Momentum Distributions and Short-Range Correlations in Few-Nucleon Systems with Local and Non-Local Interactions

Laura E. Marcucci

University of Pisa & INFN-Pisa

LNS Laboratory for Nuclear Science and the EIC² EIC Center at Jefferson Lab

3rd Workshop on Quantitative Challenges in EMC and SRC Research

- L.E. Marcucci *et al.*, Phys. Rev. C **99**, 034003 (2019)
- Work in progress in collaboration with M. Viviani (INFN-Pisa)
- Many thanks to Ronen and Rey for useful discussions (GCF)

Outline

- Which **quantities** are we interested in
- The ab-initio method of choice (**the HH method**)
- The **nuclear interaction models** adopted
- Results for $A = 3$ nuclei (mostly ^{3}He)
- Conclusions and outlook

Definitions for $A = 3$

Two-nucleon momentum distributions

$$n^{N_1 N_2}(k_{rel}, K_{c.m.}) = \int d\hat{\mathrm{k}}_{rel} \int d\hat{\mathrm{K}}_{c.m.} \Psi^\dagger(\mathrm{k}_{rel}, \mathrm{K}_{c.m.}) P_{N_1 N_2} \Psi(\mathrm{k}_{rel}, \mathrm{K}_{c.m.})$$

$$n^{N_1 N_2}(k_{rel}, K_{c.m.} = 0) \rightarrow \mathrm{back-to-back\ (BB)}$$

$$n^{N_1 N_2}(k_{rel}) = 4\pi \int_0^{K^+_{c.m.}} K^2_{c.m.}\, dK_{c.m.}\, n^{N_1 N_2}(k_{rel}, K_{c.m.})$$

$$n^{ST}(k_{rel}) = 4\pi \int d\mathrm{K}_{c.m.} \int d\hat{\mathrm{k}}_{rel}\, \Psi^\dagger(\mathrm{k}_{rel}, \mathrm{K}_{c.m.}) P_{ST} \Psi(\mathrm{k}_{rel}, \mathrm{K}_{c.m.})$$

Integrated probabilities (SRCs)

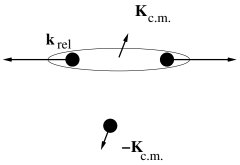
$$N^{BB}_{N_1 N_2} = 4\pi \int_0^\infty n^{N_1 N_2}(k_{rel}, K_{c.m.} = 0) k^2_{rel}\, dk_{rel}$$

$$N^{SRC,BB}_{N_1 N_2} = 4\pi \int_{k^-_{rel}}^\infty n^{N_1 N_2}(k_{rel}, K_{c.m.} = 0) k^2_{rel}\, dk_{rel}$$

$$N^{SRC}_{N_1 N_2} = 4\pi \int_{k^-_{rel}}^\infty n^{N_1 N_2}(k_{rel}) k^2_{rel}\, dk_{rel}$$

$$k^-_{rel} = 1.5\,\mathrm{fm}^{-1}$$

$$N_{N_1 N_2} = 4\pi \int_0^\infty n^{N_1 N_2}(k_{rel}) k^2_{rel}\, dk_{rel}$$



large k_{rel}
small $K_{c.m.}$ compared to k_F
$\Rightarrow$ SRCs

Generalized Contact Formalism (GCF)

The GCF in a nutshell

$$n_A^{N_1N_2,\alpha}(k) \rightarrow C_A^{N_1N_2,\alpha} \times |\phi^\alpha_{N_1N_2}(k)|^2$$
$$\rho_A^{N_1N_2,\alpha}(r) \rightarrow C_A^{N_1N_2,\alpha} \times |\phi^\alpha_{N_1N_2}(r)|^2$$

- $\phi^\alpha_{N_1N_2}(k/r) \rightarrow$ universal functions
  - **do not depend on the nucleus**
  - **depend on the interaction**
  - $\phi^\alpha_{N_1N_2}(r) \rightarrow$ solution of the nuclear zero-energy two-body problem
  - $\phi^\alpha_{N_1N_2}(k) \rightarrow$ FT of $[\phi^\alpha_{N_1N_2}(r)\,\mathrm{e}^{-(r/c)^2}]$ ($c \sim 50$ fm)
- I will consider only
  - $n_A^{N_1N_2,\alpha}(k)$
  - $N_1N_2 = pp/nn$ in ^{3}He/^{3}H, in $\alpha \equiv [s=0, t=1]$

$$C^{nn,s=0}_{^3\mathrm{H}} = n^{nn,s=0}_{^3\mathrm{H}}(k)/|\phi^{s=0}_{nn}(k)|^2$$
$$C^{pp,s=0}_{^3\mathrm{He}} = n^{\mathrm{pp},s=0}_{^3\mathrm{He}}(k)/|\phi^{s=0}_{\mathrm{nn}}(k)|^2$$

