Dark sectors at η, η' factories Sean Tulin



New gauge forces or scalar bosons beyond the minimal Standard Model





⁸Be/⁴He anomalies

Krasznahorkay et al (2016); Feng et al (2016,2017)

Dark matter physics:

- Dark sector: matter particles and forces, like the Standard Model
- Explains dark matter stability (dark charge conservation)

Dark matter physics:

 Annihilating dark matter (indirect detection anomalies, light dark matter, relic density)





Dark matter physics:

• Self-interacting dark matter





Outline: dark sectors at η,η' factories

• Dark sector particles produced in meson decays New gauge bosons, scalars, pseudoscalars (ALPs)

Precision tests of fundamental physics with η and η' mesons Gan, Kubis, Passemar, ST (2020)

Searching for new light hidden particles with η and η' mesons ST, Gatto, Kubis [Snowmass 2021 LOI]

 Dark sector particles produced directly via photoproduction (B boson)

New baryonic forces at electron-beam fixed target experiments Safa Ben Othman, Armita Jalooli, ST (in prep)

Larger η, η' samples at future facilities

Previous Experiments:

Experiment Total η		Total η'	
CB at AGS	10 ⁷	-	
CB MAMI-B	2x10 ⁷	-	
CB MAMI-C	6x10 ⁷	10 ⁶	
WASA-COSY	~3x10 ⁷ (p+d), ~5x10 ⁸ (p+p)	-	
KLOE-II	3x10 ⁸	5x10⁵	
BESIII	~107	~5x10 ⁷	

Upcoming experiments

Jefferson Eta Factory (JEF) at JLab Hall D (approved)

	η	η'	
Tagged mesons	6.5x10 ⁷	4.9×10^{7}	per 100 days

Rare Eta Decays with a TPC for Optical Photons (REDTOP) possibly at Fermilab (proposed)

Phase I (untagged mode)	2x10 ¹³	1011	
Phase II+ (tagged mode)	1x10 ¹³	1011	per year

Jefferson Eta Factory (JEF) experiment γ beam (10 GeV) on H target

GlueX + upgraded forward calorimeter at Jefferson Lab (Hall D)



Rare Eta Decays with a TPC for Optical Photons (REDTOP)

proton beam (1-3 GeV) on nuclear target (Be/D)



Rich physics program at η,η' factories

Standard Model highlights

- Theory input for light-by-light scattering for (g-2)_μ
- Extraction of light quark masses
- QCD scalar dynamics

Fundamental symmetry tests

- P,CP violation
- C,CP violation

[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]

Dark sectors (MeV—GeV)

- Vector bosons (dark photon, B boson, X boson)
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

Channel	Expt. branching rati		
$\eta \rightarrow 2\gamma$	39.41(20)%		
$\eta \rightarrow 3\pi^0$	32.68(23)%		
$\eta ightarrow \pi^0 \gamma \gamma$	$2.56(22) \times 10^{-4}$		
0.0	1.2 - 10-3		
$\eta \to \pi^{\circ} \pi^{\circ} \gamma \gamma$	$< 1.2 \times 10^{-4}$		
$\eta \rightarrow 4\gamma$	< 2.8 × 10 +		
$\eta \to \pi^+ \pi^- \pi^0$	22.92(28)%		
$\eta ightarrow \pi^+ \pi^- \gamma$	4.22(8)%		
$\eta o \pi^+ \pi^- \gamma \gamma$	$< 2.1 \times 10^{-3}$		
$\eta \rightarrow e^+ e^- \gamma$	$6.9(4) \times 10^{-3}$		
$\eta ightarrow \mu^+ \mu^- \gamma$	$3.1(4) \times 10^{-4}$		
$\eta \rightarrow e^+ e^-$	$< 7 \times 10^{-7}$		
$\eta \to \mu^+ \mu^-$	$5.8(8) \times 10^{-6}$		
0.0.			
$\eta \to \pi^0 \pi^0 \ell^+ \ell^-$			
$\eta \to \pi^+ \pi^- e^+ e^-$	$2.68(11) \times 10^{-4}$		
$\eta \to \pi^+ \pi^- \mu^+ \mu^-$	$< 3.6 \times 10^{-4}$		
$\eta \rightarrow e^+ e^- e^+ e^-$	$2.40(22) \times 10^{-5}$		
$\eta \to e^+ e^- \mu^+ \mu^-$	$< 1.6 \times 10^{-4}$		
$\eta \to \mu^+ \mu^- \mu^+ \mu^-$	$< 3.6 \times 10^{-4}$		
$\eta \to \pi^+ \pi^- \pi^0 \gamma$	$< 5 \times 10^{-4}$		
$\eta \to \pi^{\pm} e^{\mp} v_e$	$< 1.7 \times 10^{-4}$		
$\eta \to \pi^+ \pi^-$	$< 4.4 \times 10^{-6}$ [53]		
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$		
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$		

