

Tagging with polarized deuteron

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in collaboration with Ch. Weiss
1906.11119, 2006.03033
JLab LDRD project on spectator tagging
1409.5768, 1601.06665

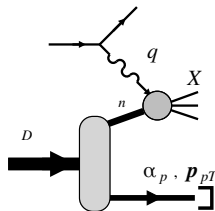


■ Deuteron

- ▶ Lightest nuclear system (loosely bound)
- ▶ Effective neutron target
- ▶ Short-range NN force → Talk W. Boeglin
- ▶ First principle calculations
- ▶ Spin 1

■ Spectator tagging

- ▶ Additional control over initial nuclear configuration
- ▶ Pole extrapolation at low momenta
→ free neutron structure [talk S. Kuhn]
- ▶ Medium modifications at high momenta [talk Kutz],
but final-state interactions
- ▶ Deuteron reaction model calculable
- ▶ Especially suited for colliders with far-forward detectors → talks Friday PM



Polarized spin 1 particle

- polarization states $m = \pm 1, 0$
- 3 by 3 density matrix in a basis of spin 1 states polarized along the collinear virtual photon-target axis

$$W_D^{\mu\nu} = \text{Tr}[\rho_{\lambda\lambda'} W^{\mu\nu}(\lambda'\lambda)]$$

- **Vector** (3) and **tensor** (5) polarization parameters

$$\rho_{\lambda\lambda'} = \frac{1}{3} \begin{bmatrix} 1 + \frac{3}{2}S_L + \sqrt{\frac{3}{2}}T_{LL} & \frac{3}{2\sqrt{2}}S_T e^{-i(\phi_h - \phi_s)} - \sqrt{3}T_{LT} e^{-i(\phi_h - \phi_{TL})} & \sqrt{\frac{3}{2}}T_{TT} e^{-i(2\phi_h - 2\phi_{TT})} \\ \frac{3}{2\sqrt{2}}S_T e^{i(\phi_h - \phi_s)} - \sqrt{3}T_{LT} e^{i(\phi_h - \phi_{TL})} & 1 - \sqrt{6}T_{LL} & \frac{3}{2\sqrt{2}}S_T e^{-i(\phi_h - \phi_s)} + \sqrt{3}T_{LT} e^{-i(\phi_h - \phi_{TL})} \\ \sqrt{\frac{3}{2}}T_{TT} e^{i(2\phi_h - 2\phi_{TT})} & \frac{3}{2\sqrt{2}}S_T e^{i(\phi_h - \phi_s)} + \sqrt{3}T_{LT} e^{i(\phi_h - \phi_{TL})} & 1 - \frac{3}{2}S_L + \sqrt{\frac{3}{2}}T_{LL} \end{bmatrix}$$

- Can be formulated in **covariant** manner

$$\rightarrow \rho^{\mu\nu} = \sum_{\lambda\lambda'} \rho_{\lambda\lambda'} \epsilon^\mu(\mathbf{p}, \lambda) \epsilon^{*\nu}(\mathbf{p}, \lambda') = \frac{1}{3} \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right) + \frac{i}{2M} \epsilon^{\mu\nu\rho\sigma} p_\rho s_\sigma - t^{\mu\nu}$$

Cfr. Spin 1/2

$$\rightarrow \rho = \sum_{\lambda\lambda'} \rho_{\lambda\lambda'} \mathbf{u}(\mathbf{p}, \lambda) \bar{\mathbf{u}}(\mathbf{p}, \lambda') = \frac{1}{2}(\not{p} + m)(1 + \gamma_5 \not{s})$$

