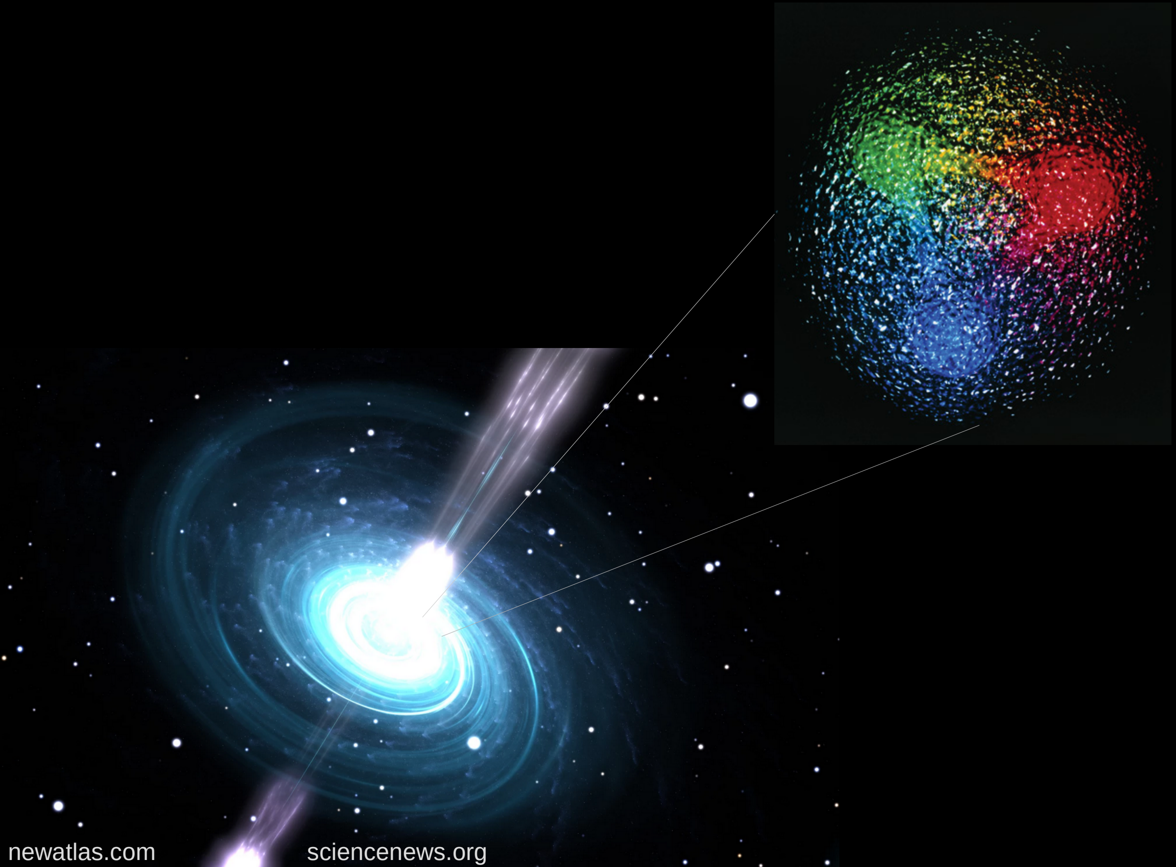
Bounds on the Equation of State of Neutron Stars from the Quantum Chromodynamics Energy-Momentum Tensor



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APS GHP meeting 2021
April 14, 2021

In collaboration with

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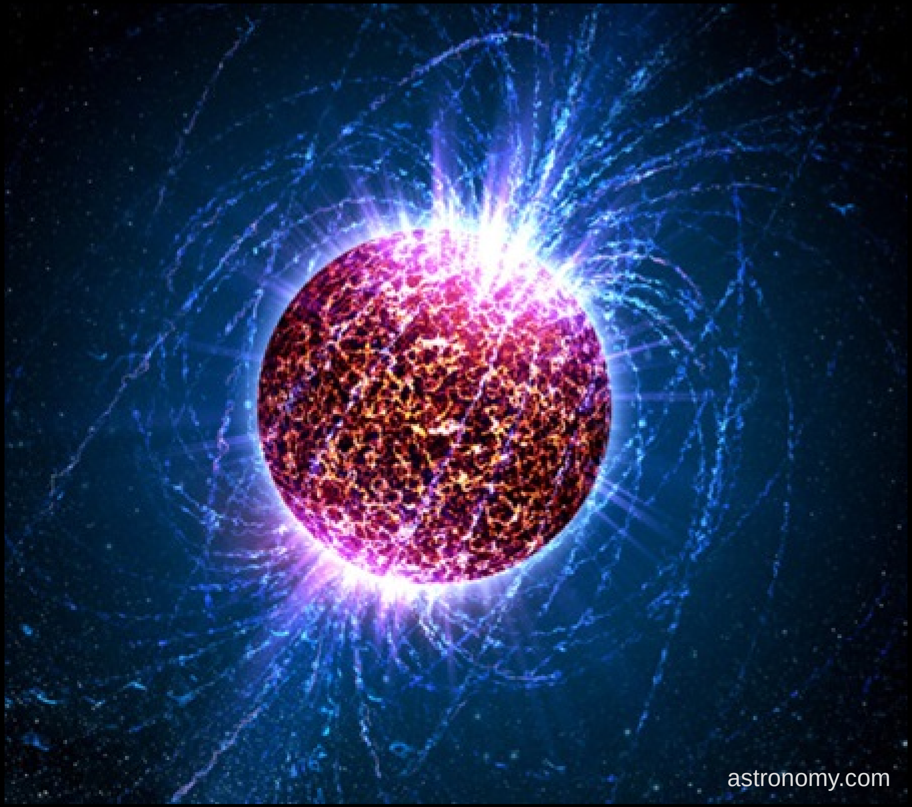

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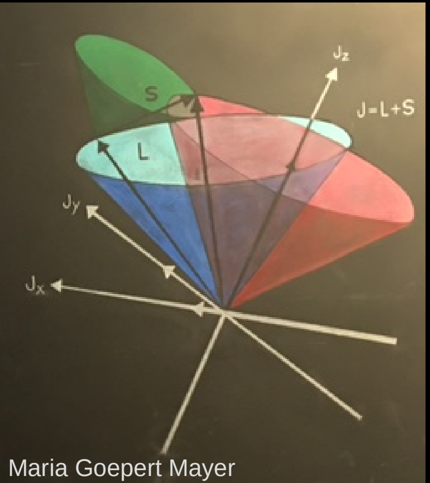
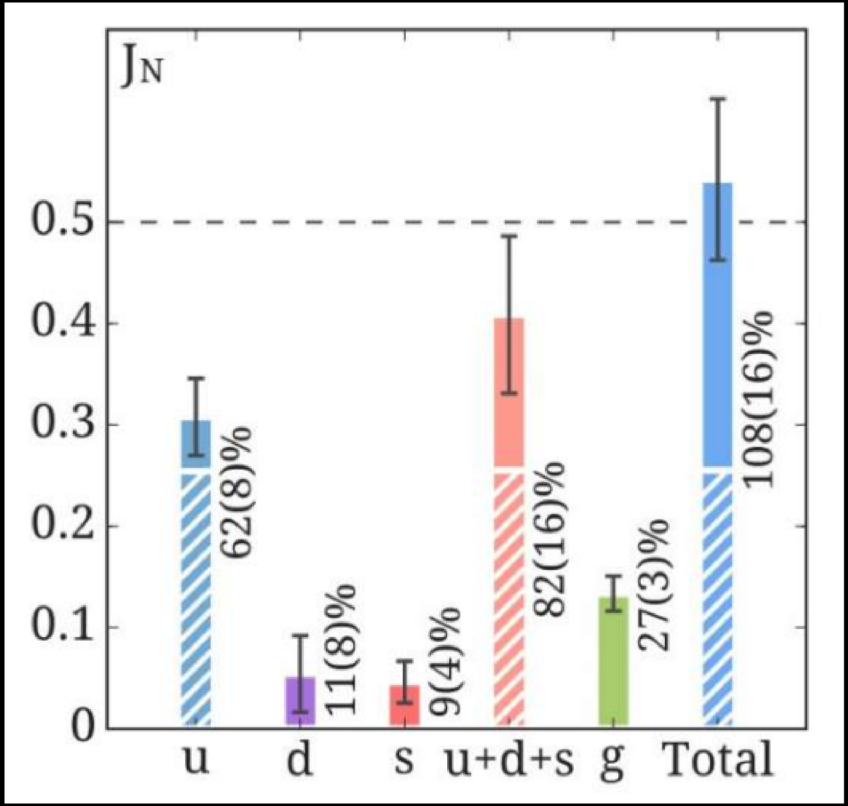
New insights into the structure of the nucleon