ratio	Discussion
	chiral anomaly, $\eta - \eta'$ mixing
	$m_u - m_d$
	χ PT at $O(p^6)$, leptophobic <i>B</i> boson, light Higgs scalars
	χ PT, axion-like particles (ALPs)
	< 10 ⁻¹¹ [52]
	$m_u - m_d$, <i>C/CP</i> violation, light Higgs scalars
	chiral anomaly, theory input for singly-virtual TFF and $(g - 2)_{\mu}$, P/CP violation
	χ PT, ALPs
	theory input for $(g - 2)_{\mu}$, dark photon, protophobic <i>X</i> boson
	theory input for $(g - 2)_{\mu}$, dark photon
	theory input for $(g - 2)_{\mu}$, BSM weak decays
	theory input for $(g - 2)_{\mu}$, BSM weak decays, <i>P/CP</i> violation
	C/CP violation, ALPs
	theory input for doubly-virtual TFF and $(g - 2)_{\mu}$, <i>P/CP</i> violation, ALPs
	theory input for doubly-virtual TFF and $(g - 2)_{\mu}$, <i>P/CP</i> violation, ALPs
	theory input for $(g-2)_{\mu}$
	theory input for $(g-2)_{\mu}$
	theory input for $(g-2)_{\mu}$
	direct emission only
	second-class current
]	<i>P</i> / <i>CP</i> violation
	<i>P/CP</i> violation <i>Gan, Kubis, Passemar, ST</i>
	<i>P/CP</i> violation [arxiv:2007.00664]

η,η' laboratory for dark sectors

- On-shell decays to new light particles in the MeV—GeV range
 - Vector bosons (hidden photons), scalar bosons, axion-like particles (ALPs)
- Leading decays of η are already suppressed $\sim \mathcal{O}\left(\alpha_{\text{em}}^2\right)$ or $\mathcal{O}\left((m_u m_d)^2\right)$
- Larger mass reach for η' but worse sensitivity (total width larger by ~100)
- Decays to light hidden particles are 2- or 3-body decays that mimic 3-, 4-, or 5body final states (often very rare)
- Search strategies (visible final states):
 - Resonance searches (bump hunting)
 - Displaced vertices (long-lived decays)
 - Rare decays new physics process mimics highly-suppressed SM channels
- Other possibilities: invisible or partially-invisible decays

Dark photon

[Fayet (2007), Reece & Wang (2009), ...]

 $\eta, \eta' \to \gamma A' \to \gamma \ell^+ \ell^-$



REDTOP sensitivities projected for FNAL/BNL (10¹⁸) or CERN (10¹⁷) POT

[Gatto (2019)]

Worthwhile to also consider

$$\eta' \to \pi^+ \pi^- A' \to \pi^+ \pi^- \ell^+ \ell^-$$

since $\mathcal{B}(\eta' \to \pi^+\pi^-\gamma) \approx 10 \times \mathcal{B}(\eta' \to \gamma\gamma)$

Protophobic X(17) vector boson to explain Atomki ⁸Be and ⁴He anomalies

[Feng et al (2016,2017)]

