

# What can we learn from gravitational waves emitted by heavy neutron stars mergers?

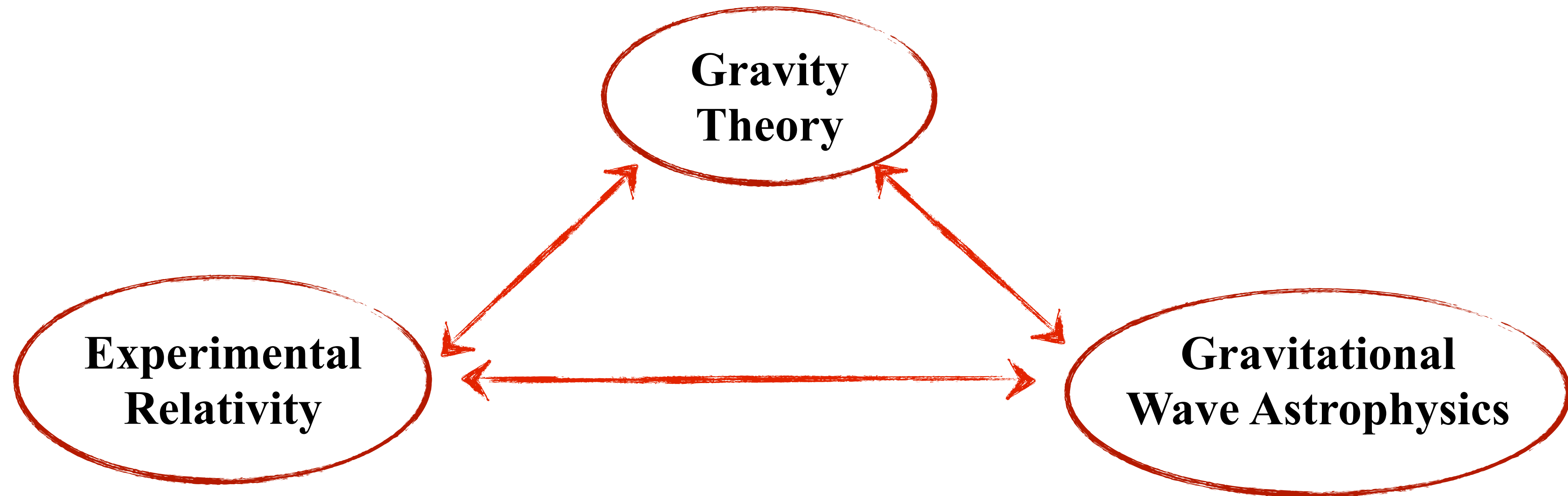
Nicolas Yunes

Illinois Center for Advanced Studies of the  
UniverseUniversity of Illinois at Urbana-Champaign

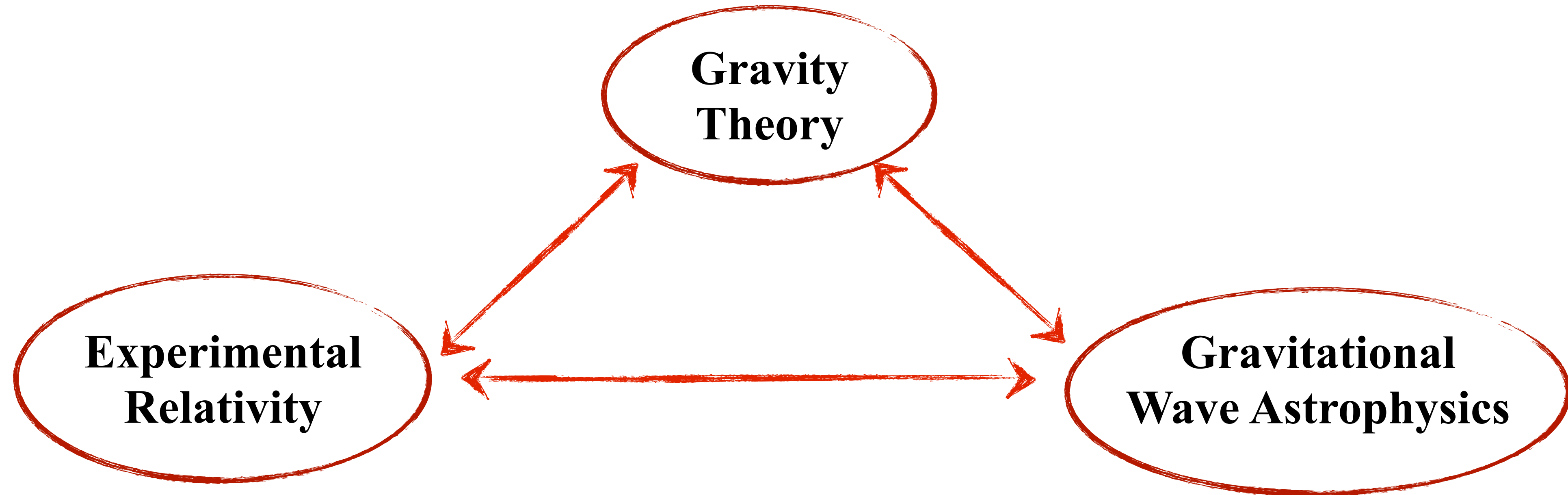
[in collaboration with **Hun Tan, Travis Dore,**  
Jaki Noronha-Hostler and Veronica Dexheimer]

DNP Workshop before April APS '21  
April 14th, 2021

# What is it that you do?



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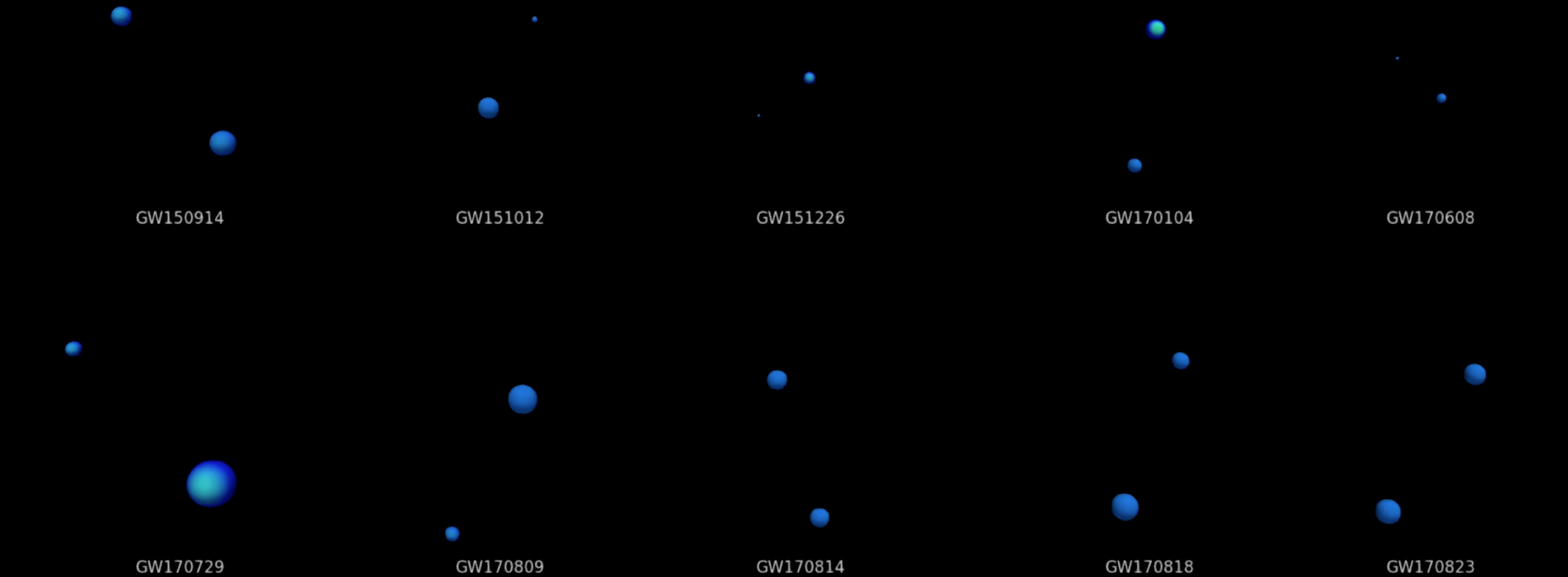
**What can we learn about nuclear physics from the  
inspiral of heavy neutron stars?**







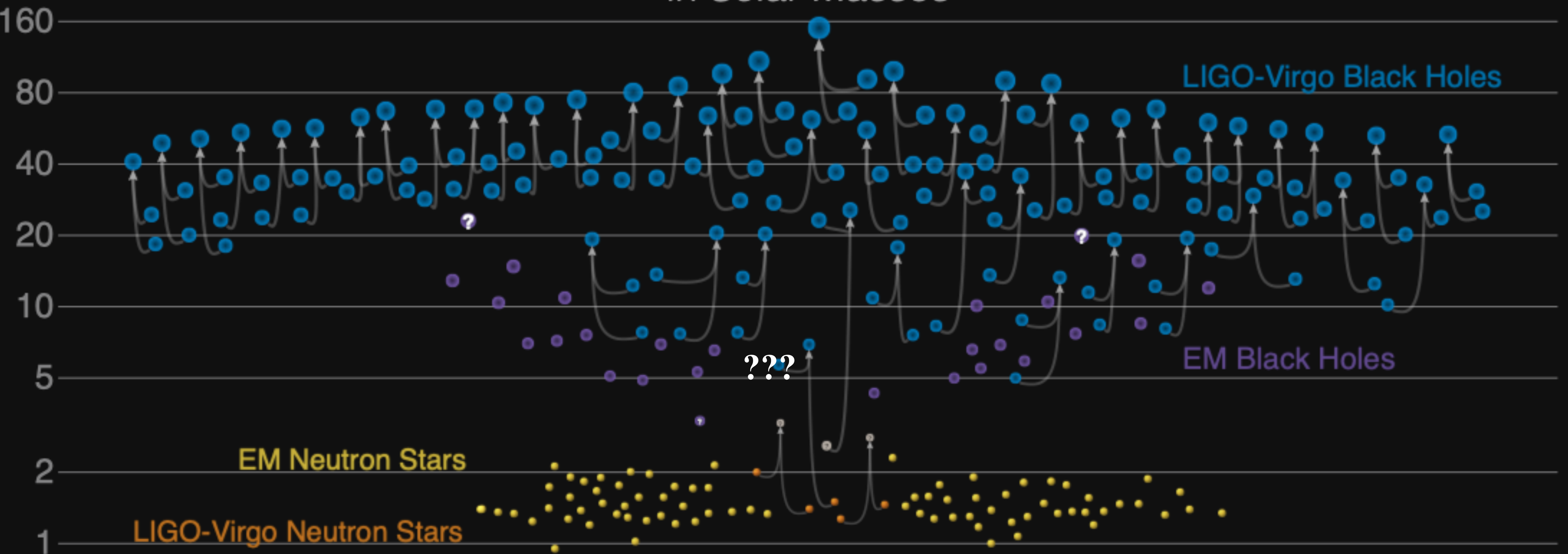
Time: -0.63 seconds



[Credit: Teresita Ramirez / Geoffrey Lovelace / SXS Collaboration / LIGO Virgo Collaboration]

