Diagnosis of information scrambling from Hamiltonian dynamics under decoherence

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OUTLINE

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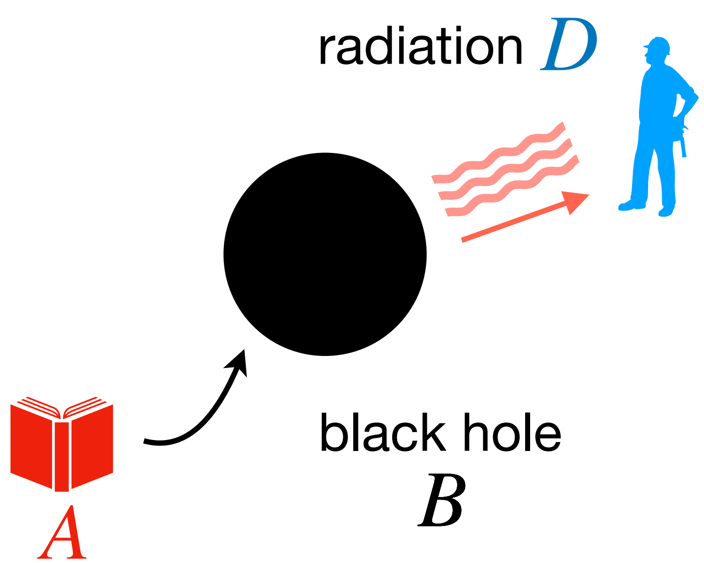
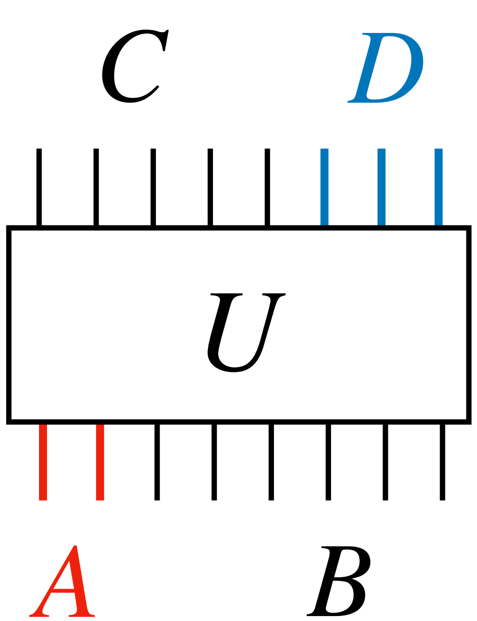
Introduction



Black hole as a fast scrambler

[Page 1993]

- U: Haar random (black hole) evolution
  (average over $2^N \times 2^N$ unitaries)
- A: input state
- B: initial black hole
- C: remaining black hole
- D: Hawking radiation (measured state)

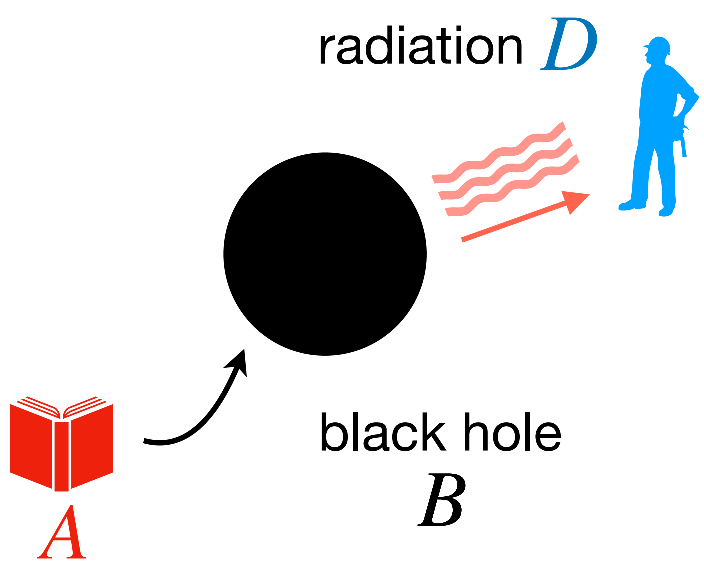


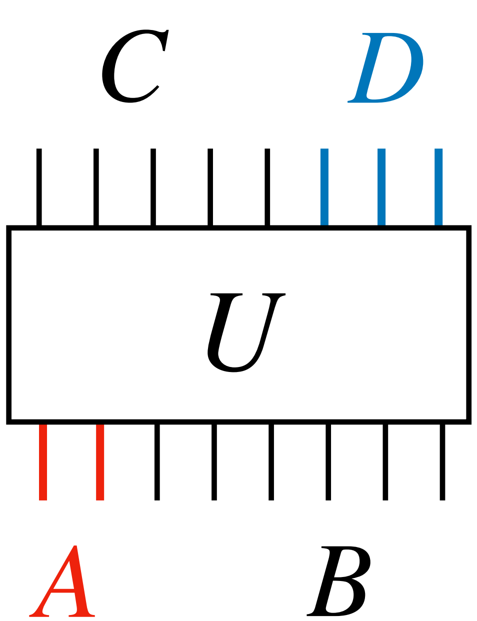
Information scrambling

- Information of input A cannot be retrieved by local measurements ($N_D \lesssim N/2$)
- Characterized by small mutual information $I(A, D)$ between input A and output D,

