Many-body factorization and position-momentum equivalence of nuclear short-range correlations

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- ✓ L. B. Weinstein @ ODU
- ✓ R. B. Wiringa @ ANL
- ✓ O. Hen @ MIT
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SciDAC Scientific Discovery through Advanced Computing

NUCLEI Nuclear Computational Low-Energy Initiative

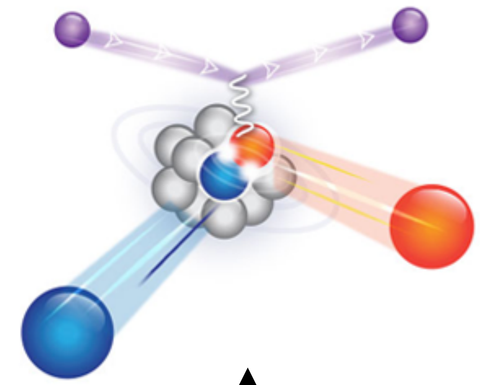
NERSC

LA-UR-21-22525 SRC EMC Workshop - March 24, 2021

Nuclear Short-Range Correlations

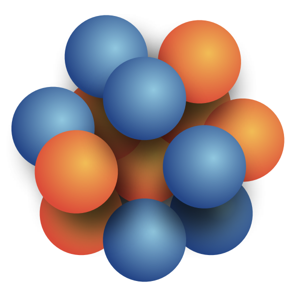
O. Hen *et al.*, Rev. Mod. Phys. **89**, 045002 (2017)

lepton scattering from nuclei



internal structure of nucleons bound in nuclei

nuclear charge radii



G. A. Miller, *et al.*, Phys. Lett. B **793**, 360-364 (2019)

SRCs

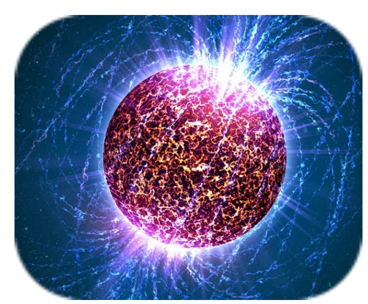
interpretation of neutrino oscillation measurements

H. Gallagher *et al.*, Annu. Rev. Nucl. Part. Sci. **61**, 355-378 (2011)

neutrinoless double beta decay matrix elements

F. Simkovic *et al.*, Phys. Rev. C **79**, 055501 (2009)
V. Cirigliano *et al.*, Phys. Rev. Lett. **120**, 202001 (2018)

symmetry energy & neutron star properties



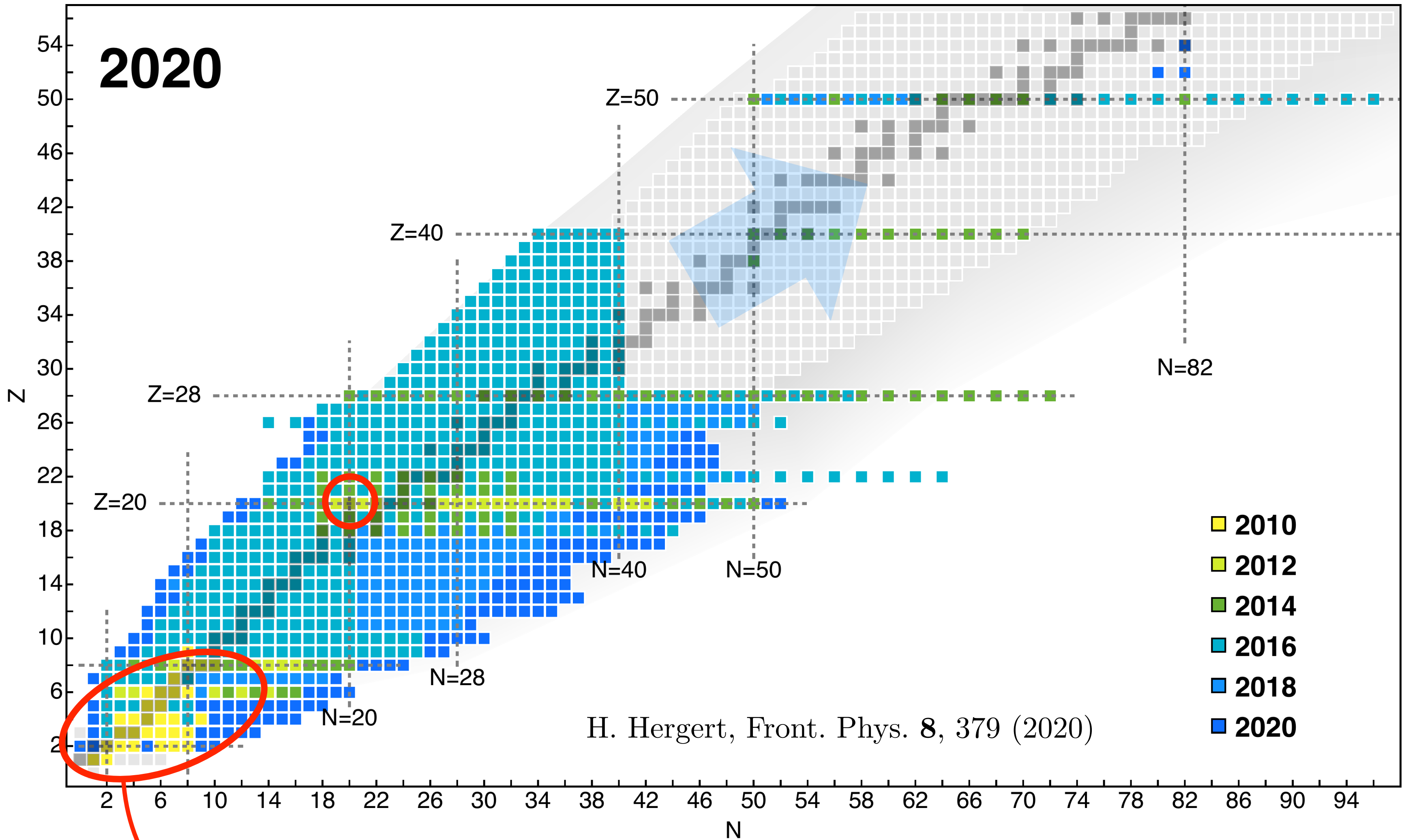
B. A. Li *et al.*, Prog. Part. Nucl. Phys. **99**, 29-119 (2018)

understanding SRC formation mechanisms and specific characteristics is required for obtaining a complete description of nuclei and nuclear matter

Nuclear Short-Range Correlations

Question: how can we *quantitatively* study SRCs and compare to observations?

Need: reliable ab-initio many-body method capable of handling SRCs



several options:

- IMSRG
- CC
- SCGF
- MBPT
- ...

notes:

- "soft" Hamiltonians
- consistent evolution of operators other than H

Quantum Monte Carlo ⟶ $A \lesssim 24$ (& 40-ish)

Nuclear Short-Range Correlations

Question: how can we *quantitatively* study SRCs and compare to observations?

Need: reliable ab-initio many-body method capable of handling SRCs + effective theory to connect ab-initio results to observables

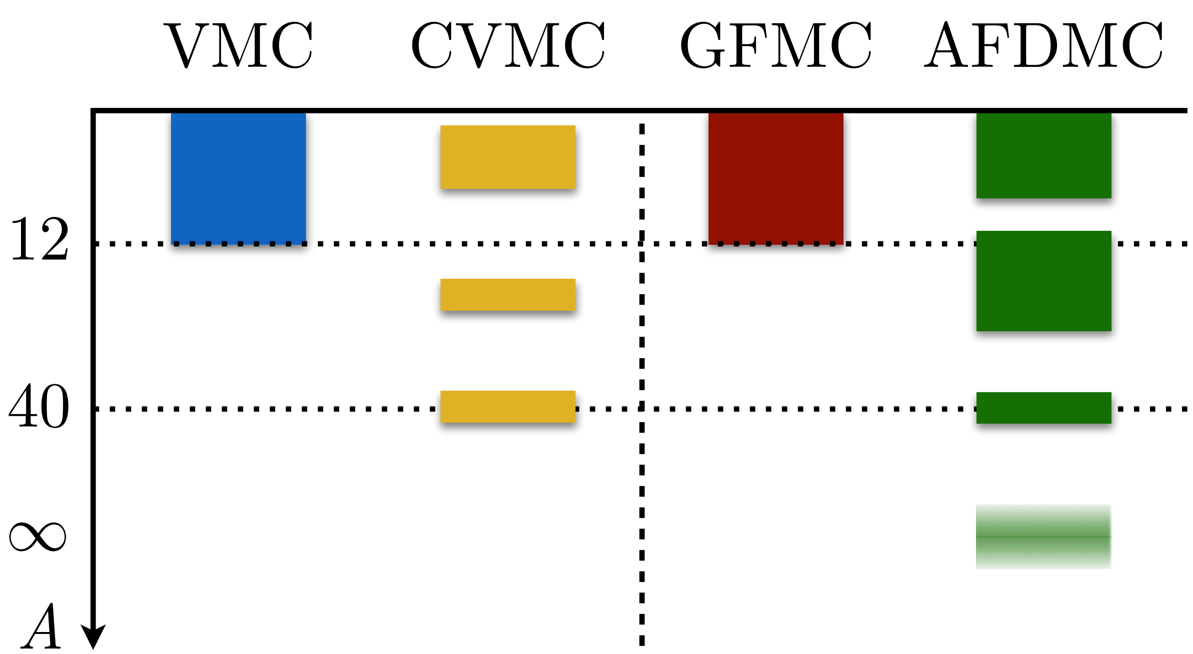
reliable ab-initio many-body method capable of handling SRCs → Quantum Monte Carlo (QMC) →