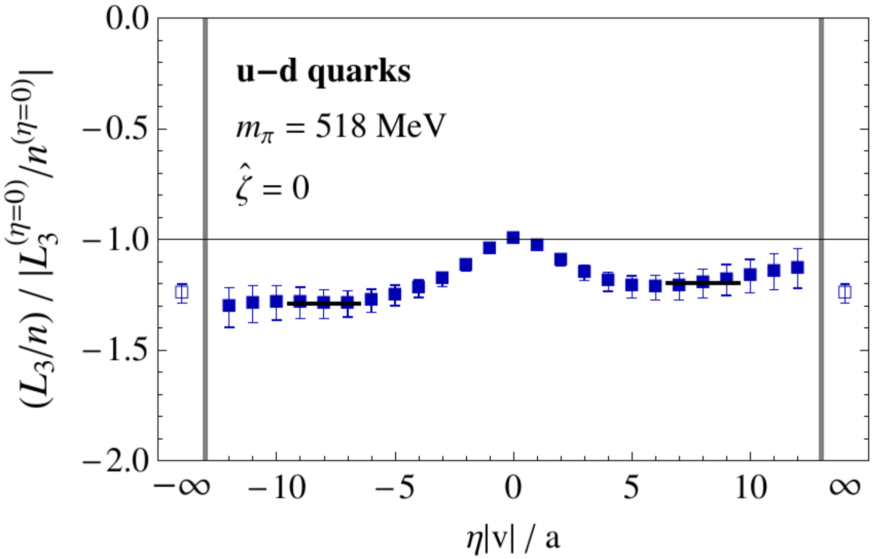
Maria Goepert Mayer



We understand a lot more about the separate contributions of quarks and gluons to the proton spin.

Alexandrou et al, PRL (2017)

New insights into the structure of the nucleon



Quark orbital angular momentum.

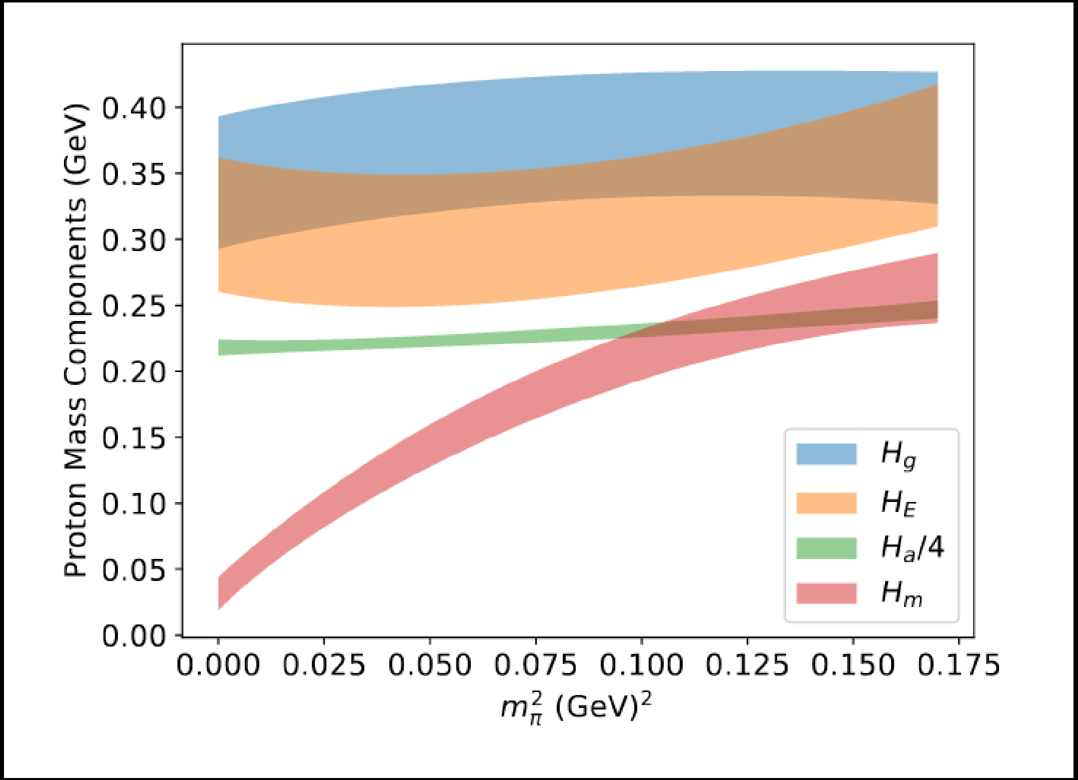
M Engelhardt PRD (2017)

x dependent OAM density

$$\int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14} = -\int_x^1 dy \left(\widetilde{E}_{2T} + H + E \right)$$

AR, M Engelhardt, S Liuti PRD (2018)

New insights into the structure of the nucleon



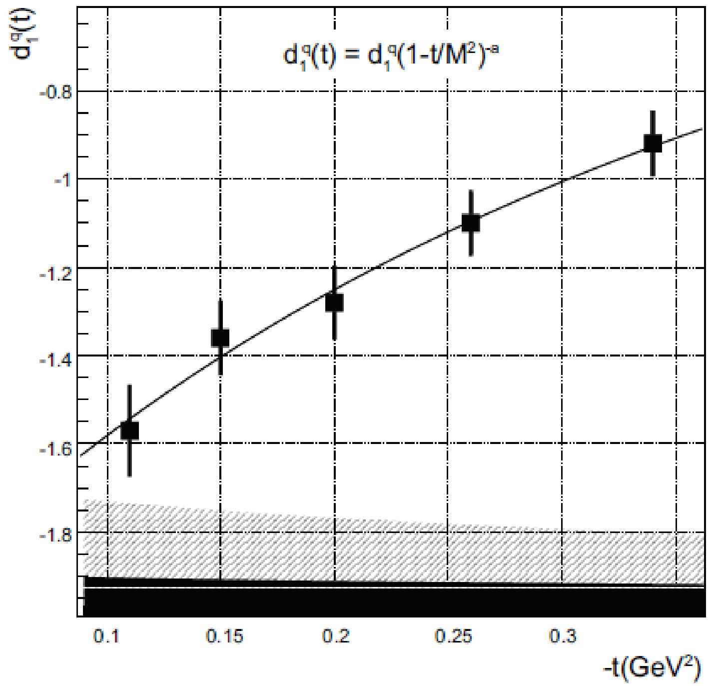
The mass decomposition of the proton.

Yang et al, PRL (2018) XQCD

New insights into the structure of the nucleon

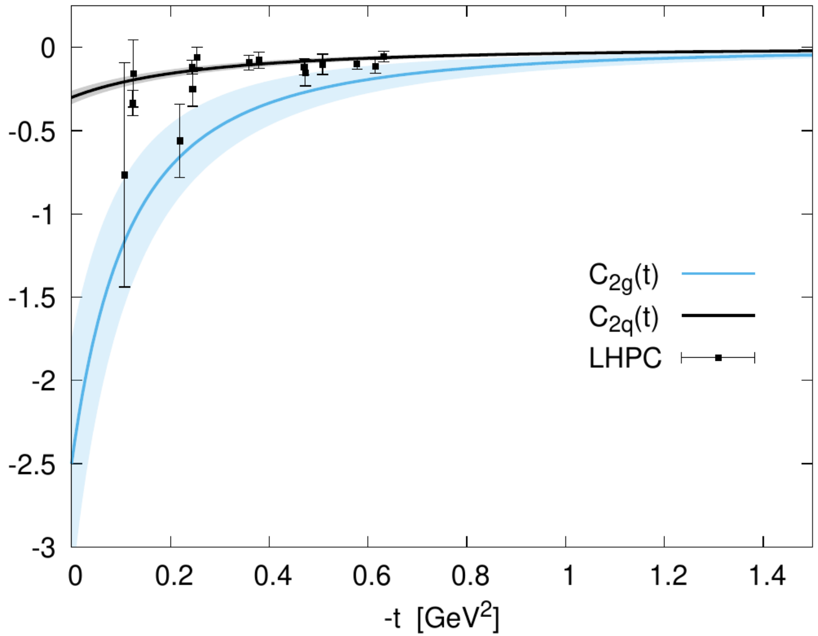
Quark and gluon pressure

Experiment



Burkert, Elouadrhiri, Girod (Nature, 2018)