$$I(A, D) = S(\rho_A) + S(\rho_D) - S(\rho_{AD})$$

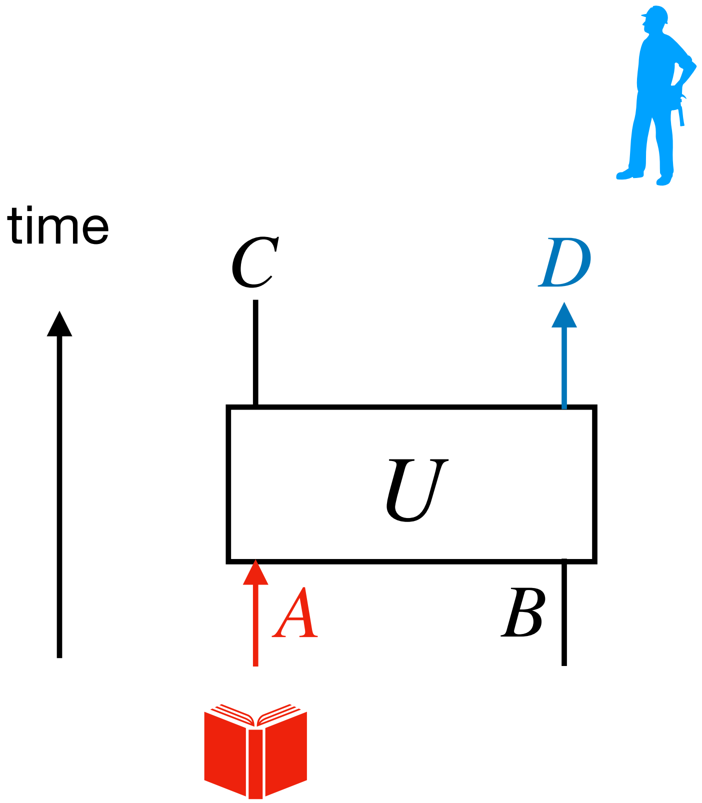
Introduction





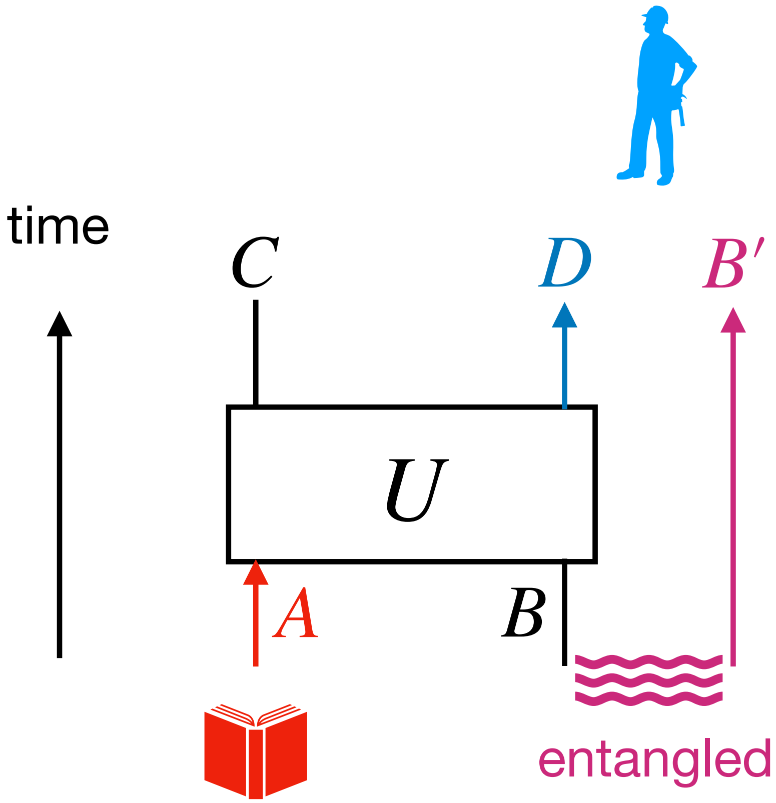
- Motivated by black hole physics (as a fast scrambler)
- Can we use it to understand information scrambling of other **physical systems**?
- Scrambled state is highly entangled
  ➡ Hard to simulate classically.
- Potential implementation on **near-term quantum devices**?
- What is the effect of **noises/errors** on some extended protocol.

Hayden-Preskill protocol



- U : **information scrambler** [Page (1993)]
  - ➡ Input state A **cannot be retrieved** by local measurements on D