- limited to relatively light nuclei
- difficult to describe relevant reactions

effective theory to connect ab-initio results to observables → short-range expansion, high-momentum expansion → Generalized Contact Formalism (GCF)

QMC + GCF ⟶ quantitative study of SRCs

Quantum Monte Carlo



$$H = \sum_i T_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

V_{ij} : fit to NN scattering & d

V_{ijk} : fit to nuclei observables

↳ phenomenological & χEFT

$|\Psi_T\rangle = \mathcal{F}\,|\Phi\rangle$

- $|\Phi\rangle$: mean-field component, quantum numbers, symmetry
- $\mathcal{F}$: correlations (2b & 3b) ⟶ induced by H

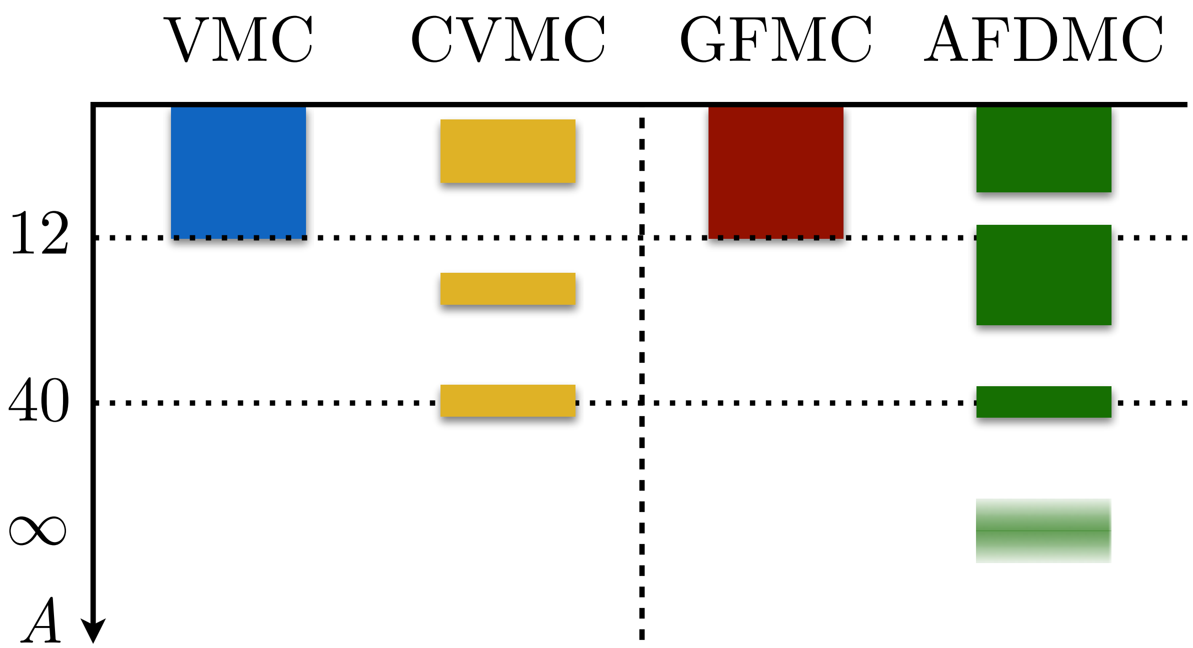
VMC: $\dfrac{\langle\Psi_T|H|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle} = E_T \geq E_0$ ⟶ $|\Psi_T\rangle$ optimization

DMC: $|\Psi_0\rangle \propto \lim\limits_{\tau\to\infty} e^{-(H-E_T)\tau}|\Psi_T\rangle$ ⟶ imaginary time propagation

Pros:
- non-perturbative
- controlled approx.
- correlated systems
- bare nuclear interactions
- uncertainty quantification
- scalability

QMC review: J. A. Carlson *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

Quantum Monte Carlo



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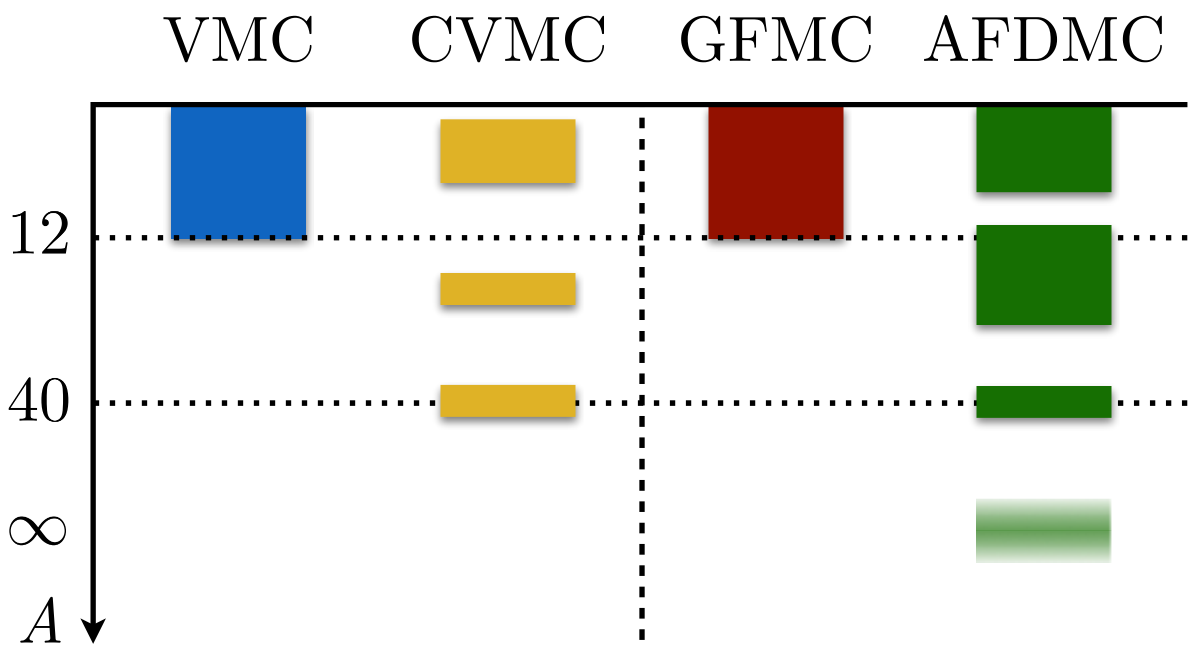
$$|\Psi_T\rangle = \mathcal{F}\,|\Phi\rangle \quad \begin{cases} |\Phi\rangle : & \text{mean-field component, quantum numbers, symmetry} \\ \mathcal{F} : & \text{correlations (2b \& 3b)} \longrightarrow \text{induced by } H \end{cases}$$

↓

- written in r-space: direct access to nucleon coordinates $(r_i,\, r,\, R)$
- spinor structure: direct access to nucleon spin/isospin projections
- transform to k-space: possible access to nucleon momenta $(k,\, q,\, Q)$
- access to the short-range and high-momentum components of the wave function

QMC review: J. A. Carlson *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

Quantum Monte Carlo



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- $|\Phi\rangle$: mean-field component, quantum numbers, symmetry
- $\mathcal{F}$: correlations (2b & 3b) ⟶ induced by H

