WILL DETMOLD

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LATTICE QCD AND EFT FOR NUCLEAR PDFS

EMC/SRC, March 24th 2021

NUCLEAR PDFS IN LQCD AND EFT

- Theoretical and experimental goal: understanding nuclear structure
  - The QCD origins of the EMC effect
  - Aspects of universality and SRC
  - Predictions for flavour, spin dependence, …
- Some old EFT, some new LQCD and some new EFT

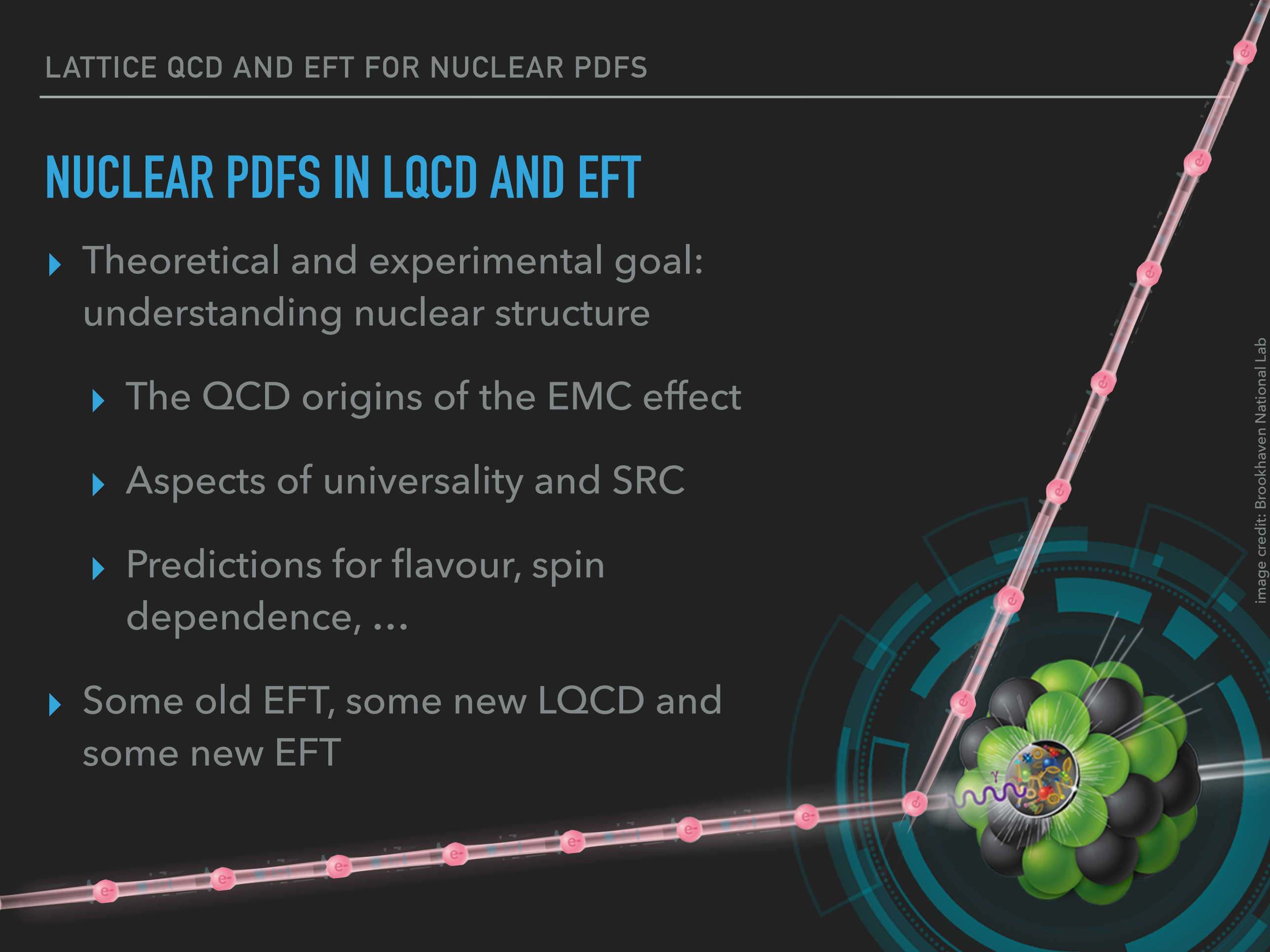


image credit: Brookhaven National Lab

NUCLEAR EFFECTIVE FIELD THEORY

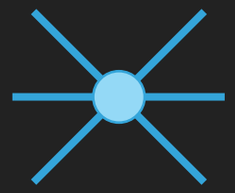
- Hadron level description of low energy properties and interactions of nuclei
- Based on separation of scales and power counting: spontaneously broken chiral symmetry
- Nuclear EFT Lagrangian (pionless for simplicity)

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots$$

$$\mathcal{L}_1 = N^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M_N} \right) N + \ldots$$

$$\mathcal{L}_2 = -\frac{1}{2} \left[C_0 \left(N^\dagger N \right)^2 + C_1 \left(N^\dagger \vec{\sigma} N \right)^2 \right] + \ldots,$$

$$\mathcal{L}_3 = -\frac{D_0}{6} (N^\dagger N)^3 + \ldots$$



PARTON PHYSICS IN EFT

- EFT provides rigorous description of low-energy QCD with quantifiable uncertainties
- Hard partonic processes not naturally described
- Operator product expansion: moments of PDFs are matrix elements of local twist-2 operators

$$\mathcal{O}^{\mu_0 \cdots \mu_n} = \bar{q}\gamma^{(\mu_0} iD^{\mu_1} \cdots iD^{\mu_n)} q$$

$$\langle A; p \,|\mathcal{O}^{\mu_0 \cdots \mu_n}|\, A; p\rangle = \langle x^n \rangle_A (Q) p^{(\mu_0} \ldots p^{\mu_n)}$$

$$\langle x^n \rangle_A (Q) = \int_{-A}^{A} x^n q_A(x, Q) dx$$

Evaluate in rest frame, EFT methods applicable

TWIST-2 OPERATORS

- EFT: match QCD operators to all possible hadronic operators with same symmetries
- Used in pion and N sectors to connect lattice PDF moments to experiment [Arndt & Savage; Chen & Ji; Detmold et al.,...]
- Isoscalar, spin independent operator matching:

$$\bar{q}\gamma^{\{\mu_1} D^{\mu_2} \dots D^{\mu_n\}} q \longrightarrow a_n \frac{1}{\Lambda^n} \mathrm{tr}\left[\Sigma^\dagger D^{\mu_1} \dots D^{\mu_n} \Sigma + h.c.\right]$$

$$+ c_n N^\dagger \mathcal{V}^{\mu_1 \dots \mu_n} N + c_n' N^\dagger S^{\{\mu_1} A^{\mu_2} \mathcal{V}^{\mu_3 \dots \mu_n\}} N + \dots$$

$$+ \alpha_n N^\dagger \mathcal{V}^{\mu_1 \dots \mu_n} N N^\dagger N + \beta_n N^\dagger \mathcal{V}^{\mu_1 \dots \mu_n} \tau_j^{\xi+} N N^\dagger \tau_j^{\xi+} N + \dots$$

