Results from the Hall A GMp12 Experiment (E12-07-108)

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Hampton University / Jefferson Lab

on behalf of the GMp collaboration

2021 Hall A Winter Workshop  
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Proton magnetic form factor

- Form factors encode electric and magnetic structure of the nucleon

→ Form factors characterize the spatial distribution of the electric charge and the magnetization current in the nucleon

\[ |\text{Form Factor}|^2 = \frac{\sigma|\text{Structured object}|}{\sigma|\text{Point like object}|} \]

- In one photon exchange approximation the cross section in \( ep \) scattering when written in terms of \( G_M^p \) and \( G_E^p \) takes the following form:

\[
\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{\epsilon\left(G_E^p\right)^2 + \tau\left(G_M^p\right)^2}{\epsilon\left(1 + \tau\right)} , \quad \sigma_{\text{Mott}} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4 E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} 
\]

Where,

\[
\tau = \frac{Q^2}{4 M^2} , \quad \epsilon = \left[ 1 + 2 \left(1 + \tau\right) \tan^2\left(\frac{\theta}{2}\right) \right]^{-1}
\]
Methods of measurements

• **Rosenbluth separation method:**
  → This method uses different beam energies and angle at fixed $Q^2$

  $$\sigma_R = \frac{d\sigma}{d\Omega} \frac{\epsilon (1+\tau)}{\tau \sigma_{\text{Mott}}} = \frac{\epsilon}{\tau} \left( G_E^p \right)^2 + \left( G_M^p \right)^2,$$

  The slope of $\sigma_R(\epsilon)$ is directly related to $G_E^p$ and the intercept to

• **Recoil polarization technique:**

  Polarized electron transfers longitudinal polarization to $G_E^p$, but transverse polarization to $G_M^p$

  $$\frac{G_E}{G_M} = \frac{-P_t}{P_l} \frac{E_e + E_e'}{2M} \tan \left( \frac{\theta_e}{2} \right)$$

  **Polarization transfer cannot determine the values of $G_E$ and $G_M$ but can determine the form factor ratio.**
→ Discrepancy in $G_E/G_M$ P-T and Rosenbluth ($\varepsilon$) separations
Resolving the Rosenbluth vs P-T discrepancy

Leading explanation is hard 2-γ exchange, not included in standard radiative corrections of Mo-Tsai, etc.


→ Expected to be relatively small for P-T method
Conclusions from combined analysis of A. Afanasev, P. G. Blunden, D. Hasell, and B. A. Raue:

→ CLAS and VEPP-3 and OLYMPUS data exclude no TPE hypothesis at >95% confidence level

→ Data of insufficient precision to distinguish calculations of 2-γ contributions

→ Renormalization of OLYMPUS results required at twice the estimated uncertainty.
Non-linearities in existing Rosenbluth data

→ Existing data indicate *no significant* non-linearities vs $\epsilon$

Fit of elastic data to quadratic form

$$\sigma_r = P_0 + P_1 (\epsilon - 0.5) + P_2 (\epsilon - 0.5)^2$$

$$\langle P_2 \rangle = 0.019 \pm 0.027$$
Precision $G_M$ is part of the 12 GeV Form Factor Program

$→$ Precision $G_M$ required to study approach of QCD scaling in Dirac $F_1$

$$F_1 = \frac{(G_F + Q^2/4M_N^2 \times G_M)}{(1 + Q^2/4M_N^2)}$$

$→$ $F_2$ provides constraint on $E(x,t)$ GPD at high-$x$, high-$t$ via sum rules

$→$ Precision $G_M$ up to $Q^2 \sim 12$ GeV$^2$ complementary to 12 GeV polarization Transfer measurements of $G_E/G_M$
GMp and other High $Q^2$ data