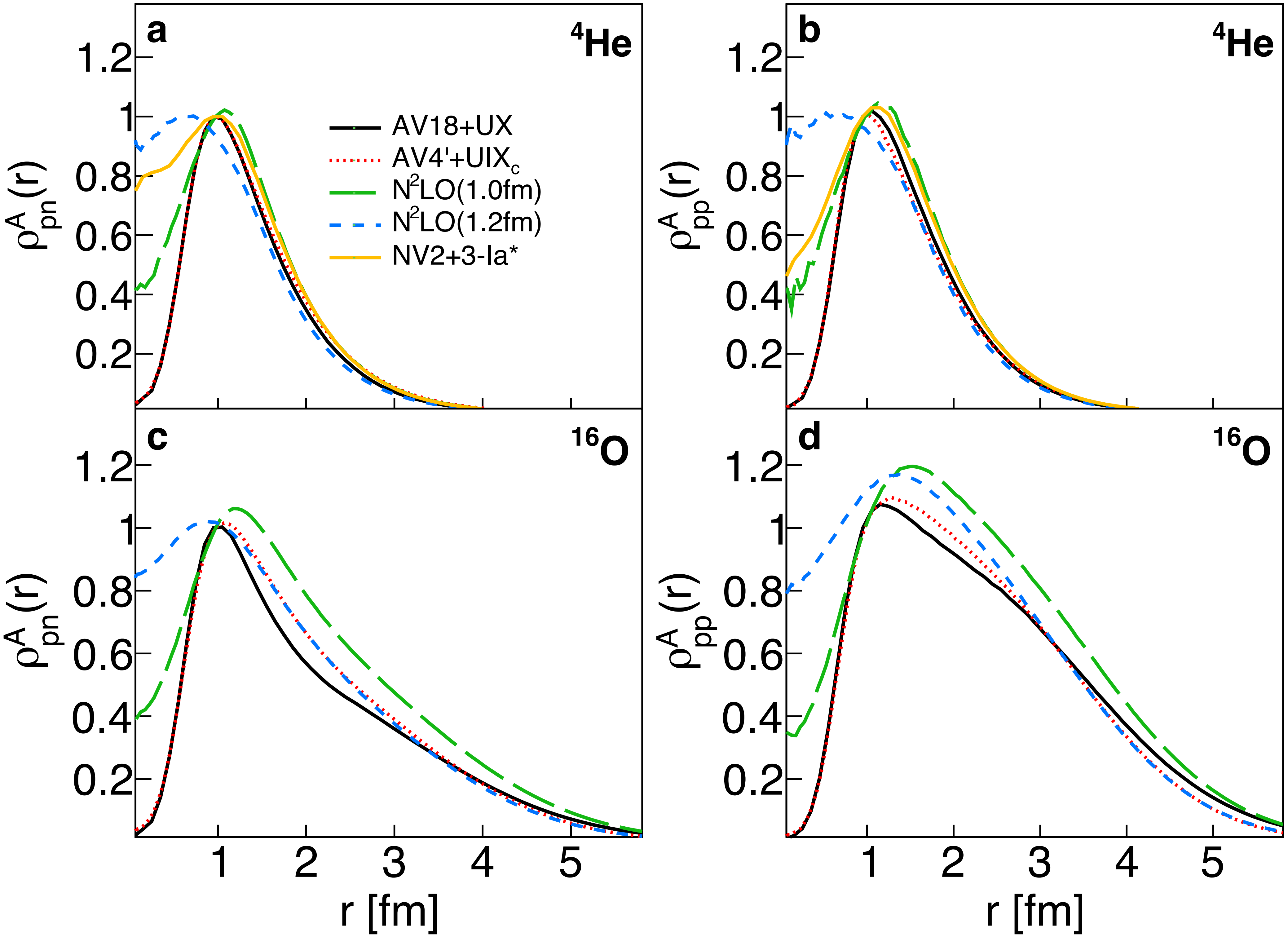
↓

$|\Psi_T\rangle$

- single- and two-nucleon coordinate-space distributions: $\rho_N(r_i)$, $\rho_{NN}(r, R)$
- single- and two-nucleon momentum-space distributions: $n_N(k)$, $n_{NN}(q, Q)$

QMC review: J. A. Carlson *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

Two-nucleon coordinate-space distributions



interactions

phenomen.:
AV18+UIX
AV4'+UIX$_c$

↓

harder

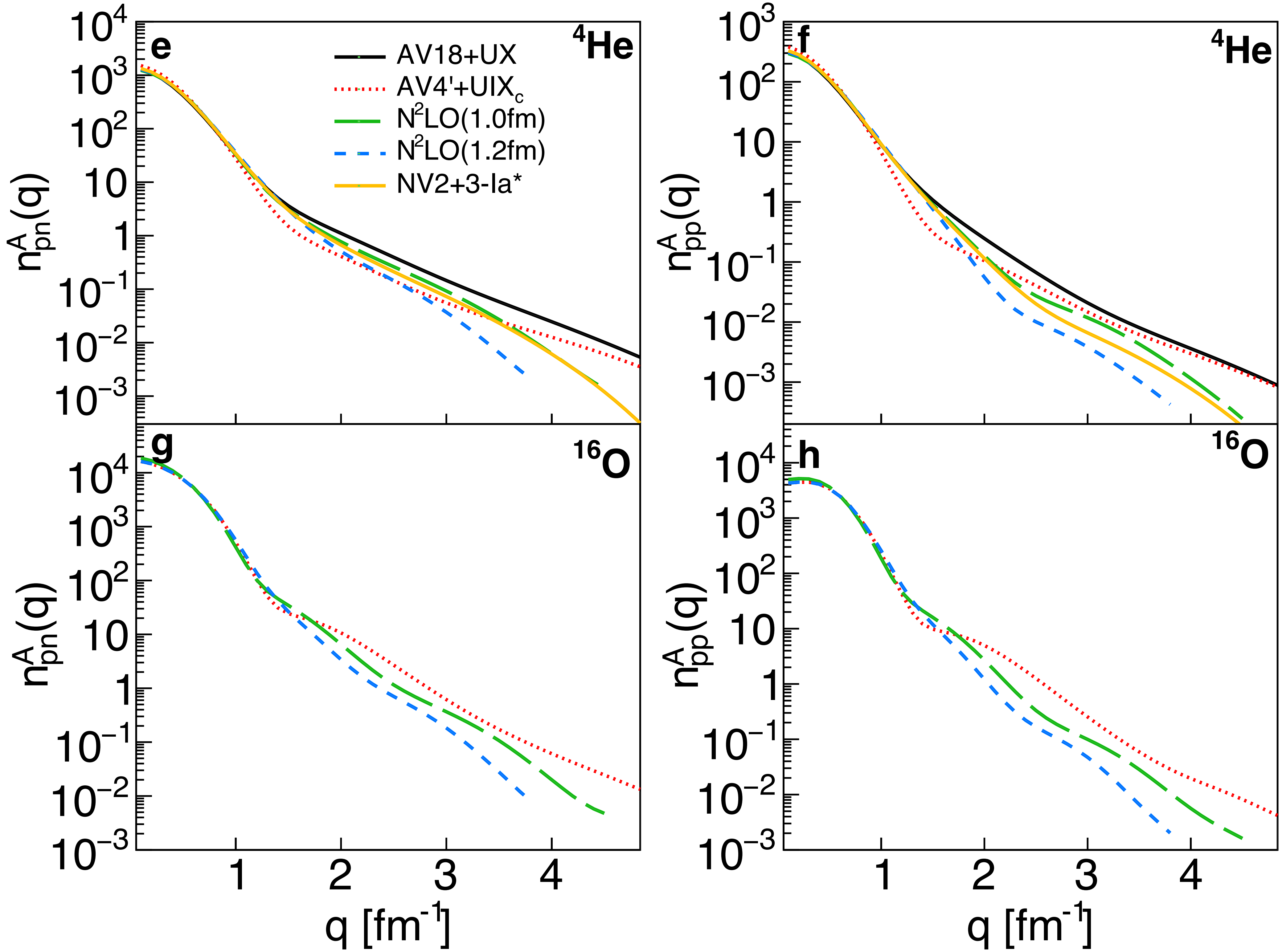
chiral EFT:
N^{2}LO(1.0fm)
N^{2}LO(1.2fm)
NV2+3-Ia*

↓

softer

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

Two-nucleon momentum-space distributions



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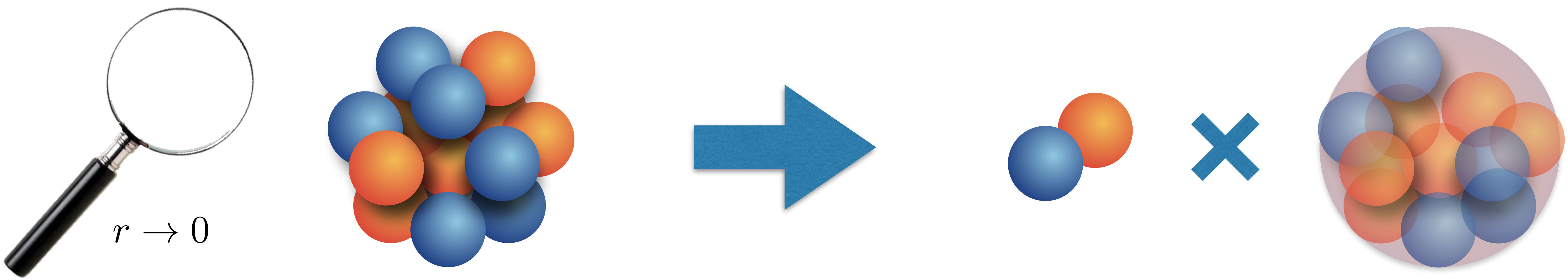
Generalized Contact Formalism

strong interaction among the nucleons in an SRC pair $\longleftarrow$ scale separation $\longrightarrow$ pair's weaker interaction with its surroundings



(r, R), (q, Q) $\quad$ (r), (q) $\quad$ (R), (Q)

$$\rho^A_{\alpha,NN}(r,R) = |\varphi^\alpha_{NN}(r)|^2 \times C^A_{\alpha,NN}(R)$$

$$n^A_{\alpha,NN}(q,Q) = |\tilde{\varphi}^\alpha_{NN}(q)|^2 \times \tilde{C}^A_{\alpha,NN}(Q)$$

$$C^A_{\alpha,NN} \equiv \int d\mathbf{R}\, C^A_{\alpha,NN}(R),$$

$$\tilde{C}^A_{\alpha,NN} \equiv \frac{1}{(2\pi)^3} \int d\mathbf{Q}\, \tilde{C}^A_{\alpha,NN}(Q),$$