# Masses in the Stellar Graveyard

*in Solar Masses*

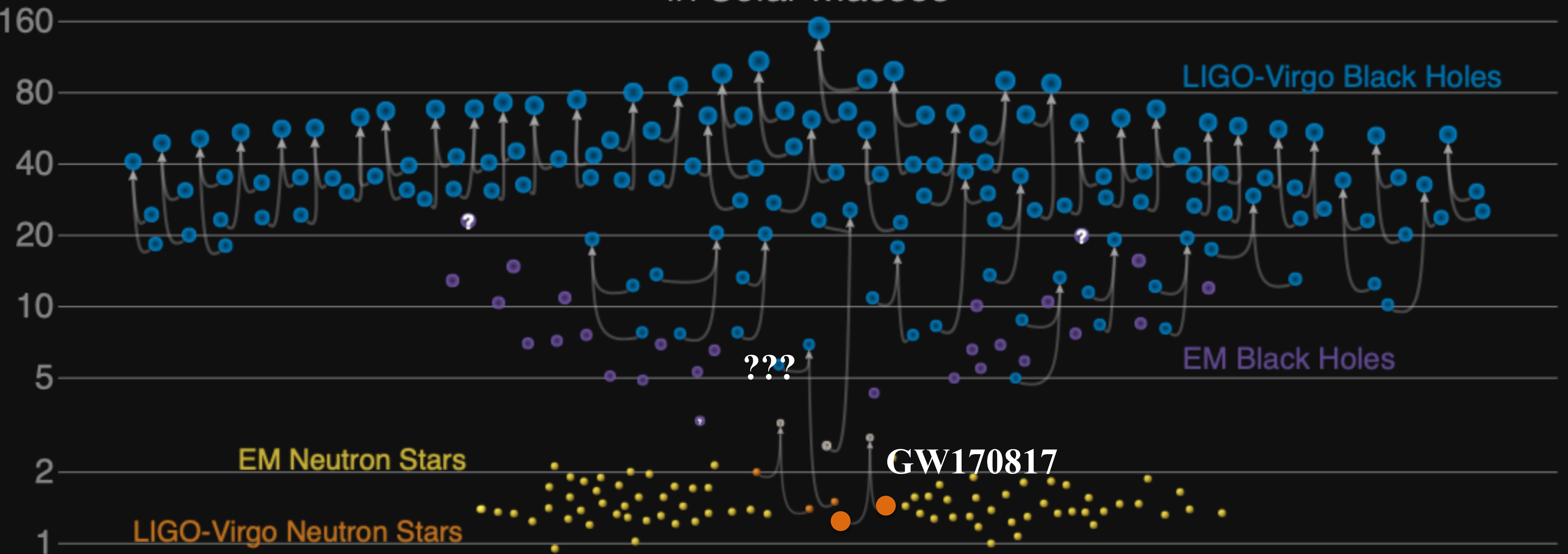


GWTC-2 plot v1.0

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

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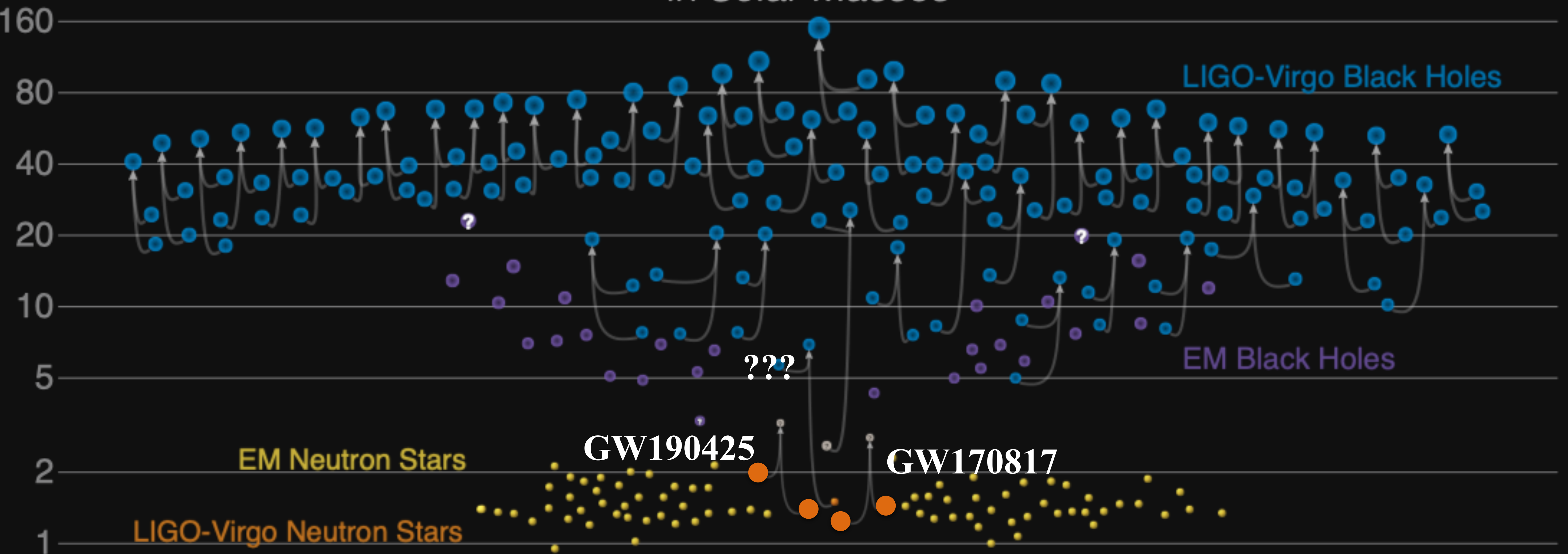
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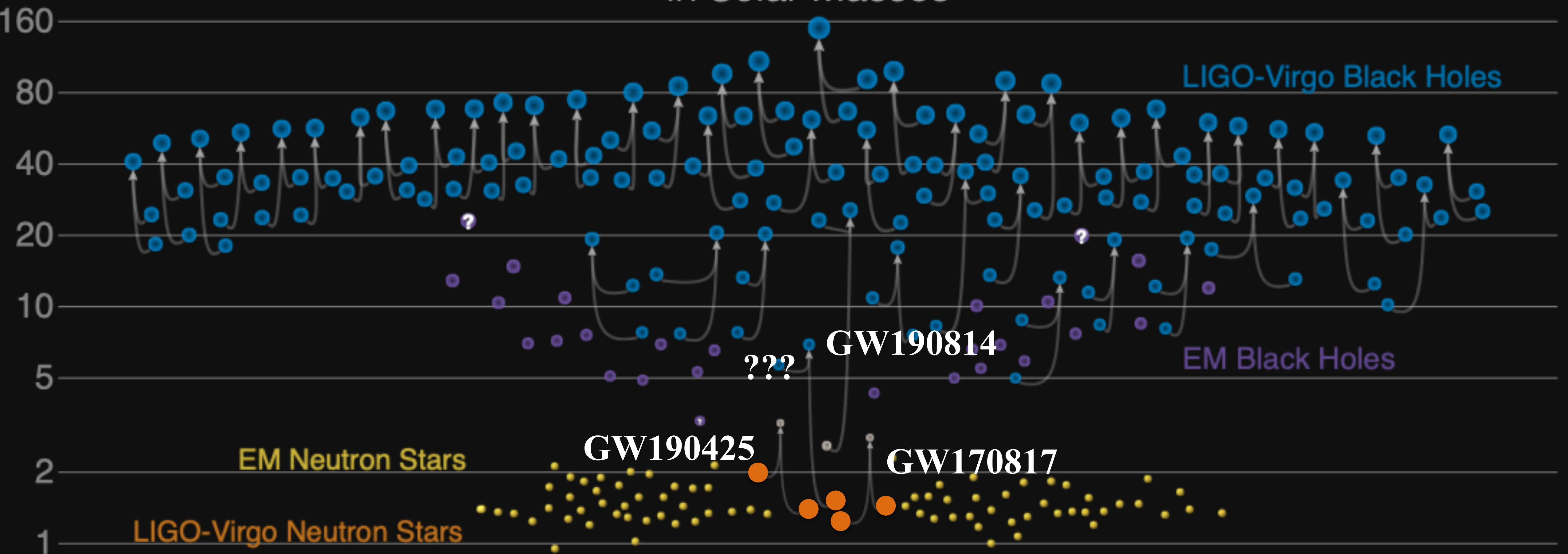


