How to do SRC calculations at low-RG resolution scales

Anderson, SKB, Furnstahl, PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)
More, SKB, Furnstahl, PRC **96** (2017)
Tropiano, SKB, Furnstahl PRC **102** (2020)
Tropiano, SKB, Furnstahl, in prep

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Michigan State University

Low and High RG Resolution Scale Pictures

NSCL FRIB

(RG) Resolution Scale $H = H(\Lambda)$ ➡ max. momenta in low-energy wf's ~ Λ

High resolution picture:

Low and High RG Resolution Scale Pictures

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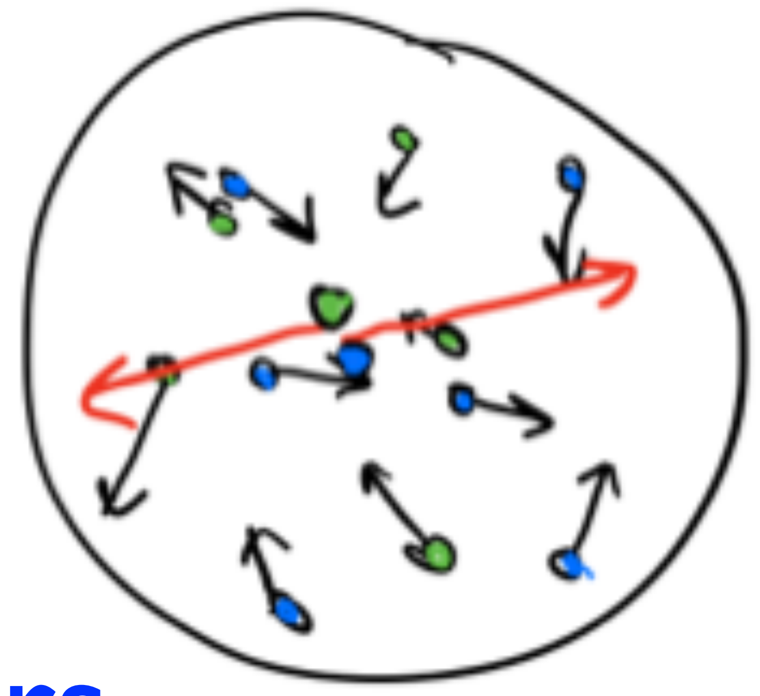
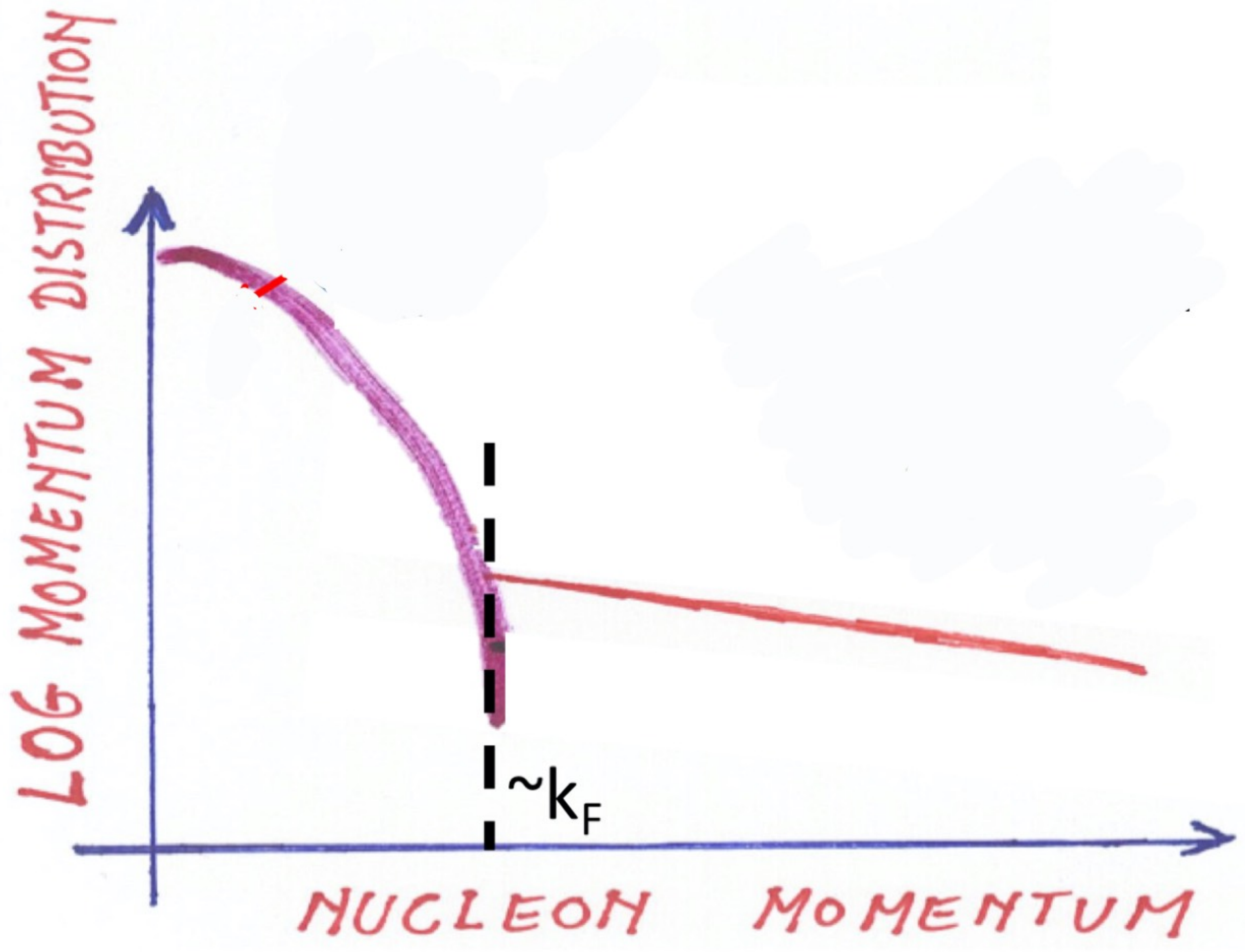
High resolution picture:



correlated SRC pairs

Hard, local interactions
AV18 etc.

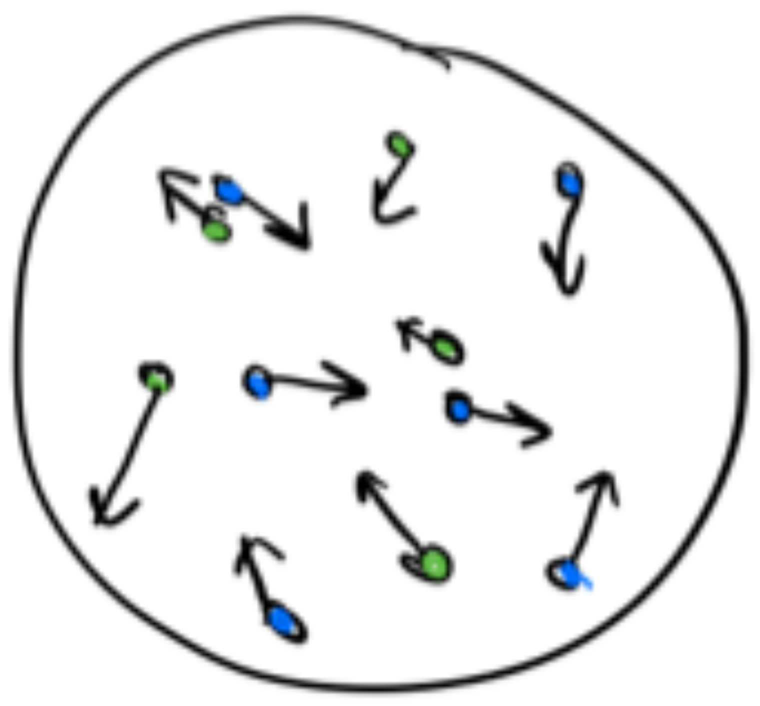


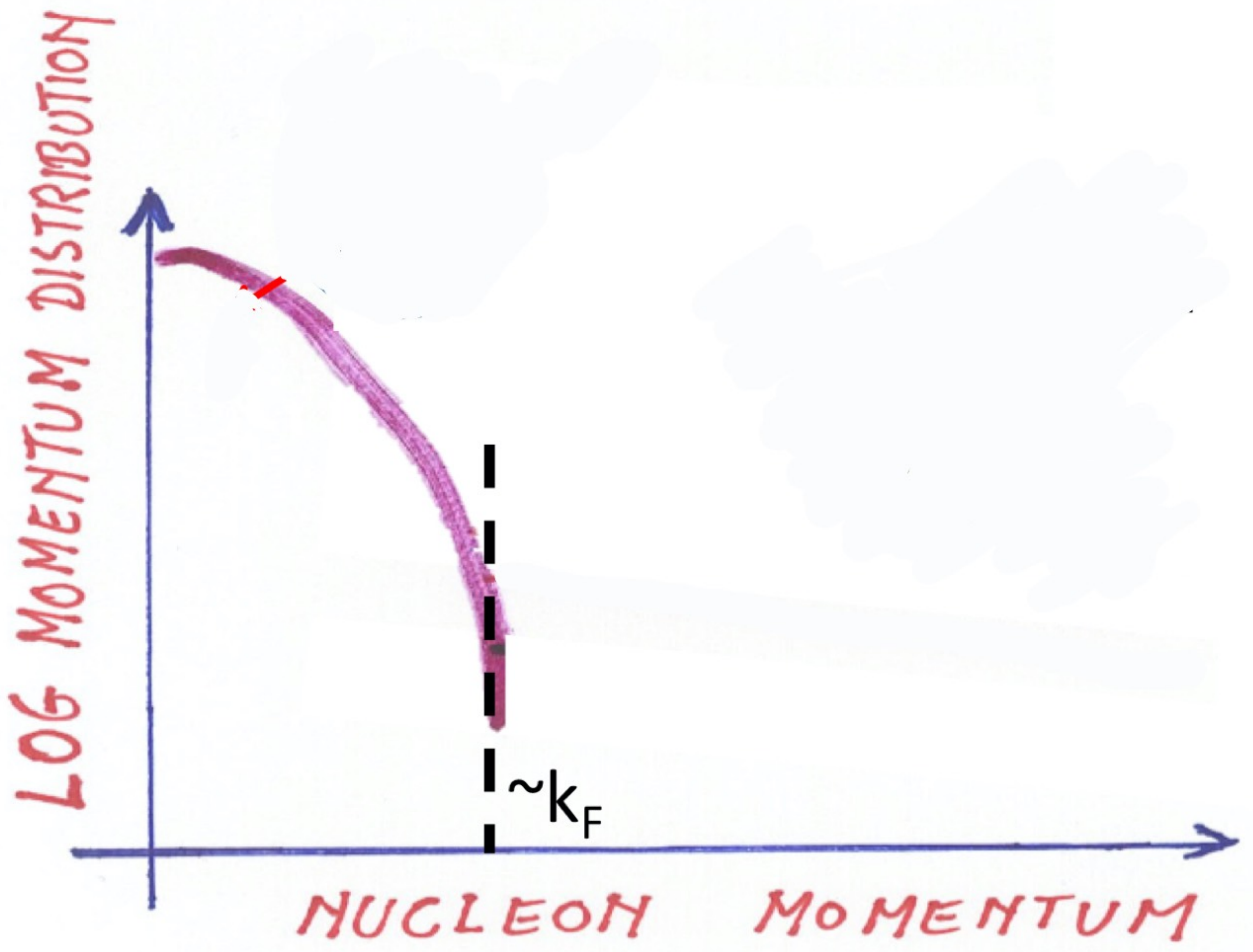
high-k tails ($k >> k_F$) present

Low and High RG Resolution Scale Pictures

(RG) Resolution Scale $H = H(\Lambda)$ ➡ max. momenta in low-energy wf's ~ Λ

Low resolution picture:





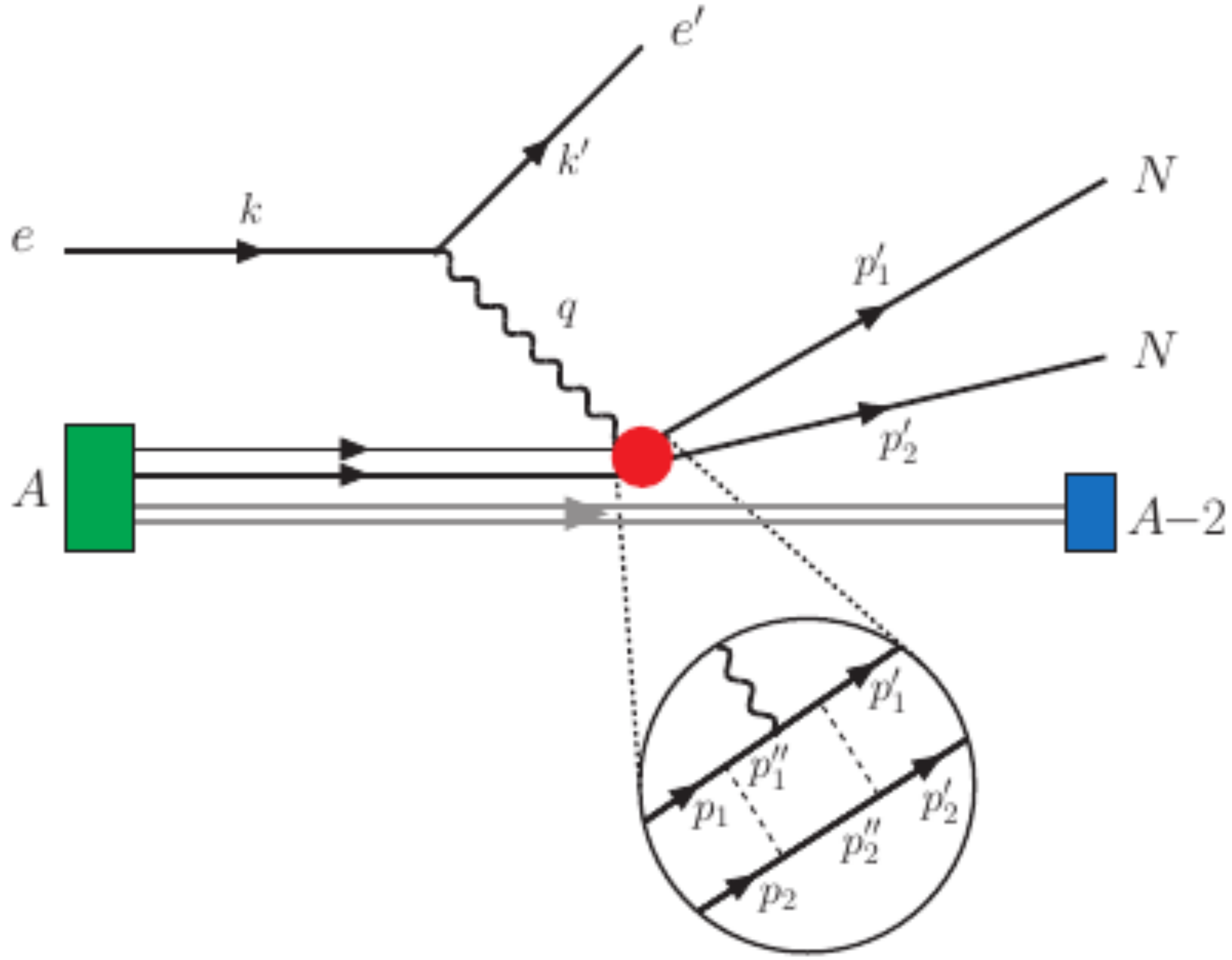
resembles "mean field" picture

chiral EFT/soft interactions/shell model/DFT

no high-k tails ($k >> k_F$)

Low and High RG Resolution Scale Pictures

SRC studies at high resolution



one-body current operators
complicated wf's

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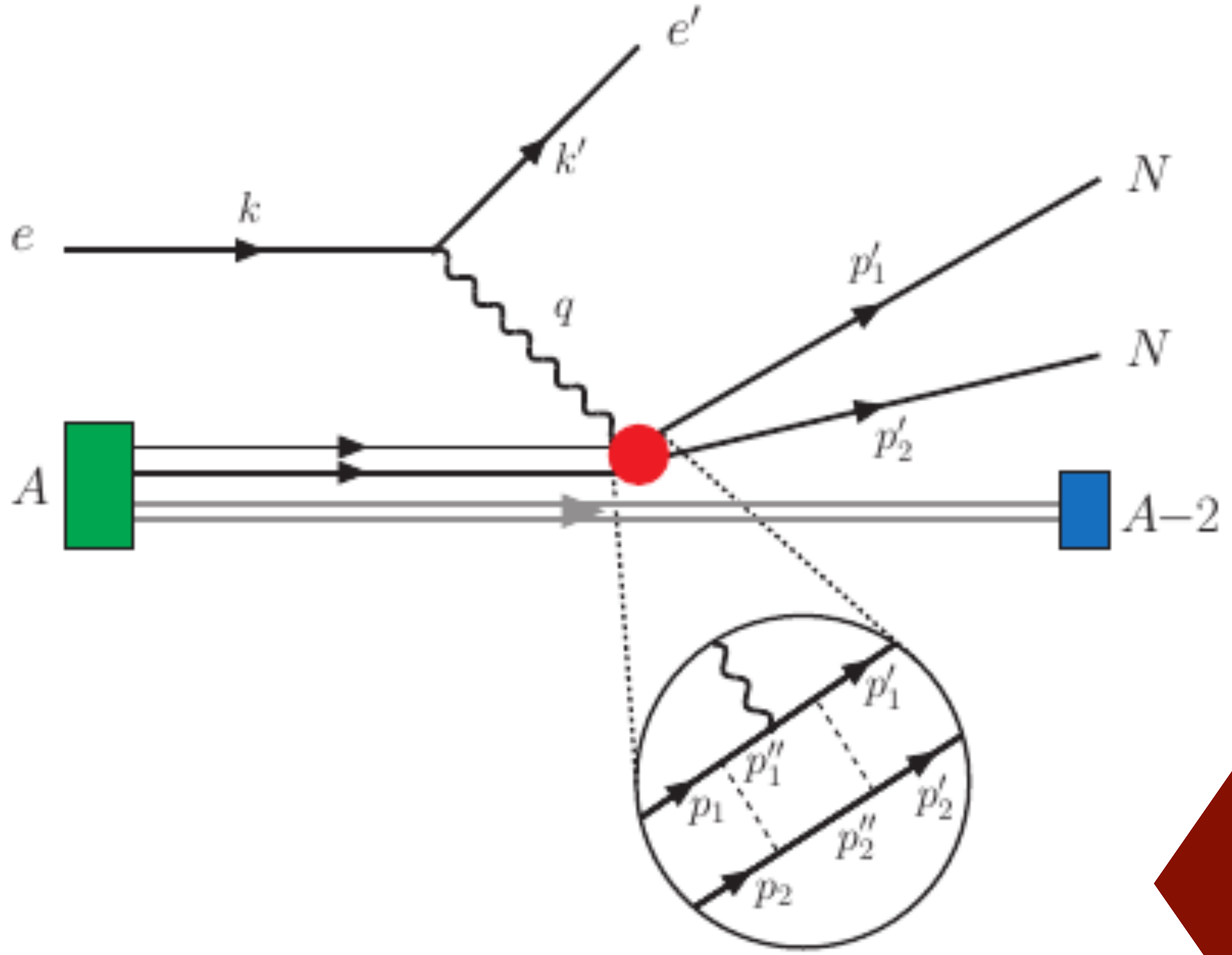
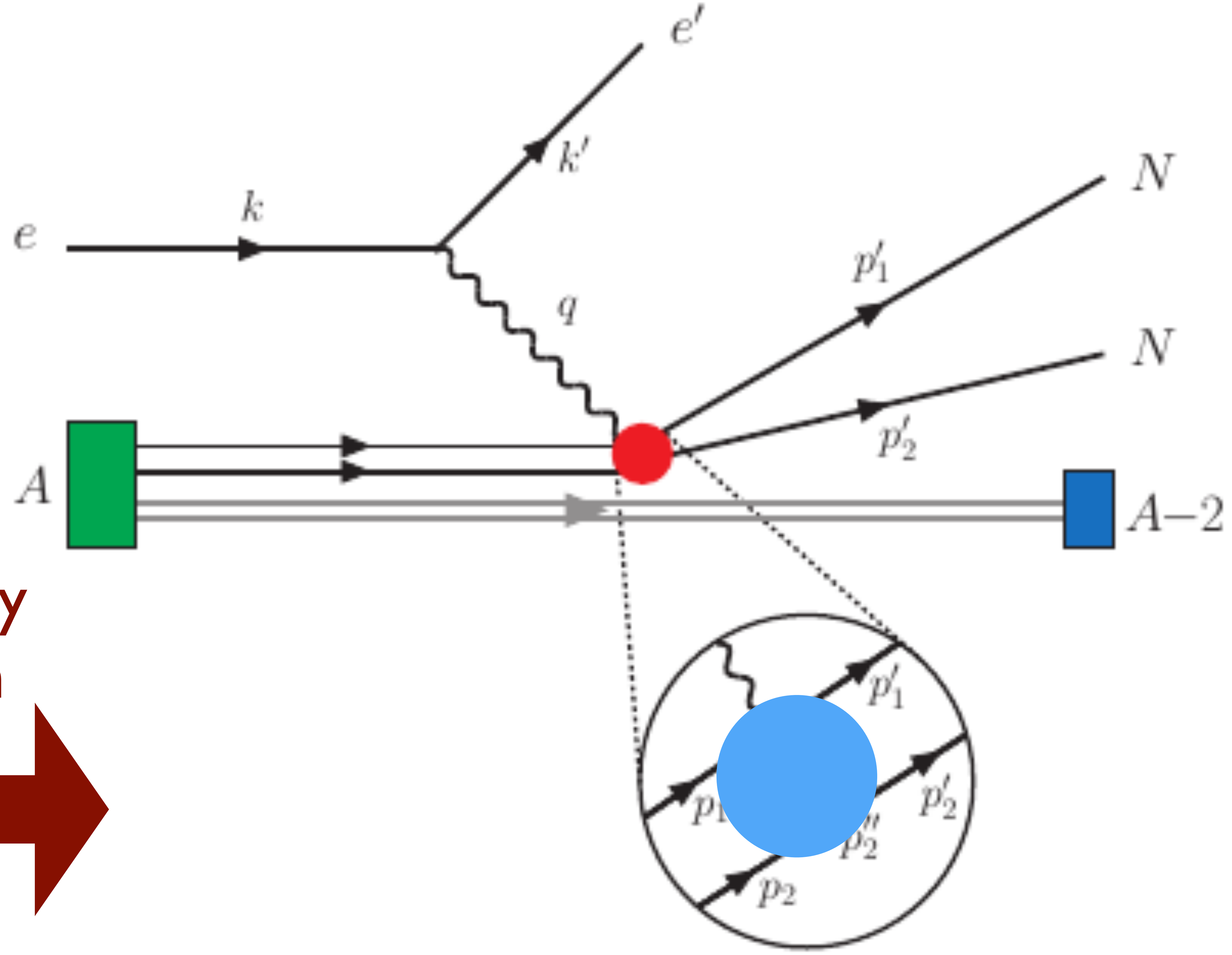


one-body current operators
complicated wf's

Connected by
RG evolution

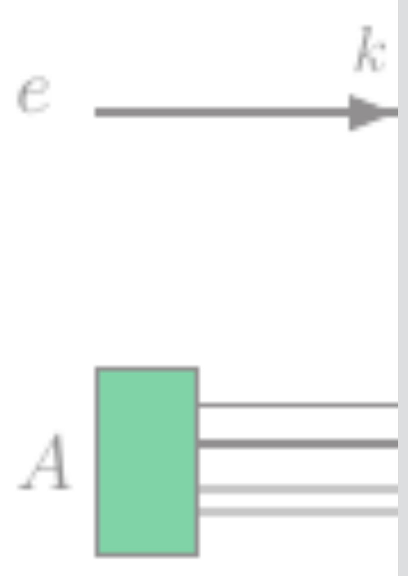
SRC studies at low resolution



two-body current operators
simple wf's

Low and High RG Resolution Scale Pictures

SRC studi



Same cross section (if done right), but different interpretations, split between structure/reaction, FSI's, etc..

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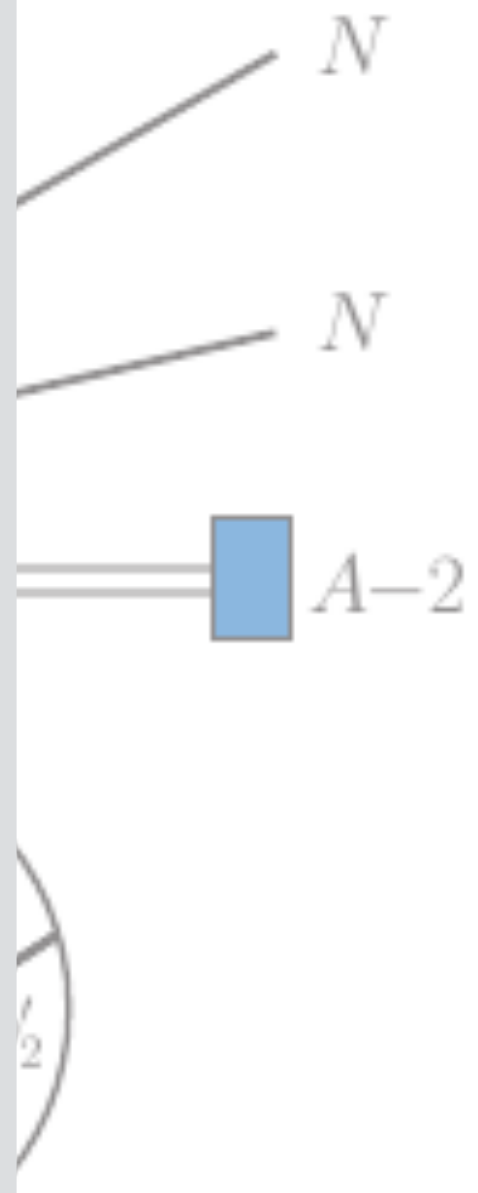

one-

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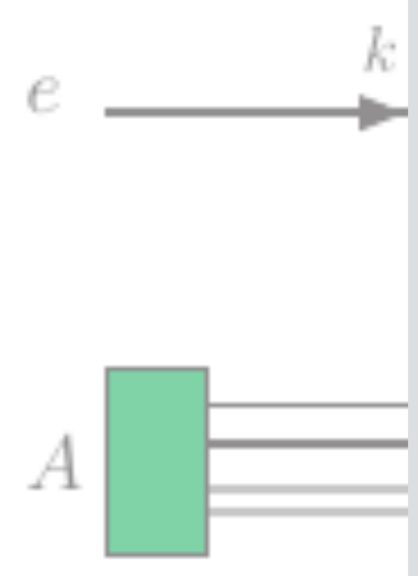
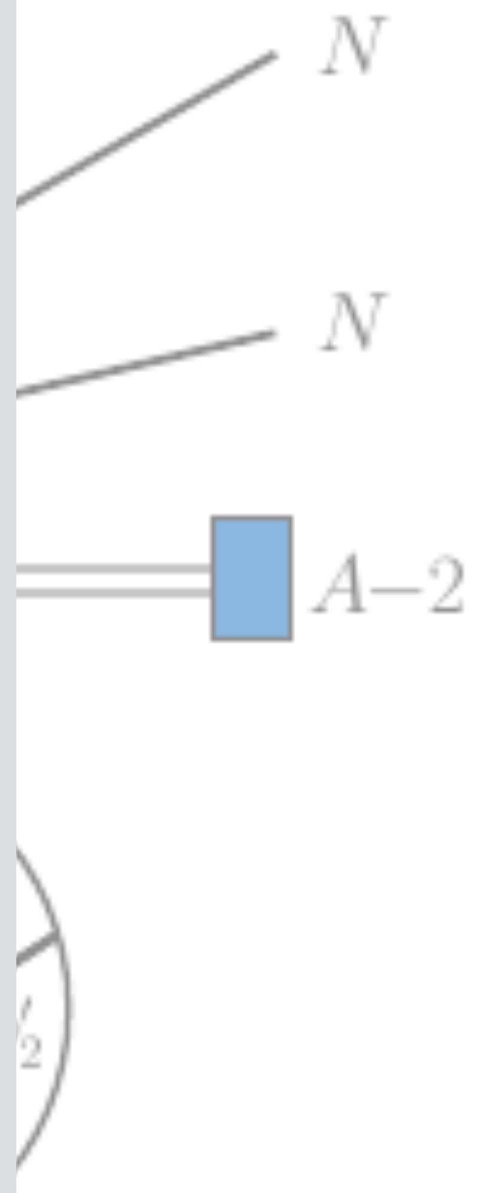
ators

Low and High RG Resolution Scale Pictures

SRC studi … lution





Same cross section (if done right), but different interpretations, split between structure/reaction, FSI's, etc..

Here:

How can SRC calculations at low RG scale be carried out in practice?

Under what approximations?

Connections to existing phenomenology (GCF/LCA)?

one-… ators
comp…

Similarity Renormalization Group

Evolve SRC physics from high to low RG resolution $(\lambda \lesssim q)$

Focus on phenomenology e.g., $n(\mathbf{q})$, $\rho(\mathbf{q}, \mathbf{Q})$ as first step

But see earlier deuteron electrodisintegration studies **More,SKB,Furnstahl PRC96 (2017)**

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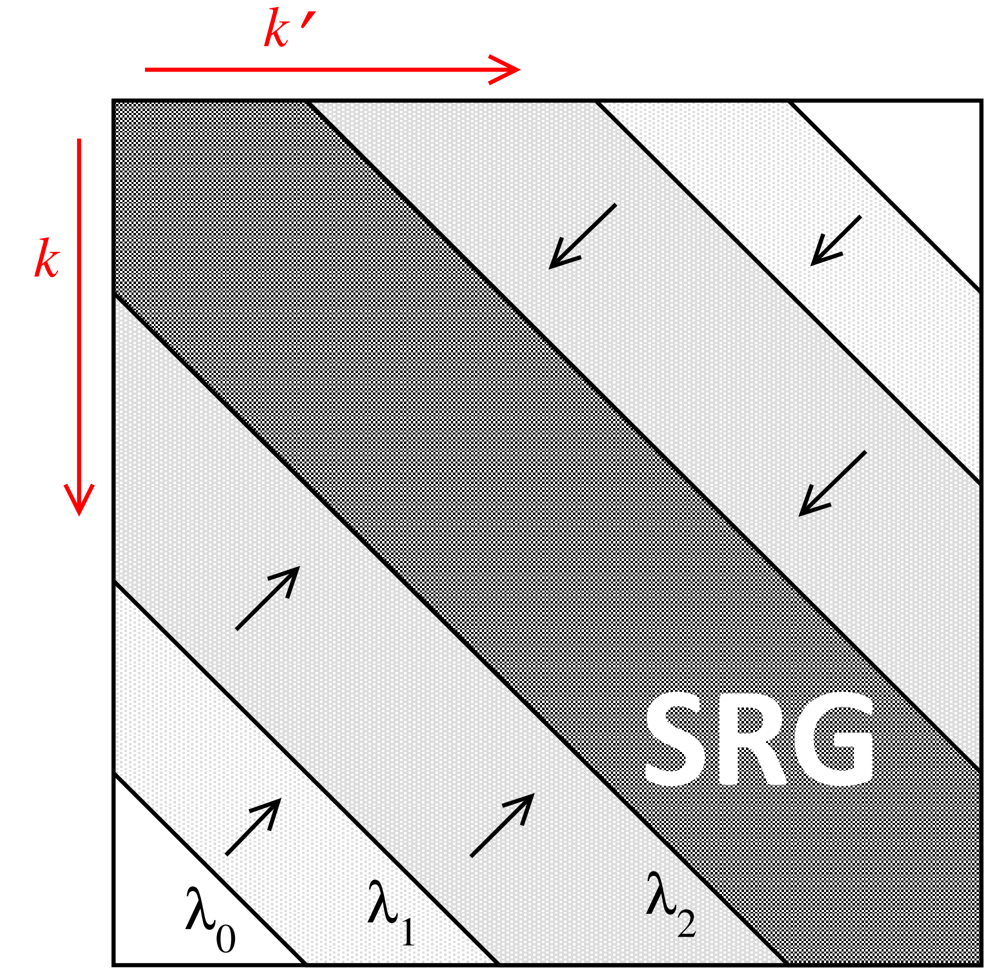
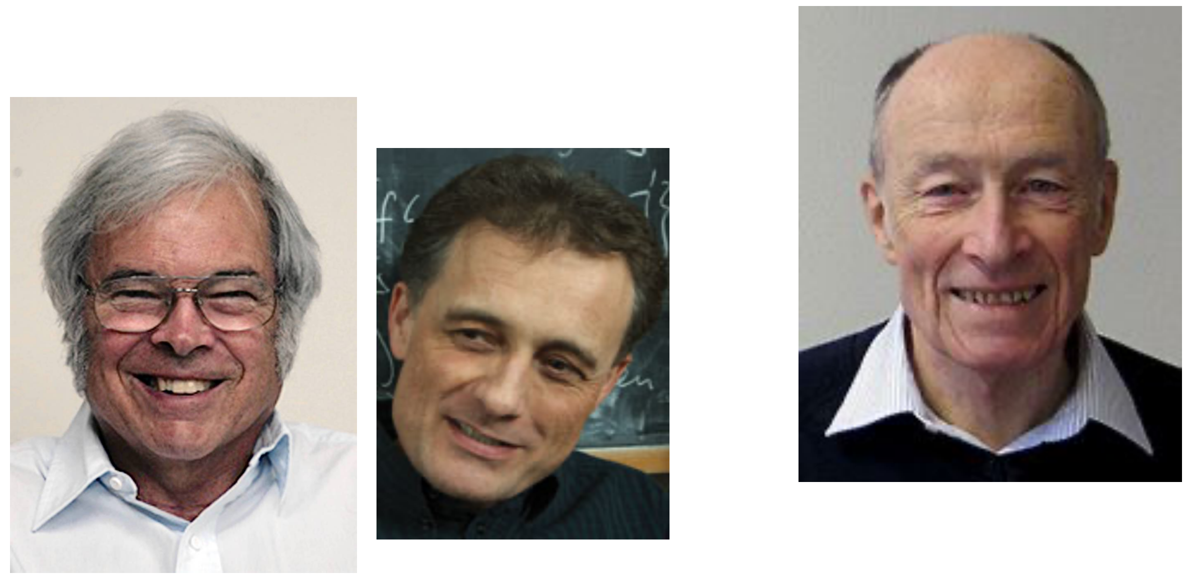
Unitary RG ("**S**imilarity **R**enormalization **G**roup")

$$H(\lambda) = U(\lambda) H U^{\dagger}(\lambda) \qquad O(\lambda) = U(\lambda) O U^{\dagger}(\lambda)$$

preserves all physics (unitary) if no approximations

low E states => $k \gtrsim \lambda$ highly suppressed





Bogner, Furnstahl, Schwenk Prog. Part. Nucl. Phys. 2010

Computing SRC operators at low-RG resolutions

$\hat{O}_q^{\rm hi}$ = operator that probes high-q components at high-RG resolution

$\langle A^{\rm hi} | \hat{O}_q^{\rm hi} | A^{\rm hi} \rangle \neq 0$

e.g., $\hat{n}^{\rm hi}(\mathbf{q}) = a_{\mathbf{q}}^\dagger a_{\mathbf{q}}$, $\hat{\rho}^{\rm hi}(\mathbf{q}, \mathbf{Q}) = a^\dagger_{\frac{Q}{2}+q} a^\dagger_{\frac{Q}{2}-q} a_{\frac{Q}{2}-q} a_{\frac{Q}{2}+q}$

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SRG evolve to $\lambda \lesssim$ q

$$\langle A^{\rm hi} | \hat{O}_q^{\rm hi} | A^{\rm hi} \rangle = \langle A^{\rm hi} | \hat{U}_\lambda^\dagger \hat{U}_\lambda \hat{O}_q^{\rm hi} \hat{U}_\lambda^\dagger \hat{U}_\lambda | A^{\rm hi} \rangle = \langle A^{\rm lo} | \hat{O}_q^{\rm lo} | A^{\rm lo} \rangle$$

wf's of **soft** $\hat{H}^{\rm lo} = \hat{U}_\lambda \hat{H}^{\rm hi} \hat{U}_\lambda^\dagger$

$\langle A^{\rm lo} | \hat{O}_q^{\rm hi} | A^{\rm lo} \rangle \approx 0$

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$$\hat{U}_\lambda = \hat{1} + \frac{1}{4} \sum_{K,k,k'} \delta U_\lambda^{(2)}(k,k')\, a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} + \frac{1}{36} \sum \delta U_\lambda^{(3)} a^\dagger a^\dagger a^\dagger a a a + \cdots$$

fixed from SRG evolution on A=2

fixed from SRG evolution on A=3

$\delta U_\lambda(\mathbf{k},\mathbf{k}')$ inherits symmetries of V_{NN} (Galilean, partial wave structure, etc.)