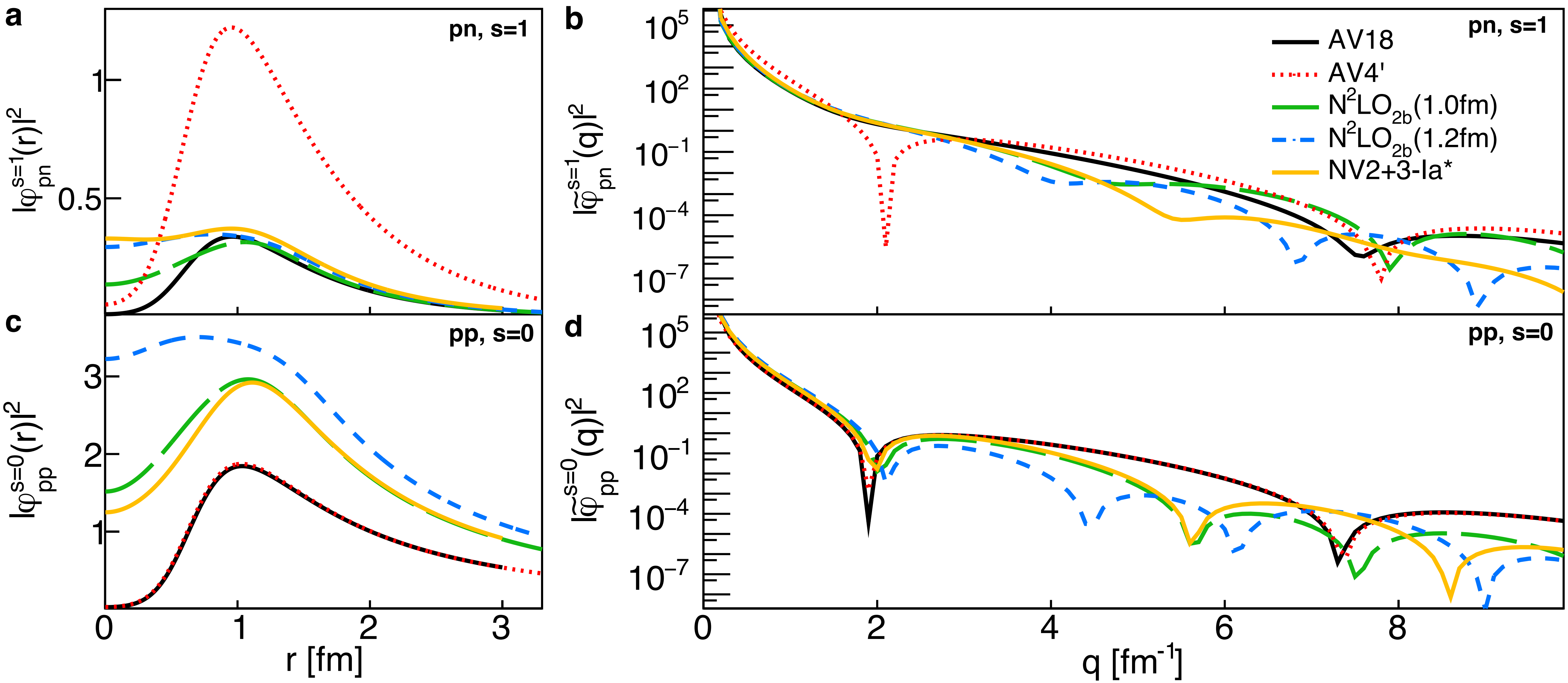
$\varphi^\alpha_{NN}(r)$, $\varphi^\alpha_{NN}(q)$ $\longrightarrow$ universal two-body functions

$C^A_{\alpha,NN}(R)$, $\tilde{C}^A_{\alpha,NN}(Q)$ $\longrightarrow$ density distributions of NN SRC pairs in a nucleus A

$C^A_{\alpha,NN}$, $\tilde{C}^A_{\alpha,NN}$ $\longrightarrow$ nuclear contacts: number of NN SRC pairs in a nucleus A

Universal functions

$\tilde{\varphi}(q)$: Fourier transform of $\varphi(r)$



interactions

phenomen.:
AV18+UIX
AV4'+UIXc
→ harder

chiral EFT:
$N^2LO(1.0fm)$
$N^2LO(1.2fm)$
NV2+3-Ia*
→ softer

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

Generalized Contact Formalism

$\varphi^{\alpha}_{NN}(r)$
$\varphi^{\alpha}_{NN}(q)$ $\longrightarrow$ model dependent
nucleus independent

$C^{A}_{\alpha,NN}$
$\tilde{C}^{A}_{\alpha,NN}$ $\longrightarrow$ nucleus dependent
model independent?

$\downarrow$

encapsulate the many-body dynamics
driving the formation of SRC pairs

$\downarrow$

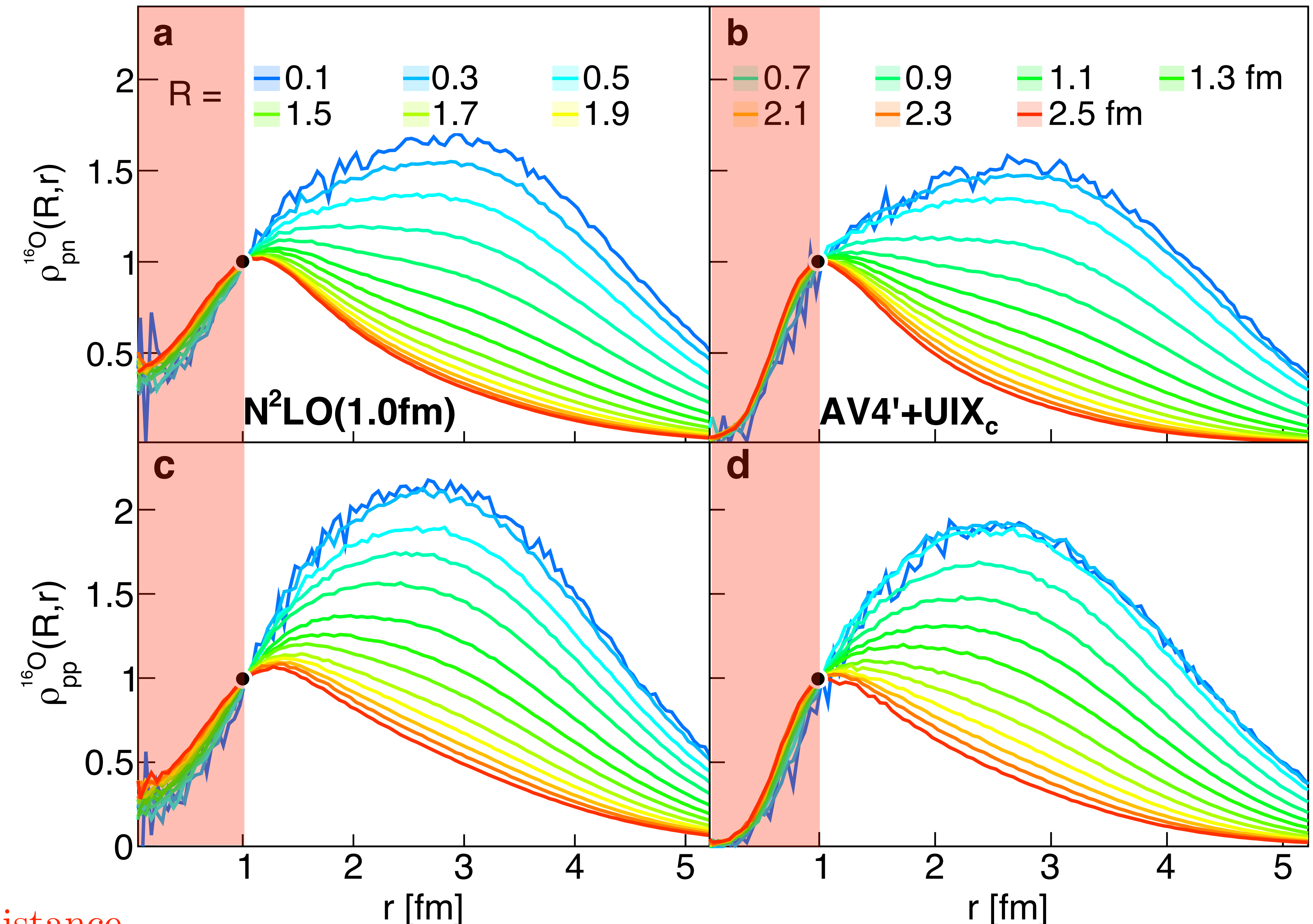
they should be:

- scale separated from the correlated two-body part
- less sensitive to the short-distance NN interaction

Questions:

- validity of the factorization?
- contacts: short-range vs mean-field?
- r-space vs k-space?
- what can we learn?