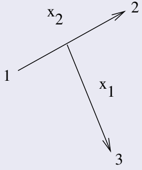
(Coulomb effect hopefully small)

The Hyperspherical Harmonics (HH) method (I)

- $|\Psi(1,\cdots,A)\rangle = \Sigma_\mu c_\mu |\Phi_\mu\rangle$
- Appropriate choice of $|\Phi_\mu\rangle$
- Appropriate variational principle $\rightarrow c_\mu \quad [\Sigma_\nu \langle\Phi_\mu|H - E|\Phi_\nu\rangle c_\nu = 0]$

The HH method in r-space

Jacobi coordinates $\rightarrow$ Hyperspherical coordinates



$$[\rho,\Omega_r],\ \ \Omega_r = [\hat{x}_1,\hat{x}_2,\phi]$$

$$\rho = \sqrt{x_1^2 + x_2^2},\ \ \tan\phi = \frac{x_1}{x_2}$$

$$\begin{aligned} |\Phi_\mu\rangle &= f_l(\rho)\, Y_{\{G\}}(\Omega_r) \\ f_l(\rho) &\equiv \text{hyperradial functions} \\ Y_{\{G\}}(\Omega_r) &\equiv \text{spin} - \text{isospin HH functions} \end{aligned}$$

The Hyperspherical Harmonics (HH) method (II)

The HH method in p-space

- Jacobi conjugate momenta $\rightarrow$ Hyperspherical coordinates

$$[Q, \Omega_p], \;\; \Omega_p = [\hat{k}_1, \hat{k}_2, \phi]$$

$$Q = \sqrt{k_1^2 + k_2^2}, \;\; \tan\phi = \frac{k_1}{k_2}$$

- Fourier transform of $f_l(\rho) Y_{\{G\}}(\Omega_r)$

$$|\Phi_\mu\rangle = g_{G,l}(Q) Y_{\{G\}}(\Omega_p)$$

$g_{G,l}(Q) =$ integral on ρ of $f_l(\rho) \times \; J_{G+2}(Q\rho)$

M. Viviani *et al.*, Few-Body Syst. **39**, 159 (2006)

$\Rightarrow$ we can use **ANY** potential (local and non-local)

$\Rightarrow$ we can calculate $n(k)$ and $\rho(r)$ directly in p- and r-space (use $|\Phi_\mu\rangle$ in p- or r-space, the c_μ are the same)

The nuclear interactions

Local interactions

- AV18/UIX (phenomenological)
- Norfolk chiral interactions: NV2+3/Ia* and NV2+3/Ib*

  I $\rightarrow$ fit for $E_{lab} \leq 125$ MeV; a/b $\rightarrow$ $R_S = 0.8/0.7$ fm

Non-local interactions

- CD-Bonn/TM (phenomenological)
- Chiral LO–N4LO / TNI N2LO ($\Lambda = 450, 500, 550$ MeV)

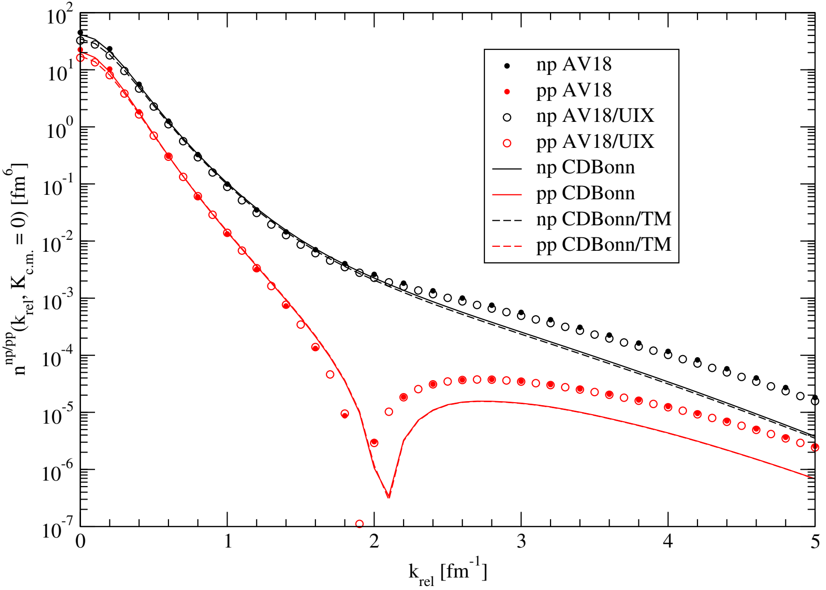
Note: Chiral interactions $\rightarrow$ $c_D - c_E$ fitted to $B(A = 3)$ and GT-tritium β decay

