# The Generalized Contact Formalism (GCF)



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# The GCF

- Effective theory for analyzing short-range correlations
- Quantitative study of SRC
- Clear definition of SRC abundances
- Comprehensive and consistent picture: Structure and Reaction
- Bridging the gap between experimental studies and ab-initio calculations (A > 3)
- Connection to the underlying interaction

• The factorization of the wave function:



 $\varphi(r) \equiv$  Solution of the **two-body** Schrodinger Eq.

• The factorization of the wave function:

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12} \to 0} \varphi(r) A(\mathbf{R}, \{\mathbf{r}_k\}_{k \neq 1, 2})$$

For any **short-range** two-body operator  $\hat{O}$  (assuming that acts on protons):



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For any **short-range** two-body operator  $\hat{O}$  (assuming that acts on protons):

$$\langle \hat{O} \rangle = \sum_{i < j} \langle \Psi | \hat{O}(\mathbf{r}_{ij}) | \Psi \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \frac{Z(Z-1)}{2} \langle A | A \rangle$$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle C_{pp}$$



# The contact formalism





# The contact formalism



Main channels:

The **deuteron** channel:  $\ell_2 = 0,2$ ;  $s_2 = 1$ ;  $j_2 = 1$ ;  $t_2 = 0$ 

The **spin-zero** channel:  $\ell_2 = 0$ ;  $s_2 = 0$ ;  $j_2 = 0$ ;  $t_2 = 1$ 

# The generalized contact formalism

• Generalization of the **atomic contact theory**  $\Psi \xrightarrow{r_{ij} \to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) A(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j})$ 

S. Tan (2008)

 $C \propto \langle A | A \rangle$ 

# The generalized contact formalism

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**S. Tan** (2008)

- $C \propto \langle A | A \rangle$
- Relies also on **significant nuclear physics works**:
  - Wave function factorization (not asymptotic form) [Levinger (1951), Heidmann (1950),
     Brueckner (1955)]
  - Universal high-momentum tail [Amado (1976), Zabolitzky (1978)]
  - Universal SRC pairs [Frankfurt and Strikman (1988)]  $\sigma_{eA}/A = a_2 \sigma_{ed}/2$
  - Asymptotic factorization with deuteron-like pairs [Ciofi degli Atti and Simula (1996)]
  - Modern ab-initio calculations, pp and nn contributions [Feldmeier (2011), Alvioli (2013),
     Wiringa (2014), Ryckebusch (2015)...]
  - Asymptotic high-momentum factorization with two-body universal function and state-dependent coefficient (the contact) [Bogner and Roscher (2012)]

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$$\Psi \xrightarrow{\boldsymbol{r}_{ij \to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j}) \qquad ; \quad \boldsymbol{C}_{ij}^{\alpha\beta} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

# The nuclear contact relations

# **Contacts:** Probability of having a correlated pair in a nucleus

**Goal:** Derive relations between the contacts and nuclear quantities

$$\langle \hat{O} \rangle = \sum_{i < j} \langle \Psi | \hat{O}(\mathbf{r}_{ij}) | \Psi \rangle = \frac{C \langle \varphi | \hat{O}(\mathbf{r}) | \varphi \rangle}{C \langle \varphi | \hat{O}(\mathbf{r}) | \varphi \rangle}$$

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# The nuclear contact relations



 $S^{p}(\boldsymbol{p_{1}}, \epsilon_{1}) = \boldsymbol{C_{pn}^{1}} S_{pn}^{1}(\boldsymbol{p_{1}}, \epsilon_{1}) + \boldsymbol{C_{pn}^{0}} S_{pn}^{0}(\boldsymbol{p_{1}}, \epsilon_{1}) + 2\boldsymbol{C_{pp}^{0}} S_{pp}^{0}(\boldsymbol{p_{1}}, \epsilon_{1})$ 

# **Two-body densities**



RW et al., PLB 780, 211 (2018) VMC data from: *R.B Wiringa et. al.*, *Phys. Rev. C* 89, 024305 (2014)

# **Two-body densities**

$$\rho_{nn}(\mathbf{r}) \xrightarrow{\mathbf{r} \to \mathbf{0}} C_{nn}^{\mathbf{0}} |\varphi_{nn}^{\mathbf{0}}(\mathbf{r})|^2$$

$$n_{nn}(k_{rel}) \xrightarrow[k \to \infty]{} C^0_{nn} |\tilde{\varphi}^0_{nn}(k_{rel})|^2$$

#### Short-range and high-momentum factorization!

$$\Psi \xrightarrow{\boldsymbol{r}_{ij \to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$



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# **One-body momentum tail**

$$n_{p}(k) \xrightarrow[k \to \infty]{} C^{d}_{pn} |\varphi^{d}_{pn}(k)|^{2} + C^{0}_{pn} |\varphi^{0}_{pn}(k)|^{2} + 2C^{0}_{pp} |\varphi^{0}_{pp}(k)|^{2}$$

# **One-body momentum tail**

 $n_{p}(k) \xrightarrow[k \to \infty]{} \frac{C_{pn}^{d}}{\varphi_{pn}^{d}(k)}^{2} + \frac{C_{pn}^{0}}{\varphi_{pn}^{0}(k)}^{2} + 2C_{pp}^{0} |\varphi_{pp}^{0}(k)|^{2}$ 



No fitting parameters!