QMC two-nucleon coordinate-space distributions (scaled)



short-distance factorization!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

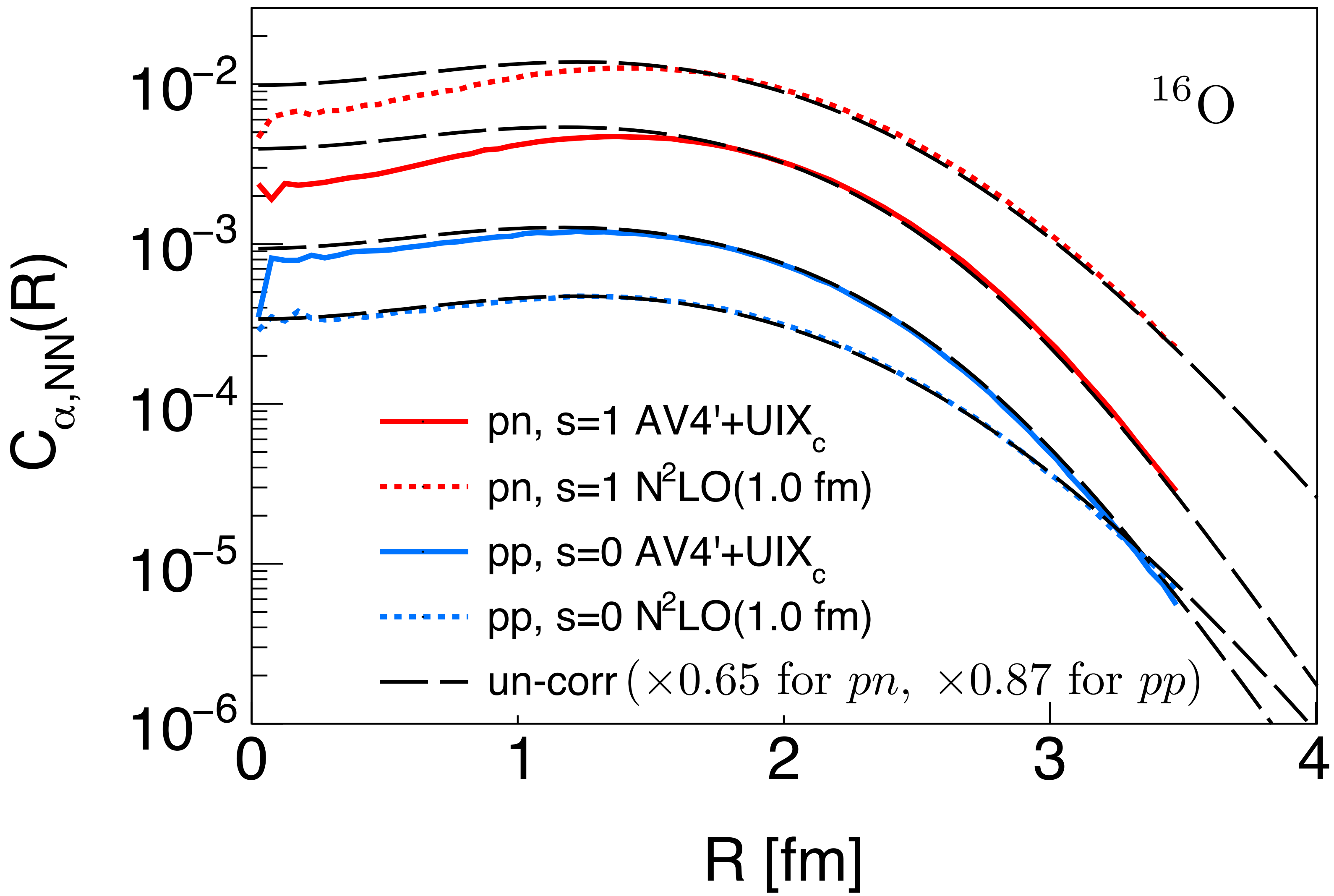
QMC two-nucleon coordinate-space distributions $\longrightarrow$ absolute contact distributions

$\rho^A_{\alpha,NN}(r,R) = |\varphi^\alpha_{NN}(r)|^2 \times C^A_{\alpha,NN}(R)$ $\longrightarrow$ integrate over r in [0-1] fm $\longrightarrow$ $C^A_{\alpha,NN}(R)$

$$C^{^{16}\mathrm{O}}_{NN,\mathrm{uncorr}}(R) = \int d\Omega_R \int_0^{1\,\mathrm{fm}} d\mathbf{r}\, \rho_N^{^{16}\mathrm{O}}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right) \rho_N^{^{16}\mathrm{O}}\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

+ Pauli exclusion principle for pp pairs

R. Cruz-Torres *et al.*, Phys. Lett. B **785**, 304-308 (2018)



agreement is insensitive to the integration range in [0-1] fm

density distribution of SRC pairs (shape of $C(R)$)

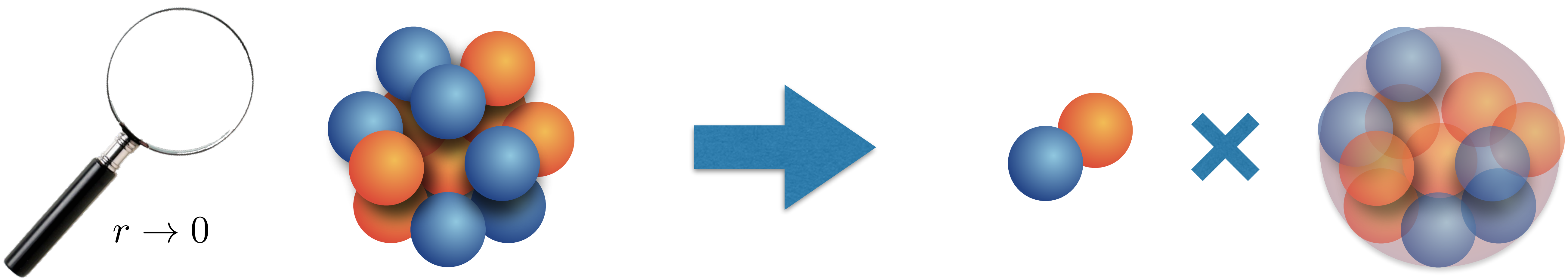
↓

mean-field property, minimal sensitivity to the short-distance NN interaction!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

strong interaction among the nucleons in an SRC pair $\longleftarrow$ scale separation $\longrightarrow$ pair's weaker interaction with its surroundings



(r, R), (q, Q) $\quad$ (r), (q) $\quad$ (R), (Q)

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$$n^A_{\alpha,NN}(q,Q) = |\tilde{\varphi}^\alpha_{NN}(q)|^2 \times \tilde{C}^A_{\alpha,NN}(Q)$$

integrate over R or Q:

$$\rho^A_{\alpha,NN}(r) = |\varphi^\alpha_{NN}(r)|^2 \times C^A_{\alpha,NN}$$

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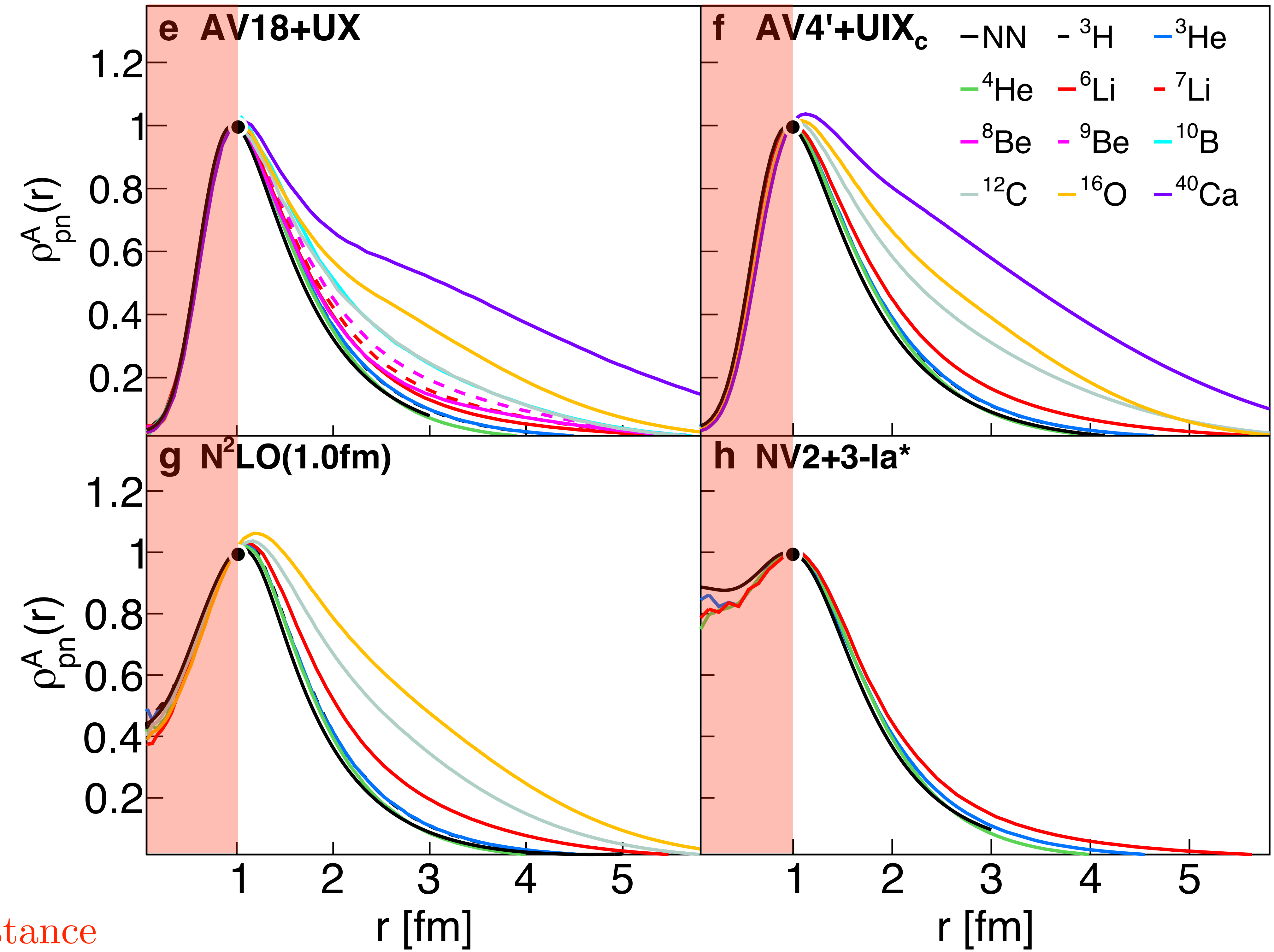
Note: ρ/n norm.: number of pairs