GMp12 data at much smaller $\varepsilon$ than Sill data

$J\text{Lab data critical for } Q^2 > 6 \text{ GeV}^2$

\[ \frac{d\sigma}{d\Omega} = \sigma_{Mott} \frac{\varepsilon \left( G_E^p \right)^2 + \tau \left( G_M^p \right)^2}{\varepsilon \left( 1 + \tau \right)}, \]

- Less sensitivity to $G_E$ in extracting $G_M$
- Lever arm in $\varepsilon$ provides sensitivity to:
  - $2\gamma$ from global fit utilizing $G_E / G_M$ from polarization transfer
E12-07-108 Experiment Overview

- Precision measurement of the elastic $ep$ cross-section over the wide range of the $Q^2$ and extraction of proton magnetic form factor

➢ To improve the precision of cross section at high $Q^2$ by a factor of 3
➢ To provide insight into scaling behavior of the form factors at high $Q^2$

GMp Uncertainties:

**Statistical:** Significant improvement over existing data for $Q^2 > 6$

**Systematic Goals:**
Point to point: 0.8-1.1%
Normalization: 1.3%

Need a good control on:
- Beam charge
- Beam position
- Scattering angle
- target density, ...

![Graph showing $G^p_M/\mu_p G_D$ vs. $Q^2$]
Experimental setup

Jefferson Lab at Newport News Virginia

CEBAF: Continuous Electron Beam Accelerator Facility

Experimental Hall A

High resolution spectrometers
Jefferson Lab Hall A

HRS Parameters:
Acceptance: \(-4.5\% < \Delta p/p < 4.5\%\), 6 msr
Resolution: \(\delta p/p \leq 2 \times 10^{-4}\)
\(\Delta x'_{\text{tar}} = 0.5\) mrad (Horizontal)
\(\Delta y'_{\text{tar}} = 1.0\) mrad (Vertical)
Data collected during GMp

**Spring 2015:**

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ (GeV)</th>
<th>HRS</th>
<th>$P_0$ (GeV/c)</th>
<th>$\Theta_{\text{HRS}}$ (deg)</th>
<th>$Q^2$ (GeV/c)$^2$</th>
<th>Events (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.06</td>
<td>R</td>
<td>1.15</td>
<td>48.7</td>
<td>1.65</td>
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**Spring 2016:**

* Surveyed angles

<table>
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<th>HRS</th>
<th>$P_0$ (GeV/c)</th>
<th>$\Theta_{\text{HRS}}$ (deg)</th>
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<td>8.84</td>
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<td>8.84</td>
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<td>11.02</td>
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<td>2.20</td>
<td>48.8*</td>
<td>16.5</td>
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</table>

**Fall 2016:**

*Most complete systematic studies during this period

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ (GeV)</th>
<th>HRS</th>
<th>$P_0$ (GeV/c)</th>
<th>$\Theta_{\text{HRS}}$ (deg)</th>
<th>$Q^2$ (GeV/c)$^2$</th>
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<td>2.17</td>
<td>48.8*</td>
<td>15.8</td>
<td>3.6</td>
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</table>
Measurement of Elastic Cross Section

- Cross section:

\[
\frac{d\sigma}{d\Omega}(\theta) = \int dE' \frac{N_{\text{det}}(E', \theta) - N_{\text{BG}}(E', \theta)}{\mathcal{L} \cdot \epsilon_{\text{eff}} \cdot \text{LT}} \cdot A(E', \theta) \cdot \text{RC}
\]

- Reduced cross section:

\[
\sigma_{\text{red}} = \frac{d\sigma}{d\Omega} \frac{\epsilon(1 + \tau)}{\sigma_{\text{Mott}}} = \frac{4E^2 \sin^4 \frac{\theta}{2}}{\alpha^2 \cos^2 \frac{\theta}{2}} \frac{E}{E'} \epsilon(1 + \tau) \frac{d\sigma}{d\Omega}
\]