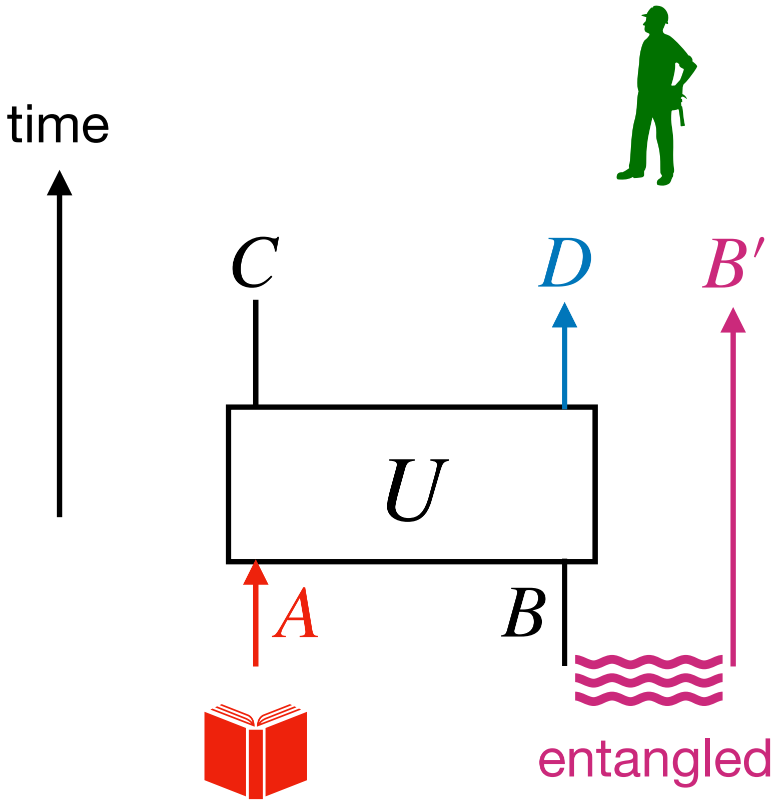
Hayden-Preskill protocol

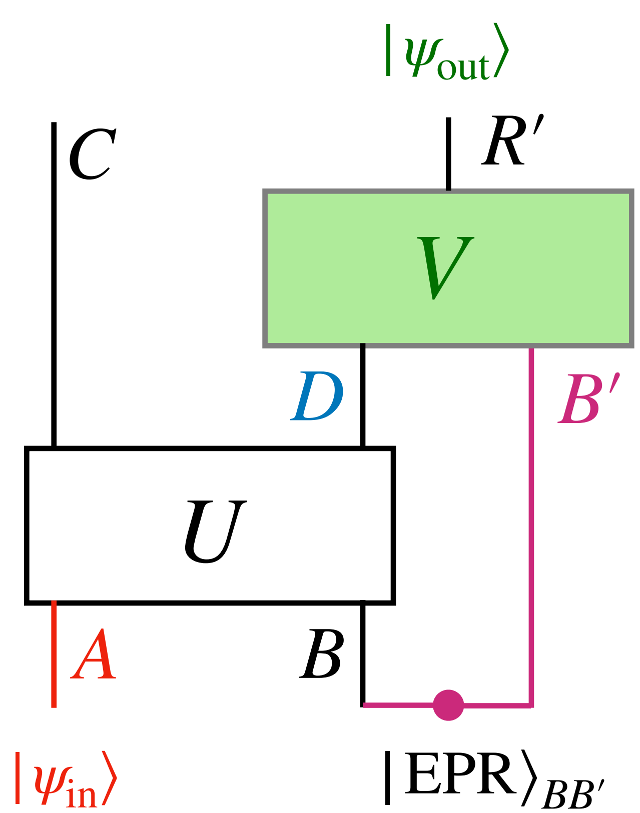


- U **: information scrambler** [Hayden, Preskill (2007)]

➡ Input state A **can be retrieved** by local measurements on D combined with initially entangled state on B'

Hayden-Preskill protocol





- U **: information scrambler** [Hayden, Preskill (2007)]
  - ➡ Input state A **can be retrieved** by local measurements on D combined with initially entangled state on B'

- V: recovery channel [Yoshida, Kitaev (2017)]
- Initial state: $|\psi_{\mathrm{in}}\rangle \otimes |\mathrm{EPR}\rangle_{BB'}$

$$|\mathrm{EPR}\rangle_{BB'} = \frac{1}{2^{N_B/2}} \sum_{i=1}^{2^{N_B}} |i_B\rangle \otimes |i_{B'}\rangle$$

- U **: information scrambler**
  - ➡ $F_{\mathrm{EPR}} \sim |\langle \psi_{\mathrm{in}} | \psi_{\mathrm{out}} \rangle|^2 \sim 1 \sim \left(d_A^2 \langle \mathrm{OTOC} \rangle\right)^{-1}$
  - ➡ $\mathrm{OTOC} = \langle O_A(0) O_D(t) O_A^\dagger(0) O_D^\dagger(t) \rangle \ll 1$

Diagnosis of information scrambling

[Hayata, Hidaka, YK (2021)]
See also [Yoshida, Yao (2018); Bao, YK (2020)]

- $F_{\text{EPR}} \sim |\langle \psi_{\text{in}} | \psi_{\text{out}} \rangle|^2 \sim 1 \Rightarrow U$: information scrambler
- This can be leveraged to diagnose how scrambling a dynamics U is
- Study the effects of decoherence with noisy evolution operator
  - Decoherence modeled by depolarizing channel on CNOT

$$\text{CNOT}\,\rho\,\text{CNOT} \to (1-p)\text{CNOT}\,\rho\,\text{CNOT} + p\frac{I}{d} \otimes \text{Tr}[\rho]$$

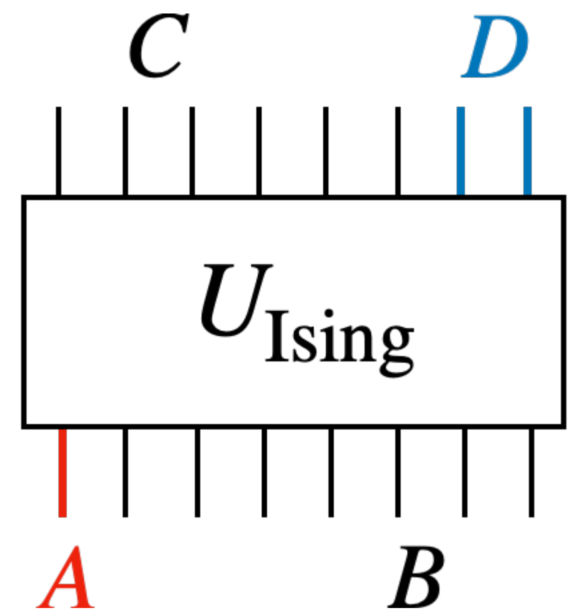
  - Ising spin chain
  - $SU(2)$ lattice Yang-Mills theory (Yang-Mills-Ising model)

Ising spin chain

[Hayata, Hidaka, YK (2021)]

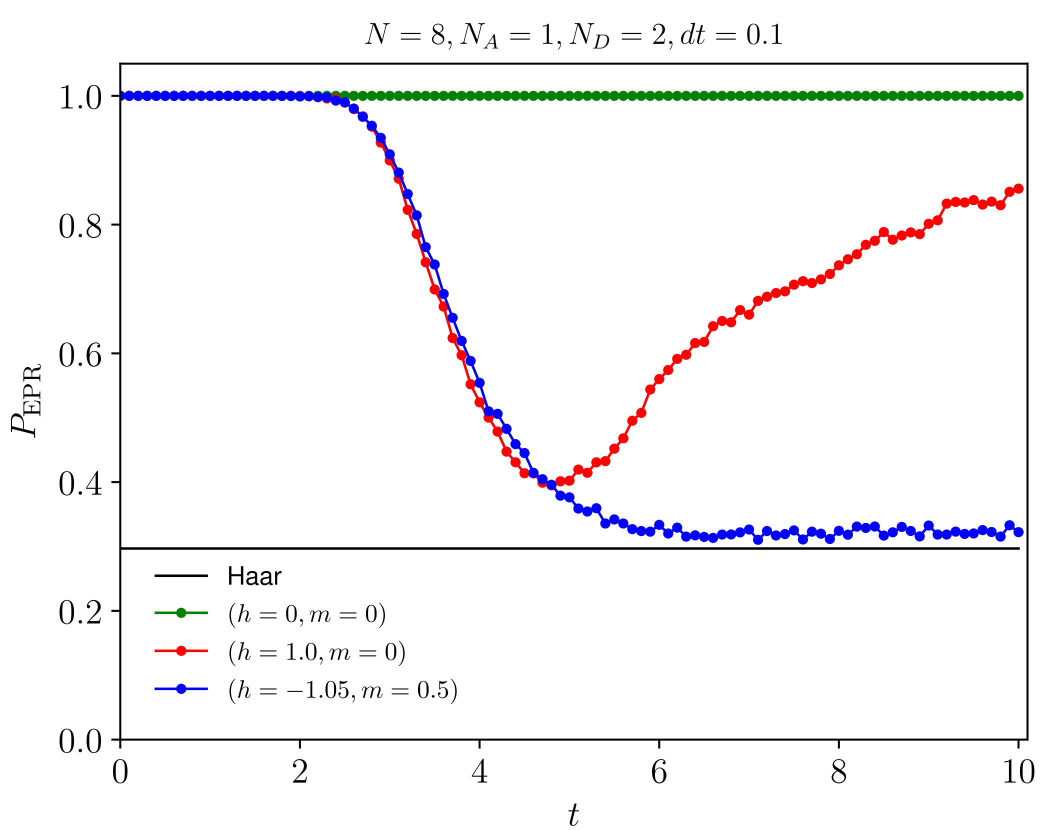
$$U_{\rm Ising} = \mathrm{e}^{-\mathrm{i}Ht}, \quad H = -\sum_i \left(Z_i Z_{i+1} + gX_i + mZ_i\right)$$