Lattice



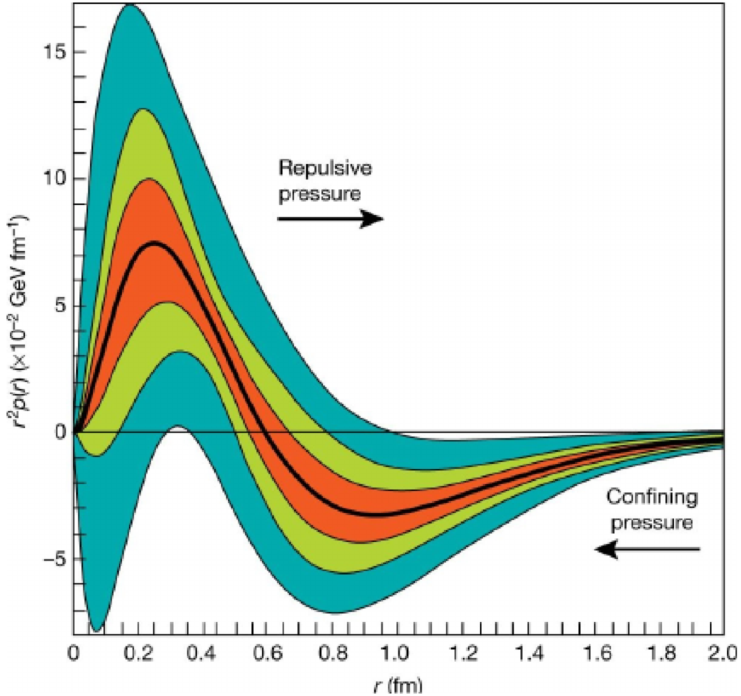
Detmold and Shanahan (PRL, 2019)

Hagler et al, (PRD, 2007)

New insights into the structure of the nucleon

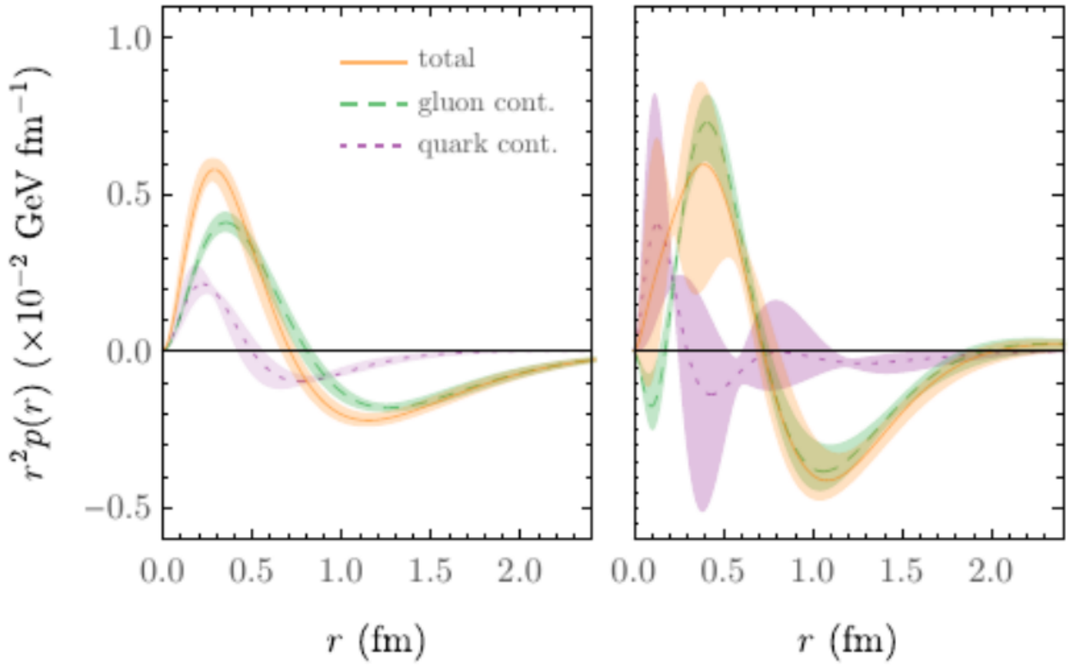
Quark and gluon pressure

Experiment



Burkert, Elouadrhiri, Girod (Nature, 2018)

Lattice



Detmold and Shanahan (PRL, 2019)

Motivation

We now have a much deeper understanding of the quark and gluon structure of hadrons.

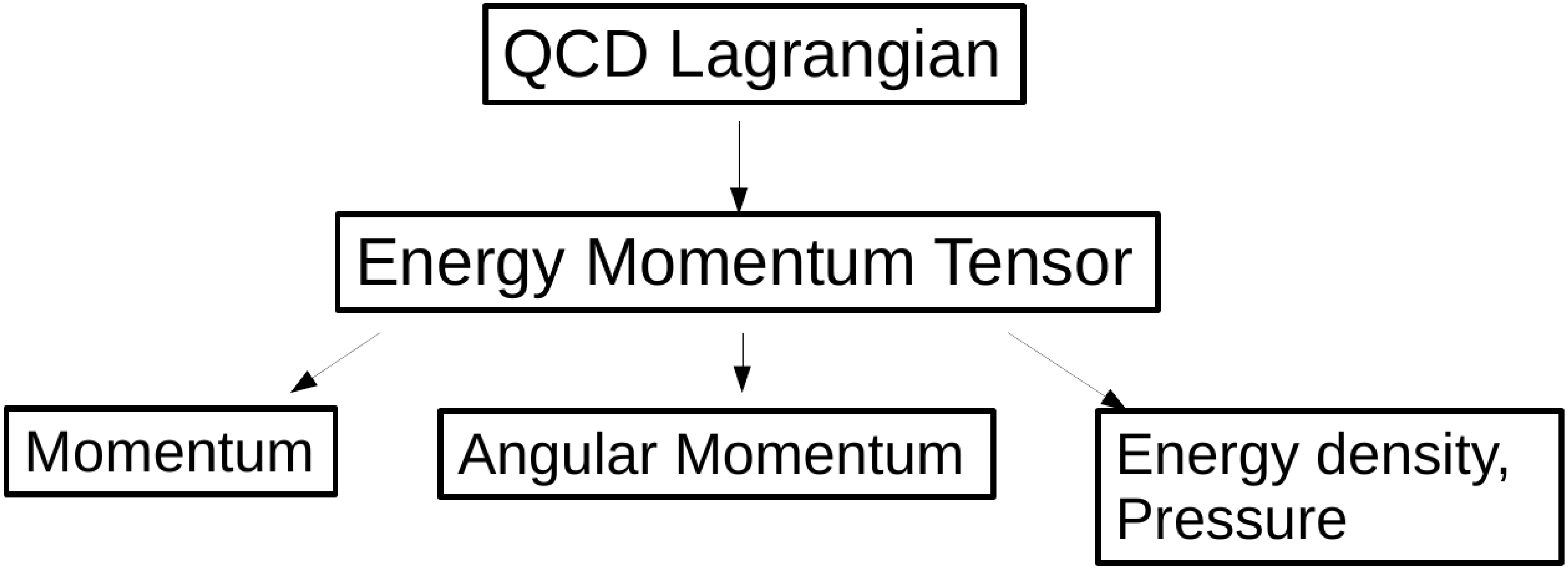
Can we use this knowledge and apply it to another place where strong interactions are crucial, the neutron stars?


riken.jp

Outline

- The QCD energy momentum tensor
- Probing QCD through deep inelastic processes
  - Exclusive scattering and observables, Generalized parton distributions
  - Radial distribution of components of the EMT
- Neutron star equation of state