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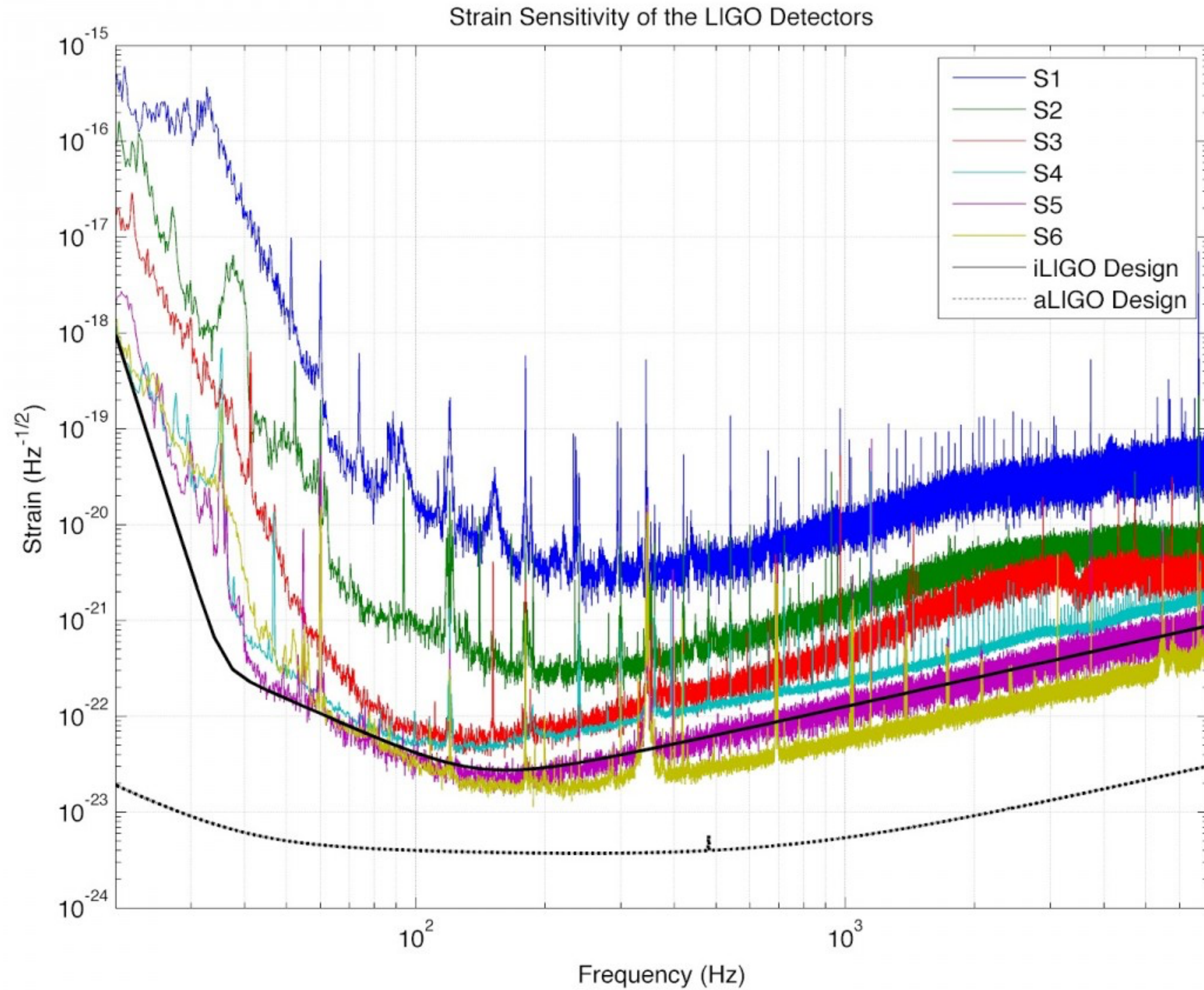


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inner product  $\rightarrow$   $(s-h | s-h)$

Fourier transform  $\rightarrow$   $\tilde{s}^*(f)$  and  $\tilde{h}^*(f, \lambda^\mu)$

template (projection of GW metric perturbation)  $\rightarrow$   $\tilde{h}^*(f, \lambda^\mu)$

detector noise (spectral noise density)  $\rightarrow$   $S_n(r)$



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template (projection of GW metric perturbation)  $\rightarrow$  (points to  $\tilde{h}^*(f, \lambda^\mu)$ )

Fourier transform  $\rightarrow$  (points to  $\tilde{s}^*(f)$ )

template param that characterize system  $\rightarrow$  (points to  $\lambda^\mu$ )

detector noise (spectral noise density)  $\rightarrow$  (points to  $S_n(r)$ )

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**The waveform model is key to extract physics information from GW data through matched filtering**

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inner product

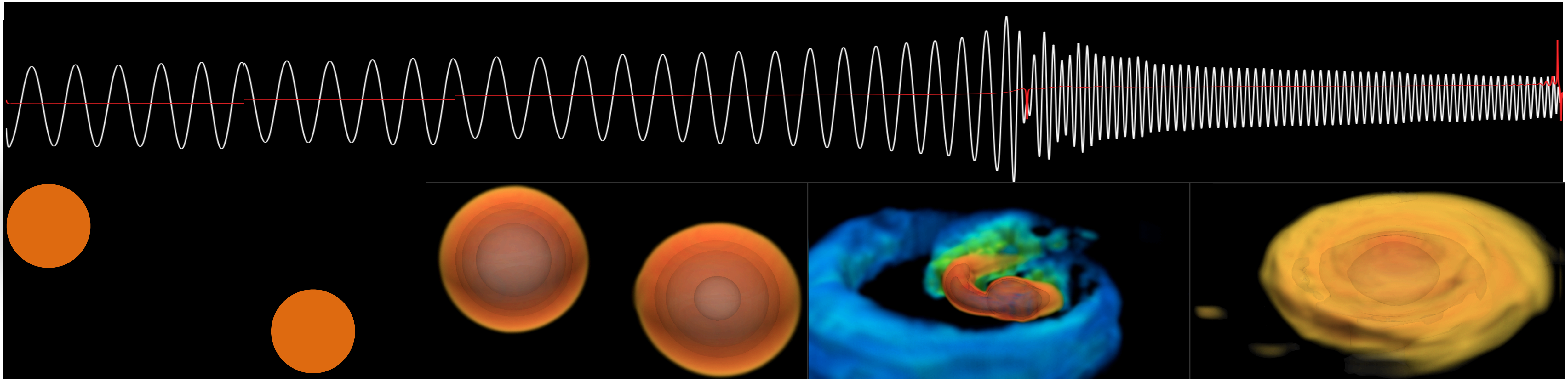
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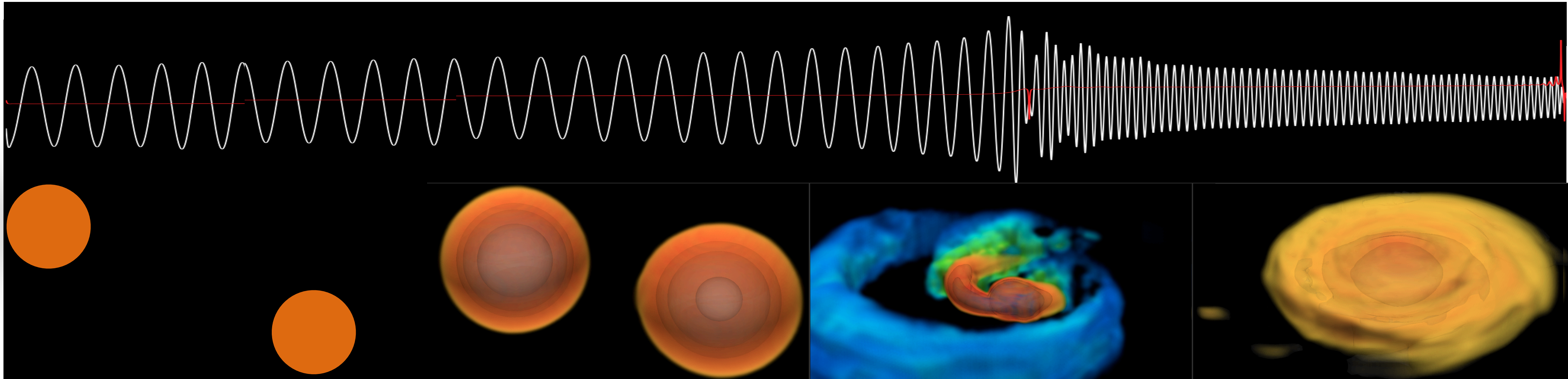
# How do you model the GWs emitted in the inspiral



