Status of, and prospects for, lattice QCD calculations of d2n

Chris Monahan
William & Mary/JLab

Pol He3 Collaboration meeting

3/25/2021

What do we want from a theory calculation?

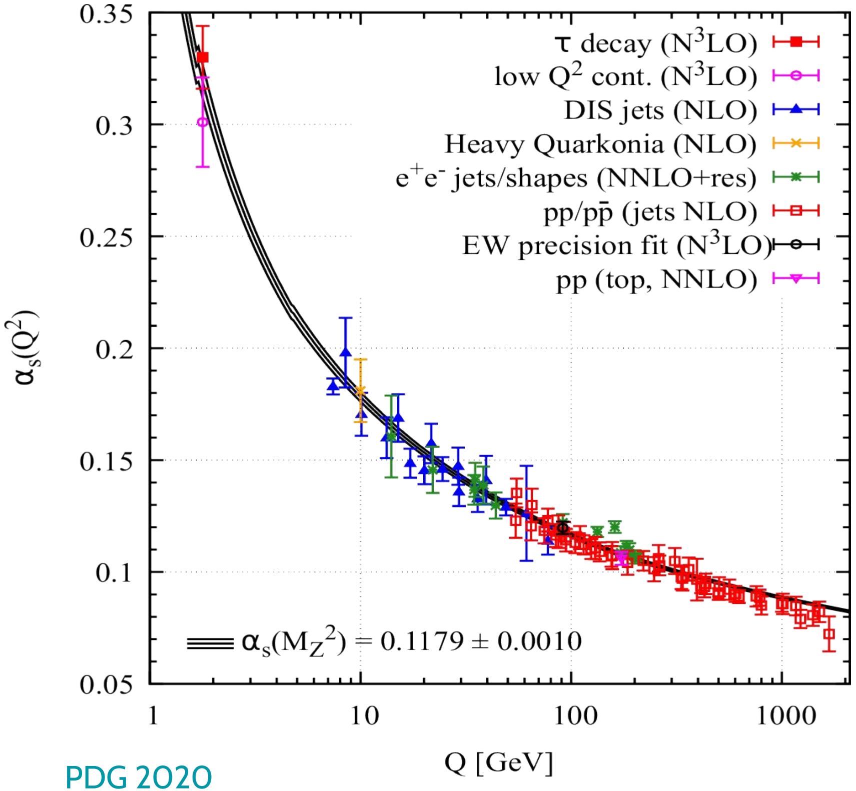
1. Calculated directly from the QCD Lagrangian
   - using quarks and gluons as our basic building blocks

2. Reliable uncertainty estimates
   - even systematic uncertainties!

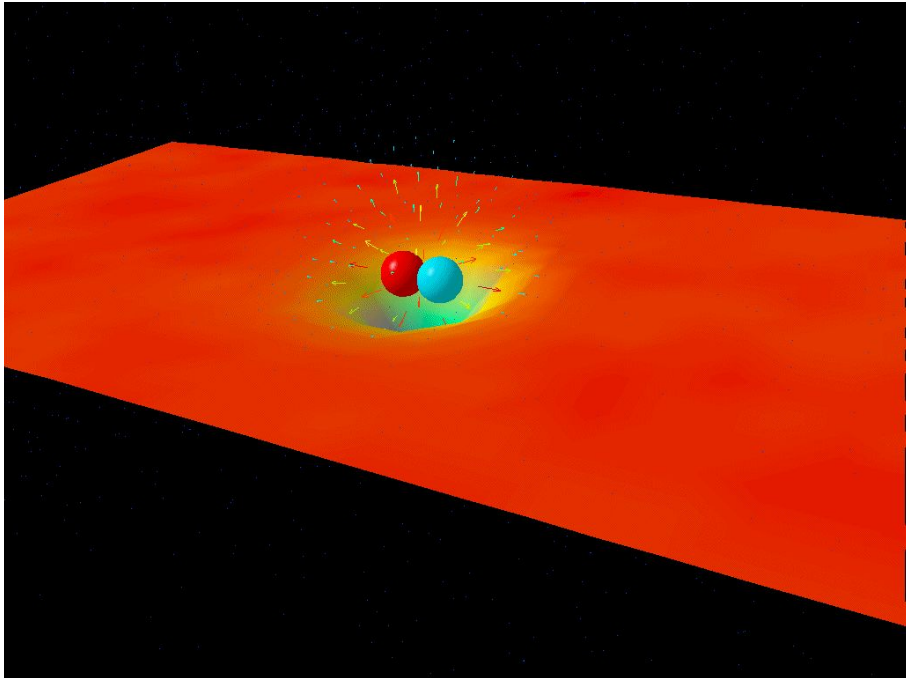
3. Systematically improvable uncertainties

What do you want from a theory calculation?