 $\eta, \eta' \to X\gamma \to e^+e^-\gamma$



Currently A2@MAMI limited to invariant mass $m_{ee} > 30$ MeV, but 17 MeV within reach of REDTOP



Leptophobic *B* boson from gauged $U(1)_B$

Model:

$$\mathcal{L}_{\text{int}} = \left(\frac{1}{3}g_B + \varepsilon e Q_q\right) \bar{q} \gamma^{\mu} q B_{\mu} - \varepsilon e \bar{\ell} \gamma^{\mu} \ell B_{\mu}$$

$$\eta \to B \gamma \to \pi^0 \gamma \gamma$$

Mimics rare decay (0.025%) plus search for $\pi^0\gamma$ resonance

$$\eta' \to B\gamma \to \pi^0 \gamma\gamma, \eta\gamma\gamma$$

 $\pi^+ \pi^- \pi^0\gamma$

 $\Upsilon(1S) \neq$ n-Pb hadrons 10^{-2} Dark photon searches A' $n' \rightarrow$ (model dependent $\eta\gamma\gamma$ $3\pi\gamma$ $\Lambda = 90 \; \mathrm{GeV}$ 10^{-4} α_B 1 TeV JEF (50) 10^{-6} JEF 10 Te 10^{-8} [Gan et al (2020)] 10^{-10} 0.20.40.60.80.0B boson mass m_B (GeV)

[Lee & Yang (1955), Pais (1973), Nelson & Tetradis (1989), ...]

Light scalar boson (S)

$$\eta, \eta' \to \pi^0 S \to \pi^0 \ell^+ \ell^-, \quad \eta' \to \eta S \to \eta \ell^+ \ell^-$$

Final states in SM arise via two- γ loop, very suppressed (single- γ process is C-violating)

$$\eta \to \pi^0 S \to \pi^0 \gamma \gamma$$

 $\gamma\gamma$ resonance in rare decay

$$\eta, \eta' \to \pi^0 S \to 3\pi, \quad \eta' \to \eta S \to \eta \pi \pi$$

Bump-hunting in Dalitz distributions

Light scalar boson (S)

- Originally considered as possible signature for (light) SM Higgs boson [Ellis et al (1976), Vainshtein et al (1980), Leutwyler & Shifman (1990)]
- Higgs-mixed scalar (Higgs portal model)

$$\mathcal{B}(\eta \to \pi^{0}S) \approx 1.8 \times 10^{-6} \sin^{2}\theta_{S} \times \lambda^{1/2} \left(1, \frac{M_{\pi^{0}}^{2}}{M_{\eta}^{2}}, \frac{m_{S}^{2}}{M_{\eta}^{2}} \right)$$

$$\mathcal{B}(\eta' \to \pi^{0}S) \approx 5.4 \times 10^{-8} \sin^{2}\theta_{S} \times \lambda^{1/2} \left(1, \frac{M_{\pi^{0}}^{2}}{M_{\eta'}^{2}}, \frac{m_{S}^{2}}{M_{\eta'}^{2}} \right)$$

$$\mathcal{B}(\eta' \to \eta S) \approx 4.7 \times 10^{-5} \sin^{2}\theta_{S} \times \lambda^{1/2} \left(1, \frac{M_{\pi^{0}}^{2}}{M_{\eta'}^{2}}, \frac{m_{S}^{2}}{M_{\eta'}^{2}} \right)$$

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[Gan et al (2020)]

• General scalar model:

 η, η' sensitive to light-quark couplings, FCNCs sensitive to top coupling

Hadrophilic scalar boson

[Batell et al (2017,2018)]



Constraints from η, η' decays

Light scalar coupling to *u*-quarks only

$$\eta \to \pi^0 S \to \pi^0 \gamma \gamma$$

 $\eta, \eta' \to \pi^0 S \to 3\pi$

More general couplings to u,d-quarks and e, μ , γ [e.g., Liu, Cloet, Miller (2018)]