[see e.g. Blanchet, Liv. Rev. in Rel.,  
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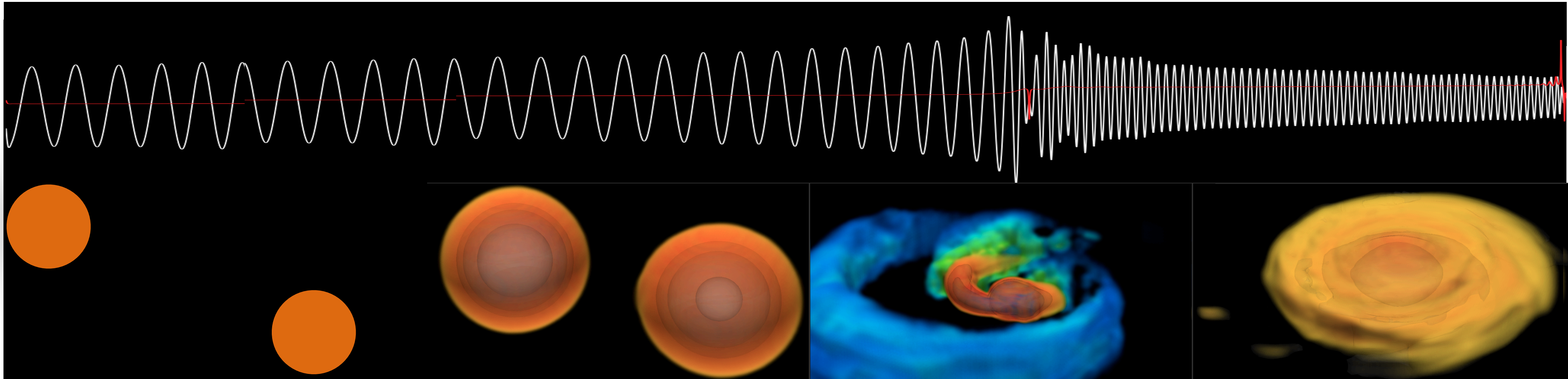
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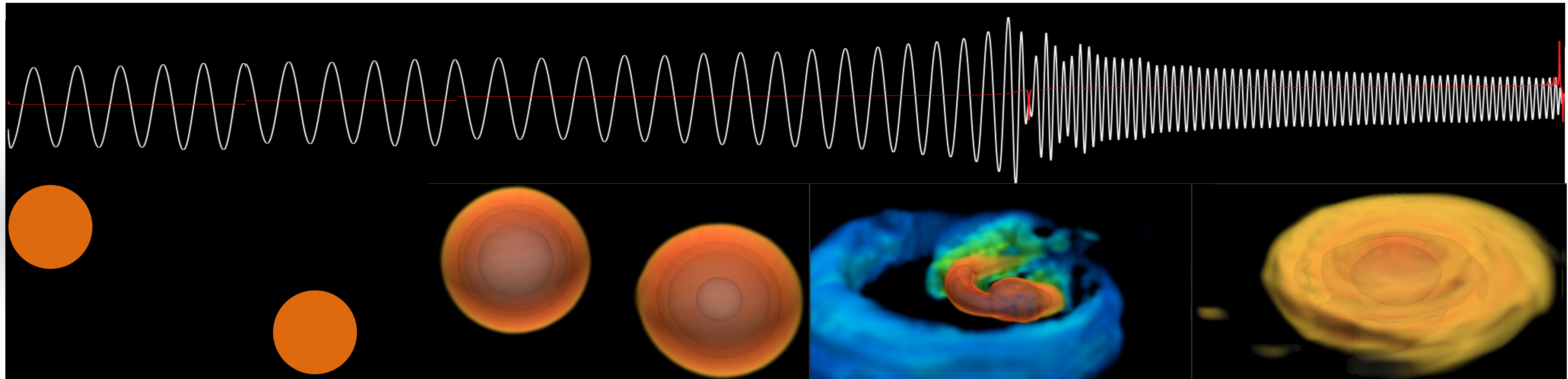
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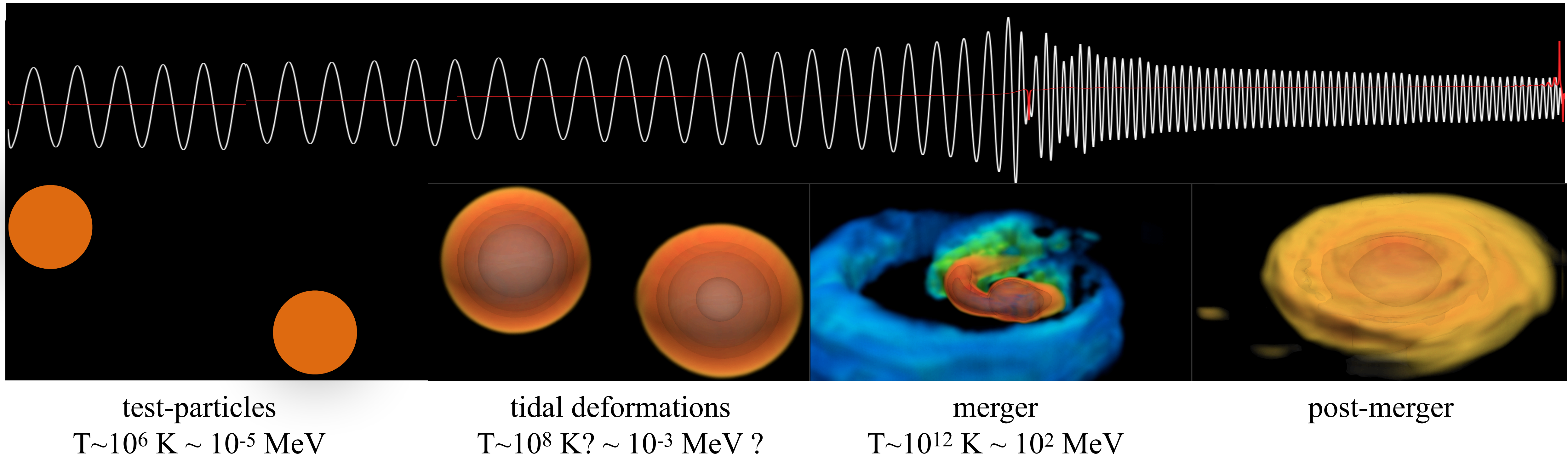
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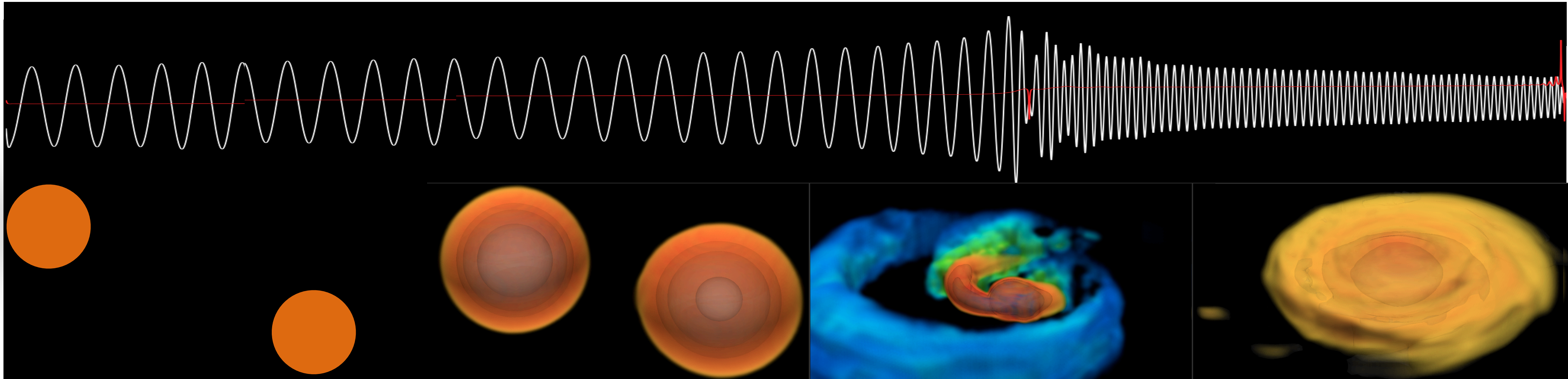
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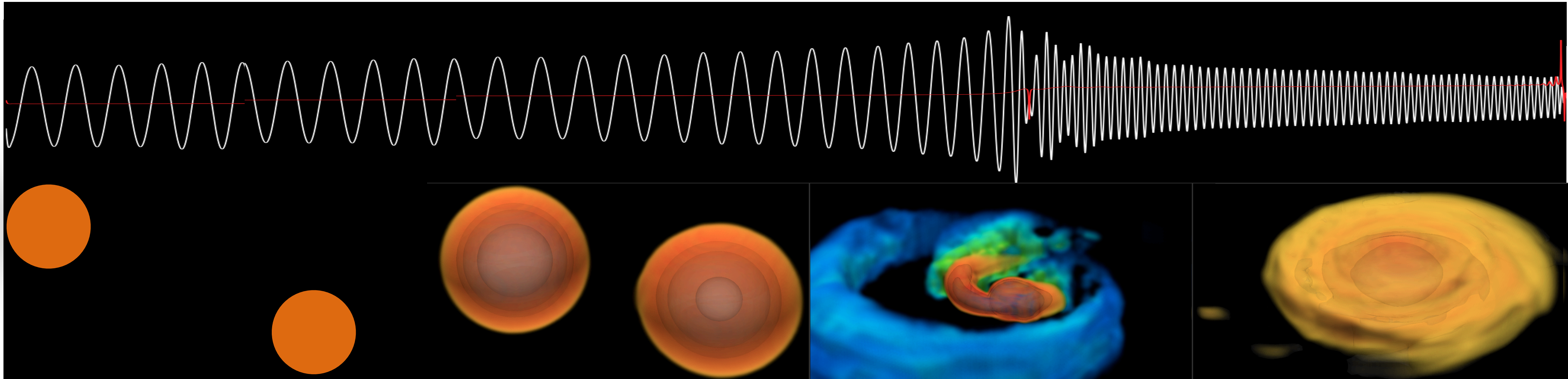
post-merger

$$\tilde{h}(f) = \mathcal{A}(f) e^{i\psi_{\text{pp}}(f) + i\psi_{\text{tidal}}(f)}$$