Tensor polarization: physics interest

- Tensor polarized observables are proportional to deuteron D -wave [Frankfurt, Strikman 80s]
 - ▶ short-range nature of the NN force
 - ▶ universality of short-range correlations for medium and heavy nuclei
- Admits gluon transversity in DIS [Jaffe, Manohar '89] → talk Shanahan
 - ▶ arises from nuclear interactions
 - ▶ $\Delta L = 2$ to compensate gluon helicity flip, not possible for nucleon
- Spin-orbit phenomena beyond the nucleon
- Exotic effects (hidden color etc.) [Miller, CR Ji, Brodsky]
- Tensor polarization of the quark sea [Kumano]
- Deuteron polarimetry

- General expression of SIDIS for a polarized spin 1 target
 - ▶ Tagged spectator DIS is SIDIS in the target fragmentation region

$$\vec{e} + \vec{T} \rightarrow e' + X + h$$

- Light-front structure of the deuteron
 - ▶ Natural for high-energy reactions as **off-shellness of nucleons** in LF quantization remains **finite**
- Dynamical model to express structure functions of the reaction
 - ▶ First step: impulse approximation (IA) model
 - ▶ Results for longitudinal, tensor spin asymmetries A_{LL}, A_{zz}
 - ▶ FSI corrections (unpolarized [Strikman, Weiss PRC '18], \rightarrow talk Weiss)

Spin 1 SIDIS: General structure of cross section

- To obtain structure functions, enumerate all possible tensor structures that obey hermiticity and transversality condition ($qW = Wq = 0$)
- Cross section has 41 structure functions,

$$\frac{d\sigma}{dx dQ^2 d\phi'} = \frac{y^2 \alpha^2}{Q^4 (1 - \epsilon)} (F_U + F_S + F_T) d\Gamma_{P_h},$$

- ▶ U + S part identical to spin 1/2 case [Bacchetta et al. JHEP ('07)]

$$F_U = F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \epsilon \cos 2\phi_h F_{UU}^{\cos 2\phi_h} + h \sqrt{2\epsilon(1-\epsilon)} \sin \phi_h F_{LU}^{\sin \phi_h}$$

$$\begin{aligned} F_S = & \mathbf{S}_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin \phi_h F_{USL}^{\sin \phi_h} + \epsilon \sin 2\phi_h F_{USL}^{\sin 2\phi_h} \right] \\ & + \mathbf{S}_L h \left[\sqrt{1-\epsilon^2} F_{LSL} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_h F_{LSL}^{\cos \phi_h} \right] \\ & + \mathbf{S}_\perp \left[\sin(\phi_h - \phi_S) \left(F_{UST,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UST,L}^{\sin(\phi_h - \phi_S)} \right) + \epsilon \sin(\phi_h + \phi_S) F_{UST}^{\sin(\phi_h + \phi_S)} \right. \\ & \left. + \epsilon \sin(3\phi_h - \phi_S) F_{UST}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UST}^{\sin \phi_S} + \sin(2\phi_h - \phi_S) F_{UST}^{\sin(2\phi_h - \phi_S)} \right) \right] \\ & + \mathbf{S}_\perp h \left[\sqrt{1-\epsilon^2} \cos(\phi_h - \phi_S) F_{LST}^{\cos(\phi_h - \phi_S)} + \right. \\ & \left. \sqrt{2\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LST}^{\cos \phi_S} + \cos(2\phi_h - \phi_S) F_{LST}^{\cos(2\phi_h - \phi_S)} \right) \right], \end{aligned}$$

Spin 1 SIDIS: General structure of cross section

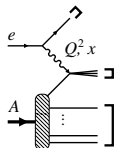
- To obtain structure functions, enumerate all possible tensor structures that obey hermiticity and transversality condition ($qW = Wq = 0$)
- Cross section has 41 structure functions,

$$\frac{d\sigma}{dx dQ^2 d\phi_{P'}} = \frac{y^2 \alpha^2}{Q^4 (1 - \epsilon)} (F_U + F_S + F_T) d\Gamma_{P_h},$$