The HH results for light nuclei

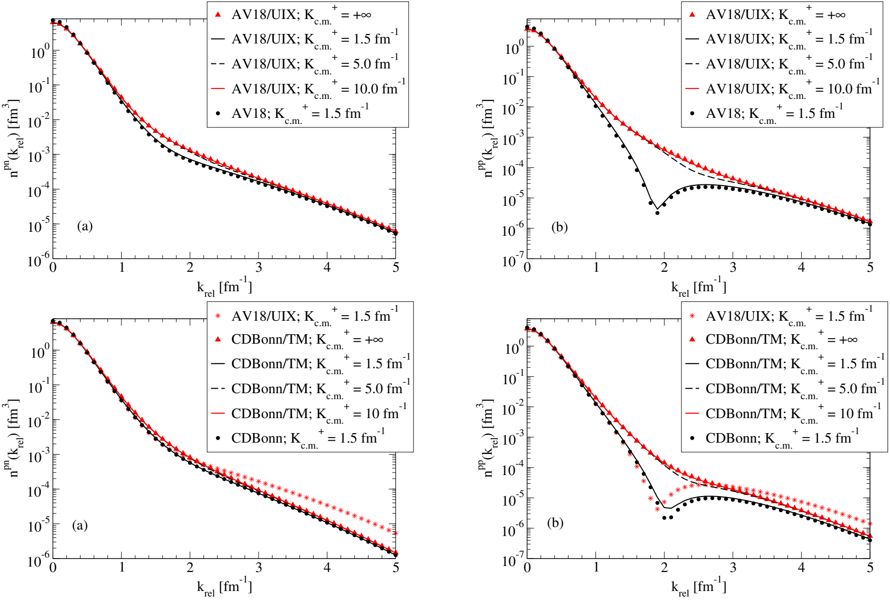
Model	$B(^3H)$	$B(^3He)$	$B(^4He)$
AV18/UIX	8.479	7.750	28.46
CDBonn/TM	8.474	7.720	29.00
NV2+3/Ia*	$\underline{8.477}$	$\underline{7.727}$	28.30
NV2+3/Ib*	$\underline{8.469}$	$\underline{7.724}$	28.21
LO500	11.091	10.409	40.09
NLO500	8.307	7.597	27.55
N2LO500/N2LO500	$\underline{8.474}$	$\underline{7.729}$	27.92
N3LO500/N2LO500	$\underline{8.477}$	$\underline{7.728}$	27.97
N4LO500/N2LO500	$\underline{8.477}$	$\underline{7.728}$	28.15
Exp.	8.475	7.725	28.30

L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

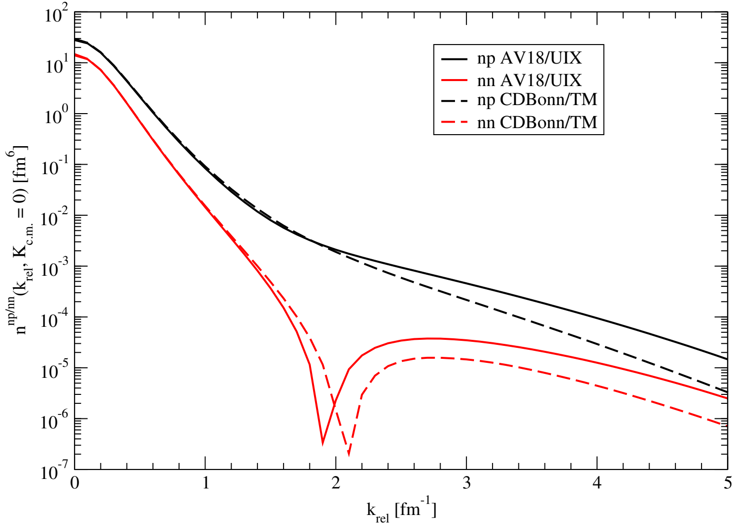
Two-N MDs in ^{3}He: phenomenological potentials (I)



Two-N MDs in ^{3}He: phenomenological potentials (II)



Two-N MDs in ^{3}H: phenomenological potentials (I)