Two body counterterms

- where

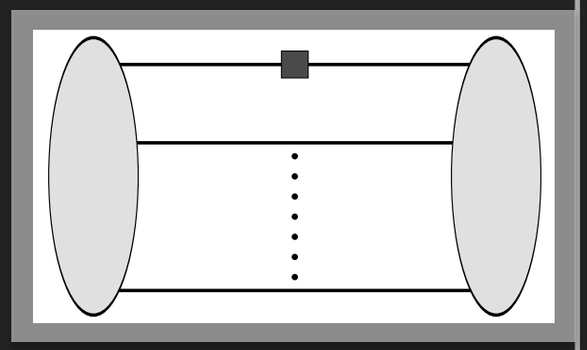
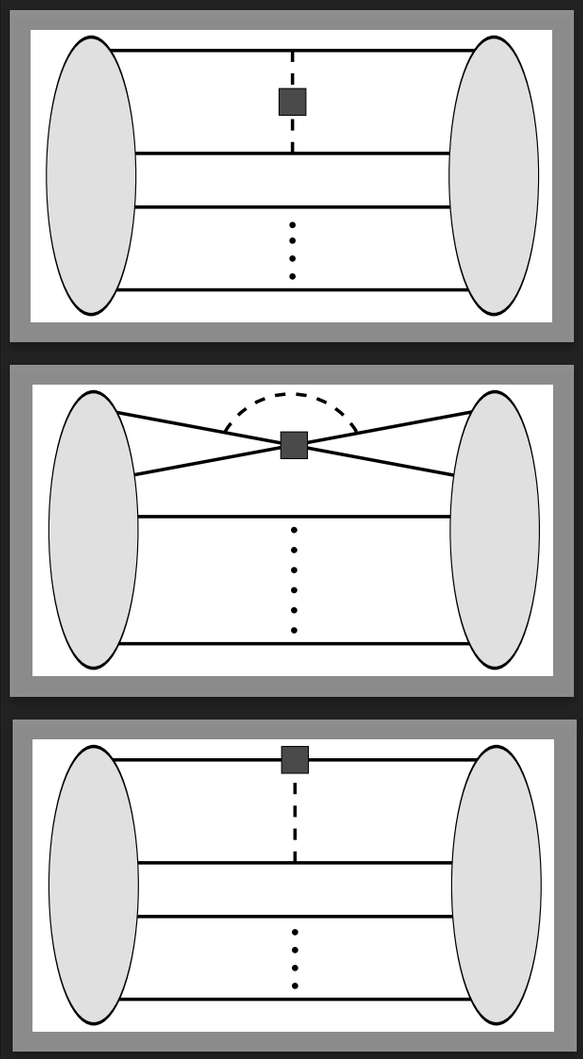
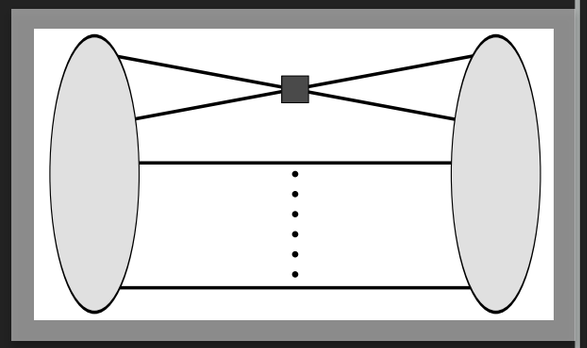
$$\mathcal{V}^{\mu_1 \dots \mu_n} = \left(v + i\frac{D}{M}\right)^{\mu_1} \dots \left(v + i\frac{D}{M}\right)^{\mu_n}$$

$$\tau_j^{\xi\pm} = \frac{1}{2}\left(\xi^\dagger \tau_j \xi \pm \xi \tau_j \xi^\dagger\right)$$

[J W Chen, WD PLB 2005]

POWER COUNTING

- Power counting in KSW scheme (Weinberg scheme similar) for small ε~p/M

ε^{-3}		higher orders 
ε^{0}		... ...

NUCLEAR PDF MOMENTS

- Nucleon matrix elements (includes pion loop effects)

$$v_{\mu_1} \dots v_{\mu_n} \langle N | \mathcal{O}^{\mu_1 \dots \mu_n} | N \rangle = \langle x^n \rangle_q$$

- Nuclear matrix elements

$$\begin{aligned} \langle x^n \rangle_{q|A} &\equiv v_{\mu_1} \dots v_{\mu_n} \langle A | \mathcal{O}^{\mu_1 \dots \mu_n} | A \rangle \\ &= \langle x^n \rangle_q \left[A + \underbrace{\boxed{\alpha_n \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle}}_{\text{Dominant term}} + \beta_n \left\langle A \middle| \left(N^\dagger \tau N\right)^2 \middle| A \right\rangle \right] + \dots \end{aligned}$$

- β_n term suppressed by N_c^2 [Kaplan & Savage 96; K & Manohar 97]
- Ellipsis includes higher-body operators, terms with derivatives: higher-order in power-counting

[J W Chen, WD PLB 2005]

FACTORISATION AND UNIVERSALITY

- Inverse Mellin transform

$$\langle x^n \rangle_{q|A} = \langle x^n \rangle_q \left[A + \alpha_n \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle \right.$$

$$\frac{f^A(x)}{A} = f^N(x) + g_2(A) f_2(x)$$

with

$$g_2(A,\Lambda) = \frac{1}{A} \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle_\Lambda \qquad \alpha_n = \frac{1}{\langle x^n \rangle_q} \int dx x^n f_2(x)$$