Nonperturbative QCD

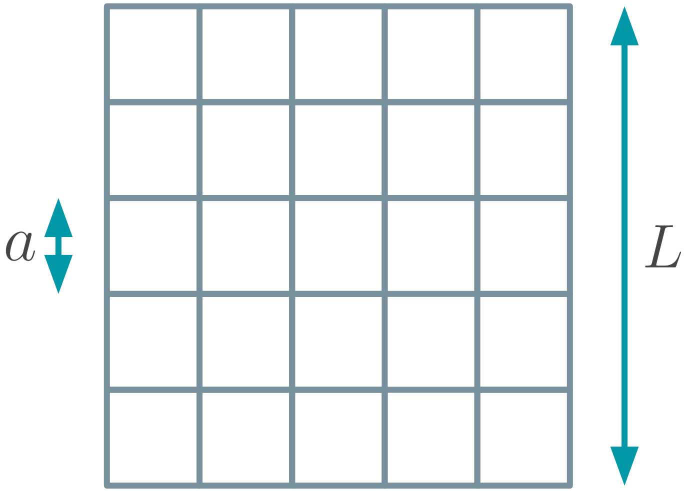


PDG 2020



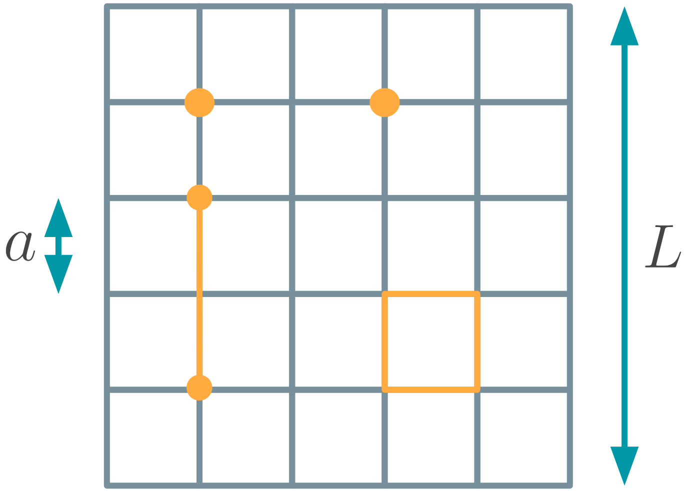
Leinweber et al., CSSM, Adelaide

Lattice QCD: the numerical recipe



1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

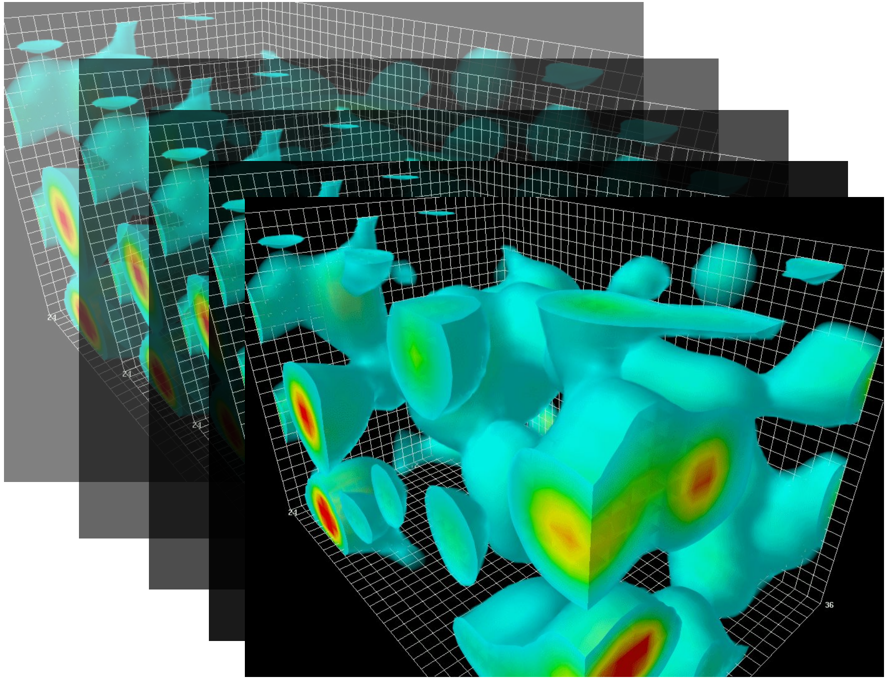
Lattice QCD: the numerical recipe



1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

Lattice QCD: the numerical recipe



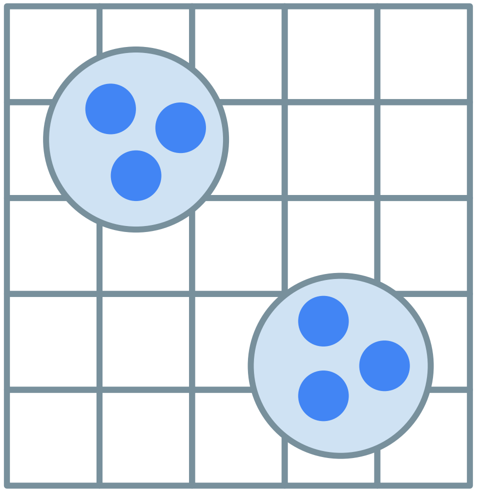
Leinweber et al., CSSM, Adelaide

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

Lattice QCD: the numerical recipe



1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

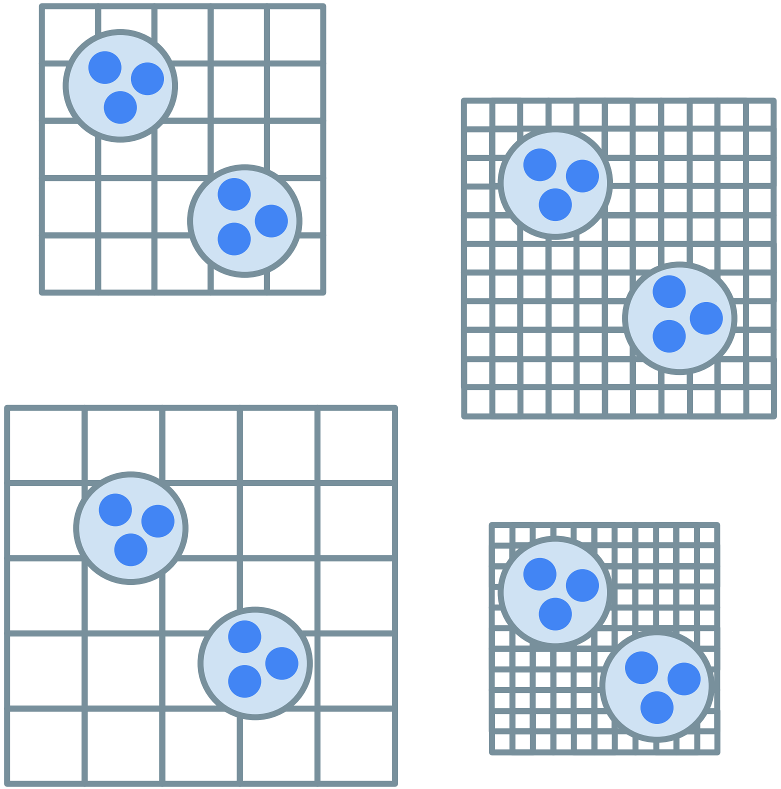
2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

Physics typically extracted from the long Euclidean time limit

Lattice QCD: the numerical recipe



1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

Continuum limit requires renormalisation!

Lattice QCD: systematic uncertainties

Real-time dynamics

Finite volume effects

Discretisation effects

Excited state contamination at short times

Statistical uncertainty

Signal-to-noise issues in long Euclidean time limit

Renormalisation prescription

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

Model dependency

Local operators

d2n

Mellin moments

“Standard” calculations

Specific observables may have specific challenges

Nonlocal operators

g2

x-dependent distributions

Novel calculations

Broadly applicable formal, computational, and data analysis challenges

Mellin moments from lattice QCD

d2n defined as the second Mellin moment of the spin-dependent structure function

$$2\int_0^1 \mathrm{d}x\, x^2 g_2(x,Q^2) = \frac{1}{3}\sum_f \left[c_{2,2}^{(f)}\left(\frac{\mu^2}{Q^2}\right) d_2^{(f)}(\mu^2) - c_{1,2}^{(f)}\left(\frac{\mu^2}{Q^2}\right) a_2^{(f)}(\mu^2) \right]$$

$$d_2^n(\mu^2) = 3\int_0^1 \mathrm{d}x\, x^2 \left[2g_1^n(x,Q^2) + 3g_2^n(x,Q^2)\right]$$

Equivalently: as the matrix element

$$\langle P,S|\mathcal{O}^{(f)}_{[\sigma\{\mu_1]\mu_2\}}|P,S\rangle = \frac{d_2^{(f)}}{3}\left[\left(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma\right)P_{\mu_2} + \left(S_\sigma P_{\mu_2} - S_{\mu_2}P_\sigma\right)P_{\mu_1} - \text{traces}\right]$$

of the local operator

