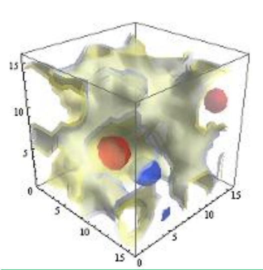
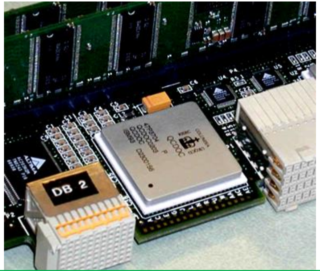
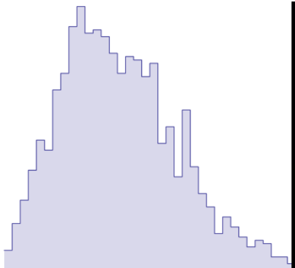
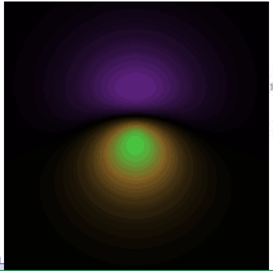
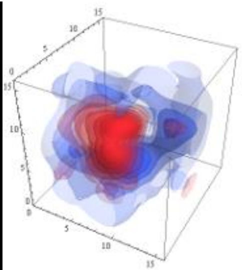
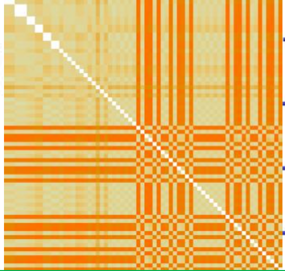
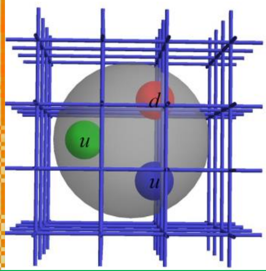
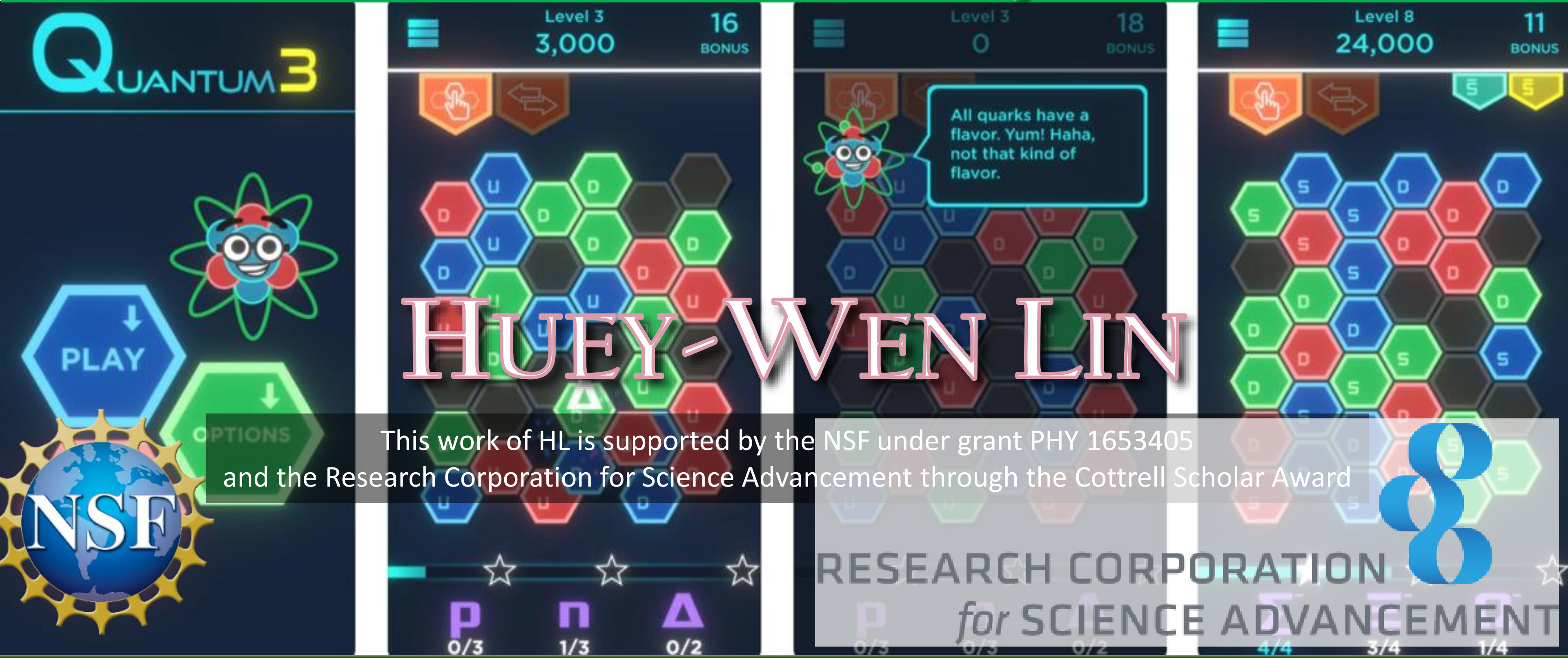








Constraints from Lattice Moments on Global Analyses



HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

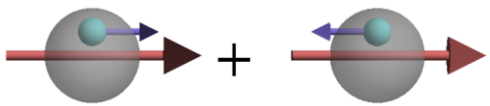
Focus on "how you used the transversity Mellin moment in the experimental analysis."

Lin, Melnitchouk, Prokudin, Sato, 1710.09858,
Phys. Rev. Lett. 120, 152502 (2018)

1D Nucleon Structure (PDFs)

§ Lattice calculations use operator product expansion to obtain moments

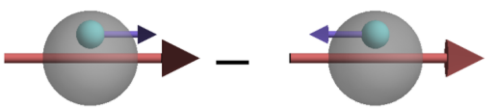
- More nucleon matrix elements with physical pion masses
- Well-studied systematics → precision structures



quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$

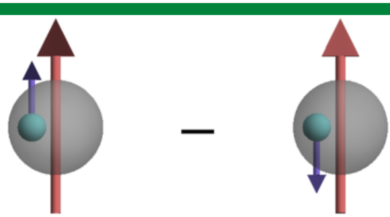
most well known



helicity

longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$



transversity

transversely polarized

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

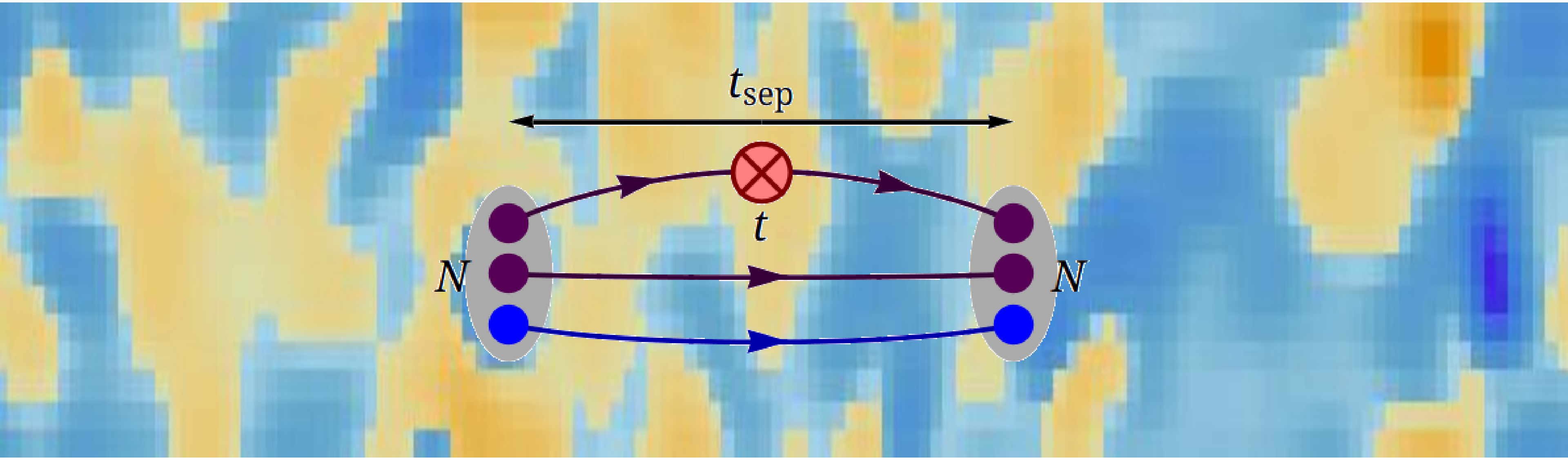
very poorly known

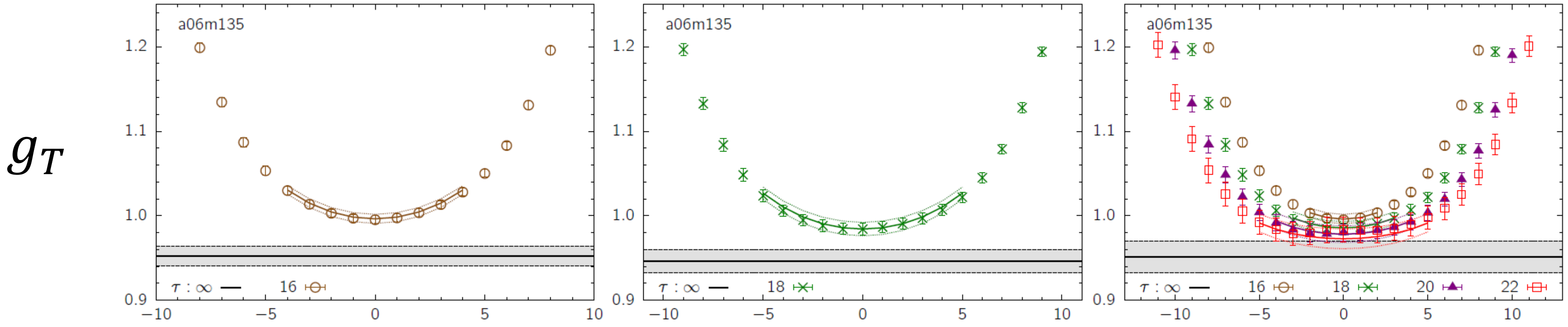
Why Tensor Charges?