- $g = m = 0$: classical Ising model
- $g = 1,\ m = 0$: critical
- $g = -1.05,\ m = 0.5$: chaotic [Banuls, Cirac, Hastings (2010)]

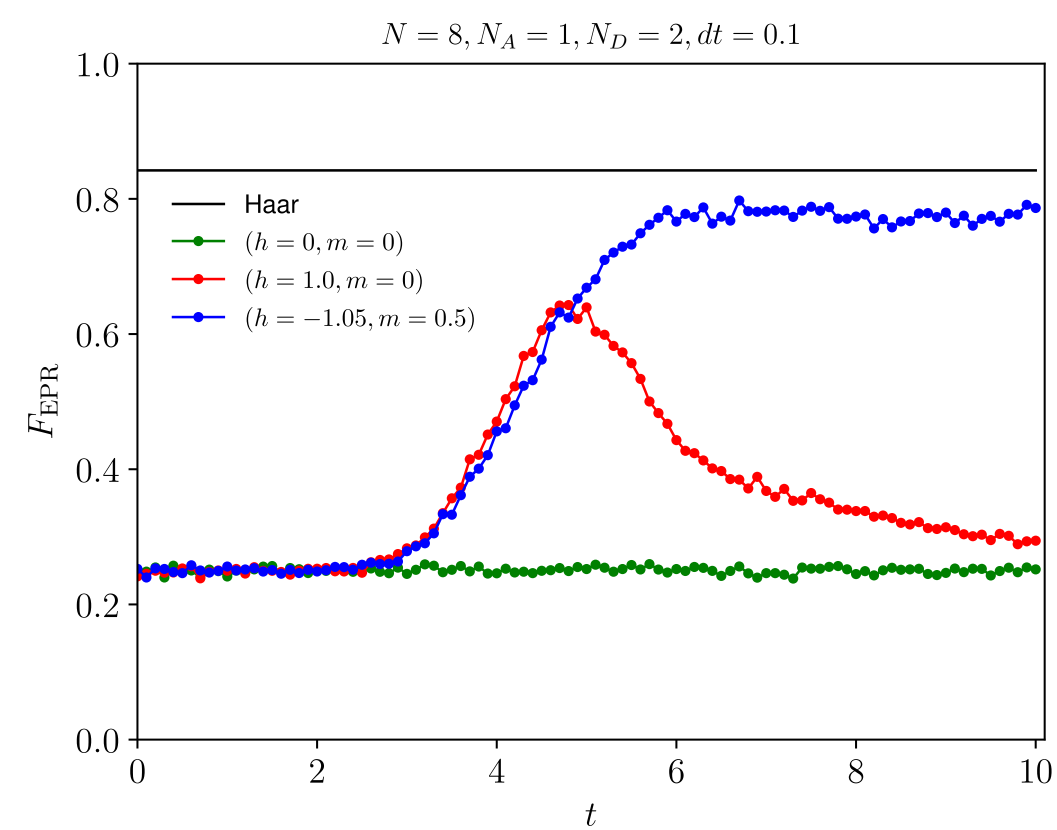


ideal

OTOC



$F_{\rm EPR}$



See also [Hosur, Qi, Roberts, Yoshida (2015)]