QCD Energy Momentum Tensor



QCD Lagrangian

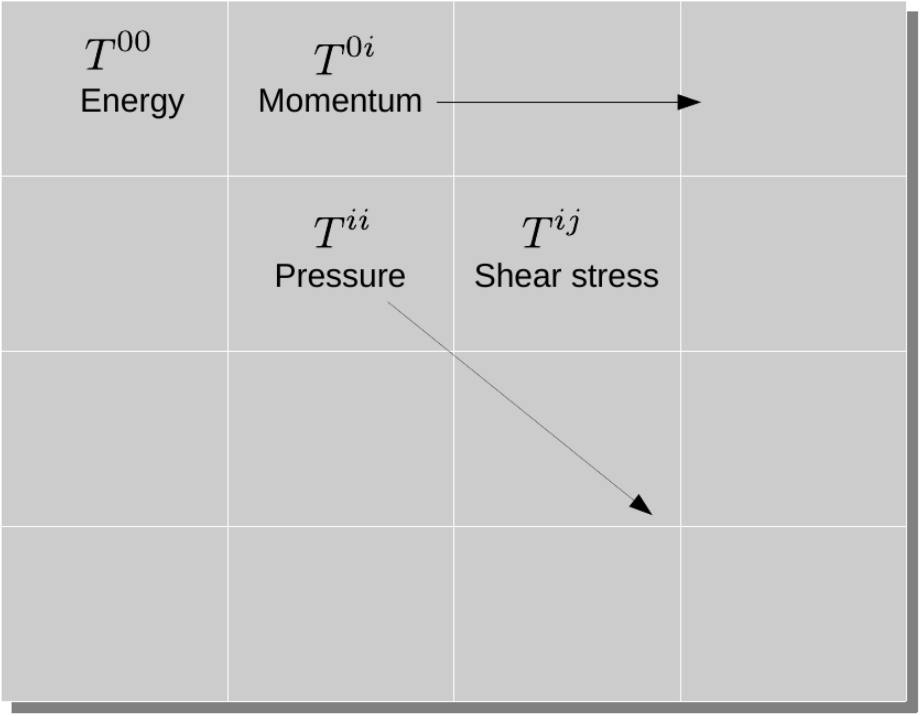
Energy Momentum Tensor

Momentum

Angular Momentum

Energy density, Pressure

QCD Energy Momentum Tensor



T^{00}
Energy

T^{0i}
Momentum

T^{ii}
Pressure

T^{ij}
Shear stress

QCD Energy Momentum Tensor

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overset{\leftrightarrow}{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

$$T^{\mu\nu} = T^{\mu\nu}_q + T^{\mu\nu}_g$$

$$T^{\mu\nu}_q = i\bar{\psi}\gamma^{(\mu}\overset{\leftrightarrow}{D}^{\nu)}\psi$$

$$T^{\mu\nu}_g = -F^{\mu\Lambda}F^{\nu}_{\Lambda} - \frac{g^{\mu\nu}}{4}F^2$$

QCD Energy Momentum Tensor

To connect to a quantity measurable in an experiment, take the matrix element of the energy momentum tensor in between hadron states.

$$\langle P| T_q^{\mu\nu} |P\rangle = \langle P| i\bar{\psi}(0)\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi(0) |P\rangle$$

QCD Energy Momentum Tensor

To connect to a quantity measurable in an experiment, sandwich the energy momentum tensor in between hadron states.

$$\langle P| T_q^{\mu\nu} |P\rangle = \langle P| i\bar{\psi}(0)\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi(0) |P\rangle$$

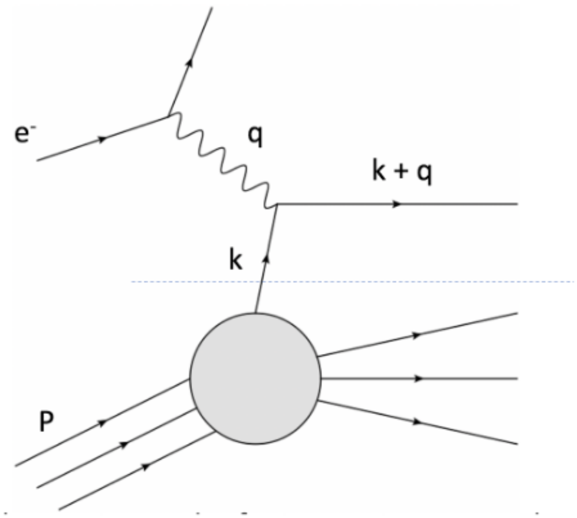
Parameterize EMT by unknown form factors

$$\langle P| T_q^{\mu\nu} |P\rangle = \bar{u}(P)A_q\gamma^{(\mu}\bar{P}^{\nu)}u(P)$$

Probing the QCD structure of hadrons

$$a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$

What do we measure in an experiment?



Deep inelastic scattering processes allow the separation of the perturbative and non-perturbative partonic structure of the hadron.

$$f(x) \quad = \quad \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \, \langle p \mid \bar{\psi}\,(0)\,\gamma^+\,\mathcal{U}(0,z^-)\psi\left(z^-\right) \mid p\rangle\big|_{z^+=0,z_T=0}$$

A parton distribution function gives the number of particles carrying momentum fraction x.

Integrating in momentum fraction x gives a local operator, the quark field separation is reduced to zero.

Probing the QCD structure of hadrons

The second moment in x of the PDF gives exactly the local operator associated with the energy carried by the partons.

$$\int dx\, x f_q(x) = A_q$$

QCD Energy Momentum Tensor

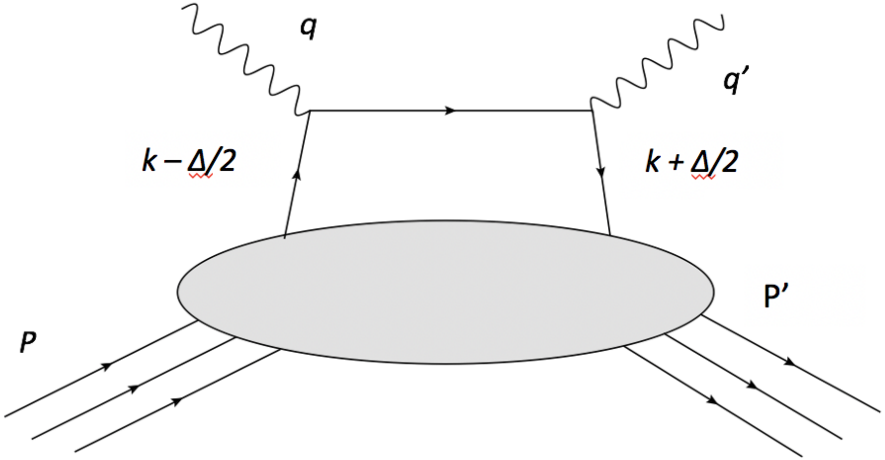
Consider taking the matrix element of the energy momentum tensor in between hadron states carrying different momenta.

$$\langle P'|\, T^{\mu\nu}_{q,g}\, |P\rangle \; = $$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}}{2M} + C_{q,g}\frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

How do we get to these?

Probing the QCD structure of hadrons – Exclusive Processes



$$H(x, \xi, t)$$

$$t = \Delta^2 \qquad \xi = -\frac{\Delta^+}{P^+ + P'^+}$$

In a deep inelastic exclusive process such as Deeply Virtual Compton Scattering, the non-perturbative partonic structure of the hadron is described by Generalized Parton Distributions.