$$\mathcal{O}^{(f)}_{\sigma\mu_1\mu_2} = -\frac{1}{4}\bar{\psi}\gamma_\sigma\gamma_5 \overleftrightarrow{D}_{\mu_1}\overleftrightarrow{D}_{\mu_2}\psi - \text{traces}$$

Gluon operators suffer significant statistical noise

d2n from lattice QCD

See also: preliminary results in Dolgov et al. hep-lat/0011010

Primary result calculated by QCDSF/UKQCD collaboration

$$d_2^n(\mu^2 = 25\,\mathrm{GeV}^2) = -0.001(3)$$

Statistical errors only

Göckeler et al., PRD 72 (2005) 054507

Improves on:
Quenched result - Göckeler et al., PRD 63 (2001) 074506
Perturbatively renormalised result - Göckeler et al., PRD 53 (1996) 2317

What about possible systematic uncertainties?

- Wilson fermions on nf = 2 Wilson-clover fermion ensembles
- Four lattice spacings: a ~ 0.7 - 0.9 fm
- Four lattice volumes: L ~ 1.7 - 1.9 fm
- Lightest pion mass ~ 600 MeV
- RI/MOM renormalisation

But $m_\pi L > 4$

See also: d1n calculated with DWF in Orginos et al., PRD 73 (2006) 094503

d2n: systematic uncertainties

Signature independent

Local operator!

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

d2n: systematic uncertainties

Finite volume effects

Likely negligible at these large pion masses, though authors "have not considered finite size effects"

Local operator!

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, "measure" your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

d2n: systematic uncertainties

Local operator!

Finite volume effects

Statistical uncertainty

Only quoted uncertainty

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

d2n: systematic uncertainties

Local operator!

Finite volume effects

Statistical uncertainty

Precise nonperturbative renormalisation removes power-divergent mixing generated by Wilson action.

Renormalisation prescription

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

d2n: systematic uncertainties

Local operator!

Lattice spacings are small, but over narrow range and “data do not yet allow us to perform a decent continuum extrapolation”

Discretisation effects

Statistical uncertainty

Renormalisation prescription

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

d2n: systematic uncertainties

Local operator!

fects

Question: is there a calculation of d2n in chiral perturbation theory?

