The treatment of divergences in parton densities

Ted Rogers
Jefferson Lab and Old Dominion University

With John Collins and Nobuo Sato: In preparation

9th Workshops of the APS Topical Group on Hadron Physics, April 14 2021

Historical: Two approaches to pdfs and factorization

- Track A:
  - Define pdf in terms of ultraviolet renormalization of bare number density operator.

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- Track A:
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- Track B:
  - Calculate higher order hard scattering amplitudes. "Absorb" collinear divergences into pdf.

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
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- Factorization (e.g., DIS):
  - Obtained from general region analysis.
  - Beyond parton model: Higher order hard scattering constructed from nested subtractions.

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$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-} \, \langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_\mathrm{T})\, |p\rangle$$

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$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}}$$

$$Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon} \right)^j$$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

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- Then: $\mathrm{d}\sigma = f \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

Track B:

- Issues:
  - Bare pdf ($f_{\text{“bare,b”}}$) of step 1 is ill-defined or inconsistent.
  - Collinear pdfs viewed as physical?

Track A vs. Track B Logic

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- Example: Track-B leads to arguments that pdf positivity is an absolute property of pdfs in certain schemes (MS-bar).

$f(x;\mu) \geq 0$ *A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377, JHEP 11 (2020) 129*

Positivity example

- Stress-test assertions about DIS factorization in other finite-range renormalizable theories.

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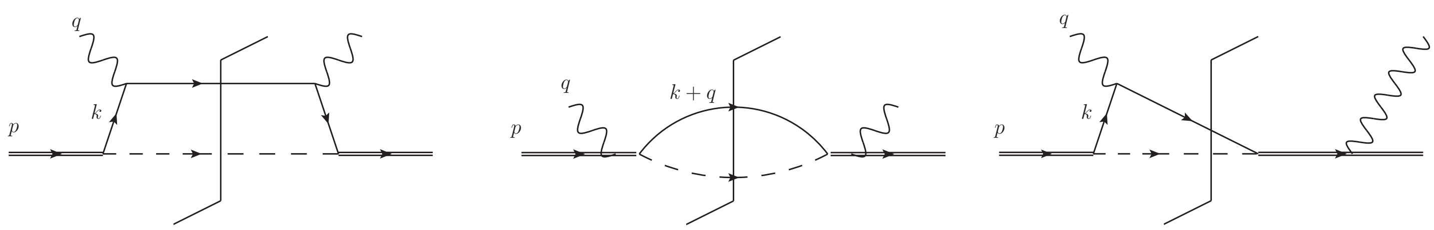

$$\mathcal{L}_{\rm int} = -\lambda \, \overline{\Psi}_N \, \psi_q \, \phi \ + \ \text{H.C.}$$

- Exact $O(\lambda^2)$ DIS cross section is easy to calculate exactly.

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Positivity example

Collinear Factorization

$$F_1(x,Q) = \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi - x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

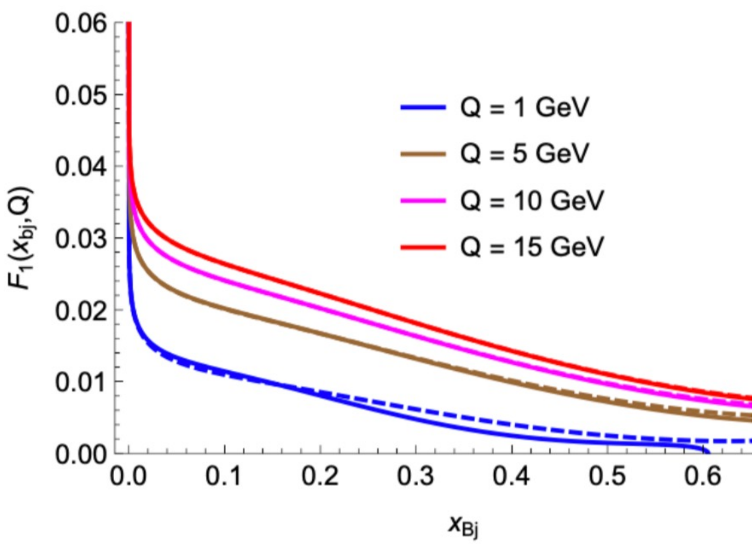
$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