RW et al., PLB 780, 211 (2018)

 $n_p(k)$ 

# **Electron-scattering experiments**

A(e, e'N) in the plane-wave impulse approximation:



# **The spectral function**

$$S(p_{1},\epsilon_{1}) = \sum_{s} \sum_{f_{A-1}} \delta(\epsilon_{1} + E_{f}^{A-1} - E_{0}) \left| \left\langle f_{A-1} \middle| a_{p_{1},s} \middle| \psi_{0} \right\rangle \right|^{2}$$

# **The spectral function**

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The initial wave function

$$\boldsymbol{\psi}_{0} \rightarrow \sum_{\alpha} \varphi_{ij}^{\alpha} (\boldsymbol{r}_{ij}) A_{ij}^{\alpha} (\boldsymbol{R}_{ij}, \{\boldsymbol{r}_{k}\}_{k \neq i, j})$$

The final wave function

$$|\psi_f^{12}\rangle = a_{p_1,s}^{\dagger}|f_{A-1}\rangle \propto |\Psi_v^{A-2}\rangle e^{ip_1\cdot r_1 + ip_2\cdot r_2}\chi_{s_1}\chi_{s_2}$$

Energy conservation:

$$\begin{split} E_{f}^{A-1} &= \epsilon_{2} + (A-2)m - B_{f}^{A-2} + \frac{P_{12}^{2}}{2m(A-2)} \\ B_{f}^{A-2} &\approx \langle B_{f}^{A-2} \rangle \end{split}$$

RW et al., PLB 791, 242 (2019)

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Energy conservation:  $E_f^{A-1} = \epsilon_2 + (A-2)m - B_f^{A-2} + \frac{P_{12}^2}{2m(A-2)}$ 

 $S^{p}(\boldsymbol{p_{1}}, \epsilon_{1}) = C^{1}_{pn}S^{1}_{pn}(\boldsymbol{p_{1}}, \epsilon_{1}) + C^{0}_{pn}S^{0}_{pn}(\boldsymbol{p_{1}}, \epsilon_{1}) + 2C^{0}_{pp}S^{0}_{pp}(\boldsymbol{p_{1}}, \epsilon_{1})$ 

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 $S_{ij}^{\alpha}$  expressed using  $\varphi_{ij}^{\alpha}$ 

# **Exclusive experiments**

Using the spectral function

$$\frac{\#pp}{\#pn} = \frac{S_{pp}^{0}(p_{1},\epsilon_{1})}{\frac{C_{pn}^{1}}{C_{pp}^{0}}S_{pn}^{1}(p_{1},\epsilon_{1}) + S_{pn}^{0}(p_{1},\epsilon_{1})}$$



Assuming isospin symmetry for symmetric nuclei

 $C^{\,0}_{pp}\approx C^{\,0}_{pn}$ 

# **Exclusive experiments**

Using the spectral function





AV18 <sup>4</sup>He



Data from: Korover et al., PRL 113, 022501 (2014)

# **GCF-based event generator**



A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, et al. (CLAS collaboration), Nature 578, 540 (2020)



A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, et al. (CLAS collaboration), Nature 578, 540 (2020)



J.R. Pybus et al., PLB 805, 135429 (2020)



I. Korover et al., arXiv:2004.07304 (2020)

**GCF-based event generator** 



**GCF-based event generator** 



# **Inclusive scattering**

![](_page_29_Picture_1.jpeg)

 $a_2 = 2/A \frac{\sigma_A(x_B, Q^2)}{\sigma_d(x_B, Q^2)} \approx 2/A \frac{C_{pn}^1(A)}{C_{nn}^1(d)}$ 

![](_page_29_Picture_3.jpeg)

![](_page_29_Figure_4.jpeg)

![](_page_30_Figure_0.jpeg)

# **Charge density**

$$r \rightarrow \infty$$
:  $\rho_{pp}(\mathbf{r}) \propto \rho_{pp}^{UC} \equiv \int d^3 R \rho_p(\mathbf{R} + \mathbf{r}/2) \rho_p(\mathbf{R} - \mathbf{r}/2)$ 

$$\rho_{pp}(\boldsymbol{r}) \xrightarrow{r \to 0} C_{pp} |\varphi_{pp}(\boldsymbol{r})|^2$$