Motivates searches for $\eta \to \pi^0 S \to \pi^0 \gamma \gamma, \ \pi^0 e^+ e^-, \ \pi^0 \mu^+ \mu^ \eta, \eta' \to \pi^0 S \to 3\pi, \quad \eta' \to \eta S \to \eta \pi \pi$

Axion-like particles (ALPs) and η,η' decays

[Aloni et al (2019), Landini and Meggiolaro (2019)]

Model:



Axion-like particles (ALPs) and η,η' decays

[Aloni et al (2019), Landini and Meggiolaro (2019)]

Model:

$$\mathcal{L}_{\rm ALP} = \mathcal{L}_{\rm QCD} + \frac{1}{2}(\partial_{\mu}a)(\partial^{\mu}a) - \frac{1}{2}m_0^2a^2 - \frac{\alpha_s}{8\pi f_a}a\,G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{\alpha_{\rm em}c_{\gamma}}{8\pi f_a}a\,F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{\partial^{\mu}a}{2f_a}\bar{q}c_q\gamma_{\mu}\gamma_5q - \frac{\partial^{\mu}a}{2f_a}\bar{\ell}c_\ell\gamma_{\mu}\gamma_5\ell$$

Signatures: many complicated 4- and 5-body final states

$$\eta \to \pi \pi a \to \pi \pi \gamma \gamma , \ \pi \pi e^+ e^- , \ \pi \pi \mu^+ \mu^- \quad \text{(and same for } \eta')$$
$$\eta' \to \pi \pi a \to \pi \pi \pi^+ \pi^- \gamma , \ 5\pi$$
$$\eta' \to \eta \pi^0 a \to \eta \pi^0 \gamma \gamma , \ \eta \pi^0 e^+ e^- , \ \eta \pi^0 \mu^+ \mu^-$$

Most of these had no motivation to be studied. Can they be searched for?

η,η' branching ratios into ALPs

Fixed effective mass scale $\Lambda/|C_{GG}| = 32\pi^2 f_a \approx 3 \text{ TeV}$



Calculated BR at leading order in χ PT, but possible large corrections at NLO

Models Theory landscape Predictions Sensitivities







Go beyond simplified models and LO predictions

Direct photoproduction of new gauge bosons

Safa Ben Othman, Armita Jalooli, ST (in prep)

See also: Fanelli and Williams (2016)

B boson model: $U(1)_{B}$ gauge boson

$$\mathscr{L}_{\rm int} = \left(\frac{1}{3}g_B + \varepsilon Q_q e\right)\bar{q}\gamma^\mu q B_\mu - \varepsilon e\bar{\ell}\gamma^\mu \ell B_\mu$$

Motivation: Benchmark model for new force coupled to quarks with suppressed lepton signatures

Direct photoproduction of new gauge bosons



B boson parameter space

Direct photoproduction of new gauge bosons



Direct photoproduction of new gauge bosons



Direct production of new gauge forces







Sub-GeV B boson: dominated by diffractive scattering (Q < GeV)

Cannot be calculated in perturbative QCD and must be modeled



Assumption I: Vector meson dominance (VMD)

External gauge fields couple by mixing with QCD vector mesons (isoscalar only)



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External gauge fields couple by mixing with QCD vector mesons (isoscalar only)

$$V(k,\lambda) \longrightarrow B(k,\lambda) = \frac{\sqrt{2}f_V m_V}{k^2 - m_V^2 + im_V \Gamma_V} \operatorname{Tr} \left[\boldsymbol{T}_V \left(\frac{1}{3} g_B \boldsymbol{I} + \varepsilon e \boldsymbol{Q} \right) \right]$$

(Function of meson mass, width, decay constant) x (group theoretic factor) x (BSM couplings)

$$\mathcal{M}(\gamma p \to Bp) = -\mathcal{M}(\gamma p \to \omega p) \left(\frac{\sqrt{2}g_B f_\omega F_\omega(m_B^2)}{3m_\omega}\right) - \mathcal{M}(\gamma p \to \phi p) \left(\frac{g_B f_\phi F_\phi(m_B^2)}{3m_\phi}\right)$$