§ Example from PNDME

PNDME, 1806.09006

- Very mild systematics: excited-state contamination



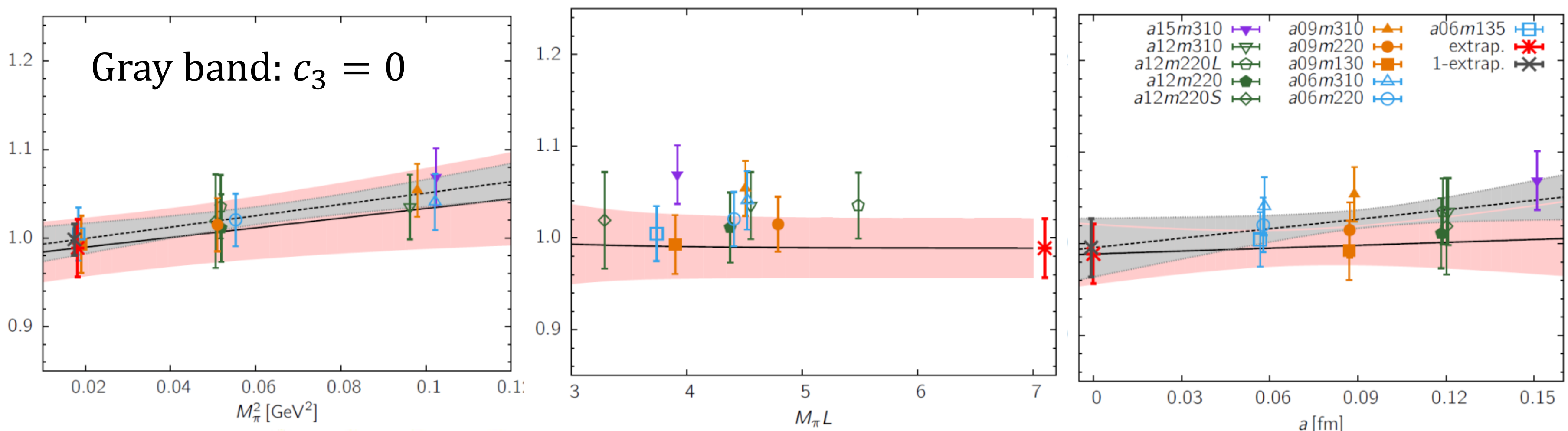


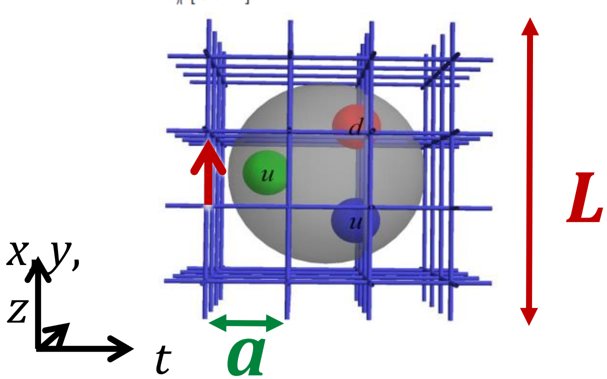
§ Example from PNDME

PNDME, 1806. 09006

- Very mild systematics: lattice spacing and volumes

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$





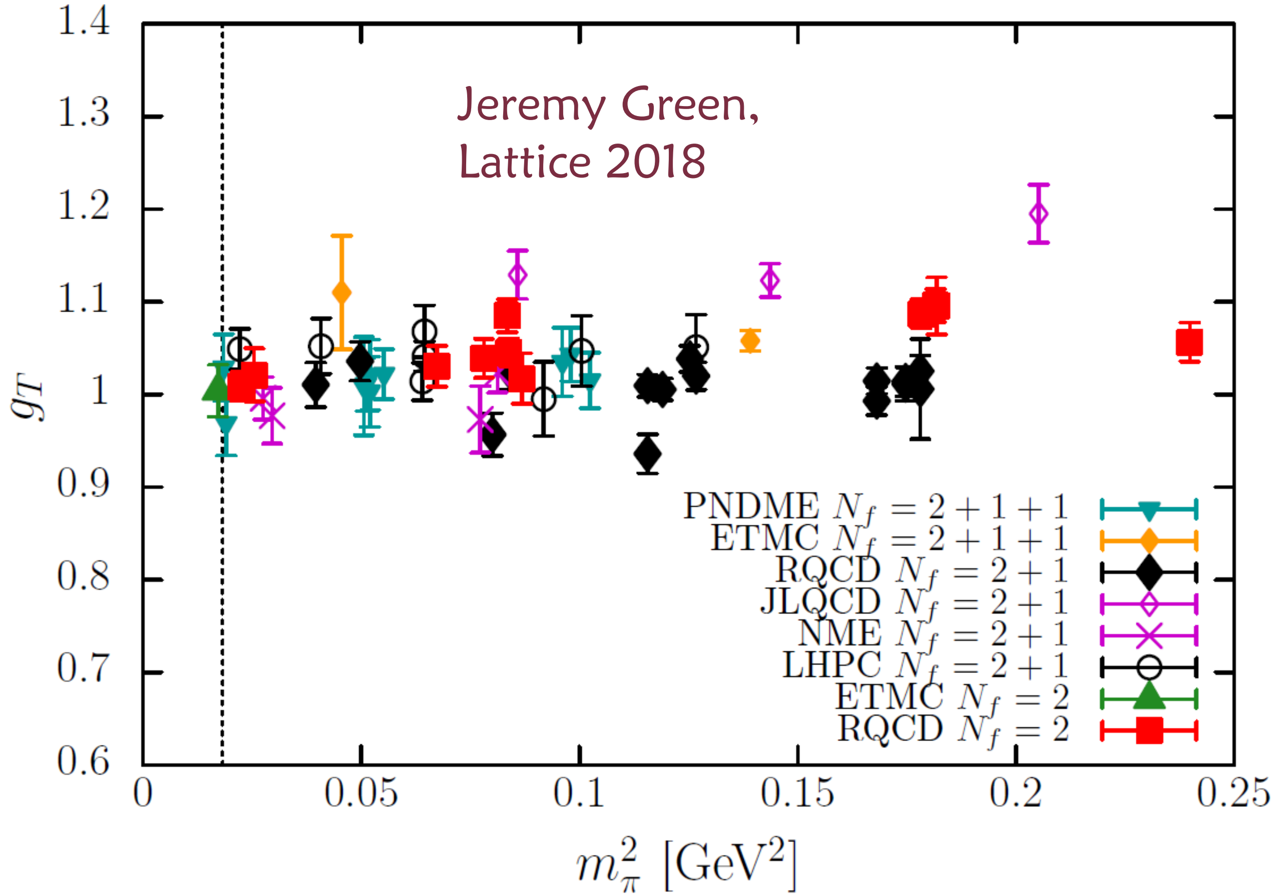
- Extrapolation to the continuum limit

$m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$, $L \to \infty$

Why Tensor Charges?

§ Mild difference among fermion/gauge actions

- Lattice results need only agree in the continuum limit



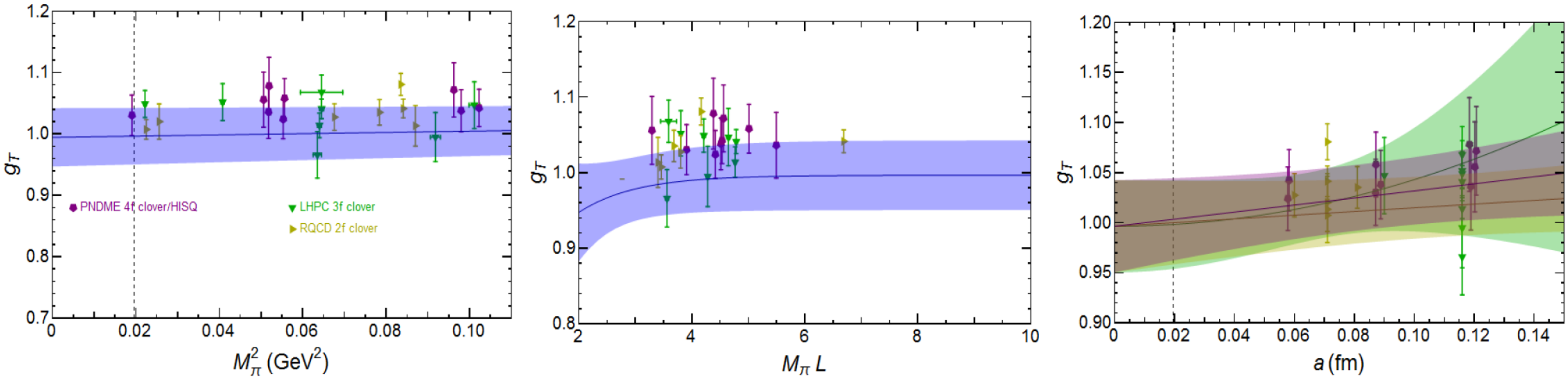
Tensor Charges Global Fit

§ A first attempt at a "global fit" of nucleon charge in 2017

§ Only take lattice data that have

- Pion masses $m_\pi^2 < 0.12\ \text{GeV}^2$ and $m_\pi L > 3$
- Multiple lattice spacings and volumes for continuum extrapolation
  - PNDME (1606.07049), RQCD (1412.7336), LHPC (1206.4527)

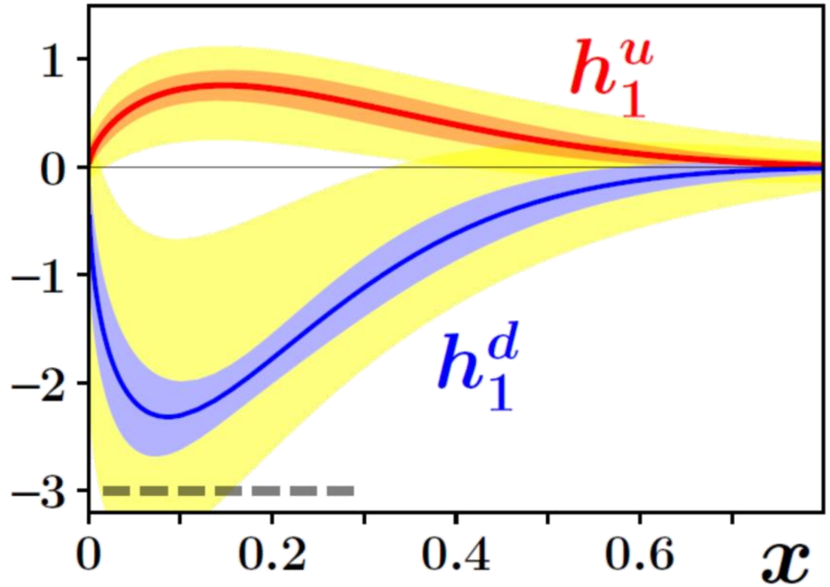
$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