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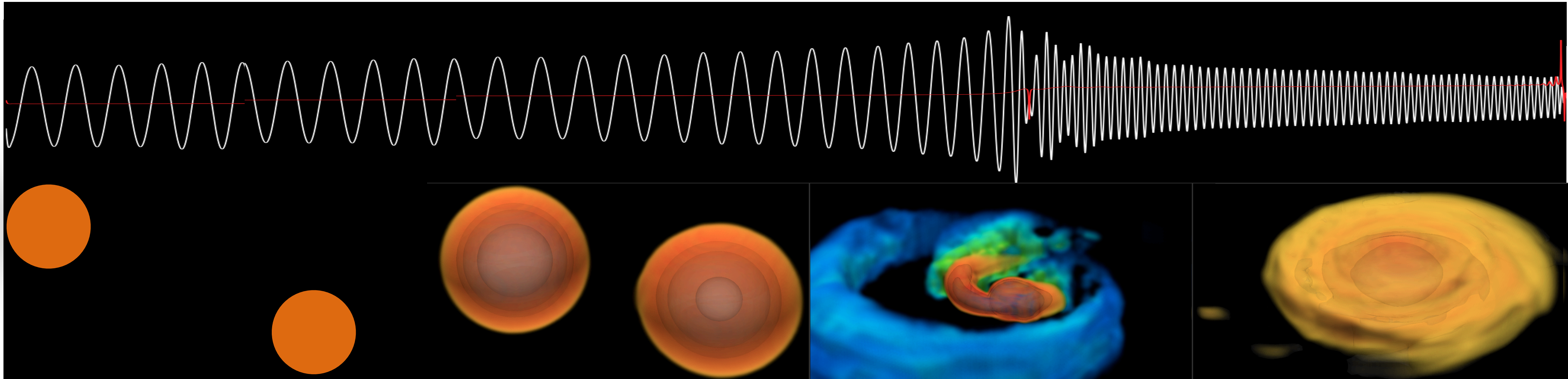
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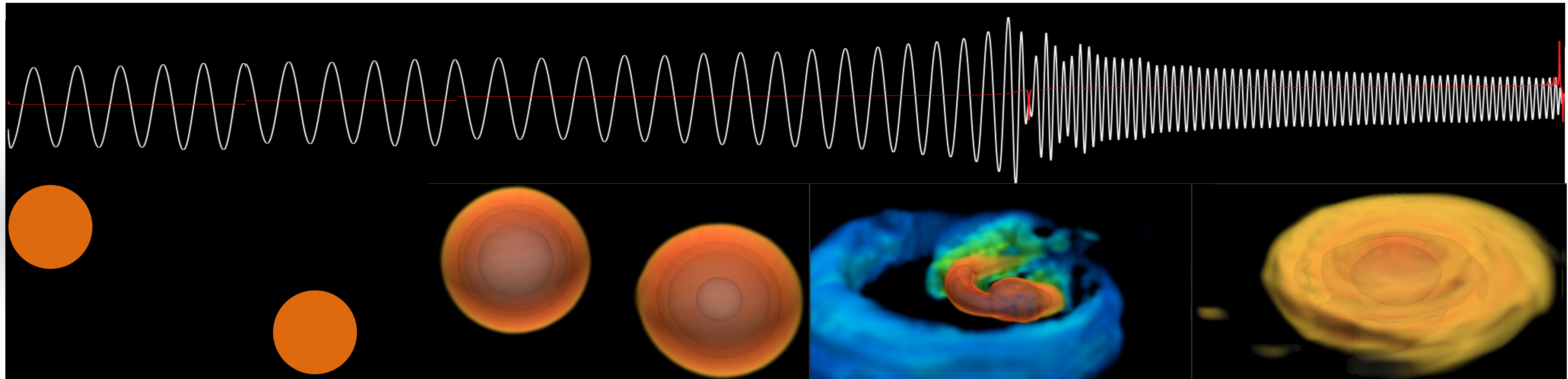
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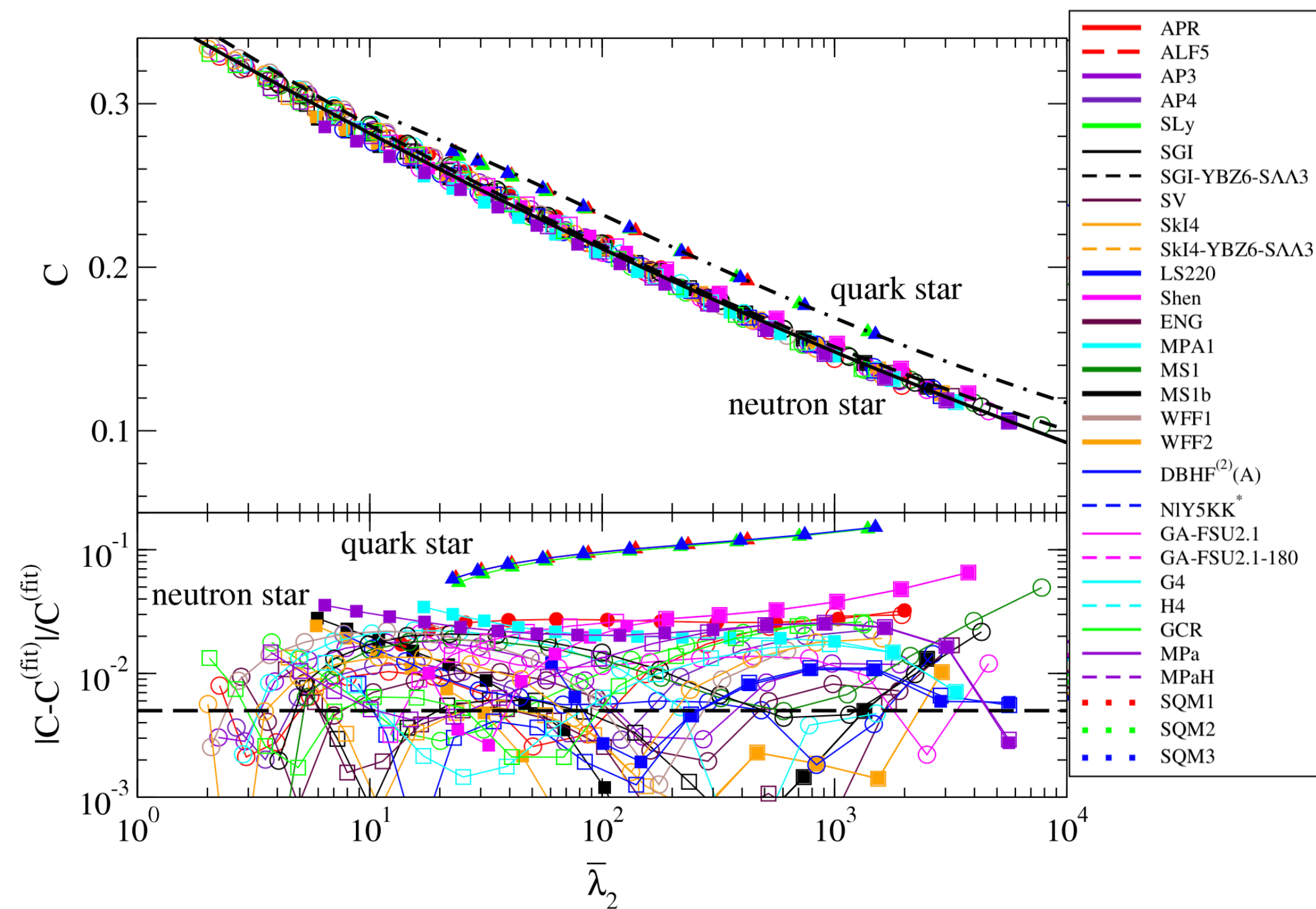
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**Gravitational waves encode the tidal deformabilities**

[see e.g. Blanchet, Liv. Rev. in Rel.,  
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# What's Love got to do with it\*?

\* “it” being nuclear physics.

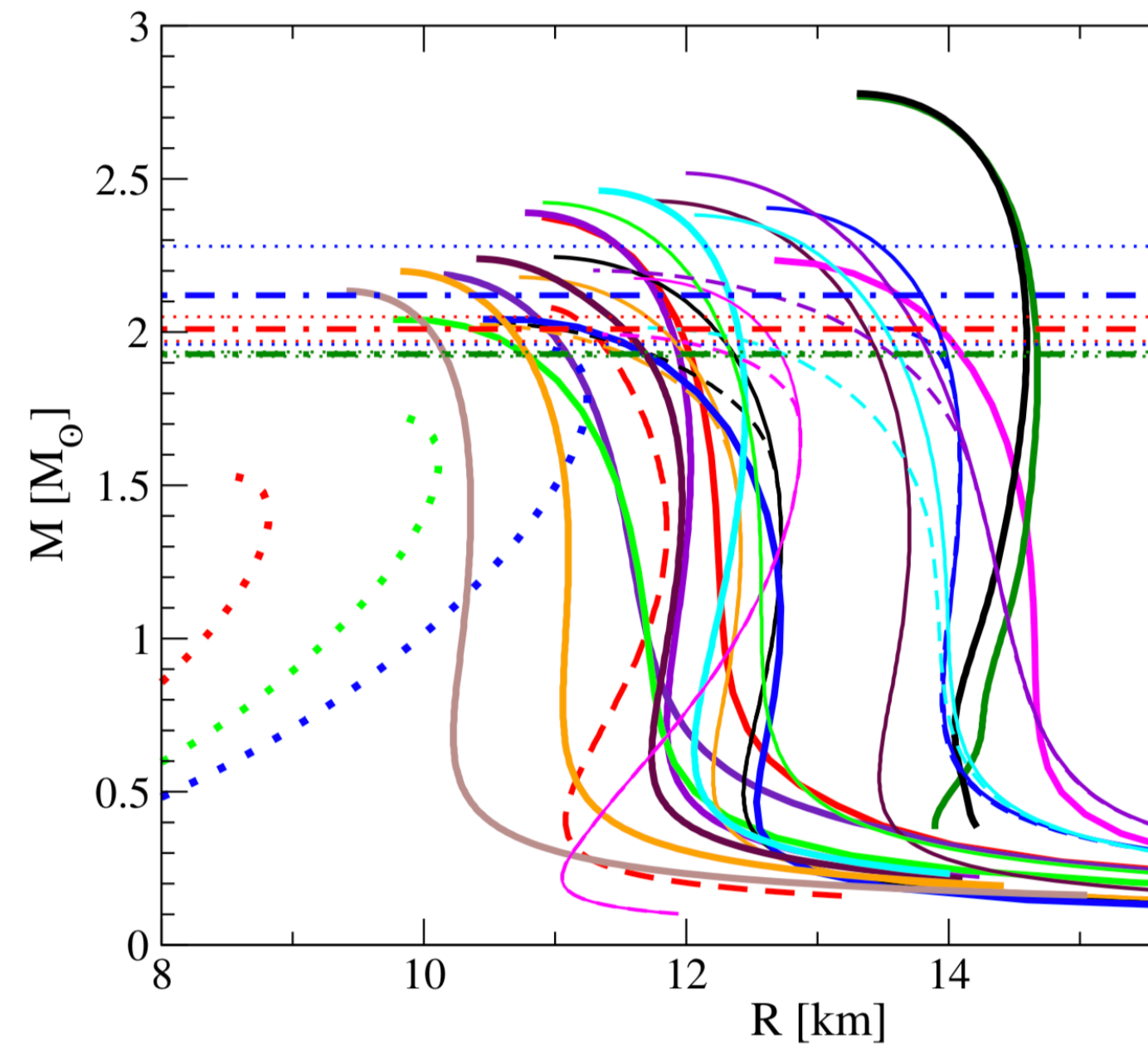
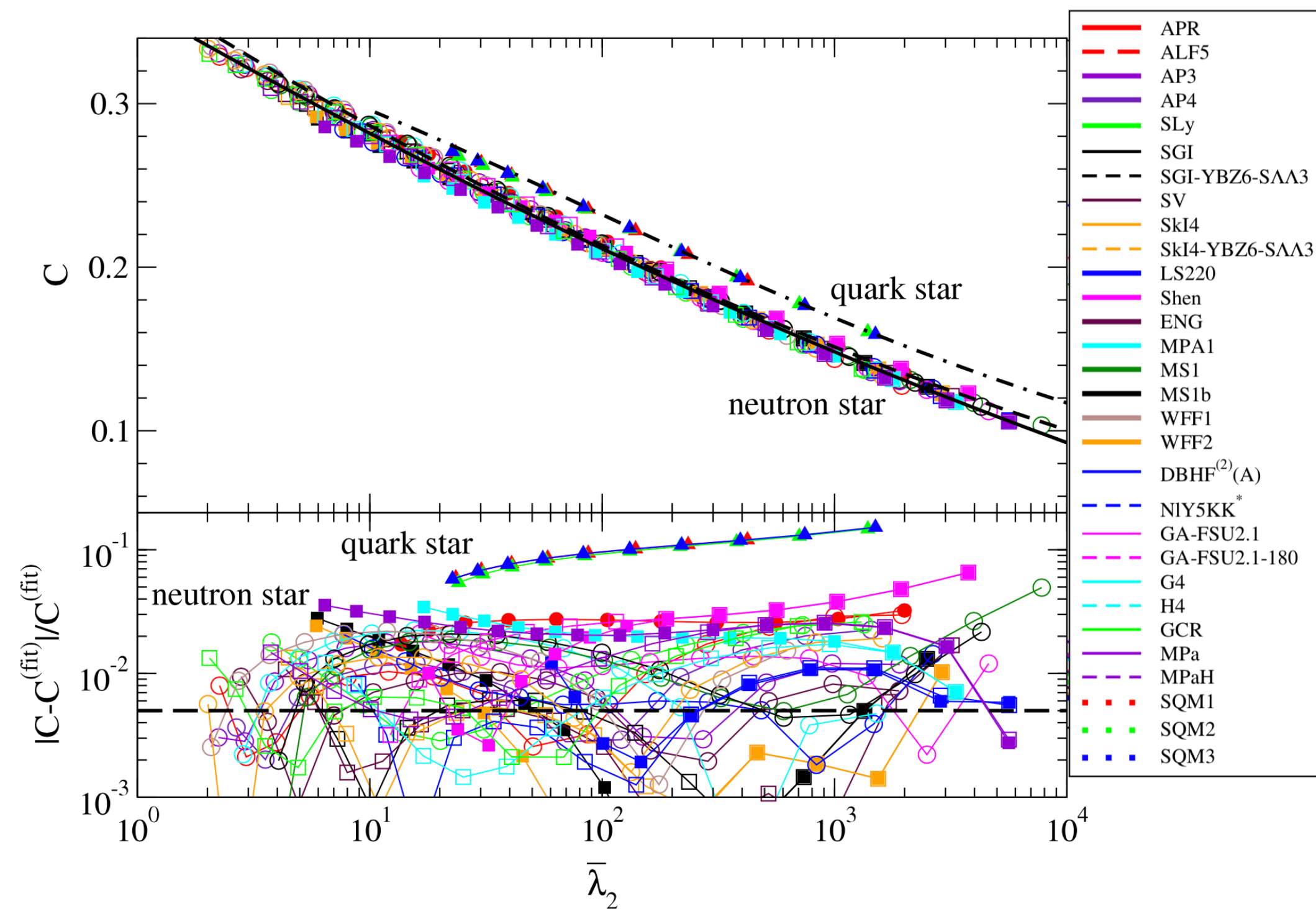