φ norm.: choice

QMC + GCF: results

QMC two-nucleon coordinate-space distributions (scaled)

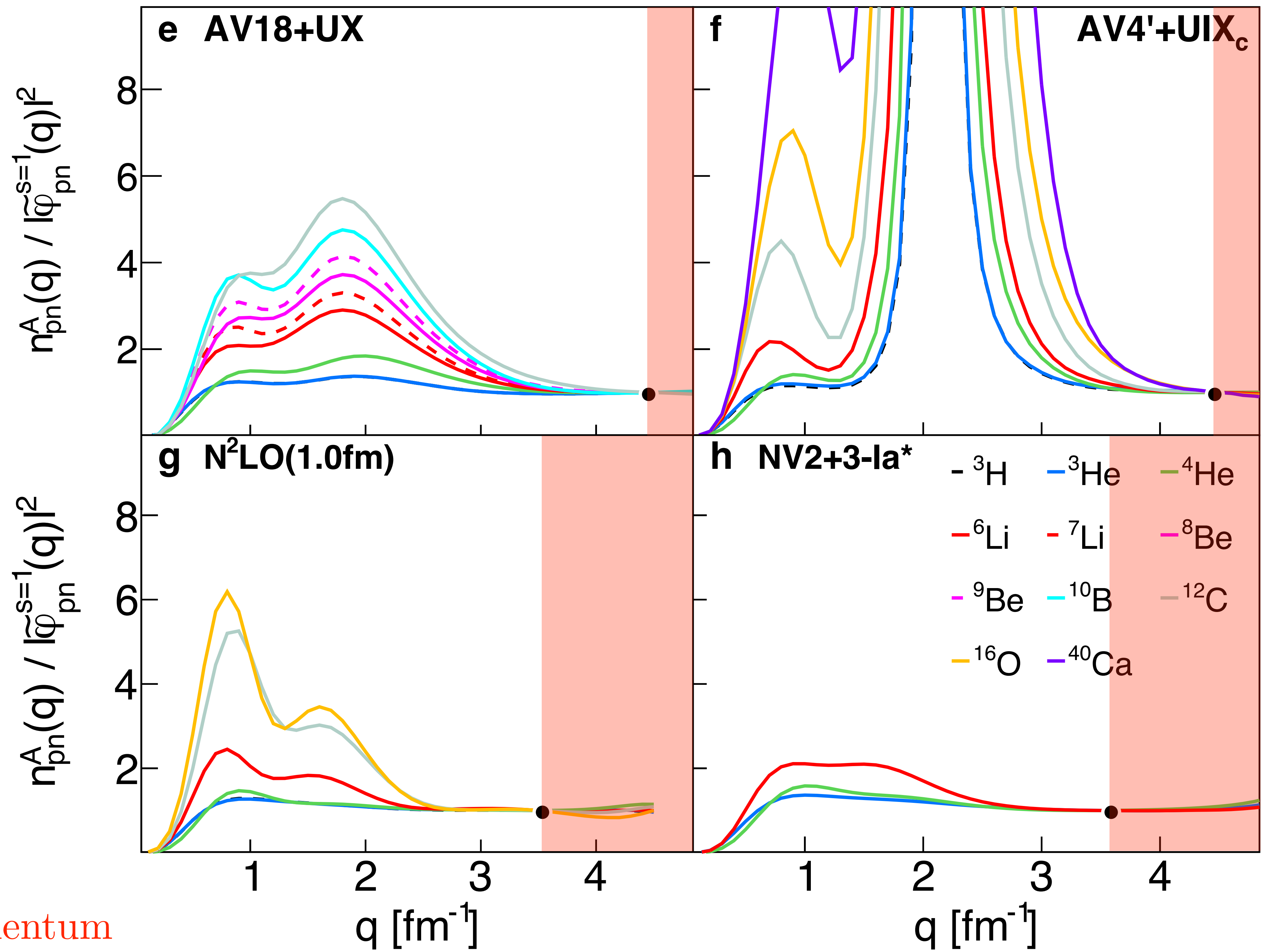


short-distance factorization!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

QMC two-nucleon momentum-space distributions (scaled)



high-momentum factorization!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

Nuclear contact ratios

$$\rho^A_{\alpha,NN}(r) = |\varphi^\alpha_{NN}(r)|^2 \times C^A_{\alpha,NN}$$

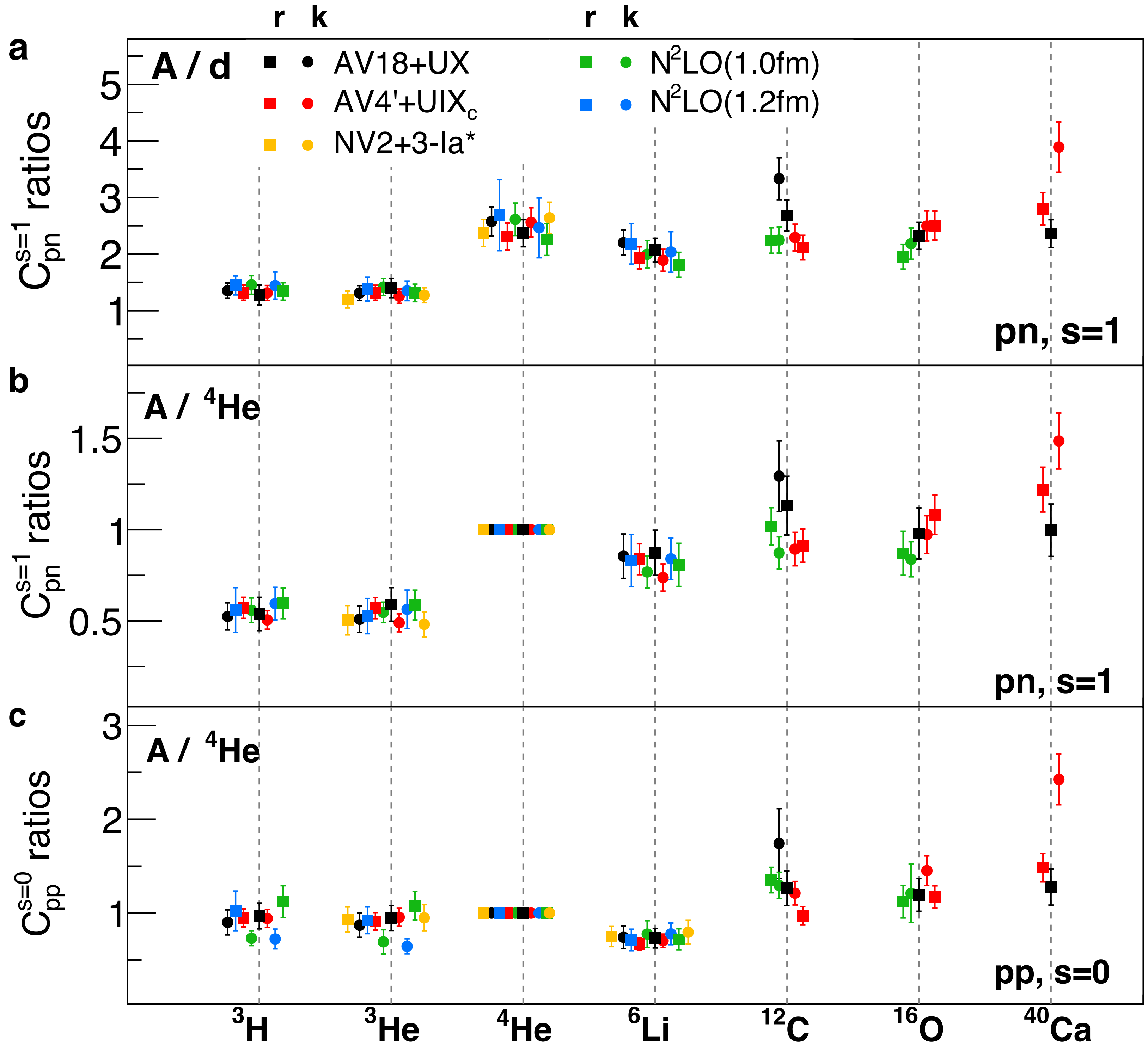
$$n^A_{\alpha,NN}(q) = |\tilde{\varphi}^\alpha_{NN}(q)|^2 \times \tilde{C}^A_{\alpha,NN}$$

↓ A to A_0

r-space: $$\frac{\rho^A_{\alpha,NN}(r)}{\rho^{A_0}_{\alpha,NN}(r)} = \frac{C^A_{\alpha,NN}}{C^{A_0}_{\alpha,NN}}$$

k-space: $$\frac{n^A_{\alpha,NN}(q)}{n^{A_0}_{\alpha,NN}(q)} = \frac{\tilde{C}^A_{\alpha,NN}}{\tilde{C}^{A_0}_{\alpha,NN}}$$

1. scale- & scheme-independent!
2. same for short-distance & high-momentum pairs!



R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

Nuclear contact ratios

Implications:

- contact ratios: determined by mean-field physics, *i.e.*, by the average field of the other A-2 nucleons
- contact ratios: insensitive to the details of the NN interaction at short distance
- contact ratios: simplify calculations for heavier nuclei!

Example:

hard interactions (such as AV18) $\longrightarrow$ difficult to use in many-body calculations of medium-mass nuclei

1. calculate $C^{A_0}_{\alpha,NN}$ for a light nucleus A_0 using the hard interaction
2. calculate the ratio $C^{A}_{\alpha,NN}/C^{A_0}_{\alpha,NN}$ using a soft interaction
3. multiply the two to get $C^{A}_{\alpha,NN}$ for the hard interaction

bonus 1: works in both r- and k-space

bonus 2: many observables have been already derived with the GCF

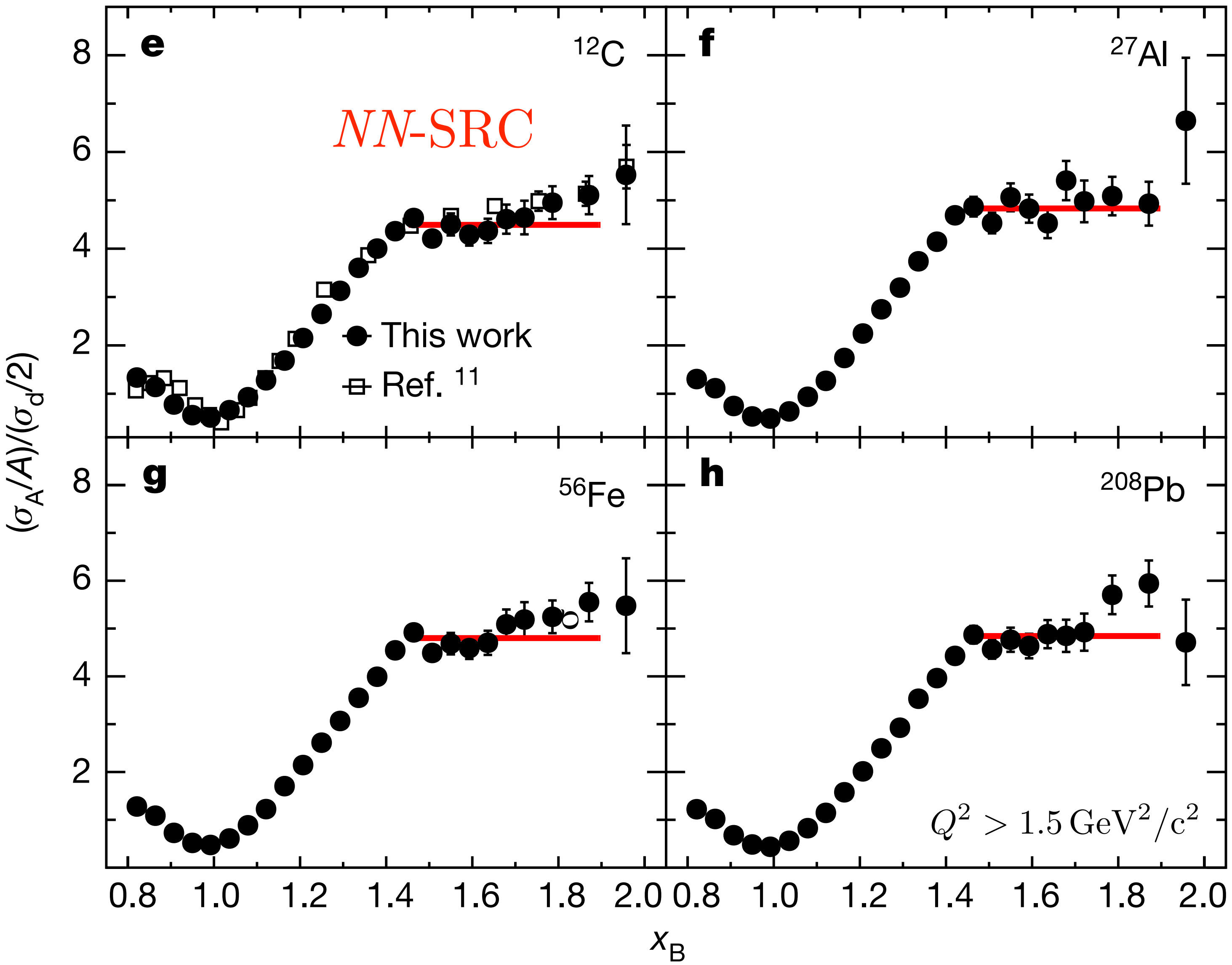
R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

Nuclear contact ratios

Implications:

- A/d (e,e') inclusive cross section ratios: observed scaling for x_B>1.5



B. Schmookler *et al.*, Nature **566**, 354-358 (2019)

extraction of SRC scaling coefficient $a_2(A/d)$

traditionally:

- interpreted as relative abundance of SRC pairs
- thought to be sensitive to the nature of the *NN* interaction
- can be computed as k- or r-space distribution ratios

contact ratios: doubts about the sensitivity of $a_2(A/d)$ to the nuclear interaction!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

Nuclear contact ratios

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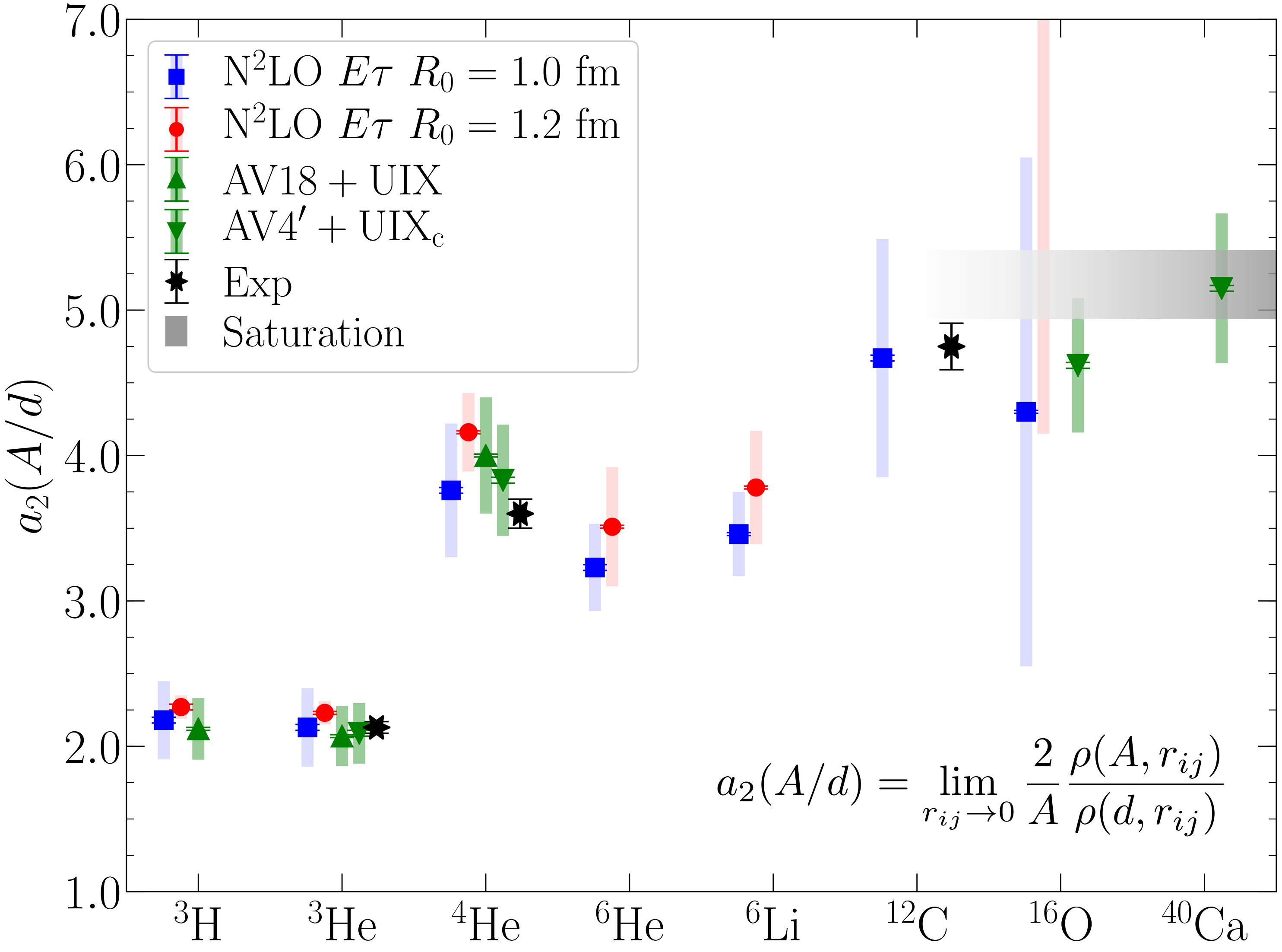

J.E. Lynn, D.L. et al., J. Phys. G: Nucl. Part. Phys. 47, 045109 (2020)