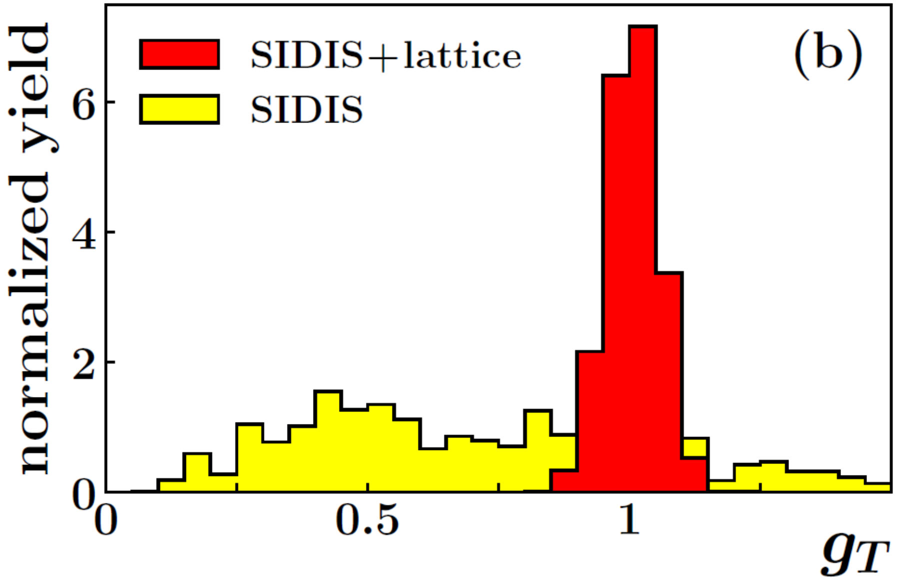


Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$

From Charge to Transversity

- Despite recent *x*-dependent lattice calculations, precision flavor PDFs remain a few years away
- Improved **transversity** distribution with LQCD g_T
  - Proof of concept: SIDIS $\pi^{\pm}$ production data from proton and deuteron targets





Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

From Charge to Transversity

§ Improved work?

- Use more experimental data and more lattice inputs
- Lattice QCD/global fit status

LatticePDF Report, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)
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$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)
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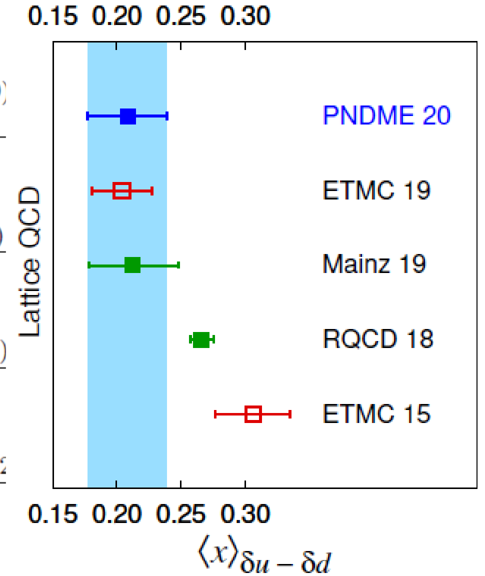
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22



- "Precision" first moment available (including all lattice errors)

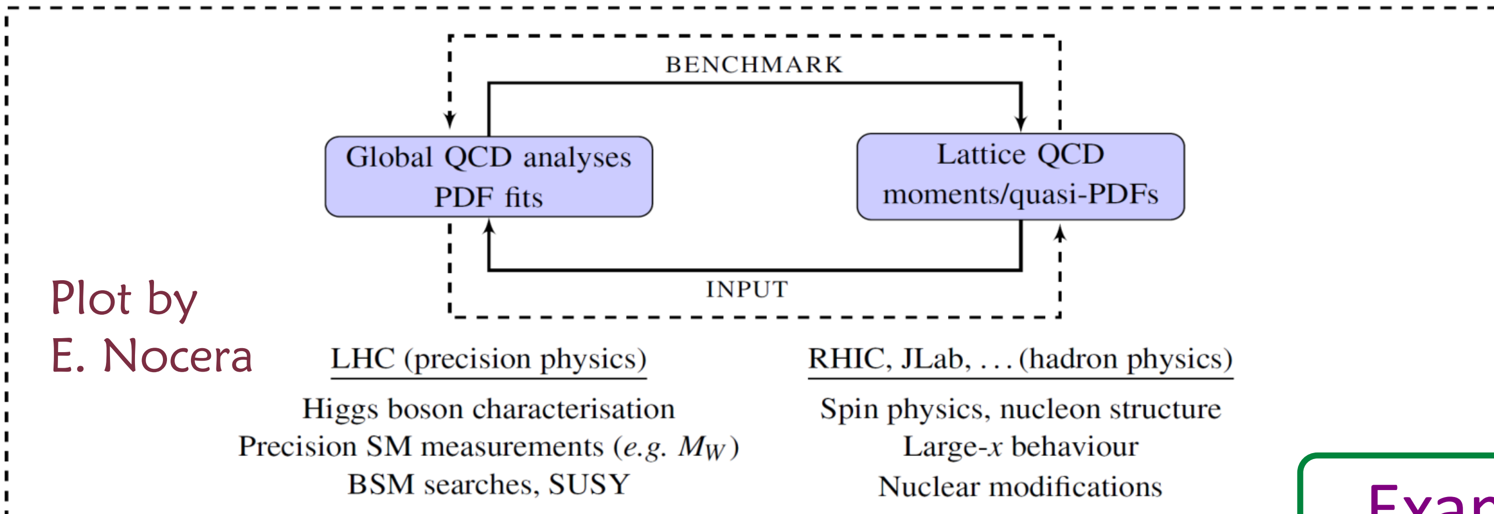
S. Mondal et al (PNDME), 2005.13779

How Can Lattice Help?

- Exciting era using LQCD moments to help study nucleon structure
  - Well-studied systematics → precision structures
  - Address neglected disconnected contributions obtaining flavor-dependent quantities
- LQCD cannot study all nucleon structure alone
  - Flavor-dependent PDFs, an example shown here
  - More can be done
- For which other quantities can LQCD achieve a precision that would make a significant difference to our understanding of nucleon structure when combined with experimental data?

How Can Lattice Help?