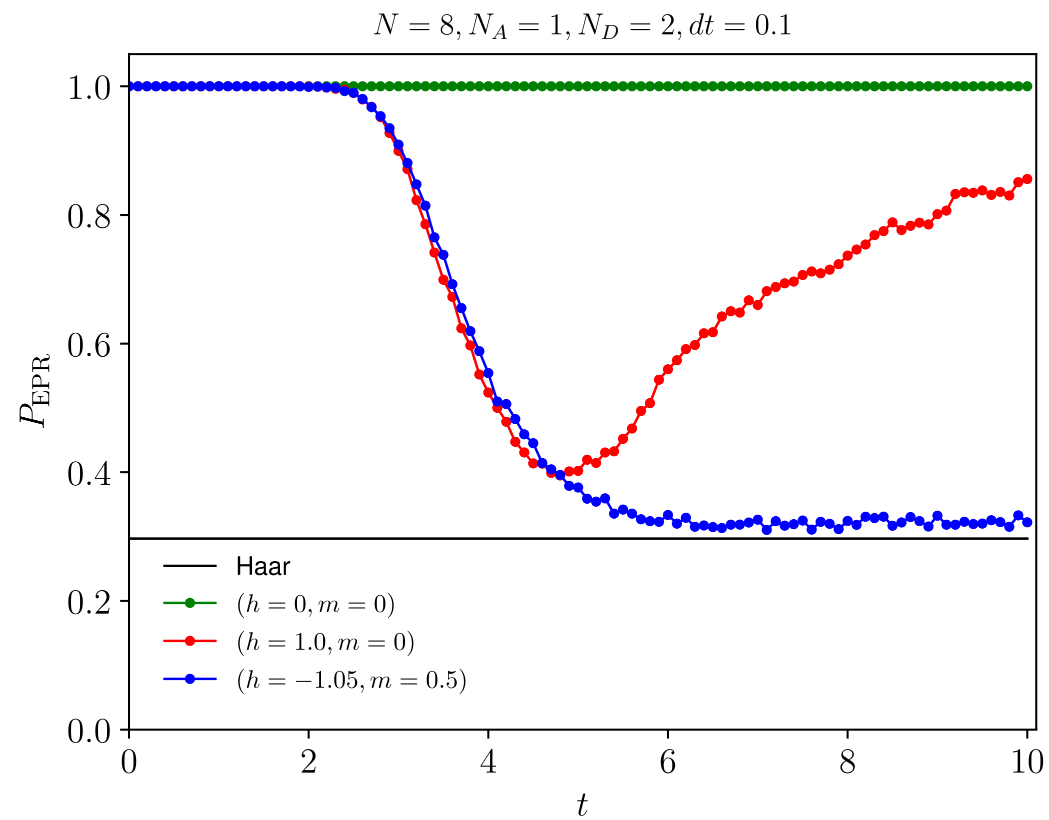
Ising spin chain

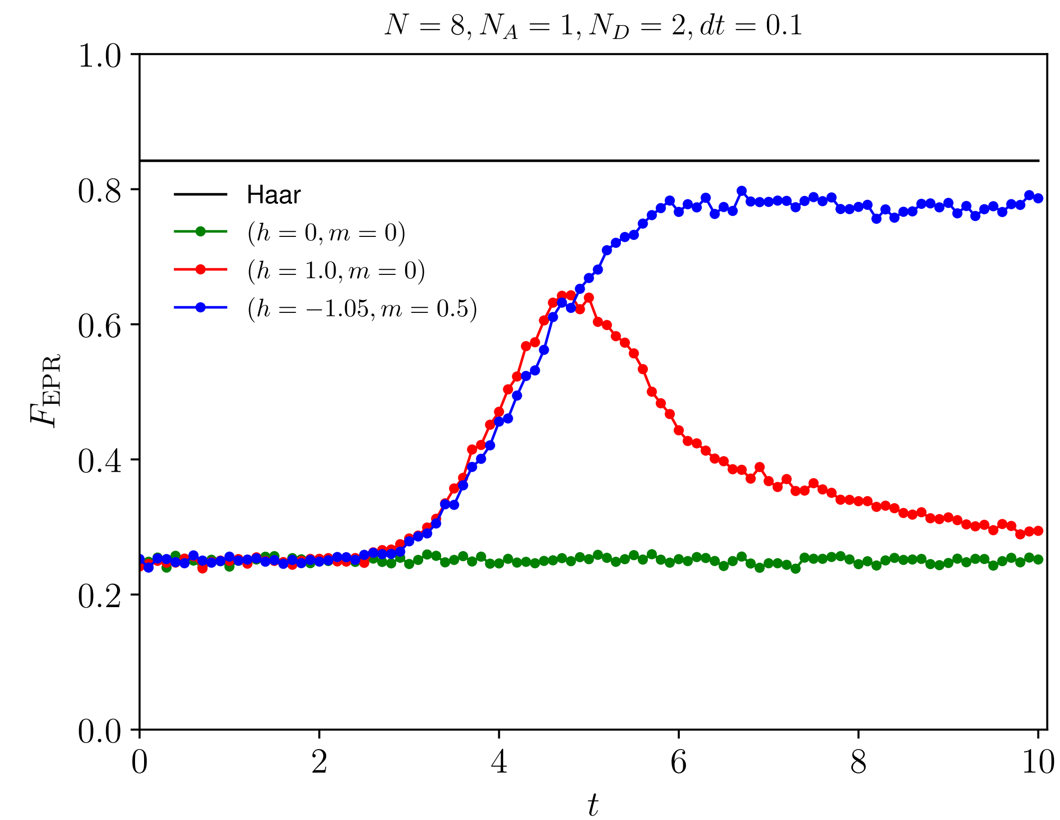
[Hayata, Hidaka, YK (2021)]