$f_2(x)$ describes two-body contributions

- Factorisation of (x,Q^2) and A dependence: universality
- Observed in data [Daté et al. 84,..., Frankfurt & Strikman 87, Gomez et al. 95]
- Requires there be only a single relevant non-trivial source of A dependence in EFT operator
- Factorisation breaks: holds to O(ε) or N_c^2: expect ~20%

[J W Chen, WD PLB 2005]

FACTORISATION AND UNIVERSALITY

- Factorised form (also holds for QE cross section)

$$\frac{f^A(x)}{A} = f^N(x) + g_2(A) f_2(x)$$

- Simple manipulations imply

$$R(x,A) = \frac{2}{A}\frac{f^A(x)}{f^d(x)} = 1 + (a_2(A) - 1)\left(1 - \frac{f^p(x)+f^n(x)}{f^d(x)}\right)$$

where (scheme independent)

$$R(1 < x < 2, A) = a_2(A)$$

$$a_2(A) = \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)} = \frac{\left\langle A \middle| (N^\dagger N)^2 \middle| A \right\rangle_\Lambda}{\left\langle d \middle| (N^\dagger N)^2 \middle| d \right\rangle_\Lambda}$$

- Consequently $R(1 < x < 2, A) = a_2(A)$ as $f_p(x > 1) = f_n(x > 1) = 0$
- EMC-SRC relation: $\boxed{dR(x,A)/dx = (a_2(A) - 1)\, h(x)}$

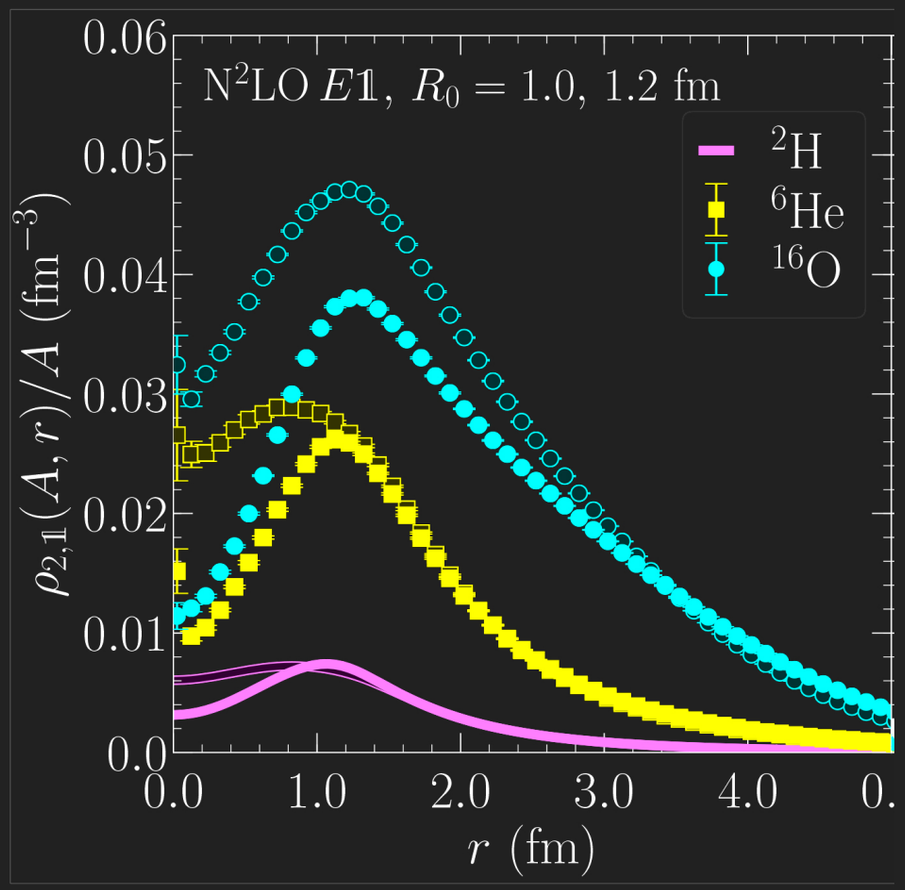
[J W Chen, WD, J Lynn, A Schwenk PRL 2017]

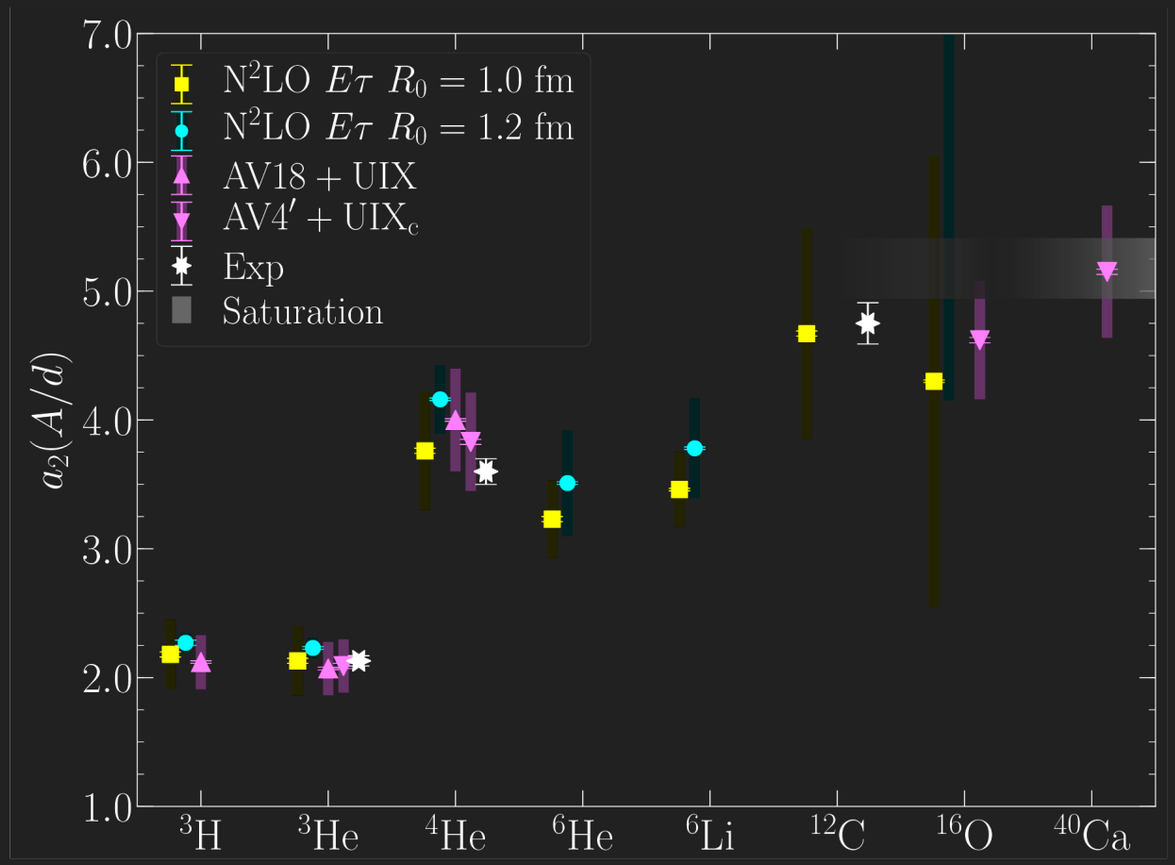
QUANTUM MONTE CARLO CALCULATIONS

- QMC calculations of two-nucleon distributions
- Postdict/predict SRC scaling factors