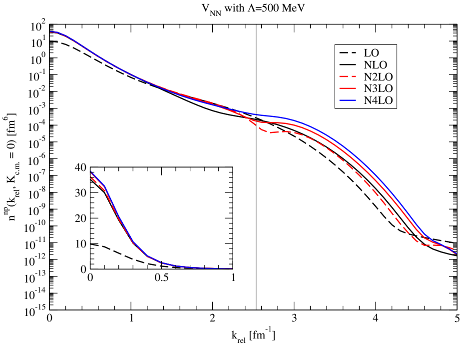
Integrated probabilities in ^{3}He

	$N_1N_2 = np$				$N_1N_2 = pp$			
	N^{BB}	$N^{SRC,BB}$	$N^{SRC}(k_{rel}^-)$	N	N^{BB}	$N^{SRC,BB}$	$N^{SRC}(k_{rel}^-)$	N
AV18	6.922	0.241	0.093	1.997	2.194	0.009	0.026	0.998
AV18/UIX	5.751	0.210	0.106	1.997	1.897	0.009	0.031	0.999
CDBonn	6.552	0.171	0.060	1.997	2.078	0.005	0.012	0.999
CDBonn/TM	5.931	0.157	0.063	1.998	1.924	0.005	0.014	0.998
AV18 - d[1]			0.042					
CDBonn - d[1]			0.032					

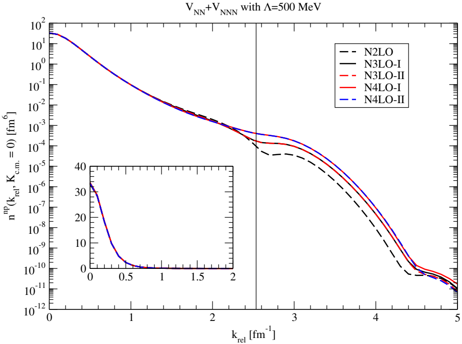
[1] F. Sammarruca, Phys. Rev. C **92**, 044003 (2015)

- Large model-dependence (see also F. Sammarruca on d^1)! $\rightarrow \neq V_{NN}$-tensor
- Very small TNI contribution

Two-N MDs in ^{3}He: chiral potentials (I)

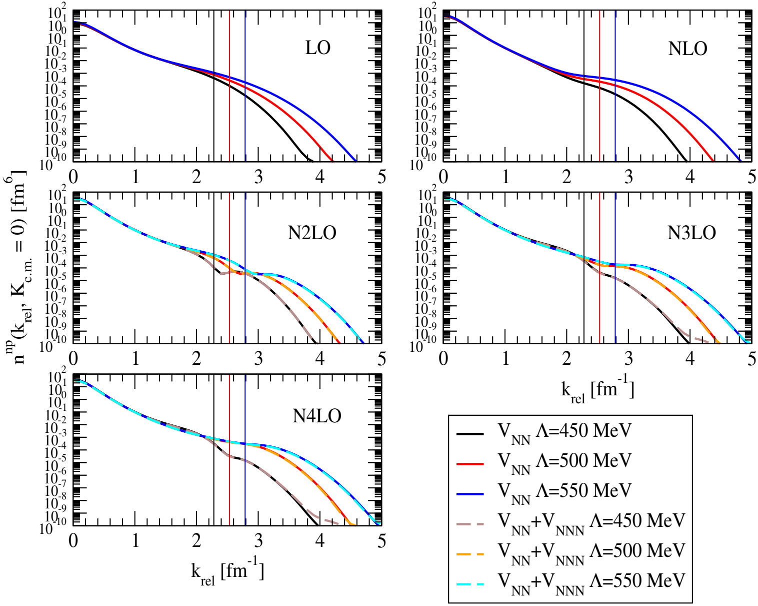


Order-by-order convergence

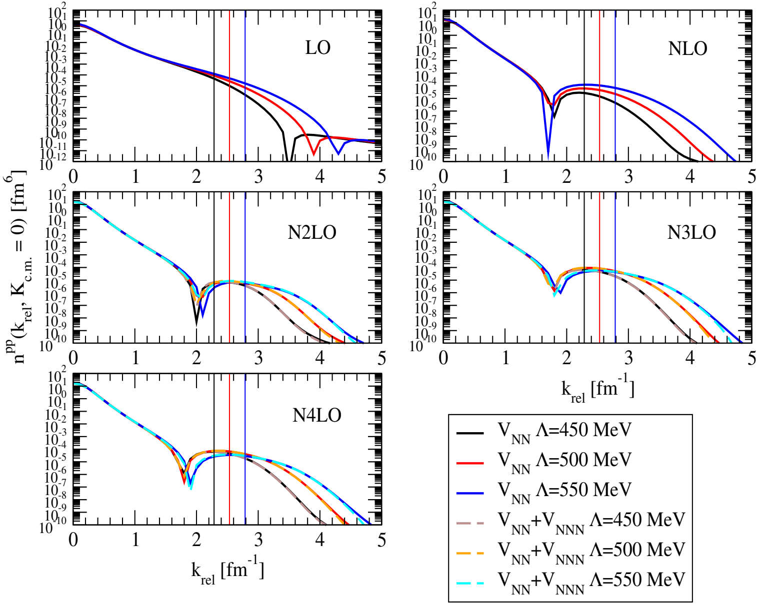


- asymptotic w.f. at LO $\neq$ NLO, N2LO ...
- Small TNI contributions
- N3LO $\neq$ N4LO for $k_{rel} \geq 2.2$ fm^{-1}

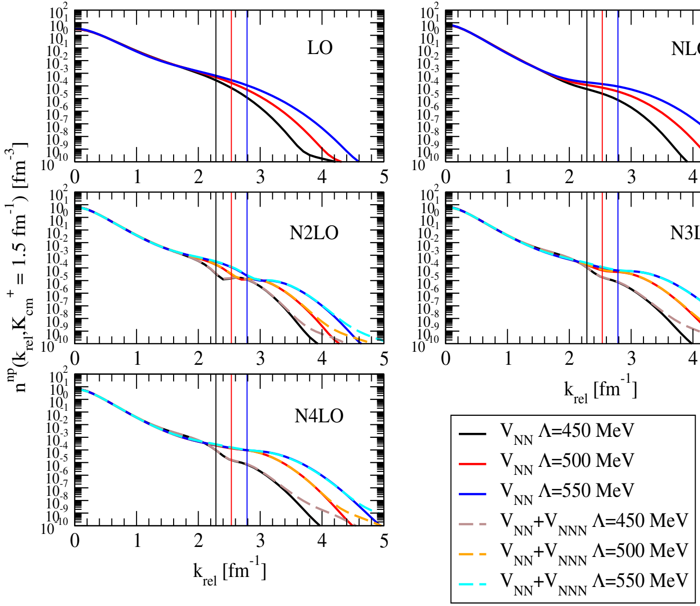
Two-N MDs in ^{3}He: chiral potentials (II)



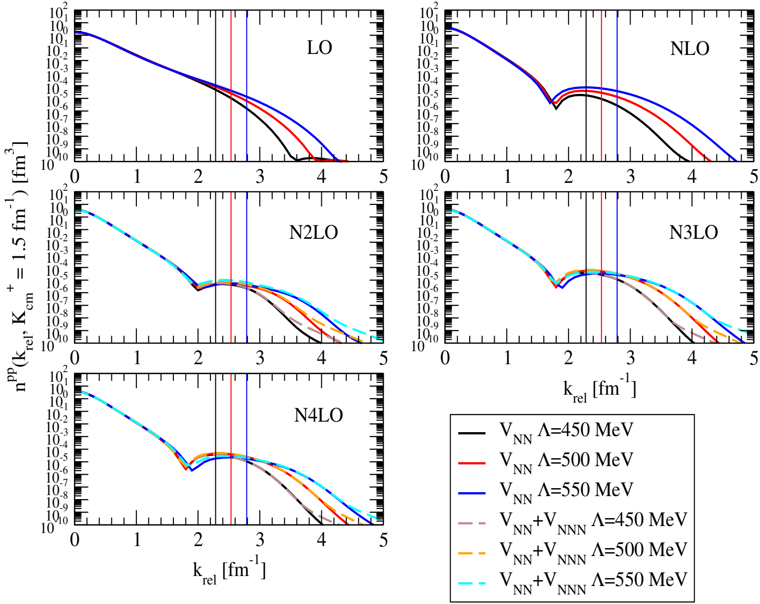
Two-N MDs in ^{3}He: chiral potentials (III)



Two-N MDs in ^{3}He: chiral potentials (IV)



Two-N MDs in ^{3}He: chiral potentials (V)



- cutoff-dependence only for $k_{rel} \geq 2.2 - 2.5$ fm^{-1}
- TNI contribution visible only for $k_{rel} \geq 3.0 - 3.5$ fm^{-1}

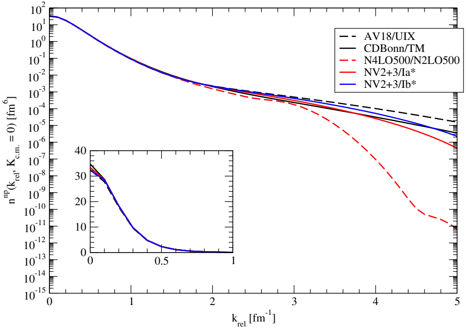
Integrated probabilities in ^{3}He (N_{np}^{SRC})

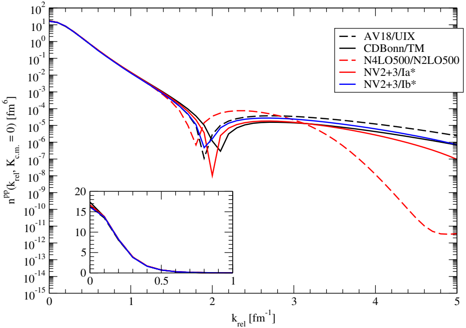
$$N_{np}^{SRC}(k_{rel}^-) = 4\pi \int_{k_{rel}^-}^{\infty} n^{np}(k_{rel}) k_{rel}^2 \, dk_{rel} \quad \text{with } k_{rel}^- = 1.5\,\text{fm}^{-1}$$

	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV	deuteron $\Lambda = 500$ MeV
LO	0.089	0.112	0.126	0.046
NLO	0.016	0.024	0.037	0.015
N2LO	0.029	0.038	0.045	0.026
N2LO/N2LO	0.030	0.040	0.050	
N3LO	0.039	0.039	0.044	0.024
N3LO/N2LO	0.042	0.043	0.050	
N4LO	0.038	0.042	0.047	0.024
N4LO/N2LO	0.040	0.045	0.051	

CDBonn/TM $\Rightarrow$ 0.063 (no TNI: 0.060)
AV18/UIX $\Rightarrow$ 0.106 (no TNI: 0.093)
$\Rightarrow$ small Λ-dependence $\Rightarrow$ small contributions from $k_{rel} \geq 2.2$ fm^{-1} region

Two-N BB MDs in ^{3}He with NV potentials





Integrated probabilities with NV potentials

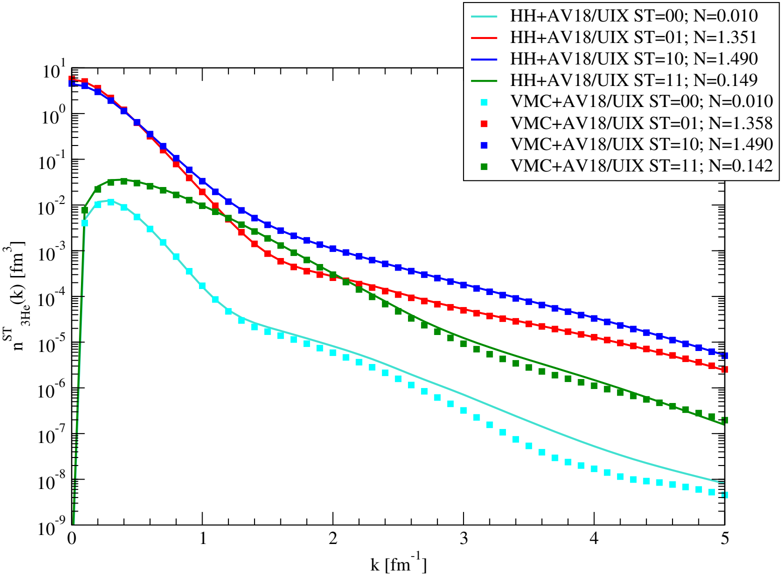
SRCs – BB ($N^{SRC,BB}_{N_1N_2}$)

$$N^{SRC,BB}_{N_1N_2} = 4\pi \int_{k^-_{rel}}^{\infty} n^{N_1N_2}(k_{rel}, K_{c.m.} = 0) k^2_{rel} \, dk_{rel}$$

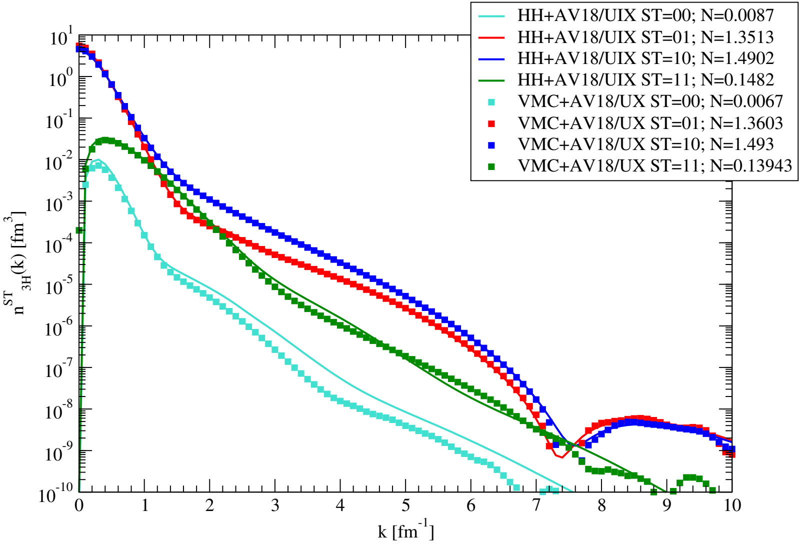
	$N^{SRC,BB}_{np}$	$N^{SRC,BB}_{pp}$
AV18/UIX	0.210	0.009
CDBonn/TM	0.157	0.005
N4LO500/N2LO500	0.113	0.006
NV2+3/Ia*	0.167	0.004
NV2+3/Ib*	0.185	0.005

⇒ again significant model-dependence for $N^{SRC,BB}_{np}$ ($N^{SRC,BB}_{pp}$ very small)

Spin-isospin projected MD: ^{3}He

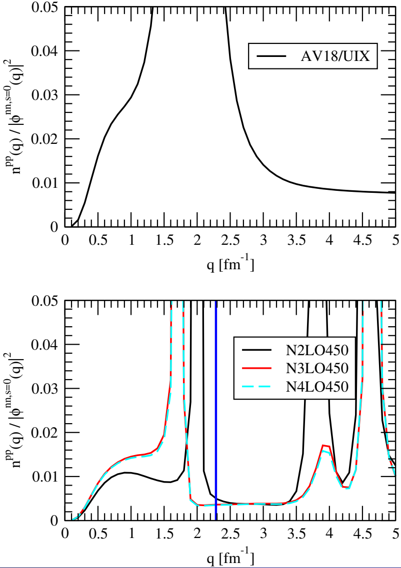


Spin-isospin projected MD: ^{3}H



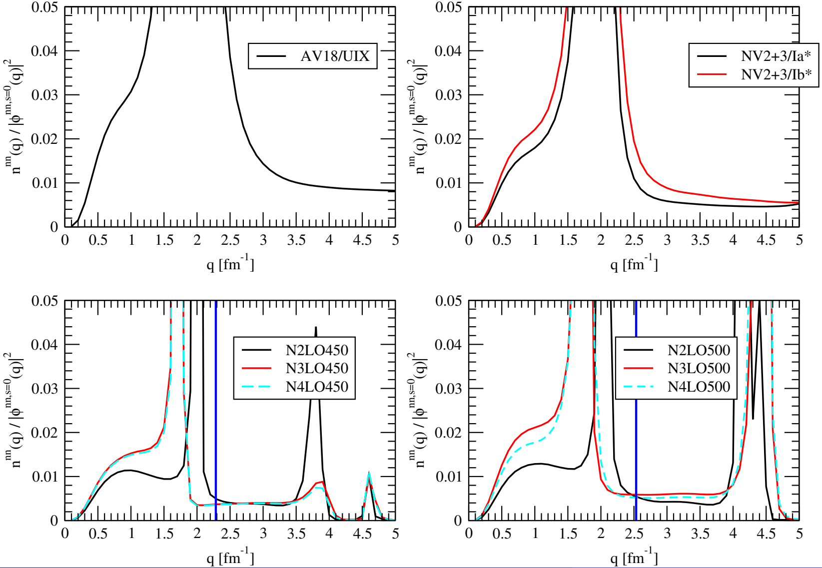
Nuclear contacts in GCF: ^{3}He

($\lim_{n\to\infty}$VERY^n PRELIMINARY RESULTS)



Nuclear contacts in GCF: ^{3}H

($\lim_{n\to\infty}$ VERY^n PRELIMINARY RESULTS)



Nuclear contacts in GCF: $C^{pp,s=0}_{^3\mathrm{He}}$ and $C^{nn,s=0}_{^3\mathrm{H}}$

$\lim_{n\to\infty}$VERY^n PRELIMINARY **ESTIMATES**

Model	$C^{pp,s=0}_{^3\mathrm{He}}$	Ref.[1]	$C^{nn,s=0}_{^3\mathrm{H}}$	Ref.[1]
AV18/UIX	0.591	0.590(70)	0.620	0.570(58)
NV2+3/Ia*	0.307	0.336(34)	0.317	
NV2+3/Ib*	0.353		0.400	
N2LO450/N2LO450	0.254		0.247	
N2LO500/N2LO500	0.277		0.284	
N2LO550/N2LO550	0.304		0.310	
N3LO450/N2LO450	0.250		0.257	
N3LO500/N2LO500	0.380		0.400	
N3LO550/N2LO550	0.367		0.387	
N4LO450/N2LO450	0.243		0.243	
N4LO500/N2LO500	0.337		0.363	
N4LO550/N2LO550	0.330		0.353	

Ref.[1]: R. Cruz-Torres *et al.*, Nature Physics **17**, 306 (2021)