Assumption II: t-channel exchange model for SM matrix elements



Assumption II: t-channel exchange model for SM matrix elements



Each coupling dressed with a form factor $F(t, \Lambda, m) = \frac{\Lambda^2 - m^2}{\Lambda^2 - t}$ Friman & Soyeur (1996)

Number of parameters: 6 couplings + 6 momentum scales $\Lambda \sim m_{o}$

Assumption II: t-channel exchange model for SM matrix elements



Natural parity amplitude (+)

Calculated from Vp \rightarrow Vp scattering + VMD for coupling initial γ *Ewerz et al (2013)*

Include additional t-dependent form factor $F_{pV}(t) = \exp(b_V t)$ Laget & Mendez-Galain (1995)

$$\frac{d\sigma_{+}(\gamma p \to \omega p)}{dt} = \frac{2\alpha_{\rm em}f_{\omega}^2 s^2}{m_{\omega}^2 (s - m_p^2)^2} \beta_{\mathbb{P}NN}^2 \beta_{\mathbb{P}\omega\omega}^2 |F_{p\omega}(t)|^2 \left(\frac{s}{s_0}\right)^{2\alpha_{\mathbb{P}}(t)-2}$$

Model parameters

fixed parameter	input value	source	fitted parameter	prior
$g_{\pi\gamma\omega}$	1.81 ± 0.03	$\omega \to \pi^0 \gamma$	$g_{\pi NN}$	13 ± 1
$g_{\eta\gamma\omega}$	0.35 ± 0.02	$\omega \to \eta \gamma$	$g_{\eta NN}$	4 ± 1
$g_{\pi\gamma\phi}$	0.137 ± 0.003	$\phi \to \pi^0 \gamma$	$\Lambda_{\pi^0 NN}$	$0.8 \pm 0.2 \text{ GeV}$
$g_{\eta\gamma\phi}$	0.704 ± 0.007	$\phi \to \eta \gamma$	$\Lambda_{\eta NN}$	$0.8 \pm 0.2 \text{ GeV}$
f_ω	$198\pm2~{ m MeV}$	$\omega \to e^+ e^-$	$\Lambda_{\pi^0\gamma\omega}$	$0.8 \pm 0.2 \text{ GeV}$
f_{ϕ}	$228 \pm 1 \text{ MeV}$	$\phi \to e^+ e^-$	$\Lambda_{\eta\gamma\omega}$	$0.8\pm0.2~{ m GeV}$
$\beta_{\mathbb{P}NN}$	$1.87~\mathrm{GeV}^{-1}$	$pp, par{p}$ data	$\Lambda_{\pi^0\gamma\phi}$	$0.8\pm0.2~{ m GeV}$
$lpha_{\mathbb{P}}(0)$	1.08	$pp, par{p}$ data	$\Lambda_{\eta\gamma\phi}$	$0.8\pm0.2~{ m GeV}$
$\alpha'_{\mathbb{P}}(0) = s_0^{-1}$	$0.25 { m GeV}^{-2}$	$pp, par{p}$ data	$\beta_{\mathbb{P}\omega\omega}$	none
			$eta_{\mathbb{P}\phi\phi}$	none
			b_{ω}	none
			b_{ϕ}	none

Model parameters

Determine model parameters using experimental data for differential cross sections for vector meson photoproduction

Data sets:Energy:• Mainly CLAS (Williams et al 2009; Dey et al 2014)up to $\sqrt{s} \approx 2.8 \text{ GeV}$

- Older data: SLAC (Ballam et al 1973), NINA (Barber et al 1984)
- High energy: ZEUS (Derrick et al 1996)

up to $\sqrt{s} \approx 4.2 \; {\rm GeV}$

 $\sqrt{s} \approx 80 \text{ GeV}$

Perform MCMC

Restricted to data points with $|t| < 1~{
m GeV^2}$ and $\sqrt{s} > 2.3~{
m GeV}$