Discretisation effects

Statistical uncertainty

Possible significant sources of extrapolation uncertainty:
1. No control over continuum extrapolation
2. Linear extrapolation down from heavy pion masses
3. Scale-setting uncertainties

Renormalisation prescription

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

Model dependency

d2n: systematic uncertainties

Finite volume effects

Not discussed, but excited state contamination now understood to be an important systematic in lattice calculations of baryon quantities

Excited state contamination at short times

Statistical uncertainty

Signal-to-noise issues in long Euclidean time limit

Derivative operators are statistically noisy

Local operator!

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (an ensemble) of this QCD vacuum

4. On each copy, “measure” your desired correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

Model dependency

Local operators

d2n

Mellin moments

"Standard" calculations

Specific observables may have specific challenges

Nonlocal operators

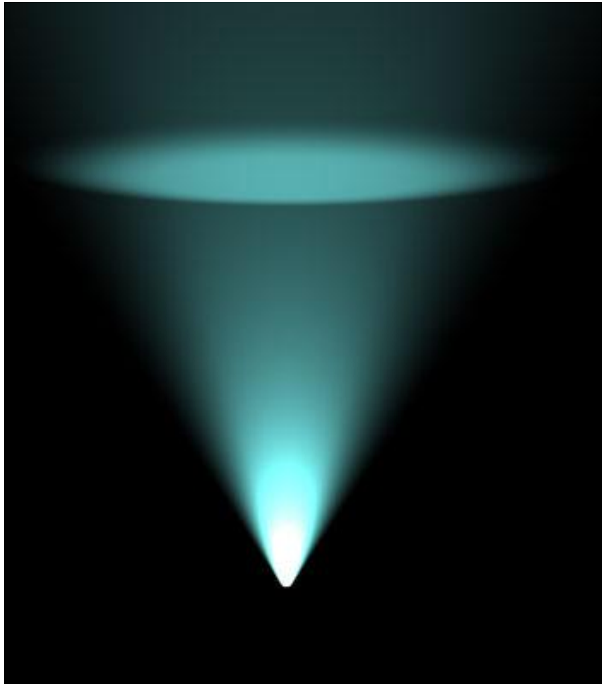
g2

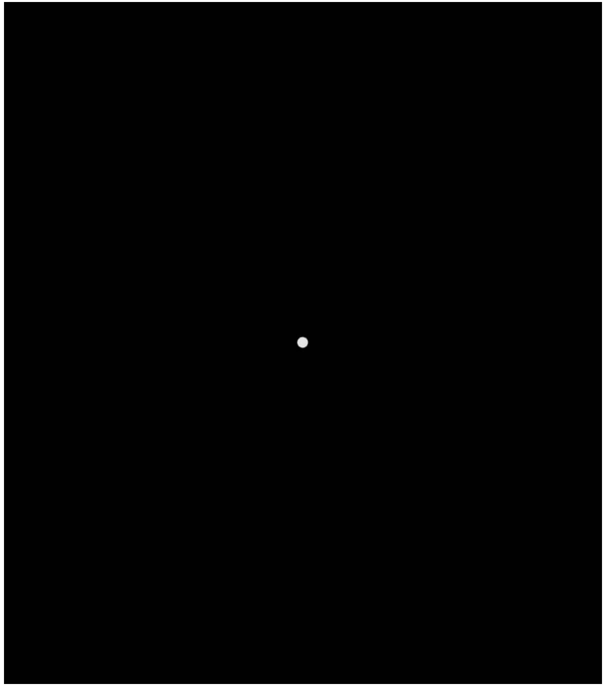
x-dependent distributions

Novel calculations

Broadly applicable formal, computational, and data analysis challenges

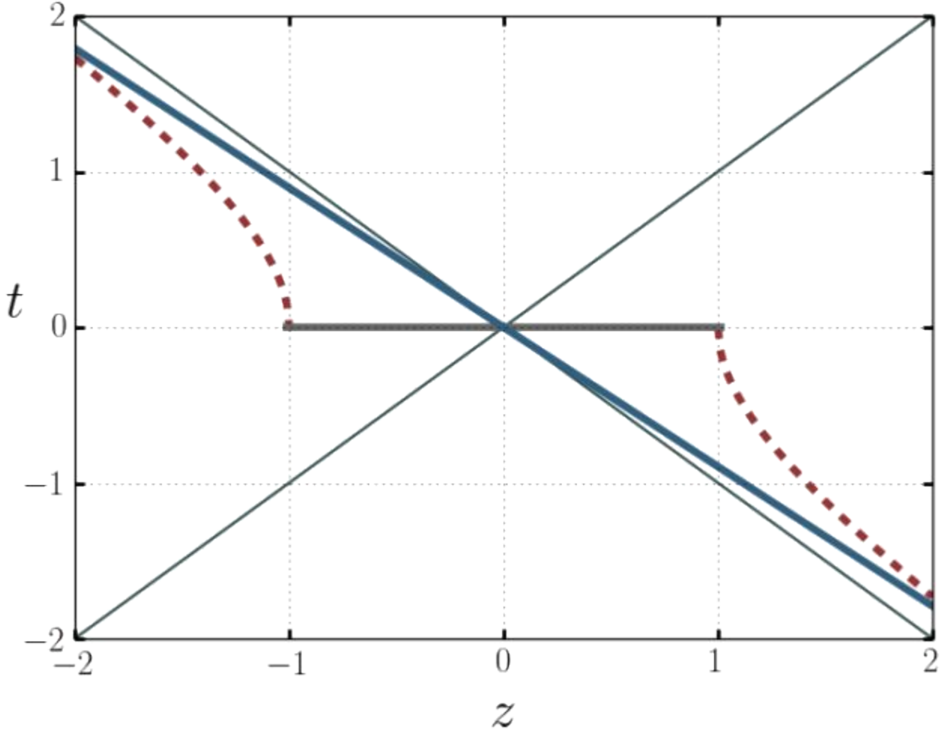
x-dependent hadron structure from lattice QCD