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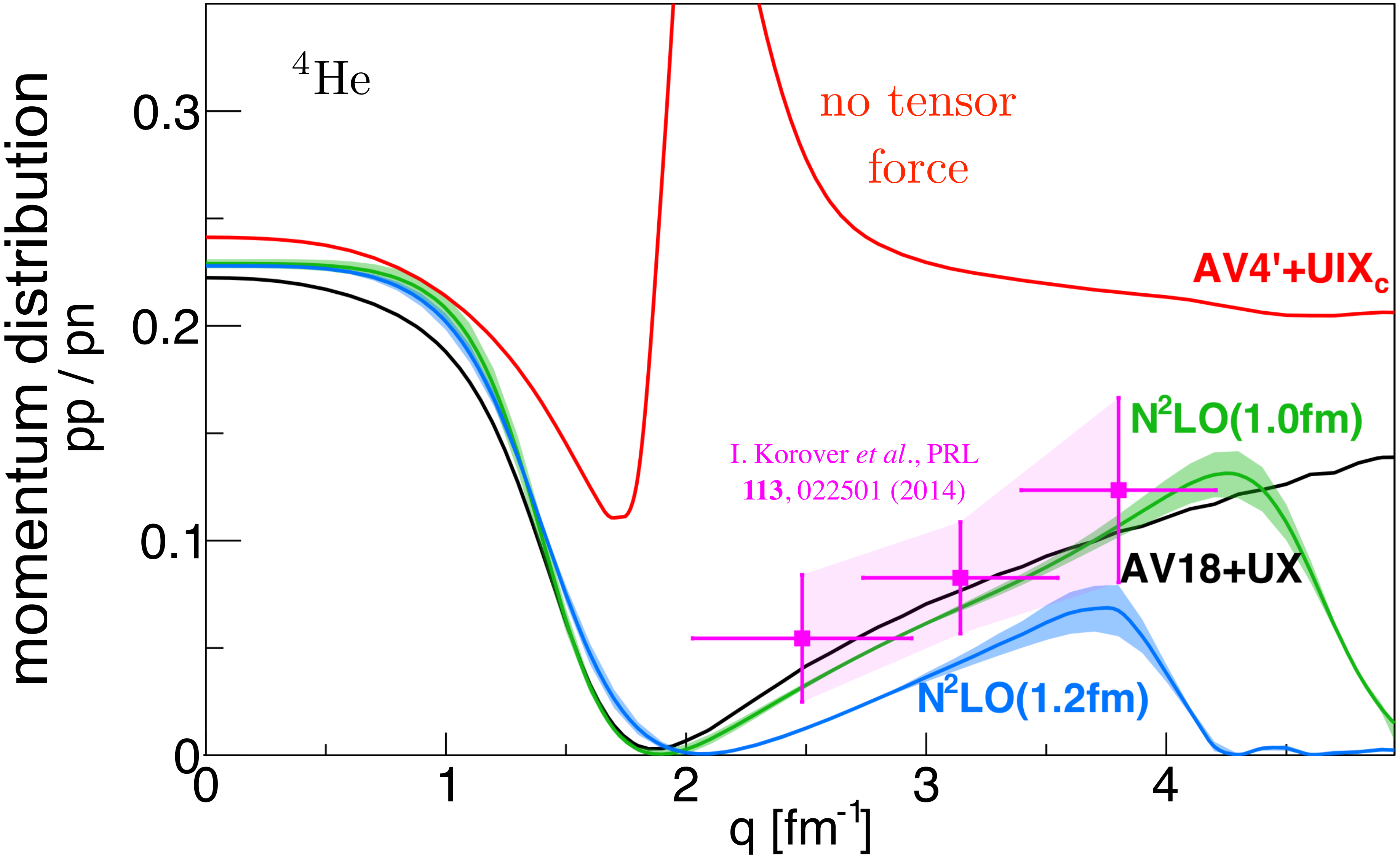
R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

Nuclear contacts

Implications:

- exclusive measurements of two-nucleon knockout $A(e,e'NN)$: sensitive to NN interaction

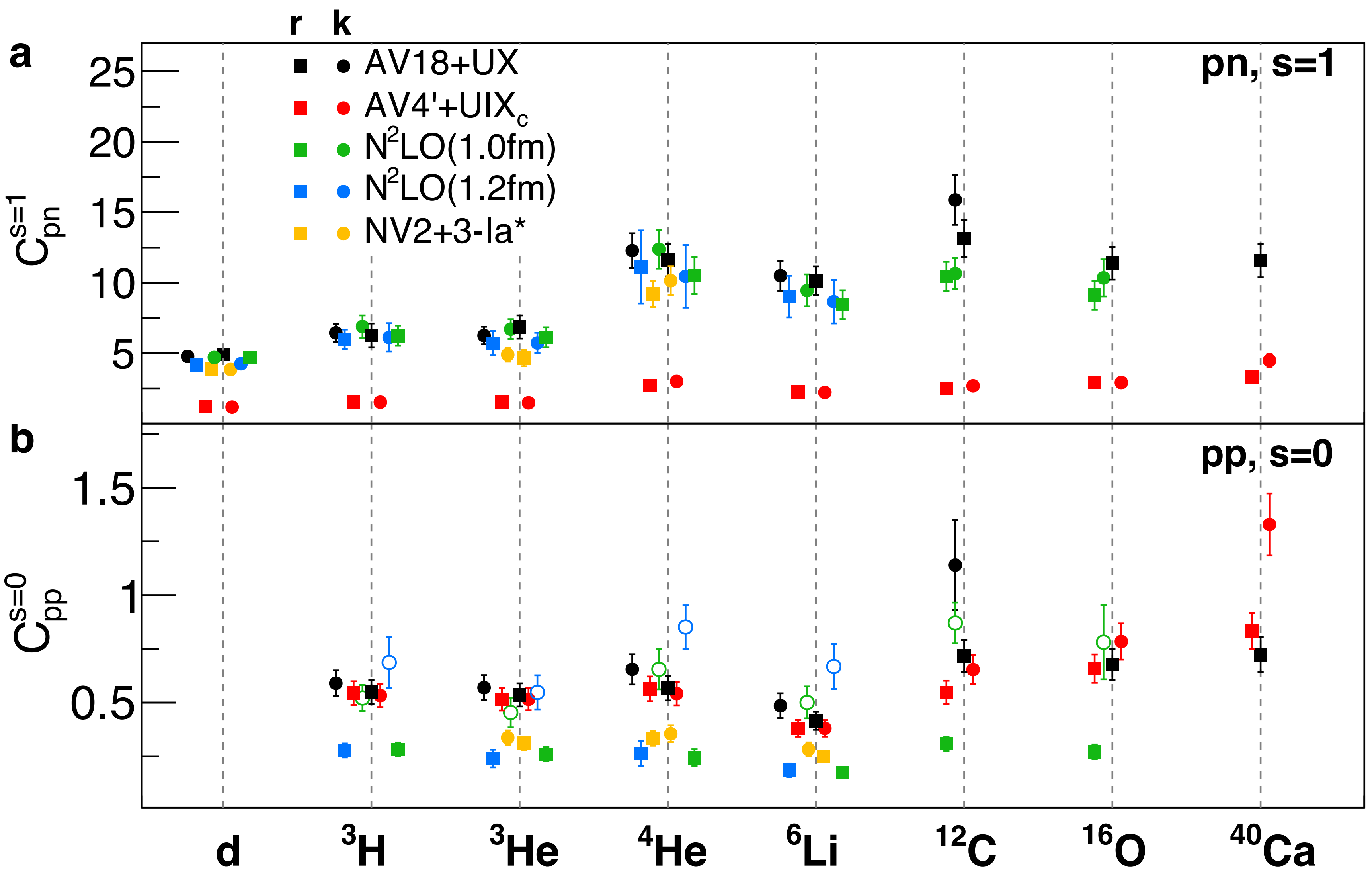


contacts: key for relating $A(e,e'NN)$ measurements to NN interaction models and ab-initio many-body calculations!

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

QMC + GCF: results

Nuclear contacts



$pn,\ s = 1$ channel

- r- to k-space equivalence
- scale & scheme independ.

↓

exception: AV4'+UIX$_c$

expected: no tensor force!

$pp,\ s = 0$ channel

- more complex
- similar conclusions

pp discrepancy:

- regulator artifacts and three-body effects: difficult to identify a clear high-momentum scaling plateau in the spin-0 *pp* channel
- A to ^{4}He contact ratios: r- to k-space equivalence ⟶ *pp* discrepancy not beyond three-body level

R. Cruz-Torres, D.L. *et al.*, Nat. Phys. (2020)

Summary & Perspectives

- QMC calculations have been performed for nuclei from ^{2}H to ^{40}**Ca** with different nuclear interaction models: phenomenological and chiral EFT
- QMC *r*- and *k*-space distributions have been analyzed using the GCF and spin/isospin-dependent nuclear contacts and contact ratios have been extracted
- Result: universal factorization of the many-body nuclear wave function at short-distance into a strongly-interacting pair and a weakly-interacting residual system
- Result: position-momentum equivalence of SRC
- Result: the relative abundance of SRC pairs in the nucleus is a long-range (mean-field) quantity that is insensitive to the short-distance nature of the nuclear force

Future work

- Many-body methods benchmark
- Applications of the nuclear contacts and contact ratios
- Derive next order corrections to the current formulation of the GCF
- Study three-nucleon correlations

Thank you!!