$$\rho_{2,1}(A,r) = \frac{1}{4\pi r^2}\left\langle \Psi \left| \sum_{i<j}^{A} \delta\left(r - r_{ij}\right) \right| \Psi \right\rangle$$

$$a_2(A/d) = \lim_{r\to 0} \frac{2}{A}\frac{\rho_{2,1}(A,r)}{\rho_{2,1}(d,r)}$$

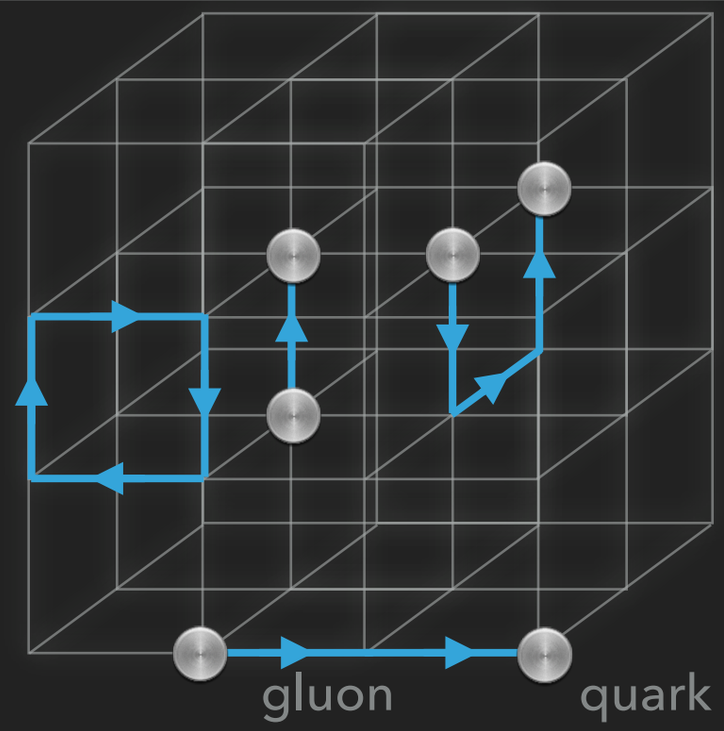


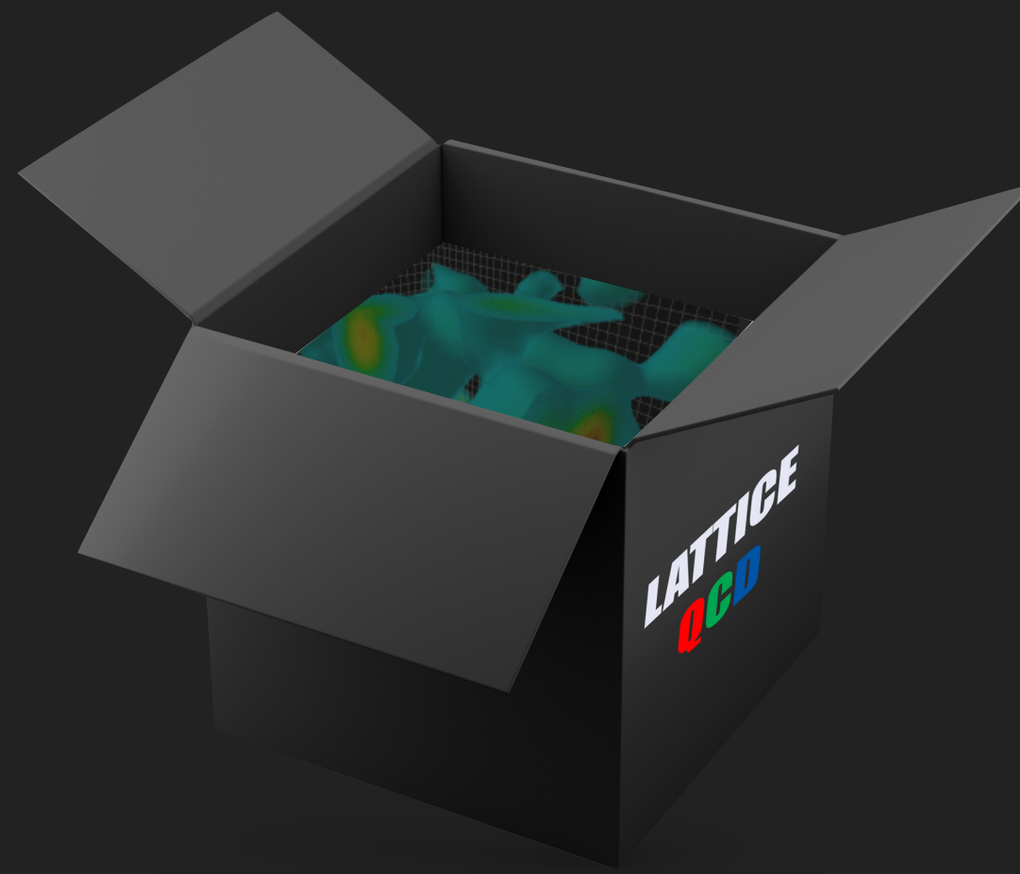


[J Lynn, D Lonardoni, WD et al JPG 2020]

