Global analysis of DVES experiments

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CNF mini-workshop: Experiment and Theory Interactions: Current Status
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Outline

1. Introduction — DVCS and GPDs
2. Method
3. Results

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I}\,.$$

- where *e. g.* interference term is

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right] \right\},$$

- where *e. g.* $c_1^{\mathcal{I}}$ harmonic for unpolarized target is

$$c_{1,\rm unpol.}^{\mathcal{I}} \propto \left[F_1\,\mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2}F_2\,\mathfrak{Re}\,\mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}}(F_1+F_2)\,\mathfrak{Re}\,\widetilde{\mathcal{H}}\right]$$

- and at leading order everything depends on four complex

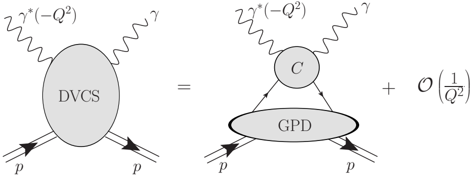
Compton form factors (CFFs)

$$\mathcal{H}(\xi,t,Q^2),\quad \mathcal{E}(\xi,t,Q^2),\quad \widetilde{\mathcal{H}}(\xi,t,Q^2),\quad \widetilde{\mathcal{E}}(\xi,t,Q^2)$$

We use formulas by [Belitsky, Müller, Ji '12].

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- CFFs are convolution:

$$^{a}\mathcal{H}(\xi, t, Q^2) = \int \mathrm{d}x \; C^{a}(x, \xi, \frac{Q^2}{Q_0^2}) \; H^{a}(x, \xi, t, Q_0^2)$$

$a{=}q, G$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

The Method

Two models

1. "Physical" CFF model
2. Neural network parametrization of CFFs

Modelling sea quark and gluon GPDs

- Instead of considering momentum fraction dependence $H(x, \ldots)$
- …it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, t) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, t)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials

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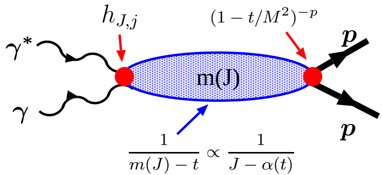
- $C_j^{3/2}(x)$ — Gegenbauer polynomials
- At LO easy multiplicative evolution
- Possible direct connection to lattice QCD results

SO(3) partial wave expansion

- To model η-dependence of GPD's $H_j(\eta, t)$ consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$ ($\frac{1}{\eta} \leftrightarrow \cos\theta$) and perform SO(3) partial wave expansion:



$$H_j(\eta, t) = \sum_{J=J_{\min}}^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2}\right)^p} \eta^{j+1-J} d^J_{0,\nu}\left(\frac{1}{\eta}\right)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer, ...)
  $\nu = 0, \pm 1$ — depending on hadron helicities
- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Modelling valence quark GPDs

- **Hybrid models** at LO
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\,t/\mathrm{GeV}^2 \quad (\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

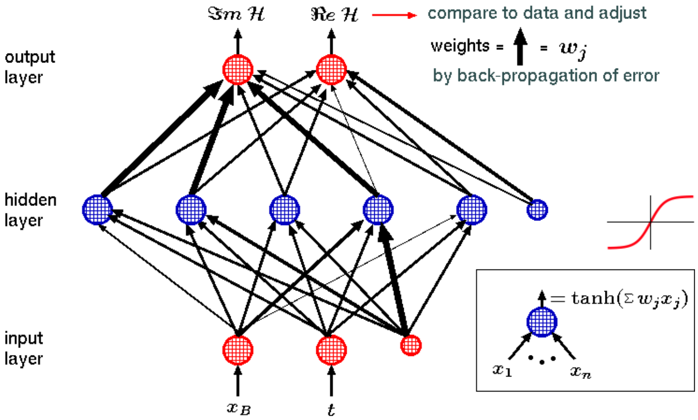
$$\mathfrak{Re}\,\mathcal{H}(\xi,t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)\mathfrak{Im}\,\mathcal{H}(\xi',t) - \frac{C}{\left(1-\frac{t}{{M_C}^2}\right)^2}$$

- Typical set of free parameters:

$M_0^{\rm sea}$, $s_{\rm sea}^{(2,4)}$, $s_G^{(2,4)}$	sea quarks and gluons H
$r^{\rm val}$, $M^{\rm val}$, $b^{\rm val}$	valence H
$\tilde{r}^{\rm val}$, $\tilde{M}^{\rm val}$, $\tilde{b}^{\rm val}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