X Ji (1997)

Probing the QCD structure of hadrons – Exclusive Processes

The first moment in x of the GPD H yields the Dirac form factor

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t)$$

Probing the QCD structure of hadrons – Exclusive Processes

The second moment in x of the GPD H involves two generalized form factors.

$$\int_{-1}^{1} dx\, x H(x,\xi,t) = A(t) + 4\xi^2 C(t)$$

energy

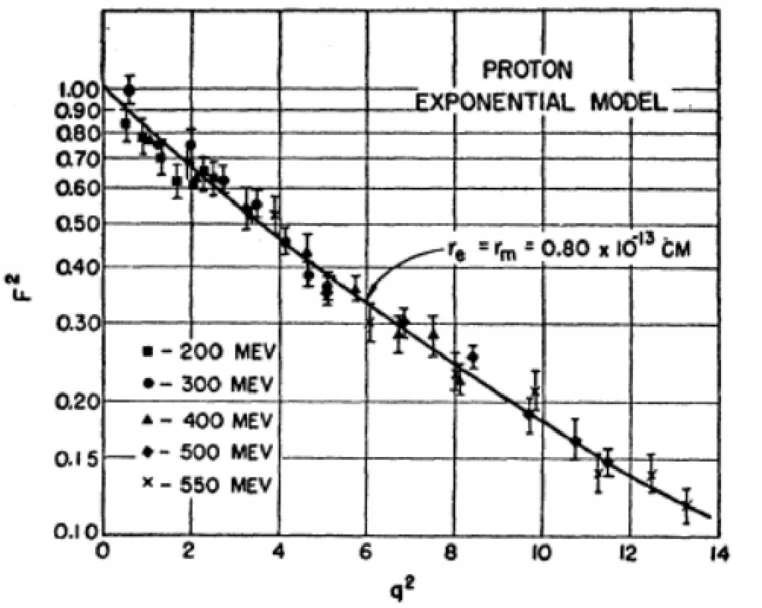
pressure

$$\langle P'|\, T^{\mu\nu}_{q,g}\, |P\rangle \;=$$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

Form factors as Fourier transforms of spatial densities

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3x$$



Hofstadter et al, (1956)

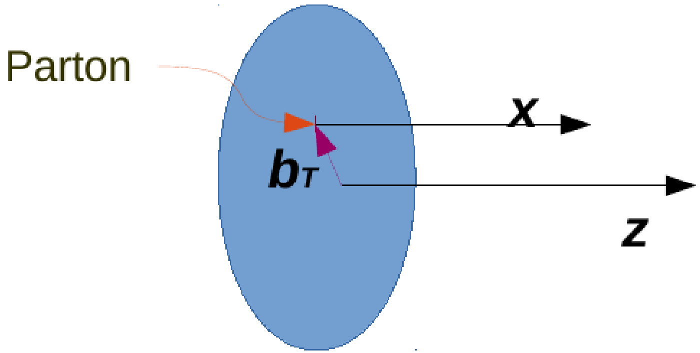
The interpretation as a probability density is problematic. It is not invariant under boosts.

Spatial densities in the infinite momentum frame

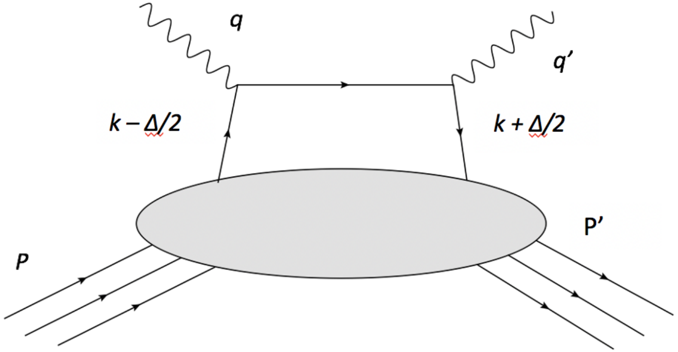
$$z^+ = \frac{z^0 + z^3}{\sqrt{2}}$$

Field operators are defined at equal ‘time’

The momentum transfer in the transverse direction allows one to define densities in the transverse plane.



$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

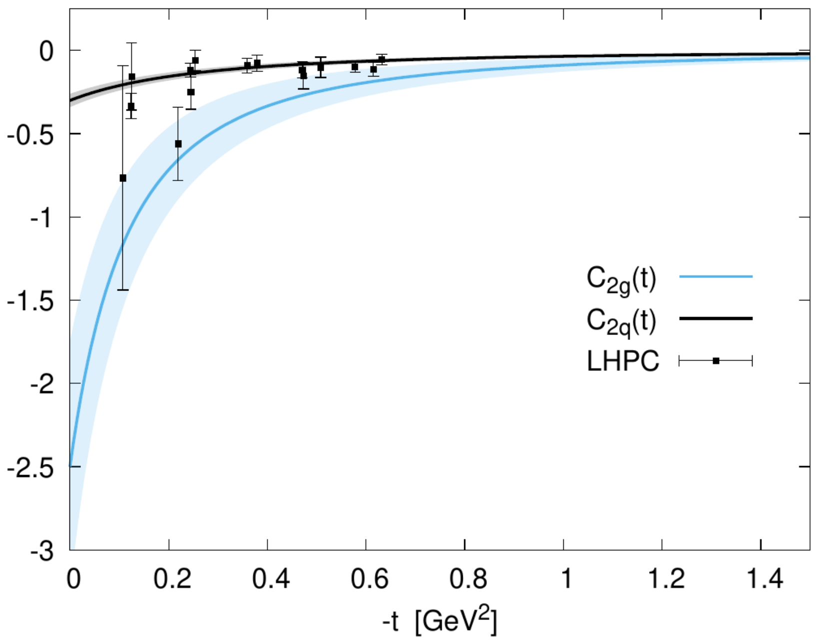


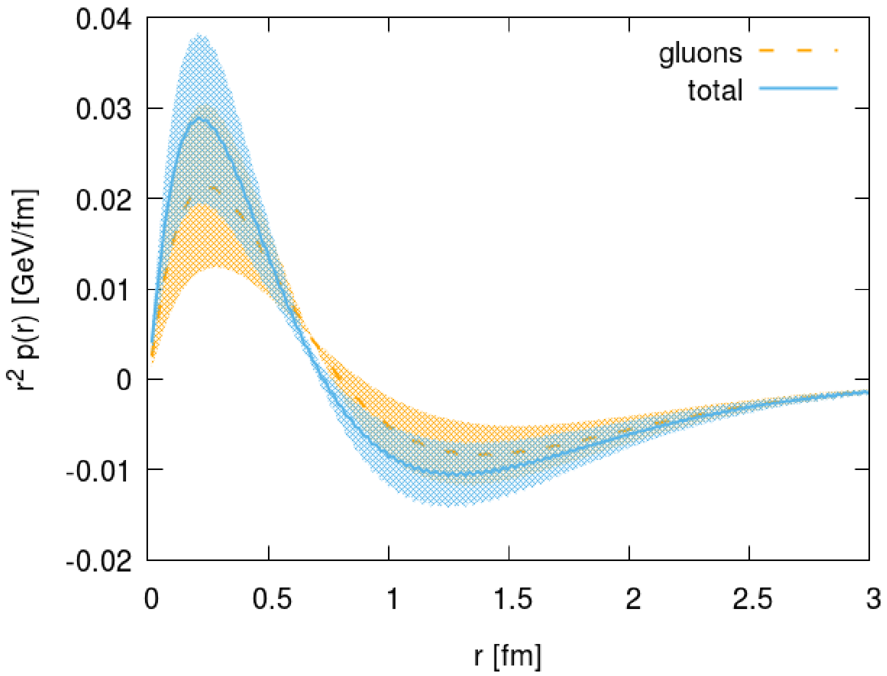
Soper (1977)
Burkardt (2003)

