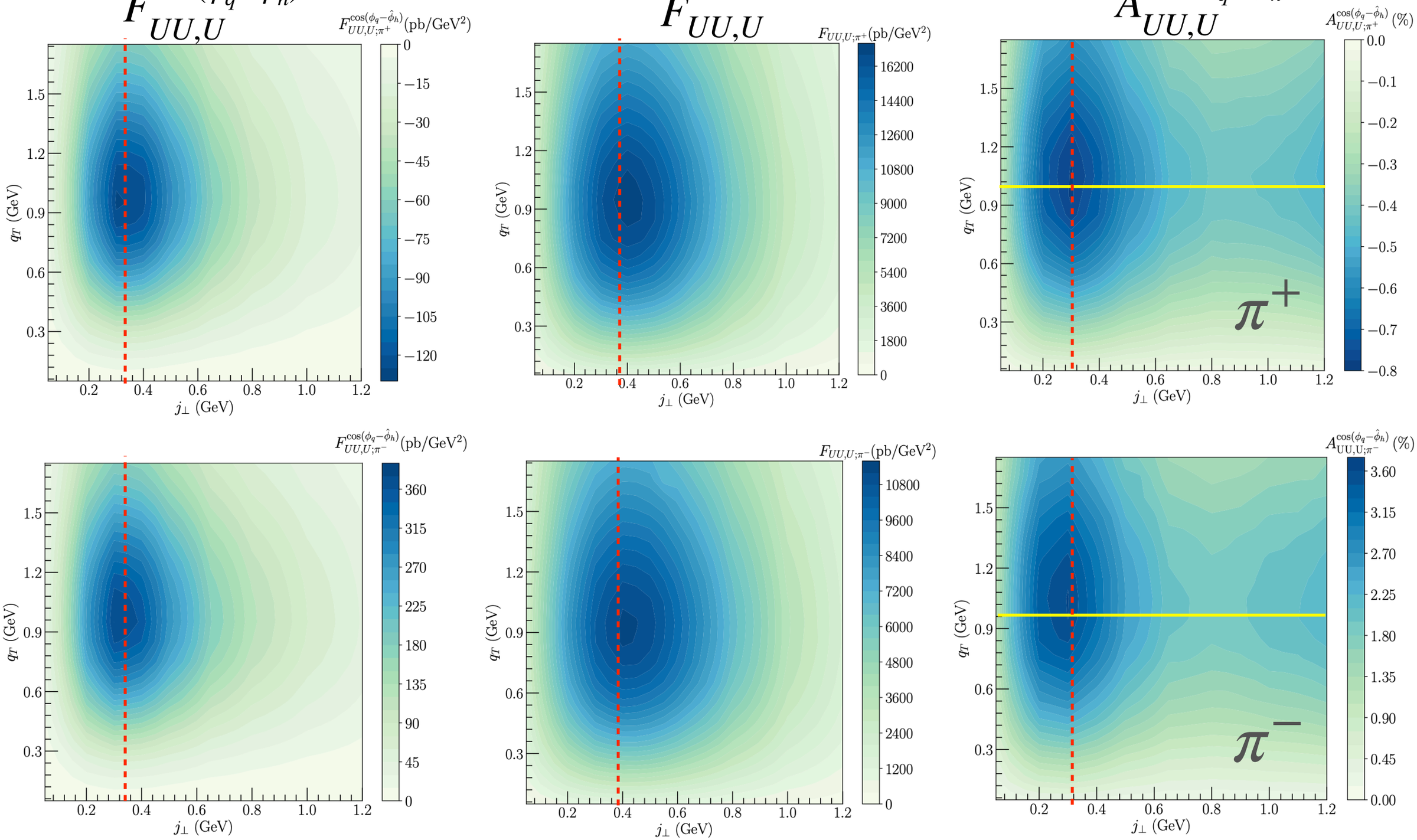

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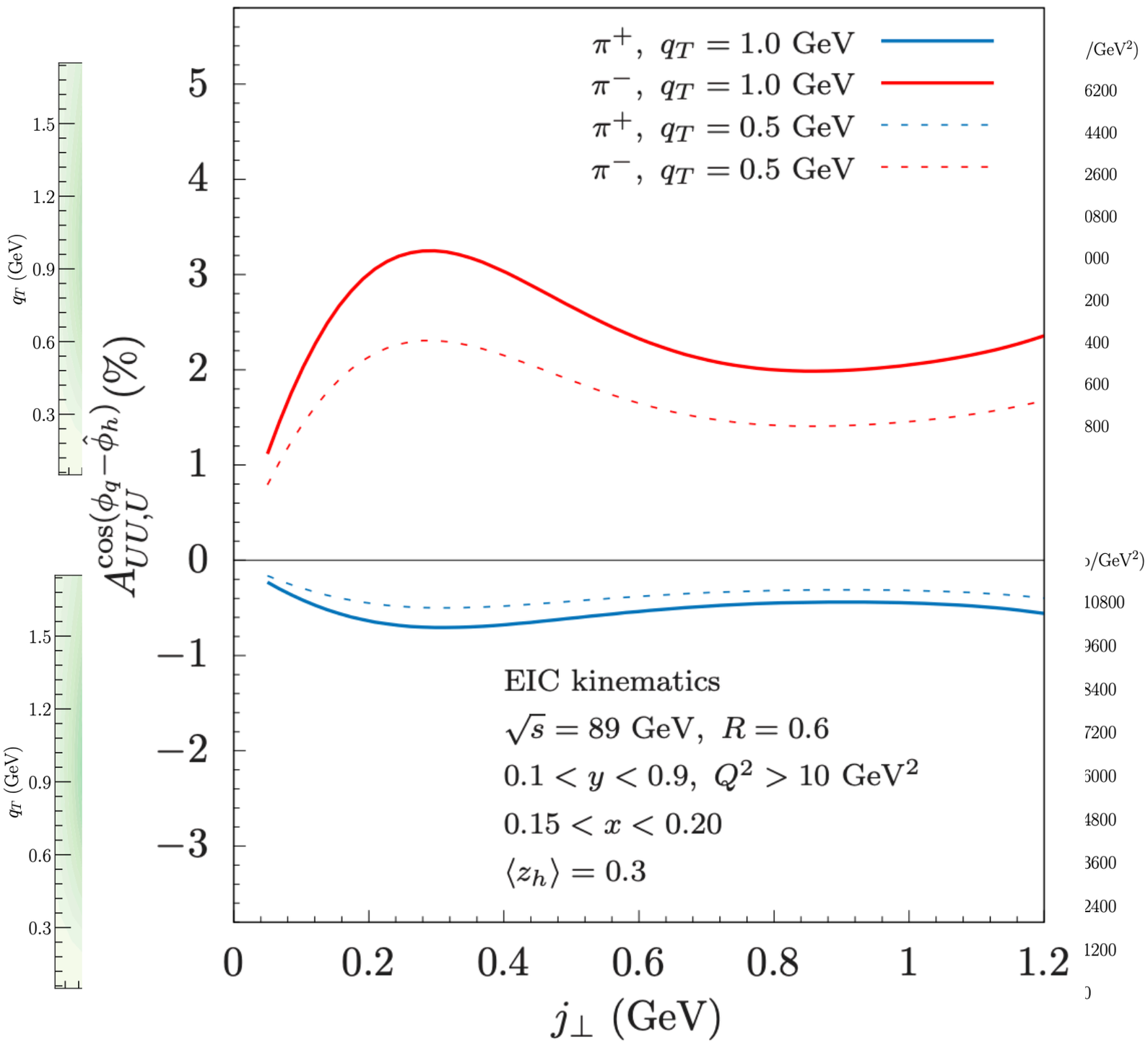
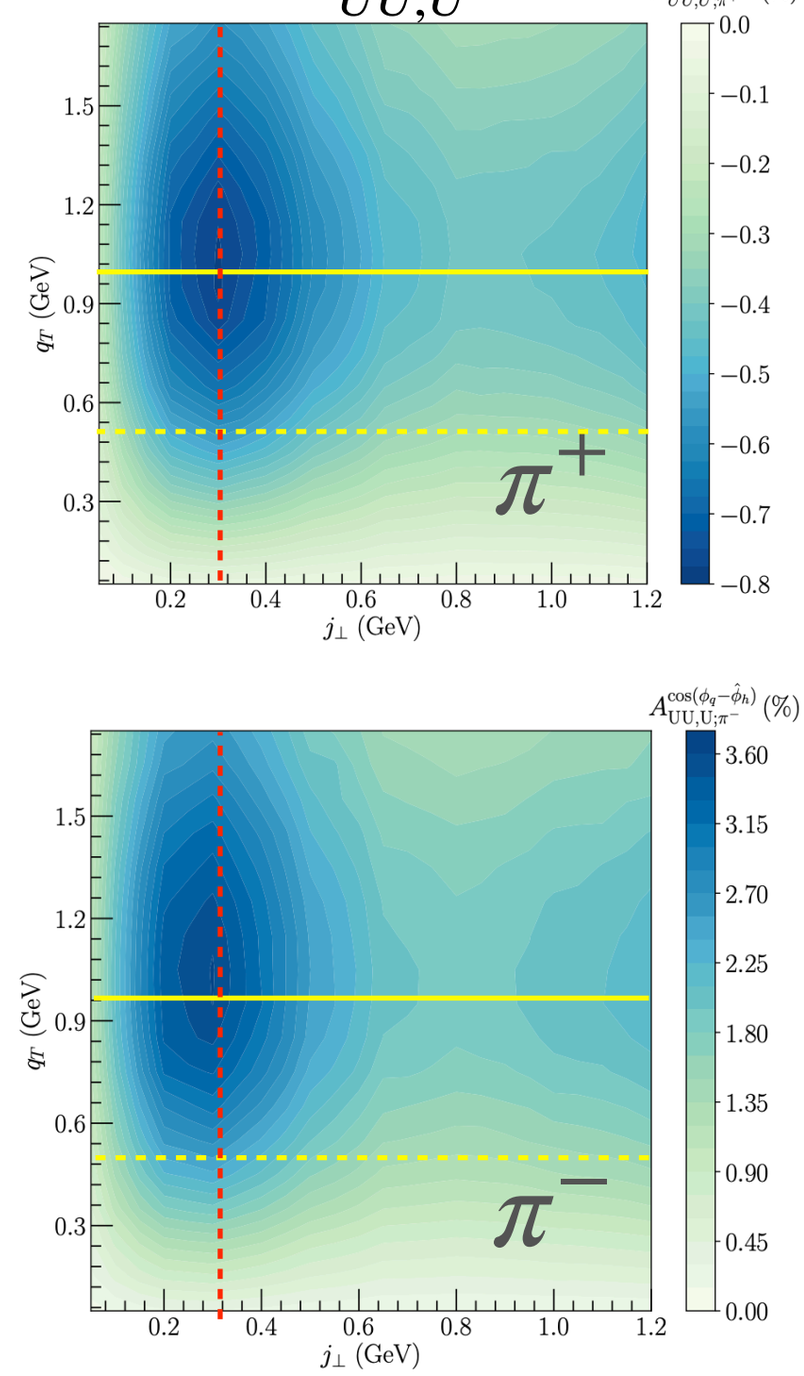
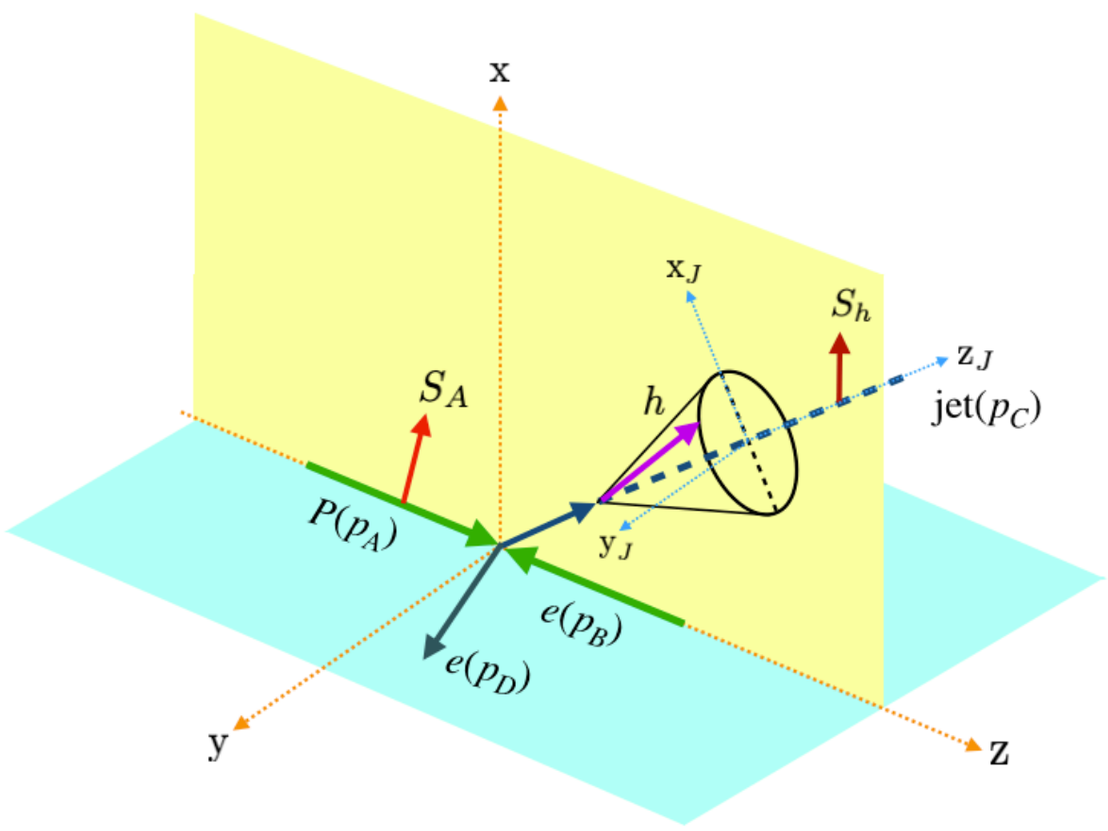


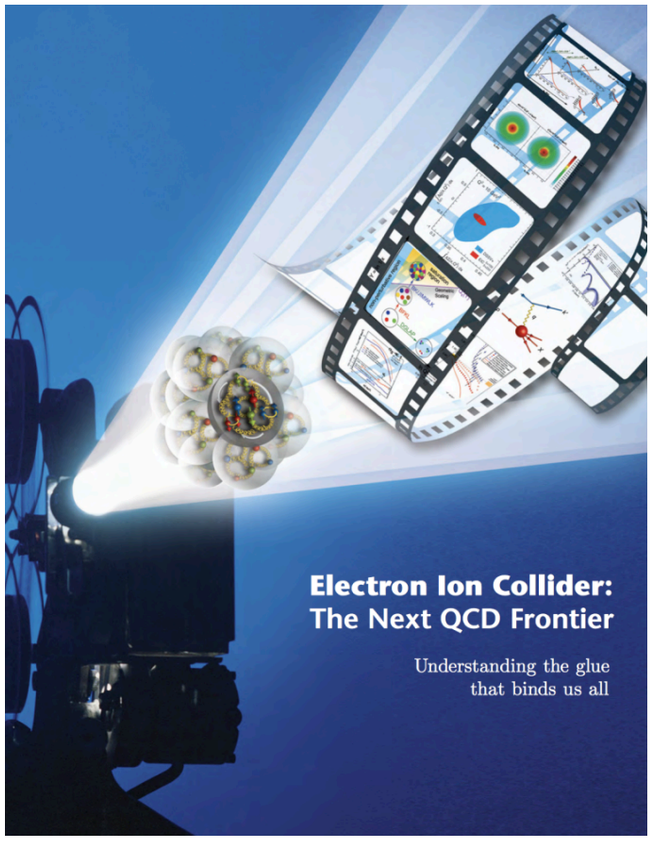
$A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}$



Summary & Outlook

- In summary, we have developed the theoretical framework for all spin asymmetries in back-to-back e+jet(h) productions.
- Sizable asymmetry can be measured with EIC kinematics.
- Open new and exciting opportunities in the direction of studying spin-dependent hadron structures





$$
\begin{aligned}
&\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet } h(S_h))+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+ \Lambda_p \Big\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \Big\} \\
&+ S_T \Big\{ \sin(\phi_q - \phi_{S_A}) F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL,U}^{\cos(\phi_q-\phi_{S_A})} \\
&\quad + \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \Big\} \\
&+ \Lambda_h \Big\{ \lambda_e F_{UL,L} + \sin(\hat{\phi}_h - \phi_q) F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \Lambda_p \Big[F_{LU,L} + \cos(\hat{\phi}_h - \phi_q) F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)} \Big] \\
&\quad + S_T \Big[\cos(\phi_q - \phi_{S_A}) F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e \sin(\phi_q - \phi_{S_A}) F_{TL,L}^{\sin(\phi_q-\phi_{S_A})} \\
&\quad + \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \Big] \Big\} \\
&+ S_{h\perp} \Big\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&\quad + \sin(\hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} \\
&\quad + \Lambda_p \Big[\cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \Big] \\
&\quad + S_T \Big[\cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})} \\
&\qquad + \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_q - \phi_{S_A}) F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})} \\
&\qquad + \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_q - \phi_{S_A})) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A})} \\
&\qquad + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad\quad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\qquad + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)} \\
&\qquad + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \Big] \Big\}
\end{aligned}
$$

Thank you

Example 2

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots \right.$$

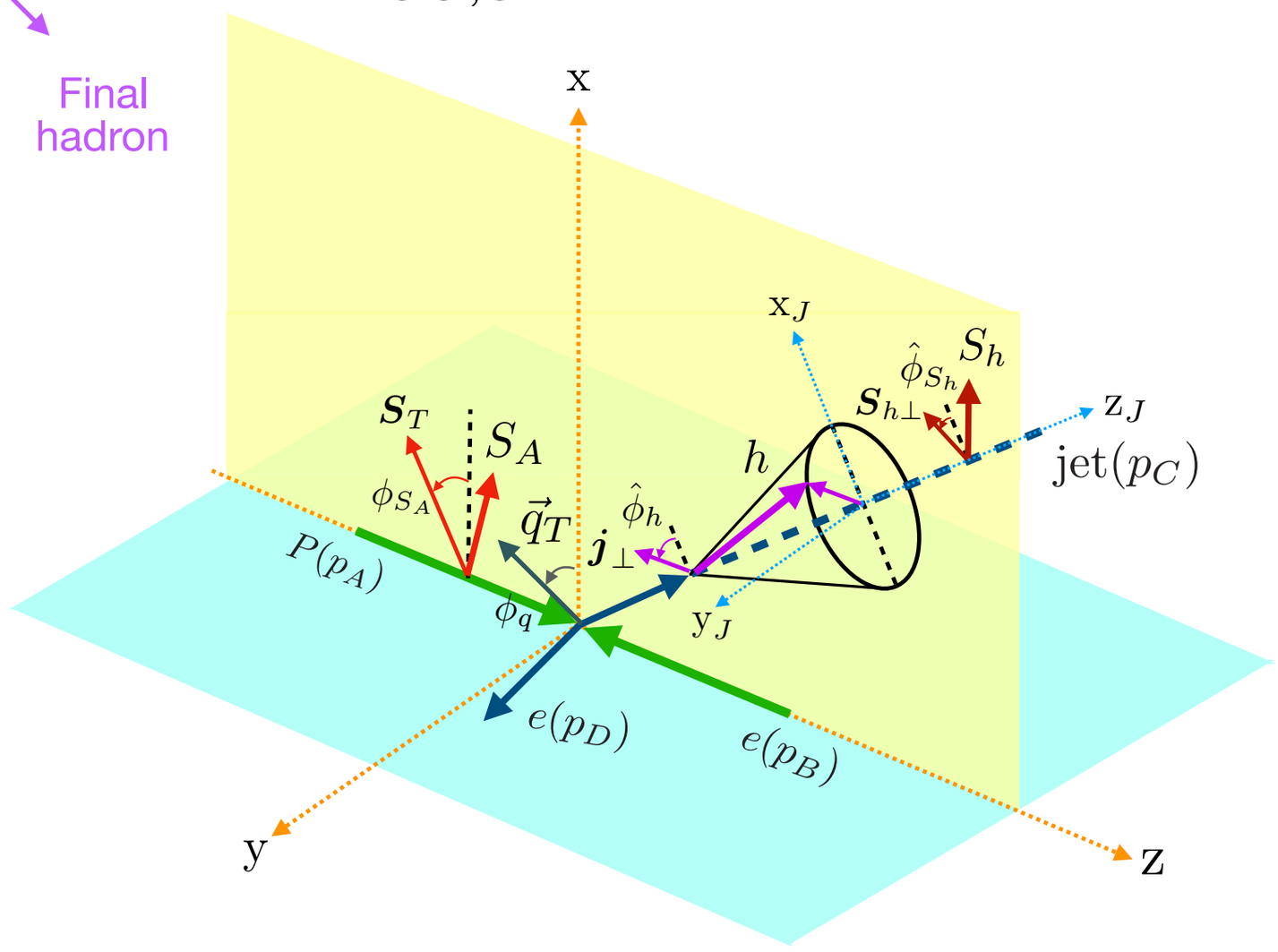
$$A_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}$$

$$ep \to e + \mathrm{jet}(h^\uparrow) + X$$

incoming proton

incoming electron

Final hadron



Example 2

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots \right.$$

$$A_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

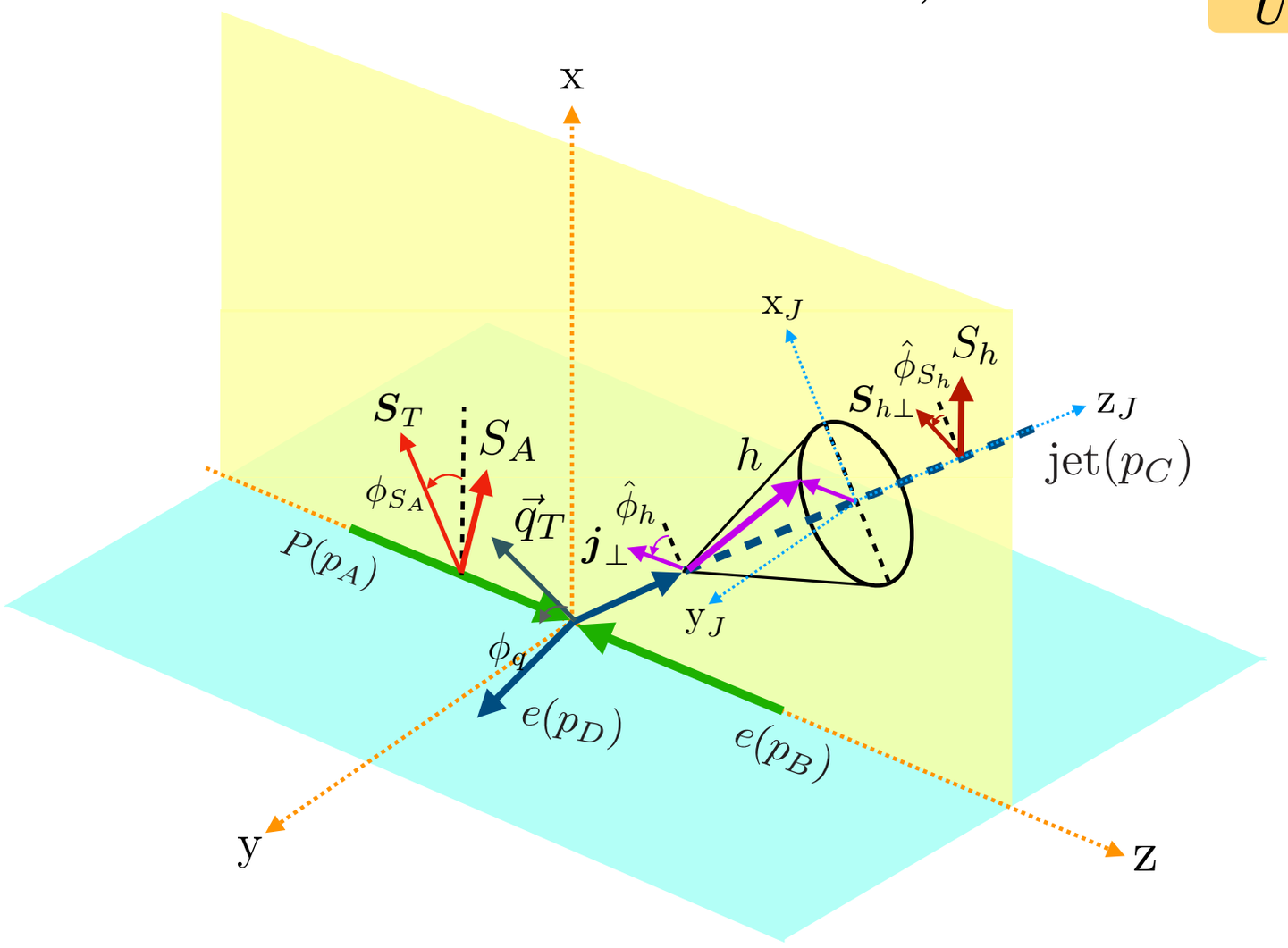
$$F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\, h/q}$$

Unpolarized TMDPDF

match to

Polarizing TMDFF

arXiv: [2003.04828]



$$ep \to e + \text{jet}(h^\uparrow) + X$$

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\, h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{h/q}$
T	$\mathcal{D}_{1T}^{\perp\, h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}$, $\mathcal{H}_{1T}^{\perp\, h/q}$

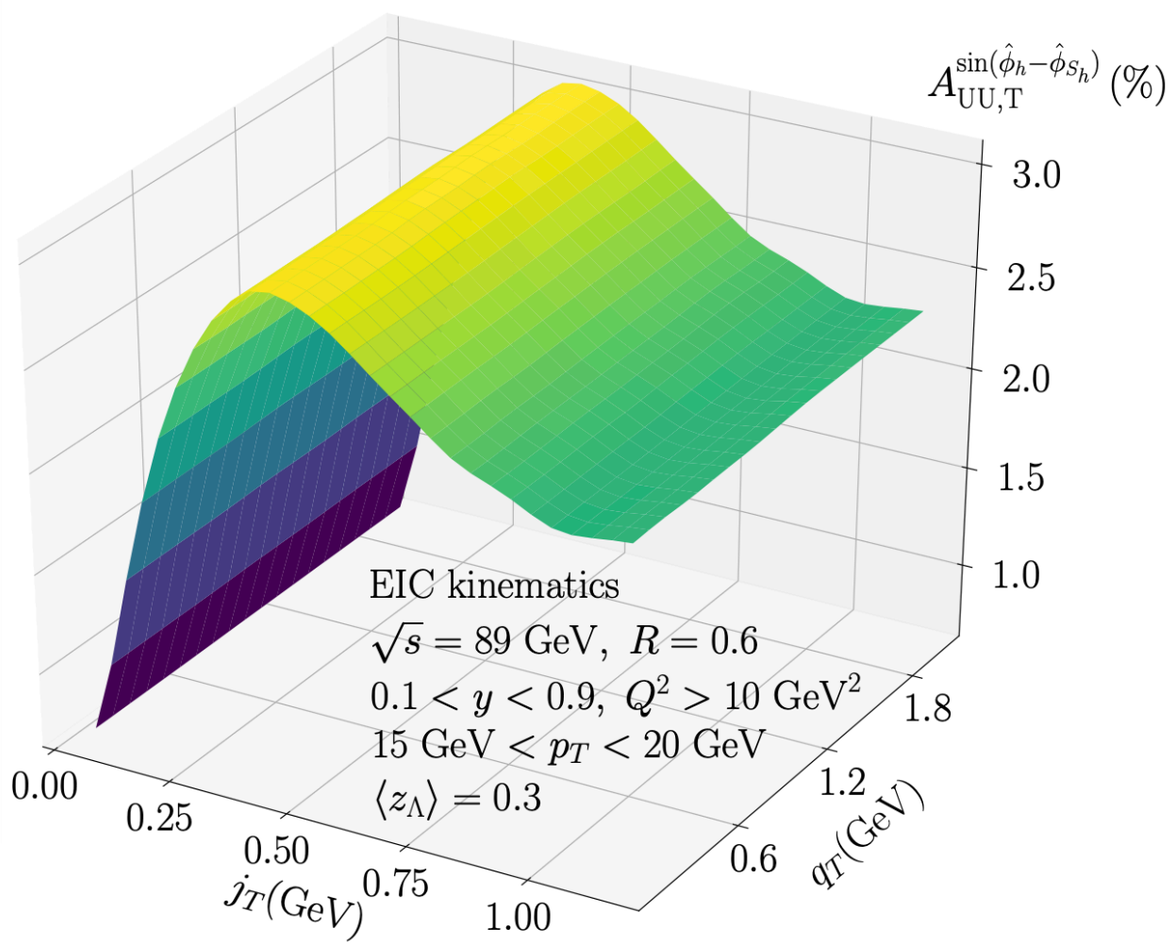
TMDJFFs for quarks.

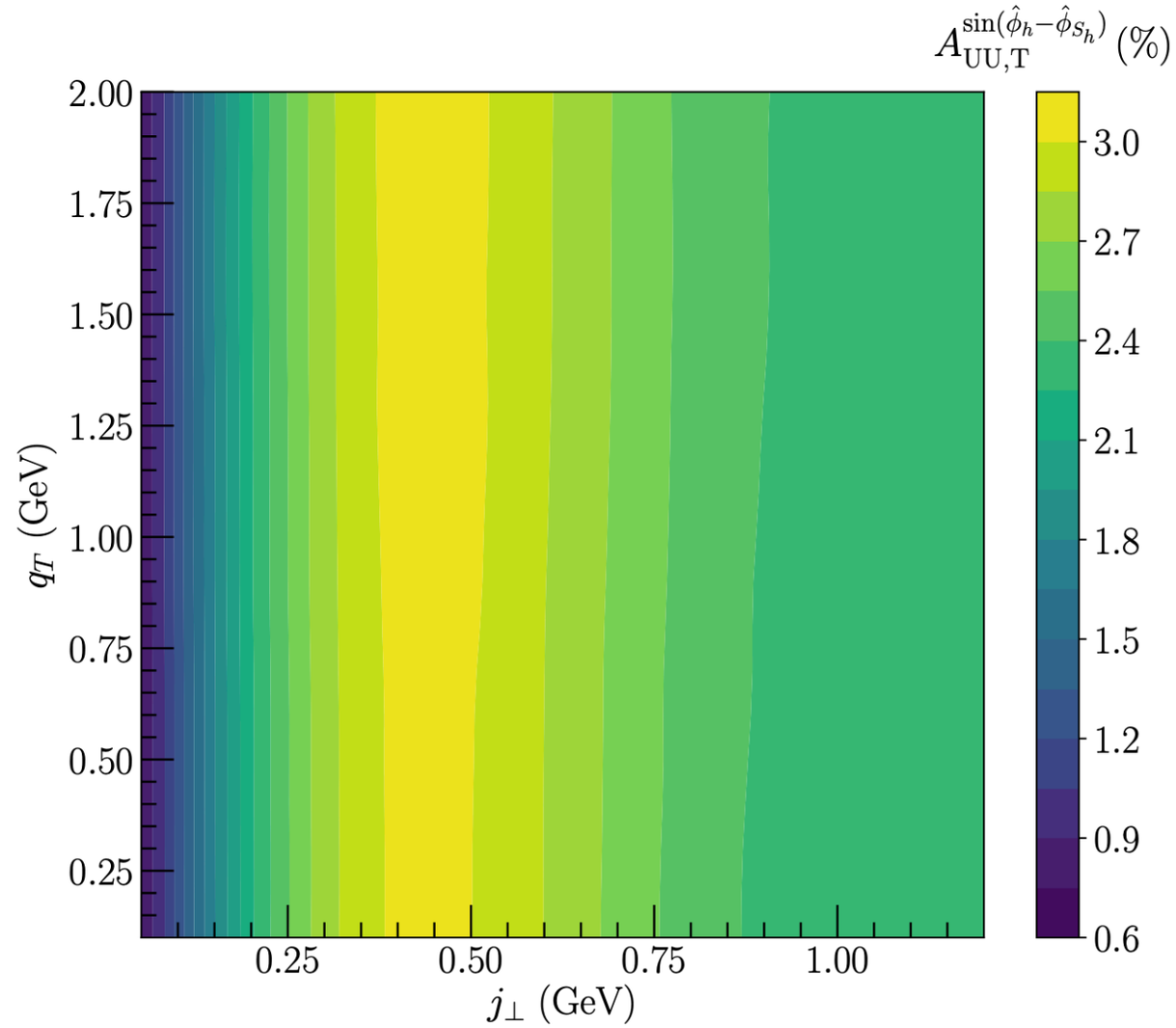
Example 1 $A_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} = \dfrac{F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})}}{F_{UU,U}}$

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{\sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cdots\right.$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

$$F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\ h/q}$$



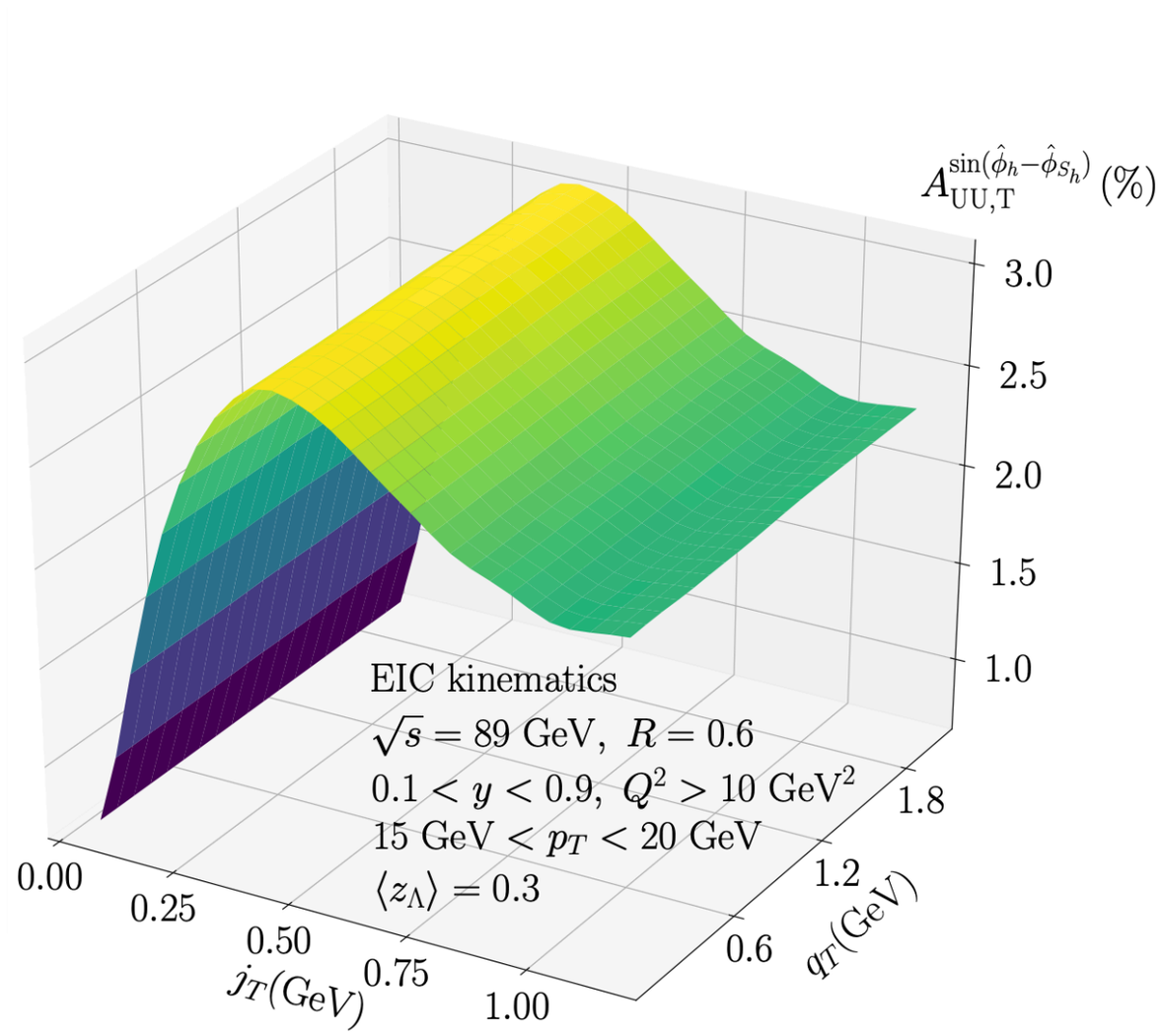


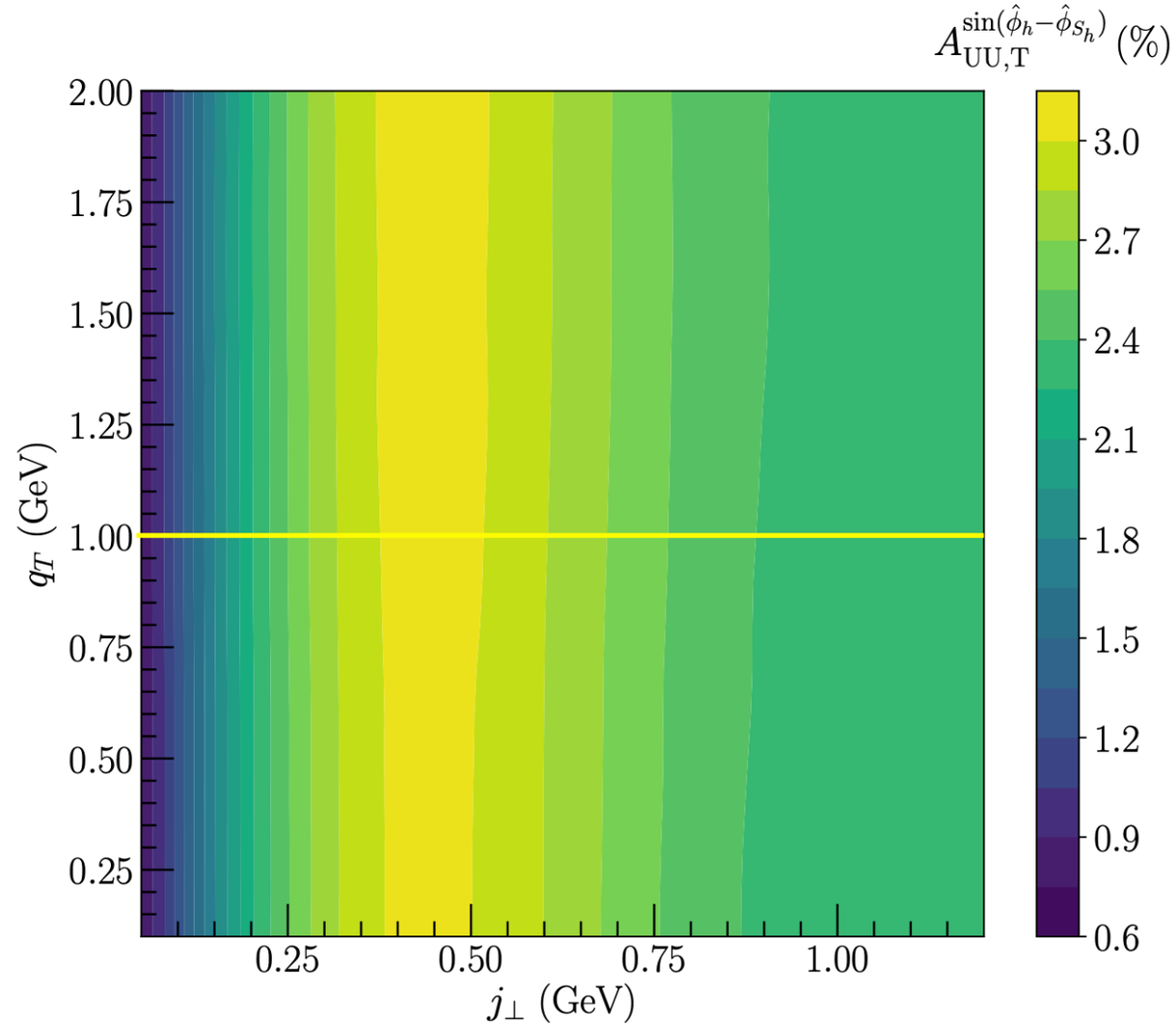
Example 1 $A_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} = \dfrac{F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})}}{F_{UU,U}}$

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{\sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cdots\right.$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

$$F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\ h/q}$$





Example 1 $A_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} = \dfrac{F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})}}{F_{UU,U}}$

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{\sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cdots\right.$$

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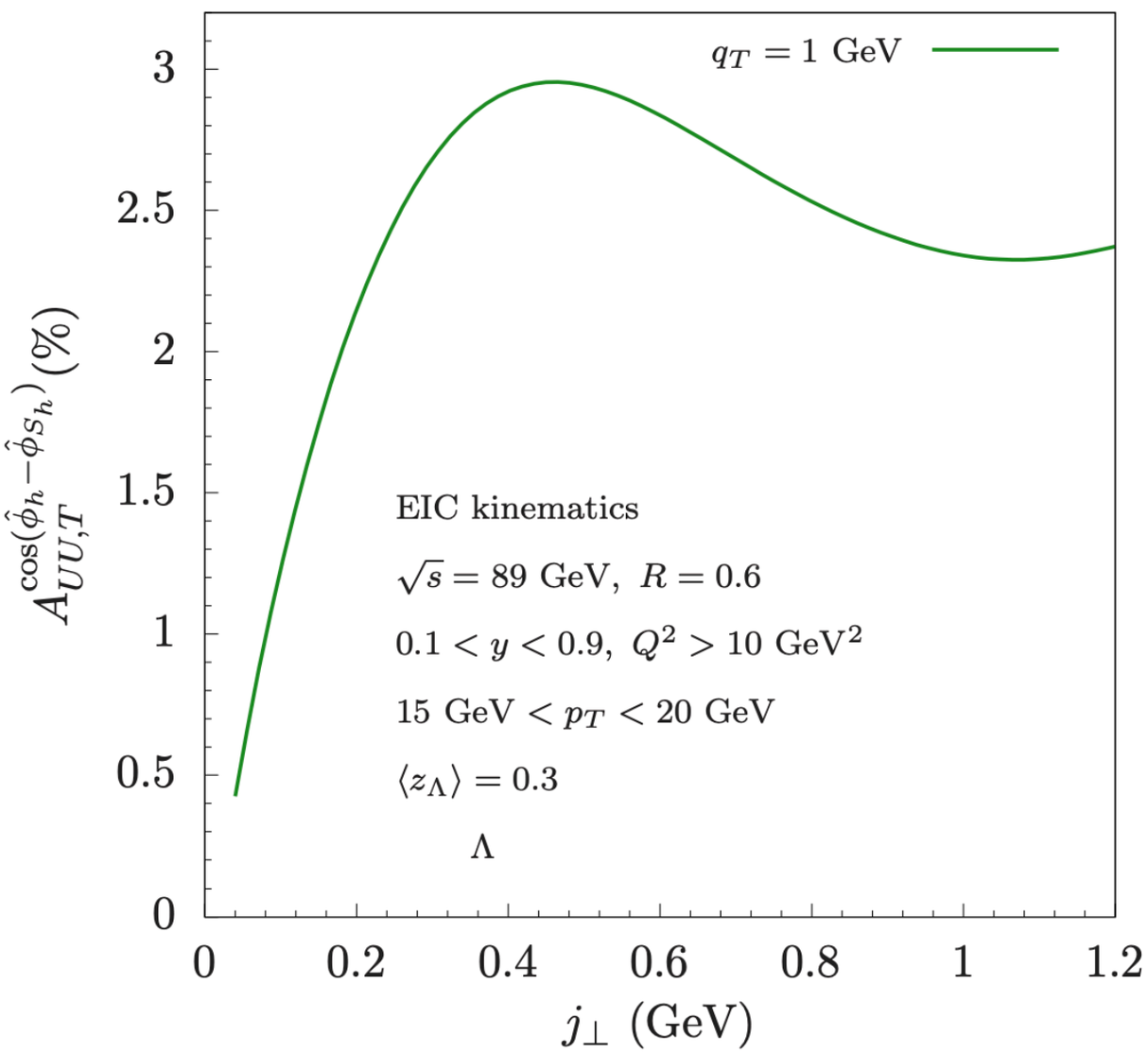
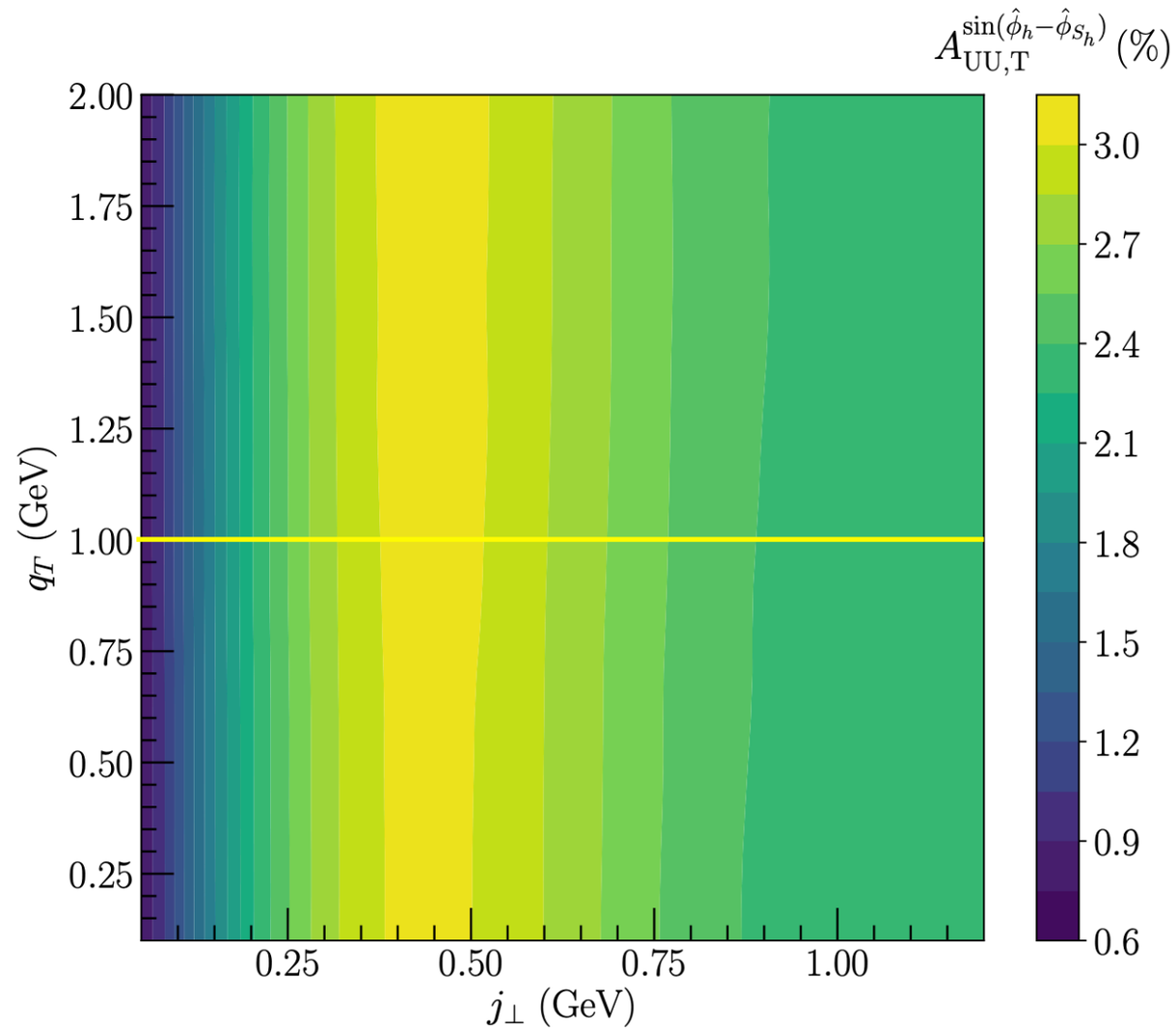
$$F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\, h/q}$$