[see e.g. Yagi & Yunes, Phys. Repts 681 (2017)]



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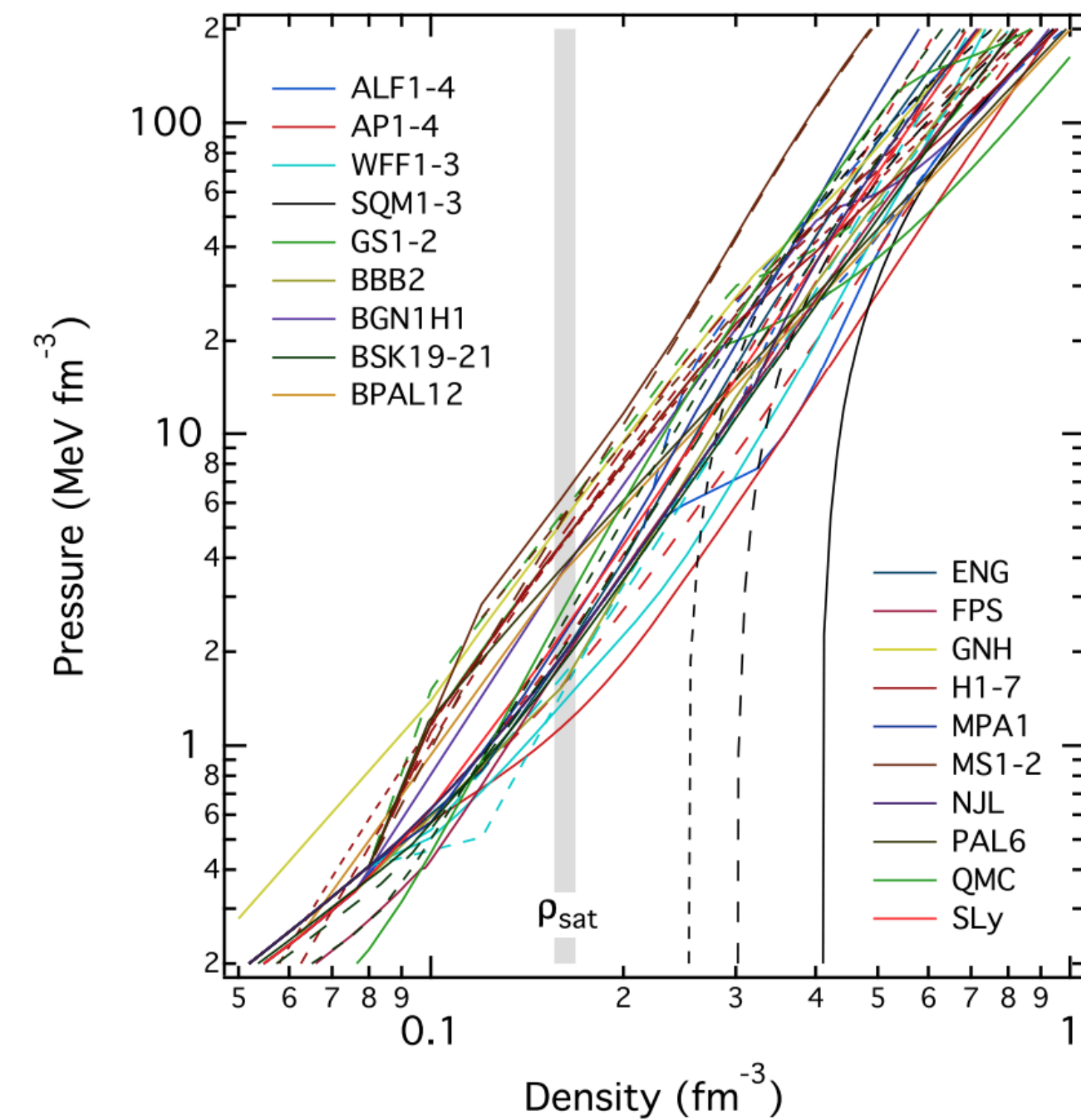
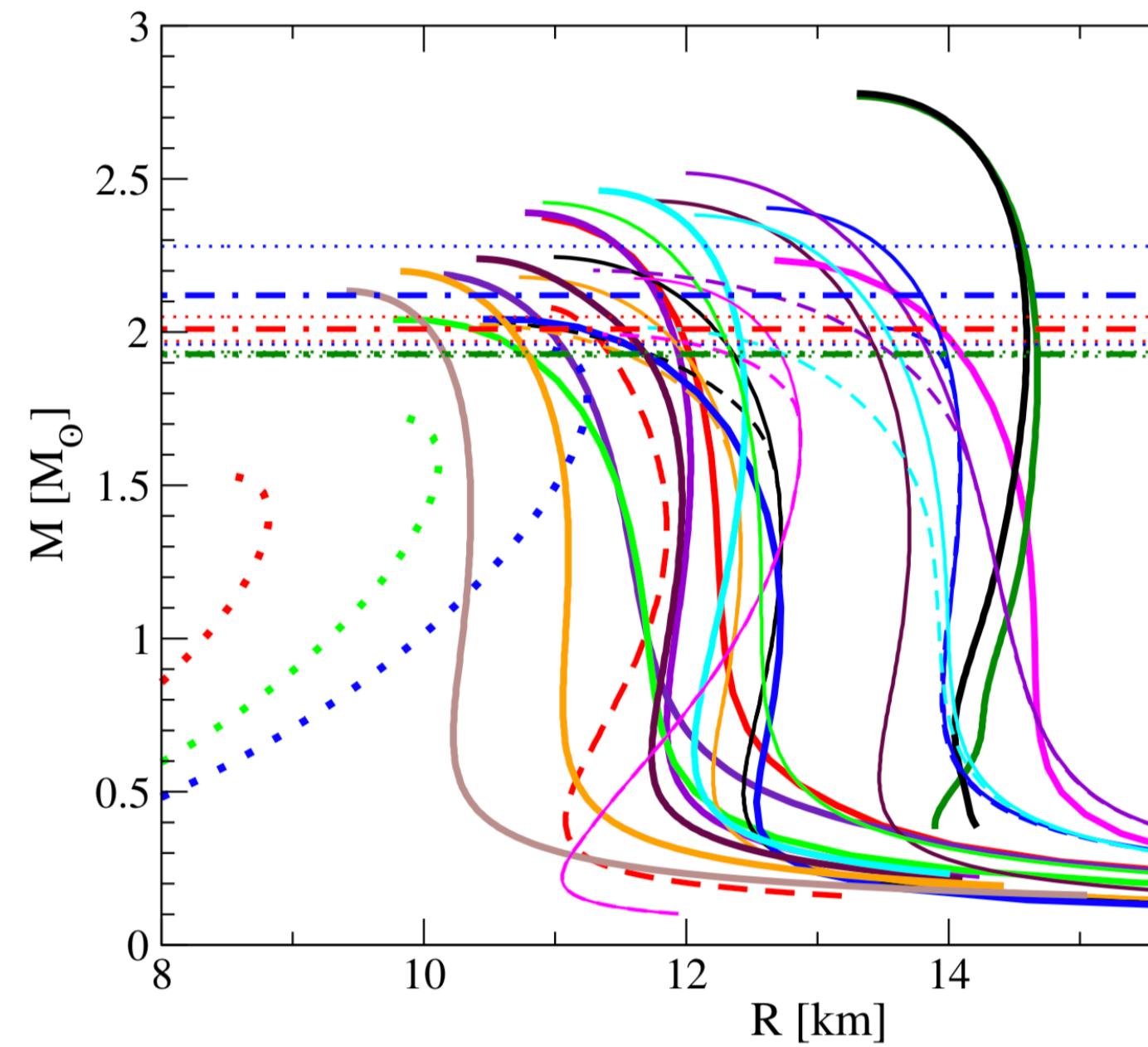