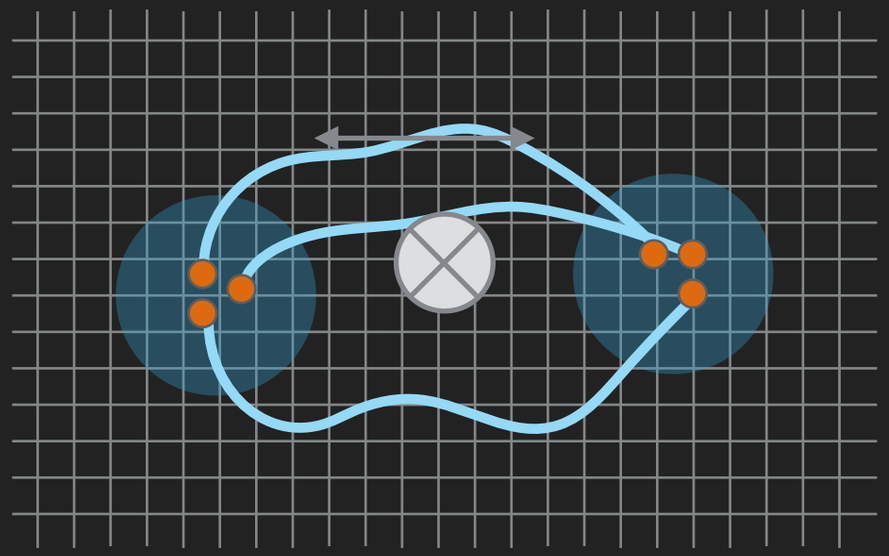
LATTICE QCD

- Strong coupling definition of QCD
- Numerical tool for nonperturbative QCD calculations
  - Discretise and compactify spacetime
  - Integration over 10^{12} degrees of freedom in current calculations using importance sampling Monte Carlo
  - Understand effects of discretisation and compatification and finite statistics



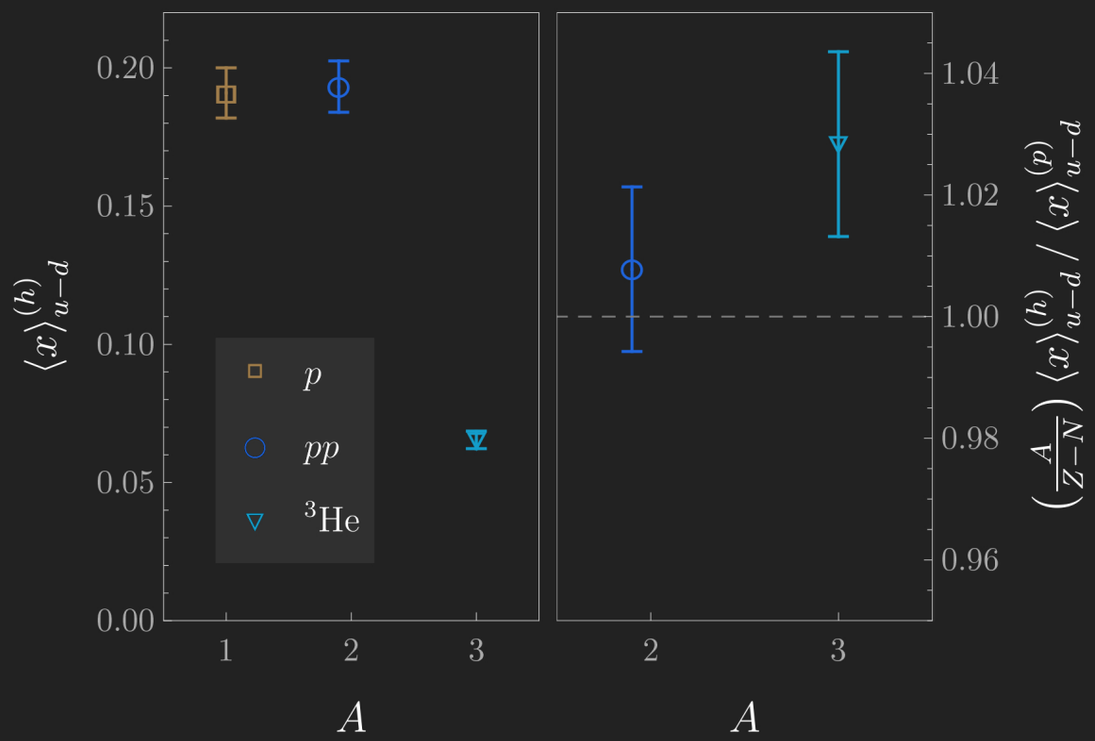


LQCD MOMENTUM FRACTIONS



- Phiala Shanahan (Monday) presented NPLQCD calculation of isovector quark momentum fraction
  - Local operator matrix element
  - Unphysical quark masses for which pion mass is 806 MeV
  - *pp* and *³He* systems