OTOC

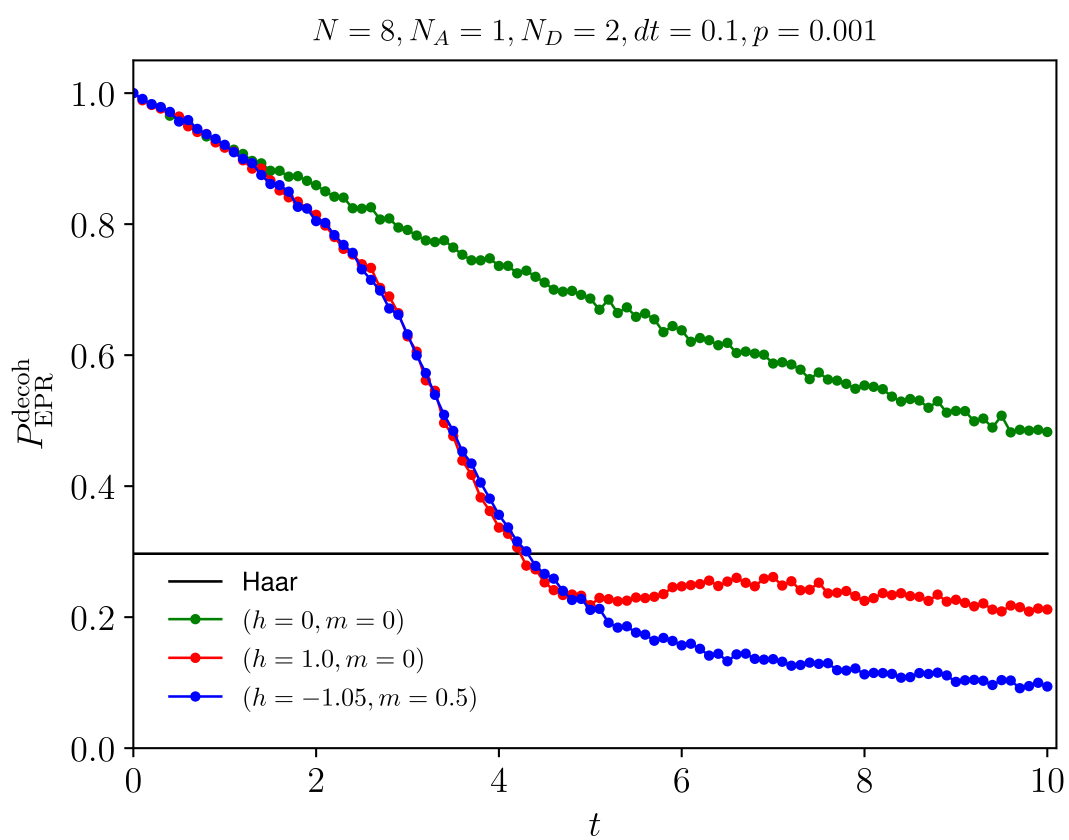
F_{EPR}

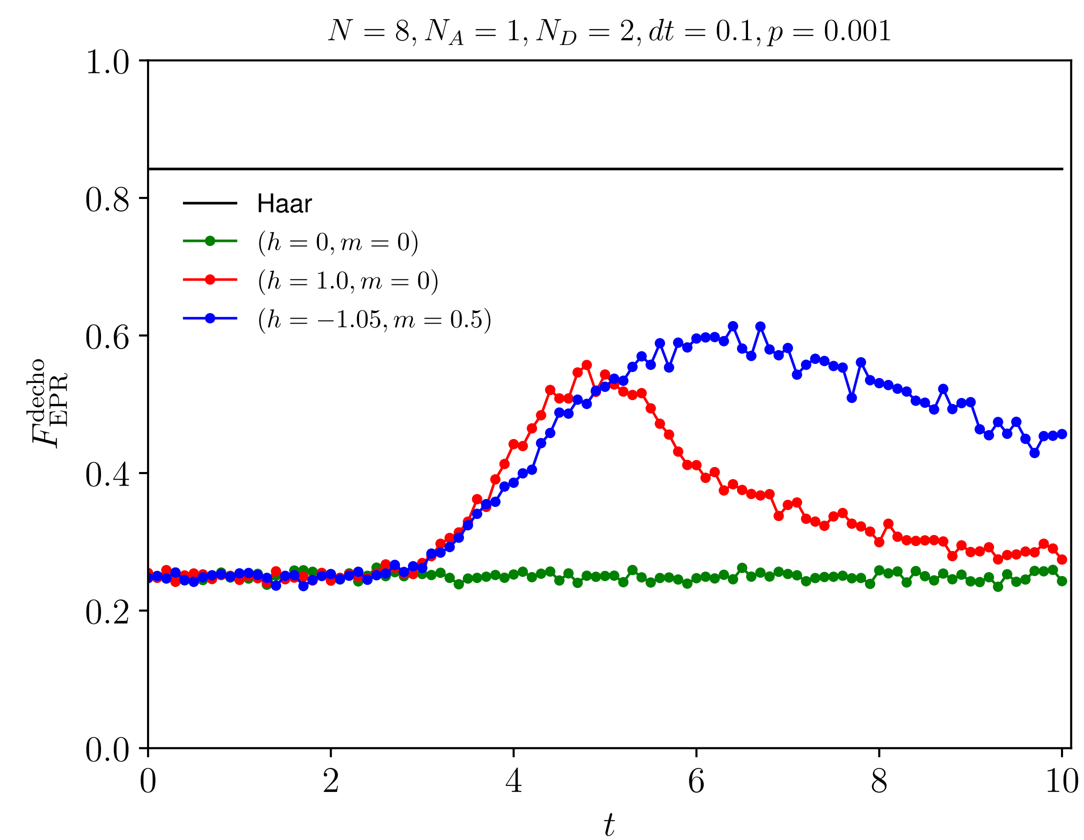
ideal





noisy



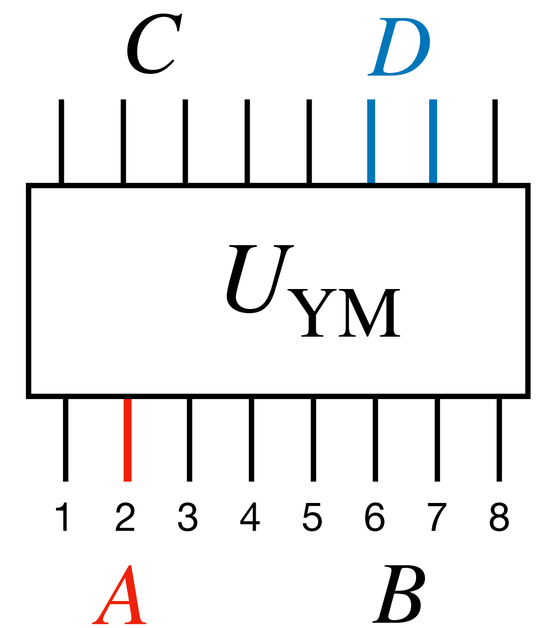


SU(2) lattice YM theory

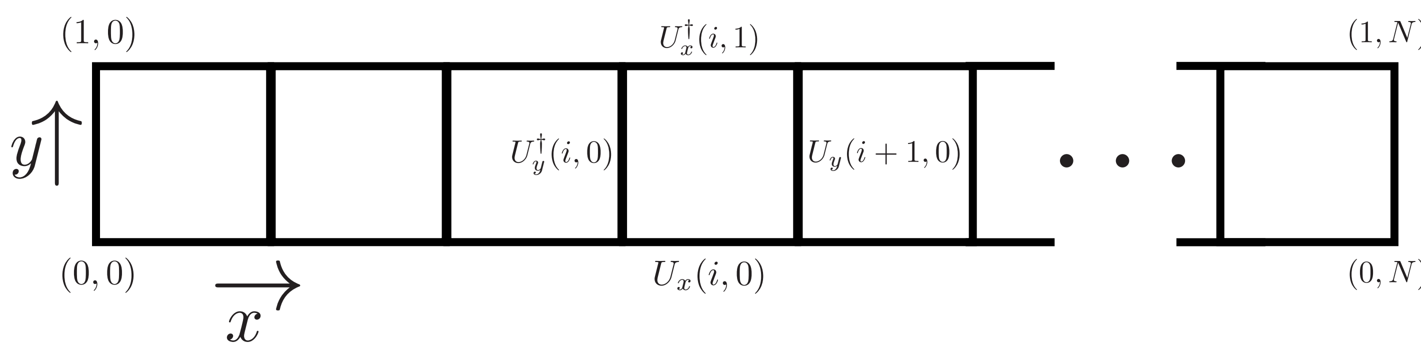
[Hayata, Hidaka, YK (2021)]

- $U_{\mathrm{YM}} = \mathrm{e}^{-\mathrm{i}Ht}$, $(K \propto 1/g)$

$$H = \frac{1}{2}\sum_{l\in L} E_l^2 - K \sum_{p \in P} \mathrm{ReTr}[UUU^\dagger U^\dagger]_p$$