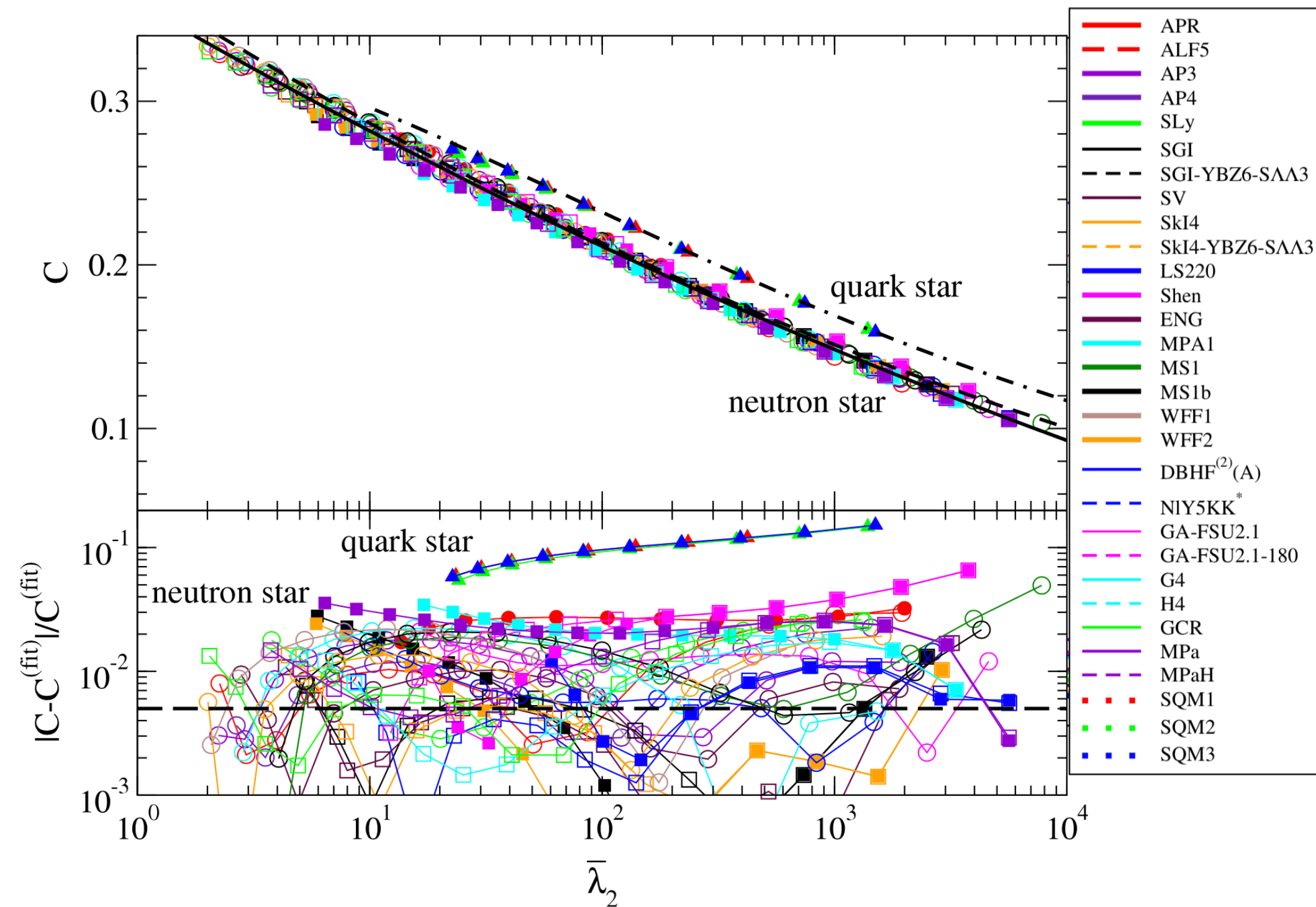
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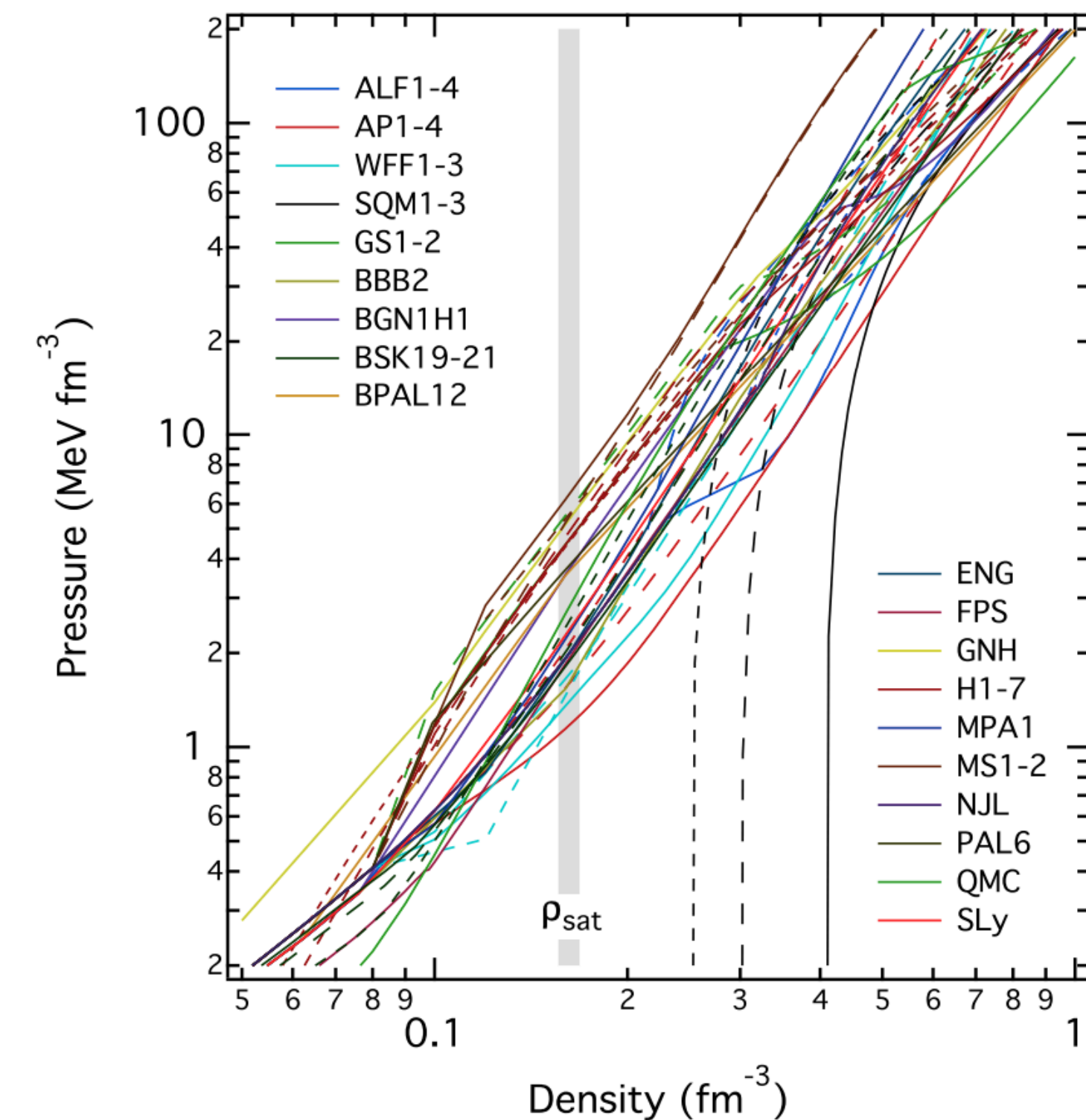
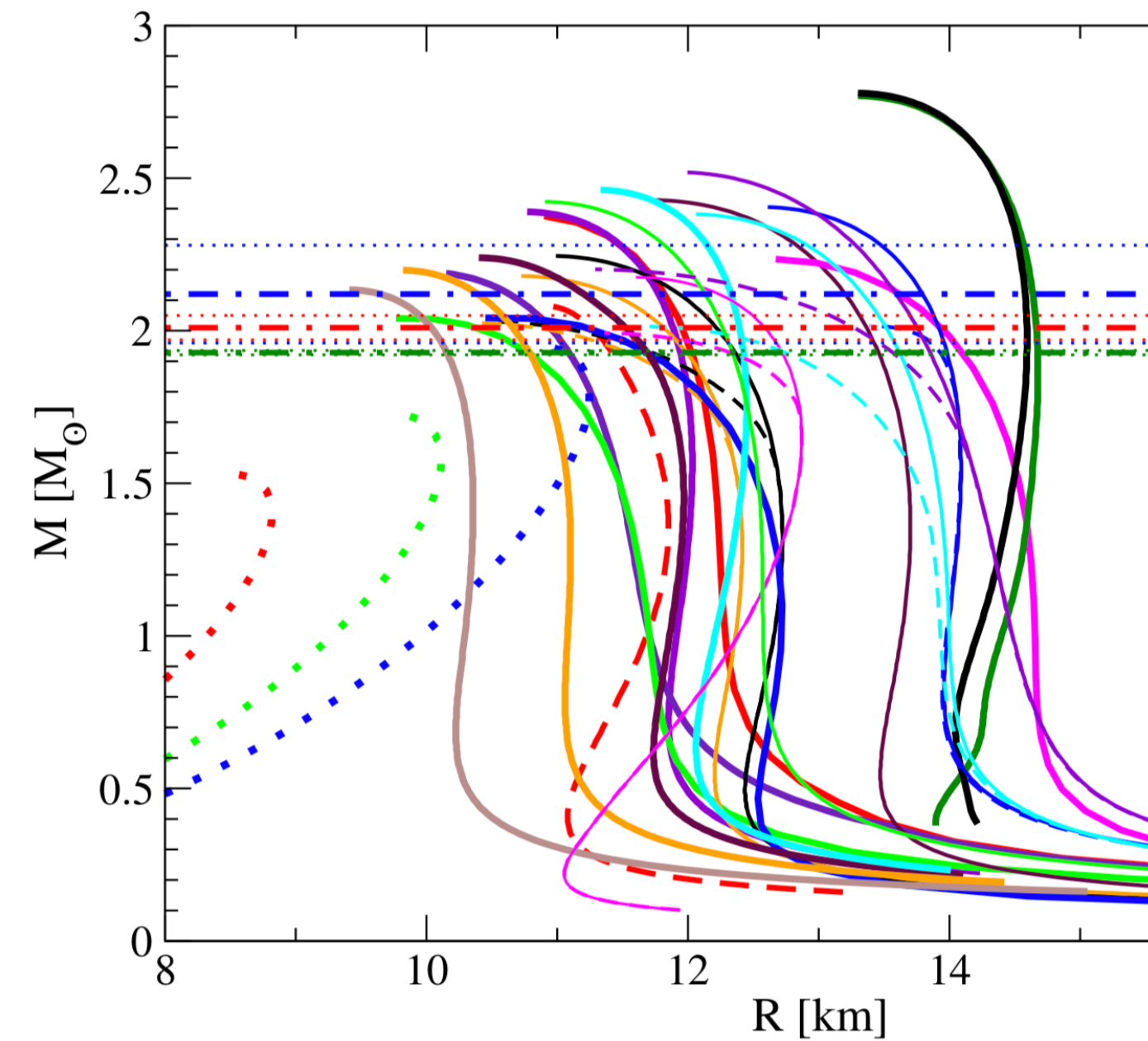
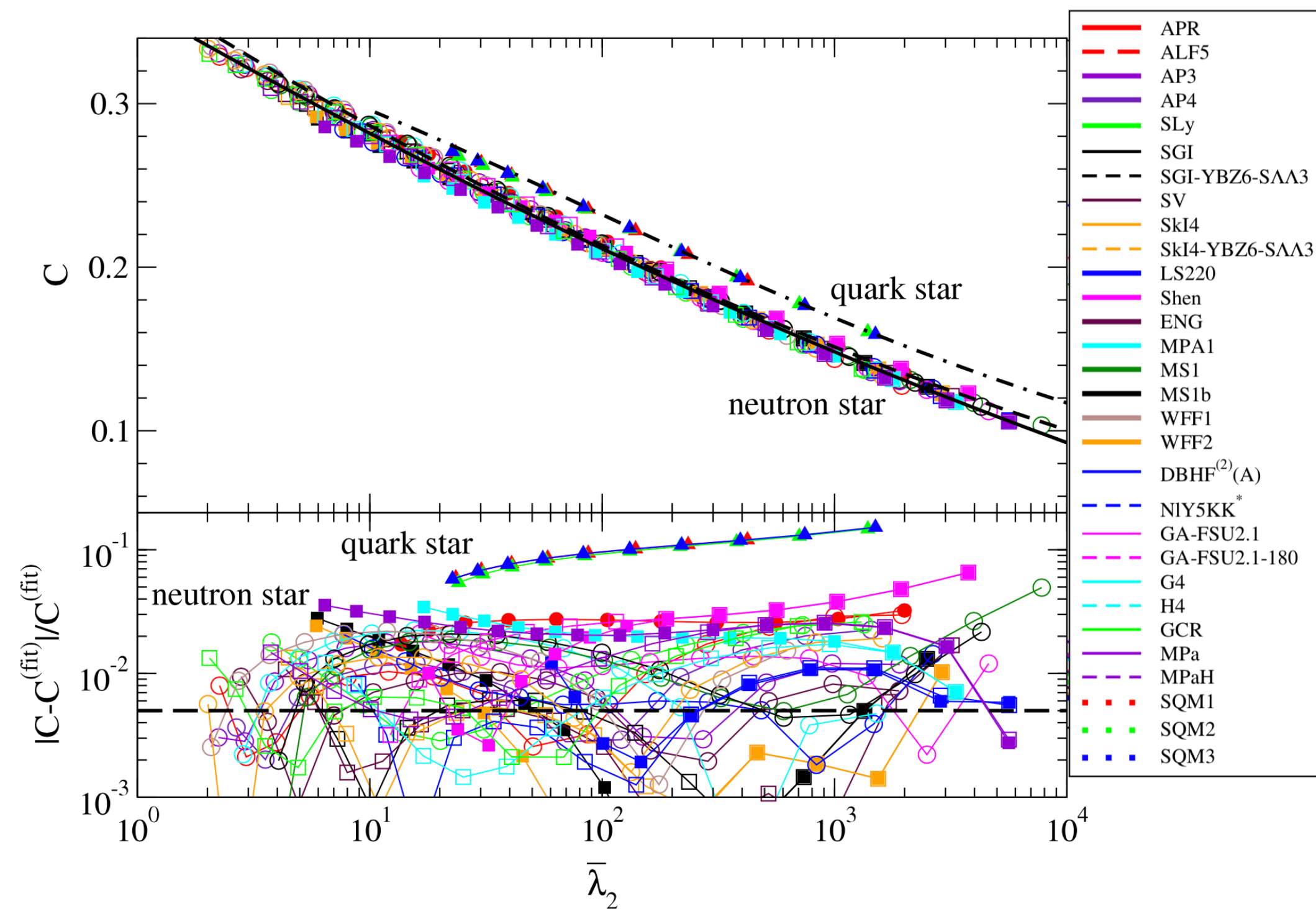