- Parameters:
  - \(N_{\text{det}}\): number of scattered elastic electrons detected
  - \(N_{\text{BG}}\): events from background processes
  - \(\mathcal{L}\): Integrated luminosity
  - \(\epsilon\): Corrections for efficiencies
  - \(\text{LT}\): live time correction
  - \(A(E', \theta)\): spectrometer acceptance
  - \(\text{RC}\): radiative correction factor
  - \(E\): beam energy
  - \(\theta\): Scattering angle

A thorough understanding of all these parameters is crucial for a precision cross section measurement
Extraction of Elastic $ep$ Cross Section

\[
\frac{d\sigma^\text{data}}{d\Omega}(\theta) = \int dE' \frac{N^\text{data}(E', \theta) - N_{BG}(E', \theta)}{L^\text{data} \epsilon L T} \cdot \frac{RC^\text{data}}{A^\text{data}(E', \theta)} \tag{1}
\]

\[
\frac{d\sigma^\text{mod}}{d\Omega}(\theta) = \int dE' \frac{N^\text{MC}(E', \theta)}{L^\text{MC}} \cdot \frac{RC^\text{MC}}{A^\text{MC}(E', \theta)}
\]

Assuming acceptance and radiative contributions are correctly modeled:

\[
\frac{d\sigma^\text{data}}{d\Omega}(\theta) / \frac{d\sigma^\text{mod}}{d\Omega}(\theta) = \frac{\int^{E_{\text{max}}} (N^\text{data}(E', \theta) - N_{BG}(E', \theta)) dE'}{\int^{E_{\text{max}}} N^\text{MC} dE'} \cdot \frac{A^\text{MC}(E', \theta)}{A^\text{data}(E', \theta)} \cdot \frac{RC^\text{data}}{RC^\text{MC}}
\]

Results were cross checked with acceptance correction method (eq 1) using Rad Cor based on code utilized for later SLAC experiments.
Detector efficiencies

$E_{\text{beam}} = 2.222$, $\theta = 42$

Detector efficiencies

$\varepsilon < 0.1\%$

$\varepsilon_{\text{cal}} > 99.8\%$

$\varepsilon_{\text{cer}} > 99.9\%$

$\varepsilon_{\text{cal}} > 99.8\%$

Number of photo-electrons

Pre-shower energy deposited

shower energy deposited

Number of photo-electrons

Cherenkov ADC sum

$\frac{\delta \varepsilon}{\varepsilon} < 0.1\%$
VDC Track Reconstruction Efficiency

➢ Standard Tracking for HRS VDCs utilizes single cluster only in each chamber
➢ GMp utilized additional Straw Chamber to perform precise checks on efficiency determination

Elastic events were reconstructed with:

1. single cluster in both VDCs
2. single cluster in 1 VDC + SC

<table>
<thead>
<tr>
<th>Kinematic</th>
<th>K3-4</th>
<th>K3-6</th>
<th>K3-7</th>
<th>K3-8</th>
<th>K4-9</th>
<th>K4-10</th>
<th>K4-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Yield ratio</td>
<td>1.0016</td>
<td>0.9994</td>
<td>0.9993</td>
<td>0.9985</td>
<td>1.0007</td>
<td>1.0021</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

Corrected yields agree to better than 0.2%

➢ A “coarse” track was formed using scintillator hit and straw chamber. This method enables us to estimate the track intercept at the focal plane without using VDC hits

Barak Schmookler (MIT)
Bashar Aljawrnhe (NC A&T)
Significant Effort to Improve Optics Calibration

- **Angle and vertex calibration**: used deep inelastic electrons from multi-foil carbon target

  A 9-foil carbon target covers a total length of 20 cm along the beam direction

- **Momentum calibration**: used elastic electrons from liquid hydrogen target

  Algorithm: Minimization of $\chi^2$ by varying the optics coefficients

  $$\chi^2(y_{tg}) = \sum_{\text{events}} (Y_{ijkl}x_{fp}^{ij}y_{fp}^{kl} \theta_{fp} - y_{tg}^{\text{survey}})^2$$
Example Data to Monte Carlo Comparison: LHRS