The PDFLattice2017 workshop

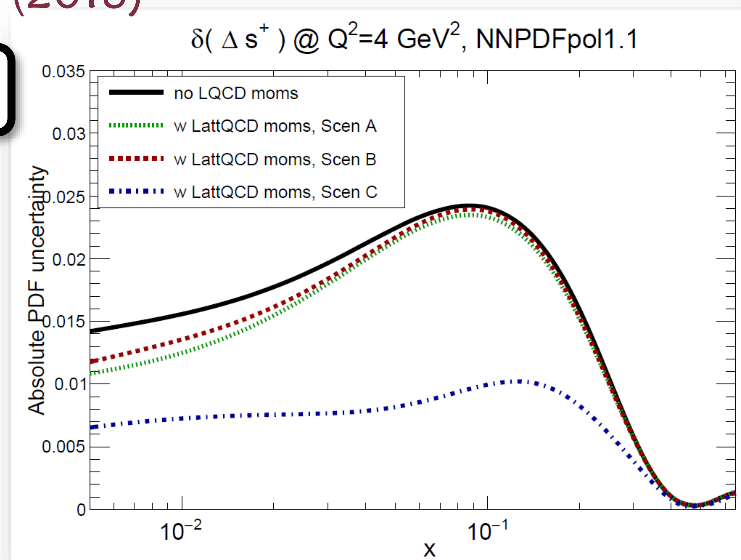


Plot by E. Nocera

Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

A: 70% B: 50% C: 20%

Example study



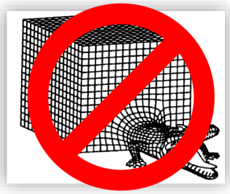
- Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?

Backup Slides

First Continuum PDF

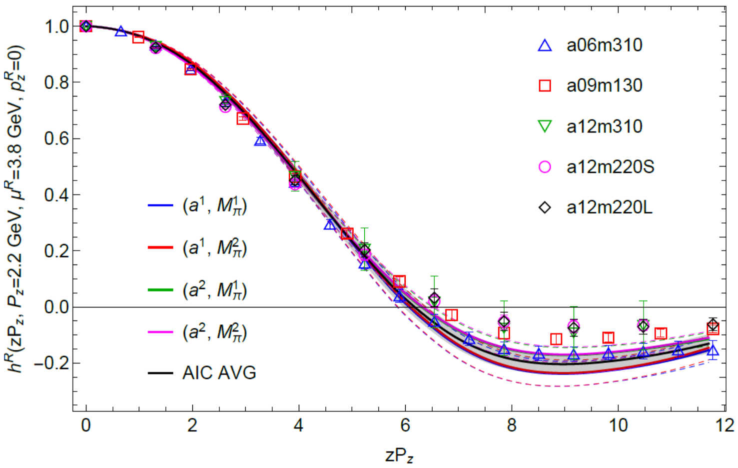
§ Nucleon PDFs using quasi-PDFs in the continuum limit

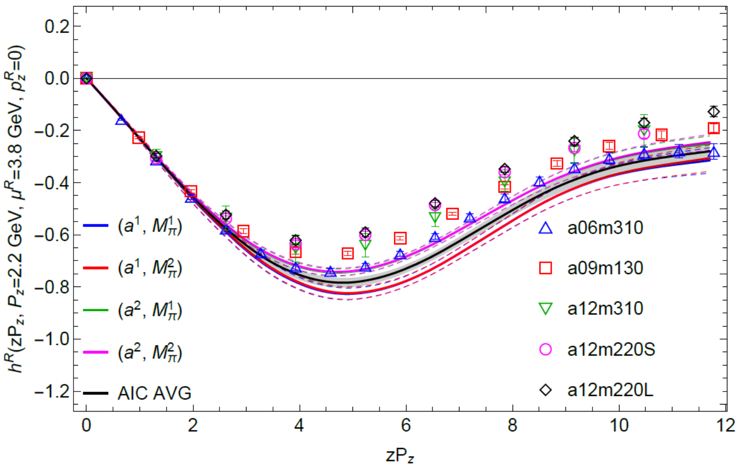
- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.09, 0.12\}$ fm
  $M_\pi \in \{135, 220, 310\}$ MeV
  $M_\pi L \in \{3.3, 5.5\}$
  $P_z \approx 2$ GeV



2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit





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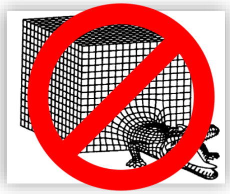

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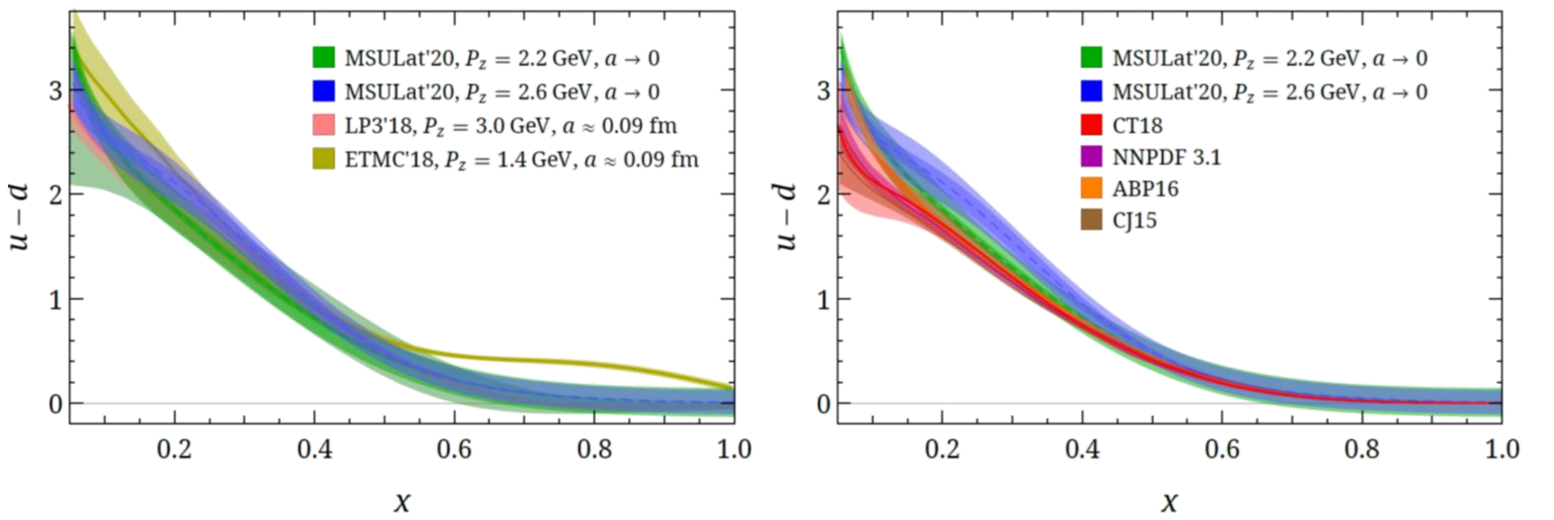
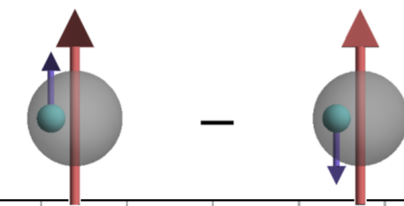