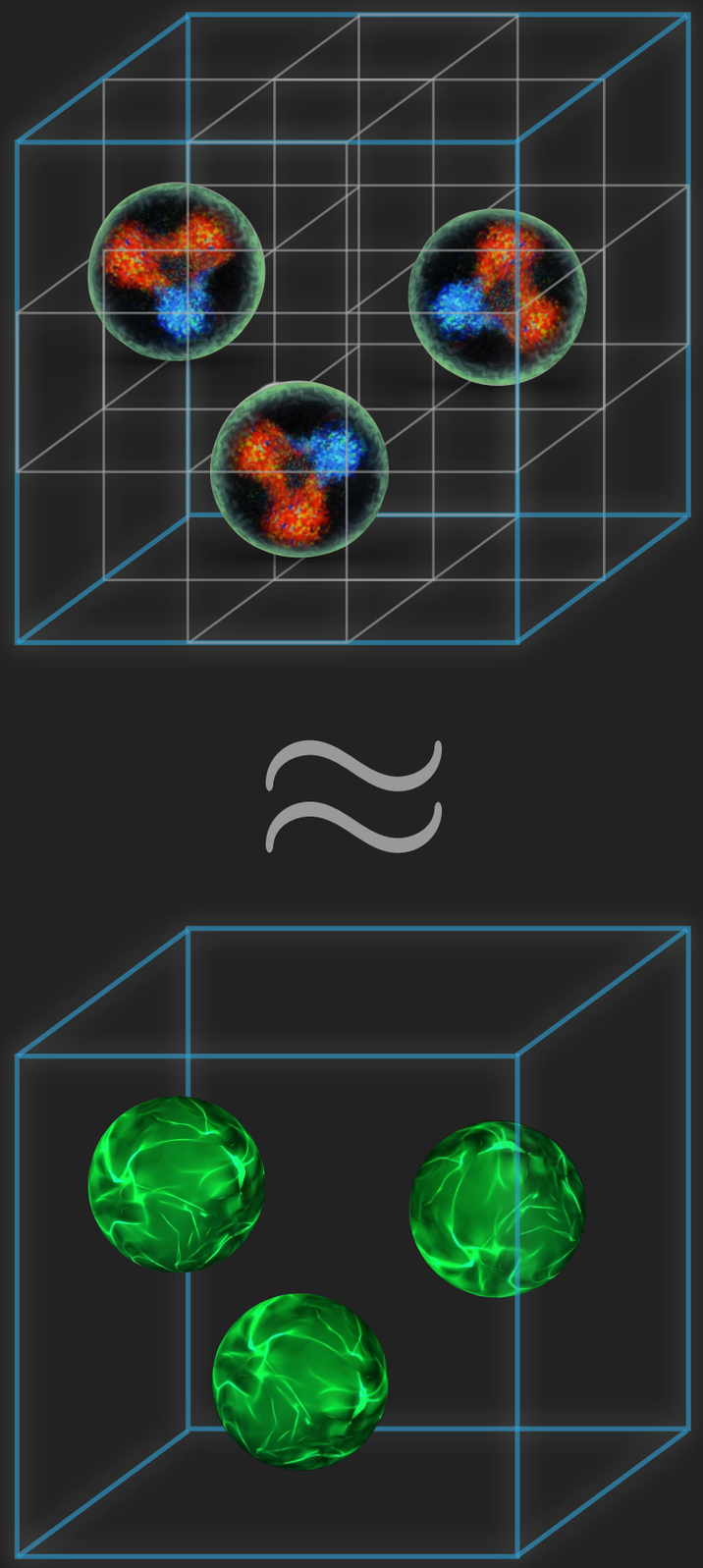
	p	pp	^{3}He
$\langle x \rangle^{(h)}_{u-d}$	0.191(1)(9)	0.194(2)(9)	0.066(1)(3)
$\left(\frac{A}{Z-N}\right) \frac{\langle x \rangle^{(h)}_{u-d}}{\langle x \rangle^{(p)}_{u-d}}$	–	1.007(14)	1.028(15)



[WD et al. 2009.05522]

EFT MATCHING

- Major issue for LQCD nuclei is finite volume (FV) effects
- Well-known Lüscher method to understand FV effect in spectrum
- Less well-developed technologies for matrix elements
- Direct matching between LQCD and EFT in same FV
  - Use EFT to extrapolate to infinite volume
  - Pionless EFT for simplicity



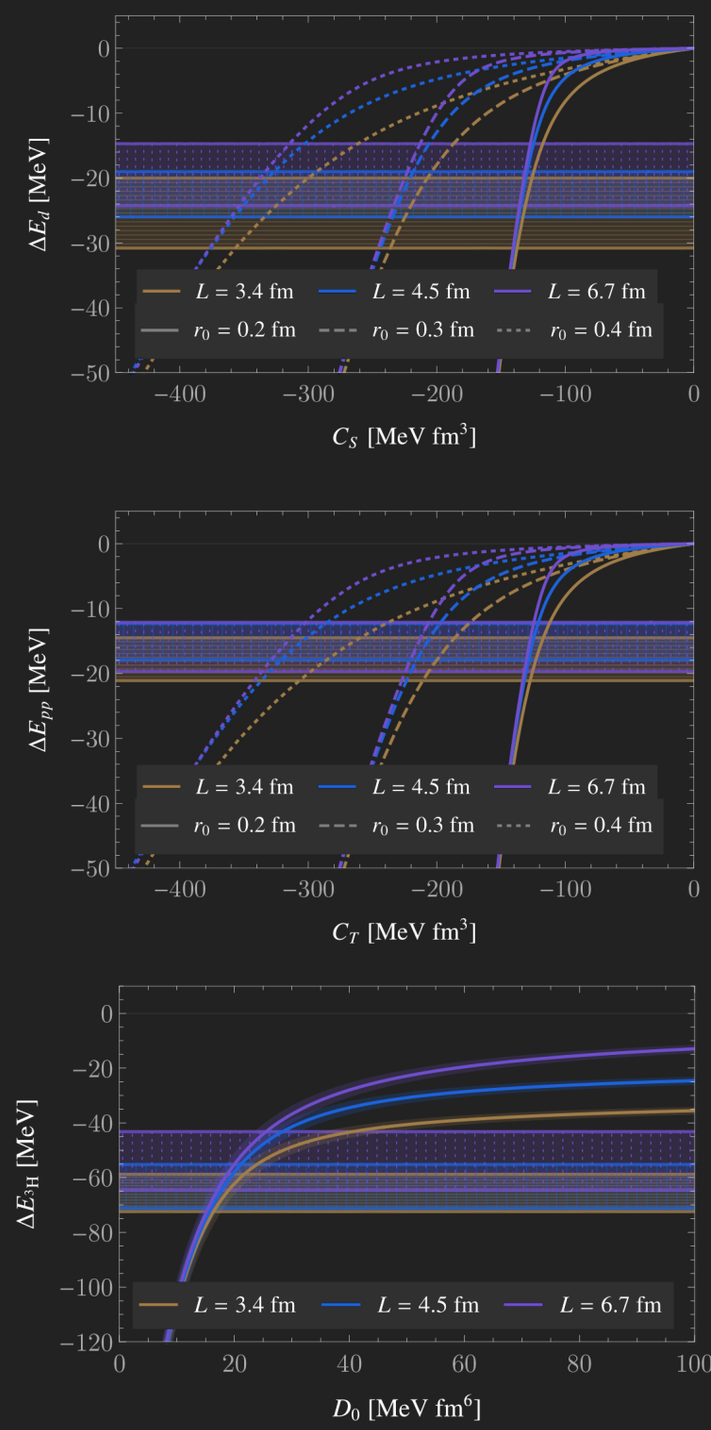
FVEFT USING THE STOCHASTIC VARIATIONAL METHOD

- SVM: variational approach using an expanding set of correlated (shifted) Gaussian trial states

  [Eliyahu, Bazak, Barnea 2019]

$$E_0^h \leq \frac{\int \Psi_h^{(N)}(\mathbf{x})^* H \Psi_h^{(N)}(\mathbf{x}) d\mathbf{x}}{\int \Psi_h^{(N)}(\mathbf{x})^* \Psi_h^{(N)}(\mathbf{x}) d\mathbf{x}}$$

- Match onto LQCD FV energies to determine nuclear wave functions
  - 2 and 3-body energies fix NN and NNN contact interactions
- Determines infinite volume energy