- ▶ **23 SF** unique to the spin 1 case (tensor pol.), 4 survive in inclusive (b_{1-4}) [Hoodbhoy, Jaffe, Manohar PLB'88]

$$\begin{aligned} F_T = & T_{LL} \left[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi_h F_{UT_{LL}}^{\cos \phi_h} + \epsilon \cos 2\phi_h F_{UT_{LL}}^{\cos 2\phi_h} \right] \\ & + T_{LL} h \sqrt{2\epsilon(1-\epsilon)} \sin \phi_h F_{LT_{LL}}^{\sin \phi_h} \\ & + T_{L\perp} [\dots] + T_{L\perp} h [\dots] \\ & + T_{\perp\perp} \left[\cos(2\phi_h - 2\phi_{T\perp}) \left(F_{UT_{TT},T}^{\cos(2\phi_h - 2\phi_{T\perp})} + \epsilon F_{UT_{TT},L}^{\cos(2\phi_h - 2\phi_{T\perp})} \right) \right. \\ & + \epsilon \cos 2\phi_{T\perp} F_{UT_{TT}}^{\cos 2\phi_{T\perp}} + \epsilon \cos(4\phi_h - 2\phi_{T\perp}) F_{UT_{TT}}^{\cos(4\phi_h - 2\phi_{T\perp})} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \left(\cos(\phi_h - 2\phi_{T\perp}) F_{UT_{TT}}^{\cos(\phi_h - 2\phi_{T\perp})} + \cos(3\phi_h - 2\phi_{T\perp}) F_{UT_{TT}}^{\cos(3\phi_h - 2\phi_{T\perp})} \right) \right] \\ & + T_{\perp\perp} h [\dots] \end{aligned}$$

Theory: high-energy scattering with nuclei



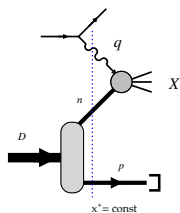
- Interplay of two scales: high-energy scattering and low-energy nuclear structure. Virtual photon probes nucleus at fixed lightcone time $x^+ = x^0 + x^3$

- Scales can be separated using methods of light-front quantization and QCD factorization

- Tools for high-energy scattering known from ep

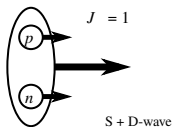
- Nuclear input: light-front momentum densities, spectral functions, overlaps with specific final states in breakup/tagging reactions

- ▶ framework known for deuteron, can be extended to ^3He
- ▶ still **low-energy** nuclear physics, just formulated differently



→ talks Strikman, Weiss

Deuteron light-front wave function



- Up to momenta of a few 100 MeV: dominated by NN
- Can be evaluated in LFQM
[Berestetsky, Frankfurt, Strikman, Terentev]
- Overlap with **on-shell** free two-nucleon state

$$P_{NN}^i = P_D^i \quad i = +, T; \quad P_{NN}^- \neq P_D^-$$

- Schrödinger (non-rel) like eq. for the wf components, rotational invariance recovered \rightarrow talk Strikman

$$\Psi_\lambda(\mathbf{k}, \lambda_p, \lambda_n) = \sqrt{E_k} \sum_{\lambda'_p \lambda'_n} \mathcal{D}_{\lambda_p \lambda'_p}^{\frac{1}{2}} [R_{fc}(k_1^\mu/m)] \mathcal{D}_{\lambda_n \lambda'_n}^{\frac{1}{2}} [R_{fc}(k_2^\mu/m)] \Phi_\lambda(\mathbf{k}, \lambda'_p, \lambda'_n)$$

- **Differences** with non-rel wave function:
 - ▶ appearance of the **Melosh rotations** to account for light-front quantized nucleon states
 - ▶ \mathbf{k} is the rel. 3-momentum in the rest frame of the on-shell NN state