- Lattice geometry



- Truncation of Hilbert space retaining SU(2) symmetry
  ➡ 3-local spin Hamiltonian (Yang-Mills-Ising model)

[Jordan 1935; Schwinger 1952; Kogut, Susskind 1975]

$$H = \sum_{i=0}^{N} \frac{3}{16}(3 - 2Z_i - Z_{i-1}Z_i) - K \sum_{i=0}^{N-1} \frac{1}{16} X_i (1 + 3Z_{i-1})(1 + 3Z_{i+1})$$

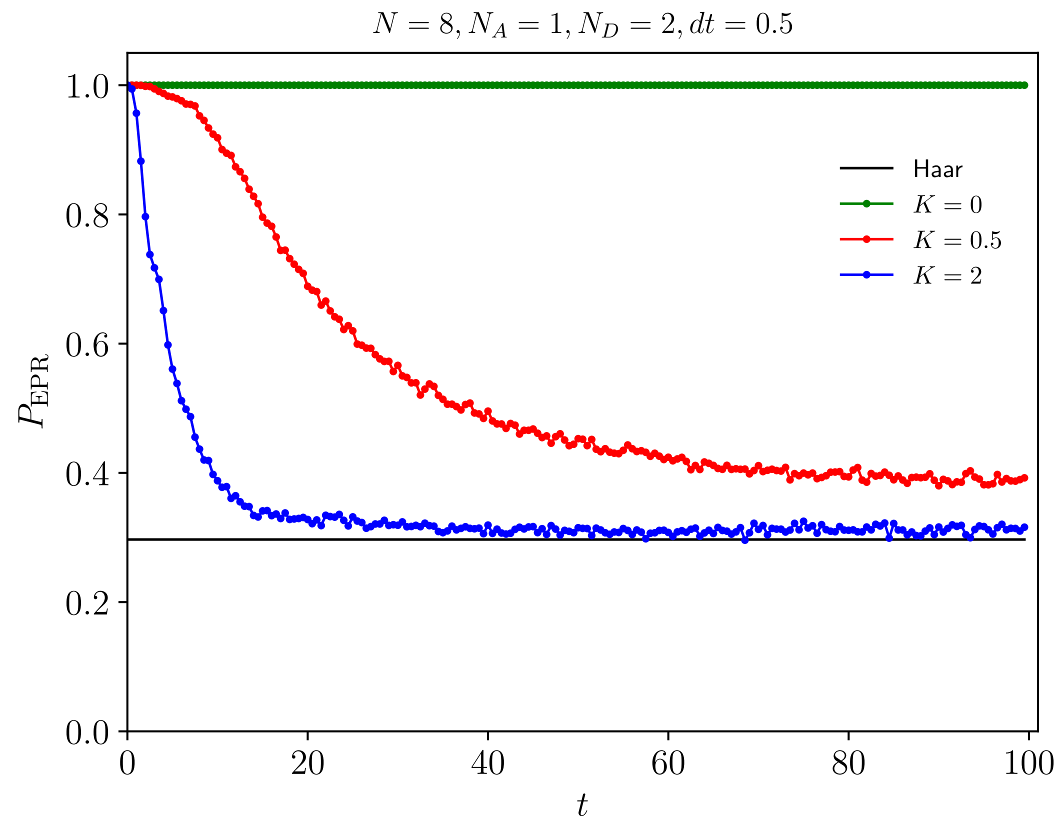
SU(2) lattice YM theory

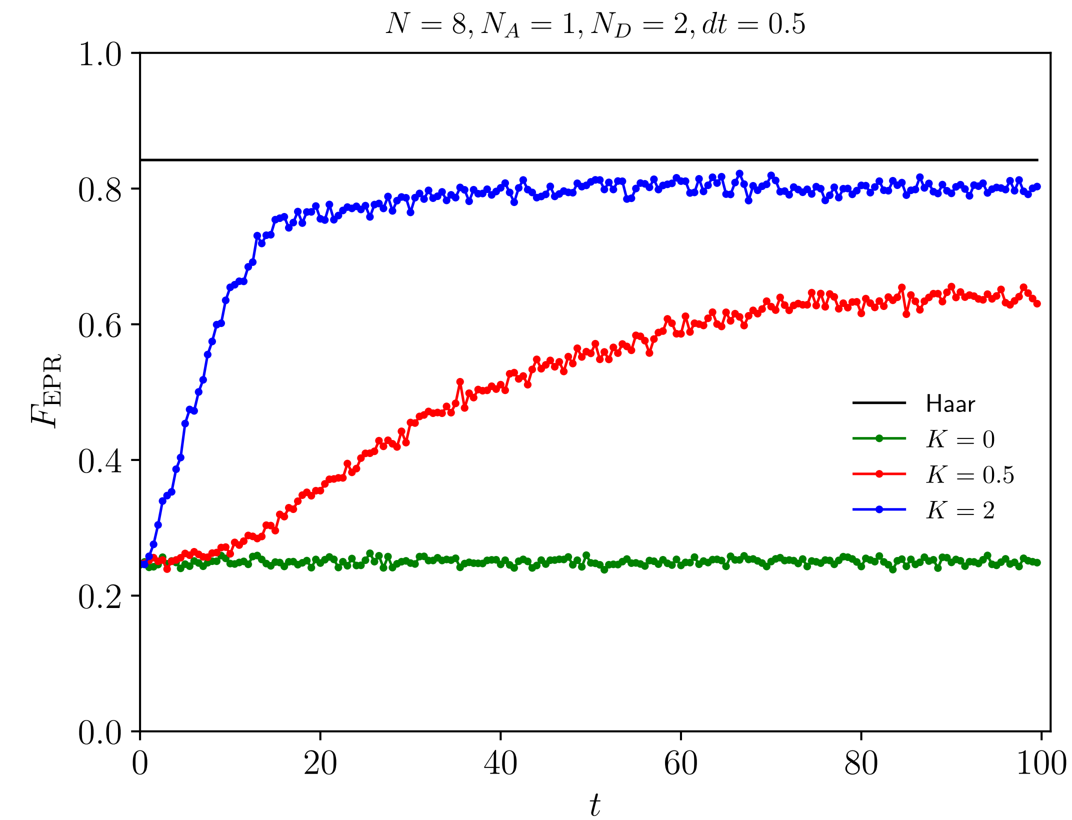
[Hayata, Hidaka, YK (2021)]