- reasonable agreement with Ref.[1]
- differences between AV18/UIX (~ 0.6) and chiral potentials ($\sim 0.3 - 0.4$)
- chiral local and non-local potentials $\rightarrow$ similar results
- similar values for $C^{pp,s=0}_{^3\mathrm{He}}$ and $C^{nn,s=0}_{^3\mathrm{H}}$

Conclusions and outlook

Conclusions

- theoretical approach to calculate $A = 3$ MD and SRC with **essentially any potential**
- results for a wide variety of $A = 3$ MD and SRC
- first **PRELIMINARY steps** to study $C^{NN,\alpha}_{A=3}$ ($C^{pp/nn,s=0}_{A=3}$)

Conclusions and outlook

Conclusions

- theoretical approach to calculate $A = 3$ MD and SRC with **essentially any potential**
- results for a wide variety of $A = 3$ MD and SRC
- first **PRELIMINARY steps** to study $C^{NN,\alpha}_{A=3}$ ($C^{pp/nn,s=0}_{A=3}$)

Short-term outlook

- complete the calculation of $C^{NN,\alpha}_{A=3}$
  - **use other potentials** (local chiral interactions)
  - calculation of **other contacts** ($C^{np,s=0}_{A=3}$, $C^{np,s=1}_{A=3}$)
  - calculation of $\rho(r)$ and $C^{NN,\alpha}_{A=3}$ extracted from r-space

Conclusions and outlook

Conclusions

- theoretical approach to calculate $A = 3$ MD and SRC with **essentially any potential**
- results for a wide variety of $A = 3$ MD and SRC
- first **PRELIMINARY steps** to study $C_{A=3}^{NN,\alpha}$ ($C_{A=3}^{pp/nn,s=0}$)

Short-term outlook

- complete the calculation of $C_{A=3}^{NN,\alpha}$
  - **use other potentials** (local chiral interactions)
  - calculation of **other contacts** ($C_{A=3}^{np,s=0}$, $C_{A=3}^{np,s=1}$)
  - calculation of $\rho(r)$ and $C_{A=3}^{NN,\alpha}$ extracted from r-space

Long-term outlook

- repeat the study of MD, SRC, etc. for ^{4}He
- extend the study to $A = 6$ nuclei (the HH method $\rightarrow A = 6$)
  A. Gnech *et al.*, Phys. Rev. C **102**, 014001 (2020)

THANK YOU FOR YOUR ATTENTION!

THANK YOU FOR YOUR ATTENTION!

Thanks to my collaborators:

- A. Kievsky, M. Viviani (INFN Pisa)
- L. Girlanda (Univ. of Salento)
- M. Piarulli and S. Pastore (WUSTL)
- R. Schiavilla and A. Gnech (ODU+JLab)
- F. Sammarruca and R. Machleidt (Idaho Univ.)

SPARES

The universal functions $\phi^{\alpha}_{N_1 N_2}(k)$ (I)

$$
\begin{aligned}
\phi^{\alpha}_{N_1 N_2}(\mathrm{r}) &= \phi^{\alpha}_{C} + \phi^{\alpha}_{A} \\
\phi^{\alpha}_{C} &= \sum_{\mu} c^{\alpha}_{\mu} |\phi_\mu\rangle \ ; \qquad |\phi_\mu\rangle = f_m(r)|LSTJ\rangle \qquad \mu \equiv \{m, LSTJ\} \\
\phi^{\alpha}_{A} &= \Omega^{R}_{\alpha}(\mathrm{r}) + \sum_{\beta} R_{\alpha,\beta}\Omega^{I}_{\beta}(\mathrm{r}) \ ; \qquad \Omega^{R/I}_{\alpha}(\mathrm{r}) = F_L/G_L(kr)|LSTJ\rangle \\
c^{\alpha}_{\mu}, R_{\alpha,\beta} &\rightarrow \text{Kohn variational principle} \\
\phi^{\alpha\equiv lsj}_{N_1 N_2}(r) &= \int d\hat{\mathrm{r}}\phi^{\alpha\equiv lsj}_{N_1 N_2}(\mathrm{r}) Y^{*}_{lm}(\hat{\mathrm{r}}) = \sum_{m} c^{\alpha}_{m,\alpha} f_m(r) + F_l(kr) + R_{\alpha,\alpha}\, G_l(kr) \\
\phi^{\alpha}_{N_1 N_2}(k) &= 2\pi \int r^2\, dr\, d\cos\theta\ \phi^{\alpha}_{N_1 N_2}(r)\, \mathrm{e}^{-r^2/c^2}\, \mathrm{e}^{ikr\cos\theta} \\
c &\simeq 50\,\mathrm{fm}
\end{aligned}
$$

The universal functions $\phi^{\alpha}_{N_1 N_2}(k)$ (II)