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Wick's theorem to evaluate $\hat{O}_q^{\rm lo} = \hat{U}_\lambda \hat{O}_q^{\rm hi} \hat{U}_\lambda^\dagger = \hat{O}_{1b}^{\rm lo} + \hat{O}_{2b}^{\rm lo} + \cancel{\hat{O}_{3b}^{\rm lo}} + \cdots$

Computing SRC operators at low-RG resolutions

$\hat{O}^{\rm hi}_q$ = operator

SRG $H^{\rm lo}_\lambda$ a "cluster" hierarchy $V^{2N}_\lambda \gg V^{3N}_\lambda \gg V^{4N}_\lambda \ldots$

cancellations of KE/PE "amplify" the importance of 3N for bulk energies

$_{+q} a^\dagger_{\frac{Q}{2}-q} a_{\frac{Q}{2}-q} a_{\frac{Q}{2}+q}$

SRG evol

$\hat{U}_\lambda \hat{H}^{\rm hi} \hat{U}^\dagger_\lambda$

$\langle A^{\rm hi} | \hat{O}^{\rm hi}_q | A$

≈ 0

$\hat{U}_\lambda = \hat{1}$

Wick's

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For high-q operators ($\lambda \lesssim q$), evidence that

$\hat{O}_q^{1b}(\lambda) \ll \hat{O}_q^{2b}(\lambda)$ **BUT** $\hat{O}_q^{2b}(\lambda) \gg \hat{O}_q^{3b}(\lambda) \gg \cdots$

$_{+q} a^\dagger_{\frac{Q}{2}-q} a_{\frac{Q}{2}-q} a_{\frac{Q}{2}+q}$

$\hat{U}_\lambda \hat{H}^{\text{hi}} \hat{U}_\lambda^\dagger$

≈ 0

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Can assess SRG truncations by varying λ (observables don't change if no approximation made)

Computing SRC operators at low-RG resolutions

$\hat{O}^{\rm hi}_q$ = operator

$+q \, a^\dagger_{\frac{Q}{2}-q} \, a_{\frac{Q}{2}-q} \, a_{\frac{Q}{2}+q}$

SRG evol

$\langle A^{\rm hi} | \hat{O}^{\rm hi}_q | A$

$\hat{U}_\lambda \hat{H}^{\rm hi} \hat{U}^\dagger_\lambda$

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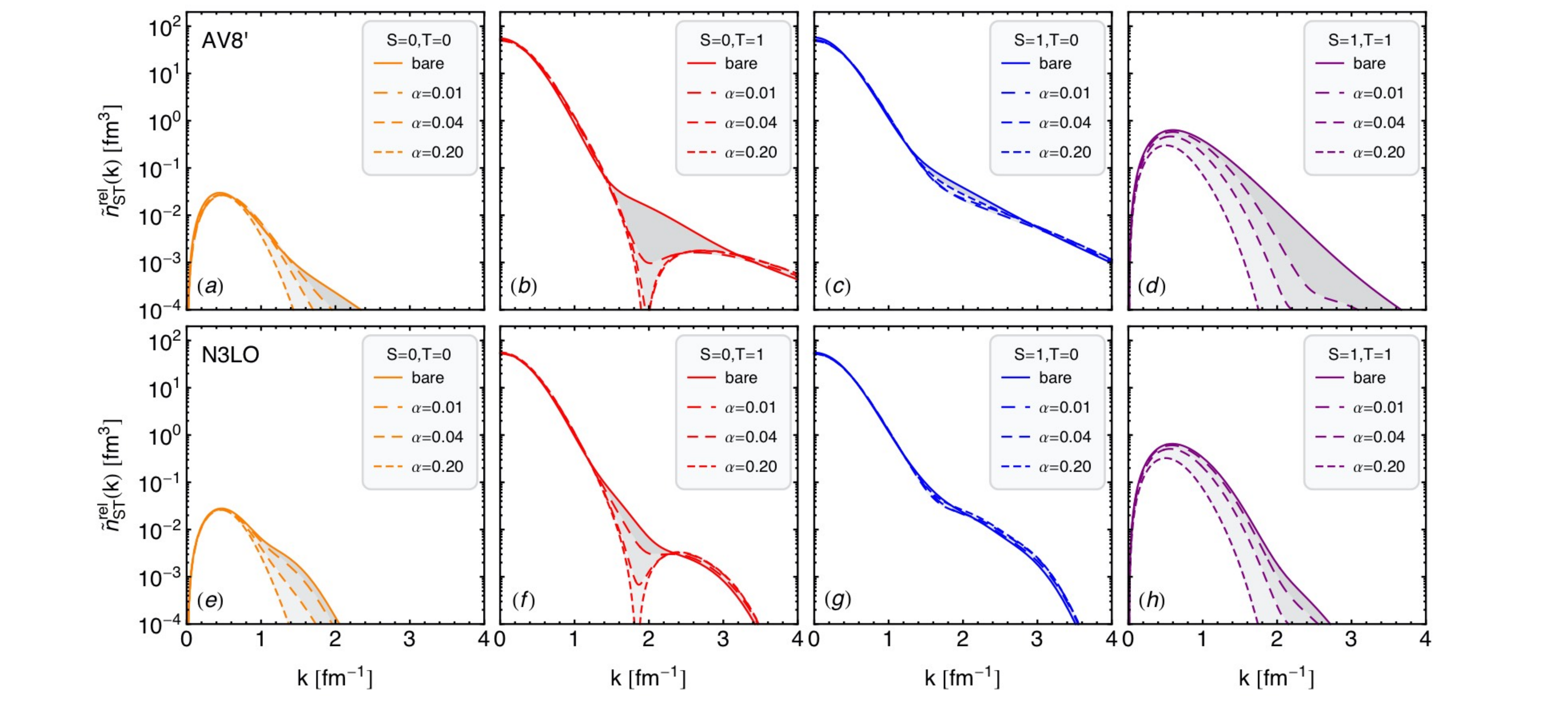
Wick's

Neff, Feldmeier, Horiuchi PRC 92 (2015)



Some λ-dependence for relative momentum dist.

integral over sizable CM ==> non-SRC physics; sensitive to induced 3-body

Computing SRC operators at low-RG resolutions

$\hat{O}^{\text{hi}}_q$ = operator

$+q \; a^{\dagger}_{\frac{Q}{2}-q} \; a_{\frac{Q}{2}-q} \; a_{\frac{Q}{2}+q}$

SRG evol

$\langle A^{\text{hi}} | \hat{O}^{\text{hi}}_q | A$

$\hat{U}_\lambda \hat{H}^{\text{hi}} \hat{U}^{\dagger}_\lambda$

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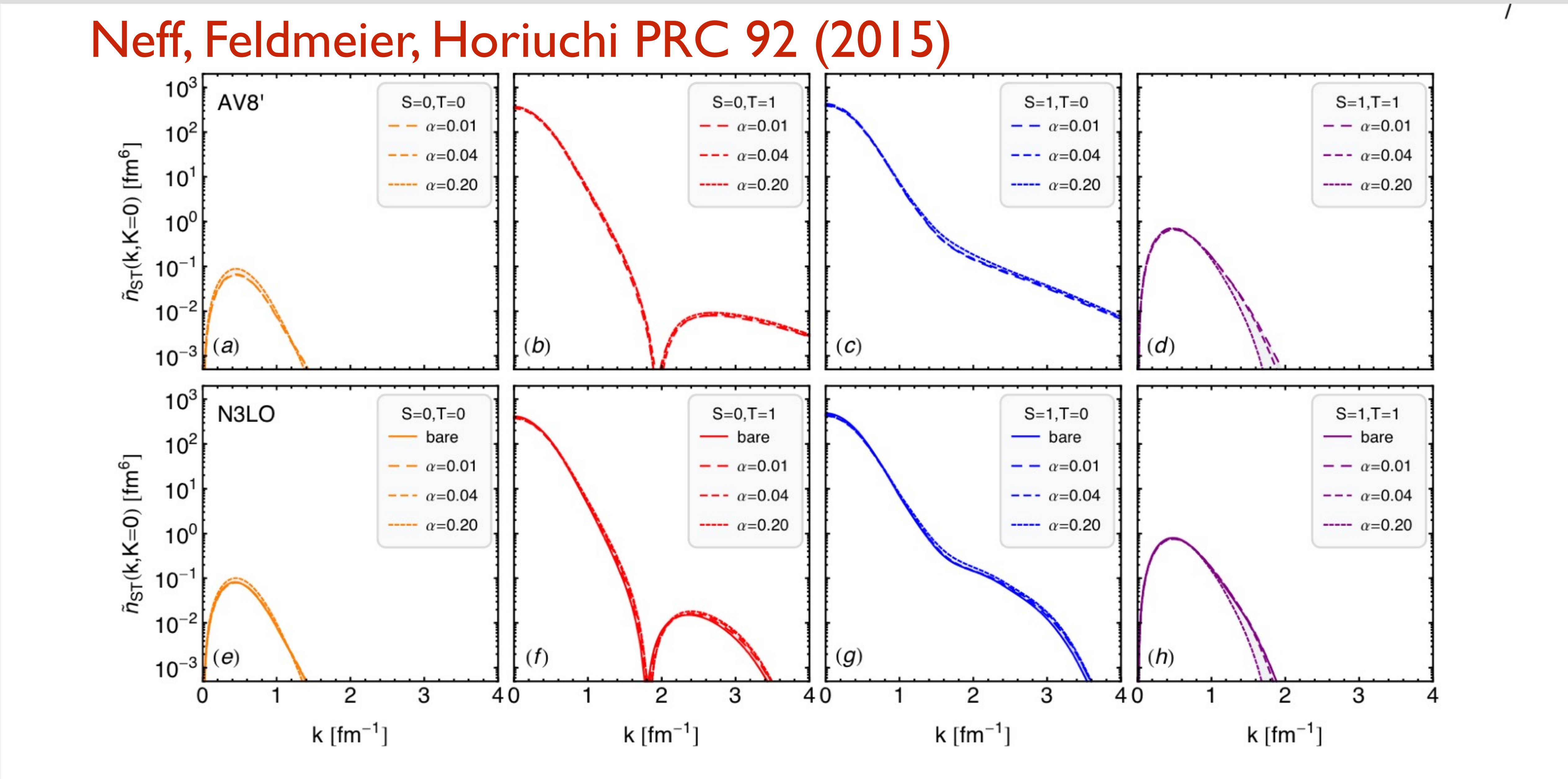

reduced λ-dependence for K=0 pair momentum dist.

induced 3-body negligible <==> SRC pairs 2-body physics

cf. LCA, GCF, leading-order Brueckner, …

Computing SRC operators at low-RG resolutions

momentum distribution

$$\hat{n}^{\mathrm{hi}}(\mathbf{q}) = a^\dagger_{\mathbf{q}} a_{\mathbf{q}}$$

$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U^{(2)}_\lambda\right) a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \left(\hat{1} + \delta U^{\dagger(2)}_\lambda\right)$$

Computing SRC operators at low-RG resolutions

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$$\Downarrow$$

$$\hat{n}^{\text{lo}}(\mathbf{q}) = a^\dagger_{\mathbf{q}} a_{\mathbf{q}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{k}')\, a^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{k}'} a^\dagger_{\mathbf{q}+\mathbf{k}-\mathbf{k}'} a_{\mathbf{q}-2\mathbf{k}'} a_{\mathbf{q}} + h.c.$$

$$+ \frac{1}{4}\sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{q}-\mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{q}-\mathbf{K}/2,\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

$$+ (\cdots)\, a^\dagger a^\dagger a^\dagger a a a + (\cdots) a^\dagger a^\dagger a^\dagger a^\dagger a a a a \cdots$$

Computing SRC operators at low-RG resolutions

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momenta $\gg \lambda$ absent in $|A^{\rm lo}\rangle$

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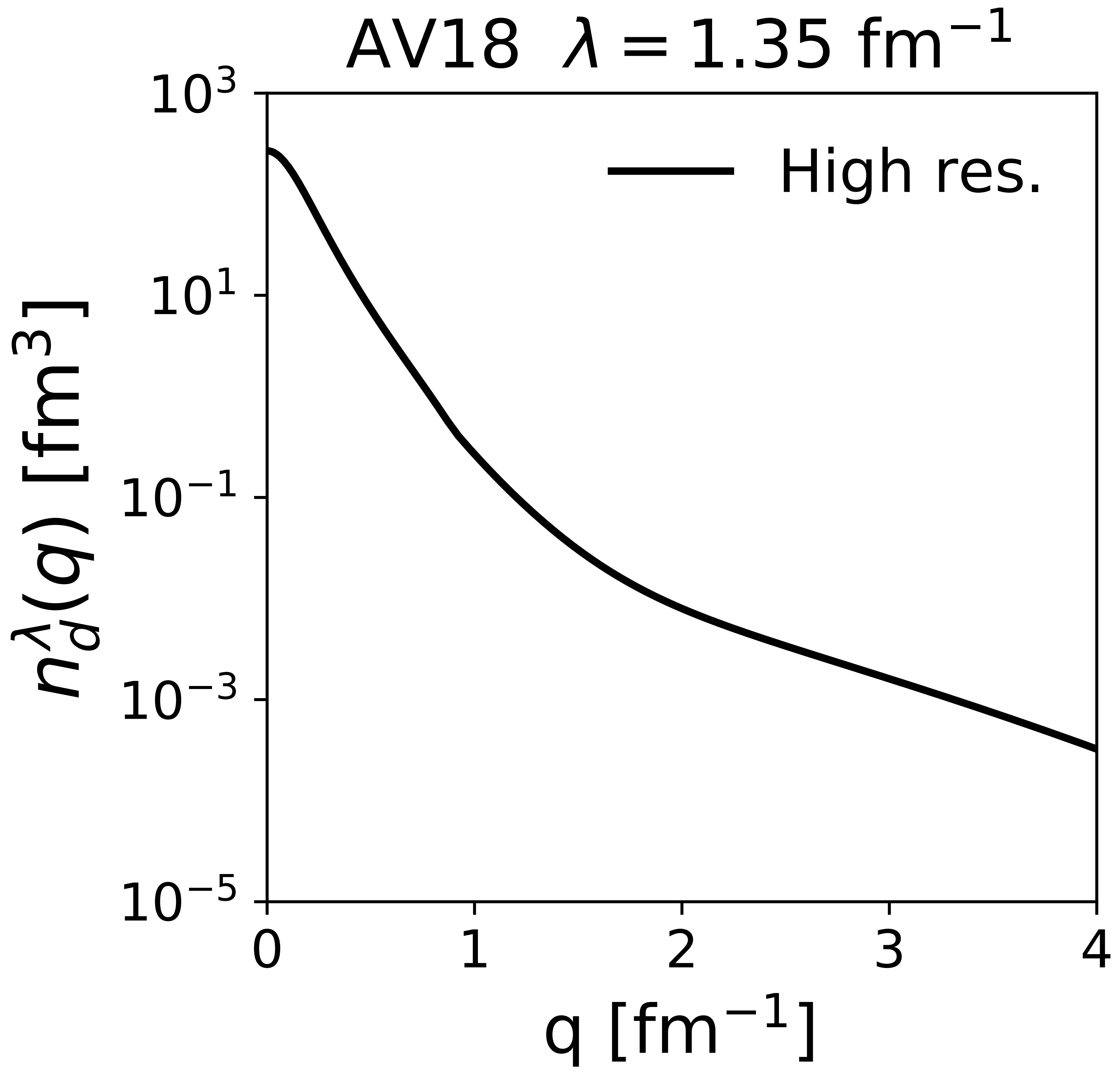
$$+ \cancel{(\cdots)\, a^\dagger a^\dagger a^\dagger aaa} + \cancel{(\cdots) a^\dagger a^\dagger a^\dagger a^\dagger aaaa} \;\cdots$$

Computing SRC operators at low-RG resolutions

Deuteron illustration



$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U_\lambda^{(2)}\right) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \left(\hat{1} + \delta U_\lambda^{\dagger(2)}\right)$$

$$\langle D^{\mathrm{hi}} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\mathrm{hi}} \rangle$$

Computing SRC operators at low-RG resolutions

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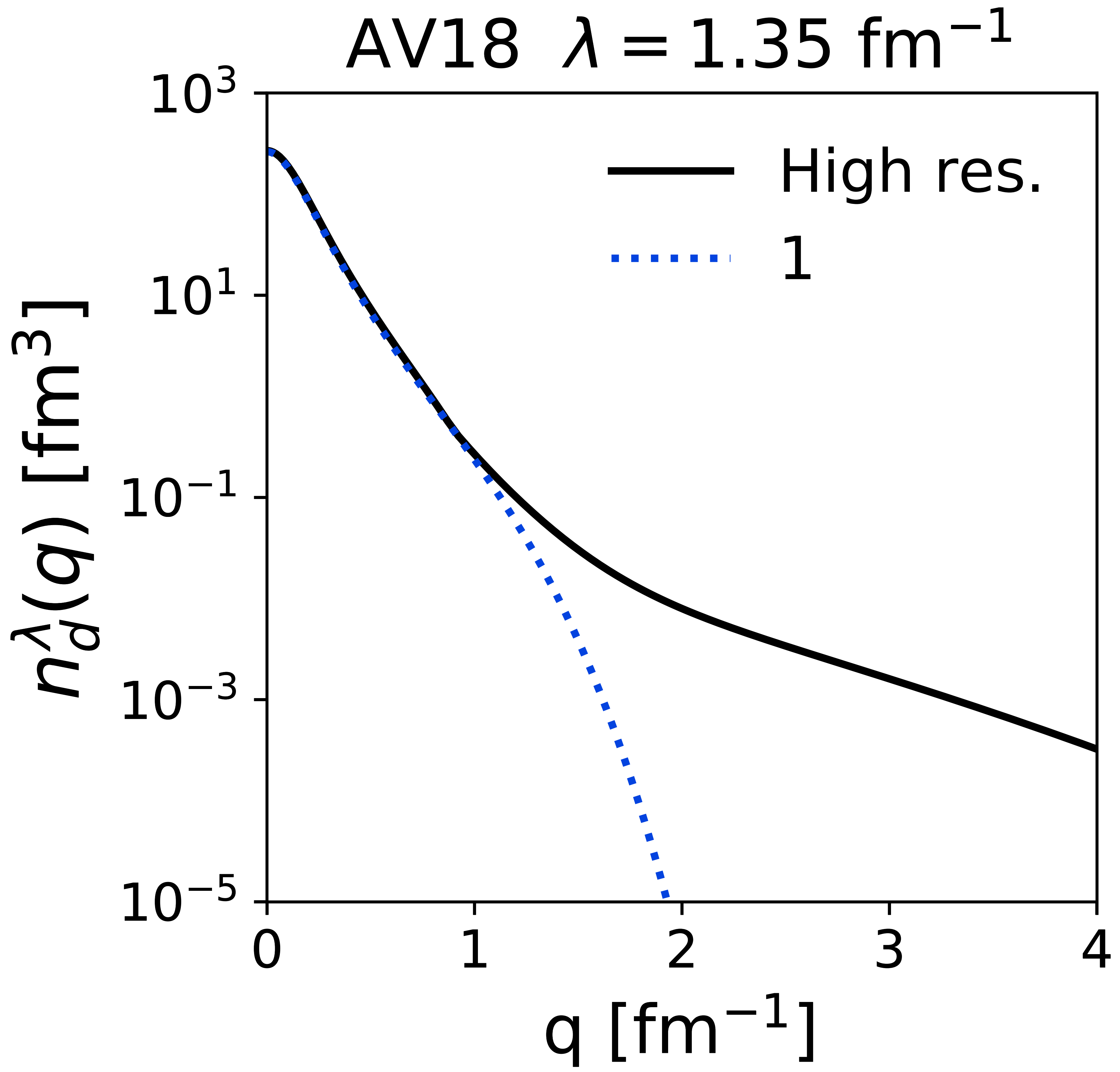
$$\hat{n}^{\text{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U_{\lambda}^{(2)}\right) a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \left(\hat{1} + \delta U_{\lambda}^{\dagger(2)}\right)$$



$$\langle D^{\text{hi}} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | D^{\text{hi}} \rangle$$

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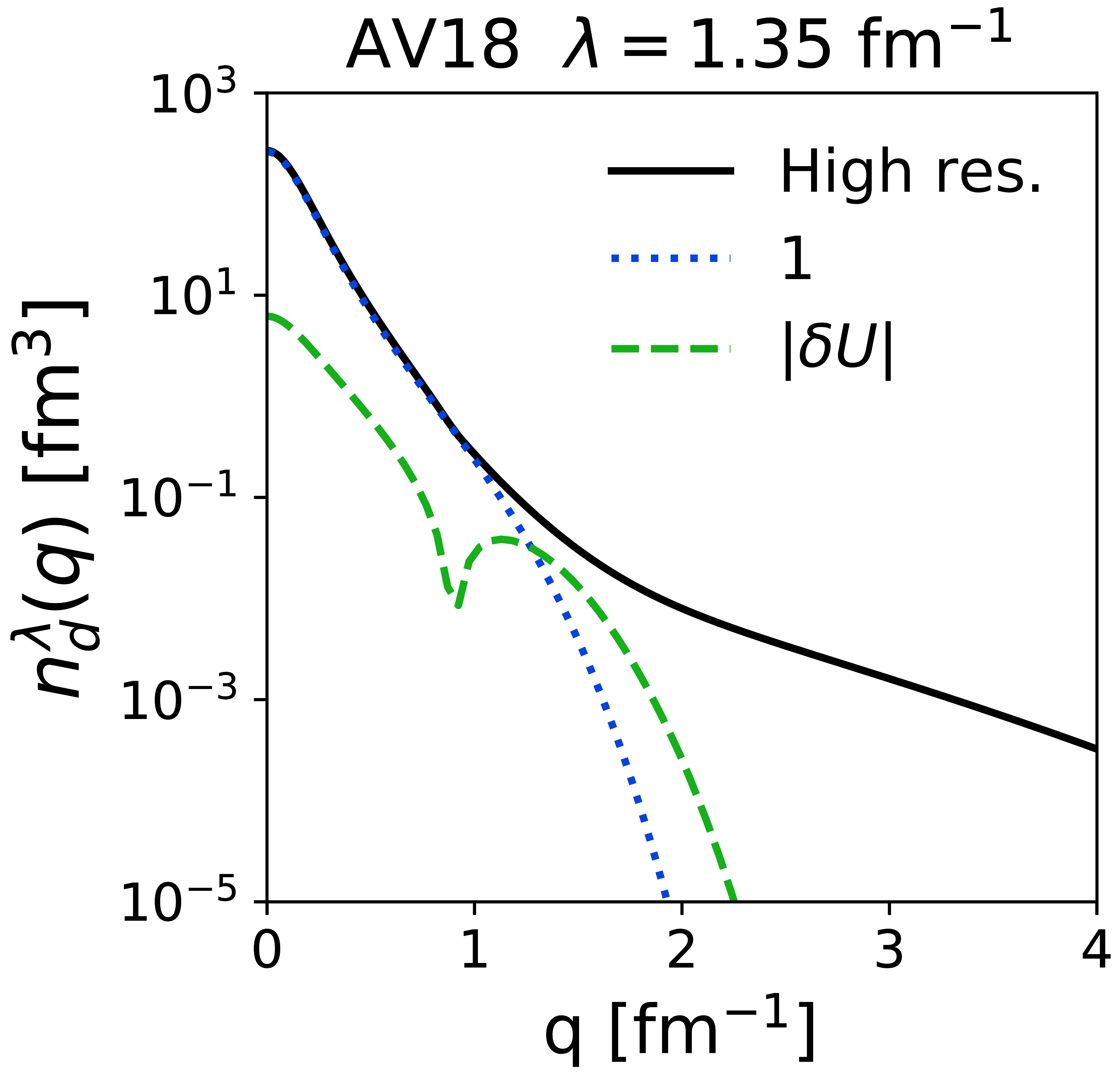

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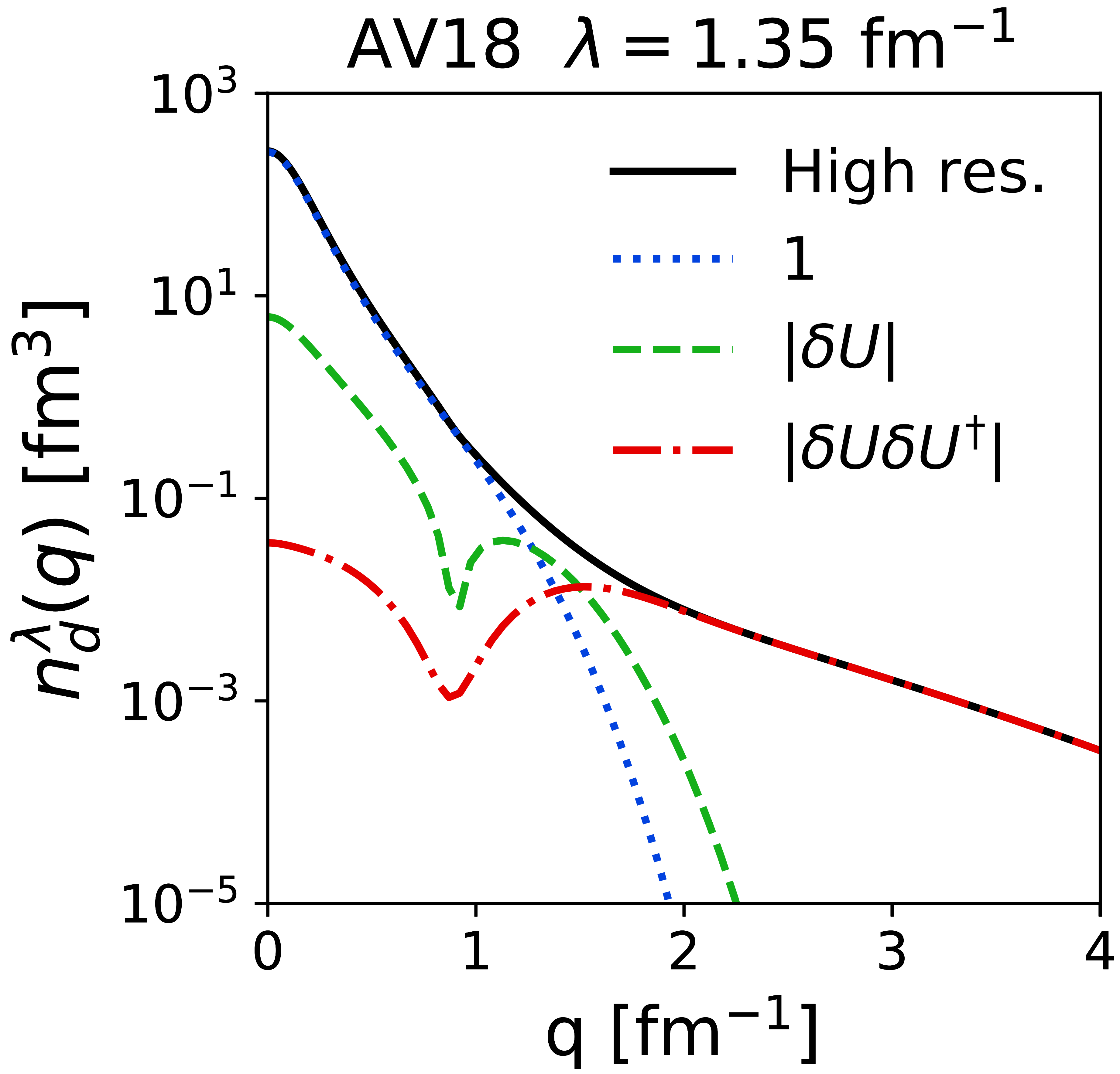

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$$\langle D^{\text{lo}} | \delta U a_\mathbf{q}^\dagger a_\mathbf{q} \delta U^\dagger | D^{\text{lo}} \rangle$$

Computing SRC operators at low-RG resolutions

Consider $\mathbf{q} \gg \lambda$

momenta $\gg \lambda$ absent in $|A^{\mathrm{lo}}\rangle$

$$n^{\mathrm{lo}}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q} - \mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{q} - \mathbf{K}/2, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

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Expectation value in $|A^{\mathrm{lo}}\rangle$ ==> only "soft" $\mathbf{K}, \mathbf{k}', \mathbf{k} \lesssim \lambda$ contribute

$$\approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q})\, \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

$K \ll q$

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Consider $\mathbf{q} \gg \lambda$

momenta $\gg \lambda$ absent in $|A^{\mathrm{lo}}\rangle$

$$n^{\mathrm{lo}}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q}-\mathbf{K}/2)\, \delta U_\lambda^\dagger(\mathbf{q}-\mathbf{K}/2, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Expectation value in $|A^{\mathrm{lo}}\rangle$ ==> only "soft" $\mathbf{K}, \mathbf{k}', \mathbf{k} \lesssim \lambda$ contribute

$$\approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q})\, \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} \qquad K \ll q$$

Scale separation
$k, k' \ll q$

$$\delta U_\lambda(k, q) \approx F_\lambda^{\mathrm{lo}}(k) F_\lambda^{\mathrm{hi}}(q)$$

Computing SRC operators at low-RG resolutions

Consider $\mathbf{q} \gg \lambda$

momenta $\gg \lambda$ absent in $|A^{\rm lo}\rangle$

$$n^{\rm lo}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q} - \mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{q} - \mathbf{K}/2, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

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$K \ll q$

$$\approx (F^{\rm hi}(q))^2 \sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} F^{\rm lo}(\mathbf{k}) F^{\rm lo}(\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Computing SRC operators at low-RG resolutions

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$K \ll q$

Leading-order
Operator Product Expansion

smeared local operator
low-k physics
A-dependence in ME's

$$\approx (F^{\mathrm{hi}}(q))^2 \sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} F^{\mathrm{lo}}(\mathbf{k})F^{\mathrm{lo}}(\mathbf{k}')\,a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Universal (A-indep)
Wilson Coeff, fixed by A=2
depends on operator

Computing SRC operators at low-RG resolutions

NSCL FRIB

Similar factorized forms for other SRC operators

$$\hat{\rho}^{\text{hi}}(\mathbf{q},\mathbf{Q}) = a^{\dagger}_{\frac{\mathbf{Q}}{2}+\mathbf{q}} a^{\dagger}_{\frac{\mathbf{Q}}{2}-\mathbf{q}} a_{\frac{\mathbf{Q}}{2}-\mathbf{q}} a_{\frac{\mathbf{Q}}{2}+\mathbf{q}} \xrightarrow{\mathbf{q} \gg \lambda} \approx (F^{\text{hi}}(q))^2 \sum_{\mathbf{k},\mathbf{k}'}^{\lambda} F^{\text{lo}}(\mathbf{k}) F^{\text{lo}}(\mathbf{k}')\, a^{\dagger}_{\frac{\mathbf{Q}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{Q}}{2}-\mathbf{k}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}'} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}'}$$

Computing SRC operators at low-RG resolutions

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Scaling of high-q tails

$$\frac{\langle A^{\text{hi}} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | A^{\text{hi}} \rangle}{\langle D^{\text{hi}} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | D^{\text{hi}} \rangle} \approx \cancel{\frac{|F^{\text{hi}}(q)|^2}{|F^{\text{hi}}(q)|^2}} \times \frac{\sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} \langle A^{\text{lo}} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | A^{\text{lo}} \rangle}{\sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} \langle D^{\text{lo}} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | D^{\text{lo}} \rangle}$$

Computing SRC operators at low-RG resolutions

NSCL FRIB

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$$F^{\mathrm{hi}}(\mathbf{q}) \propto \Psi^{\mathrm{hi}}_{A=2}(\mathbf{q})$$

ratio of (smeared) contacts
only sensitive to low-k/mean-field physics
approx. independent of resolution scale

(see GCF talks of Diego/Ronan)

Computing SRC operators at low-RG resolutions

Si

RG-evolved SRC operators

links few- and A-body systems (Operator Product Expansion)

RG "derivation" of the GCF

Correlations/scaling for 2 observables w/same leading OPE

Subleading OPE ==> deviations from scaling calculable in principle?

(see GCF talks of Diego/Ronan)

Options for treating wf's at low-RG resolutions

All the hard q physcis factorized in A-indep Wilson Coeffs

SRC calculations amount to computing matrix elements of

$$\sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} F^{\rm lo}(\mathbf{k}) F^{\rm lo}(\mathbf{k}') \left\langle A^{\rm lo} \middle| a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} \middle| A^{\rm lo} \right\rangle$$

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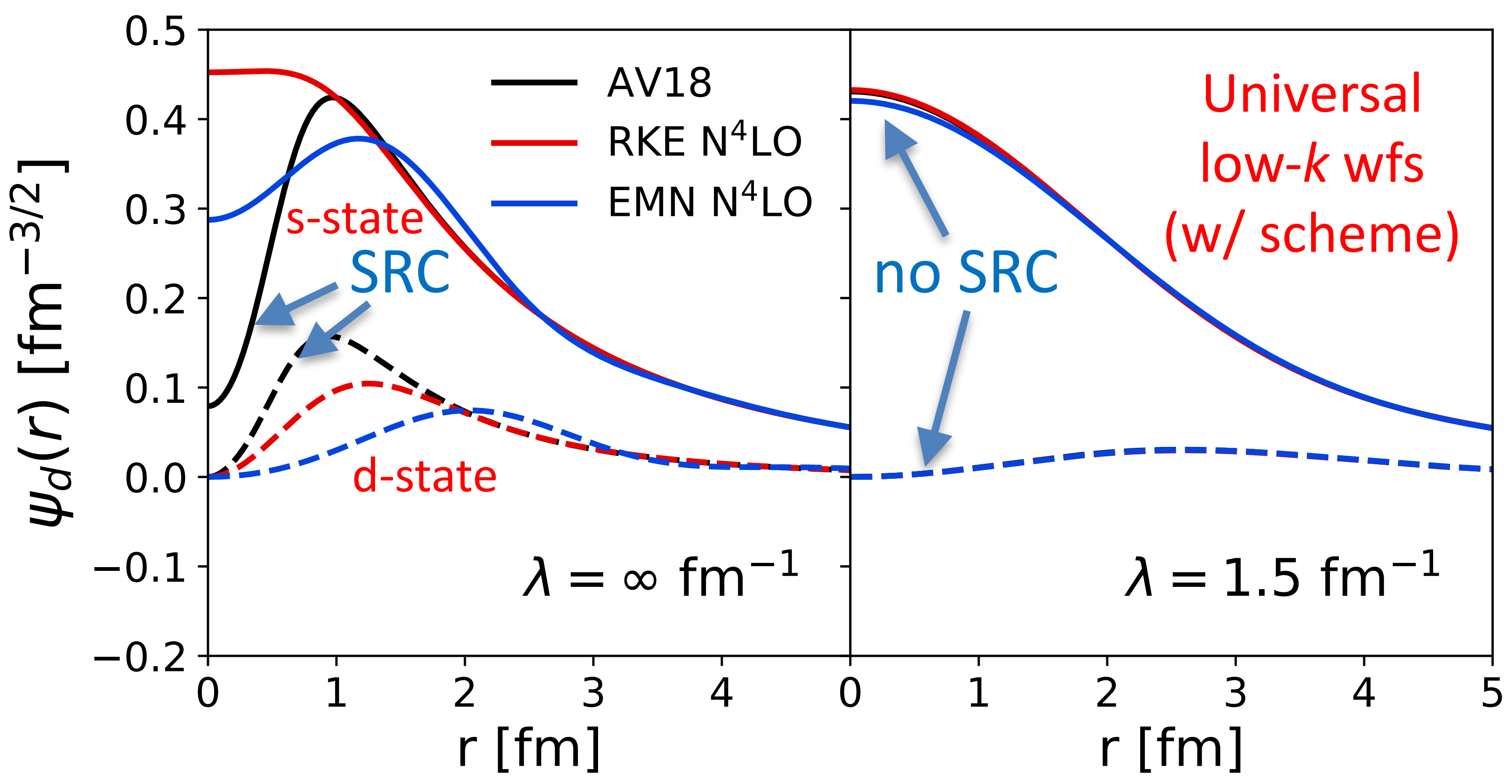
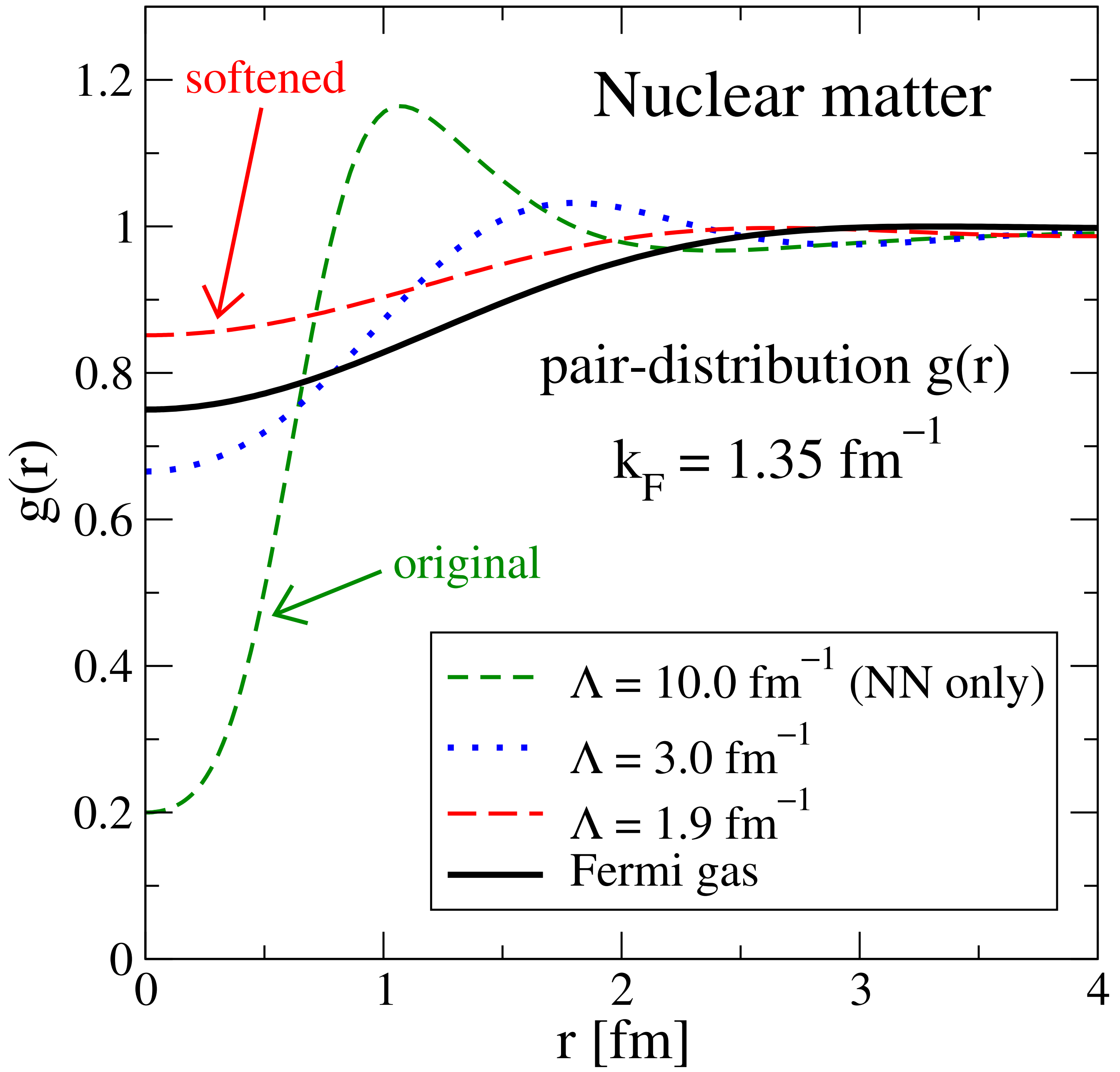
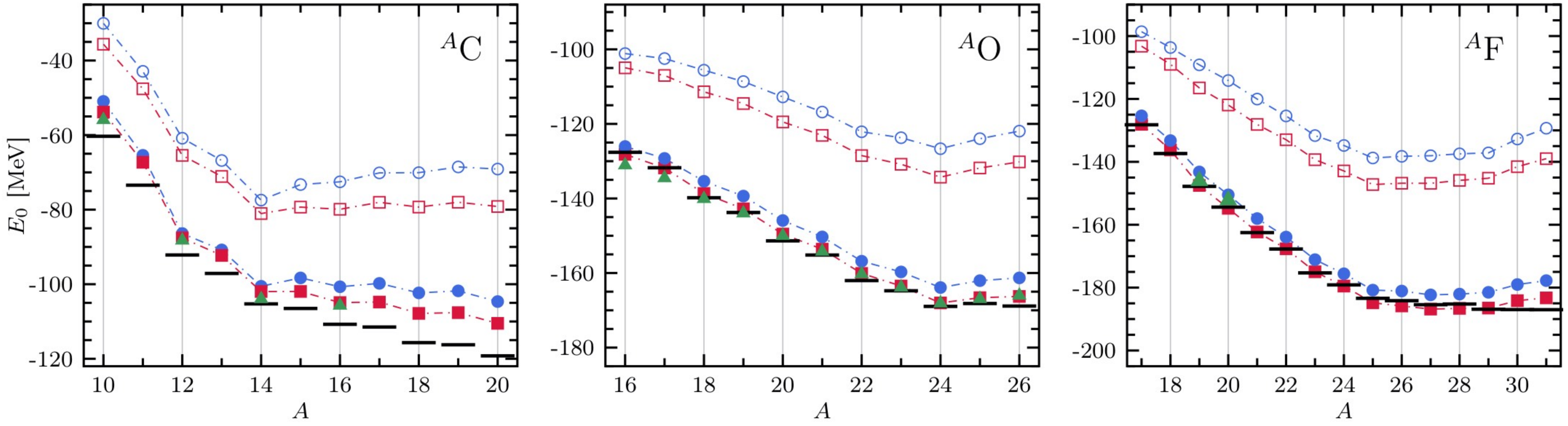


no explicit SRCs at low resolution



Options for treating wf's at low-RG resolutions



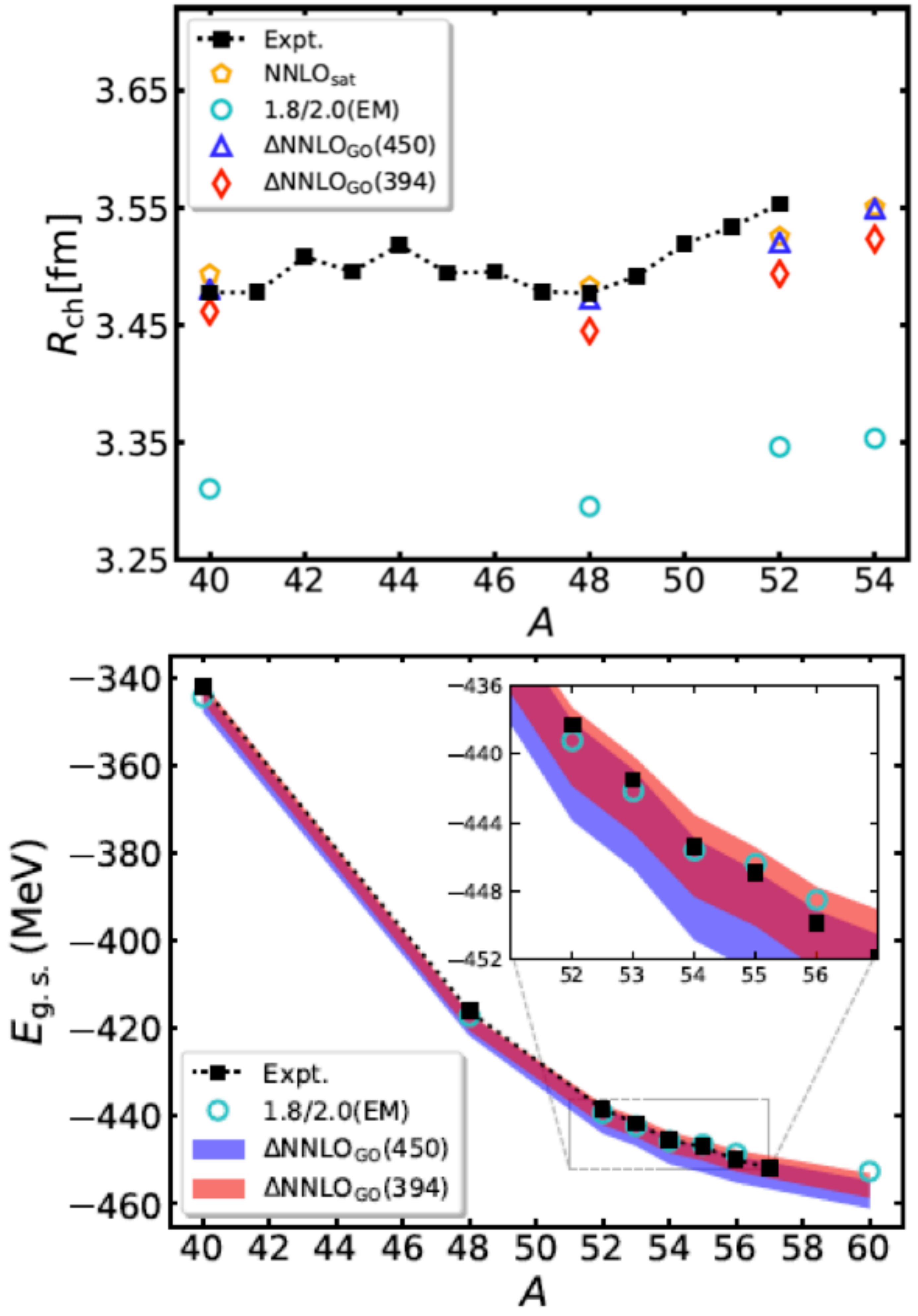
Tichai et al., Frontiers in Physics (2021)

Figure 7. Reference (○/□) and second-order NCSM-PT (●/ ■) energies with $N_{\mathrm{max}}^{\mathrm{ref}} = 0$ and 2, respectively, for the ground states of $^{11-20}$C, $^{16-26}$O and $^{17-31}$F using the Hamiltonian described in Sec. 3. All calculations are performed using 13 oscillator shells and an oscillator frequency of $\hbar\omega = 20\,\mathrm{MeV}$. The SRG parameter is set to $\alpha = 0.08\,\mathrm{fm}^4$. Importance-truncated NCSM calculations (▲) are shown for comparison. Experimental values are indicated by black bars. Figure taken from Ref. [36].

Simple methods "work"

- MBPT
- shell model
- polynomially scaling methods (IMSRG, CC, SCGF, etc.)

Options for treating wf's at low-RG resolutions



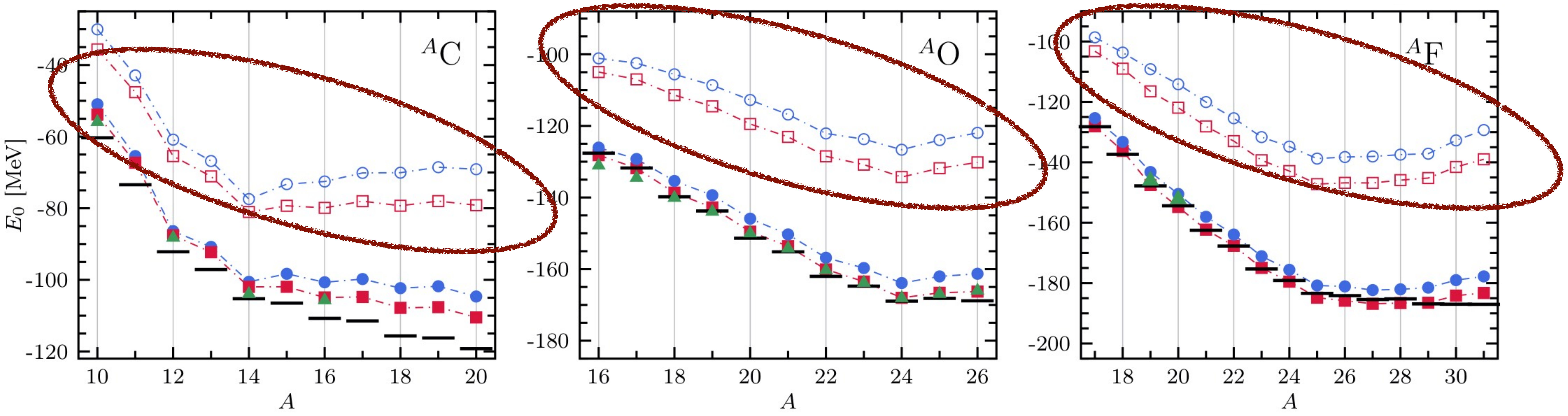
Charge radii (top) and ground-state energies (bottom) of calcium isotopes with *A* nucleons computed with new potentials ΔNNLO$_{GO}$.

Ongoing developments:

"soft" interactions w/good saturation properties in medium mass

e.g., ΔNNLO$_{GO}$ chiral EFT (with Δ's)

Options for treating wf's at low-RG resolutions



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Need beyond HF for precision energetics/radii

Can we use HF for SRC studies at low resolution?

Or HF treated in LDA? Let's find out!...

Options for treating wf's at low-RG resolutions

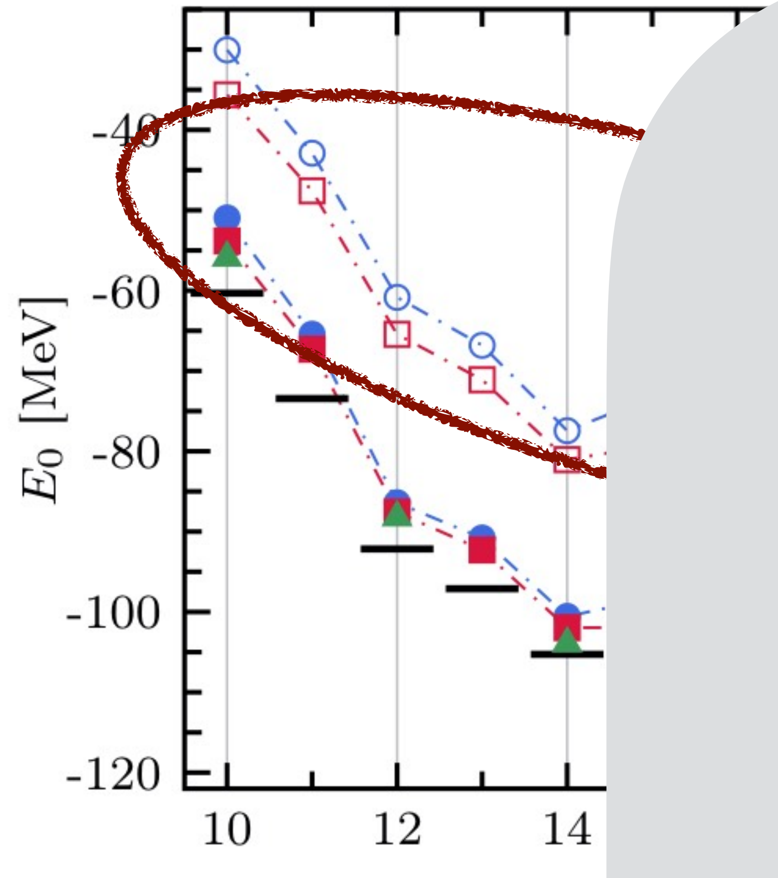


Figure 7. Referen
respectively, for the
All calculations are
The SRG paramete
comparison. Exper

Strategy for SRC calcs. at low-RG scales $\lambda \ll q$

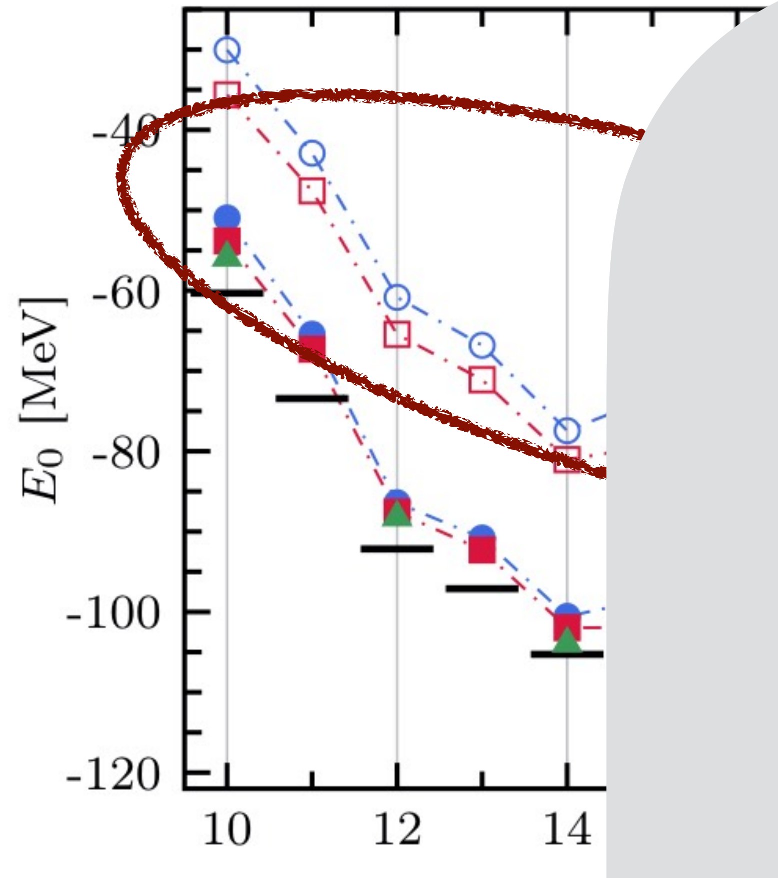
$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q} - \mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{k}', \mathbf{q} - \mathbf{K}/2)\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

adii

lution?

out!...

Options for treating wf's at low-RG resolutions



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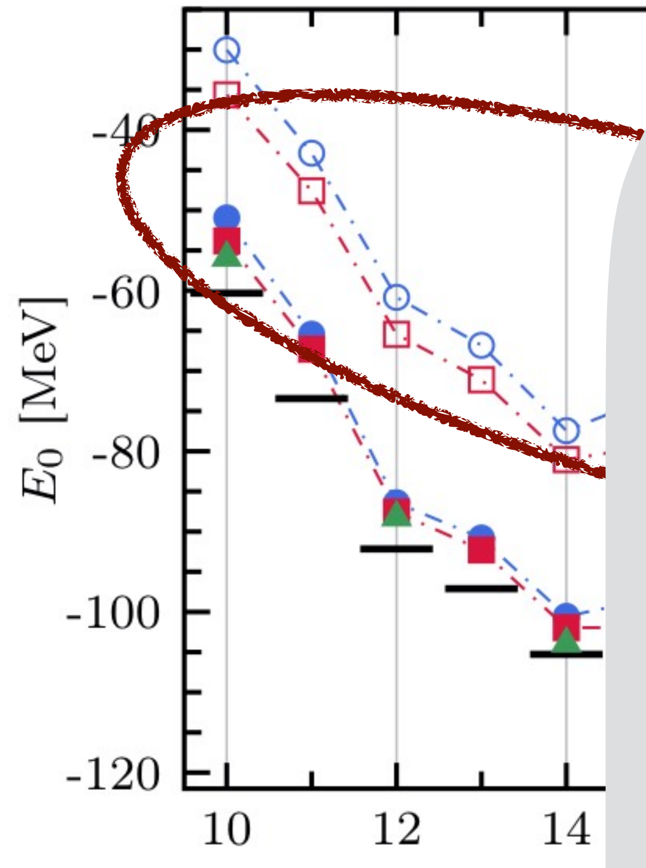
fixed from A=2

adii

lution?

out!...

Options for treating wf's at low-RG resolutions



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fixed from A=2

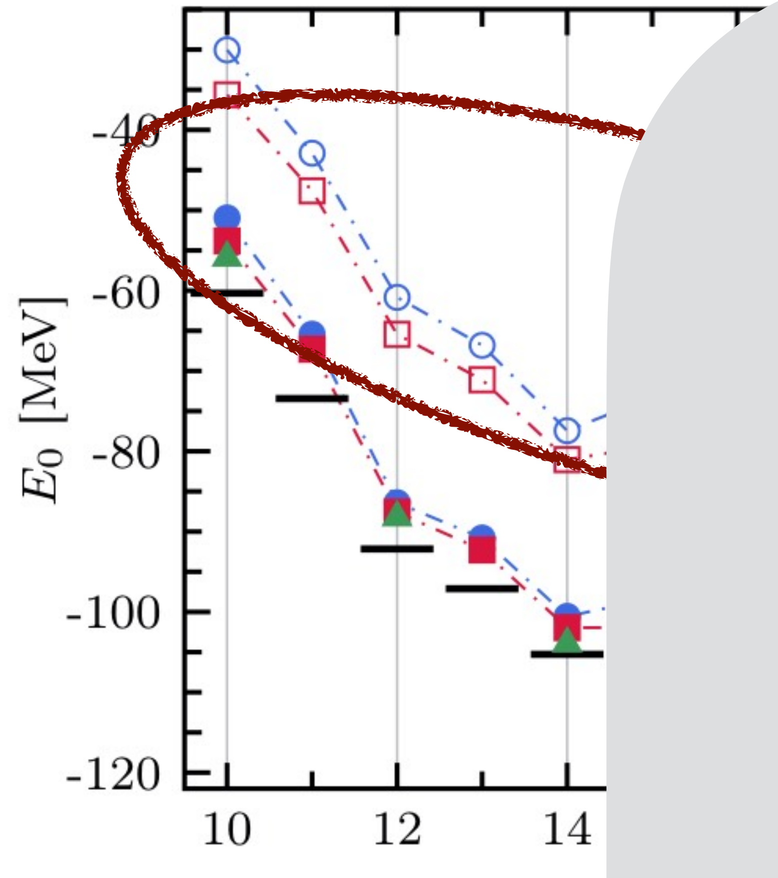
evaluate matrix elements in A-body states using LDA (free fermi gas)

adii

lution?

out!...

Options for treating wf's at low-RG resolutions



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fixed from A=2

What SRC phenomenology can this (ridiculously) simple approach reproduce?

evaluate matrix elements in A-body states using LDA (free fermi gas)

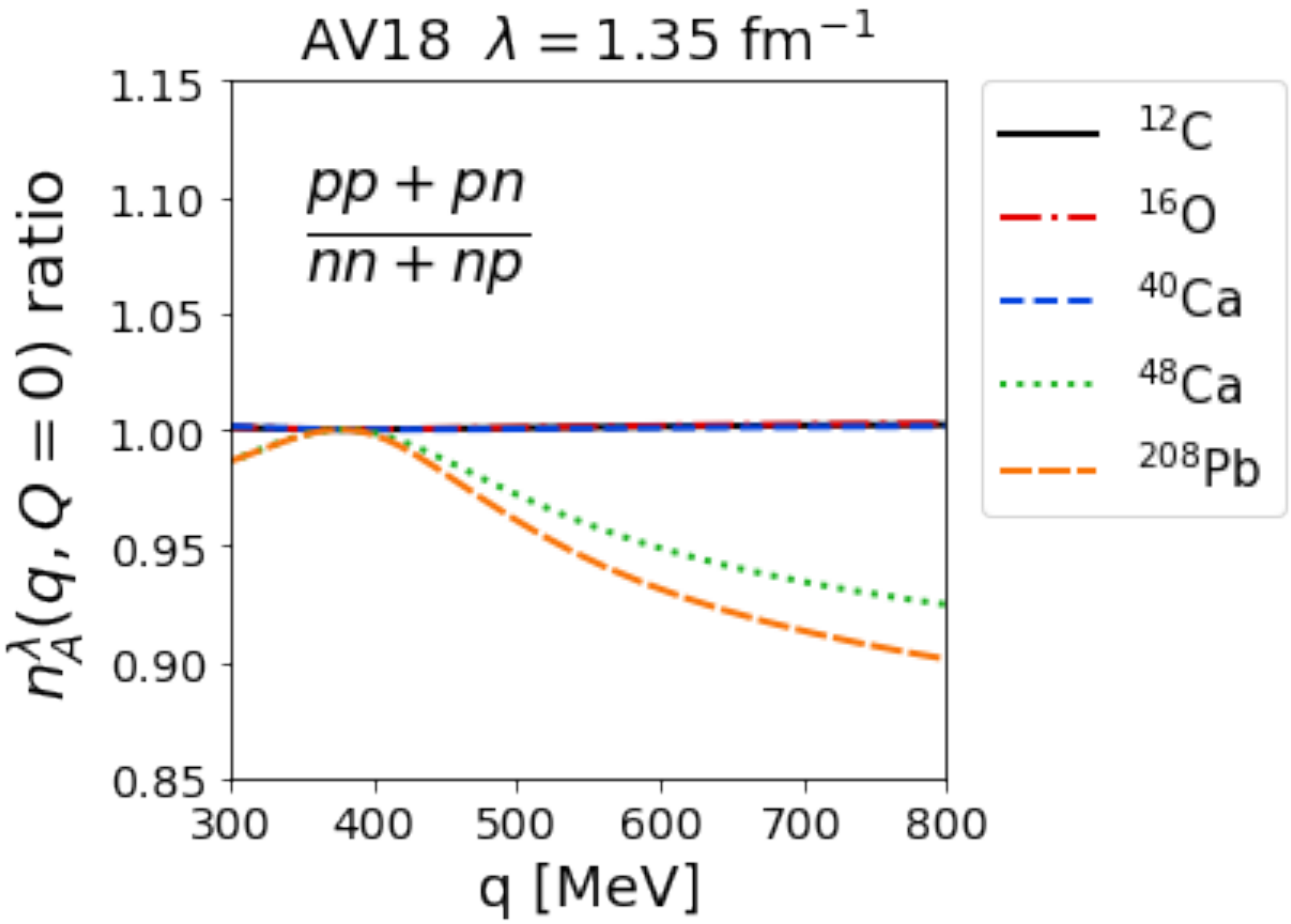
adii

lution?

out!...

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



Preliminary SRC low-resolution LDA calculations

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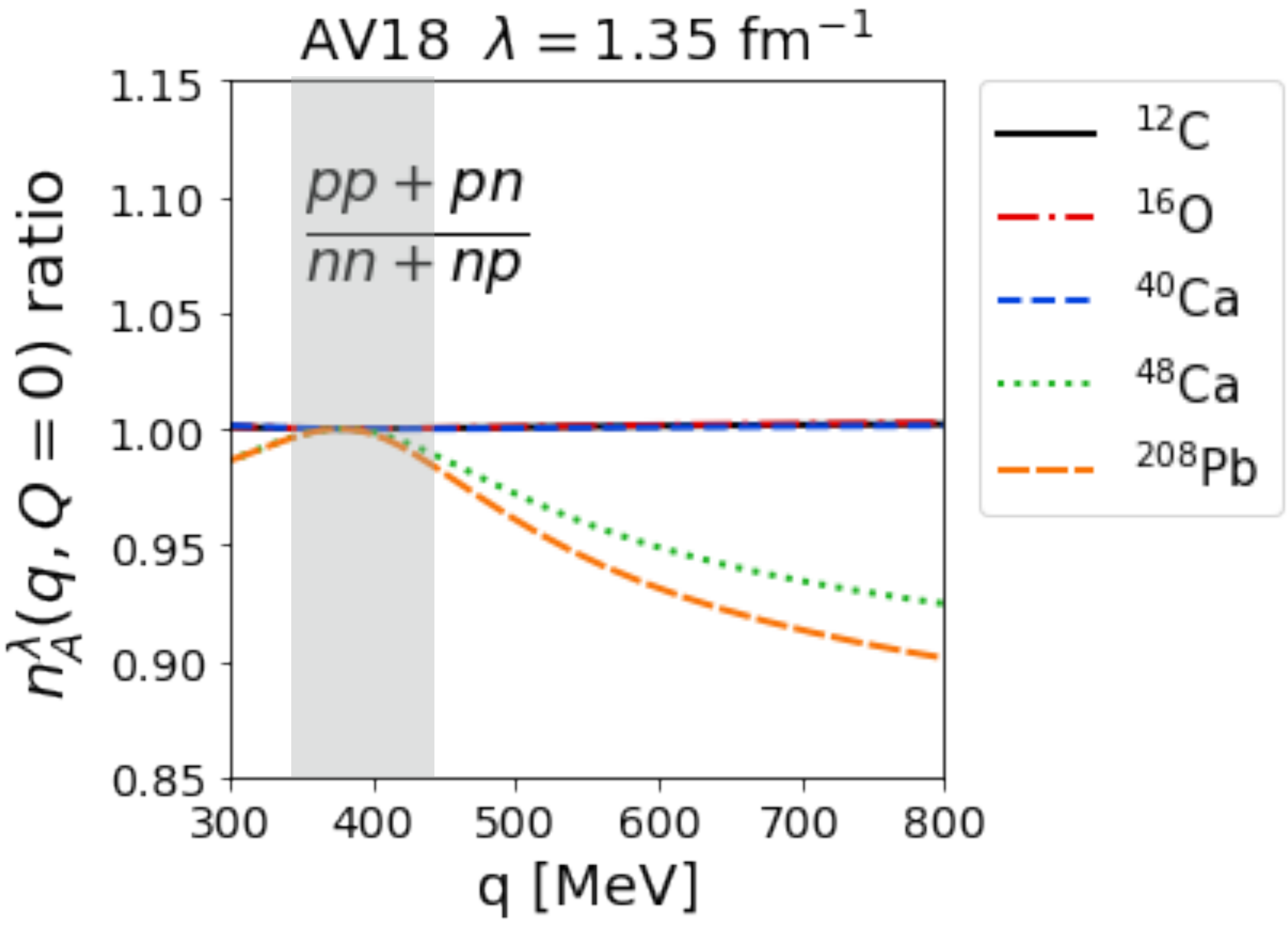

np dominance => ratio should be ~ 1 irrespective of N/Z

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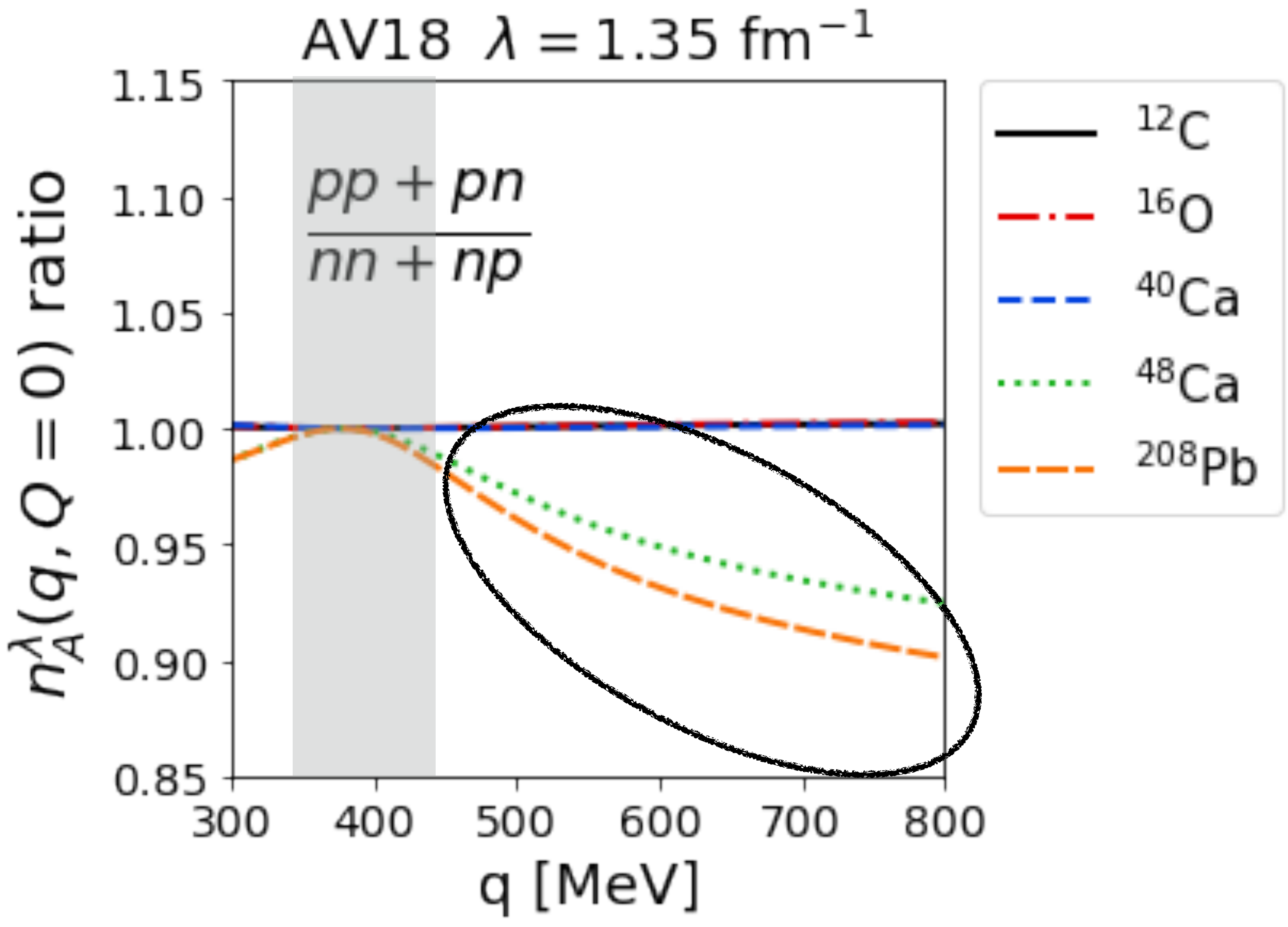
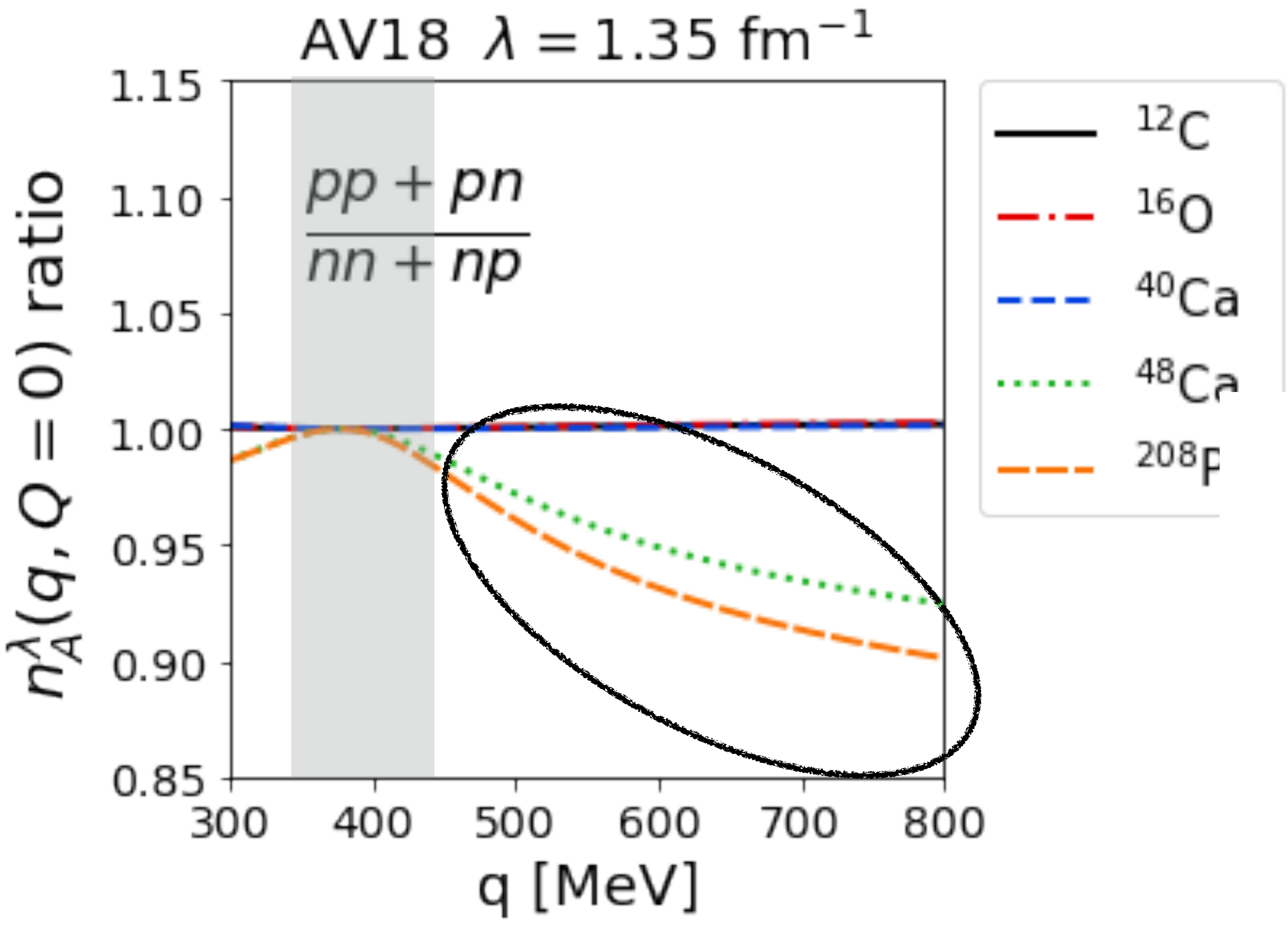


np dominance => ratio should be ~ 1 irrespective of N/Z

transition towards scalar counting at higher relative q

Preliminary SRC low-resolution LDA calculations

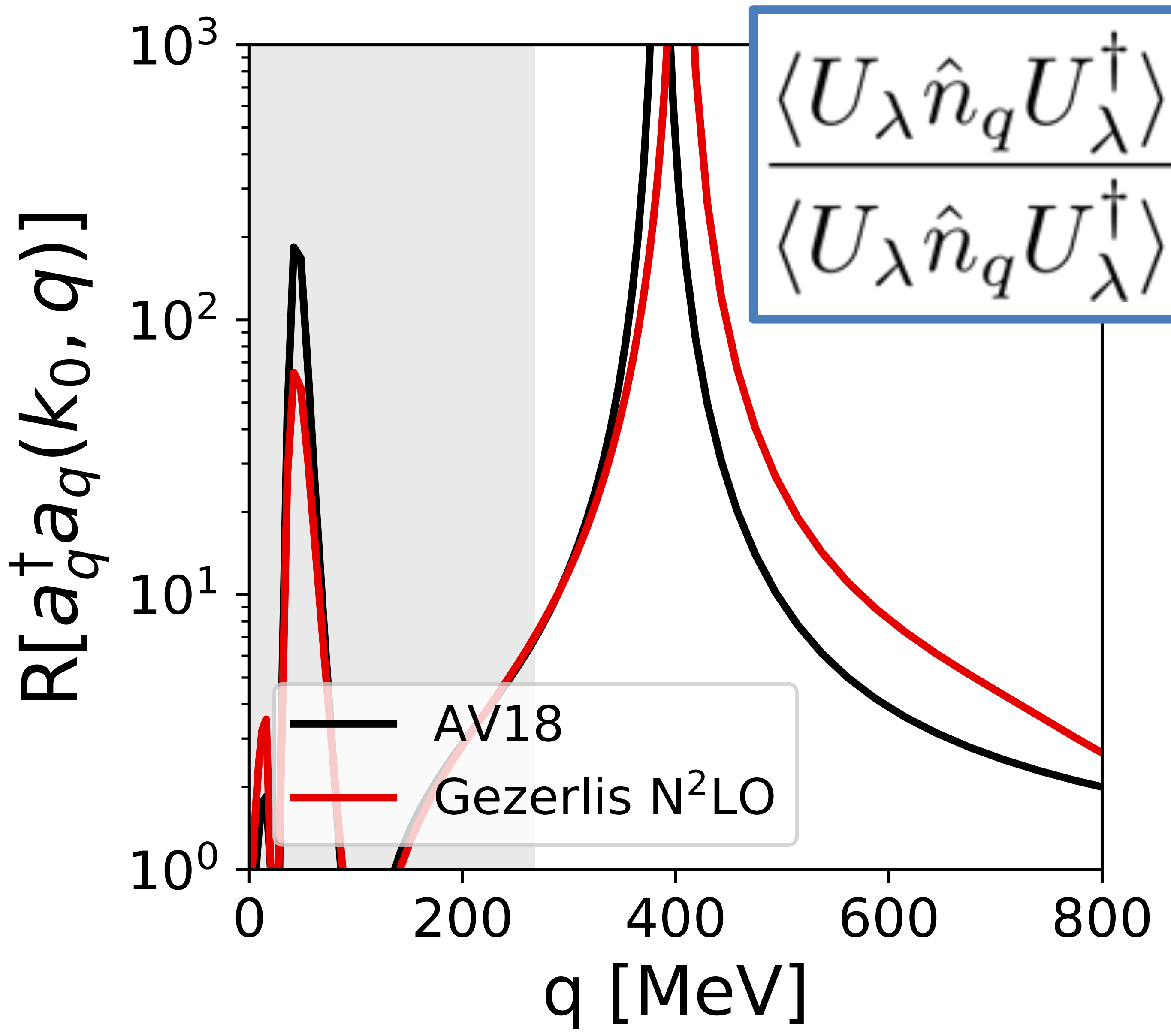


UNDER CONSTRUCTION

Tropiano, SKB, Furnstahl (in progress)

np dominance => ratio should be ~ 1 irrespective of N/Z

Ratio of *evolved* high-mom. distributions in a low-mom. state (insensitive to details!)

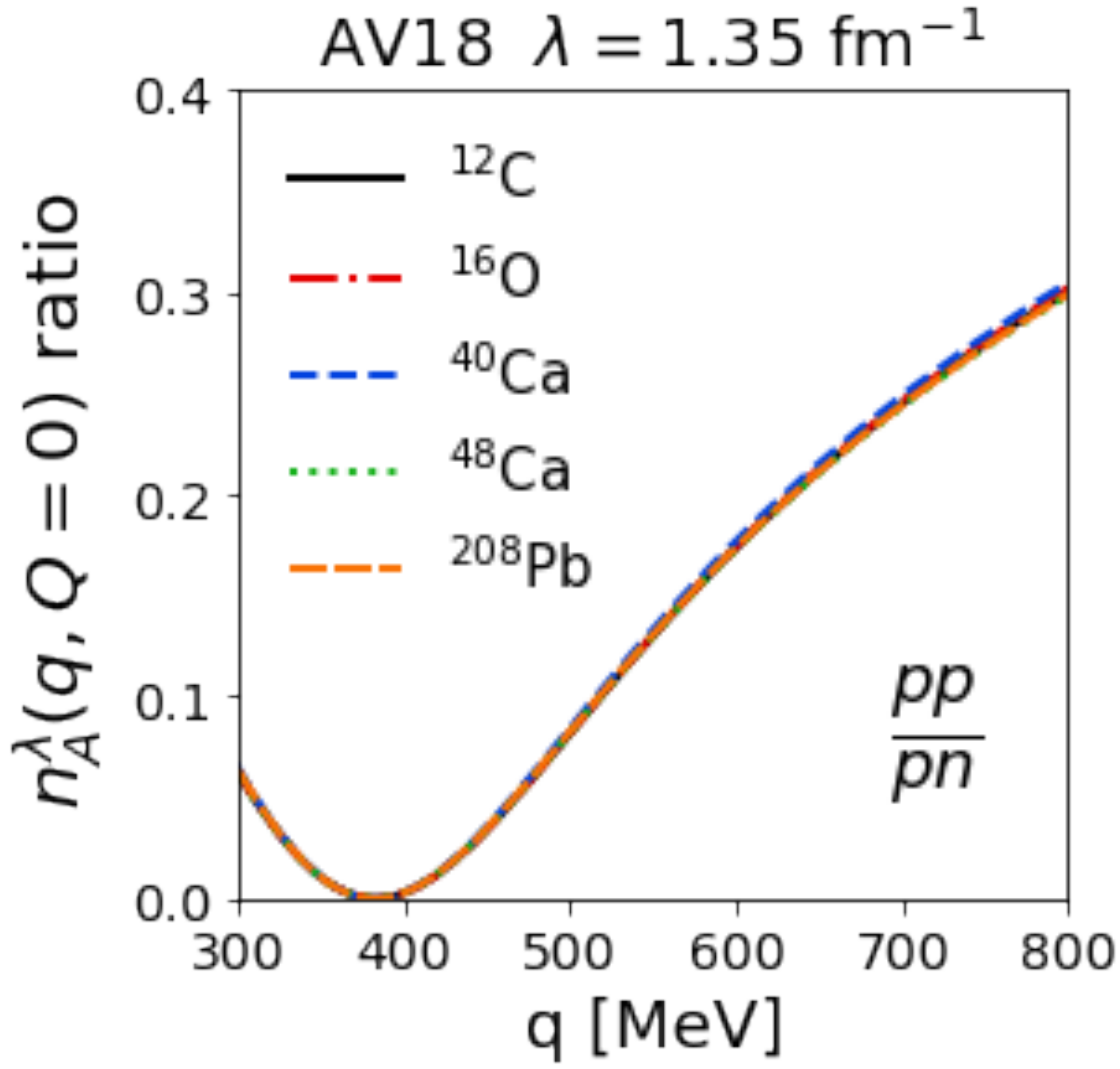


$$\frac{\langle U_\lambda \hat{n}_q U_\lambda^\dagger \rangle [^3\mathrm{S}_1]}{\langle U_\lambda \hat{n}_q U_\lambda^\dagger \rangle [^1\mathrm{S}_0]}$$

Preliminary SRC low-resolution LDA calculations

UNDER CONSTRUCTION

Tropiano, SKB, Furnstahl (in progress)



Preliminary SRC low-resolution LDA calculations

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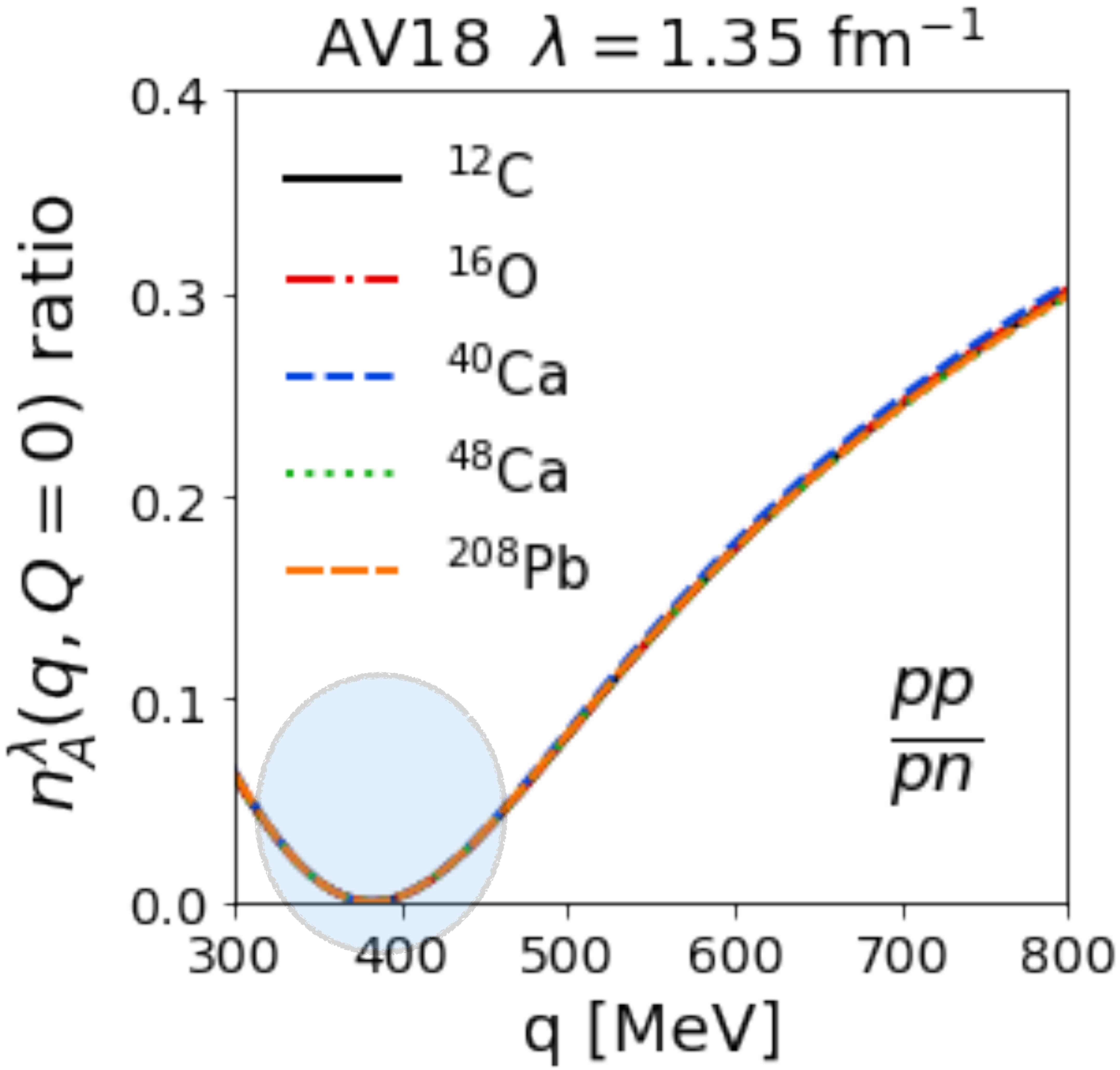
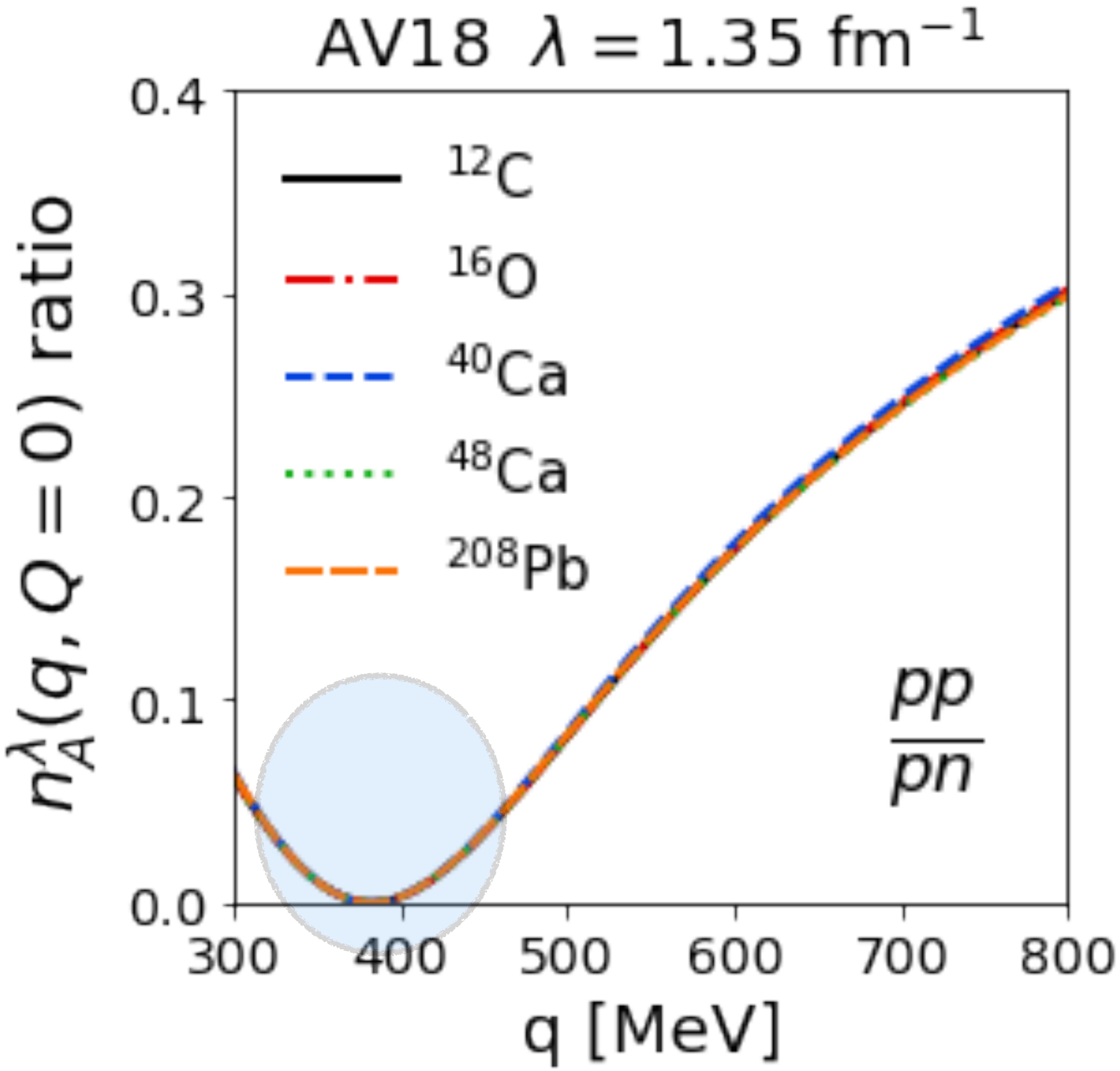


np pair (tensor force) dominance

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)

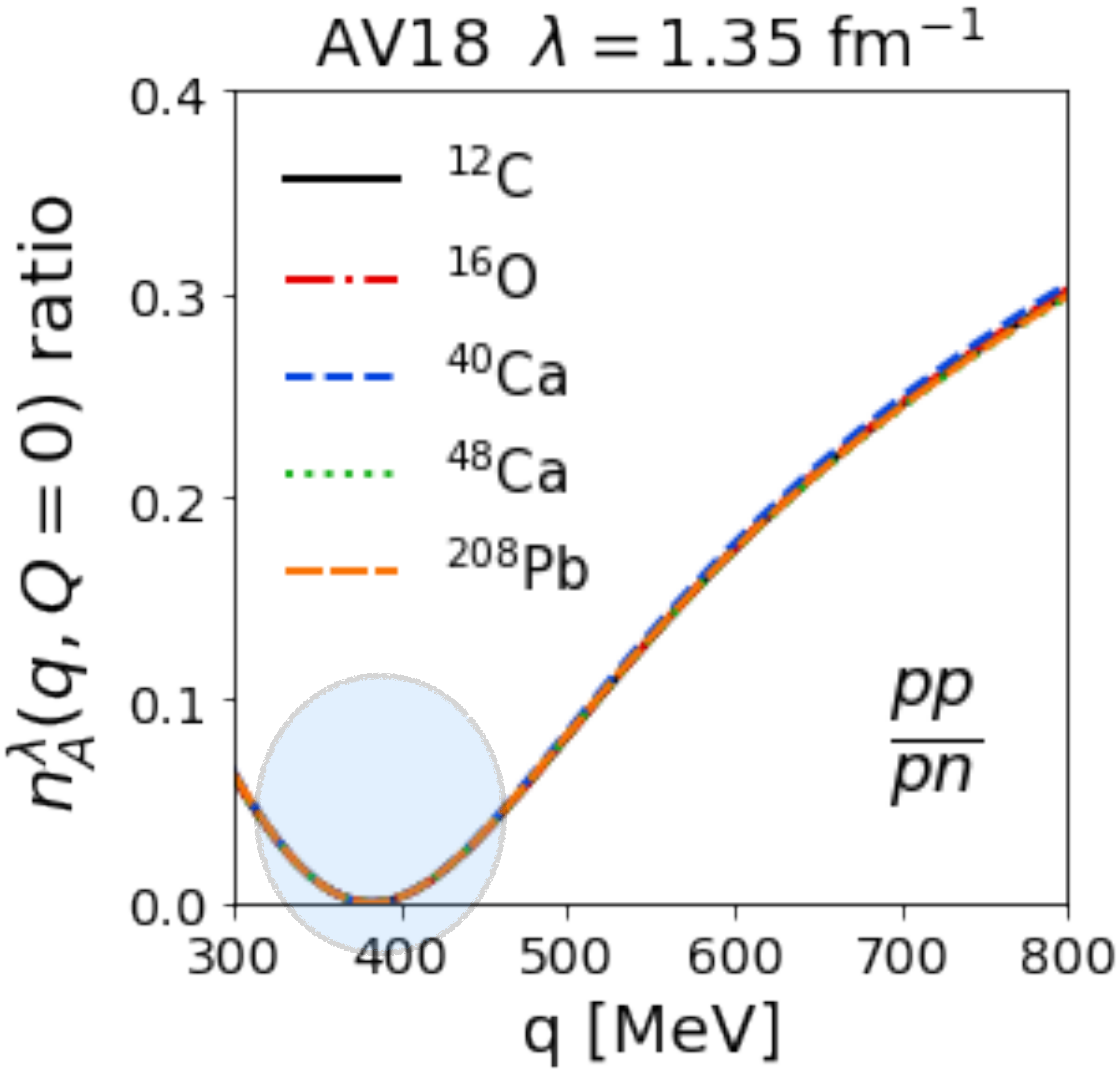


np pair (tensor force) dominance

weak nucleus dependence follows from factorization

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



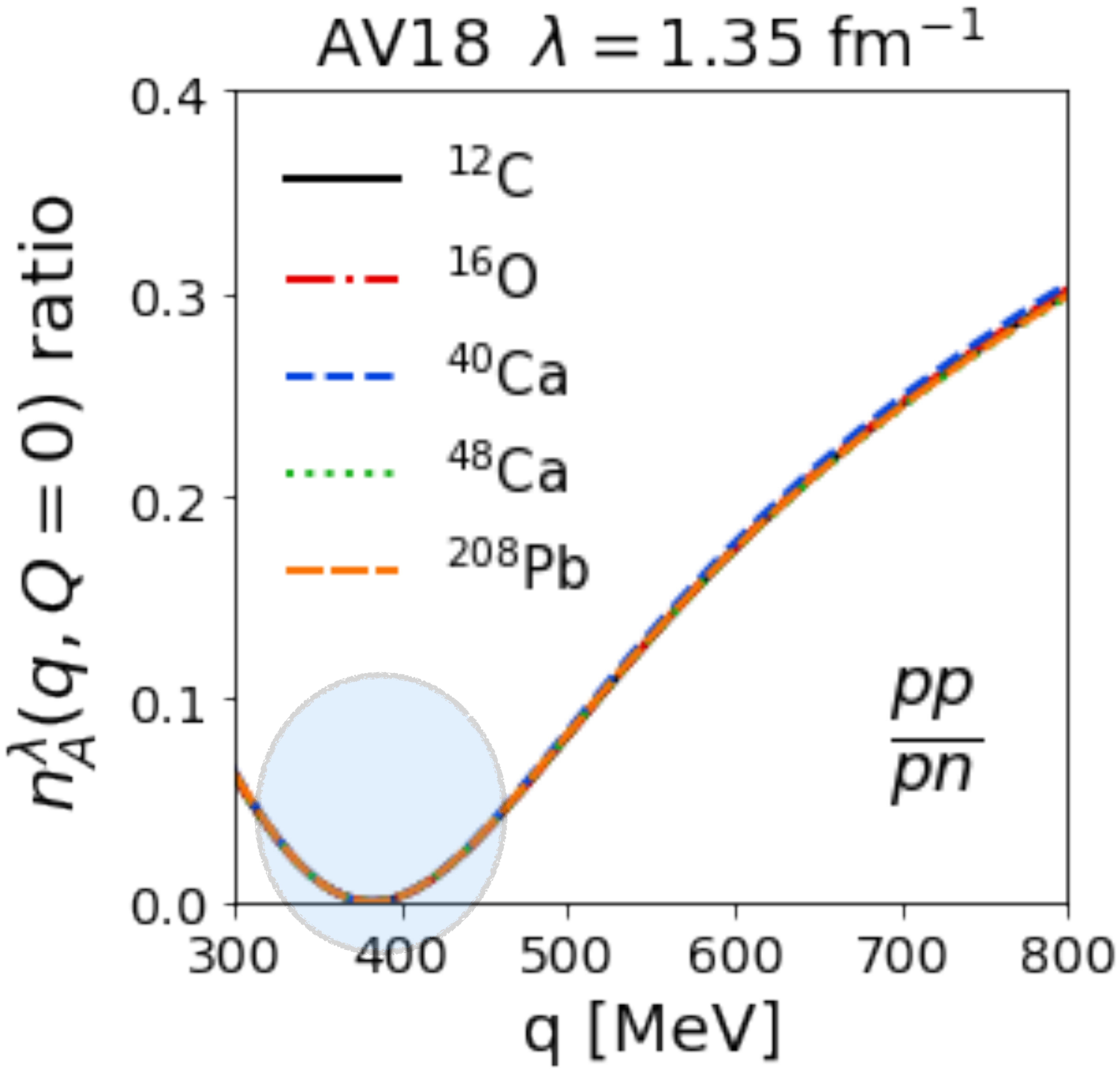
np pair (tensor force) dominance

weak nucleus dependence follows from factorization

$$\text{Ratio} \approx \frac{(F^{hi}_{pp}(q))^2 \left\langle A^{lo} \middle| \sum^{\lambda}_{\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{Q}{2}+\mathbf{k}} a^\dagger_{\frac{Q}{2}-\mathbf{k}} a_{\frac{Q}{2}-\mathbf{k}'} a_{\frac{Q}{2}+\mathbf{k}'} \middle| A^{lo} \right\rangle}{(F^{hi}_{np}(q))^2 \left\langle A^{lo} \middle| \sum^{\lambda}_{\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{Q}{2}+\mathbf{k}} a^\dagger_{\frac{Q}{2}-\mathbf{k}} a_{\frac{Q}{2}-\mathbf{k}'} a_{\frac{Q}{2}+\mathbf{k}'} \middle| A^{lo} \right\rangle}$$

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



np pair (tensor force) dominance

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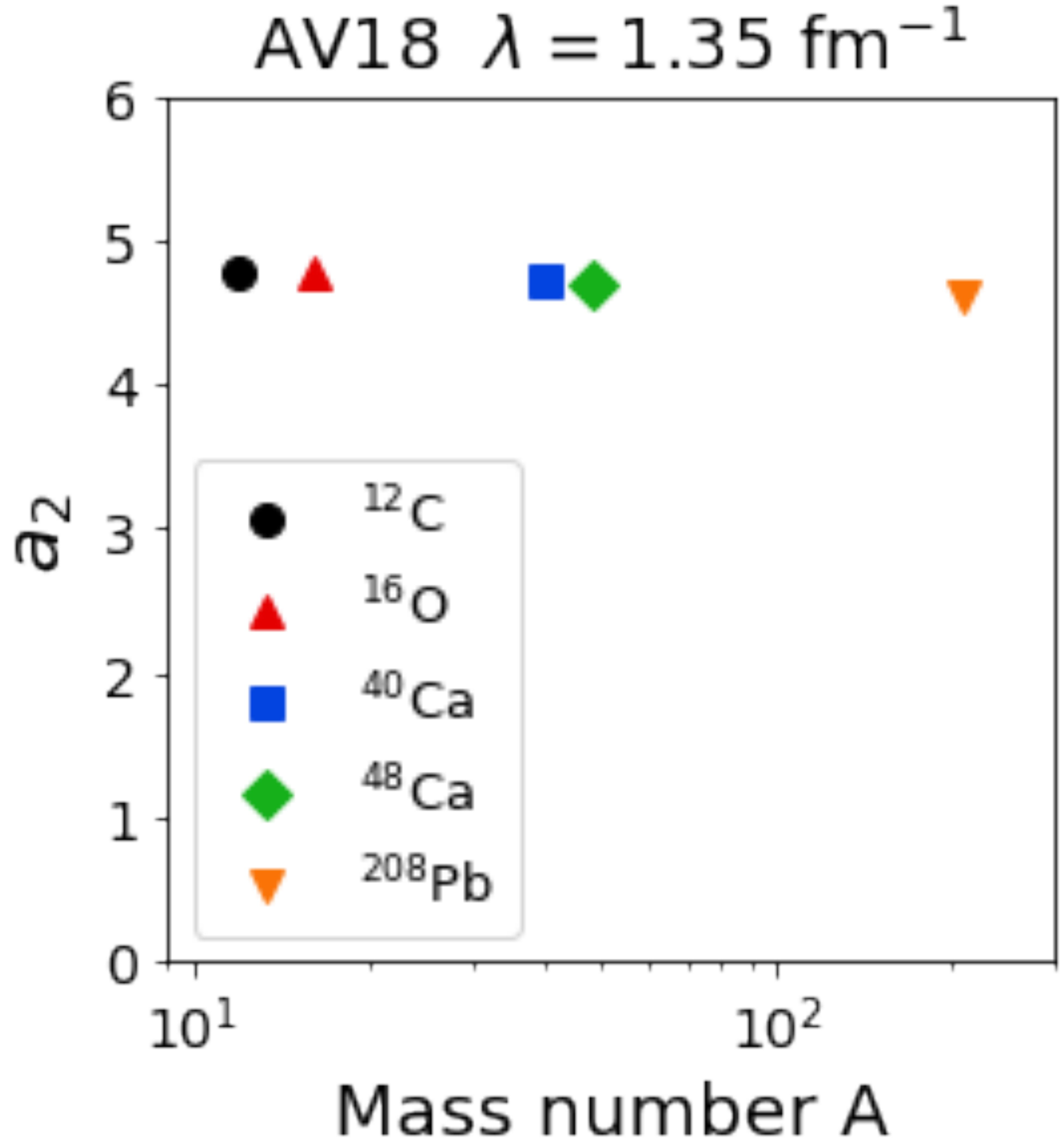
$$\text{Ratio} \approx \frac{(F^{\mathrm{hi}}_{pp}(q))^2}{(F^{\mathrm{hi}}_{np}(q))^2}$$

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



Followed Ryckebusch et al. prescription

$$a_2(A) = \lim_{\text{high } p} \frac{P^A(p)}{P^d(p)} \approx \frac{\int_{\Delta p^{\text{high}}} dp\, P^A(p)}{\int_{\Delta p^{\text{high}}} dp P^d(p)}.$$

$$\Delta p^{\text{high}} = [3.8 \ldots 4.5] \text{ fm}^{-1}$$

Decent agreement w/LCA calcs (flatter A-dependence)

But systematics need to be explored more!

Looking ahead

Can we use low-RG scale pictures to directly compute cross sections, etc?

$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger \widehat{O}(q) U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

Looking ahead

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cf deuteron electrodisintegration studies **More, SKB, Furnstahl PRC96 (2017)**

$$\frac{d}{d\mu_F}\left[\boxed{\sigma} = \boxed{\text{reaction}} \underbrace{\otimes}_{\mu_F} \boxed{\text{structure}}\right] = 0$$

Factorization is scale-dependent (not unique)!!

Looking ahead

Can we use low-RG scale pictures to directly compute cross sections, etc?

$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger\widehat{O}(q)U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

cf deuteron electrodisintegration studies **More, SKB, Furnstahl PRC96 (2017)**

$$\frac{d}{d\mu_F}\left[\sigma = \text{reaction} \underbrace{\otimes}_{\mu_F} \text{structure}\right] = 0$$

Factorization is scale-dependent (not unique)!!

scale/scheme dependence of extracted properties? (e.g., SFs)

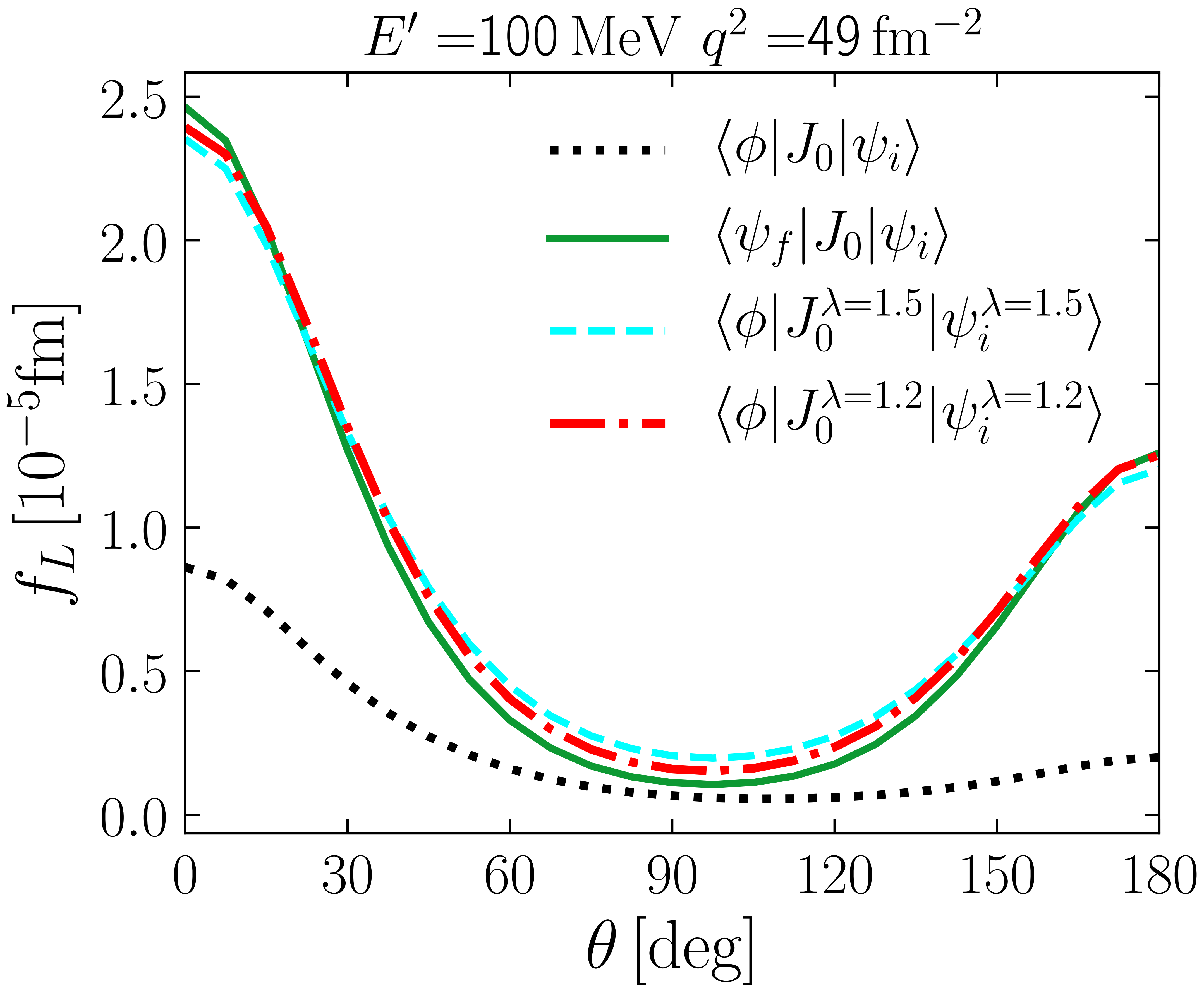
extract at one scale, evolve to another? (like PDFs)

how do FSIs, physical interpretations, etc. depend on RG scale?

Scale Dependence of Final State Interactions

NSCL FRIB

Deuteron Electrodisintegration



x_B=1.64, Q^2= 1.78 GeV^2

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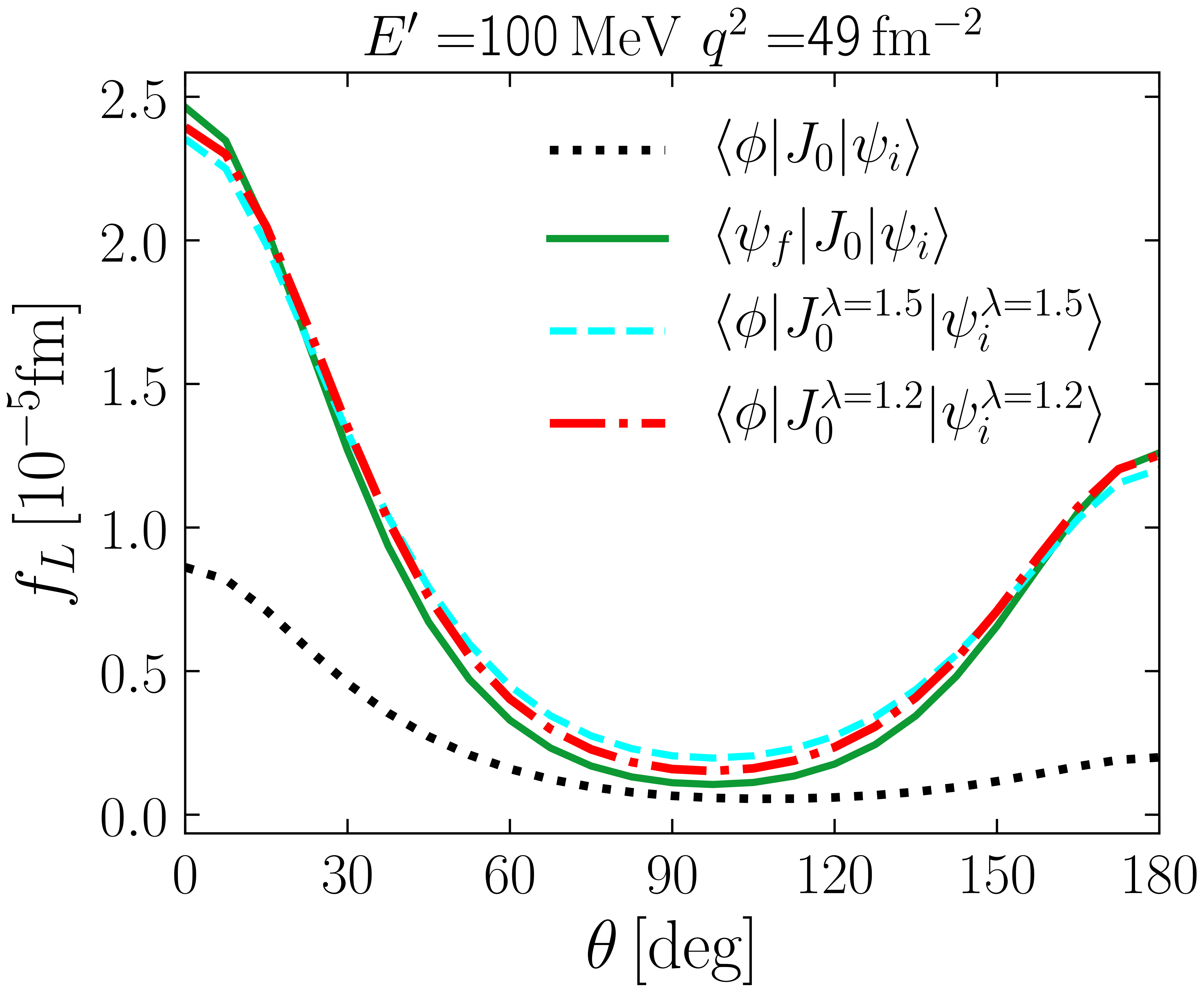


x_B=1.64, Q^2= 1.78 GeV^2

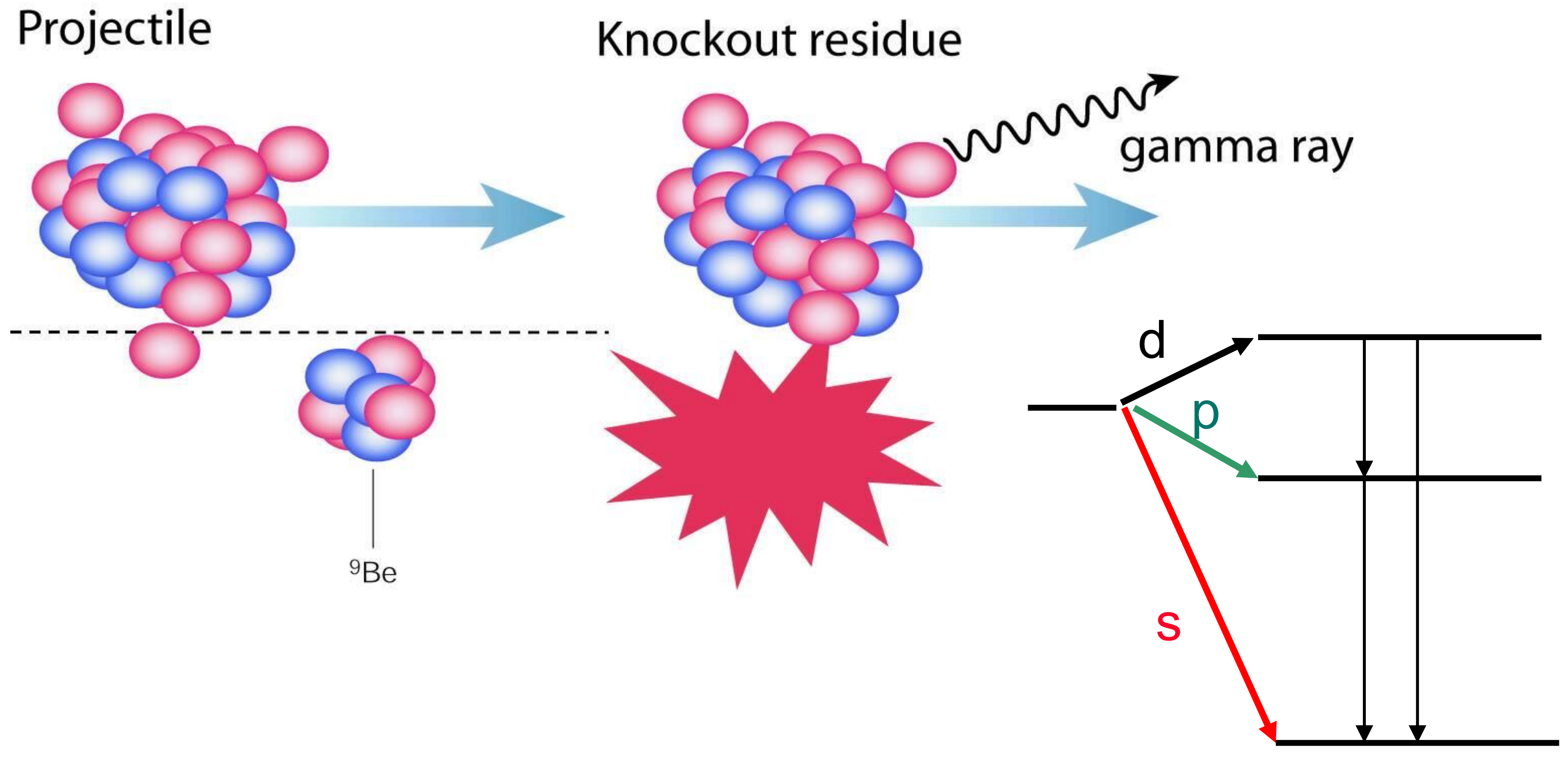
FSI sizable at large λ
but negligible at low-resolution!

Takeaway point:

Size of FSI depends on RG scale/scheme

Ditto physical interpretations

Other exclusive knock-out reactions [pictures from A. Gade]



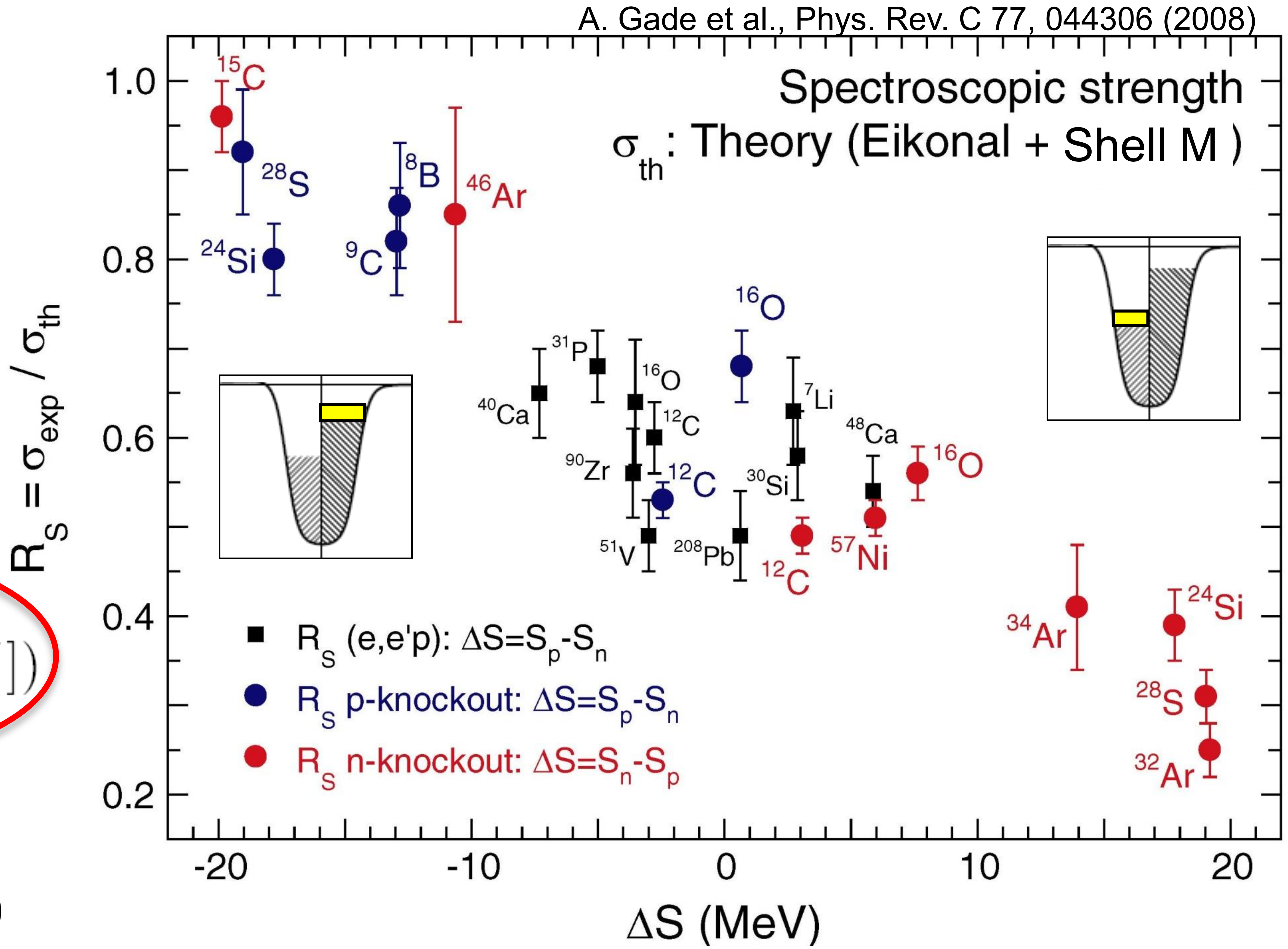
Exclusive reactions, theory vs. experiment

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2S(j^\pi)\,\sigma_{sp}(j, S_N + E_x[j^\pi])$$

Structure theory

Reaction theory

Origin and systematics of R = σ_{exp} / σ_{th} < 1 are not understood (includes e,e'p results)



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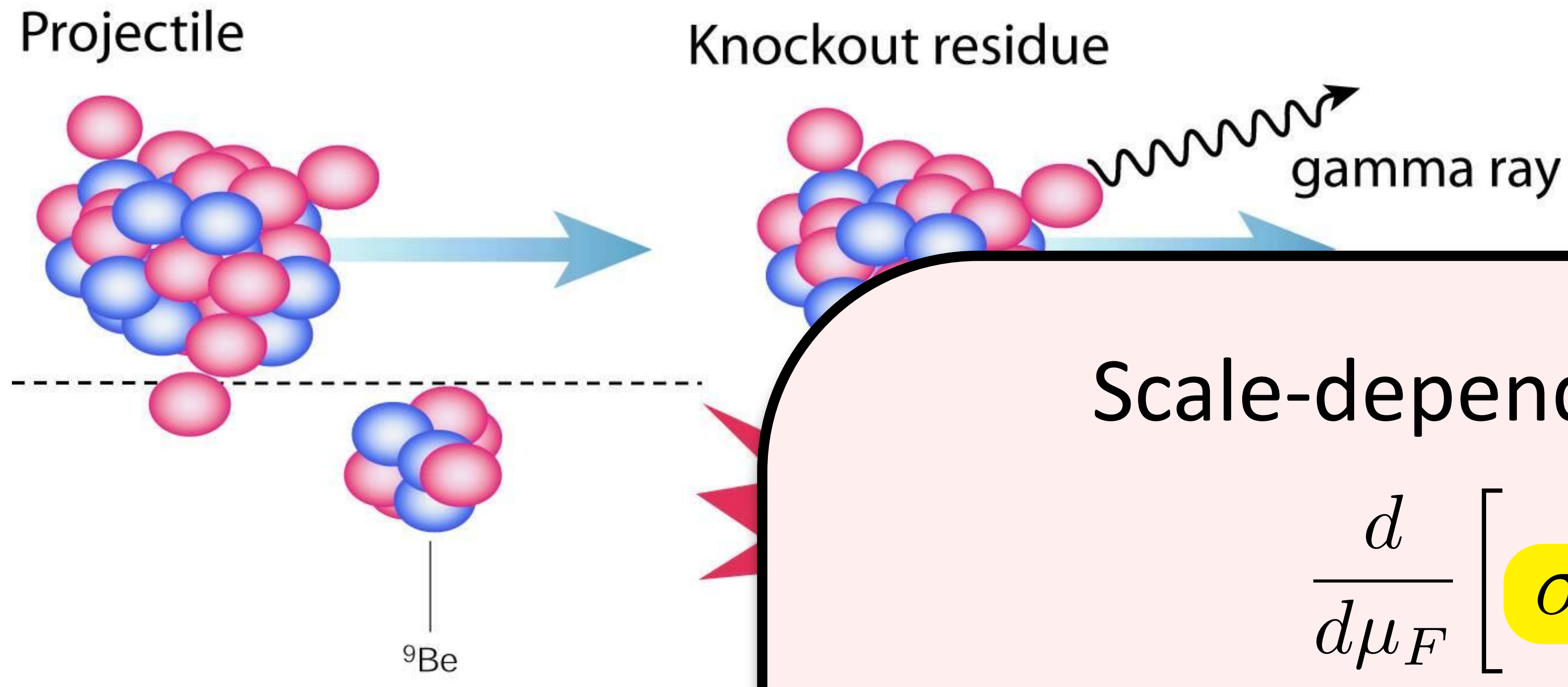


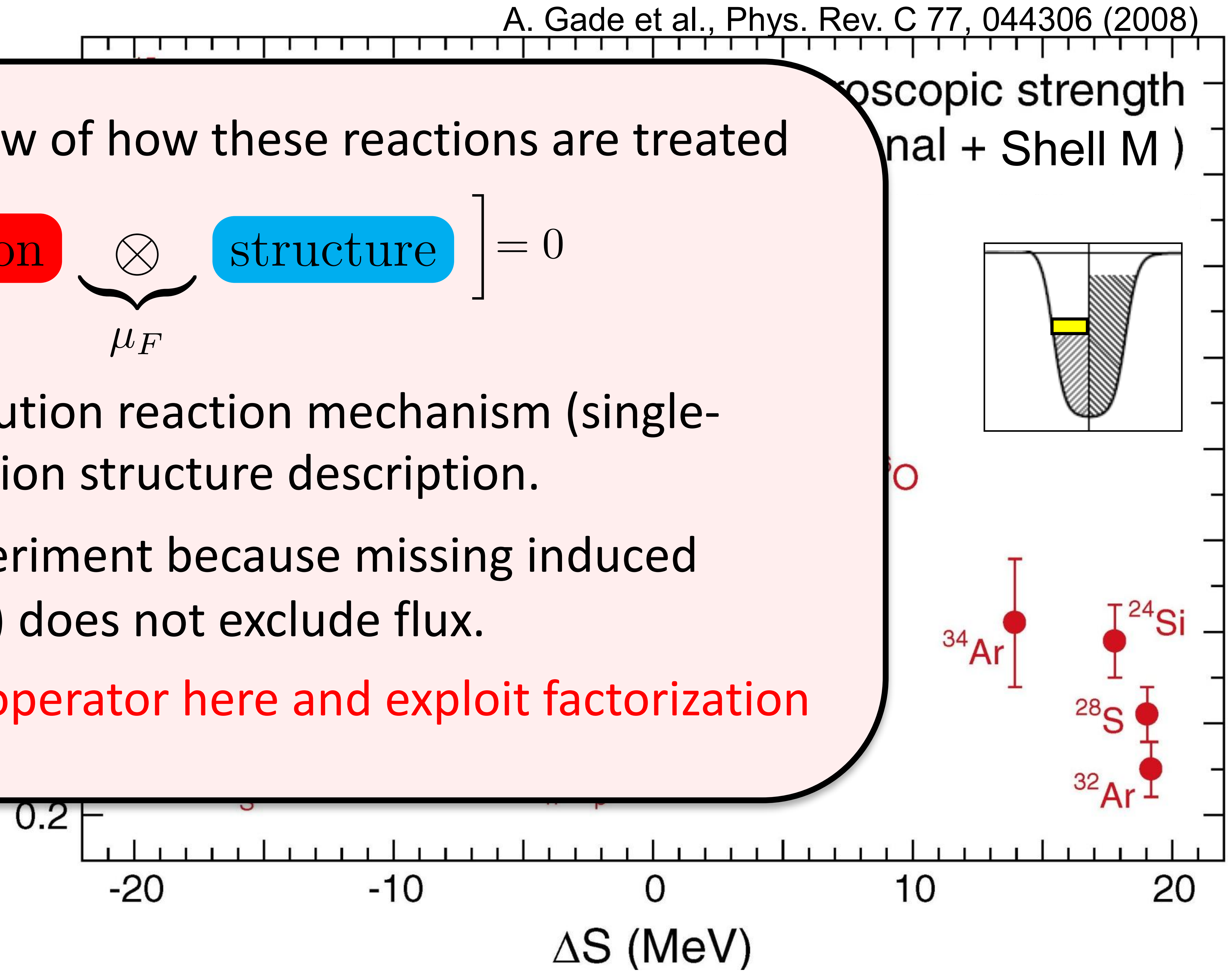
Scale-dependent (RG) view of how these reactions are treated

$$\frac{d}{d\mu_F}\left[\sigma = \text{reaction} \underbrace{\otimes}_{\mu_F} \text{structure}\right] = 0$$

- Analysis mixes a high-resolution reaction mechanism (single-particle) with a low-resolution structure description.
- Theory is greater than experiment because missing induced current (e.g., 2-body for e^-) does not exclude flux.
- Plan: use SRG on reaction operator here and exploit factorization

Exclusive reactions

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^{N}$$



Origin and systematics of R = $\sigma_{exp} / \sigma_{th} < 1$ are not understood (includes e,e'p results)

Summary

RG smoothly connects high- and low-resolution pictures. There is no "correct" picture in an absolute sense (e.g., can reproduce SRC phenom. in a low resolution picture, or can do many-body calculations in high-resolution picture)

Unexpected simplifications for calculating SRC quantities at low-RG resolution ($\mathbf{q} \gg \lambda$)

- factorization of q-dependence into A-indep Wilson coeffs (few-body physics)
- simple many-body calculations due to low-k wf's
- LDA (free fermi gas) seems to be sufficient (a-la LCA) at reproducing some of the usual SRC phenomenology

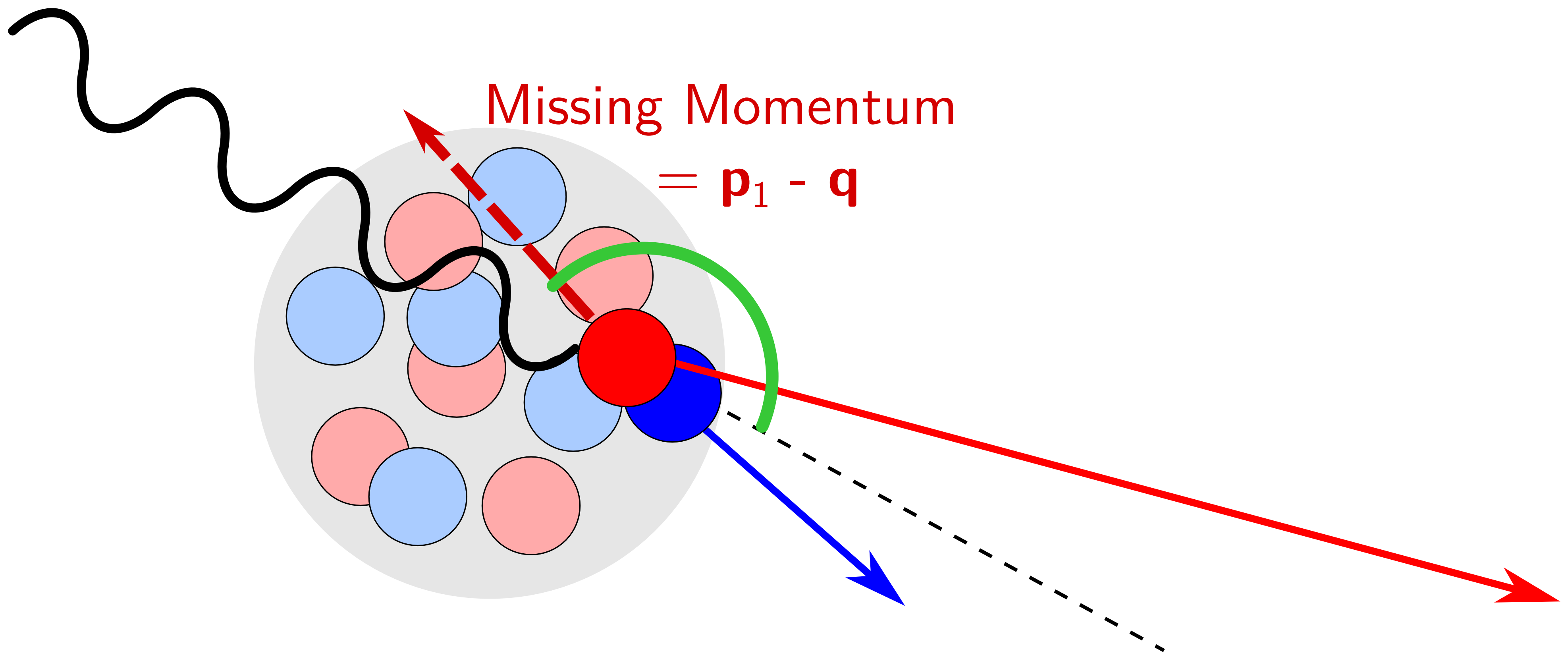
Natural connections to GCF/LCA approaches; RG/OPE machinery ==> corrections to scaling, 3N SRCs, etc. possible

Interpretations, FSIs, etc. depend on RG scale for deuteron electrodisintegration. Can we exploit this in more complicated knock-out reactions by treating structure/reaction consistently at the same resolution scale?

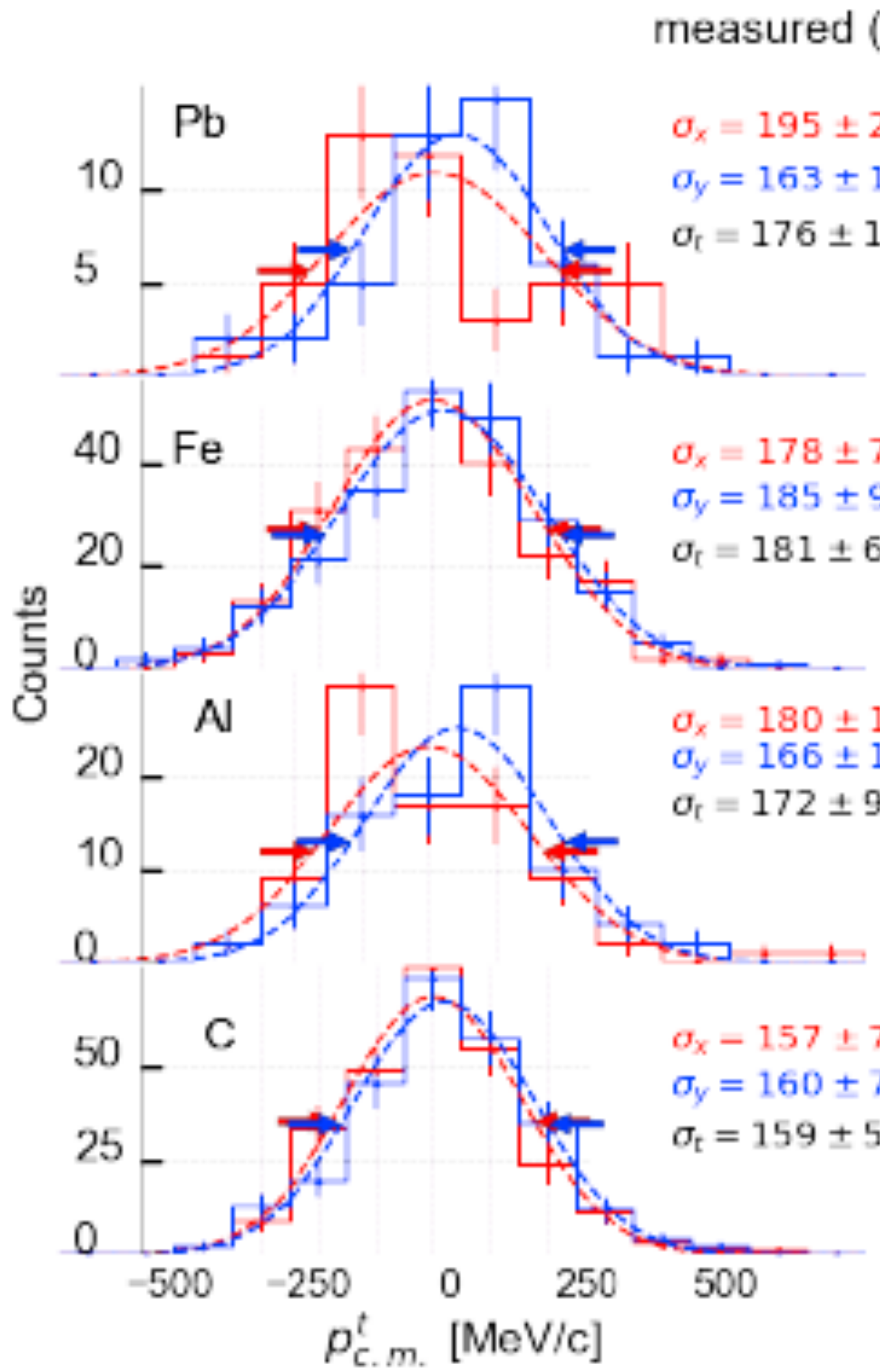
Extras

Tropiano, SKB, Furnstahl (in progress)

2) Kinematics of knocked-out nucleons



knocked out SRC nucleons fly out almost back-to-back (relative s-wave pairs)



pair CM momentum distribution gaussian of width ~ k_F

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

2) Kin

evolved pair momentum distribution $(\lambda \sim k_F < < q)$

$$\rho_{NN,\alpha}(Q,q) \sim \gamma_\alpha^2(q;\Lambda) \sum_{k,k'} |\langle \psi^A(\Lambda)| \left[a^\dagger_{\frac{Q}{2}+k} a^\dagger_{\frac{Q}{2}-k} a_{\frac{Q}{2}-k'} a_{\frac{Q}{2}+k'}\right]_\alpha |\psi^A(\Lambda)\rangle$$

kn

alm

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ution

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

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m.e. of smeared contact operator ==>
high q pairs dominated relative s-waves

evolved $\psi(\Lambda)$ "soft", dominated by MFT configs ==>
CM Q distribution smooth/gaussian with width ~ k_F

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ution

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

3) np dominance at intermediate (300-500 MeV) relative momenta

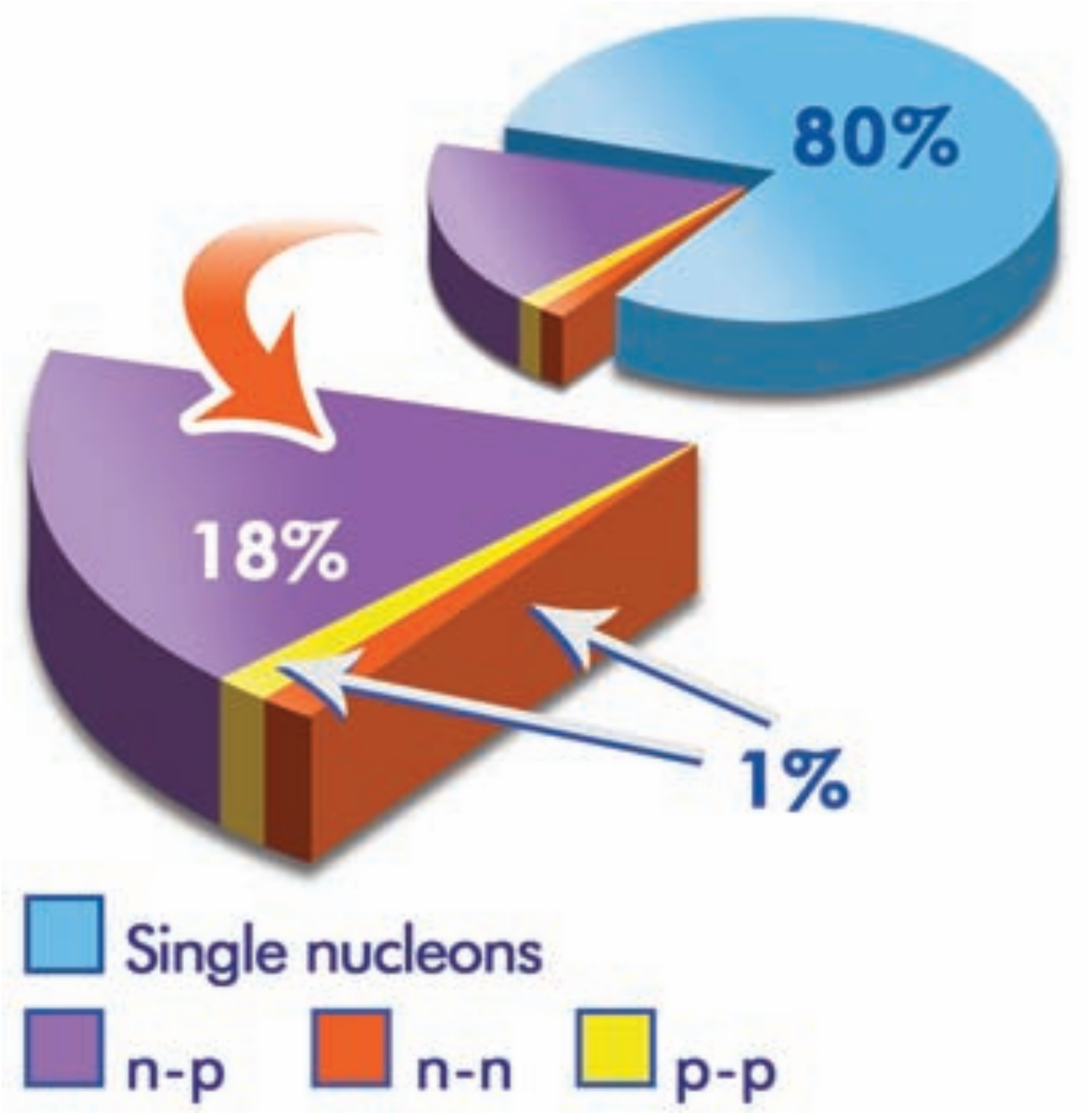

Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C.

R. Subedi et al., Science (2008)

20% of nucleons in SRC pairs
but mostly neutron-proton

SRC phenomenology revisited (low-res picture)

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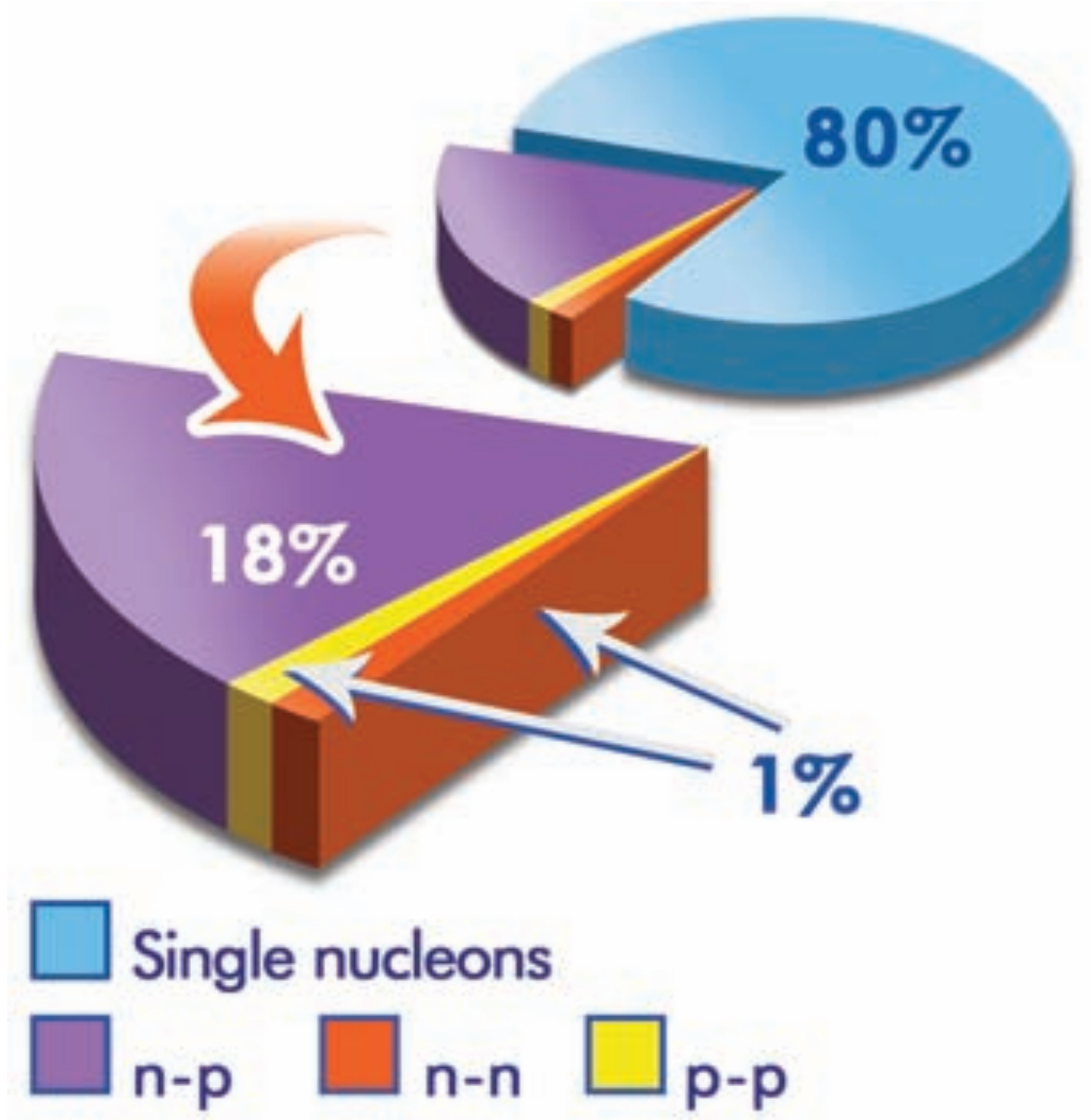

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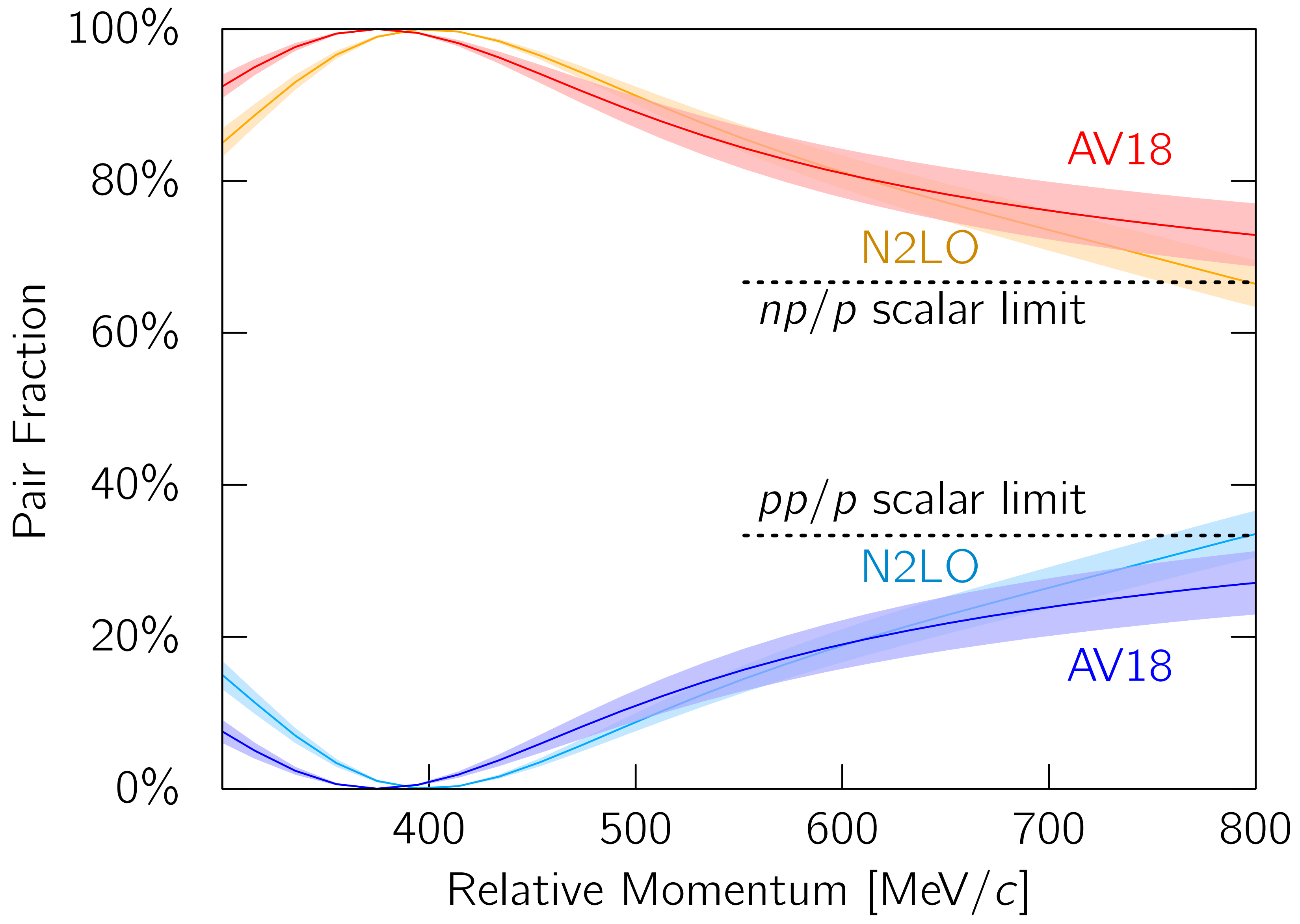

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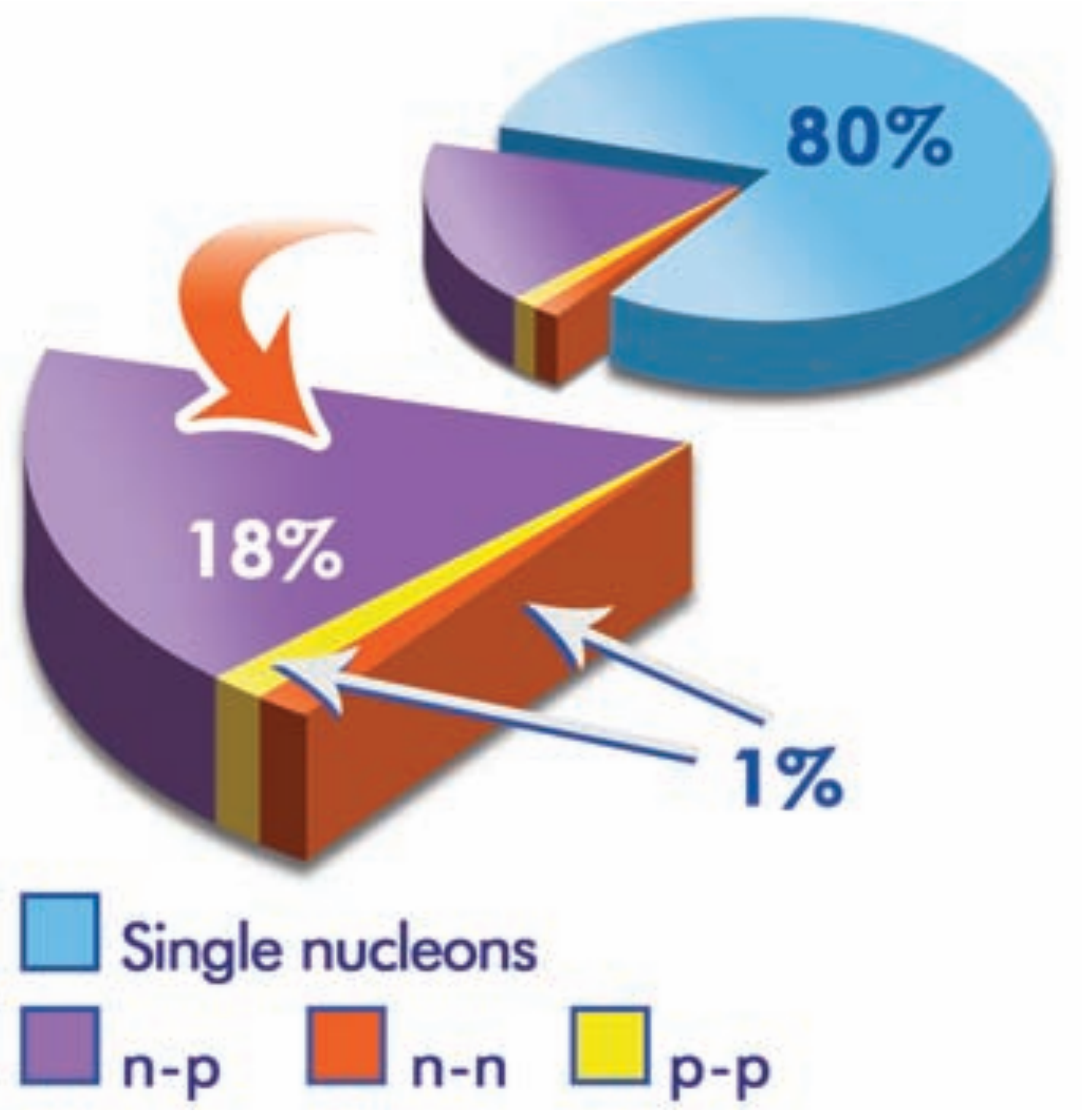

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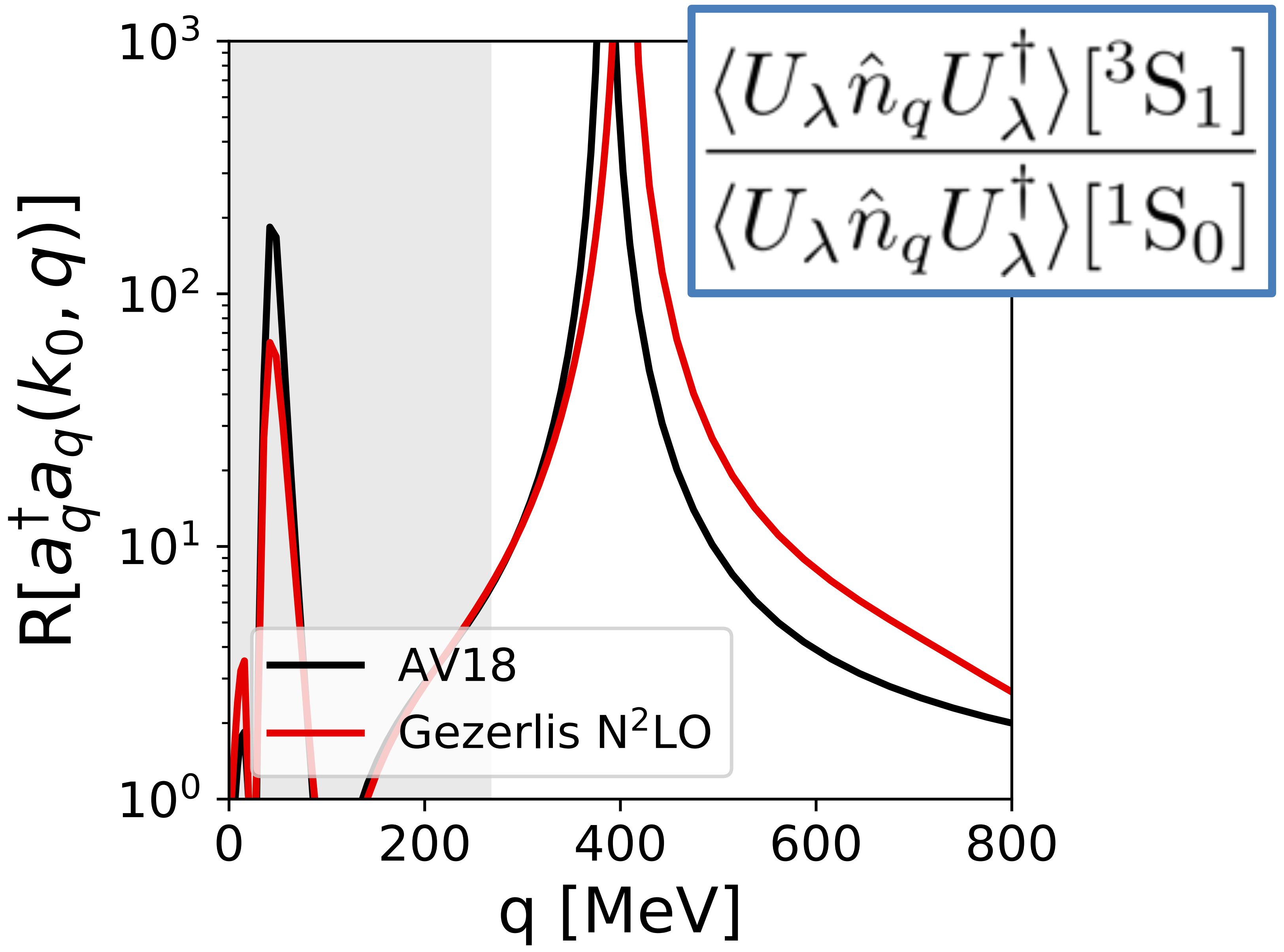
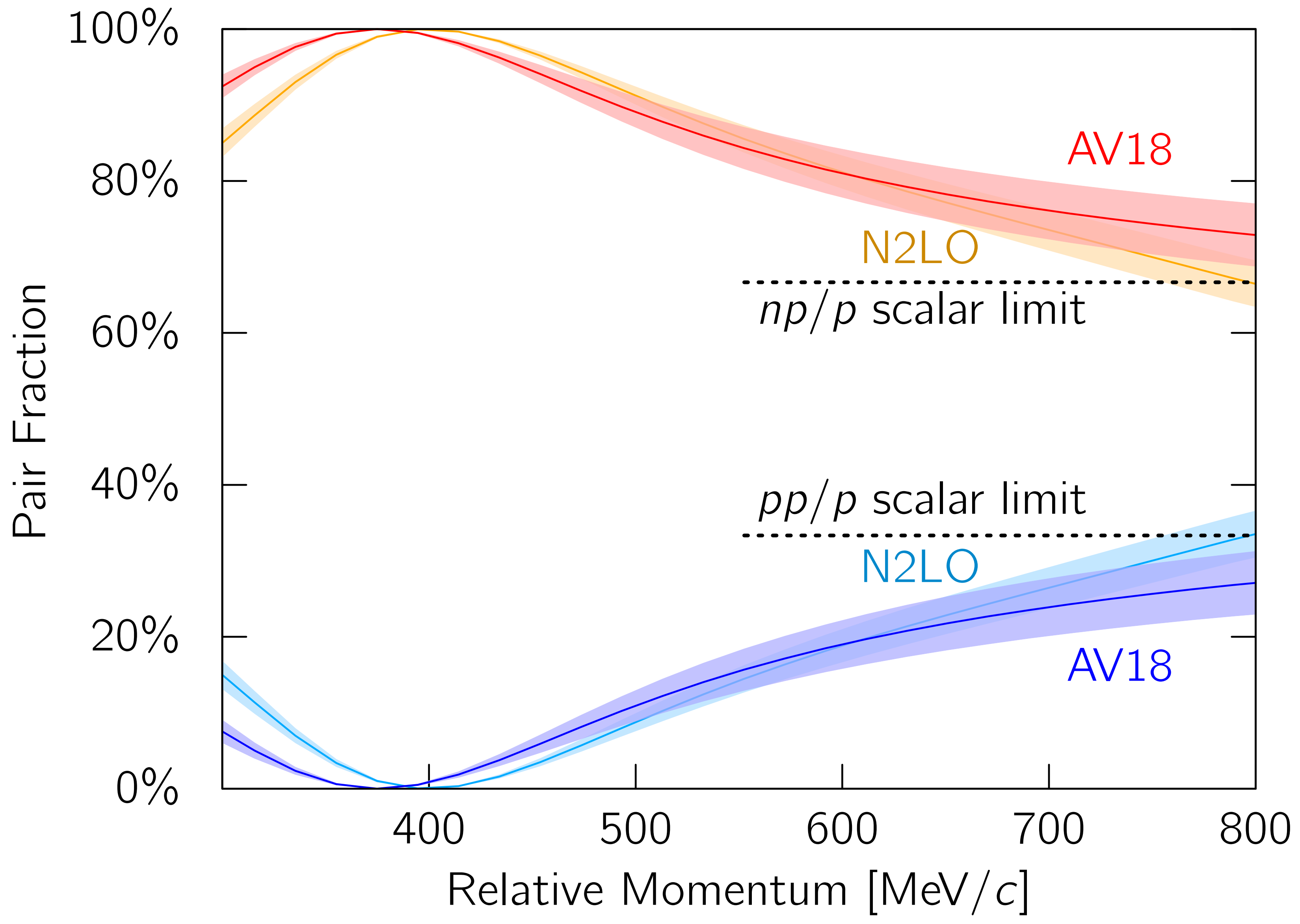
Ratio of *evolved* high-mom. distributions in a low-mom. state (insensitive to details!)





SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

$$\rho_A^{NN,\alpha}(r) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(r)|^2$$
$$n_A^{NN,\alpha}(q) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(q)|^2$$

A-dep scale factors (“nuclear contacts”) $C_A \sim <\chi|\chi>$

Universal (same all A, **not** V_{NN}) shape from
two-body zero energy wf ϕ

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

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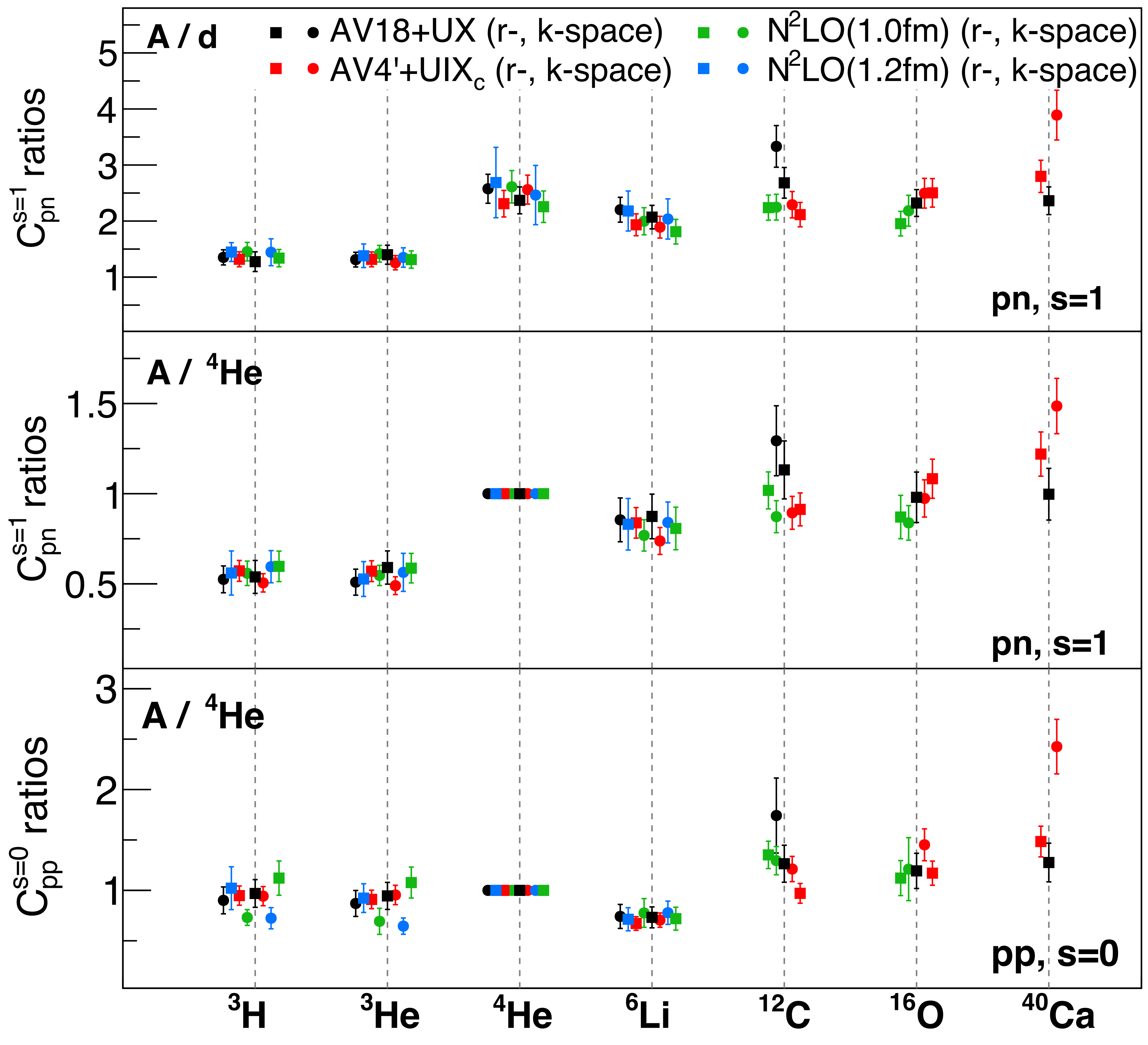
A-dep scale factors ("nuclear contacts") $C_A \sim <\chi|\chi>$

Universal (same all A, **not** V_{NN}) shape from two-body zero energy wf ϕ

But φ_{NN} is scale and scheme dependent. Ratios are independent but only probe "mean field" part



SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Gene

$\rho_A^{NN,\alpha}($

$n_A^{NN,\alpha}($

A-dep scale

Universal (s
two-body z

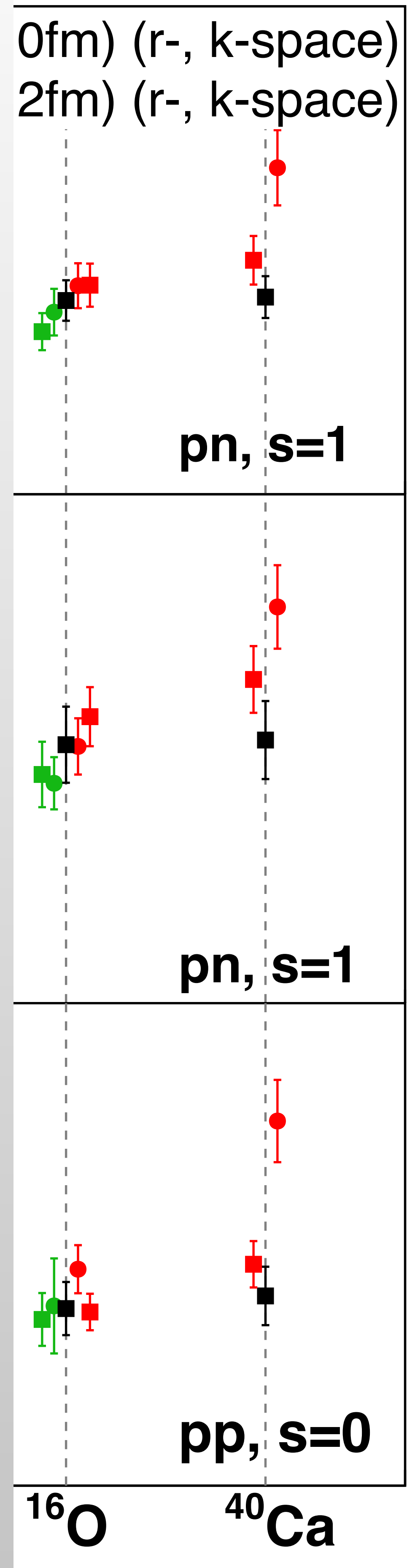
**But
schem
are in
probe**

Contacts **not** RG invariant

$$C_A = \sum_{K,k',k}^{\Lambda_0} \langle \psi^A_{\Lambda_0} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} \left| \psi^A_{\Lambda_0} \right\rangle \quad \Rightarrow \quad f(\Lambda) \sum_{K,k',k}^{\Lambda} \langle \psi^A_{\Lambda} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} \left| \psi^A_{\Lambda} \right\rangle$$

A-independent

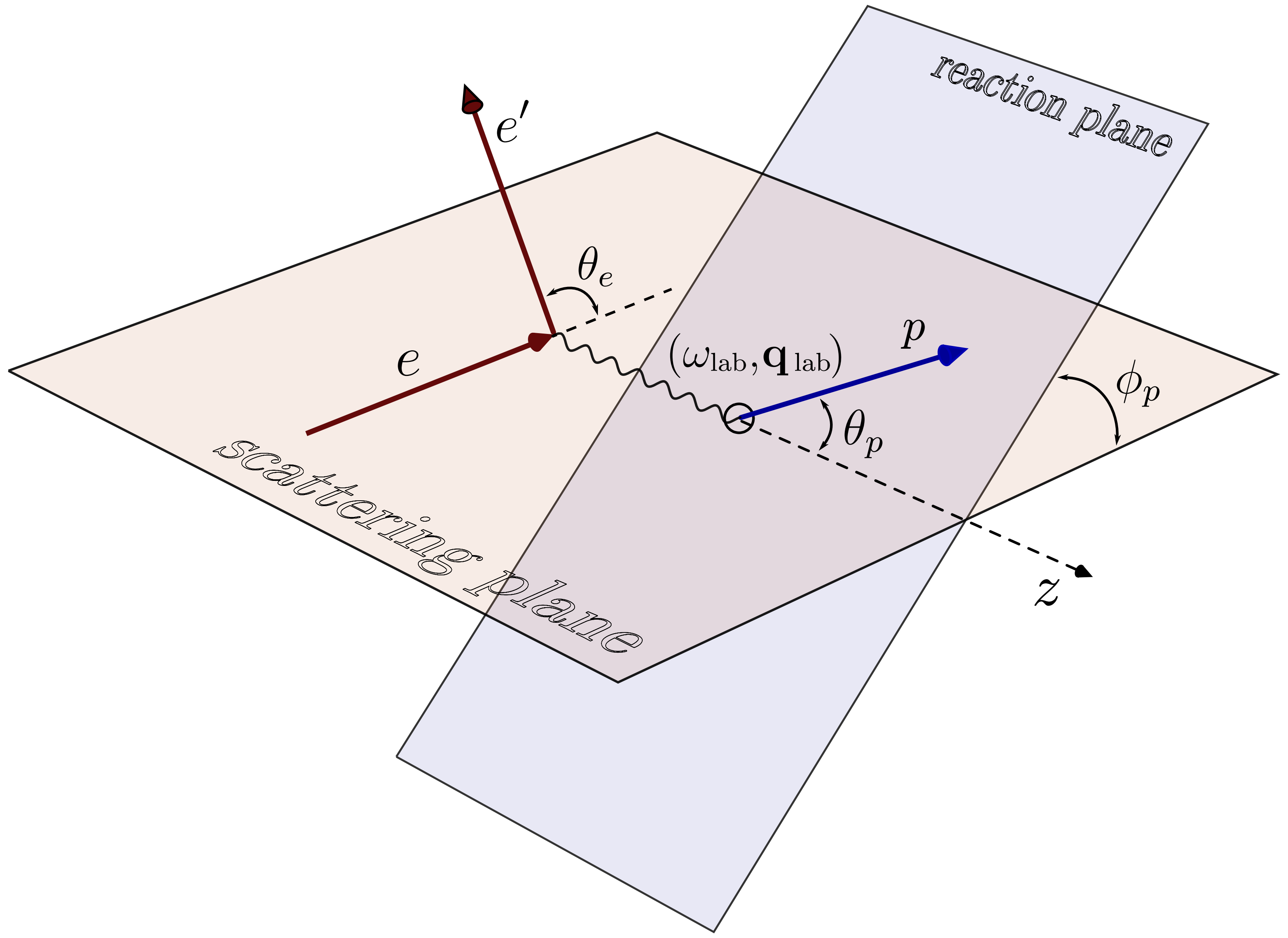
…But ratios in different A approx. RG invariant



Test ground: $^{2}H(e,e'p)n$

- Simplest knockout process (no induced 3N forces/currents)
- Focus on longitudinal structure function f_L

$$f_L \sim \sum_{m_s, m_J} \left|\langle \psi_f | J_0 | \psi_i \rangle\right|^2$$

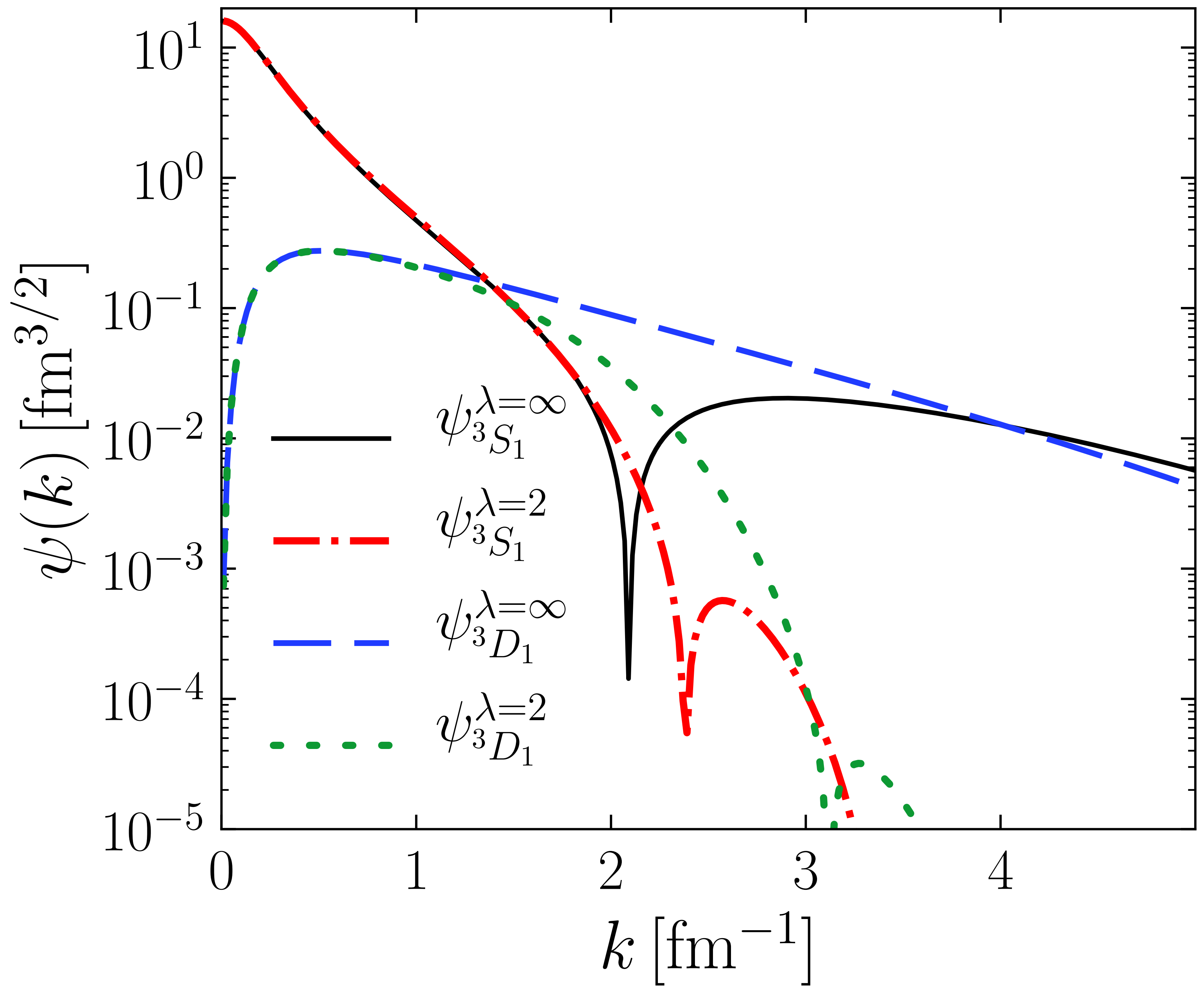


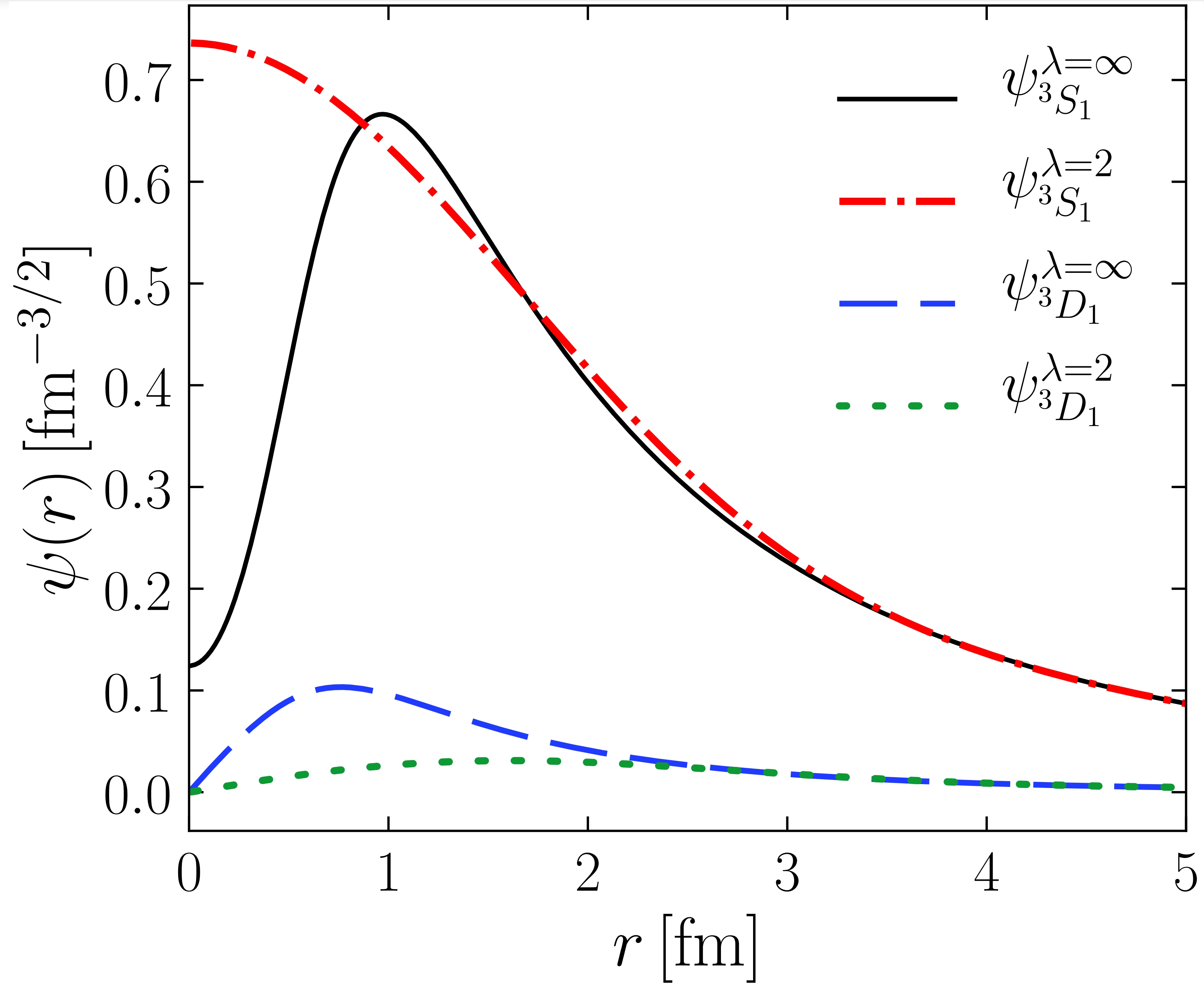
- $$f_L^\lambda \sim \Big|\underbrace{\langle \psi_f | U_\lambda^\dagger}_{\psi_f^\lambda} \underbrace{U_\lambda J_0 U_\lambda^\dagger}_{J_0^\lambda} \underbrace{U_\lambda | \psi_i \rangle}_{\psi_i^\lambda}\Big|^2; \quad U_\lambda^\dagger U_\lambda = I; \quad f_L^\lambda = f_L$$

- Components (deuteron wf, transition operator, FSI) scale-dependent, total is not.

- Are some resolutions "better" than others? E.g., in a given kinematics, can FSI be minimized with different choices of λ?? How do interpretations change with scale?

Deuteron wave function evolution





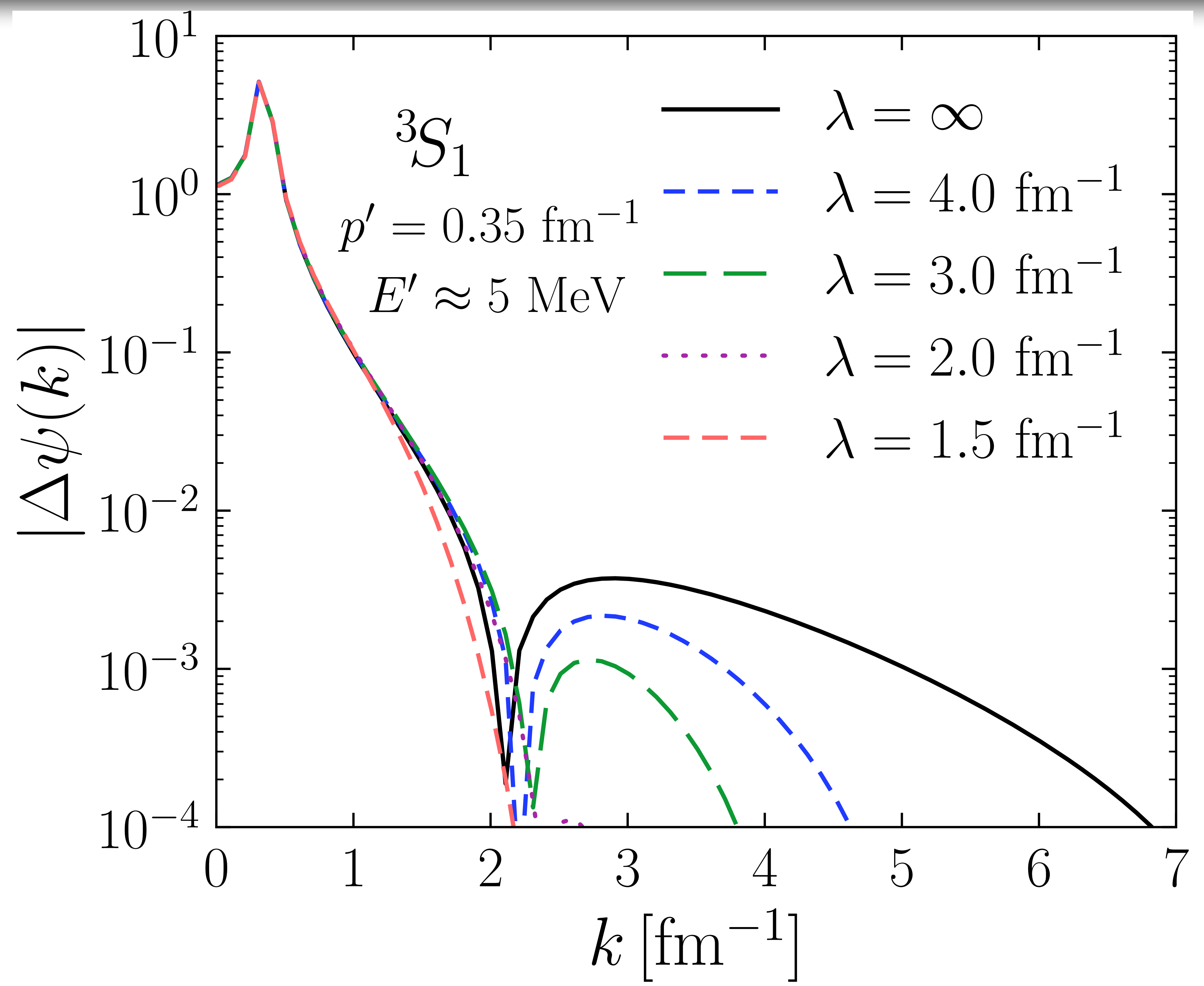
k < λ components invariant <==> RG preserves long-distance physics

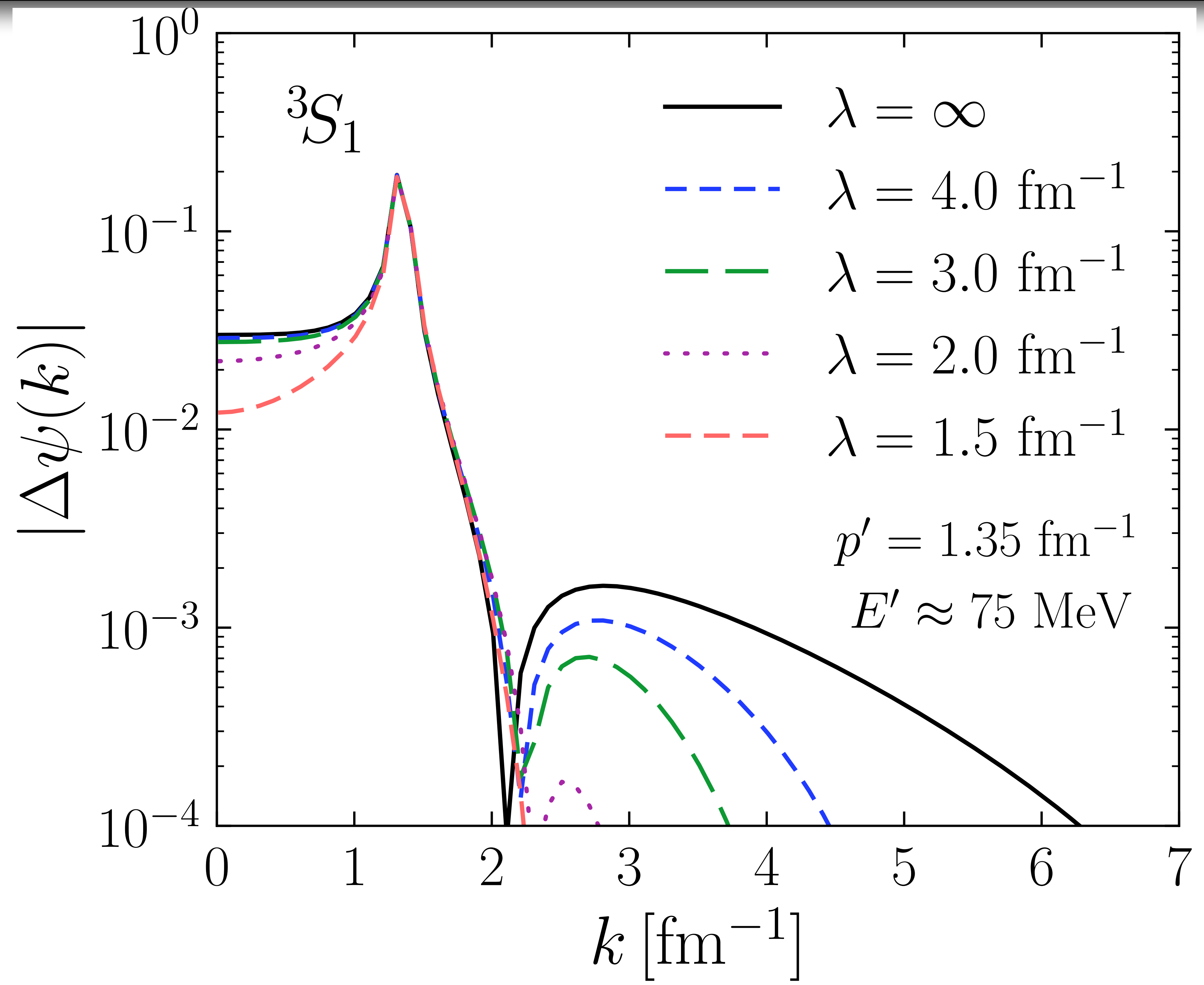
k > λ components suppressed <==> short-range correlations blurred out

Folklore: Simple wave functions at low λ <==> more complicated operators?

especially for high-q processes?

Final-state wave function evolution

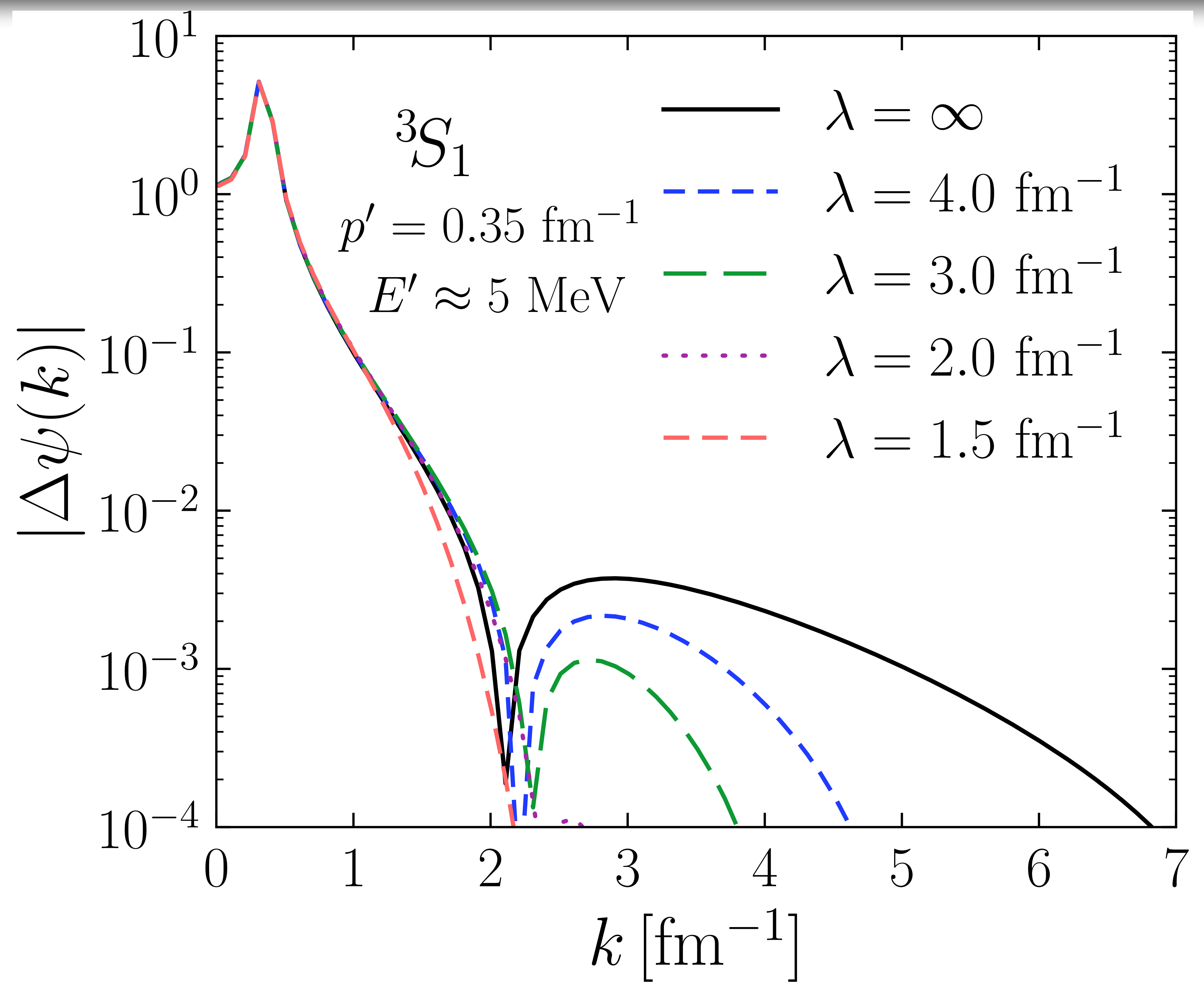


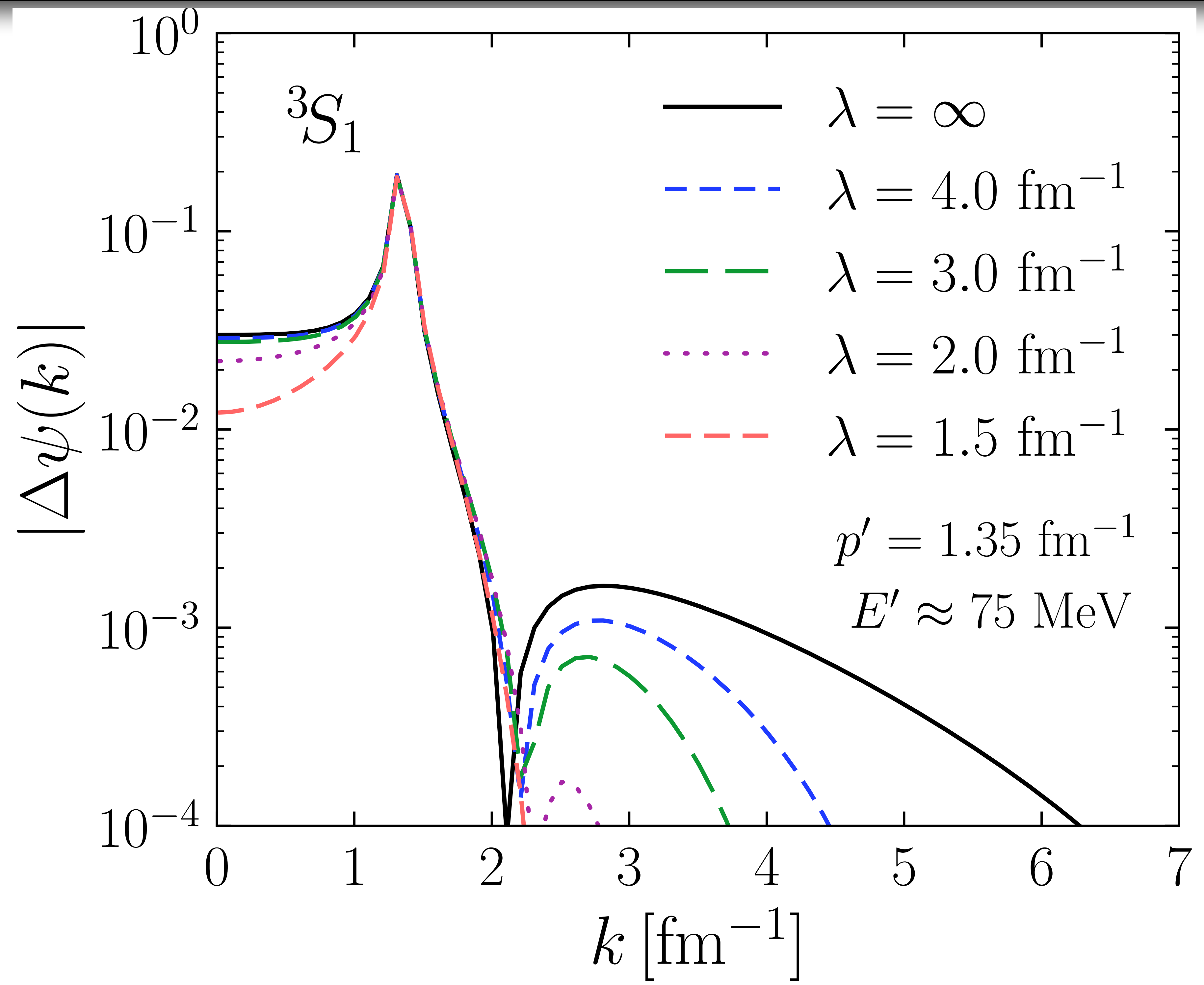


$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

Final-state wave function evolution

NSCL FRIB



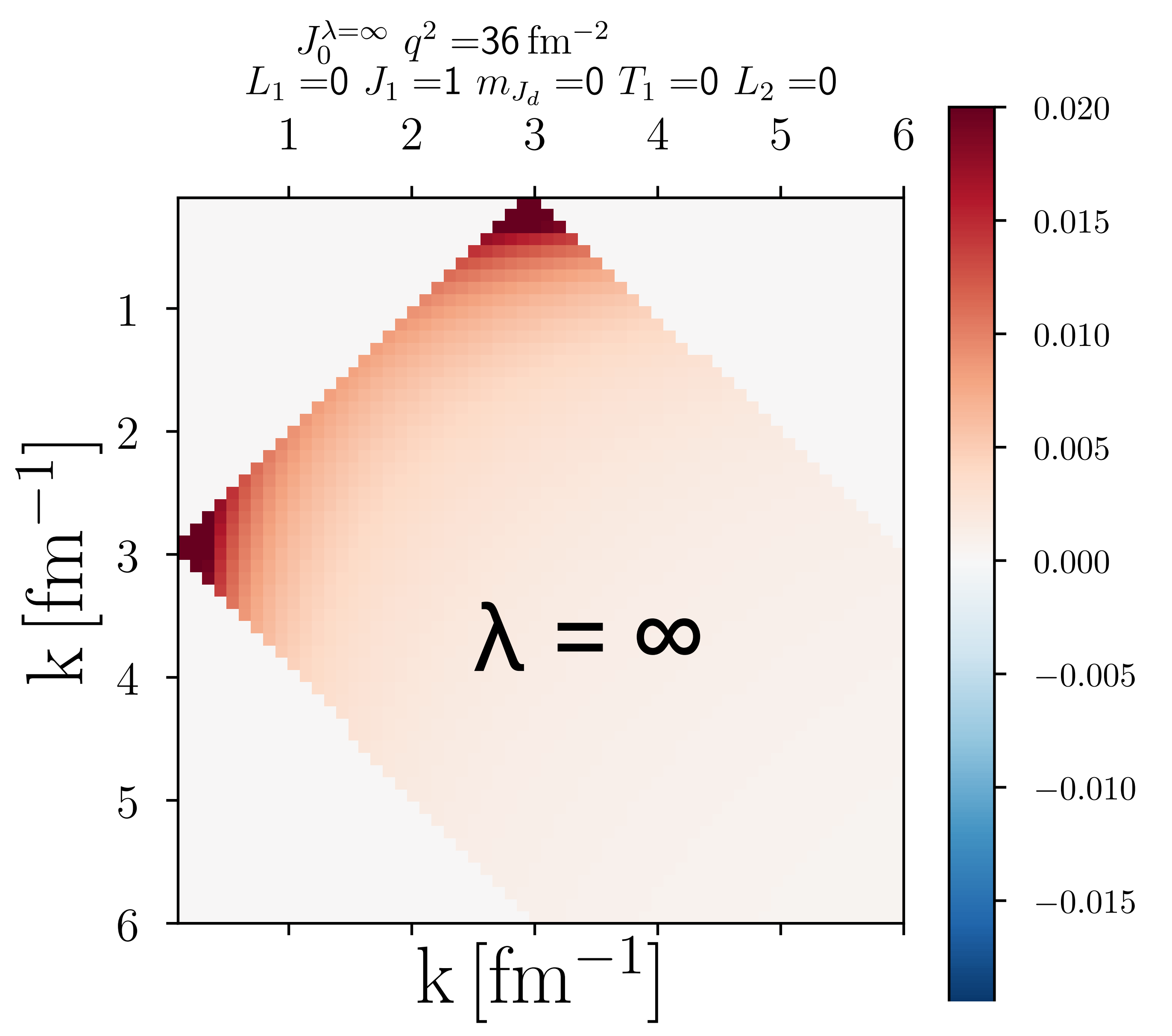


$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

- High-k tail suppressed with evolution
- For $p' \gtrsim \lambda,\ \Delta\psi_f^\lambda(p';k)$ localized around outgoing p′

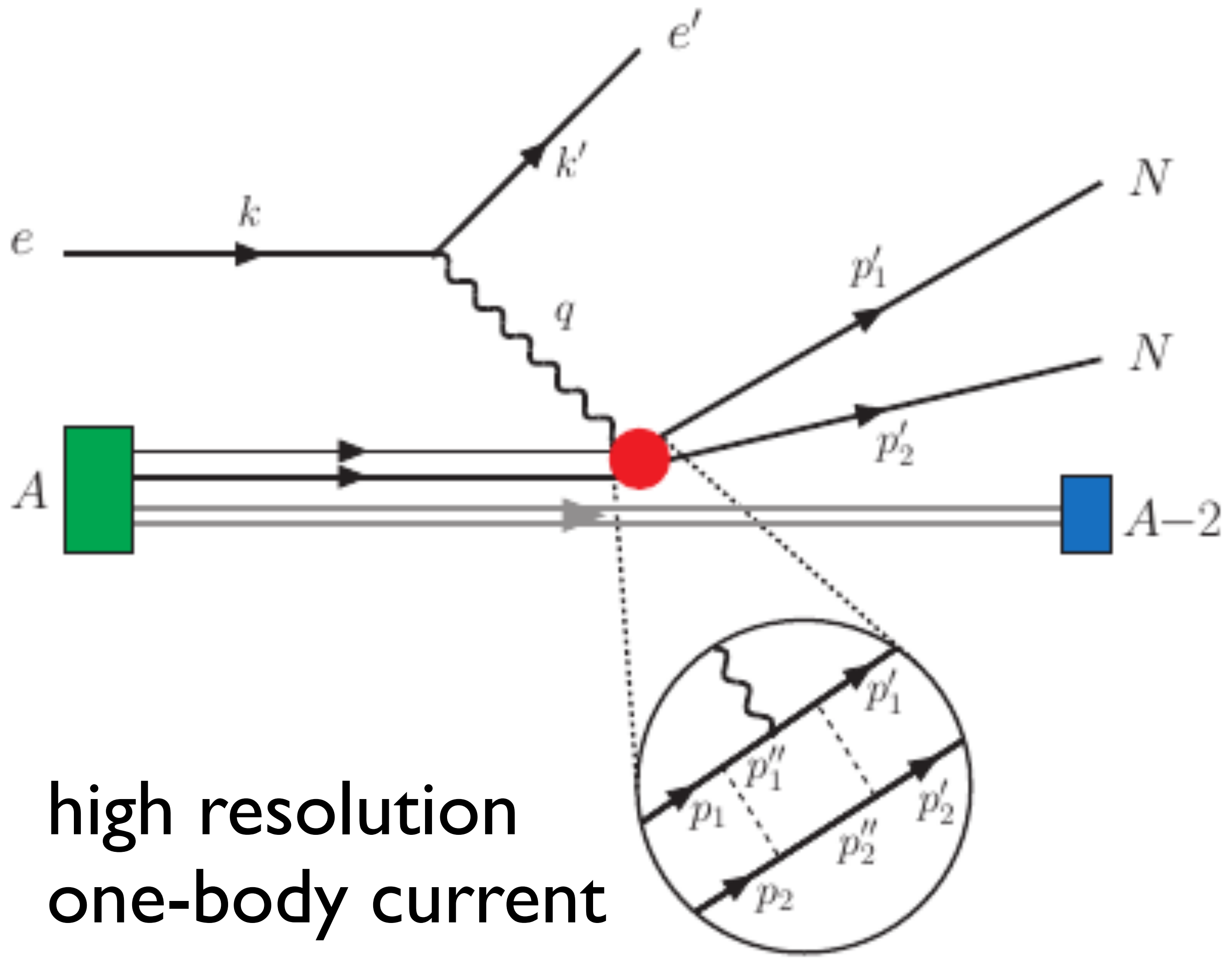
"local decoupling" Dainton et al. PRC 89 (2014)

Current operator evolution



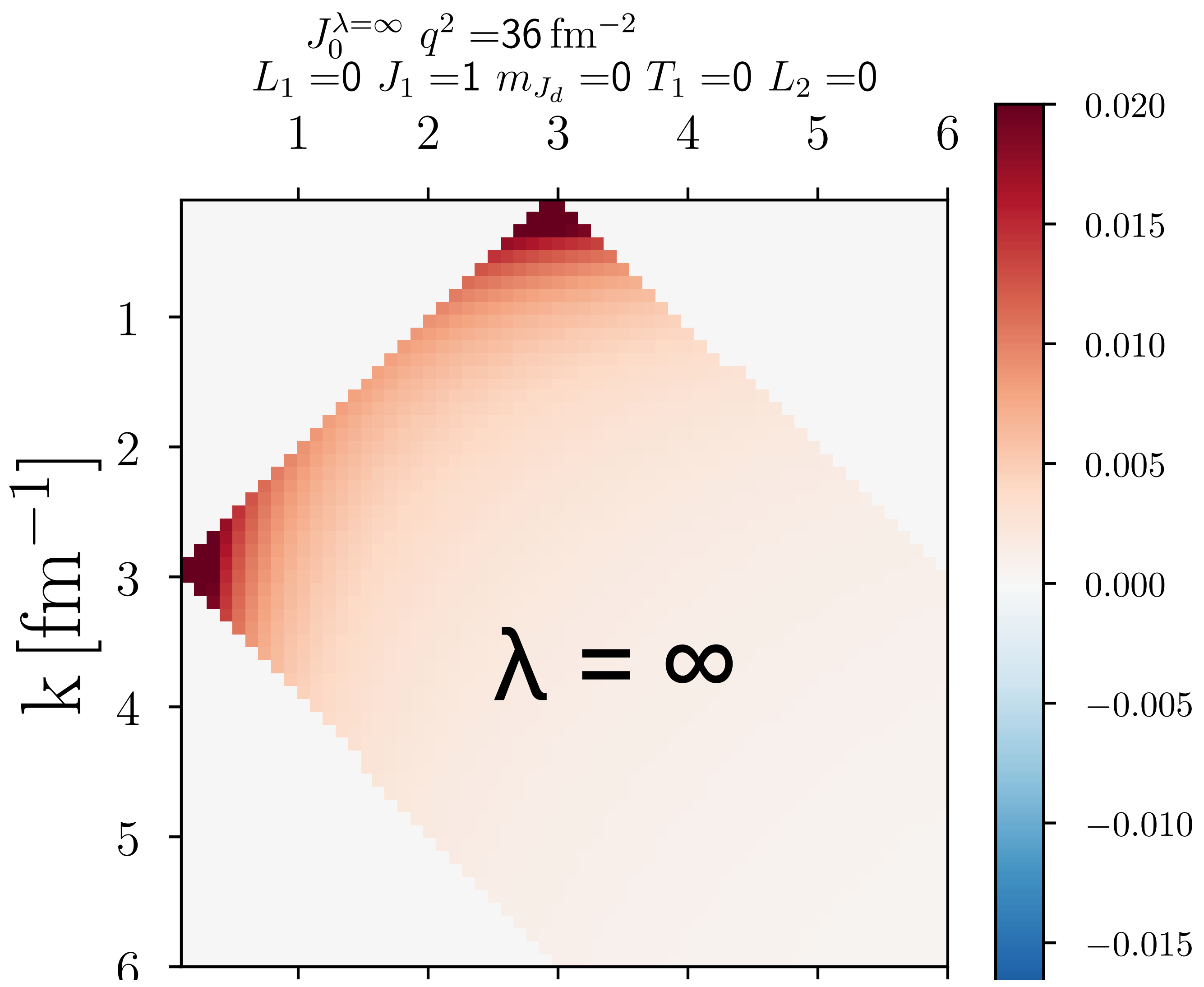
$^3S_1$ channel

$q^2 = 36\ fm^{-2}$



high resolution
one-body current

Current operator evolution



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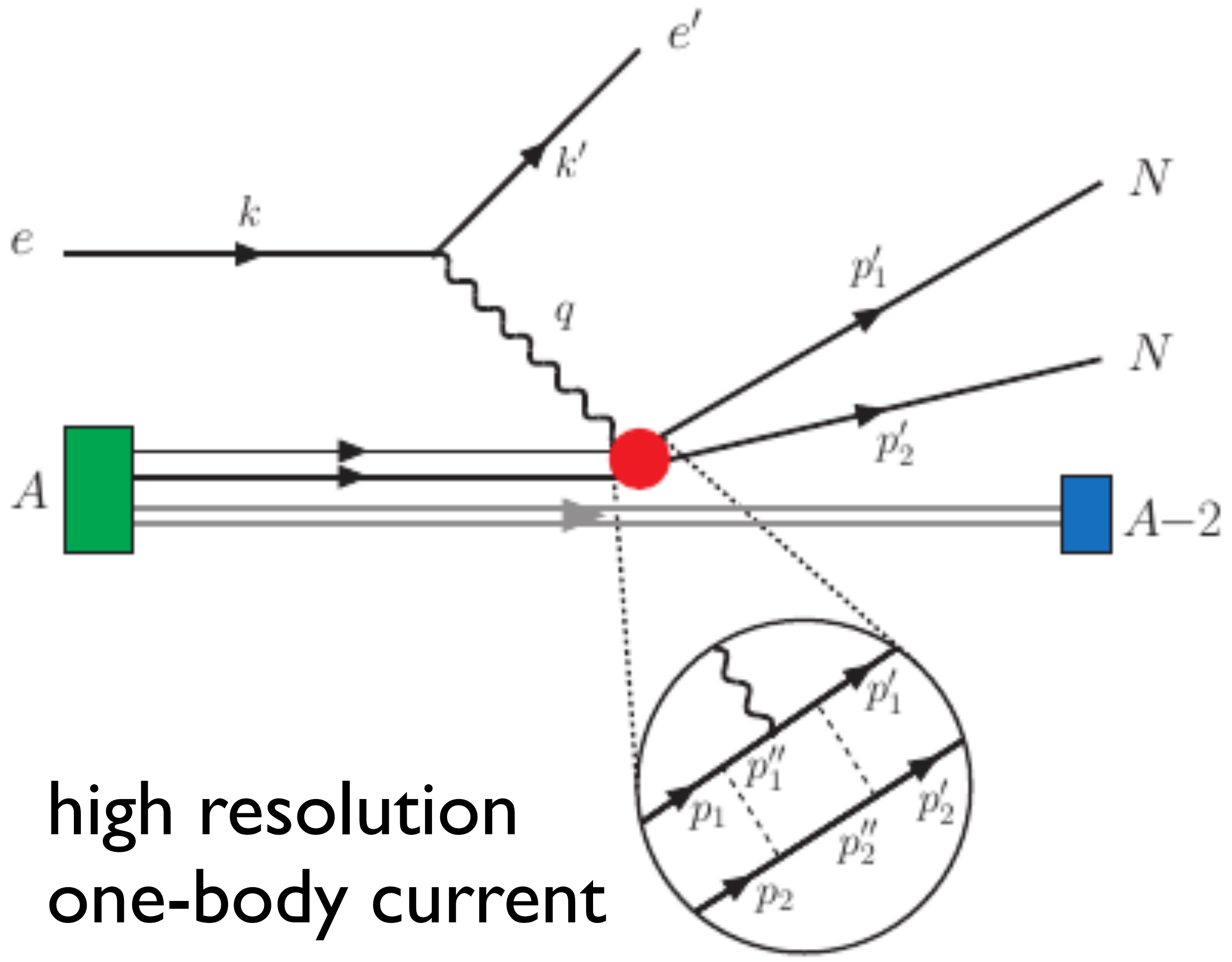
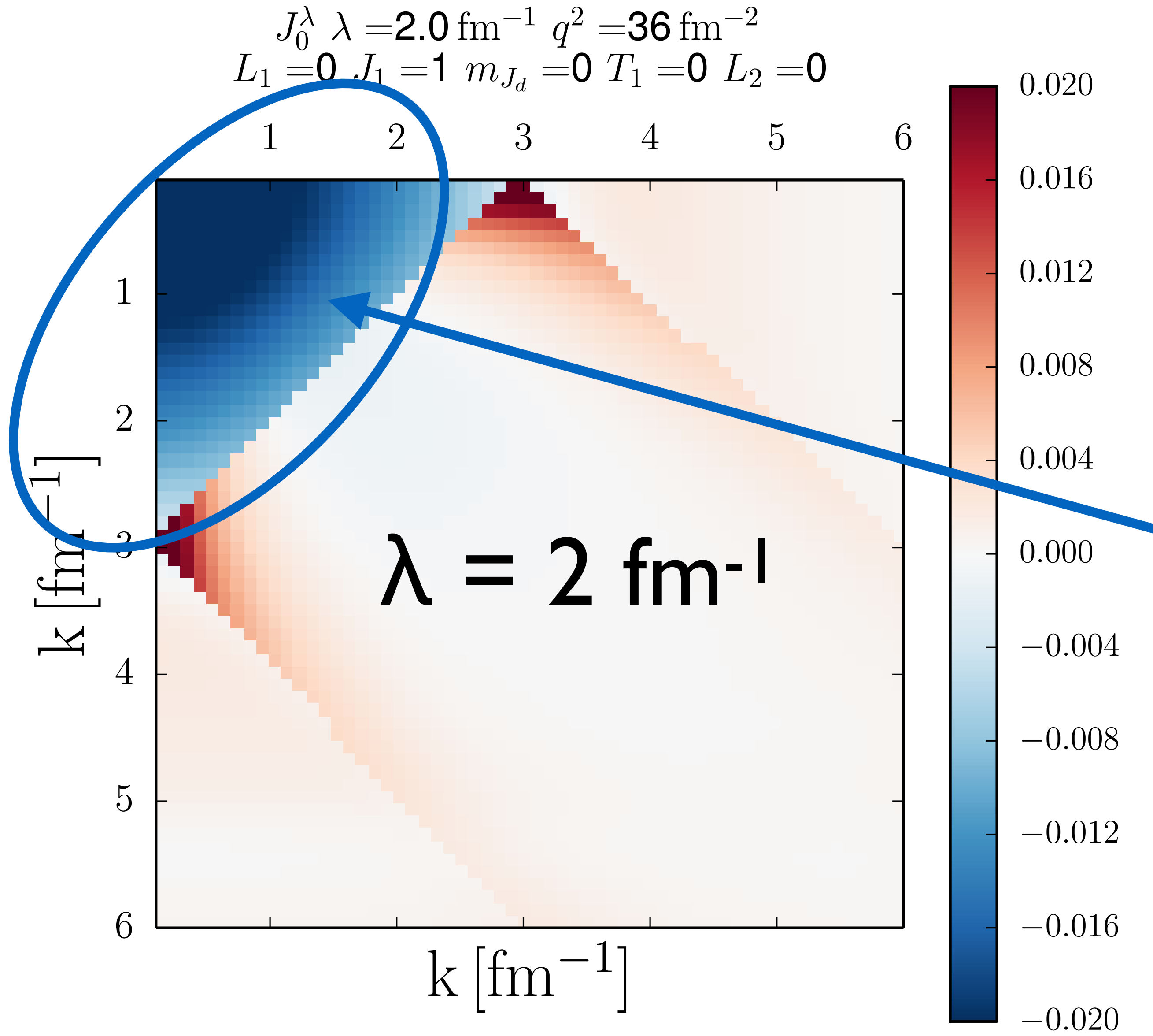
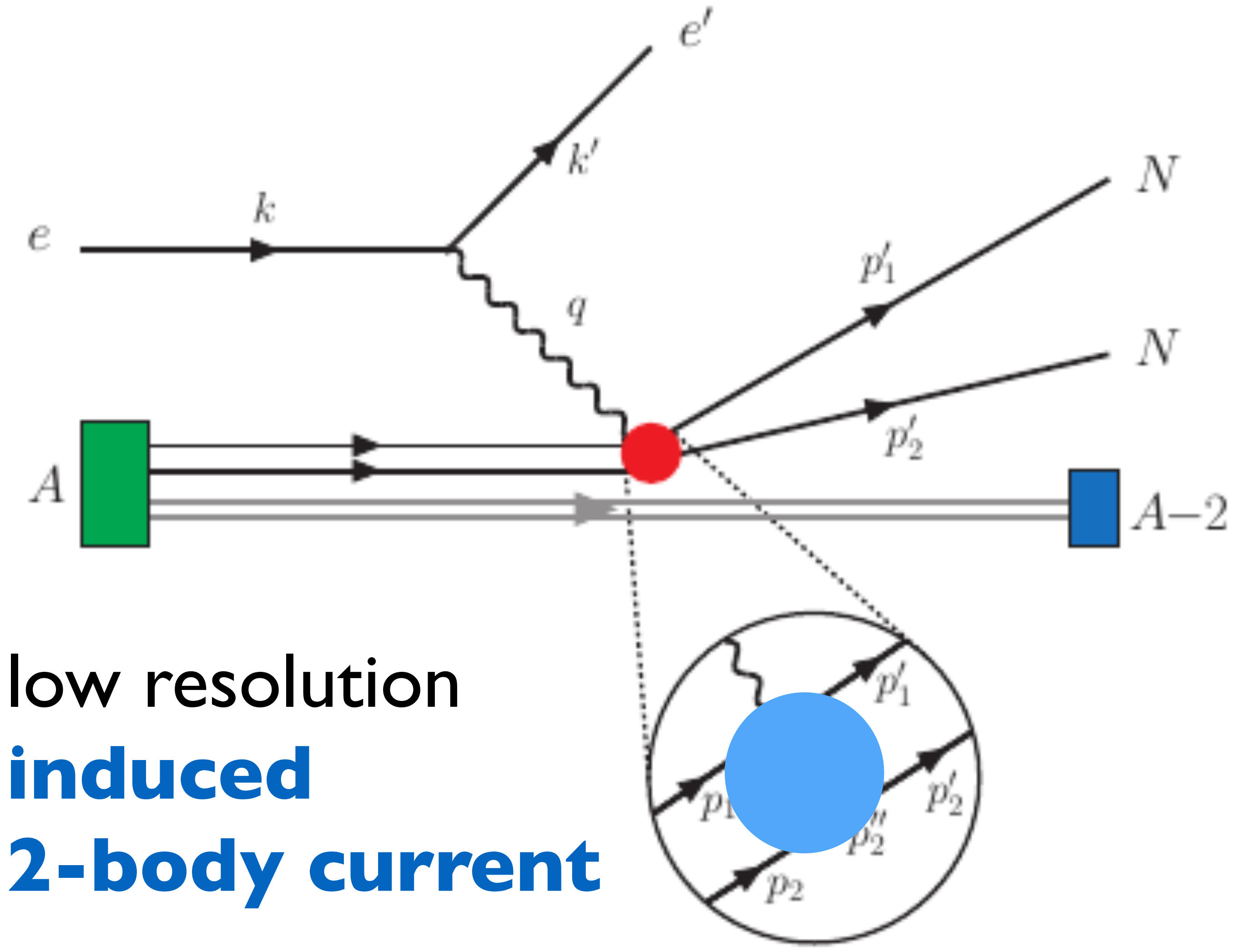
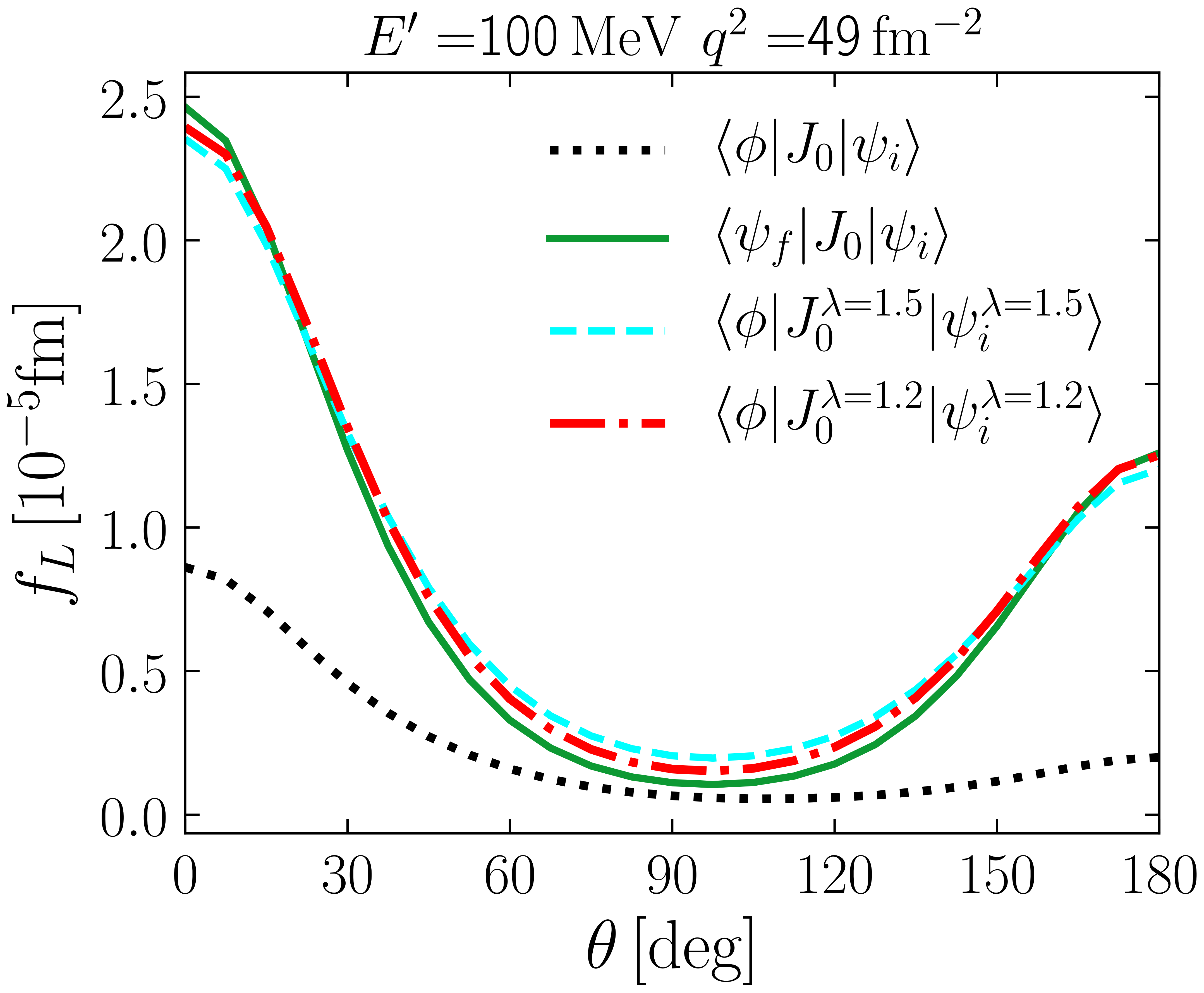






Scale Dependence of Final State Interactions

Look at kinematics relevant to SRC studies



x_B=1.64, Q^2= 1.78 GeV^2

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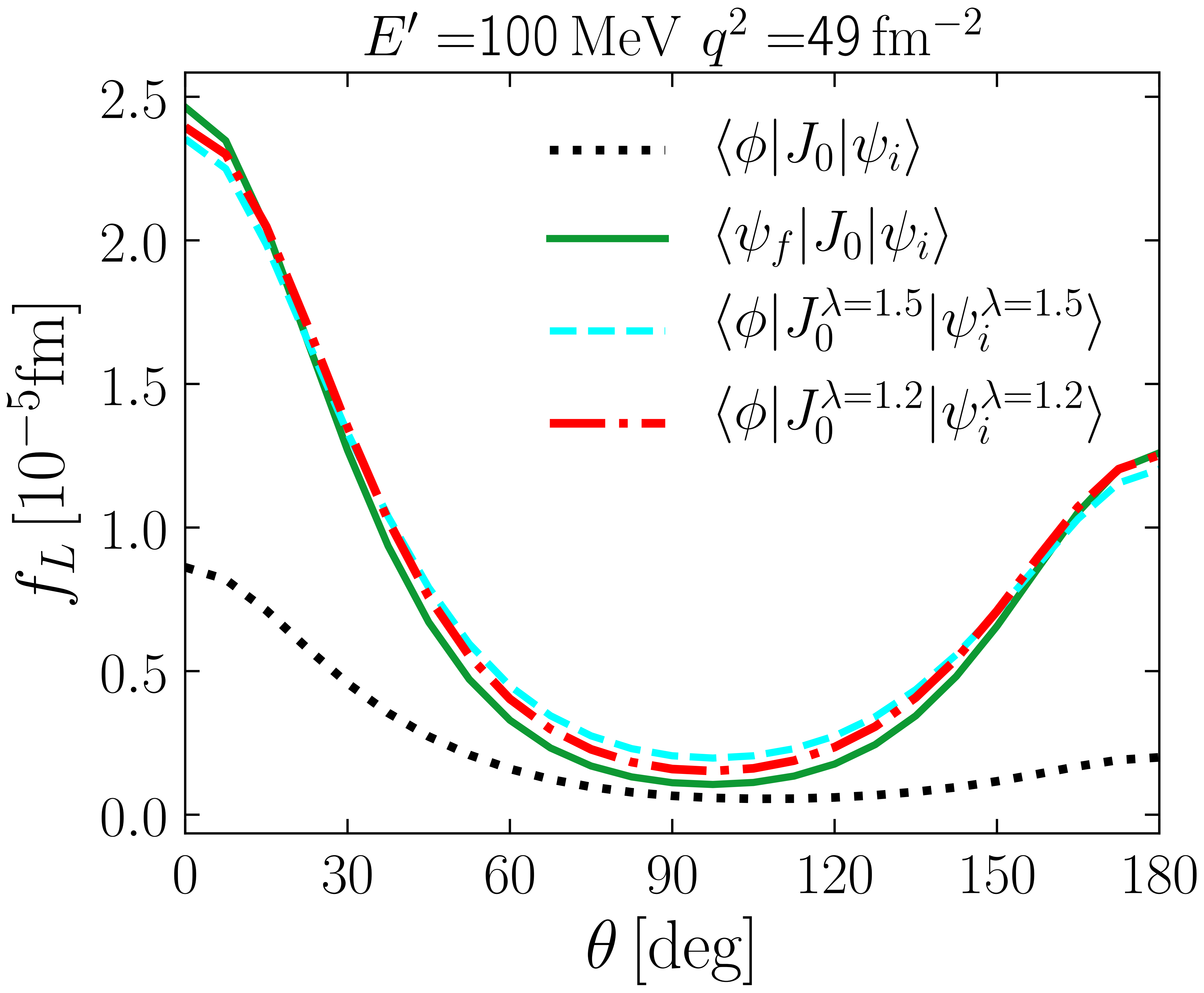


x_B=1.64, Q^2= 1.78 GeV^2

FSI sizable at large λ
but negligible at low-resolution!

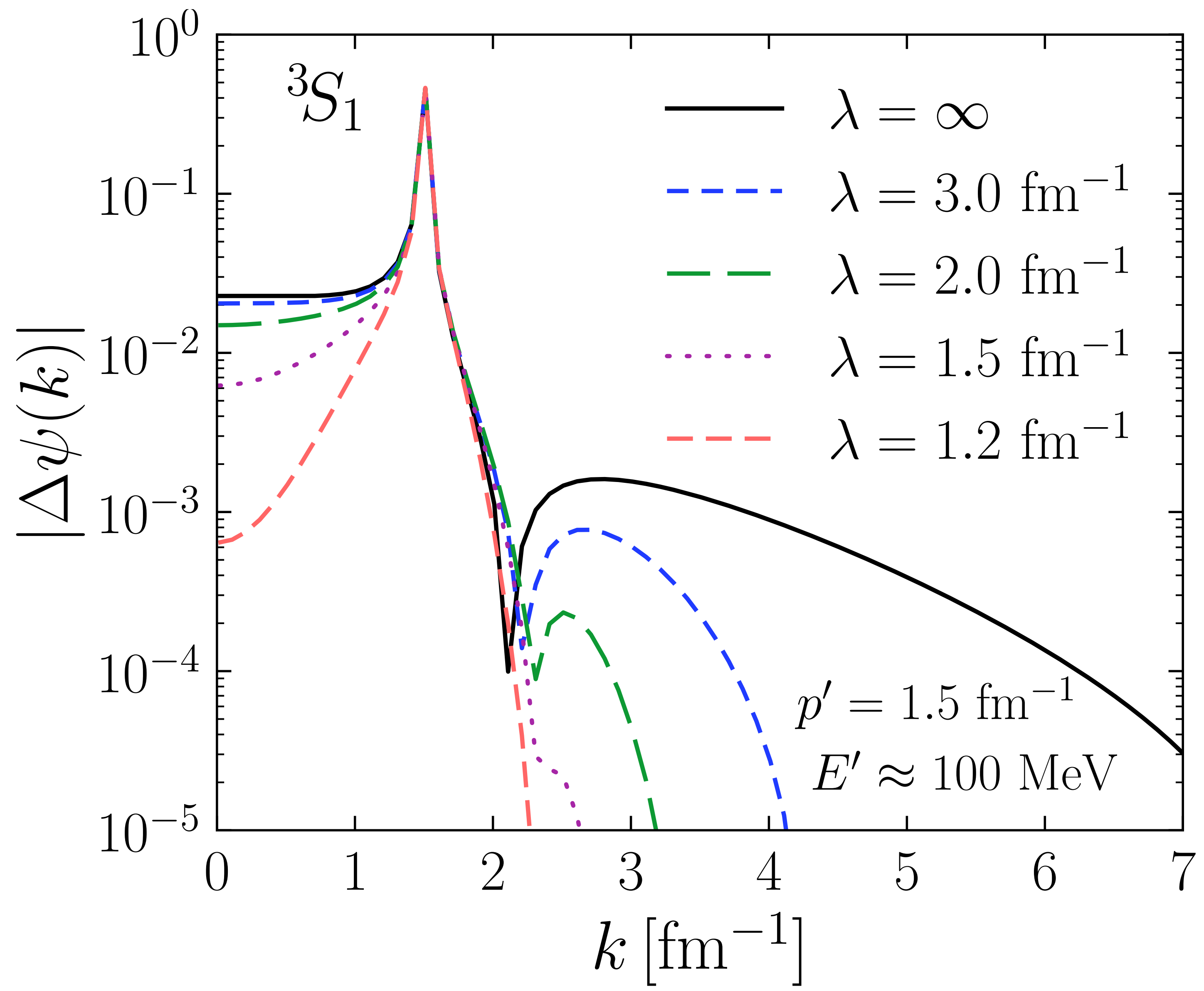
Folklore:

shouldn't hard processes
be complicated in low resolution
(λ << q) pictures?

Why are FSI so small at low λ in these kinematics ?

Scale Dependence of Final State Interactions

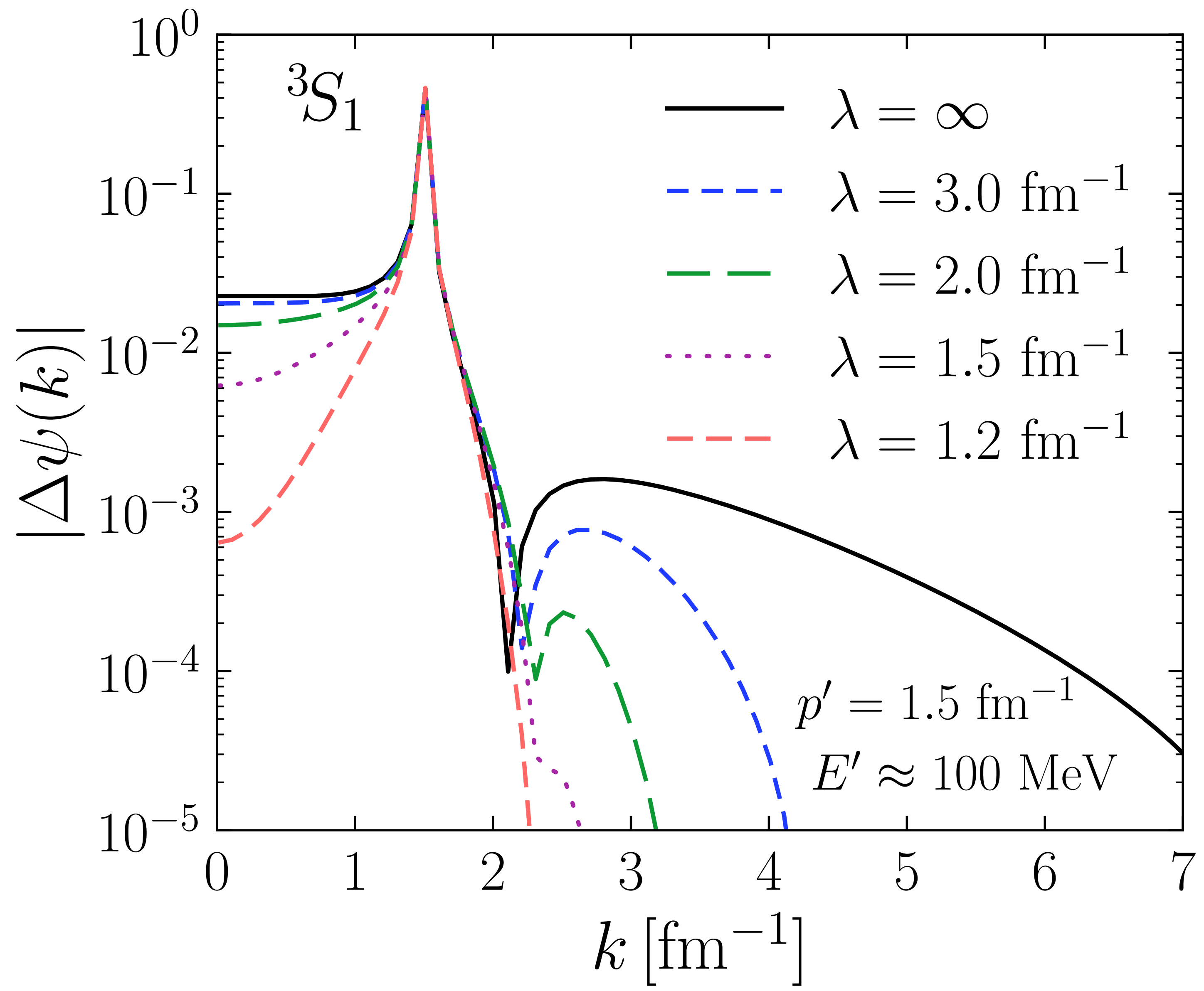
final state wf (FSI piece)



For $p' \gtrsim \lambda$, interacting part of final state wf localized at $k \approx p'$

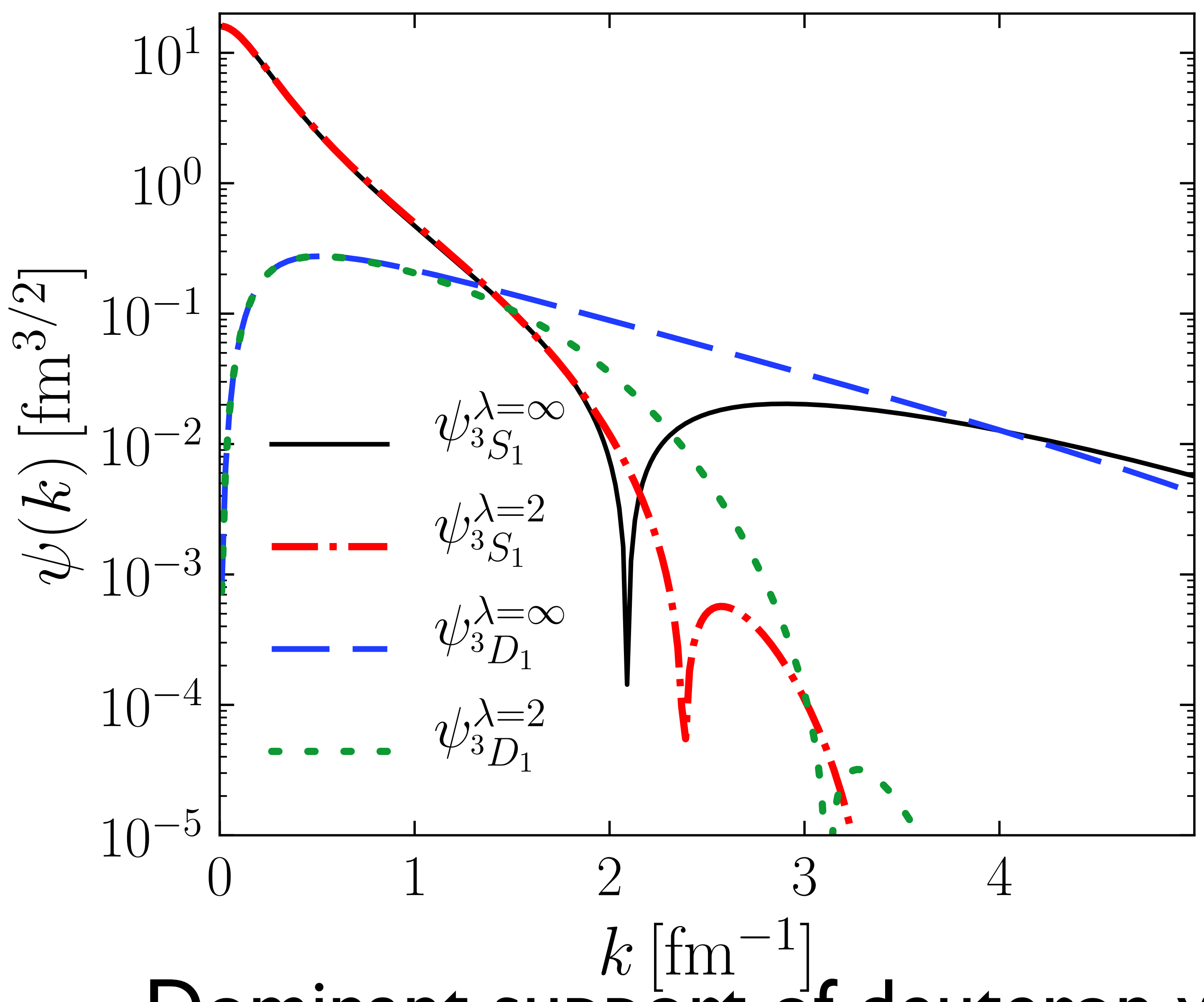
Scale Dependence of Final State Interactions

final state wf (FSI piece)



For p′ ≳ λ, interacting part of final state wf localized at k ≈ p′

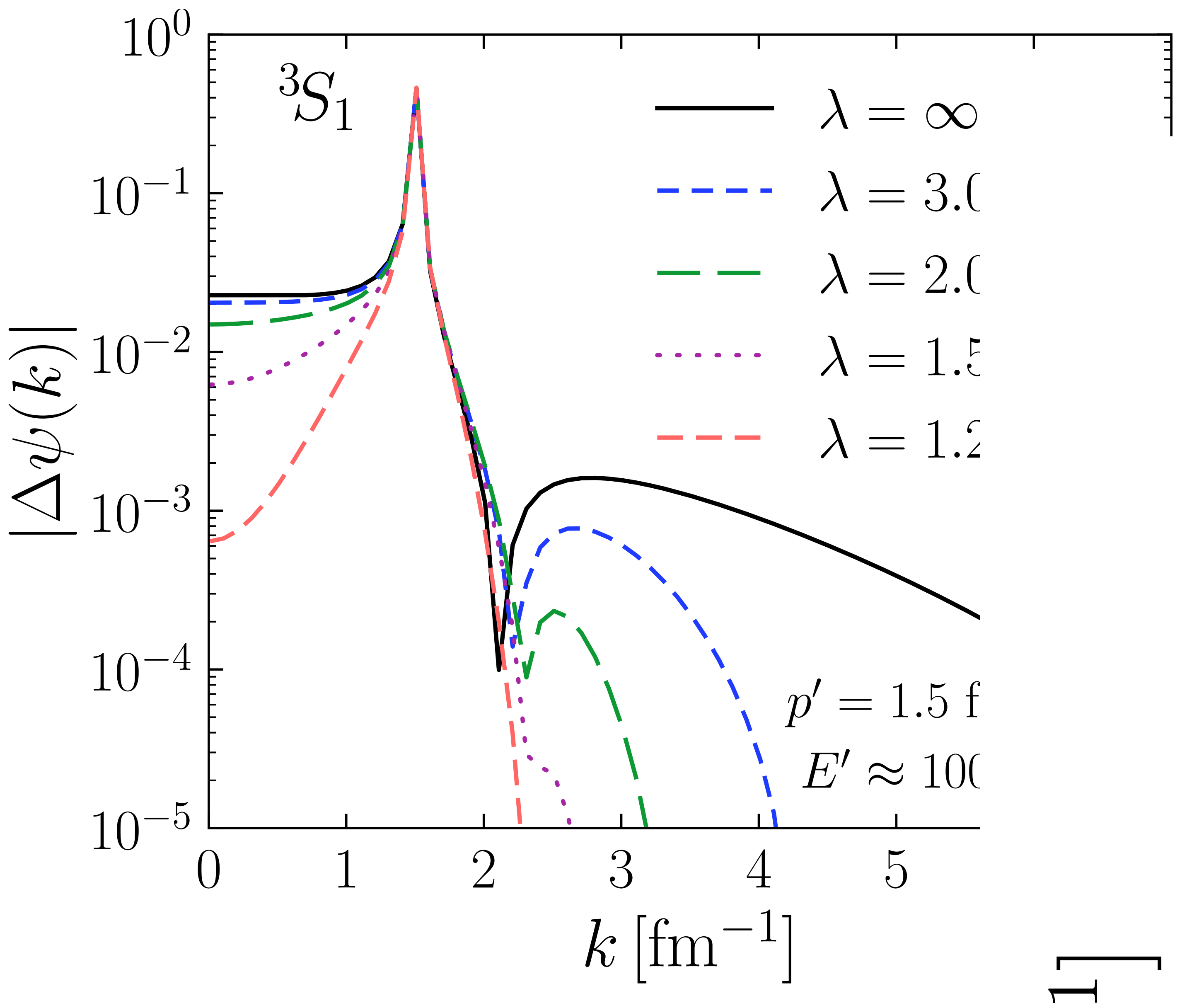
initial state (deuteron) wf

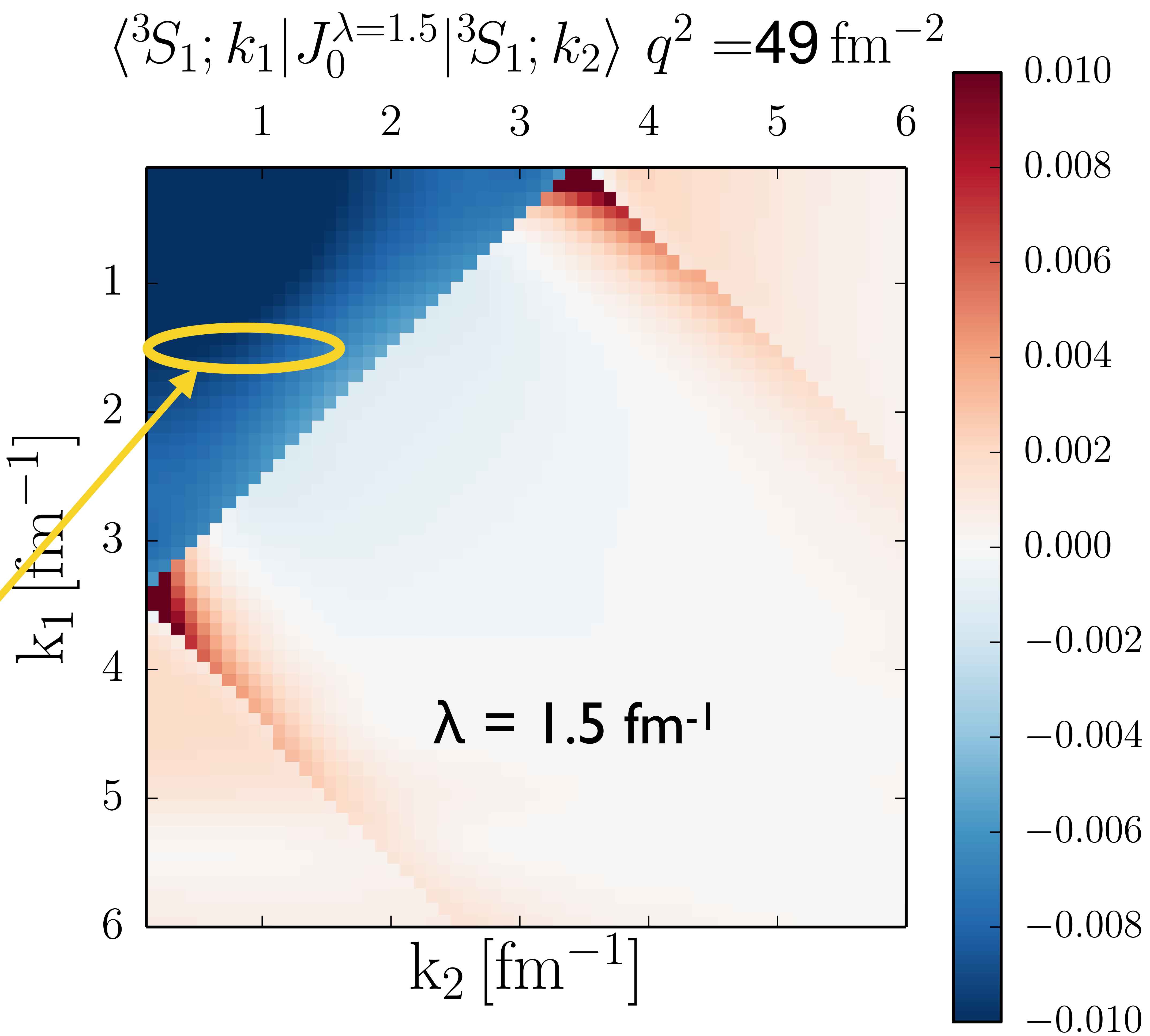


Dominant support of deuteron wf at k ≲ λ

Scale Dependence of Final State Interactions

final state wf (FSI piece)





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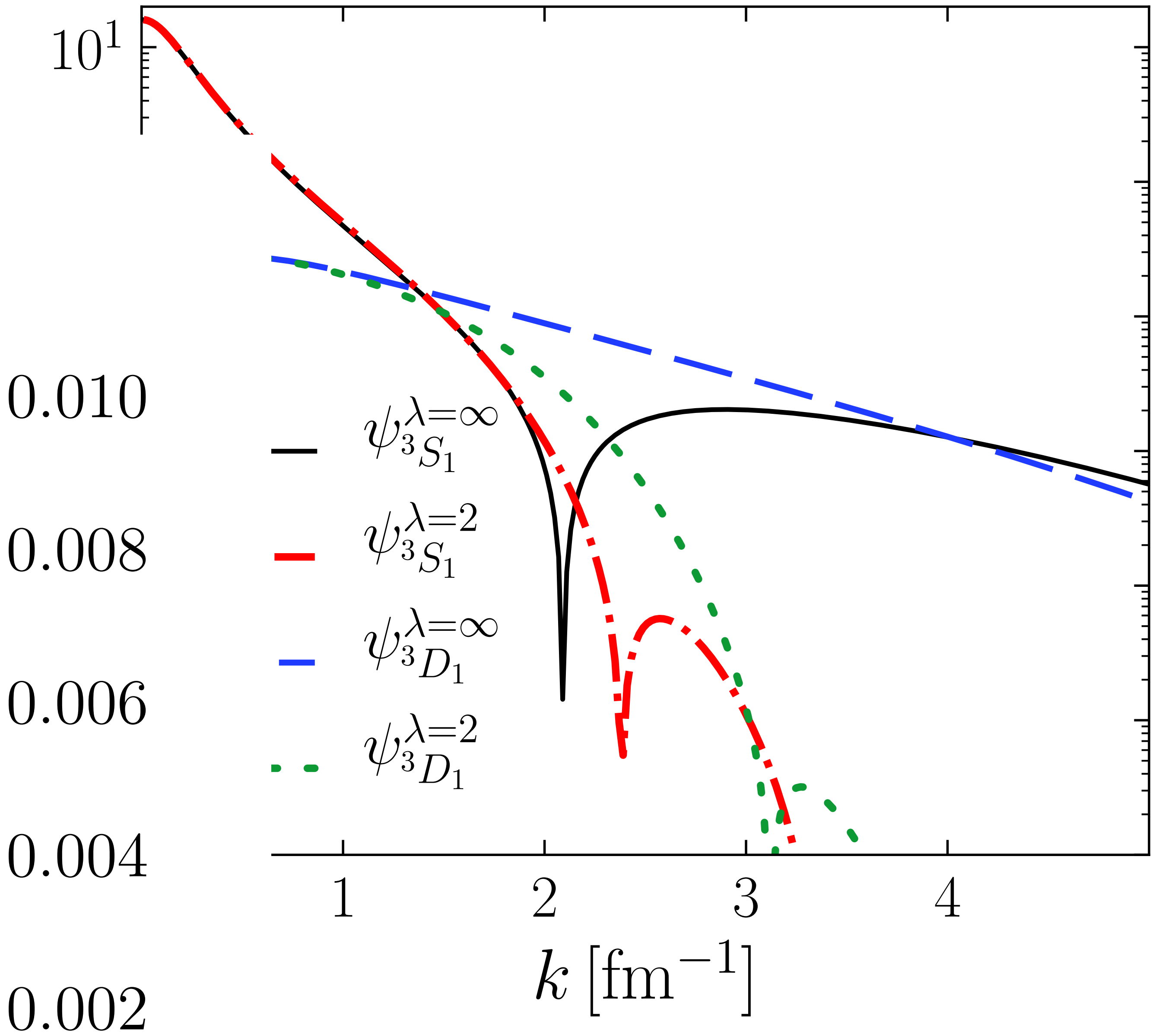


$J^{\lambda}_{\mathbf{q}}(k', k)$

probed by transition

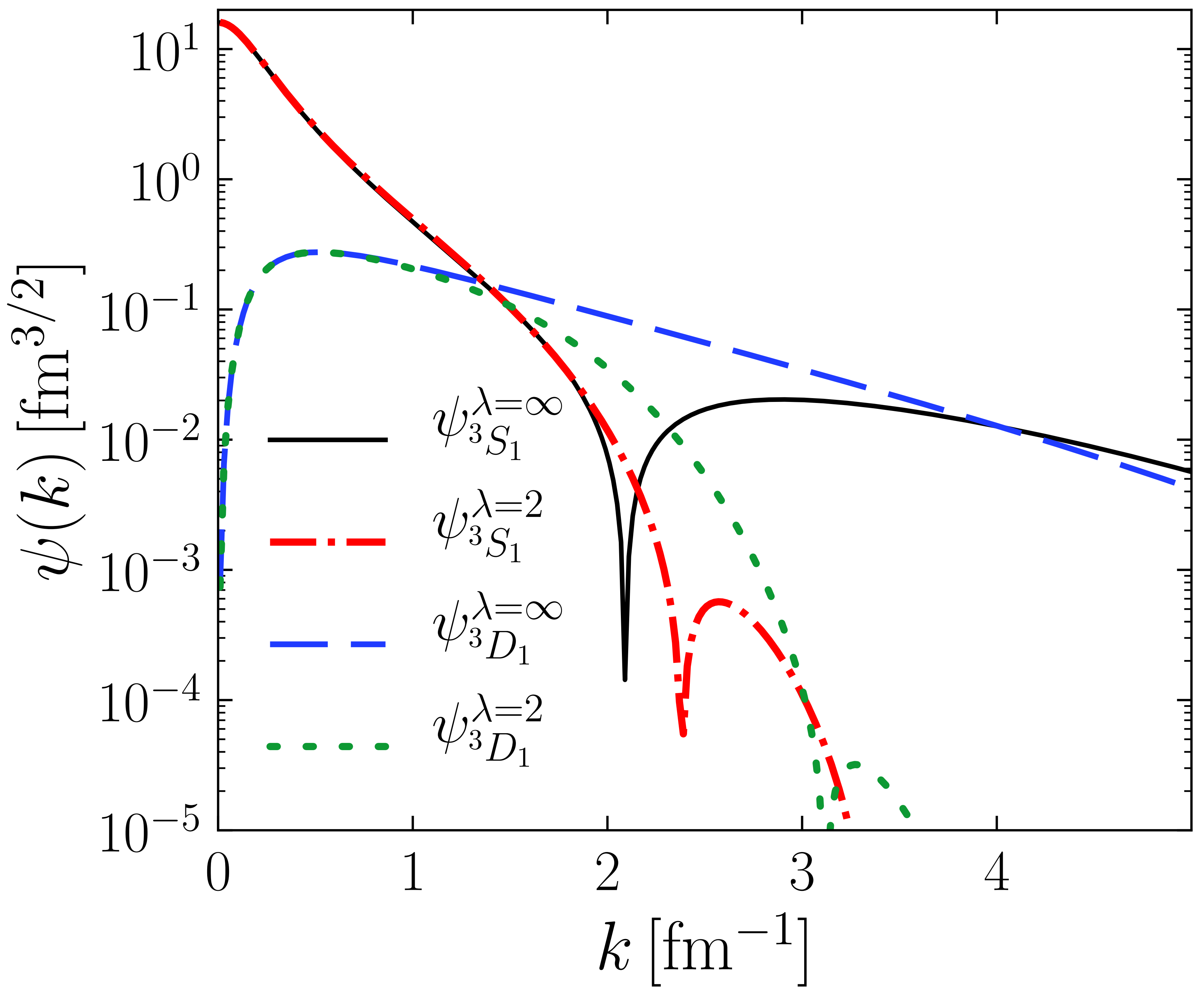
(smooth and non-singular)

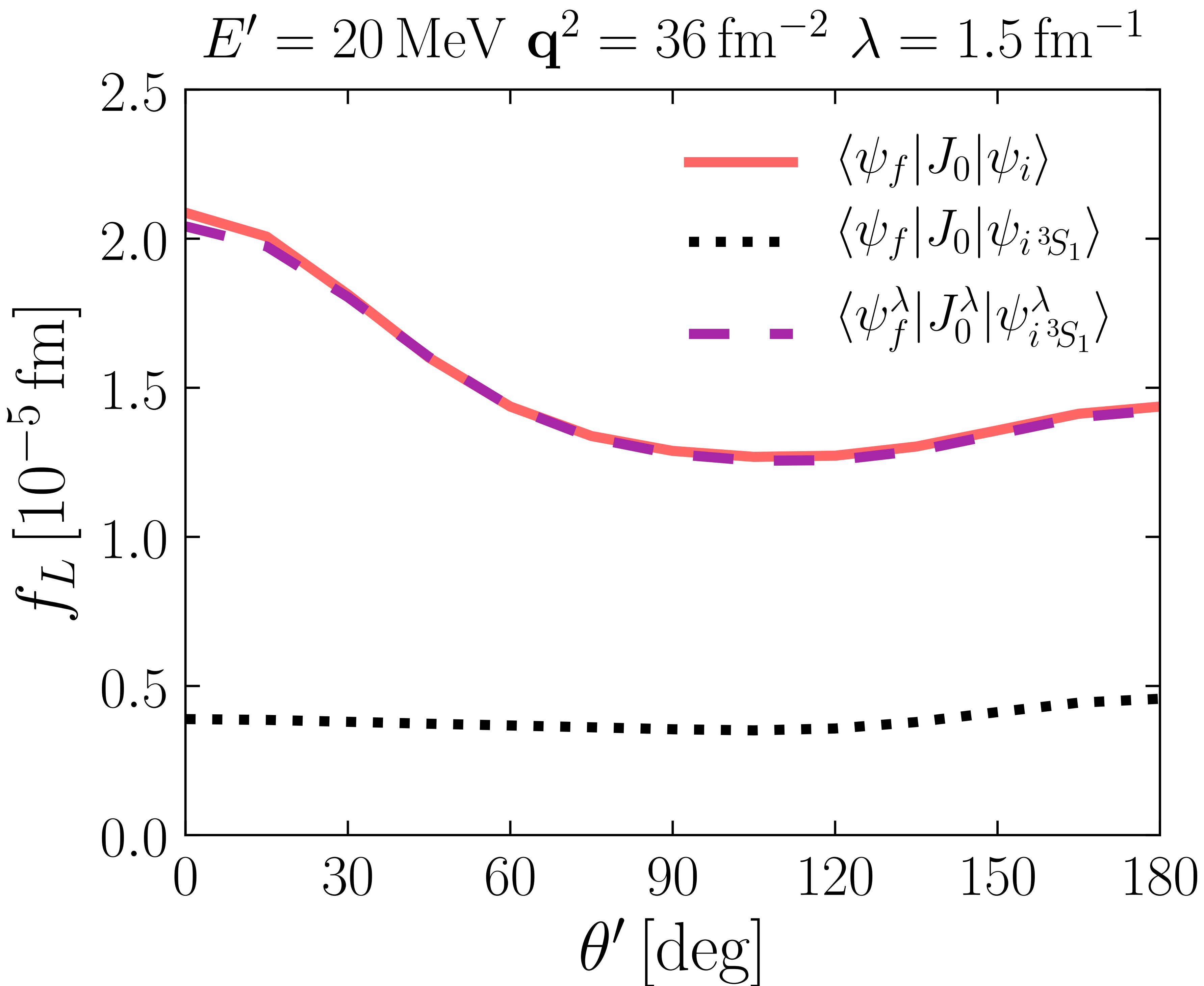
∴ FSI ~ T(p’,p’)

(small!)

Scale Dependence of Interpretations

- Analysis/interpretation of a reaction involves understanding which part of wave functions probed (highly scale dependent!)
- E.g., sensitivity to D-state w.f. in large $\mathbf{q}^2$ processes





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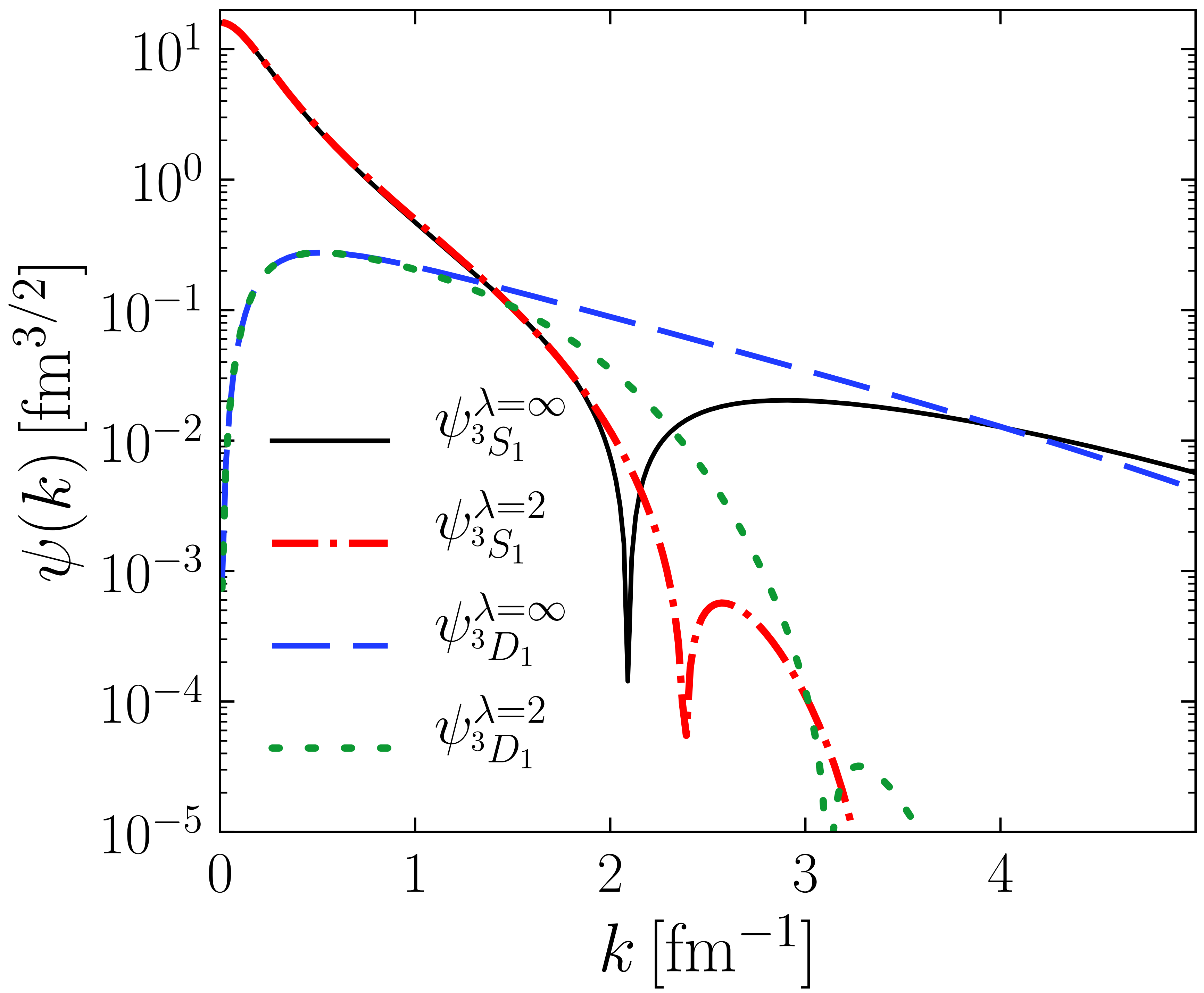
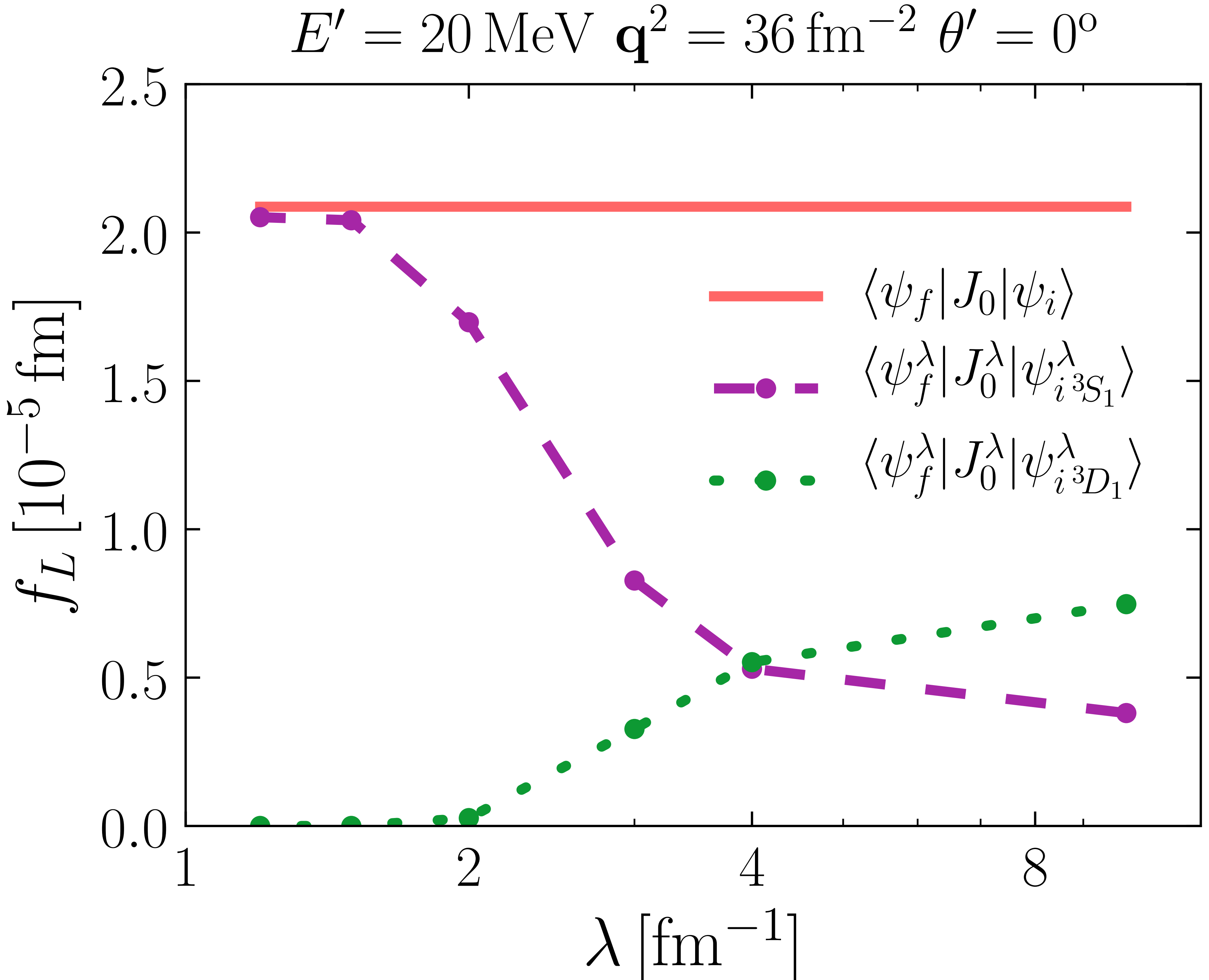
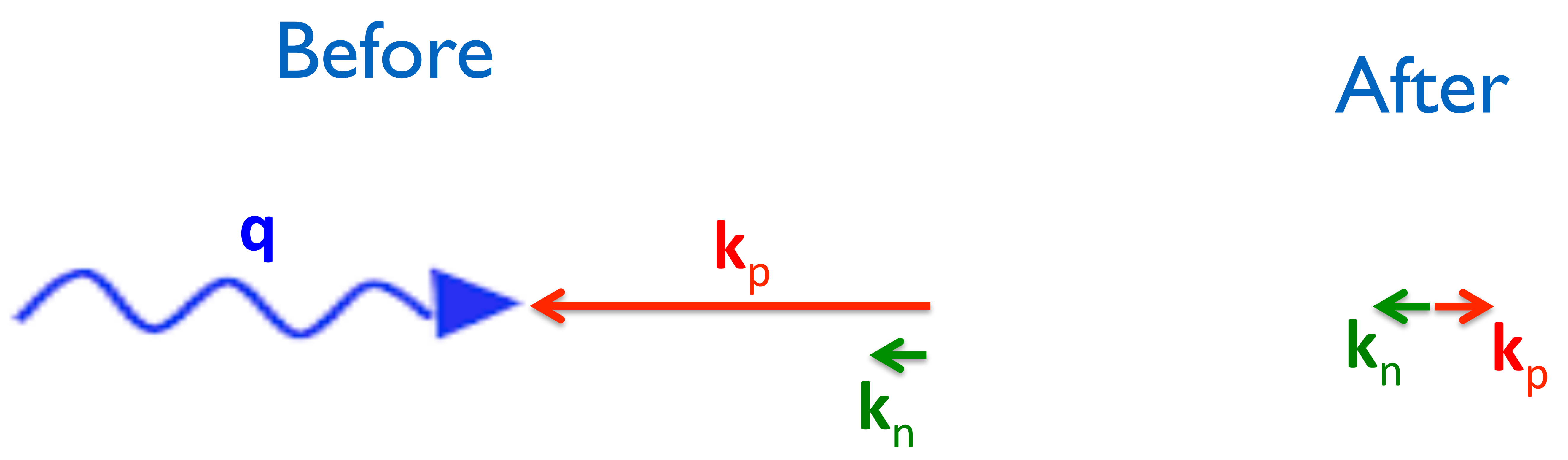




Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p′) for θ=0 in **high-resolution** picture (COM frame of outgoing np)



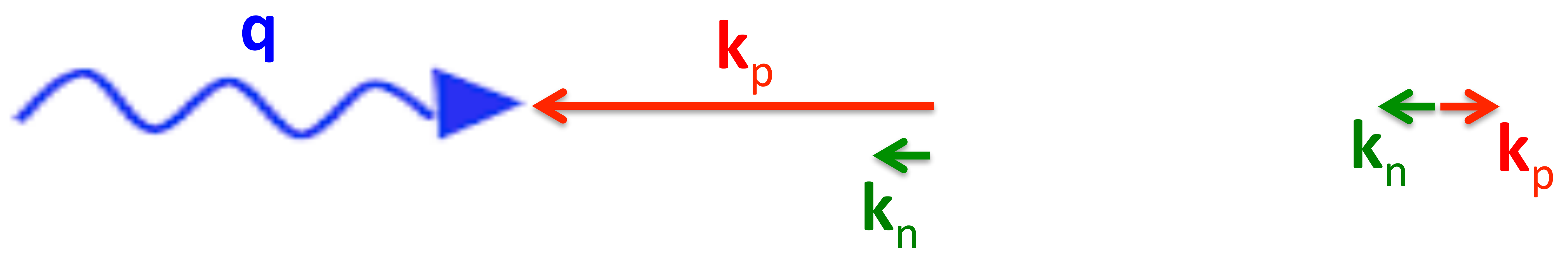
photon only couples to proton

Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p') for $\theta=0$ in **high-resolution** picture (COM frame of outgoing np)

Before

After



photon only couples to proton

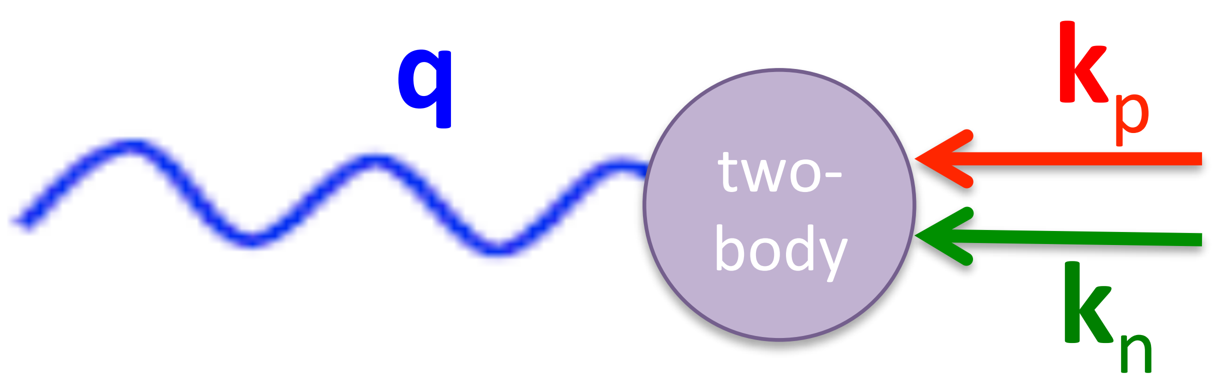
$\therefore$ proton has large momentum => initial large relative momentum

(i.e., SRC pair)

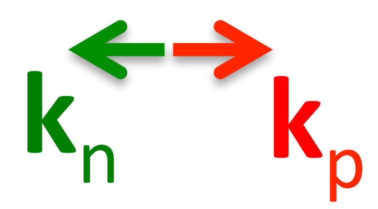
Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p') for $\theta=0$ in **low-resolution** picture (COM frame of outgoing np)

Before

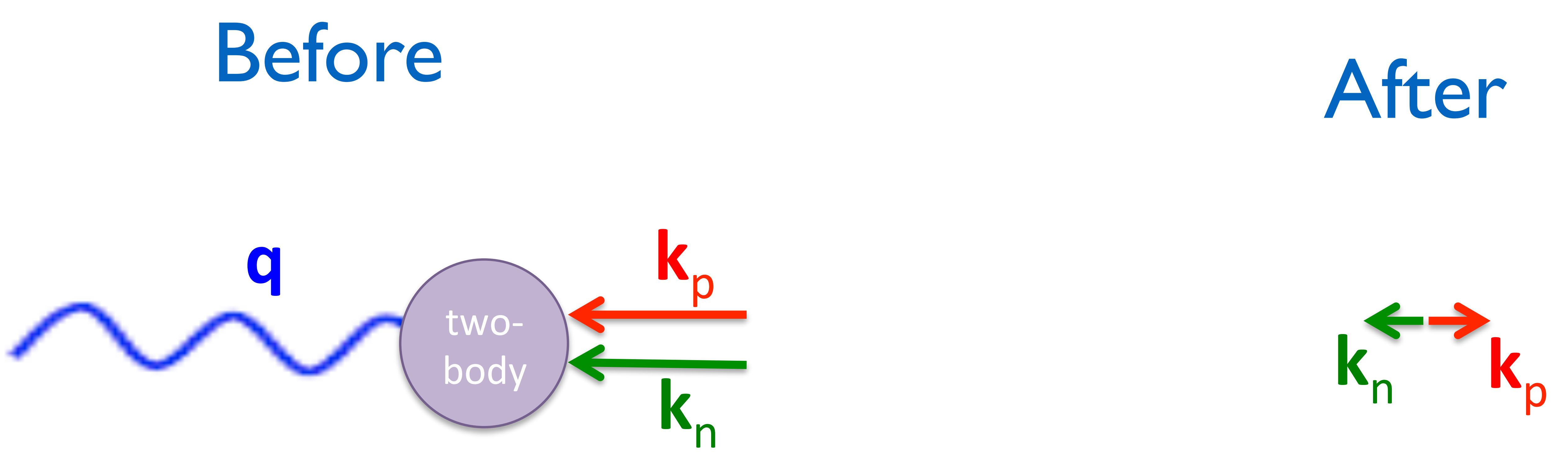


After



Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p') for $\theta=0$ in **low-resolution** picture (COM frame of outgoing np)



no large relative momentum in evolved deuteron wf

1-body current makes no contribution

∴ 2-body current mostly stops the low-relative momentum np pair