Effective neutron spin density matrix

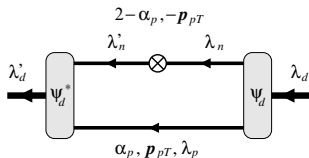
- Deuteron LF wavefunction:

$$\Psi_{\lambda_d}(\mathbf{k}, \lambda_p, \lambda_n) = \sqrt{E_k} \sum_{\lambda'_p \lambda'_n} \mathcal{D}_{\lambda_p \lambda'_p}^{\frac{1}{2}} [R_{fc}(k_1^H/m)] \mathcal{D}_{\lambda_n \lambda'_n}^{\frac{1}{2}} [R_{fc}(k_2^H/m)] \Phi_{\lambda}(\mathbf{k}, \lambda'_p, \lambda'_n)$$

- 4D covariant formulation: [Kondryatchuk, Strikman '83]

$$\Psi_{\lambda_d}(\alpha_p, \mathbf{p}_{pT}, \lambda_p, \lambda_n) = \bar{u}_{LF}(\mathbf{p}_n \lambda_n) \Gamma_{\alpha}(\mathbf{p}_p, \mathbf{p}_n) v_{LF}(\mathbf{p}_p, \lambda_p) \epsilon_{pn}^{\alpha}(\mathbf{p}_{pn}, \lambda_d)$$

- Matrix elements of nucleon operators



$$\langle \hat{O}_n \rangle = \int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} \frac{2 \text{tr}[\Gamma_n \Gamma_n]}{(2 - \alpha_p)} \quad \alpha_p = 2p_p^+ / p_d^+$$

- Effective neutron spin density matrix (cfr. parton correlators in QCD)

$$\Pi_n = (\rho_{pn})^{\alpha\beta} (\not{p}_n + m) \Gamma_{\alpha}(\not{p}_p - m) \Gamma_{\beta}(\not{p}_n + m)$$

Nucleon LF momentum distributions

- Can be split into unpolarized, vector and tensor polarization terms:

$$\Pi_n[\text{unpol}] = \frac{1}{2}(\not{p}_n + m)(f_0^2 + f_2^2),$$

$$\Pi_n[\text{vector}] = \frac{1}{2}(\not{p}_n + m)\not{\epsilon}_n(\mathbf{S}_d, \mathbf{k})\gamma_5,$$

$$\Pi_n[\text{tensor}] = -\frac{1}{2}(\not{p}_n + m)(\mathbf{k} T_d \mathbf{k}) \frac{3}{k^2} \left(2f_0 + \frac{f_2}{\sqrt{2}} \right) \frac{f_2}{\sqrt{2}}.$$

- Allows for the definition of **nucleon** light-front momentum distributions

$$\text{Helicity independent} \quad S_d(\alpha_p, \mathbf{p}_{pT}) = \frac{\text{tr}[\Pi_n \gamma^+]}{(2 - \alpha_p)^2 p_d^+},$$

$$\text{Helicity dependent} \quad \Delta S_d(\alpha_p, \mathbf{p}_{pT}) = \frac{\text{tr}[\Pi_n (-\gamma^+ \gamma_5)]}{(2 - \alpha_p)^2 p_d^+},$$

$$\text{Transversity} \quad \Delta_T S_d(\alpha_p, \mathbf{p}_{pT}) = \frac{\text{tr}[\Pi_n (i\sigma^{i+} \gamma_5)]}{(2 - \alpha_p)^2 p_d^+},$$

- S_d receives contributions from $\Pi_n[\text{unpol}]$ and $\Pi_n[\text{tensor}]$
 $\Delta S_d, \Delta_T S_d$ receives contributions from $\Pi_n[\text{vector}]$
- Tensor polarization does not induce nucleon helicity dependence

Nucleon LF momentum distributions (II)

■ LF momentum distributions obey sum rules

▶ baryon

$$\int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} S_d(\alpha_p, \mathbf{p}_{pT})[\text{unpol}] = 1,$$
$$\int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} S_d(\alpha_p, \mathbf{p}_{pT})[\text{tensor}] = 0,$$

▶ momentum

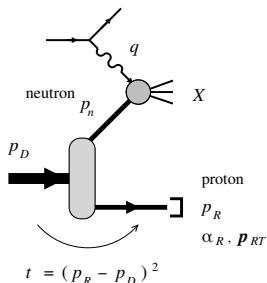
$$\int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} (2 - \alpha_p) S_d(\alpha_p, \mathbf{p}_{pT})[\text{unpol}] = 1,$$
$$\int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} (2 - \alpha_p) S_d(\alpha_p, \mathbf{p}_{pT})[\text{tensor}] = 0$$

▶ axial

$$\int \frac{d\alpha_p}{\alpha_p} d^2 p_{pT} \Delta S_d(\alpha_p, \mathbf{p}_{pT})[\text{vector}] = S_d^z \frac{g_{Ad}}{2g_A},$$
$$1 - \frac{3}{2} \omega_2 = \frac{g_{Ad}}{2g_A}.$$

→ cfr correction in inclusive polarized *ed* DIS

Tagged DIS with deuteron: model for the IA



- Hadronic tensor can be written as a product of nucleon hadronic tensor with deuteron light-front densities

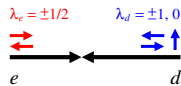
$$\sum_{\lambda, \lambda'} \rho_{\lambda \lambda'} W_D^{\mu\nu}(\lambda', \lambda) = 4(2\pi)^3 \frac{\alpha_R}{2 - \alpha_R} \sum_i W_{N,i}^{\mu\nu} S_{d,i}$$

All SF can be written as

$$F_{ij}^k = \{\text{kin. factors}\} \times \{F_{1,2}(\tilde{x}, Q^2) \text{ or } g_{1,2}(\tilde{x}, Q^2)\} \\ \times \{\text{bilinear in deuteron radial } f_0(k) [\text{S-wave}], f_2(k) [\text{D-wave}]\}$$

- In the IA the following structure functions are **zero** → sensitive to FSI
 - ▶ beam spin asymmetry [$F_{LU}^{\sin \phi_h}$]
 - ▶ target vector polarized single-spin asymmetry [8 SFs]
 - ▶ target tensor polarized double-spin asymmetry [7 SFs]

Polarized structure function: longitudinal asymmetry



■ Pole extrapolation of double spin asymmetry

- ▶ Nominator

$$d\sigma_{||} \equiv \frac{1}{4} \left[d\sigma\left(+\frac{1}{2}, +1\right) - d\sigma\left(-\frac{1}{2}, +1\right) - d\sigma\left(+\frac{1}{2}, -1\right) + d\sigma\left(-\frac{1}{2}, -1\right) \right]$$

- ▶ Denominators: 2-state

$$d\sigma_2 \equiv \frac{1}{4} \sum_{\Lambda_e} [d\sigma(\Lambda_e, +1) + d\sigma(\Lambda_e, -1)]$$

- ▶ Asymmetries: **tensor polarization** enters in 2-state one

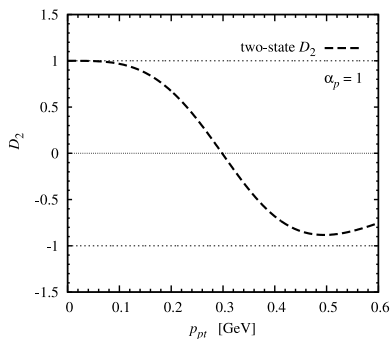
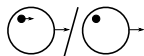
$$A_{||,2} = \frac{d\sigma_{||}}{d\sigma_2} [\phi_h \text{ avg}] = \frac{F_{LS_L}}{F_T + \epsilon F_L + \frac{1}{\sqrt{6}} (F_{T_{LL}T} + \epsilon F_{T_{LL}L})}$$

■ Impulse approximation yields in the Bjorken limit $[\alpha_p = \frac{2p_p^+}{p_D^+}]$

$$A_{||,i} \approx \mathcal{D}_i(\alpha_p, |\mathbf{p}_{pT}|) A_{||n} = \mathcal{D}_i(\alpha_p, |\mathbf{p}_{pT}|) \frac{D_{||} g_{1n}(\tilde{x}, Q^2)}{2(1 + \epsilon R_n) F_{1n}(\tilde{x}, Q^2)}$$

Nuclear structure factor \mathcal{D}_2

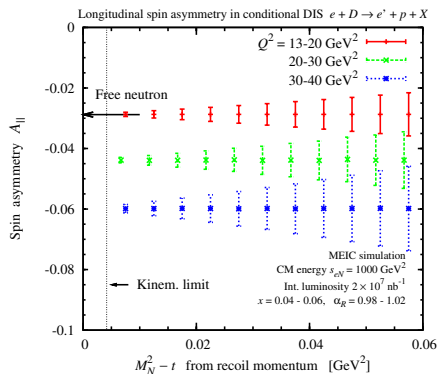
- Quantifies neutron depolarization due to nuclear structure
- Depends on spectator kinematics α_p, p_{pT}
- $\mathcal{D}_2 = \Delta S_d[\text{pure } +1]/S_d[\text{pure } +1]$ has **probabilistic interpretation**



WC, C. Weiss, PLB ('19); PRC ('20)

- Bounds: $-1 \leq \mathcal{D}_2 \leq 1$
- Due to lack of OAM $\mathcal{D}_2 \equiv 1$ for $p_T = 0$
- Clear contribution from D-wave at finite recoil momenta
- \mathcal{D}_2 close to unity at small recoil momenta
- 3-state (unpol) denominator less "nice"
- 2-state asymmetry is also easier experimentally!!

Tagging: simulations of $A_{||}$



JLab LDRD arXiv:1407.3236, arXiv:1409.5768
<https://www.jlab.org/theory/tag/>

- EIC simulations, spectator can be detected in far-forward detectors
- D-wave suppr. at on-shell point \rightarrow neutron $\sim 100\%$ polarized
- Precise measurements of neutron spin structure
- Systematic uncertainties cancel in ratio (momentum smearing, resolution effects)
- Statistics requirements
 - ▶ Physical asymmetries $\sim 0.05 - 0.1$
 - ▶ Effective polarization $P_e P_D \sim 0.5$
 - ▶ Luminosity required $\sim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

Tensor polarized observable A_{ZZ} [Frankfurt, Strikman '83]

- Analogue of $A_{LL} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$ for vector polarization ($\sim S_L$) is the tensor asymmetry

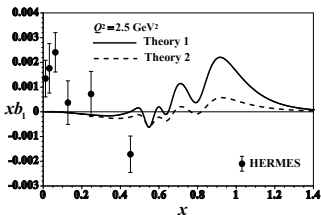
$$A_{ZZ} = \frac{\sigma^+ + \sigma^- - 2\sigma^0}{\sigma^+ + \sigma^- + \sigma^0}$$

→ **no** electron polarization required.

- Requires all three polarization states; $A_{ZZ} \in [-2, 1]$ and $\sim T_{LL}$
- ed : A_{ZZ} measured in different ranges of Bjorken $x = 2 \frac{Q^2}{2(p_d q)}$, $x \in [0, 2]$:
 - ▶ elastic $x = 2$ [T_{20}]: NIKHEF ('98), ...
 - ▶ quasi-elastic $x > 1$: NIKHEF ('99), BLAST @ MIT-Bates ('17), future JLab12
 - ▶ DIS inclusive $x < 1$ [b_1]: Hermes @ DESY ('05), future JLab12

A_{ZZ} in inclusive DIS

- A_{ZZ} is proportional to tensor pol. structure function b_1 in scaling limit [Hoodbhoy, Jaffe, Manohar '89]
 - ▶ partonic density interpretation $b_1 = \frac{1}{2} \sum_q e_q^2 (q^0 - q^1)$
 - ▶ leading twist, QCD operator as in F_2 , same evolution etc.
 - ▶ encodes dependence of unpol. quarks on nuclear interactions
→ “donut” vs “dumbbell”



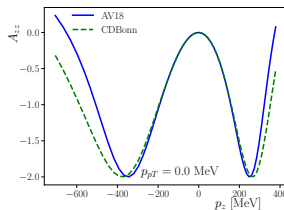
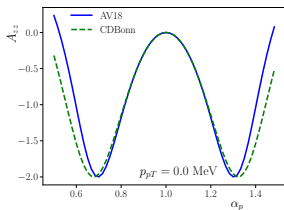
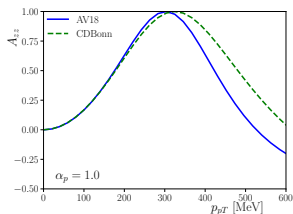
Cosyn, Dong, Kumano,
Sargsian PRD'17

- Small A_{ZZ} values
→ averaging over initial nuclear confs.
- Mismatch theory / data → HT effects?
- Shadowing corrections at small x
[Frankfurt, Strikman; Nikolaev, Schäfer '97;
Edelmann, Piller, Weise '97]
- Hidden color + pions match data [Miller '16]

A_{zz} with spectator tagging

- Tensor polarization is sensitive to unpolarized quark distributions, partonic factor cancels out \rightarrow ratio of LF densities remains

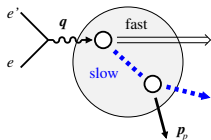
$$A_{zz}(\alpha_p, \mathbf{p}_T) = -\frac{\frac{f_0(k)f_2(k)}{\sqrt{2}} + \frac{f_2^2(k)}{4}}{f_0^2(k) + f_2^2(k)} (3 \cos 2\theta_k + 1) \quad \alpha_p = \left(1 + \frac{k^3}{\sqrt{m^2 + k^2}}\right); \quad \mathbf{p}_{pT} = \mathbf{k}_T$$



- Maximal A_{zz} at $f_2(k) = \sqrt{2}f_0(k)$, not the S wave node!
- $A_{zz} = 1$ at $\alpha_p = 1$ ($\theta_k = \pi/2$) \rightarrow pure $m = \pm 1$
 $A_{zz} = -2$ at $p_{pT} = 0$ ($\theta_k = 0, \pi$) \rightarrow pure $m = 0$
- Needs quantification of FSI effects

\rightarrow Constraints on deuteron D -wave

Final-state interactions in tagging



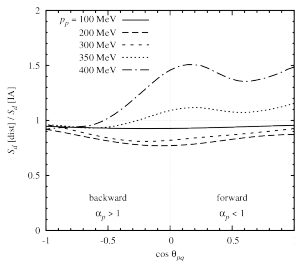
- **Issue** in tagging: DIS products can interact with spectator \rightarrow rescattering, absorption

- Dominant contribution at intermediate $x \sim 0.1 - 0.5$ from "**slow**" hadrons that hadronize inside nucleus

- Measure nucleon fracture functions [EIC]

- Features of the FSI of slow hadrons with spectator nucleon are similar to what is seen in quasi-elastic deuteron breakup.

- FSI vanish at the pole \rightarrow pole extrapolation **still feasible**



Strikman, Weiss, PRC7 035209 ('18)

Conclusions

- Unique observables with **polarized deuteron**: free neutron spin structure, tensor polarization
- Natural and intuitive interpretation through effective neutron spin density matrix [deuteron polarization, nuclear structure]
- Extraction of **nucleon spin structure** in a wide kinematic range at EIC
- Tagged tensor asymmetry A_{zz} can be made maximal (-2,1) compared to small inclusive one
→ constraints on **D-wave**.
- **Final-state interactions** need to be further quantified (but drop out for pole extrapolation)
- Lots of extensions to be explored...