

Shining a Light on the QGP -Experimental Summary of Photon Measurements at RHIC and LHC

Friederike Bock, Oak Ridge National Laboratory 9th Workshop of the APS Topical Group on Hadronic Physics.

Run:26533

Probing the QGP with Direct Photons



Can we determine the point where the QGP switches on?

ALICE

Direct Photon in $pp(\bar{p})$ collisions



Let's start with the base-line!

- Large variety of results available from 19.4 GeV 13 TeV for (isolated) direct photons
 → New results at √s = 13 TeV
- Decent agreement at large \sqrt{s} & high $p_{\rm T}$ between pQCD & data
- All pp data seem to align on a common x_T -curve within $\pm (20 50)\%$, if scaled with $(\sqrt{s})^n$ with n = 4.5
- Intriguing number:
- ightarrow Pure vector gluon exchange: n=4
- ightarrow Scale breaking effects in QCD could increase this number
- → Closer look needed if data could be described even better by slightly different **n** - could help pin down prompt photon contribution even at low p_{τ}



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(Virtual) Direct Photons in pp at low p_{T}





- New: First results on virtual photon measurement in pp collisions at 7 TeV & 13 TeV
- No large thermal component expected O(0.1-1%) in pp
- Similar size of uncertainties of real & virtual photon measurements (O(5%)) at LHC at low p_{T}
- Measuring $\gamma_{\rm dir}$ for low $p_{\rm T}$ @ LHC energies very challenging @ RHIC energies possible for $p_{\rm T} > 1.5~{\rm GeV}/c$

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Direct Photons in p–Au at RHIC at low p_{T}



d-Au MB



Increasing the system size

- Measured direct photon excess ratio in MB & 0-5% p–Au collisions at $\sqrt{s_{NN}} = 200$ GeV
- Reevaluated the pp reference data including external conversions in fit
- No clear excess yield at low p_{τ} seen in d-Au MB & p-Au MB collisions with respect to pp, well described by pQCD calculation
- Excess of low $p_{\rm T}$ direct photon with respect to pp seen for 0-5% central collisions
- Indication for thermal contribution also in central p-Au collisions



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Photons



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Direct Photons in p–Pb at LHC at low $p_{\text{\tiny T}}$



How about at LHC?

- Combination of 4 reconstruction techniques via BLUE method
- Individual sys uncertainties O(5-10%), combined total O(4-5%)
- Upper limits at 90% C.L. (arrows) determined where R_{γ} with total uncertainties consistent with unity
- 0-20% central collisions don't show a significant excess
- NLO & thermal (Shen et al.) calculations consistent with measurements

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$\label{eq:calculations} Theory calculations from: $$W$. Vogelsang (CT10,nCTEQ15,EPPS16/GRV), J.F. Paquet (CTEQ6.1M/BFG), C. Shen $$W$. Solution of the second sec$



Photons

Shen et al. arXiv:1609.02590



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Theory calculations from:

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Direct Photon Spectra at RHIC - BES & Cu-Cu





- Direct photon yield in Au-Au at $\sqrt{s_{_{\rm NN}}} = 39,62.4,200$ GeV & Cu-Cu at $\sqrt{s_{_{\rm NN}}} = 200$ GeV & Pb-Pb $\sqrt{s_{_{\rm NN}}} = 2.76$ TeV follow similar behavior at low $\rho_{_{\rm T}}$
- Spectra normalized by $(dN_{ch}/d\eta)^{\alpha}$, where $\alpha = 1.25 \pm 0.02$ obtained from simultaneous fit to N_{coll} vs $dN_{ch}/d\eta$ for all collision systems

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Photons

Direct Photon Spectra at RHIC - BES & Cu-Cu





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Direct Photon Spectra at RHIC - BES & Cu-Cu









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- Story not as clear, when looking at STAR data in addition
- \bullet Theoretically not easy to understand scaling across different $\sqrt{s_{\rm NN}}$
- Prompt and thermal photons should scale with different slopes at one $\sqrt{s_{\rm NN}}$
- Can we learn something about admixture from different p_{τ} cuts?









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Thermal photons

 $p_T > 0.6 \text{GeV}$





Slopes





Direct Photon Yield and Flow - At the LHC



• Central points for direct photon yield and $v_2^{\gamma, \rm dir}$ underestimated by most theoretical calculations by factors of 2-5



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Direct Photon Yield and Flow - At the LHC





Direct Photon Yield and Flow - Comparison to PHENIX





p_ (ĞeV/c)

Direct Photon Yield and Flow - Comparison to PHENIX





p_ (ĞeV/c)

Direct Photon Yield and Flow - Comparison to PHENIX





Photons as probes for the initial state & scaling properties

What can we learn about the scaling properties when going from $pp \rightarrow p-A \rightarrow A-A$ from γ spectra?