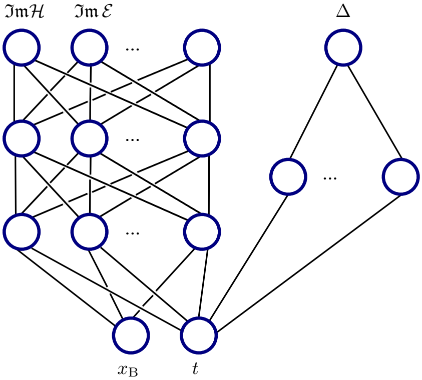
- [K.K., Müller '09] ("KM model")

Unconstrained neural networks



- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Networks constrained by dispersion relations



- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets
- Real parts are then fixed by dispersion relations

Results

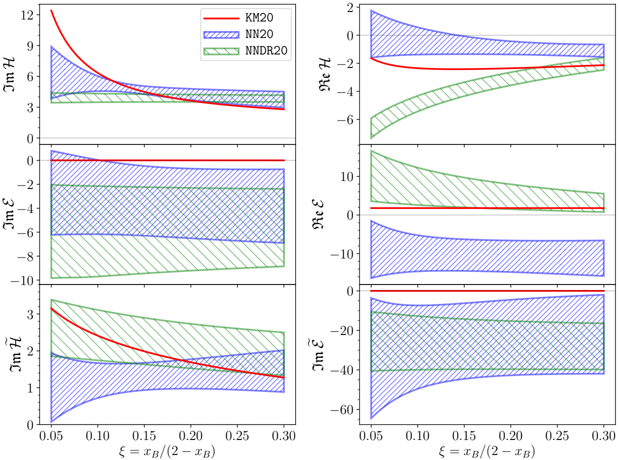
Fitting both models to JLab fixed target data

- [Čuić, K.K., Schäfer '20]
- First, using only proton data, and not attempting flavor separation
- NN = unconstrained neural nets, NNDR = neural nets + dispersion relations
- Values for χ^2/n_{pts} are reasonable:

Observable	n_{pts}	KM20	NN20	NNDR20
# CFFs + Δs		3 + 1	6	4 + 1
Total (harmonics)	277	1.3	1.6	1.7
CLAS [20] A_{LU}	162	0.9	1.0	1.1
CLAS [20] A_{UL}	160	1.5	1.7	1.8
CLAS [20] A_{LL}	166	1.3	3.9	0.8
CLAS [21] $d\sigma$	1014	1.1	1.0	1.2
CLAS [21] $\Delta\sigma$	1012	0.9	0.9	1.0
Hall A [22] $d\sigma$	240	1.2	1.9	1.7
Hall A [22] $\Delta\sigma$	358	0.7	0.8	0.8
Hall A [23] $d\sigma$	450	1.5	1.6	1.7
Hall A [23] $\Delta\sigma$	360	1.6	2.2	2.2
Hall A [8] $d\sigma_n$	96			
Total (ϕ-space)	4018	1.1	1.3	1.3

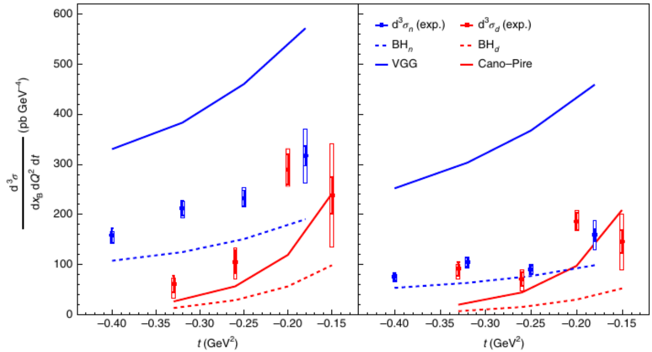
Extraction of 6 (out of 8) CFFs

- Witness the power of dispersion relations:



Hall A neutron DVCS measurement

- [Benali et al. '20], DVCS off a deuterium target:



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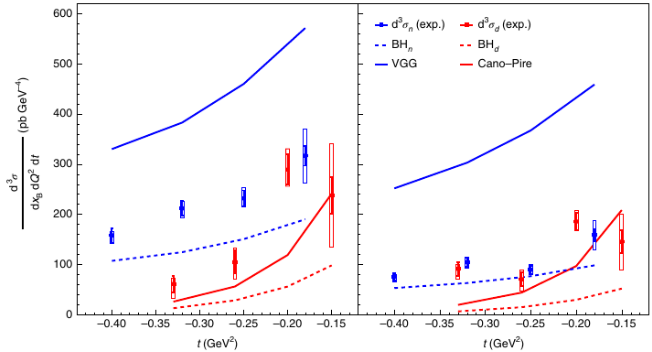
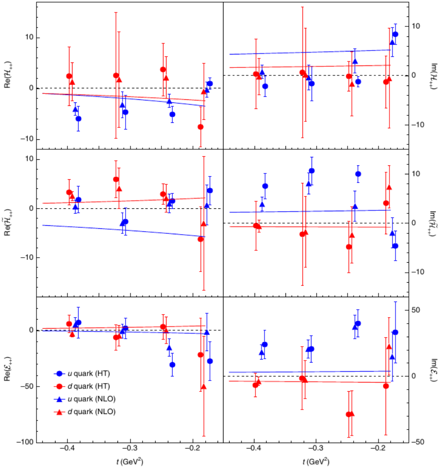