![](_page_31_Figure_3.jpeg)

In collaboration with Javier Menendez and Alessandro Lovato

• Matrix elements

$$M_{\alpha}^{\beta} = \left\langle \Psi_{f} \left| O_{\alpha}^{\beta} \right| \Psi_{i} \right\rangle$$

$$\alpha = F, GT, T$$
  
$$\beta = V, AA, AP, PP, MM$$

(Vector, Axial, Pseudoscalar, Magnetic)

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$$\alpha = F, GT, T$$
  
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$$O_F^\beta = (4\pi R_A) \sum_{a\neq b} V_F^\beta(r_{ab}) \tau_a^+ \tau_b^+$$

$$O_{GT}^{\beta} = (4\pi R_A) \sum_{a \neq b} V_{GT}^{\beta}(r_{ab}) (\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b) \tau_a^+ \tau_b^+$$

$$R_A = 1.2A^{1/3} \qquad \qquad O_T^\beta = (4\pi R_A) \sum_{a \neq b} V_T^\beta(r_{ab}) S_{ab} \tau_a^+ \tau_b^+$$

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• SRC are commonly accounted for using correlation functions

<sup>(</sup>Vector, Axial, Pseudoscalar, Magnetic)

• Matrix elements

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- SRC are commonly accounted for using correlation functions
- Instead: Use directly GCF expressions!

<sup>(</sup>Vector, Axial, Pseudoscalar, Magnetic)

• Transition densities: (Fermi as an example)

$$\rho_F(r) = \frac{1}{4\pi r^2} \left\langle \Psi_f \left| \sum_{a < b} \delta(r - r_{ab}) \tau_a^+ \tau_b^+ \right| \Psi_i \right\rangle$$
$$M_F^\beta = (4\pi R_A) \int V_F^\beta(r_{ab}) \rho_F(r) 4\pi r^2 dr \equiv \int \rho_F^\beta(r) dr$$

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$$M_F^\beta = (4\pi R_A) \int V_F^\beta(r_{ab}) \rho_F(r) 4\pi r^2 dr \equiv \int \rho_F^\beta(r) dr$$

• Short distances:

$$\rho_F(r) \xrightarrow{r \to 0} |\varphi^0(r)|^2 C^0_{pp,nn}(f,i) \qquad C(f,i) \propto \langle A_f | A_i \rangle$$

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$$M_{F}^{\beta} = \int_{0}^{r_{0}} \rho_{F}^{\beta}(r) dr + \int_{r_{0}}^{\infty} \rho_{F}^{\beta}(r) dr$$

$$\int_{\text{GCF}} \text{Shell Model}$$

![](_page_40_Figure_1.jpeg)

![](_page_41_Figure_1.jpeg)

 $^{12}Be \rightarrow ^{12}C$ 

#### Merging option 1: Continuity with SM

0.001 0.000 VMC -0.001GCF+SM 1 Preliminary -0.002 $ho_F^V$ -0.003-0.004WSS psd VMC5 -0.005WSS merge 1 2 6 0 4 8 10 r [fm]

 $^{12}Be \rightarrow ^{12}C$ 

Merging option 2: Assuming contact value is known

0.001 VMC 0.000 -0.001GCF+SM 2 Preliminary -0.002 $ho_F^V$ -0.003-0.004WSS psd VMC5 -0.005WSS merge 2 2 8 4 6 10 r [fm]

 $^{12}Be \rightarrow ^{12}C$ 

Matrix element results

$$^{12}Be \rightarrow ^{12}C$$

 VMC:
  $M_F^V = -0.22$  

 Shell-Model (WSS psd):
  $M_F^V = -0.41$  

 GCF Merge 1:
  $M_F^V = -0.29$  

 GCF Merge 2:
  $M_F^V = -0.24$ 

#### **Collaborators**

Jerusalem: B. Bazak, N. Barnea

**Experiment:** O. Hen, E. Piasetzky, L. B. Weinstein , D. W. Higinbotham, A. Schmidt, R. Cruz-Torres, I. Korover, M. Duer, J. R. Pybus, A.W. Denniston

**Theory:** G. A. Miller, R. B. Wiringa, D. Lonardoni, M. Piarulli, E. Pazy, A. Lovato, J. Menendez

# **Summary**

- The GCF provides a framework for quantitative studies of SRC pairs.
- > Describes the effects of SRCs on nuclear structure and reactions.
- Good agreement with ab-initio structure calculations.
- Used to analyze, guide and design experiments (event generator)
- Allows confronting data with ab-initio structure calculations and

different NN interaction models

![](_page_46_Figure_7.jpeg)

![](_page_46_Figure_8.jpeg)