Isolated direct photon measurement in p–Pb collisions at $\sqrt{s_{_{\rm NN}}}=8~{\rm TeV}$ by ATLAS

- *N_{coll}* scaling works at mid rapidity
- Prompt photon production at large p_τ in forward and backward region could constrain nPDFs & energy loss scenarios significantly
- Current precision not yet sufficient to do so
- Slight preference for no energy loss in p–Pb collisions



Isolated Photons as calibration & tagging objects for jet modification studies in p-A and A-A collisions



$\gamma\text{-h}$ and $\gamma\text{-jet}$ correlations in p(d)-A & A-A collisions



- Base-line measurements in pp & p-Pb 5 TeV (ALICE)
- Access to intermediate photon p_{T} triggered correlation (10-40 GeV/c) functions even @ LHC energies
- No significant modification of jet fragmentation observed in p-A collisions
- $\gamma_{\rm dir}$ +jet and π^0 +jet show similar level of suppression of recoil jet, stronger for R=0.2 than for R=0.5

Modification of jet properties in Pb-Pb collisions



Constraining quark-jet modification



γ +jet p_{T} -balance & γ -tagged jet FF

- pp-like peaked $x_{J\gamma}$ in peripheral Pb-Pb, smeared in central Pb-Pb
- $\rightarrow\,$ Variation in jet-by-jet E-loss
- $\bullet \ \gamma\text{-tagged}$ jet frag. functions different modification in central evts. than inclusive jets
- ξ^{γ}_{T} & gamma-tagged Jet shape
 - Central PbPb collisions \rightarrow enhancement of low- p_{τ} part. and a depletion of high- p_{τ} part. ξ_{T}^{γ} modified stronger compared to ξ_{jet}
 - Larger enhancement at large r & Smaller depletion at intermediate r compared to di-jets
- \rightarrow Increased quark fraction (70-80%)?
- \rightarrow Lower jet p threshold (higher fraction of quenched jets)?

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$\xi^{\gamma}_{\rm T}$ & gamma-tagged Jet shape from CMS

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ATLAS: PLB 789 (2019) 167 CMS: PLB 785 (2018) 14, PRL 121, 242301 (2018)

Thanks to all speakers & the organizers for making this conference possible!





BACKUP



Direct Photons in pp at LHC at low p_{T}



• Systematic uncertainties of individual meas.

 \rightarrow dominated by $p_{\rm T}\text{-}independent$ material unc. of 4.5% PCM, 2.8% EMC & global E-scale unc. 3% PHOS

- Combination of 3 reconstruction techniques via BLUE method
- NLO prediction plotted as

 $\textit{R}_{ extsf{NLO}} = 1 + (\gamma_{ extsf{dir}}^{ extsf{NLO}} \cdot \textit{N}_{ extsf{Coll}}) / \gamma_{ extsf{dec}}$

• Upper limits at 90% C.L. (arrows) determined where R_{γ} with total uncertainties consistent with unity



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Constraints to FF from RHIC





- High precision data from PHENIX further constrains FF
- Data favor BFG II FF over BFG I and GLV
- $\rightarrow\,$ BFG II FF has largest gluon contribution

25

20

15

p_{_} (GeV)

- BEG

5

BFG II GRV NLO

PHENIX with stat, error

10

10⁻¹⁰

10⁻¹

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Direct Photons in pp at LHC at high $p_{\rm T}$





- More differential data available from ATLAS & CMS for inclusive direct photon production at 7,8 & 13 TeV (isolated)
- Reasonable agreement with different pQCD calculations & event generators
- New results on isolated γ + N jet production test pQCD up to O($\alpha_{em} \alpha s^4$)



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Direct Photons in pp at LHC at high p_{T}



• New results on isolated γ + N jet production test pQCD up to O($\alpha_{\it em}\alpha s^4)$

Photons

600 700 900

(c)

m^{γ-jet1} [GeV]

ILO/Da

0.6

(d)



I/N, dN₃/dx

0.5

Direct Photons in pp at LHC at high p_{τ}



• New results on isolated $\gamma + N$ jet production test pQCD up to $O(\alpha_{em}\alpha s^4)$

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 $\Delta \phi^{\gamma-\text{jet3}}$ [rad]



Direct Photons in d-Au at RHIC







- Measured direct photon excess ratio in d–Au collisions at $\sqrt{s_{_{\rm NN}}}=200~{\rm GeV}$ over wide $p_{_{\rm T}}$ range
- $\bullet\,$ Small hint at suppression at high $p_{\rm T},$ statistical precision not sufficient
- $\rightarrow R_{dA}$ slightly better described if Cronin, isospin and shadowing effect are included
- No significant low $p_{T} R_{dA}$

Direct Photon Spectra in Au-Au at RHIC - 200 GeV (I)





• 20-30% reduction of direct photon R_{AA} expected due to energy loss

12 14 16 18

p_T (GeV/c)

8 10

ALICE

Direct Photon Spectra in Au-Au at RHIC - 200 GeV (II)





- Nearly no centrality dependence in R_{γ} , peripheral still $\sim 5\%$ excess, although not statistically significant anymore
- Excess $\approx 20\%$ in 0-20% Au–Au, systematic uncertainties O(5%)
- Strong excess above extrapolated pp measurement (green curve) seen in all centrality classes
- Slope of excess depends very little on centrality ($T_{e\!f\!f}\approx 235\pm40~{\rm MeV}/c)$