$$\xi = -\frac{\Delta^+}{P^+ + P'^+}$$

taken to be zero

Extracting Generalized Form Factors from lattice QCD

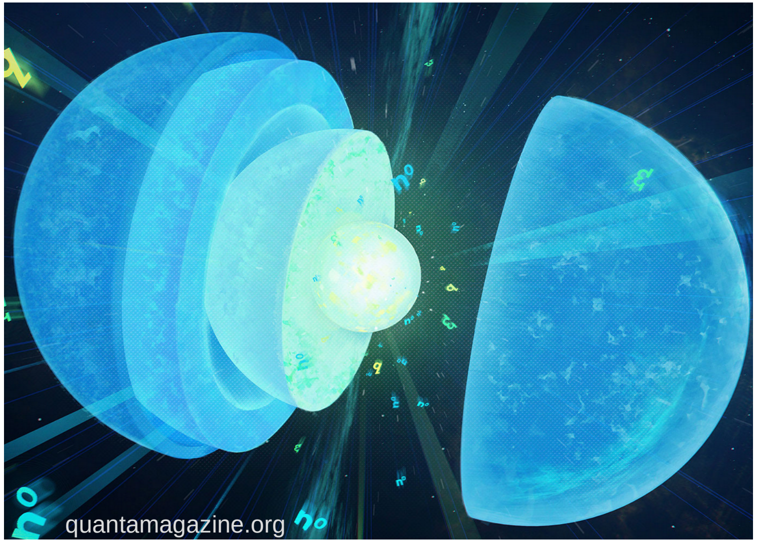




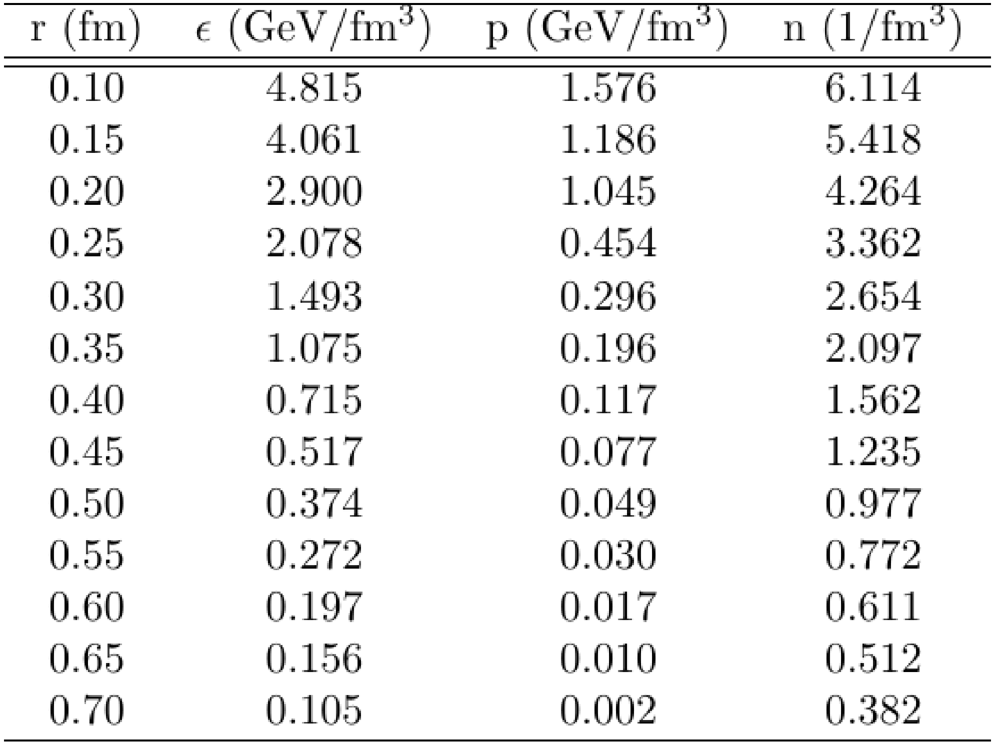
Detmold and Shanahan (PRL, 2019)

Hagler et al, (PRD, 2007)

What goes on inside a neutron star?

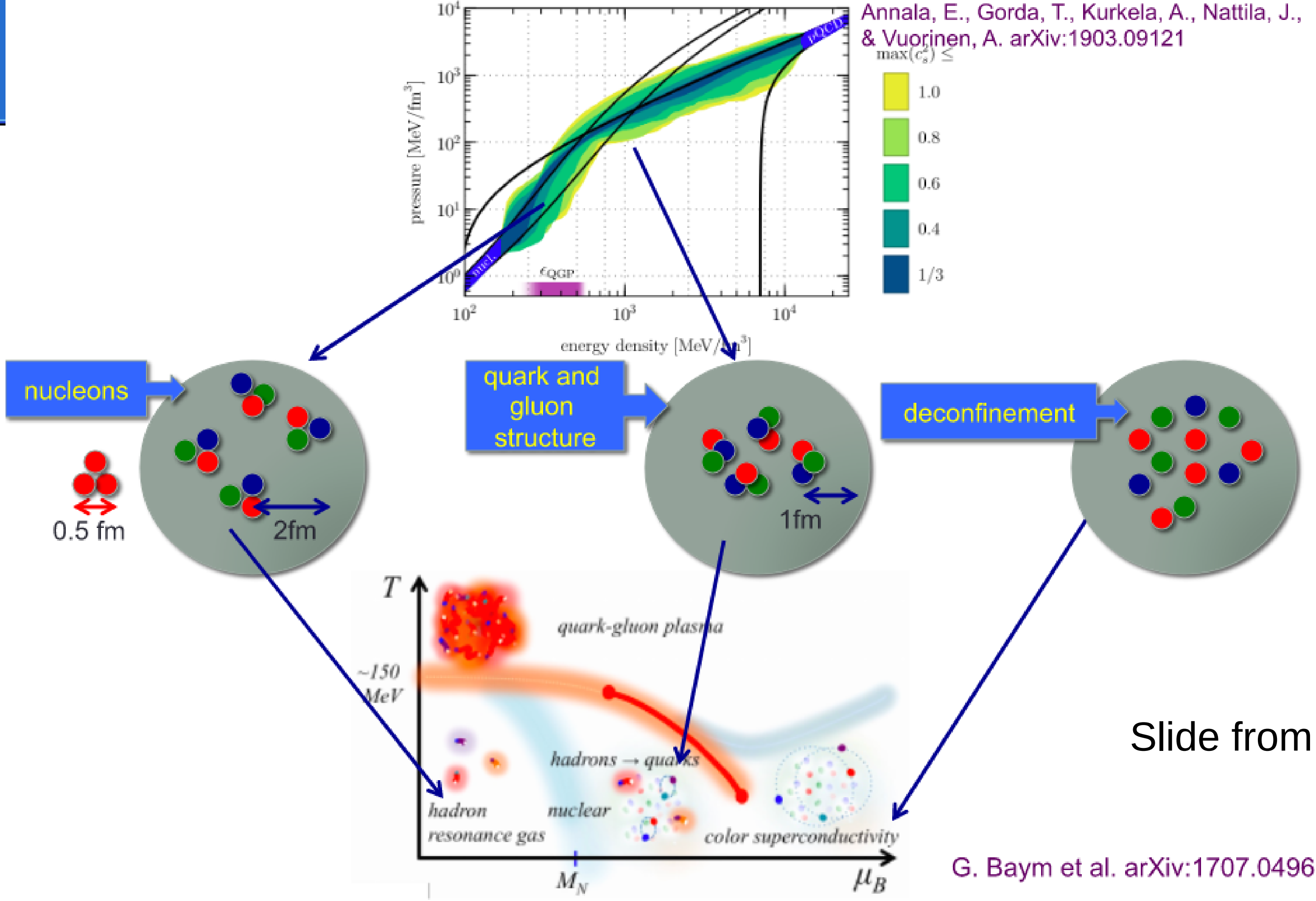

quantamagazine.org

Spatial distribution of components of the QCD EMT



r (fm)	ϵ (GeV/fm^3)	p (GeV/fm^3)	n (1/fm^3)
0.10	4.815	1.576	6.114
0.15	4.061	1.186	5.418
0.20	2.900	1.045	4.264
0.25	2.078	0.454	3.362
0.30	1.493	0.296	2.654
0.35	1.075	0.196	2.097
0.40	0.715	0.117	1.562
0.45	0.517	0.077	1.235
0.50	0.374	0.049	0.977
0.55	0.272	0.030	0.772
0.60	0.197	0.017	0.611
0.65	0.156	0.010	0.512
0.70	0.105	0.002	0.382