Moments of PDFs

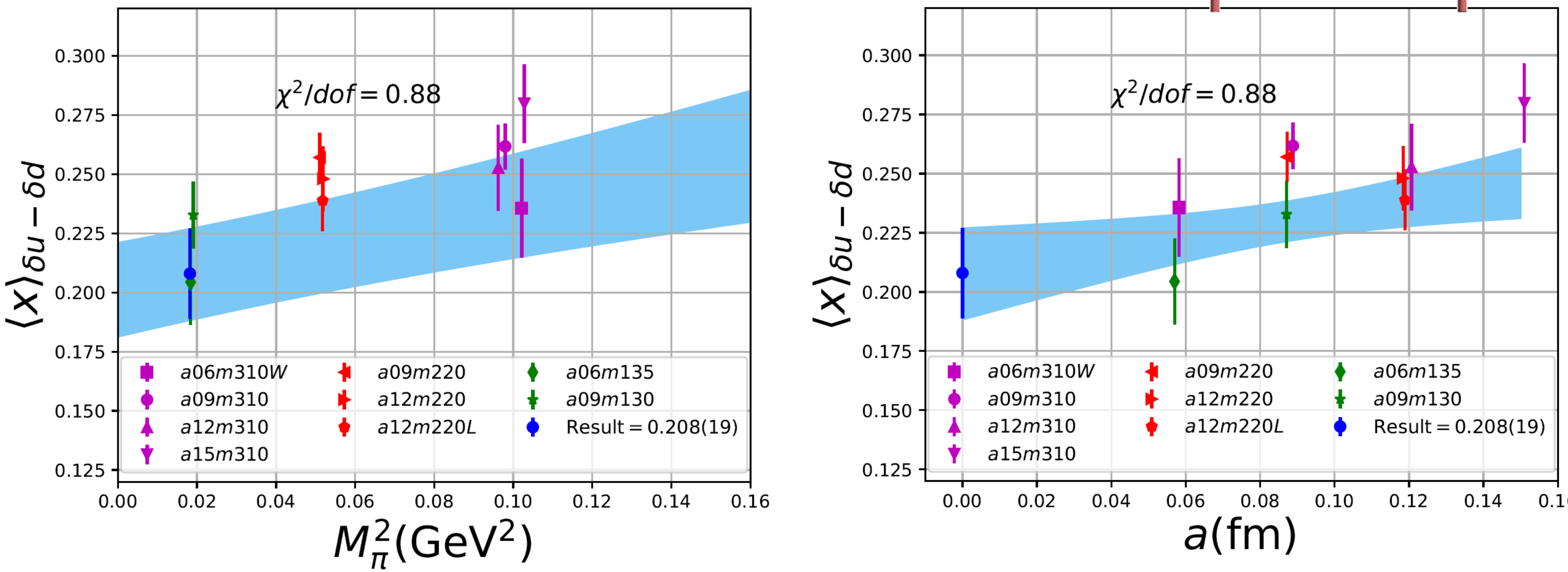
- Only lowest few moments

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



- State-of-the art example
  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779



- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
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	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
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	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
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	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
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	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
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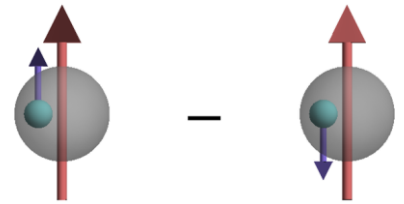
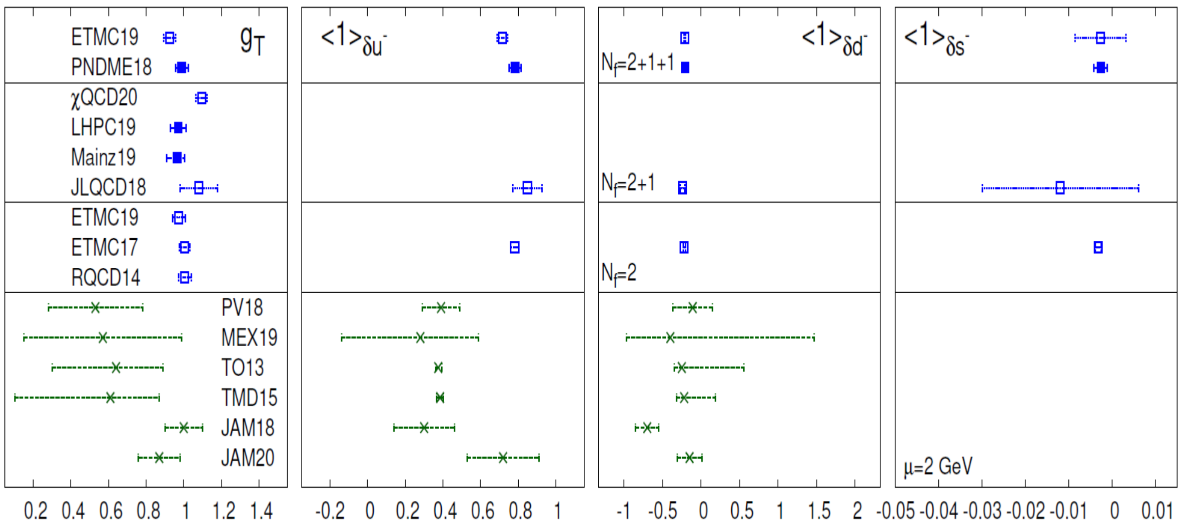
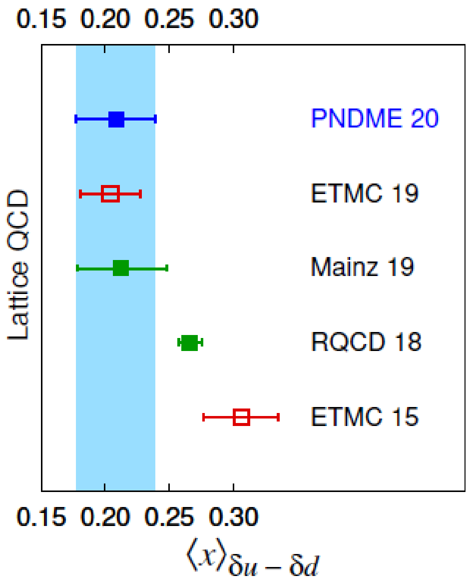