- Idea: combine proton and neutron DVCS data using isospin symmetry and get separate results for up and down quark contributions to CFFs

Flavor separation of CFFs

- [Benali et al. '20] attempt local flavor separation — large uncertainties:



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Maybe, with the little help from

1. Global fit (instead of local)
2. Dispersion relations constraints

$$\mathfrak{Re}\,\mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx \frac{2x}{\xi^2 - x^2}\,\mathfrak{Im}\,\mathcal{H}(x, t)$$

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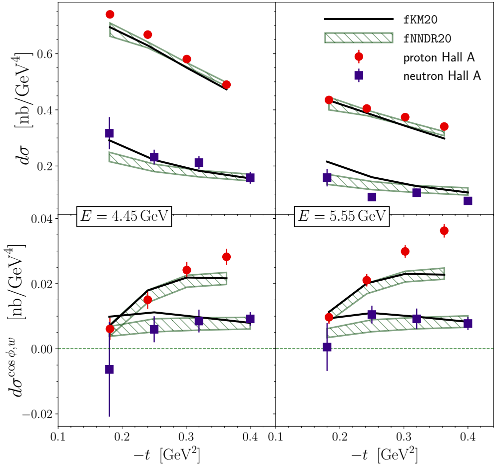
More constrained model generally leads to smaller uncertainties of the results. (*Bias-variance trade-off*)

Including the neutron DVCS data

- Separate model for each flavor: $\mathcal{H} \to \mathcal{H}_u, \mathcal{H}_d$, etc.
- Flavored models: fKM ("physical"), fNNDR (neural nets + dispersion relations)



Including the neutron DVCS data

- Values for $\chi^2/n_{\rm pts}$ are reasonable:

Observable	$n_{\rm pts}$	KM20	NN20	NNDR20	fKM20	fNNDR20
# CFFs + Δs		$3+1$	6	$4+1$	$5+2$	$8+2$
Total (harmonics)	277	1.3	1.6	1.7	1.7	1.8
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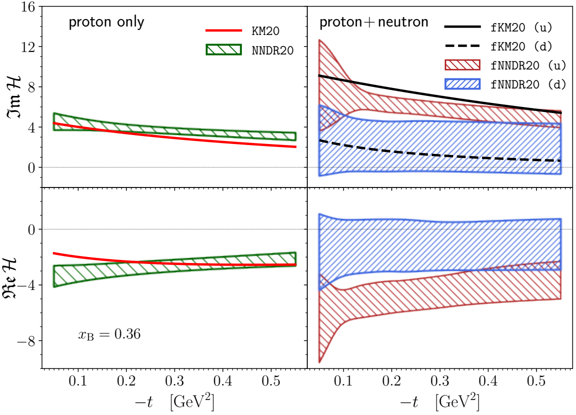
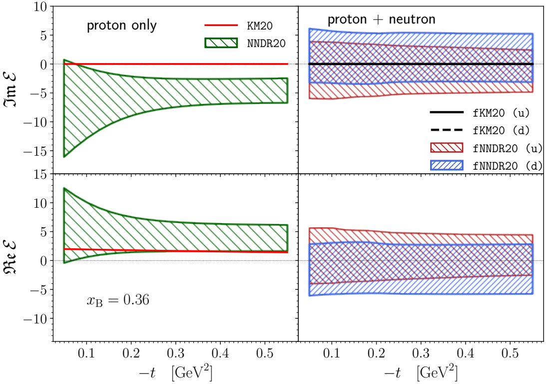
Separating flavored CFFs

- Contributions of u and d quarks to CFF $\mathcal{H}$ are cleanly separated:



Sanity check

- For other CFFs, there is no visible separation. E. g. $\mathcal{E}$:



Outlook

- Further flavor decomposition will likely be possible with inclusion of DVMP data in the analysis — work in progress
- Going from hybrid to complete conformal-space GPD model — work in progress
- All the components for the NLO analysis are available — work in progress

The End