Densities and distance scales



Annala, E., Gorda, T., Kurkela, A., Nattila, J., & Vuorinen, A. arXiv:1903.09121

nucleons

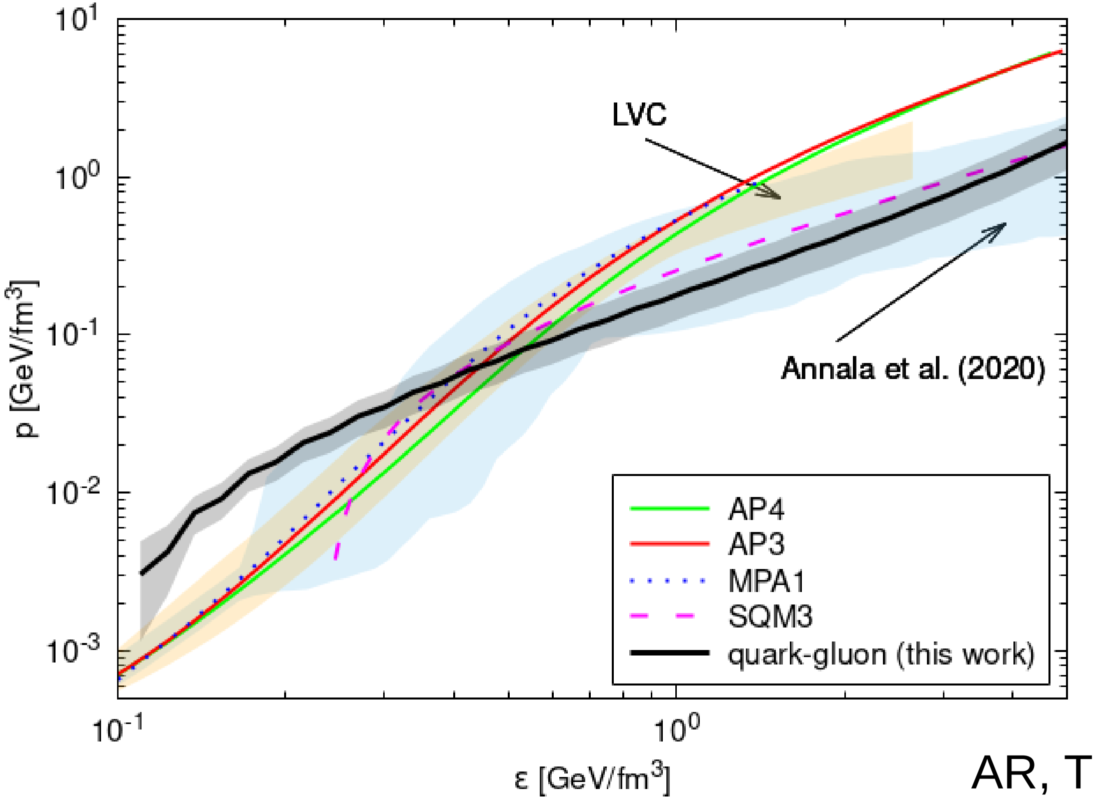
quark and gluon structure

deconfinement

Slide from Simonetta Liuti

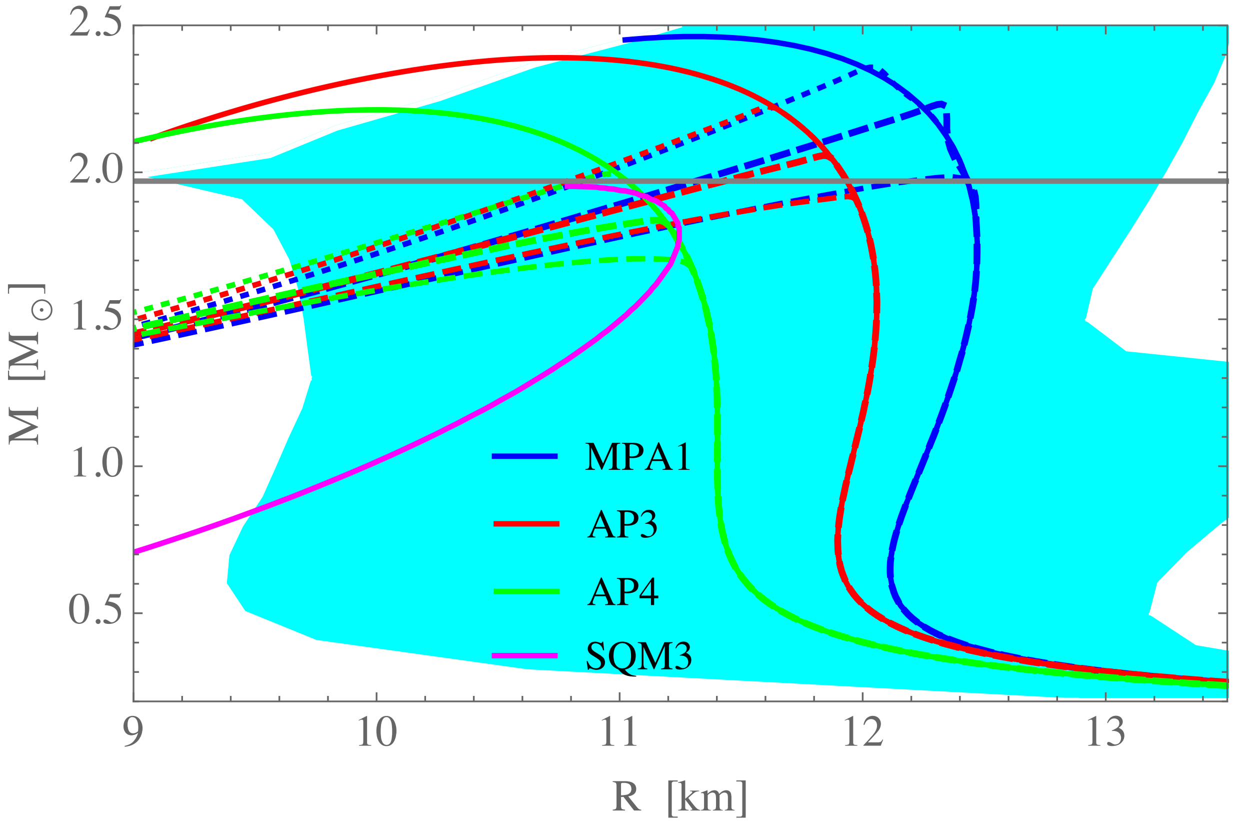
G. Baym et al. arXiv:1707.04966

EoS of neutron stars



AR, T Gorda, S Liuti, K Yagi 1812.01479

Mass radius relation



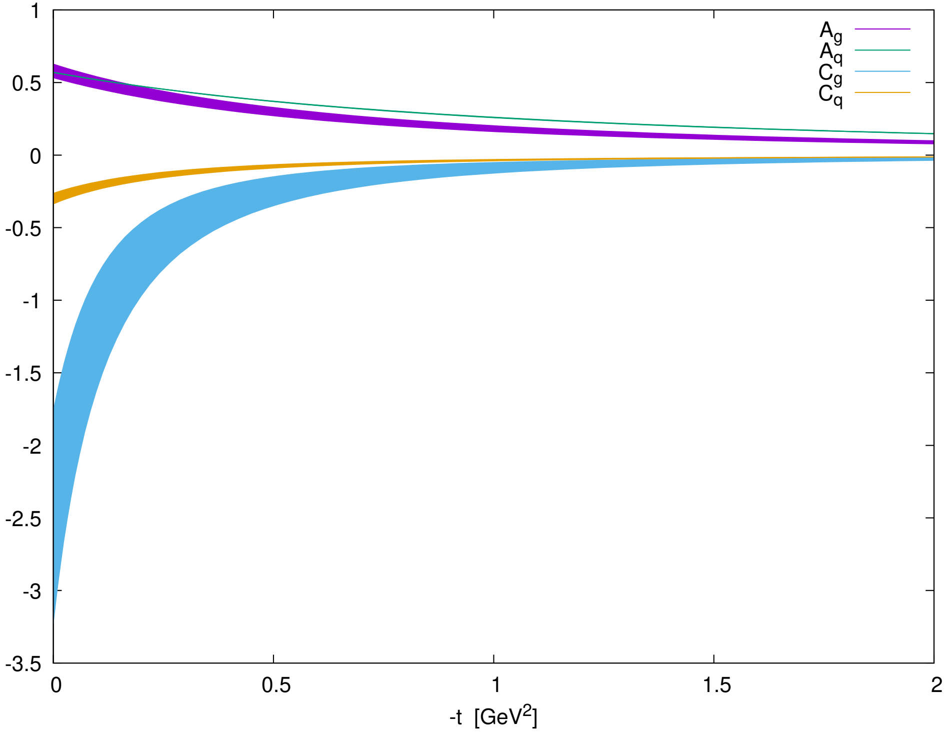
Transition pressures for hybrid EoS

thin	0.15 GeV/fm^3
thick	0.2 GeV/fm^3
dotted	0.3 GeV/fm^3

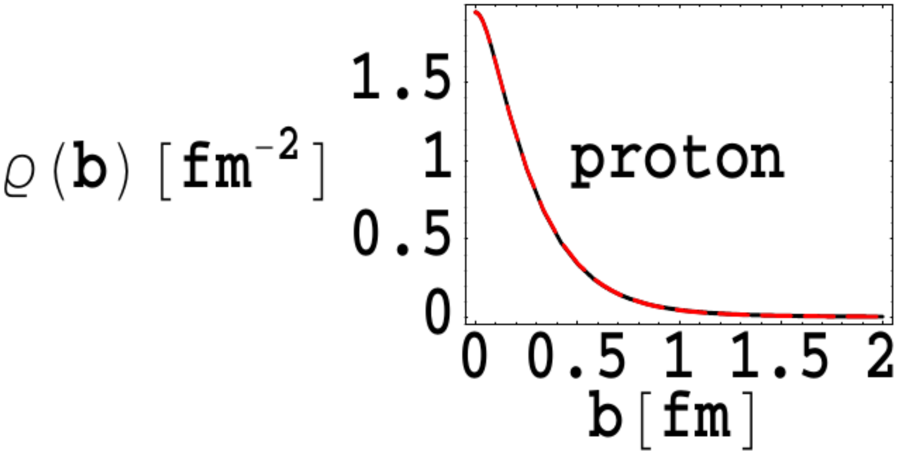
Summary

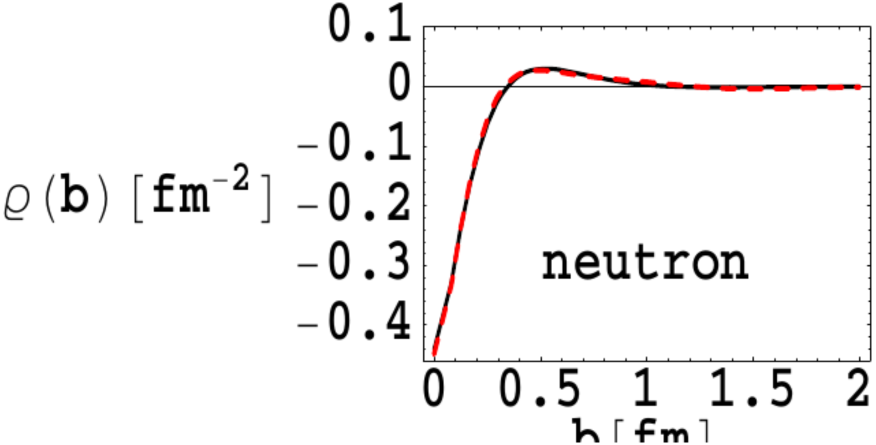
- Recent advancements on both the theoretical and experimental sides have given us a plethora of information on the QCD structure of hadrons.
- Much better understanding on the mechanism of confinement.