LatticePDF Report, 1711.07916, 2006.08636





S. Mondal et al (PNDME), 2005.13779

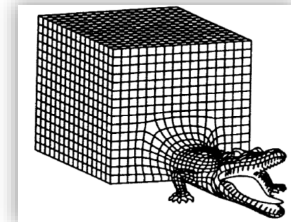
Transversity

§ Summary of physical pion mass results

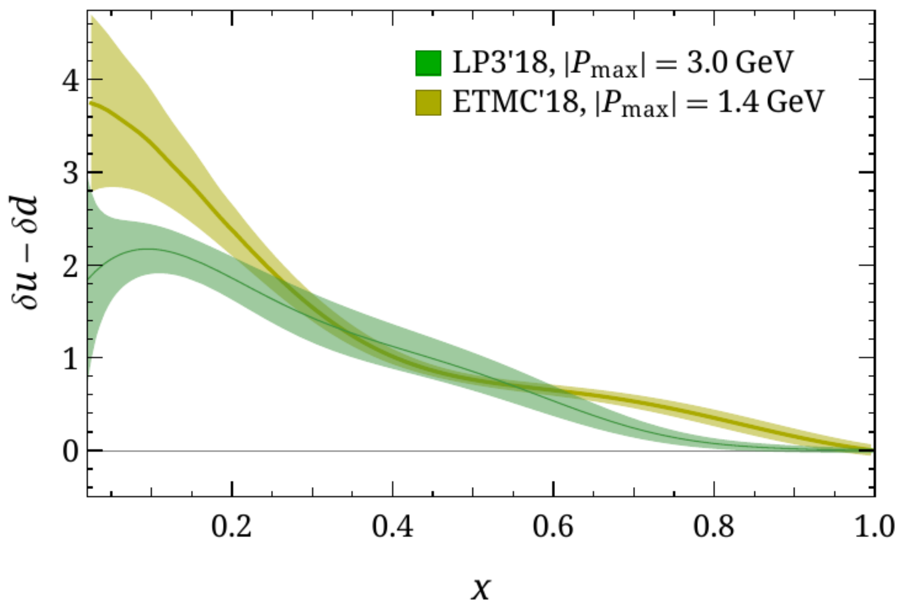
- Quasi-PDF method only

$$\boldsymbol{\delta u(x) - \delta d(x)}$$

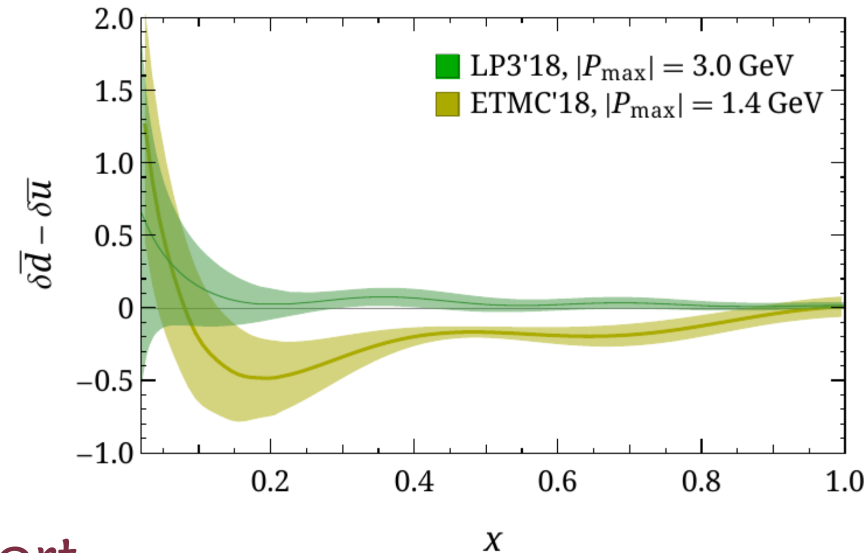
Transversity



Finite volume,
Discretization,
…



$$\boldsymbol{\delta\,\bar{d}(x) - \delta\,\bar{u}(x)}$$



2006.08636, PDFLattice2019 report

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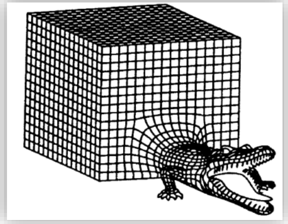

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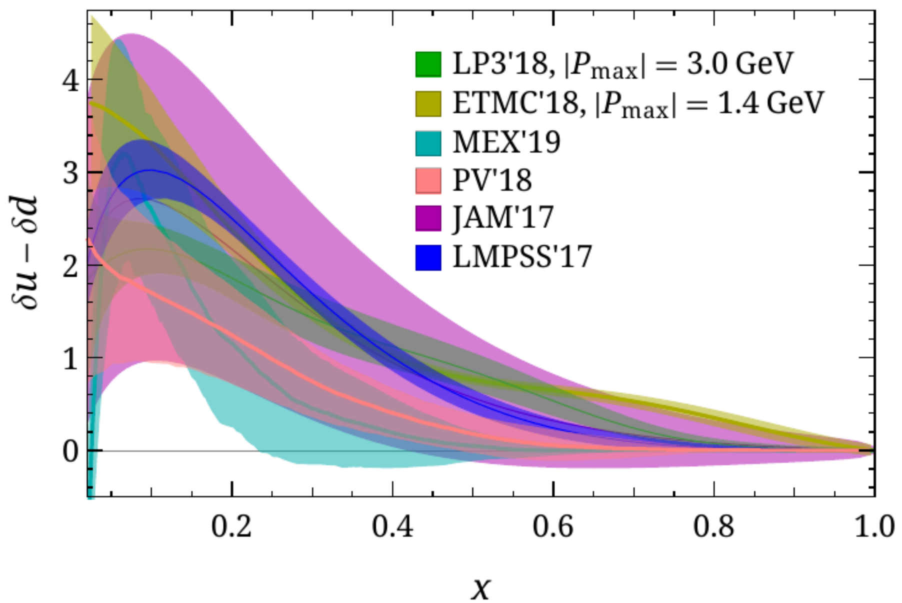
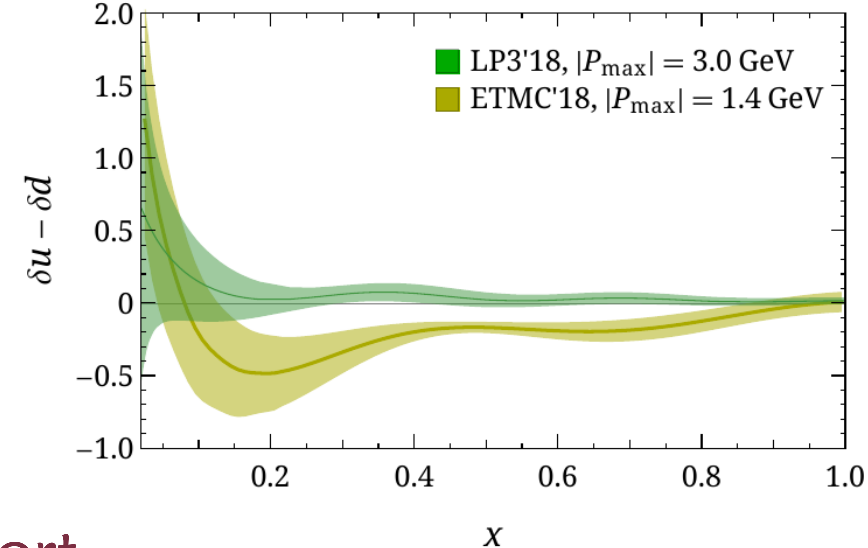


