Soft and hard aspects of QCD dynamics in hadronic form factors

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Workshop “The Future of Color Transparency and Hadronization Studies at Jefferson Lab and Beyond”
Jefferson Lab, 7-8 June 2021

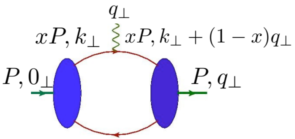
Hadronic form factors

- Hadronic form factors: Observation: $(1/Q^2)^{n_q-1}$ counting rules for a hadron made of n_q quarks
- Exclusive-inclusive connection: Observation: Parton distributions behave like $(1-x)^{2n_q-3}$
- Expectation: some fundamental/easily visible reason
- Warning: For every complex problem there is an answer that is clear, simple, and wrong

Soft mechanism

- Early idea: Feynman mechanism/Drell-Yan formula [PRL 70]

$$F(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \Psi^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\Psi(x, \mathbf{k}_\perp)$$



- Take region where both $\Psi_M(x, \mathbf{k}_\perp)$ and $\Psi^*_M(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)$ are maximal
  - $|\mathbf{k}_\perp| \sim \Lambda$ is small and
  - $\bar{x} \equiv 1 - x$ is close to 0, so that $|\bar{x}\mathbf{q}_\perp| \sim \Lambda$
- If $|\Psi(x, \Lambda)|^2 \sim (1-x)^{2n_q - 3}$ then

$$F(Q^2) \sim \int_0^{\Lambda/Q} \bar{x}^{2n_q-3} \, d\bar{x} \sim (1/Q^2)^{n_q - 1}$$

$\Rightarrow$ Causal relation: Form of $f(x)$ determines $F(Q^2)$

Hard mechanism

- Another region in DY formula

$$F(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \Psi^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\Psi(x, \mathbf{k}_\perp)$$

  • finite x and small $|\mathbf{k}_\perp|$, e.g., region $|\mathbf{k}_\perp| \ll \bar{x}|\mathbf{q}_\perp|$, where $\Psi(x, \mathbf{k}_\perp)$ is maximal. Then

$$F_M(Q^2) \sim 2\int_0^1 dx \, |\Psi^*(x, \bar{x}\mathbf{q}_\perp)\, \varphi(x)|$$

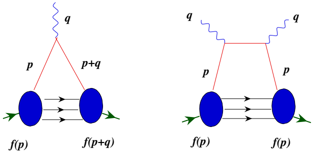
  $\Rightarrow$ form factor repeats large-$\mathbf{k}_\perp$ behavior of WF

- Distribution amplitude

$$\varphi(x) = \int d^2\mathbf{k}_\perp \Psi(x, k_\perp)$$

- Mechanism was proposed by G.B. West [PRL 70] (though in covariant BS-type formalism)

West's model



$$F(Q^2) \sim \int d^4p f(p) f(p+q)$$

- $f(p) = f(t, M^2)$ is a function of $t \equiv p^2$ and spectator mass M^2
- If $f(t, M^2) \sim t^{-n} g(M^2)$, then $F(Q^2) \sim (1/Q^2)^n$

$$\nu W_2(x) \sim \int_{t_{\min}}^{t_{\max} \sim -2\nu} dt f^2(t, M^2) \sim (t_{\min})^{2n-1}$$

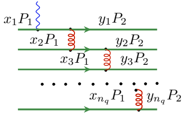
where $t_{\min} = \left(\frac{-x}{1-x}\right)\left[M^2 - (1-x)M_N^2\right]$

$$\Rightarrow \nu W_2(x) \sim (1-x)^{2n-1}$$

DY vs West's model

- DY: Active parton is "on-shell" $p^2 \sim \Lambda^2$
- $F(Q^2)$ reflects the size of phase space in which $1 - x \sim \Lambda/Q$
- West's model: Active parton is highly virtual
- $F(Q^2)$ reflects shape of WF for large virtualities
  $\Rightarrow$ Two mechanisms are completely different
- Surpise:
  $(1/Q^2)^n \Leftrightarrow (1-x)^{2n-1}$ holds in both models!
- NB: In DY model, n is not necessarily integer
- NB: In West's model, $(1/Q^2)^n$ and $(1-x)^{2n-1}$
  have the same cause, but not "causing" each other

Hard mechanism & pQCD



- Integer n naturally appear in hard model: reflects number of hard propagators
- Hard exchange in a theory with dimensionless coupling constant gives $n = n_q - 1$ [BF 73]
- Consequence of scale invariance [MMT 73]
- QCD: $(\alpha_s/Q^2)^{n_q-1}$
- Suppression: $F_\pi(Q^2) \to (2\alpha_s/\pi)s_0/Q^2$ $[s_0 = 4\pi^2 f_\pi^2 \approx 0.7\,\mathrm{GeV}^2]$
- Known: $\alpha_s/\pi \sim 0.1$ is penalty for an extra loop
- VMD: $F_\pi(Q^2) \to m_\rho^2/Q^2$ with $m_\rho^2 \approx 0.6\,\mathrm{GeV}^2 \approx s_0$

Perturbative QCD for Pion EM Form Factor

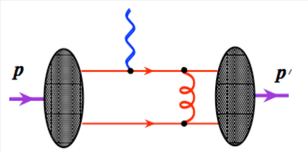
- First application of pQCD to exclusive processes (AR (1977); Jackson PhD Thesis (1977) for "Abelian QCD")
- Light-cone pion Distribution Amplitude $\varphi_\pi(\xi) = f_\pi a(\xi)$ introduced

$$F_\pi^{\text{lead}}(Q^2) = F_\pi^A(Q^2) = 16\pi\alpha_s(Q^2)\,\frac{f_\pi^2}{Q^2}\,\frac{C_2(R)}{N_c}\,|\gamma(Q^2)|^2,$$

$$\gamma = \int_0^1 a(\xi, Q^2)\,d\xi/(1-\xi^2) \qquad \left|\int_0^1 a(\xi, Q^2)\,d\xi\right| = 1.$$

- Hard scenario: small-size configurations dominate large-Q^2 behavior
- ERBL evolution results in $a(\xi, Q^2) \to \frac{3}{2}(1-\xi^2)$ as $Q^2 \to \infty$
- $f_\pi = 93$ MeV

Hard gluon exchange diagram

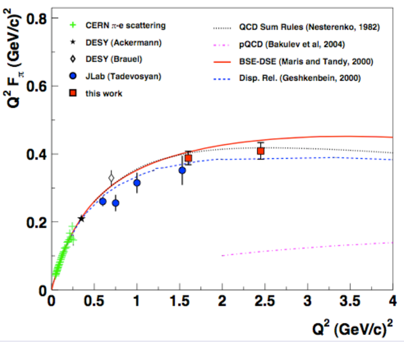


$$F_\pi^{\text{as}}(Q^2) = \frac{16\pi f_\pi^2 \alpha_s(Q^2)}{Q^2}$$

Pion Form Factor Data and pQCD

- With asymptotic pion DA $\varphi_\pi^{as}(\alpha)$, hard pQCD contribution to $F_\pi(Q^2)$ is less than 1/3 of experimental value
- Nonperturbative soft mechanism dominates for available Q^2

Experimental Data on Pion Form Factor



QCD Sum Rules for Pion Form Factor

- Ioffe and Smilga (1982); V. A. Nesterenko and AR (1982)

- a) pQCD diagram
- b) LO QCD SR diagram
- c) 2-loop QCD SR diagram
- (c) Converts into pQCD diagram for large Q^2
- (c) is $\sim \alpha_s/\pi$ of (b) for small and moderate Q^2

- Dashed line: Borel parameter $M^2 = 1.8\,\mathrm{GeV}^2$
- Solid line: $M^2 = \infty$
- Corresponds to local duality

QCD Sum Rules for Pion Form Factor

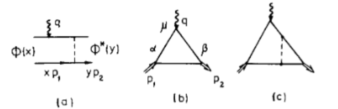


Fig. 1. Diagrams relevant to calculation of the pion form factor in QCD: (a) asymptotic perturbative QCD diagram; (b) lowest-order diagram of the QCD sum rule approach; (c) one of two-loop diagrams of the QCD sum rule approach.

Comparison with data

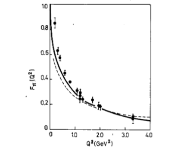


Fig. 3. Comparison between experimental data (taken from ref. [17]) and our theoretical predictions based on eq. (10): $M^2 = \infty$ (solid line); $M^2 = 1.8\ \mathrm{GeV}^2$ (broken line).

Local Quark-Hadron Duality

- The lowest state (pion) is dual to perturbative spectral density

$$f_\pi^2 F_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2\, \rho^{\rm pert}\left(s_1, s_2, q^2\right)$$

- Analytic expression (NR (1982))

$$F_\pi^{\rm local\ duality}(Q^2) = 1 - \left(1 + 6s_0/Q^2\right) / \left(1 + 4s_0/Q^2\right)^{3/2}$$

- Asymptotically $F_\pi^{\rm LD,LO} = 6s_0^2/Q^4$, but it behaves like $1/Q^2$ at moderate Q^2
- Estimate for NLO local duality term

$$\frac{1}{\pi^2 f_\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2\, \Delta\rho^{\rm pert}(s_1, s_2, q^2) = \frac{\alpha_s}{\pi} \left(1 + Q^2/2s_0\right)^{-1}$$

- NLO term has $\sim s_0/Q^2$ behavior suppressed by $\sim \alpha_s/\pi$ factor

Local Quark-Hadron Duality for Proton

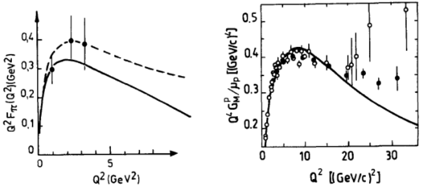


Fig. 5: Pion form factor. *Fig. 6: Proton magnetic form factor*

- LD for the proton magnetic form factor (Nesterenko & AR, (1983))

$$G_M^p(Q^2) = \frac{8}{3}\sqrt{T^2-1}\left\{(4T^2-1)(T^2-1) + (4T^2-3)\,T\sqrt{T^2-1}\right\}^{-1}$$

- $T = 1 + Q^2/2s_0^{\rm N}$ with $s_0^{\rm N} = 2.3\,{\rm GeV}^2$ taken from QCD SR analysis of 2-point correlator
- LD may be written in the form of Drell-Yan form factor formula with LC wave function

$$\Psi(x_i, k_{i\perp}) \sim \theta\left[\sum_i \frac{k_{i\perp}^2}{x_i} \leq s_0\right]$$

From Local Duality to GPD representation

- DY formula has the structure

$$F(Q^2) = \sum_{\text{flavors}} e_a \int_0^1 \mathcal{F}_a(x, Q^2)\, dx$$

- $\mathcal{F}_a(x, Q^2)$ is GPD at zero skewness $\xi = 0$
- Note that $\mathcal{F}_a(x, Q^2 = 0) = f_a(x)$ is the parton density (PDF)

Changing sharp cut-off to wave function with Gaussian $k_\perp$ dependence

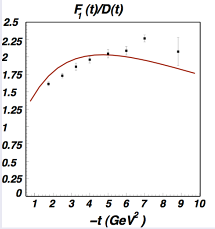
$$\Psi(x_i, k_{i\perp}) \sim \exp\left[-\frac{1}{\lambda^2}\sum_i \frac{k_{i\perp}^2}{x_i}\right]$$

suggests

$$\mathcal{F}_a(x,t) = f_a(x)e^{(1-x)t/2x\lambda^2}$$

$f_a(x)$=experimental densities

Adjusting λ^2 to provide $\langle k_\perp^2 \rangle \approx (300\text{MeV})^2$



Regge-type models for GPDs

"Regge" improvement:

$$f(x) \sim x^{-\alpha(0)}$$
$$\Rightarrow \mathcal{F}(x,t) \sim x^{-\alpha(t)}$$
$$\Rightarrow \mathcal{F}(x,t) = f(x) x^{-\alpha' t}$$

Accomodating quark counting rules:

$$\mathcal{F}(x,t) = f(x) x^{-\alpha' t(1-x)}|_{x \to 1}$$
$$\sim f(x) e^{\alpha'(1-x)^2 t}$$

Does not change small-x behavior but provides

$f(x)|_{x\to 1}$ vs. $F(t)|_{t\to\infty}$ interplay:

$$f(x) \sim (1-x)^n \Rightarrow F_1(t) \sim t^{-(n+1)/2}$$

Note: no pQCD involved in these counting rules!

Extra $1/t$ for $F_2(t)$

can be produced by taking

$$\mathcal{E}_a(x,t) \sim (1-x)^2 \mathcal{F}_a(x,t)$$

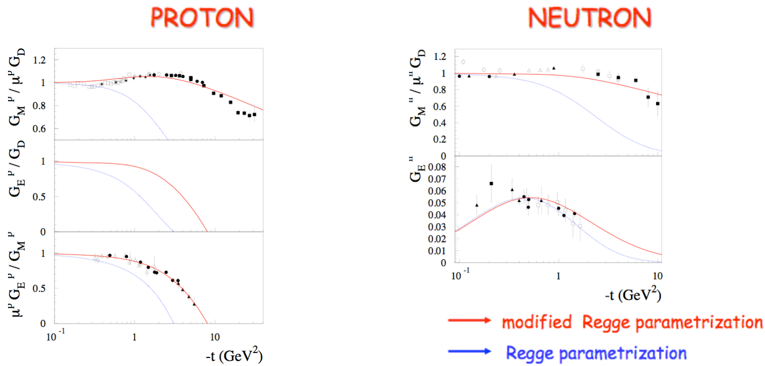
for "magnetic" GPDs

More general:

$$\mathcal{E}_a(x,t) \sim (1-x)^{\eta_a} \mathcal{F}_a(x,t)$$

Fit : $\eta_u = 1.6$, $\eta_d = 1$

Fit to All Four Nucleon $G_{E,M}$ Form Factors



- "Nucleon form-factors from generalized parton distributions"
  M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen
  Phys.Rev.D 72 (2005) 054013;
  304 citations in INSPIRE database
- "Generalized parton distributions from nucleon form-factor data"
  M. Diehl, Th. Feldmann, R. Jakob, P. Kroll
  Eur.Phys.J.C 39 (2005) 1-39;
  274 citations in INSPIRE database

Conclusions

- To induce a small-size configuration in the nucleon, one needs 2 hard gluon exchanges
- Each exchange is suppressed by $\alpha_s/\pi \approx 0.1$ factor
- Total suppression of hard mechanism is $\sim 1/100$
- Soft contributions are capable to describe data on nucleon form factors for all available Q^2's
- Soft mechanism does not involve formation of small-size quark configurations
- I was not surprised that small-size configurations have not been seen in nucleon CT experiments

Work is supported by Jefferson Science Associates, LLC
under U.S. DOE Contract #DE-AC05-06OR23177
and by U.S. DOE Grant #DE-FG02-97ER41028