- As a result, we have precisely the tools necessary to explore the transition region from low density to high density in the equation of state of neutron stars.

Extracting Generalized Form Factors from lattice QCD



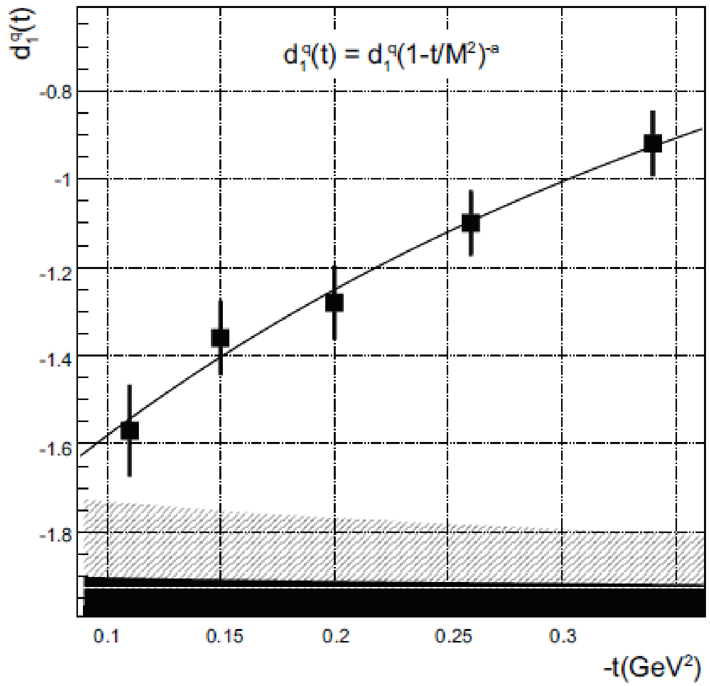
The curious case of the neutron

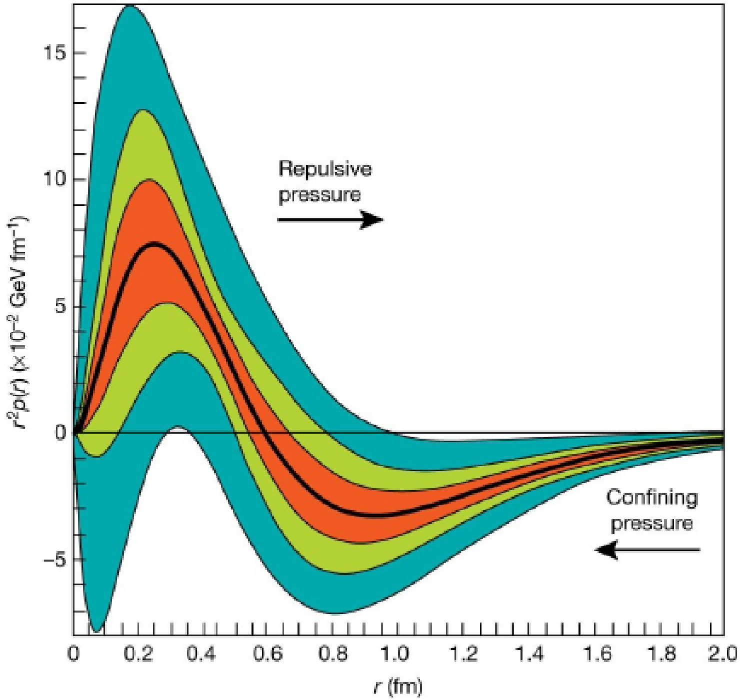




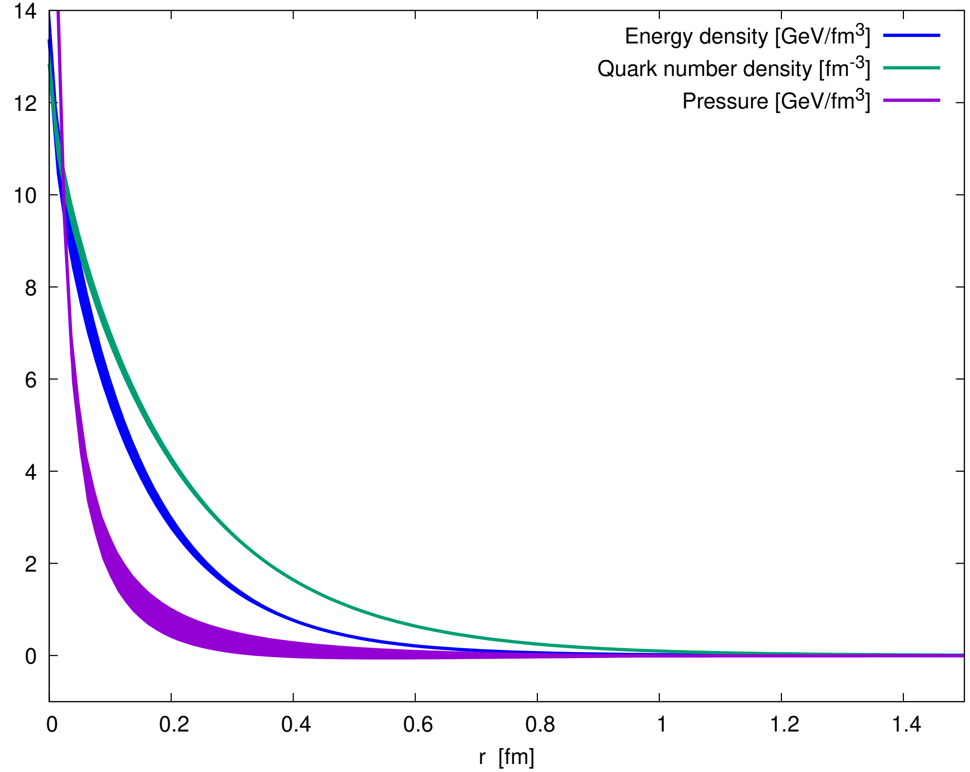
G Miller (2007)

Extracting Generalized Form Factors from experiment





Burkert, Elouadrhiri, Girod (Nature, 2018)


Energy density [GeV/fm^3]
Quark number density [fm^-3]
Pressure [GeV/fm^3]
14
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
r [fm]

$$\int \frac{d^4z}{2\pi} e^{ik.z}\langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel,
JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T}\langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-}\langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs