Change Detection Based on Multiscale Basis

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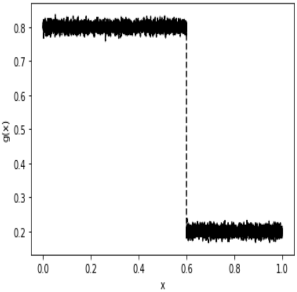
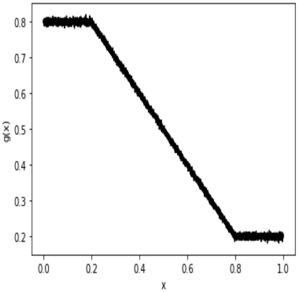
Two kinds of change

1. Change happens suddenly
2. Change happens gradually.



(a)



(b)

Noise and change

Noise : randomness, with mean zero. Noise happens everywhere, the noise can not change the mean of data.

Change (Suddenly) : change can reflect some information. If the change happens, the local mean of the data will change.

Basic idea

Problem : detect when and where the change happens.

Key observation : when no change happens, the local mean of the data will not change; when changes happens, the local means of the data will change obviously.

Question

How to measure the difference of local mean?

We can use the Multiscale basis to measure the difference of local mean.

A Simple Example

Our target function is

$$g(x) = \begin{cases} 0.8, & 0 \leqslant x \leqslant 0.6 \\ 0.2, & 0.6 < x \leqslant 1 \end{cases} \tag{1}$$

For this target function $g(x)$, the changes happens at $x = 0.6$.
Original test function f is

$$f(x) = \begin{cases} 1, & 0 \leqslant x \leqslant 1/2 \\ -1, & 1/2 < x \leqslant 1 \end{cases} \tag{2}$$

A sample Example

To detection the change, we check

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx$$
$$= \int_0^{1/2} g(x)dx - \int_{1/2}^1 g(x)dx$$

It can be viewed as the difference between the mean of $g(x)$ in $(0, 1/2)$ and the mean of $g(x)$ in $(1/2, 1)$.

A sample Example

We use the test function f to obtain the sequence of Multiscale test basis. Given level n, the Multiscale test basis is defined as

$$f_{n,j}(t) = 2^{n/2} f\left(2^n t - j\right), \quad j = 0, 1, \ldots, 2^n - 1 \tag{3}$$

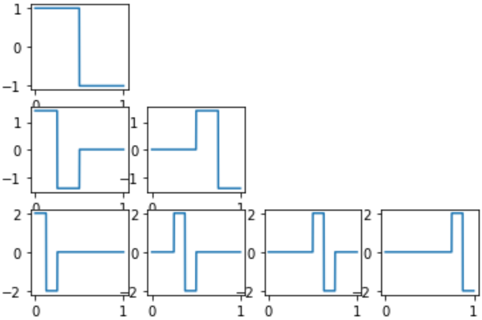


Figure 1: The Multiscale basis for scale level n = 0, 1, 2

A Sample Example

$$\begin{aligned}\langle f_{2,0}, g\rangle &= \int_0^1 f_{2,0}(x)g(x)dx \\ &= \int_0^{1/4} f_{2,0}(x)g(x)dx \\ &= 2\Big(\int_0^{1/8} g(x)dx - \int_{1/8}^{1/4} g(x)dx\Big)\end{aligned}$$

It can be viewed as the difference between the mean of $g(x)$ in $(0, 1/8)$ and the mean of $g(x)$ in $(1/8, 1/4)$. Compare with the original test function, we obtain the difference of local mean.

A Sample Example

Given an level n and a threshold value ϵ, We can check the

$$A_{n,j} = \langle f_{n,j}, g \rangle, \quad j = 0, 1, 2, \ldots, 2^n - 1$$

The the integration of g and $f_{n,j}$ measures the difference of local mean of g. This is our conclusion

1. if $|A_{n,j}| < \epsilon$, no change happens.
2. if $|A_{n,j}| > \epsilon$, change happens in the support of $f_{n,j}$. The support can be obtain from the index j.

1. The accuracy of change detection comes the length of the support, which is control by the level n.
2. The position information comes for the index of the basis.
3. The threshold ϵ is an experience parameters.

Change Detection For Functions

Suppose $g(x)$ defined on [0, 1] is the target function we will detect. The test function function f need satisfies

1. Vanishing moments [1] property of order k, that is

$$\langle f, (\cdot)^j \rangle = 0, j = 0, 1, \ldots, k-1.$$

2. Local support.
3. Identity norm,

$$\langle f, f \rangle = 1$$

To detect the anomoly, we integrate g and f.
Property 1 ensures that when there is no change happens, the result is sufficiently small, when the changes happens, the result is large, this is the key to distinct whether the changes happens or not.

Since the test function has local support, we can shift the support, property 2 can help to find where the changes happens and the length of the support determined the accuracy.

If we wish to get a higher accuracy, we can shrink the support of the test function, property 3 ensures that if we shrink the test function, the integration will not be very if the changes happens.

Algorithm 1: Multiscale Detection for Function

Target function $g(x)$, Initial test function $f(x)$. Level n, the Multiscale test basis $f_{n,j}(x)$ is defined by (3) , thredhold ϵ.;

for j from 0 to $2^n - 1$;

Calculate

$$A_{n,j} = \langle f_{n,j}, g \rangle$$

If $|A_{n,j}| \leqslant \epsilon$, no change happens;

If $|A_{n,j}| > \epsilon$, change happens, store the change index j;

End;

Calculate the change interval from the change index.;

Return the change interval.

Change Detection for Discrete Data

For the discrete data, we use the same idea with the continuous case. We partition the dataset into several set uniformly, and the use the test function to detect each set.

For a fixed test function, we use the test function to test each set independently. So the algorithm is easy to extend to test the online data.

Algorithm 2: Online detection for Discrete Data

Given period length n, the test function f, threshold ϵ, empty set W;

for each $t > 0$;

1. $W := W \cup \{x_n\}$;

2. If the length of W is n, then we use the test function f to test the set W. If change happens we we store the change interval $[t - n, t]$;

3. clear the set W to empty and return to step 1;

End;

Return the change interval.

Advantage

1. High computational efficiency. If the size data is n, then the computation is $O(n)$
2. The test function is based on the type of change. If the change happens suddenly, the test function with vanish moment order 1 is good enough, we only use the mean information. If the change happens gradually, we can use the higher moment information, the test function has least order 2 vanishing moment property.
3. Easy to extend to the online detect.

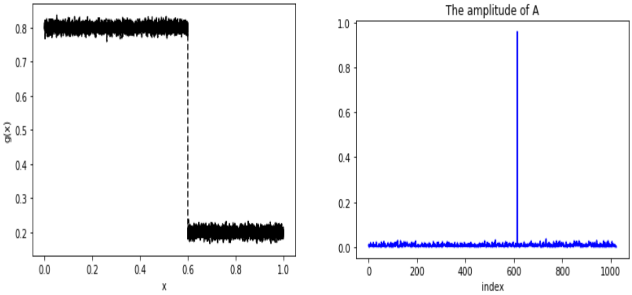
The advantage and the disadvantage of the Algorithm

Disadvantage

1. The threshold ϵ an experience parameter.
2. If the period is small, then the local means will might change if no change happens. The result may not be accurate.

Numerical Example 1

In this example, we detect the function (1) and the original test function is (2). We choose level $n = 10$, threshold $\epsilon = 0.1$. Add gaussian noise with mean 0 and variace 0.01. The result is : changes happens between (0.5996, 0.6005)



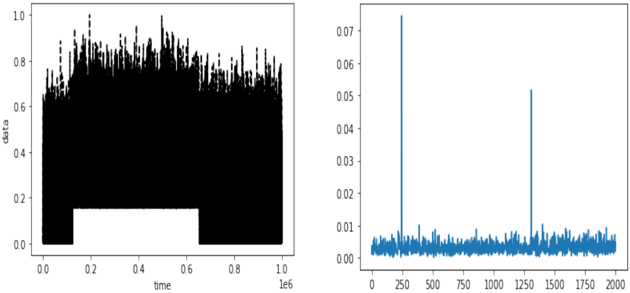
(a) (b)

Figure 2: (a) the target function $g(x)$, (b) the magnitude of A.

Numerical Example 2

We use a sample of ZEUS Monte Carlo simulations in the context of Inclusive Deep Inelastic Scattering to represent the data stream. In particular, we observe a stream Q^2 reconstructions based on the measurement of the (x, y, z) position and energy E of the outgoing lepton in the calorimeter. We use a sample of 1000000 events.

The data set we use is length = 1000000, change1 = 123456, change2 = 654321. We set the length of the result is 500, we can get
change happen at (123250, 123750)
change happen at (654000, 654500)
change happen at (654000, 654500) time : 4.853628158569336
We set the change index is the mean of the interval, that is, [123500, 654300].



(a) (b)

Figure 3: (a) the data set, (b) the detect result.

Z. Chen, C. A. Micchelli, and Y. Xu, *Multiscale methods for Fredholm integral equations*, vol. 28, Cambridge University Press, 2015.