Model parameters

fixed parameter	input value	source	fitted parameter	prior	best-fit value
$g_{\pi\gamma\omega}$	1.81 ± 0.03	$\omega \to \pi^0 \gamma$	$g_{\pi NN}$	13 ± 1	13.0 ± 0.4
$g_{\eta\gamma\omega}$	0.35 ± 0.02	$\omega \to \eta \gamma$	$g_{\eta NN}$	4 ± 1	4 ± 1
$g_{\pi\gamma\phi}$	0.137 ± 0.003	$\phi \to \pi^0 \gamma$	$\Lambda_{\pi^0 NN}$	$0.8 \pm 0.2 \text{ GeV}$	0.8 ± 0.1
$g_{\eta\gamma\phi}$	0.704 ± 0.007	$\phi \to \eta \gamma$	$\Lambda_{\eta NN}$	$0.8 \pm 0.2 \text{ GeV}$	0.7 ± 0.3
f_ω	$198\pm2~{ m MeV}$	$\omega \to e^+ e^-$	$\Lambda_{\pi^0\gamma\omega}$	$0.8\pm0.2~{ m GeV}$	0.8 ± 0.1
f_{ϕ}	$228\pm1~{ m MeV}$	$\phi \to e^+ e^-$	$\Lambda_{\eta\gamma\omega}$	$0.8\pm0.2~{ m GeV}$	0.8 ± 0.2
$eta_{\mathbb{P}NN}$	$1.87 { m ~GeV^{-1}}$	$pp, par{p}$ data	$\Lambda_{\pi^0\gamma\phi}$	$0.8\pm0.2~{ m GeV}$	0.5 ± 0.2
$lpha_{\mathbb{P}}(0)$	1.08	$pp, par{p}$ data	$\Lambda_{\eta\gamma\phi}$	$0.8\pm0.2~{ m GeV}$	0.6 ± 0.3
$\alpha'_{\mathbb{P}}(0) = s_0^{-1}$	$0.25 { m GeV}^{-2}$	$pp, par{p}$ data	$eta_{\mathbb{P}\omega\omega}$	none	2.0 ± 0.1
			$eta_{\mathbb{P}\phi\phi}$	none	0.620 ± 0.005
			b_{ω}	none	3.8 ± 0.1
			b_{ϕ}	none	1.40 ± 0.01

B boson production cross section



Straightforward to calculate differential cross section as well

B boson photoproduction

B boson model predictions are in good shape

$$\gamma p \to B p \to \pi^0 \gamma \, p \,, \, \pi^+ \pi^- \pi^0 \, p$$

- Questions remain:
 - What are the expected backgrounds?
 - What do experimentalists need to interface model predictions with their simulations?

• ...

Summary

- η,η' factories offer complementary probes of dark sectors and synergy with Standard Model η,η' decay studies
- Dark sector models well known but much remains

χ PT theorists Phenomenologists Can the BSM processes be better What is the landscape of quantified (NLQ corrections/form factors)? interesting parameter space? What are the connections with other searches? **Experimentalists** What final states are most accessible? What input is needed from theory?

Backup slides



Need vector meson photoproduction matrix elements (SM process)

Previous approach: Fanelli & Williams (2016) $\mathcal{M}(\gamma p \to V p) \sim \sqrt{\sigma(\gamma p \to V p)}$

$$\sigma_{\pm}(\gamma p \to Bp) = \frac{4\alpha_B \Phi(m_B)}{27} \left(\frac{|F_{\omega}(m_B^2)|^2 \sigma_{\pm}(\gamma p \to \omega p)}{\Phi(m_{\omega})} + \frac{|F_{\phi}(m_B^2)|^2 \sigma_{\pm}(\gamma p \to \phi p)}{2\Phi(m_{\phi})} + \frac{\cos\varphi_{\pm}|F_{\omega}(m_B^2)||F_{\phi}(m_B^2)|\sqrt{2\sigma_{\pm}(\gamma p \to \omega p)\sigma_{\pm}(\gamma p \to \phi p)}}{\sqrt{\Phi(m_{\omega})\Phi(m_{\phi})}} \right)$$

(Φ = phase space factors)