x-dependent hadron structure from lattice QCD

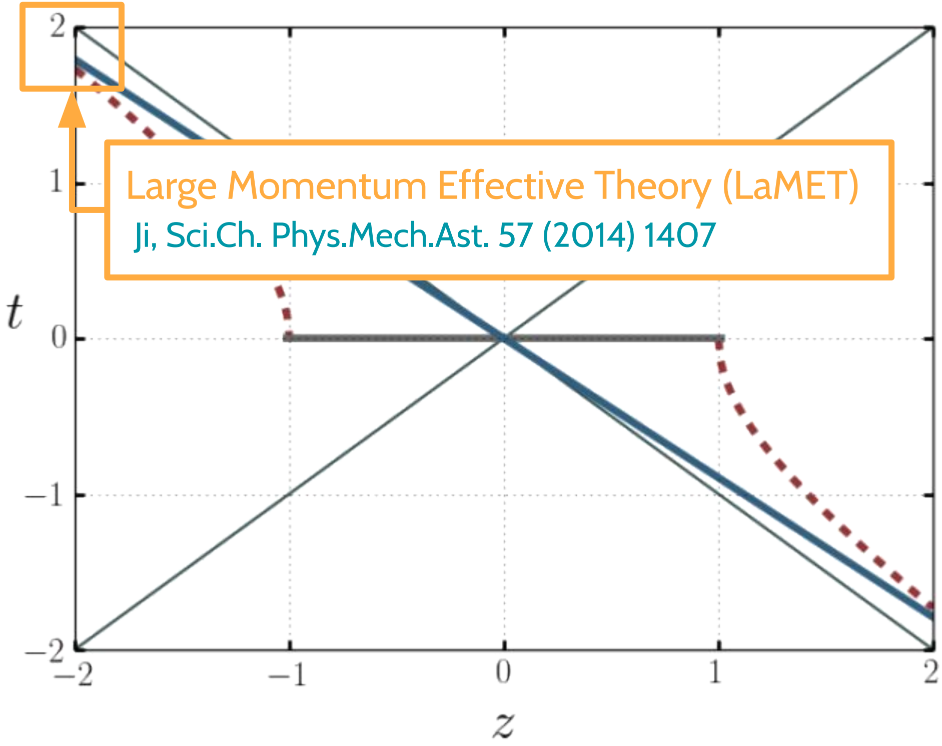
Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025



Note: Davoudi & Savage, PRD 86 (2012) 054505
Musch et al., PRD 83 (2011) 094507
Braun & Müller, EPJC 55 (2008) 349
Detmold & Lin, PRD 73 (2006) 014501
Liu & Dong, PRL 72 (1994) 1790

x-dependent hadron structure from lattice QCD

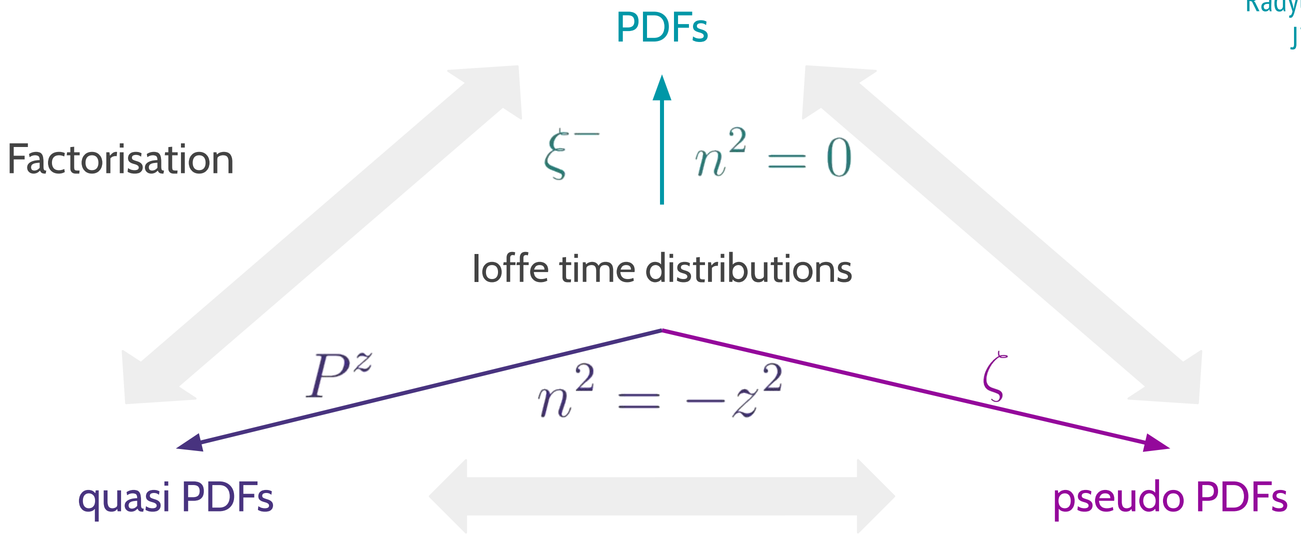
Ji, PRL 110 (2013) 262002
Radyushkin, PRD 96 (2017) 034025



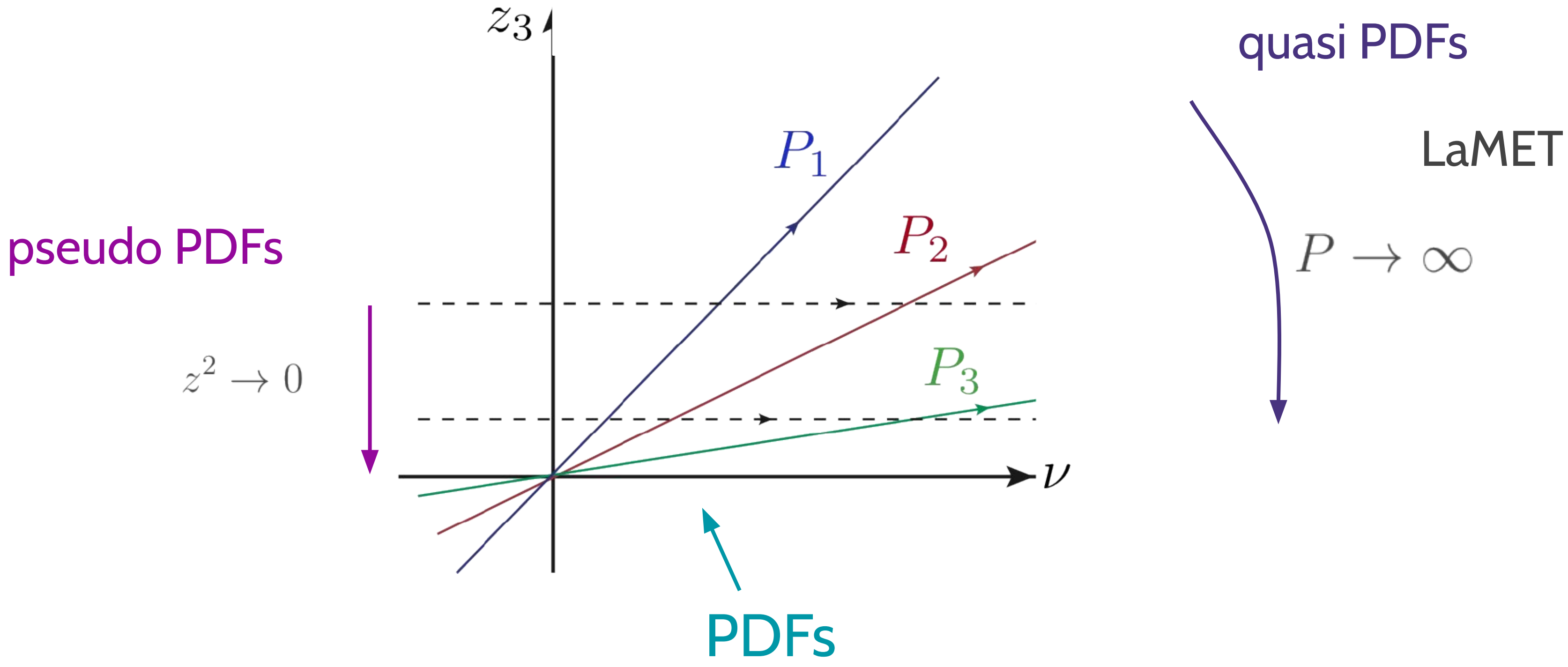
Note: Davoudi & Savage, PRD 86 (2012) 054505
Musch et al., PRD 83 (2011) 094507
Braun & Müller, EPJC 55 (2008) 349
Detmold & Lin, PRD 73 (2006) 014501
Liu & Dong, PRL 72 (1994) 1790

x-dependent hadron structure from lattice QCD

Del Debbio, Giani & CJM, JHEP 09 (2020) 021
Izubuchi et al., PRD 98 (2018) 056004
Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002

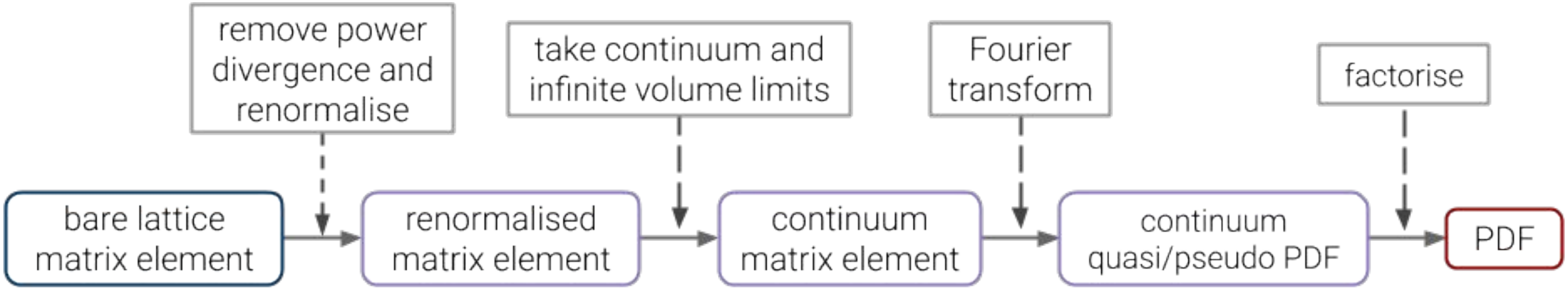


x-dependent hadron structure from lattice QCD



Based on Radyushkin, QCD Evolution 2017

x-dependent hadron structure from lattice QCD



Recall: lattice QCD is QCD formulated on a discrete Euclidean hypercube; quark fields live on the lattice nodes and gluons on the links.

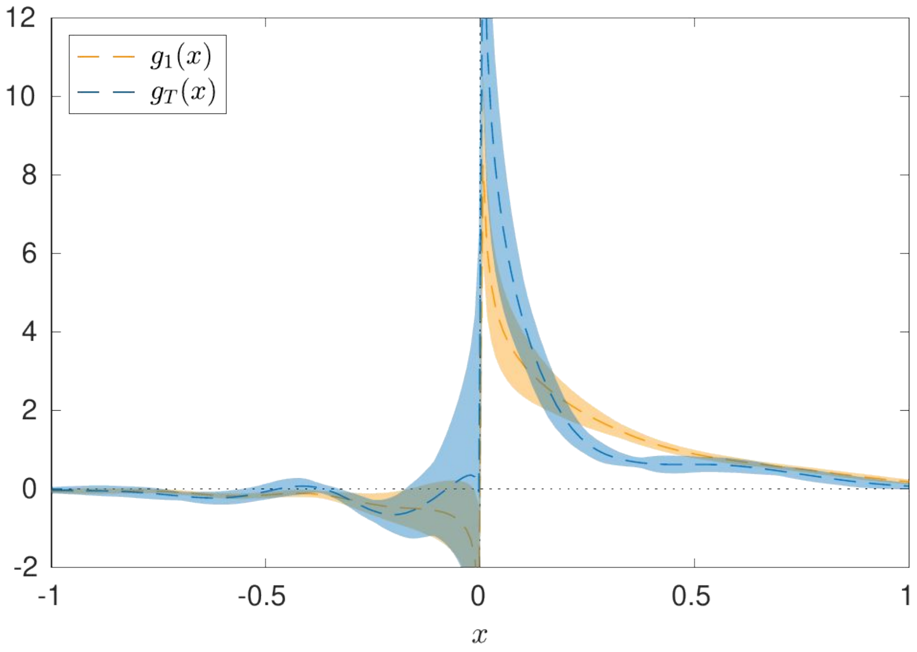
Based on CJM, POS(LATTICE2018) 018, 1811.00678

gT from lattice QCD

First calculation of a twist-3 distribution, $g_T(x)$

What about possible systematic uncertainties?

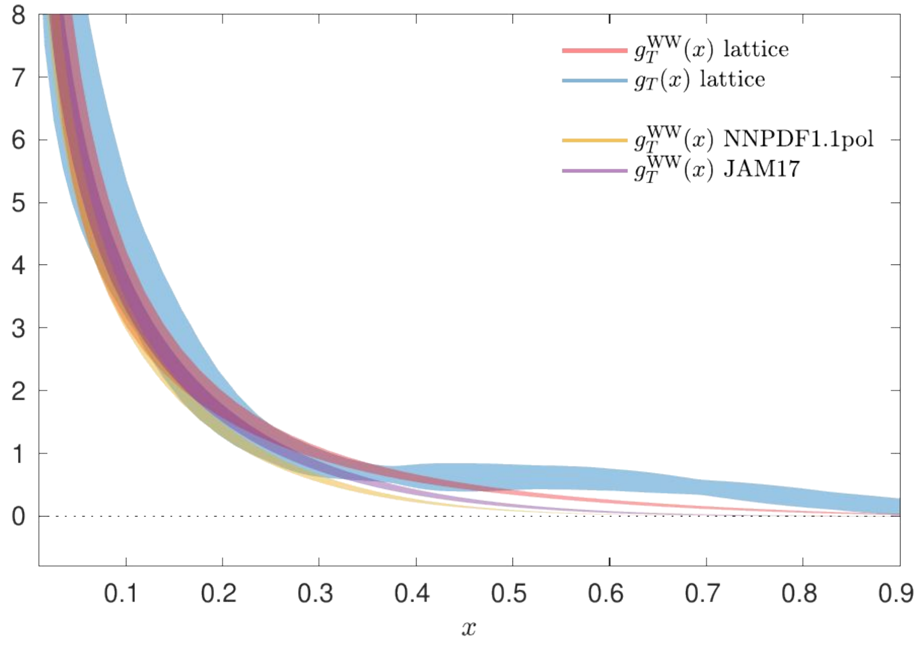
- Twisted mass Wilson fermions on nf = 2+1+1 twisted mass ensembles
- One lattice spacing: a ~ 0.9 fm
- One lattice volume: L ~ 3 fm
- One pion mass ~ 260 MeV
- Three nucleon boosts ~ 0.9 - 1.67 GeV
- RI/MOM renormalisation