Direct Photon Spectra in Au-Au at RHIC - 200 GeV (III)



- Virtual direct photon spectrum measured by STAR at low p_T disagrees between 1-3 GeV/c by a factor 2
- BUT: Large syst. errors due to unmeasured eta contribution at low p_T





- \bullet Direct photon excess measured with combined PCM + PHOS in 3 centrality classes with 2010 Pb–Pb data
- R_{γ} excess at high p_{T} for all centralities
- $\gamma^{\rm dec}$ suppressed by $\approx R_{\rm AA}^{\pi^0}$ \rightarrow larger excess in central collisions
- Low $p_{\rm T}~\sim 15\%$ excess in 0-20% and $\sim 9\%$ in 20-40%
- In agreement with NLO pQCD, JETPHOX above 5 ${\rm GeV}/c$
- No low $p_{\rm T}$ excess seen in pp collisions at same center-of-mass energy
- Scaled pp spectrum & upper limits fully consistent with Pb-Pb results





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PHENIX Direct Photon ν_2/ν_3 Results - Au-Au



arXiv:1509.07758



- \bullet Direct photon ν_2 & ν_3 comparable to that of other hadrons
- Two independent methods give comparable result
- \bullet Theory not able to reproduce large ν_2 and even less ν_3



Direct Photon Yield and Flow - At RHIC

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- Large yield and large anisotropy have been observed in Au–Au at 200 GeV by PHENIX
- Challenge for theory to describe both measurements simultaneously
- Large yield from early emission?
- Large v₂ from late emission?

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\Rightarrow Direct Photon Puzzle





Cocktail Simulation of Decay Photon v_2



Decay photon v_2 :

- KE_T scaling: v_2 of mesons scales with KE_T $KE_T = m_T - m = \sqrt{p_T^2 + m^2} - m$
- $\Rightarrow v_2^{\pi^0} \approx v_2^{\pi^{\pm}} (m^{\pi^0} \approx m^{\pi^{\pm}})$
- $\rightarrow~v_2$ of various mesons (X) calculated via ${\cal K} E_{\cal T}$ (quark number) scaling from $v_2^{{\cal K}^\pm}$

$$v_2^X(p_T^X) = v_2^{K^{\pm}} \left(\sqrt{(KE_T^X + m^{K^{\pm}})^2 - (m^{K^{\pm}})^2} \right)^2$$

• Decay photon v₂ from different mesons obtained from cocktail calculation



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\textit{v}_2^{γ} Inclusive and Decay

- $v_2^{\gamma,\text{inc}}$ measured with PCM & PHOS
- \rightarrow Corrected for BG flow from impurities $_{\rm [JPG~44~(2917)~no.~2,~025106]}$
- \rightarrow Assumed to be independent
- \rightarrow Consistent, *p*-values of 0.93 (0-20%) & 0.43 (20-40%)





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- $p_{\scriptscriptstyle T}~<3~GeV/{\it c:}~v_2^{\gamma,inc}=v_2^{\gamma,dec}$
- $\Rightarrow \begin{array}{l} \mbox{Either no contribution of } \gamma_{\rm dir} \\ \mbox{or } v_2^{\gamma, \rm inc} \approx v_2^{\gamma, \rm dec} \end{array}$
- ightarrow Theory $\sim 30-40\%$ too high
- $p_{\tau}~>3~GeV/\mathit{c}:~v_{2}^{\gamma,inc} < v_{2}^{\gamma,dec}$
- $\rightarrow \mbox{Direct photon } v_2 \mbox{ contribution with } v_2^{\rm direct} < v_2^{\rm decay}$
- $\rightarrow\,$ Mainly prompt photons







Direct photon v_2 :

$$v_2^{\gamma,\mathsf{dir}} = rac{R_\gamma \cdot v_2^{\gamma,\mathsf{inc}} - v_2^{\gamma,\mathsf{dec}}}{R_\gamma - 1}$$

- $\bullet\,$ Measured ${\it R}_{\gamma}$ often less than $2\sigma_{\rm sys}$ deviation from 1
- ⇒ Central value & unc. calculated using MC simulation following Bayesian approach with probability distributions of true values of $R_{\gamma}^{t}(p_{T}), v_{2}^{\gamma, \text{dec}, t}(p_{T}), v_{2}^{\gamma, \text{inc}, t}(p_{T})$ assuming R_{γ} can't be smaller unity & partially p_{T} correlated unc.
- Large direct photon v_2 for $p_T < 3 \text{ GeV}/c$ measured
- Magnitude of $v_2^{\gamma, dir}$ comparable to hadrons
- Result points to late production times of direct photons after flow is established



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Jet observables: a quick reminder



$$\begin{aligned} \xi^{jet} &= \ln \frac{|\mathbf{p}^{jet}|^2}{\mathbf{p}^{track} \cdot \mathbf{p}^{jet}} \tag{1} \\ \xi^{\gamma}_T &= \ln \frac{-|\mathbf{p}^{\gamma}_T|^2}{\mathbf{p}^{track}_T \cdot \mathbf{p}^{\gamma}_T} \tag{2} \end{aligned}$$