$$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$$



2006.08636, PDFLattice2019 report

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PNDME, 1806.09006

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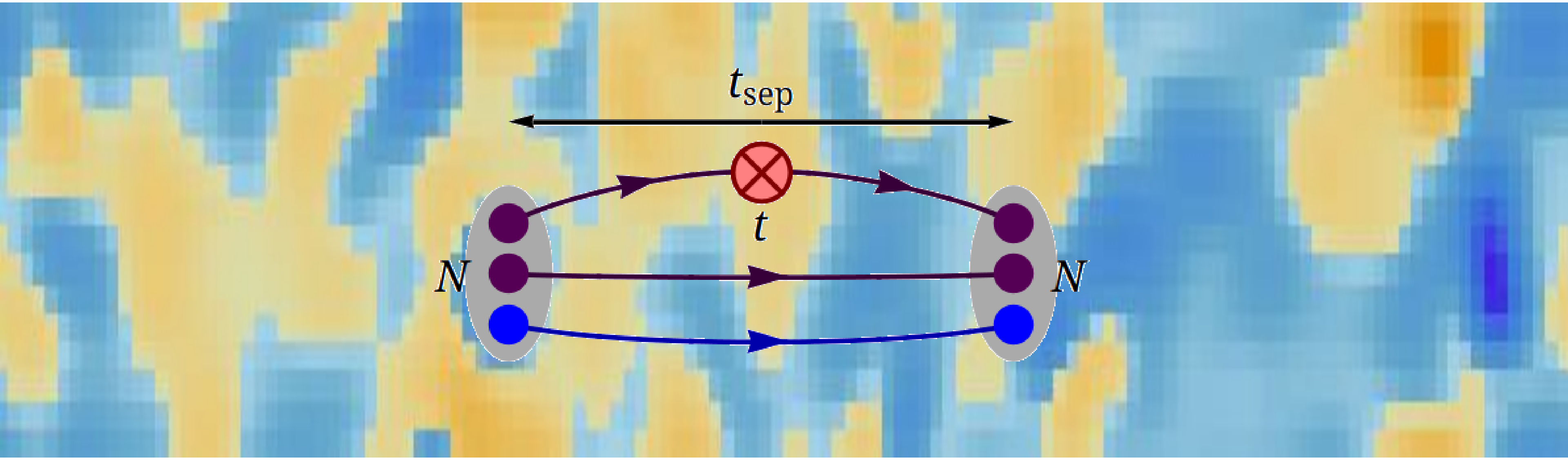
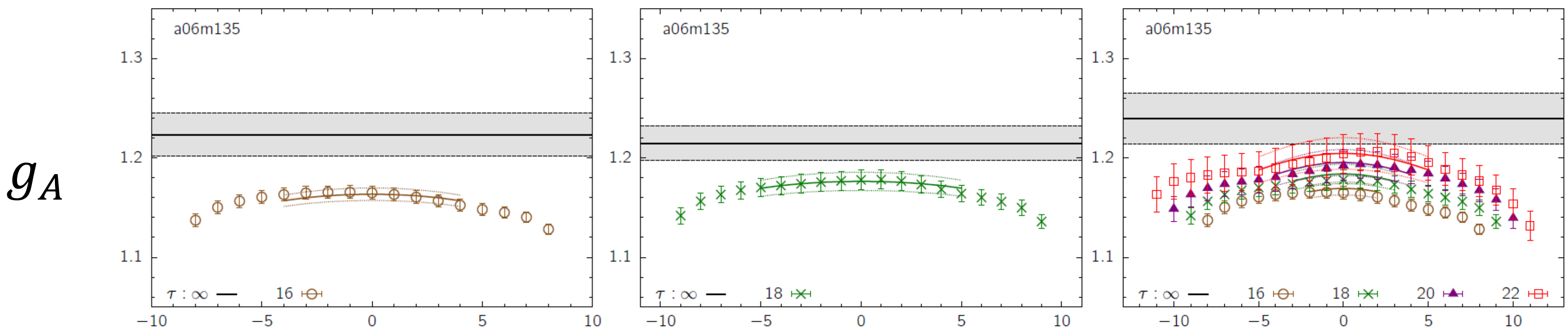
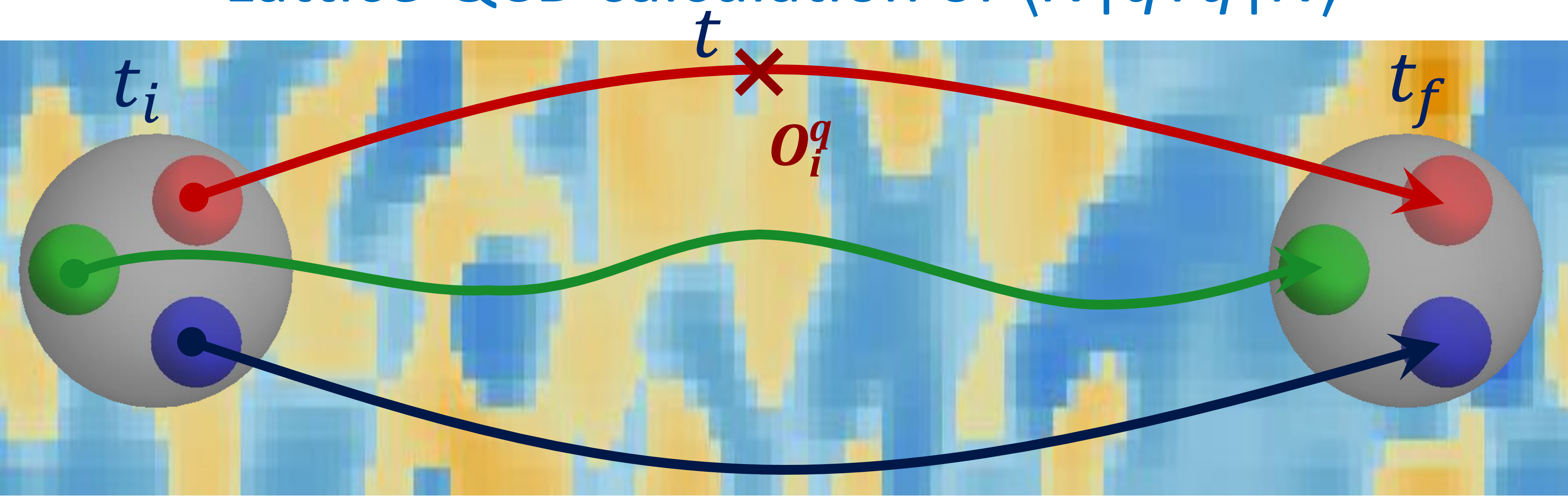




Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N | \bar{q}\Gamma q | N \rangle$



§ Systematic uncertainty (nonzero a, finite L, etc.)

- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  ($m_\pi \to m_\pi^{\text{phys}}$, $L \to \infty$, $a \to 0$)