[WD, P Shanahan. 2102.04329]

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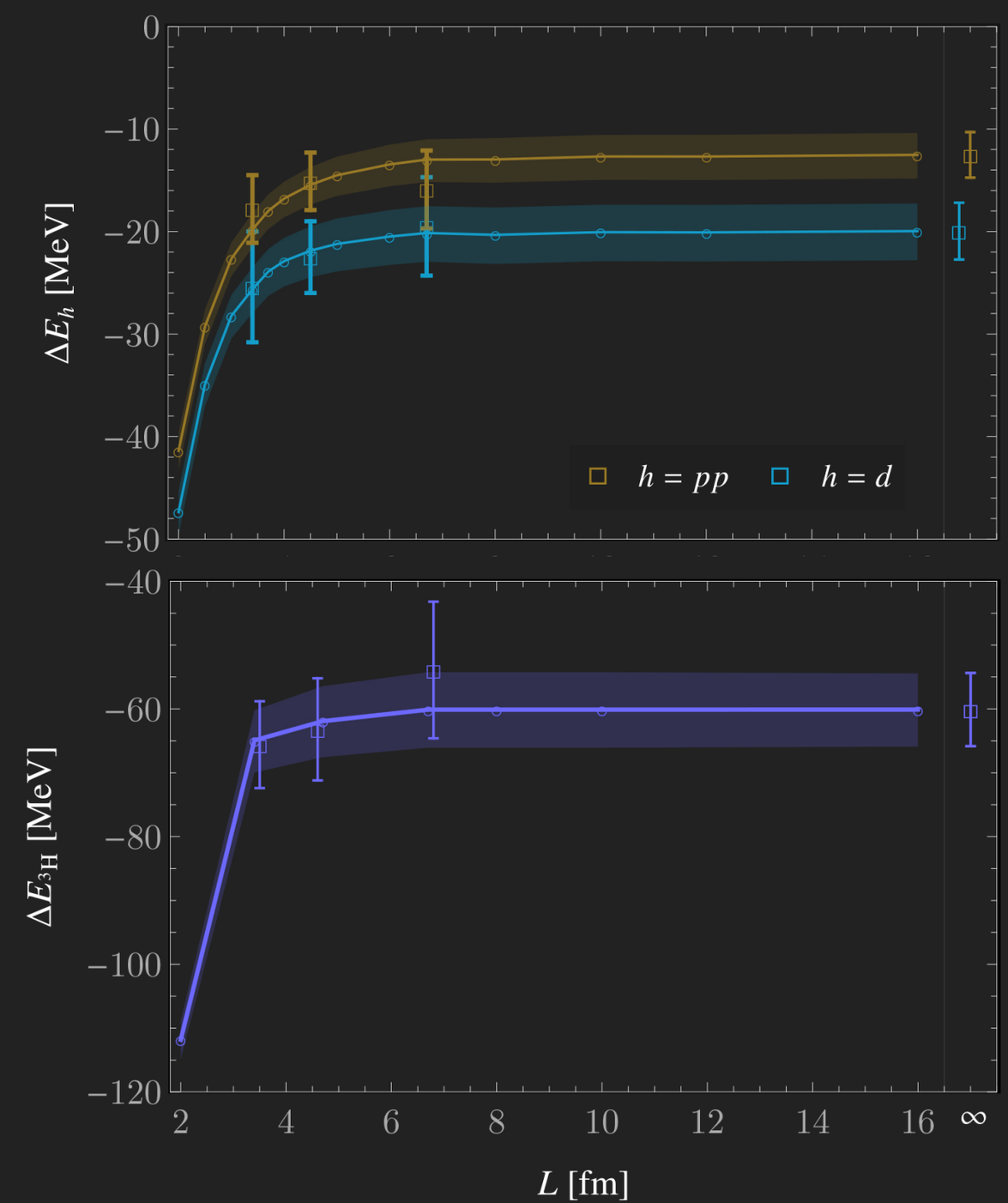

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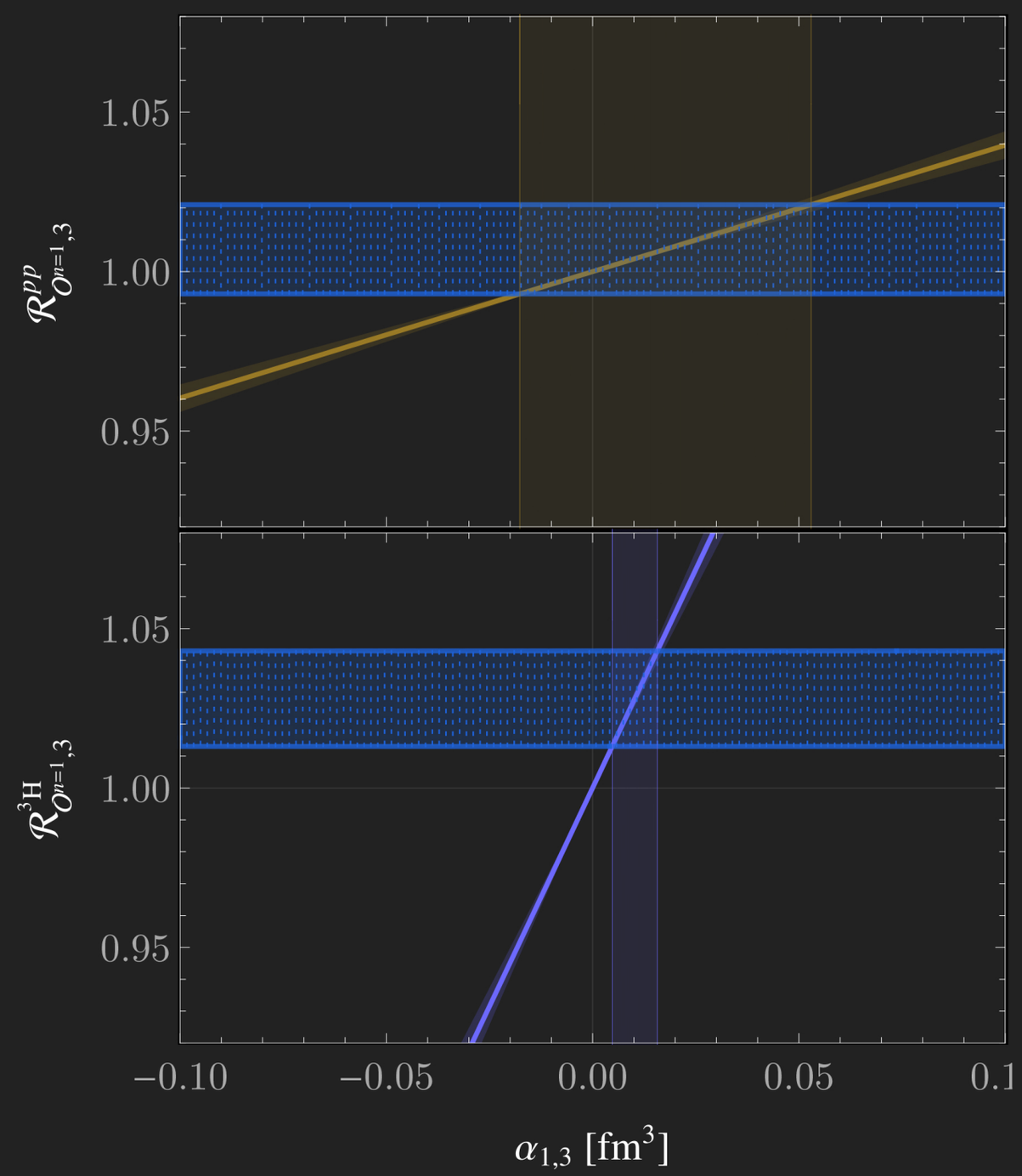
FVEFT USING THE STOCHASTIC VARIATIONAL METHOD

- Given EFT wavefunctions, matrix elements are easily computed
- LQCD matching determines EFT counterterms
  - Enables infinite volume prediction for matrix elements
  - Example: isovector momentum fraction

$$\mathcal{R}^h_{\mathcal{O}^n,3} \equiv \frac{A^h}{(Z^h - N^h)\langle x^n \rangle_3} \frac{\left\langle \Psi_h \middle| \mathcal{O}_3^{(n)} \middle| \Psi_h \right\rangle}{\langle \Psi_h \mid \Psi_h \rangle}$$
$$= \left(1 + \frac{\alpha_{n,3}}{(Z^h - N^h)\langle x^n \rangle_3} h_h(\Lambda, L)\right)$$

Two body counterterm



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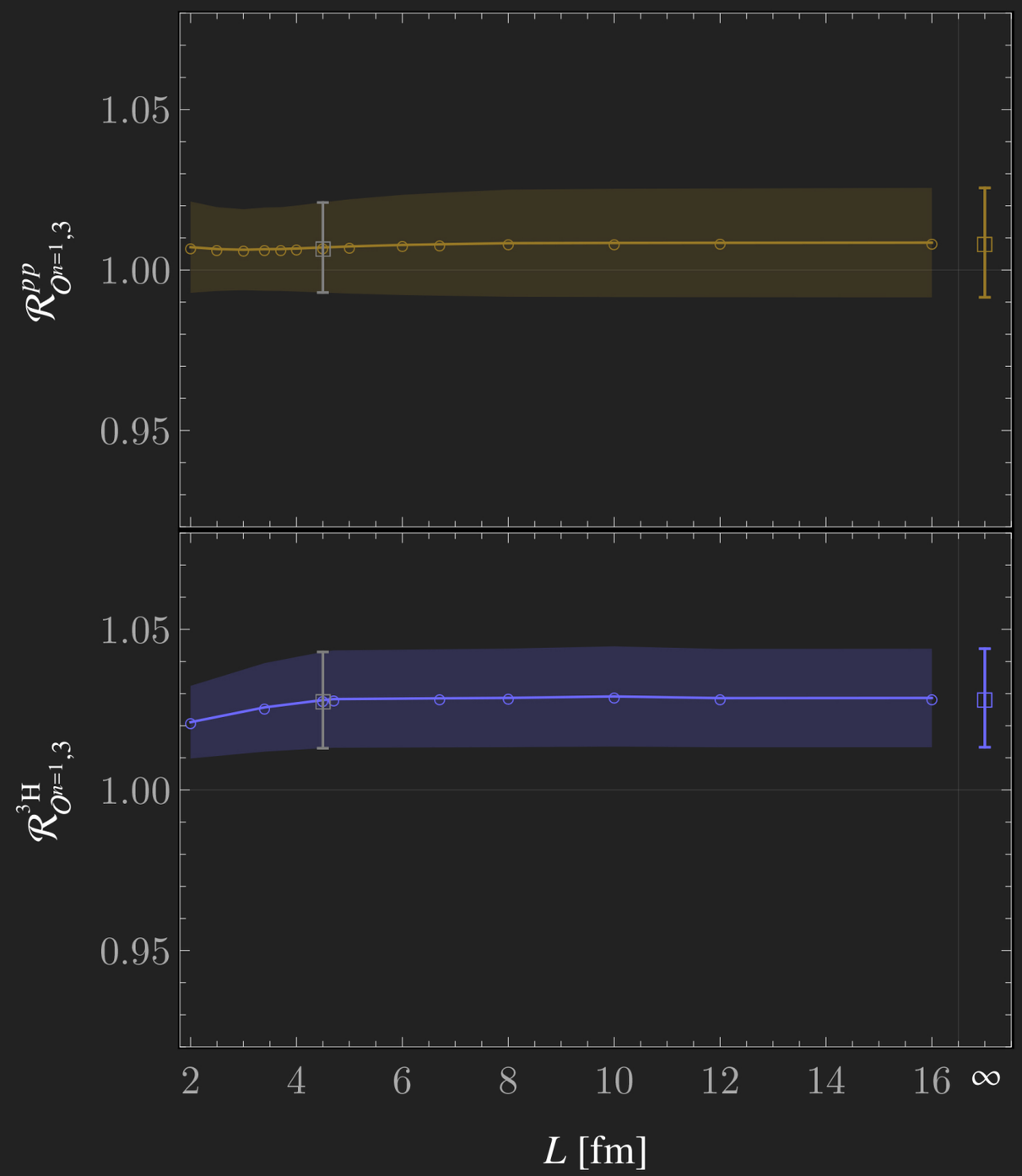
Two body counterterm



[WD, P Shanahan. 2102.04329]

OUTLOOK

- First LQCD constraints on nuclear PDFs via EFT matching
- Other flavour and spin structures underway
- Larger nuclei: might require different many-body techniques
  - Nuclear lattice EFT
- x-dependence of nuclear PDFs
  - Reconstruct from enough moments
  - Use new lattice methods based on large momentum effective theory (LaMET)