Theoretical framework

TMDJFFs

$$
\begin{aligned}
&\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+ \Lambda_p \Big\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \Big\} \\
&+ S_T \Big\{ \sin(\phi_q - \phi_{S_A}) F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL,U}^{\cos(\phi_q-\phi_{S_A})} \\
&\quad + \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \Big\} \\
&+ \Lambda_h \Big\{ \lambda_e F_{UL,L} + \sin(\hat{\phi}_h - \phi_q) F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \Lambda_p \Big[F_{LU,L} + \cos(\hat{\phi}_h - \phi_q) F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)} \Big] \\
&\quad + S_T \Big[\cos(\phi_q - \phi_{S_A}) F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e \sin(\phi_q - \phi_{S_A}) F_{TL,L}^{\sin(\phi_q-\phi_{S_A})} \\
&\quad + \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \Big] \Big\} \\
&+ S_{h\perp} \Big\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&\quad + \sin(\hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} \\
&\quad + \Lambda_p \Big[\cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \Big] \\
&\quad + S_T \Big[\cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})} \\
&\quad + \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_q - \phi_{S_A}) F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})} \\
&\quad + \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_q - \phi_{S_A})) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A})} \\
&\quad + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\quad + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)} \\
&\quad + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \Big] \Big\},
\end{aligned}
$$

$$ep \to e + \text{jet}(h) + X$$

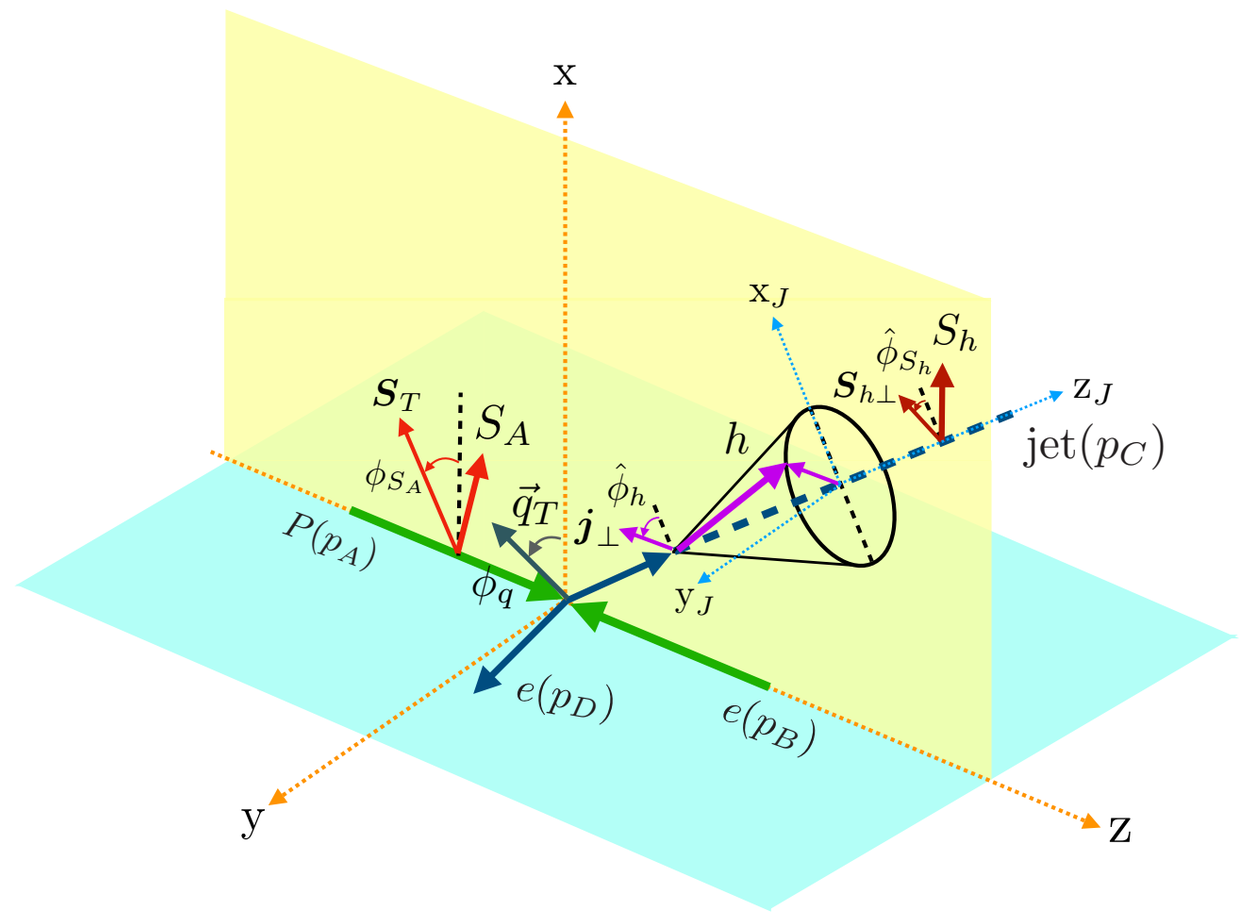


Illustration for the distribution of hadrons inside jets produced with an electron in the collisions of a polarized proton and en electron.