OTOC

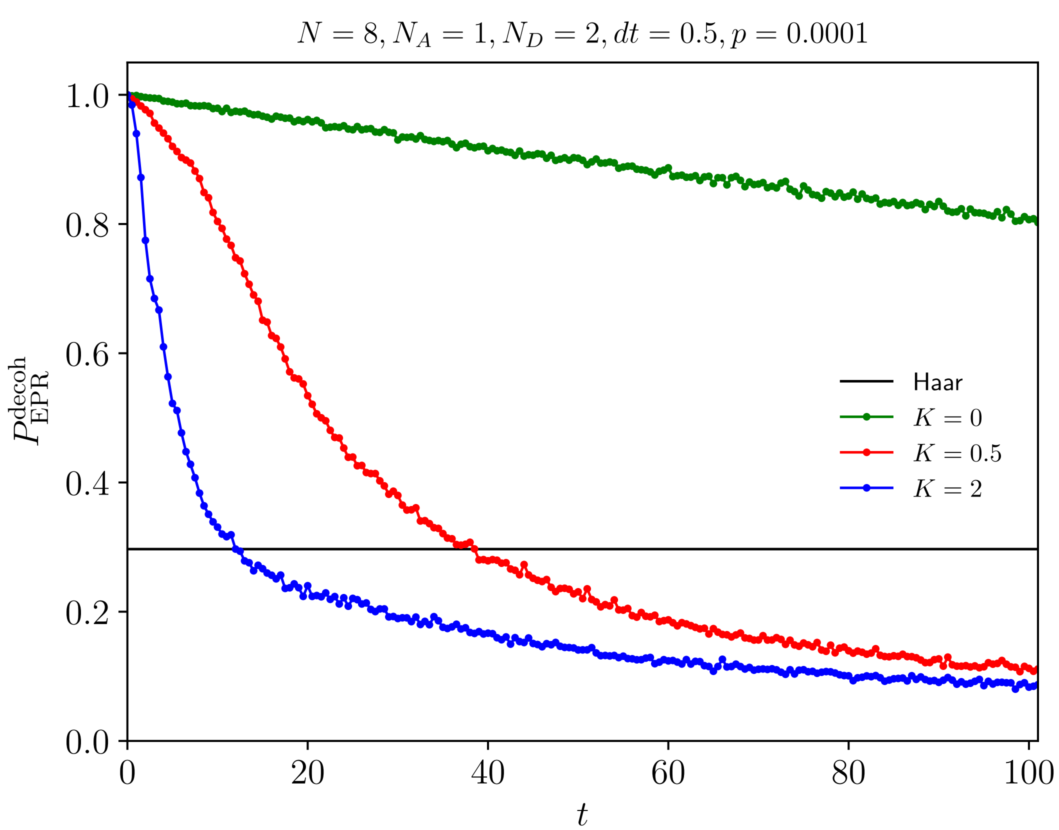
F_{EPR}

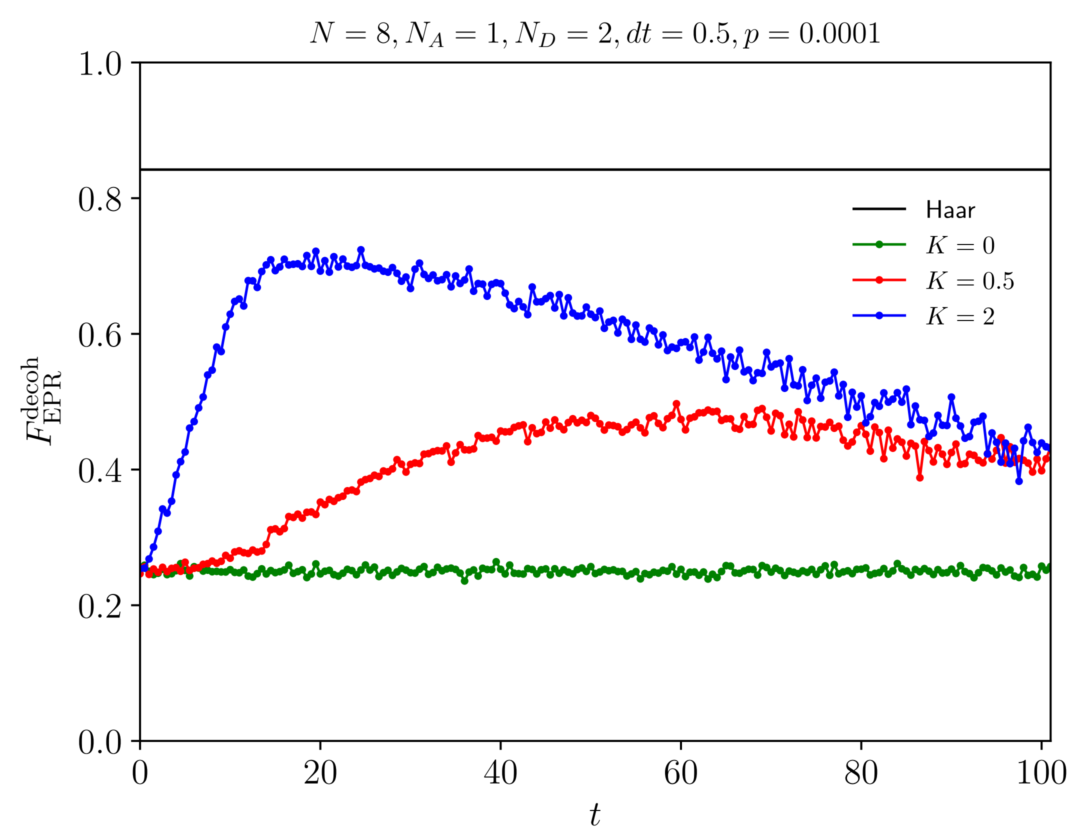
ideal





noisy





Summary

- Noise (decoherence) induces decay of OTOC
  ➡ hard to distinguish from OTOC decay due to scrambling

- In Hayden-Preskill protocol, scrambling/noise results in increase/ decrease in F_{EPR}
  ➡ Easy to distinguish their effects on noisy quantum devices

- $SU(2)$ lattice Yang-Mills theory
  ➡ Converted to simple spin model under truncation with SU(2) retained
  ➡ Rise in the fidelity observed (decay in OTOC)
  ➡ What is the effect of truncation?