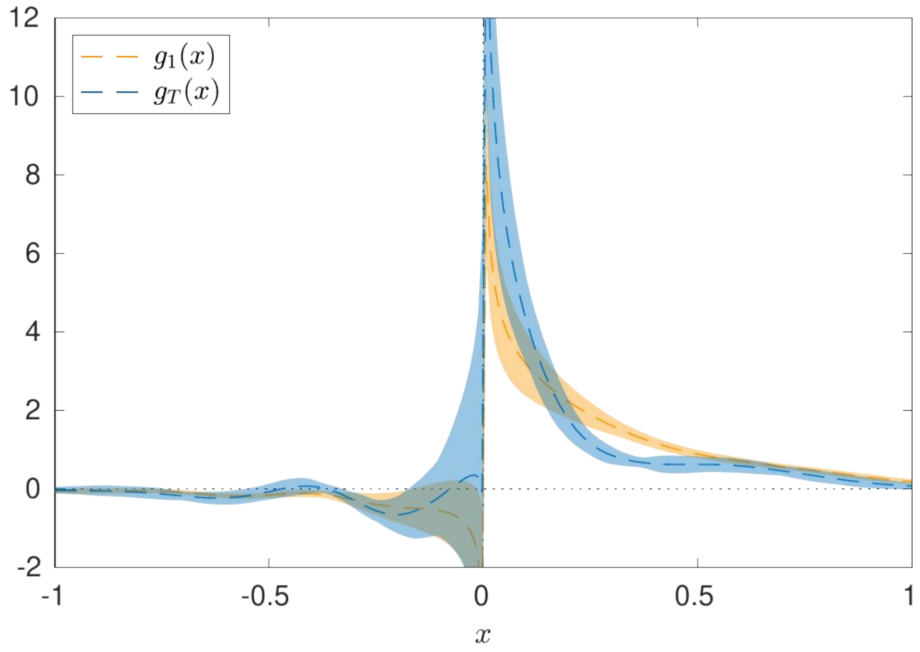


Bhattarcharya et al., PRD 102 (2020) 111501

gT from lattice QCD

First calculation of a twist-3 distribution, $g_T(x)$





Bhattarcharya et al., PRD 102 (2020) 111501

HadStruc and the JLab theory lattice structure effort

A newly coalesced collaboration

Colin Egerer	W&M
Christos Kallidonis	W&M
Joe Karpie	Columbia/BNL
Tanjib Khan	W&M
Wayne Morris	ODU
Kostas Orginos	W&M/JLab
Anatoly Radyushkin	ODU/JLab
David Richards	JLab
Raza Sufian	W&M
Jianwei Qiu	JLab
Savvas Zafeiropoulos	Marseille

So what are the prospects?

d2n is a conceptually straightforward, but technically challenging, lattice calculation

We could significantly improve the statistical precision and some sources of systematic uncertainties

- ensembles with wider range of lattice spacings, larger volumes and lighter pion masses
- new techniques make calculations using gluon operator feasible

We would still use Wilson-clover fermions

- requires nonperturbative renormalisation to remove power-divergent mixing

Summary:

- formalism is in place and HadStruc has the relevant expertise
- new nonperturbative results required, for both matrix elements and renormalisation parameters
- HadStruc is limited by person power

My guess: this is a two year project?

So what are the prospects?

g2 is a conceptually and technically challenging lattice calculation

Existing calculation of gT uses state-of-the-art lattice techniques

- many similarities with our approaches and resources
- technical differences would help shed light on, and possibly quantify, systematic uncertainties

Summary:

- formalism is only partially in place, but HadStruc has the relevant expertise
- new formalism and nonperturbative results required
- HadStruc is (still) limited by person power

My guess: this is a two year project to obtain results similar to 2004.04130. Five years for control over systematics

Summary

d2n is a conceptually straightforward, but technically challenging, lattice calculation

Single lattice result available that quotes only statistical uncertainties

$$d_2^n(\mu^2 = 25\,\mathrm{GeV}^2) = -0.001(3)$$

Göckeler et al., PRD 72 (2005) 054507

Potential for significant improvement in control over systematic uncertainties

- highly unlikely to be completed in less than a year

Great potential for complementary lattice calculations in the longer term

- via gluon operator
- via g2

What do you want from a theory calculation?

Thank you!

Chris Monahan

cjmonahan@wm.edu

x-dependent hadron structure from lattice QCD

$$f^{(0)}_{j/H}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \langle H(P) \left| \overline{\psi}(0, \omega^-, \mathbf{0}_\mathrm{T}) W(\omega^-, 0) \Gamma_j \psi(0) \right| H(P) \rangle$$

PDFs

ξ^- $\quad n^2 = 0$

$$h^{(0)}_{j/H}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle H(P) | \overline{\psi}(n) W(n, 0) \Gamma^\mu_j \psi(0) | H(P) \rangle$$

P^z $\quad n^2 = -z^2 \quad$ ζ

quasi PDFs

pseudo PDFs

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \langle H(P) \left| \overline{\psi}(0, z, \mathbf{0}_\mathrm{T}) W(z, 0) \Gamma_j \psi(0) \right| H(P) \rangle$$

Ji, PRL 110 (2013) 262002

$$\widetilde{p}^{(0)}_{j/H}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \langle H(P) \left| \overline{\psi}(0, z, \mathbf{0}_\mathrm{T}) W(z, 0) \Gamma_j \psi(0) \right| H(P) \rangle$$

Radyushkin, PRD 96 (2017) 034025