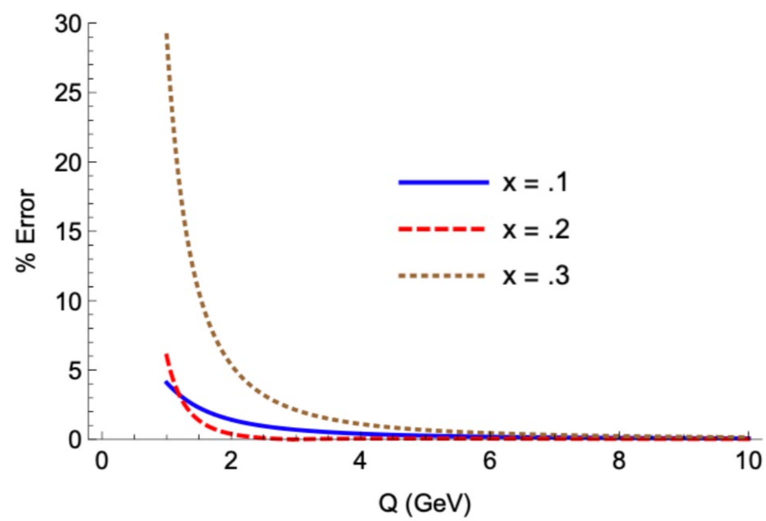
Partonic structure function

Parton Distribution

Positivity example

Collinear Factorization





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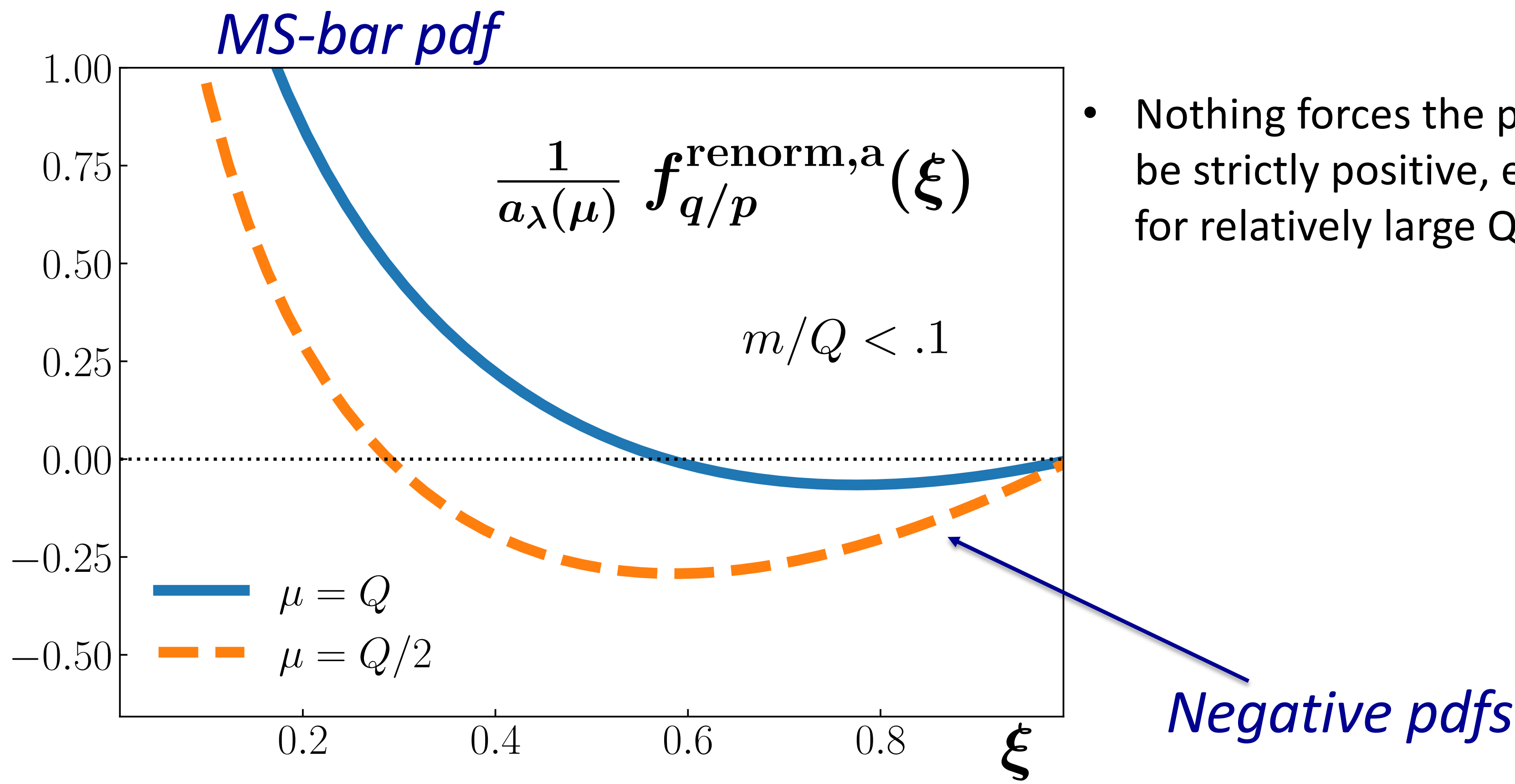

- Nothing forces the pdf to be strictly positive, even for relatively large Q.

Track B:

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Summary

- Historically two alternative ways of viewing divergences and their role in pdf definitions.
  - Track A: UV renormalization – no collinear divergences
  - Track B: Collinear absorption – absorb collinear divergences
- Track A is the more logically consistent approach.
- Most practical calculations are unaffected, but there are interesting exceptions:
  - Positivity *(Constraints on pdfs at low-ish Q?)*
  - Soffer bound
  - Heavy quarks

Not discussed today