- Excellent comparison after subtraction of target cell endcaps via dummy (~3%)
- Small offsets in $W$ consistent with estimated kinematic uncertainties

Data to MC ratio: 1.0102
$P_0$: 2.6720 GeV/c
Beam energy = 6.427 GeV
Scattering angle = 37.01 deg
$Q^2 = 6.99$ (GeV/c)$^2$
Cross section = $2.89 \times 10^{-6}$ fb/sr
## Error Budget (LHRS Fall 2016)

<table>
<thead>
<tr>
<th>Source</th>
<th>$d\sigma/\sigma$ (%) (pt-pt)</th>
<th>$d\sigma/\sigma$ (%) (Norm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam charge ($\Delta I = 0.06 \mu A$)</td>
<td>0.6 (at 10 $\mu A$) - 0.1 (at 65 $\mu A$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Scattering angle ($\Delta \theta = 0.2$ mrad)</td>
<td>0.1 - 0.4</td>
<td>0.1 - 0.4</td>
</tr>
<tr>
<td>Beam energy ($\Delta E = 5 \times 10^{-4}$)</td>
<td>0.3</td>
<td>0.3</td>
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<tr>
<td>Boiling</td>
<td>$&lt;0.35$ (at 10 $\mu A$) - 0 (at 60 $\mu A$)</td>
<td>0.35 (at 60 $\mu A$)</td>
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<td>Track Reco</td>
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<td>Target Length</td>
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<td>Radiative correction</td>
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<td>Background subtraction</td>
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<td>0.2</td>
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<tr>
<td>Cross section model</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.2 - 1.3%</td>
<td>1.4 - 1.6%</td>
</tr>
</tbody>
</table>
Cross section relative to 1-γ cross section calculated with $G_E = G_M/\mu = G_{\text{dip}}$

Significant improvement in precision for $Q^2 > 6$.

Systematic uncertainties on Fall 2016 LHRS data ~1.3% (pt-pt), 1.5% (norm)

RHRS (additional 2% from optics)
Status of 1st paper

- A draft of an intended PRL Letter was circulated Summer 2020.

- It was decided that we needed to address the impact of updated radiative corrections to older data studied by Gramolin et. al. Utilizing the formalism of Maximon & Tjon

=> This was found to reduce the tension with the P-T results.

- A. Gramolin provided corrections for data set of higher $Q^2$ data for which enough information on external materials was available.

=> This data set of 121 data points were included in a global fit And Rosenbluth separations were updated.
Global fit: Maximizing information from the data

A global fit to the modern higher $Q^2$ cross section data (> 0.5) to:

- Provide good description of cross section
- Utilize for analysis of global Rosenbluth separations at large $Q^2$
- Study signal of $2\gamma$ contributions at larger $Q^2$ by comparing to P-T data

Fit was performed to the reduced cross section utilizing:

- Rosenbluth form
  \[ \sigma_R = \frac{d \sigma}{d \Omega} \frac{\varepsilon^{1+\tau}}{\tau \sigma_{Mot}} = \frac{\varepsilon}{\tau} \left( G_E^p \right)^2 + \left( G_M^p \right)^2, \]
- Updated radiative corrections from Gramolin et. al applied for ‘modern’ data (121 data points with $Q^2 > 0.4$).
- Normalization factors determined as nuisance parameters.
We have studied the systematics for different fit choices:

1. **Form Factor fit to full Q^2 data set** (467 data points):

   \[
   \sigma_R = \frac{d \sigma}{d \Omega} \frac{\varepsilon (1+\tau)}{\tau \sigma_{\text{Mott}}} = \left( G_M^p \right)^2 + \frac{\varepsilon}{\tau} \left( G_E^p \right)^2
   \]

   Form factors utilize Kelly-like form

2. **G_M / RS fit to restricted data set with new RCs** (121 data points):

   \[
   \sigma_R = \frac{d \sigma}{d \Omega} \frac{\varepsilon (1+\tau)}{\tau \sigma_{\text{Mott}}} = \left( G_M^p \right)^2 \left( 1 + \frac{\varepsilon}{\mu^2} RS \right)
   \]

   \[
   RS = 1 + c_1 \tau + c_2 \tau^2
   \]
Global fitting results: Impact of GMp12 data

\[ \sigma_R = \frac{d\sigma}{d\Omega} \frac{\varepsilon(1+\tau)}{\tau \sigma_{\text{Mott}}} = \left( G_M^p \right)^2 \left( 1 + \frac{\varepsilon}{\mu^2} RS \right) \]

GMp12 data reduces FF ratio uncertainty by factor of \(~2\) for \(Q^2 > 8\)

For large fraction of JLab 12 GeV
Region GMp12 data reduces \(G_M^p\)
uncertainty by \(~40\%\)
Rosenbluth separations with GMp12

For $Q^2 > 7$, GMp12 data:
- uncertainties typically 2-4 times smaller than existing data.
- increases lever arm in $\varepsilon$ of existing data, allowing Rosenbluth separations for 1st time.

To combine different experiments:
- use global fit to center boxed data to same $Q^2$
- normalize each data set using global fit results.
- include normalization uncertainties in pt-pt errors for all sets except GMp12.

\[ \sigma_R = \frac{d\sigma}{d\Omega} \frac{\varepsilon(1+\tau)}{\tau} \sigma_{Mott} = \frac{\varepsilon}{\tau} \left( G_E^p \right)^2 + \left( G_M^p \right)^2, \]
Rosenbluth separations with GMp12

$$\sigma_R = \frac{d \sigma}{d \Omega} \frac{\varepsilon (1+\tau)}{\tau \sigma_{Mott}} = \frac{\varepsilon}{\tau} \left( G_E^p \right)^2 + \left( G_M^p \right)^2,$$

where $$Q^2 = \frac{-4 \pi \alpha^2}{m^2} \frac{\sigma_R}{\sigma_M}.$$
Rosenbluth separations with GMp12

$$\sigma_R = \frac{d \sigma}{d \Omega} \frac{\varepsilon (1+\tau)}{\tau \sigma_{Mott}} = \frac{\varepsilon}{\tau} \left( G_E^p \right)^2 + \left( G_M^p \right)^2$$

Significant tension with P-T results is observed for 1st time for $Q^2 > 7 \text{ GeV}^2$
Summary

- GMp12 provided benchmark for precision inclusive cross sections using HRS spectrometers.
  - Final Cross sections for Fall2016 data to be published soon with uncertainties of
    - 1.2 - 2% pt-pt
    - 1.5% normalization
  - Uncertainty on GMp for 12 GeV kinematics reduced by ~40% or more.
  - important for JLab 12 GeV Form Factor and GPD program
  - provides precision normalization for upcoming 12 GeV experiments at JLab

- Significant evidence for continuing tension with P-T data for $Q^2 > 6$ signaling evidence of 2-$\gamma$ contributions for 1st time.

- Updated draft of 1st paper coming very soon.
GMp (E12-07-108) Analysis Team

- Spokesperson:
  - John Arrington
  - Eric Christy
  - Shalev Gilad
  - Vincent Sulkosky
  - Bogdan Wojtsekhowski

- Postdoc:
  - Kalyan Allada

- Ph.D students (all have defended):
  - Thir Gautam (Hampton U.)
  - Longwu Ou (MIT)
  - Barak Schmookler (MIT)
  - Yang Wang (William & Mary)
  - Bashar Aljahrneh (NCA&T)

Thanks to JLab accelerator team, Hall A target group, and all shift takers for their tremendous effort to make the GMp run successful

Thanks!

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