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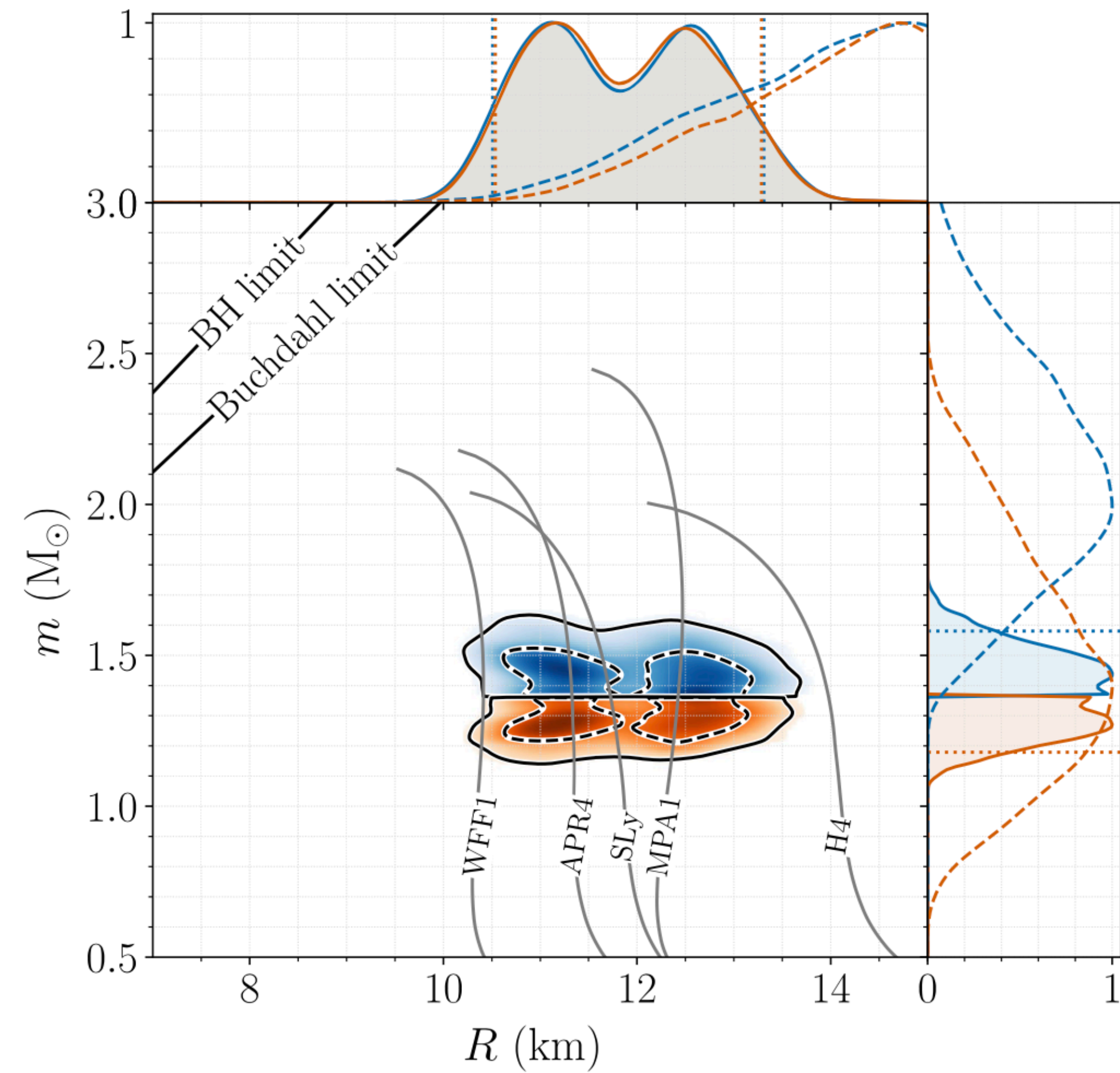
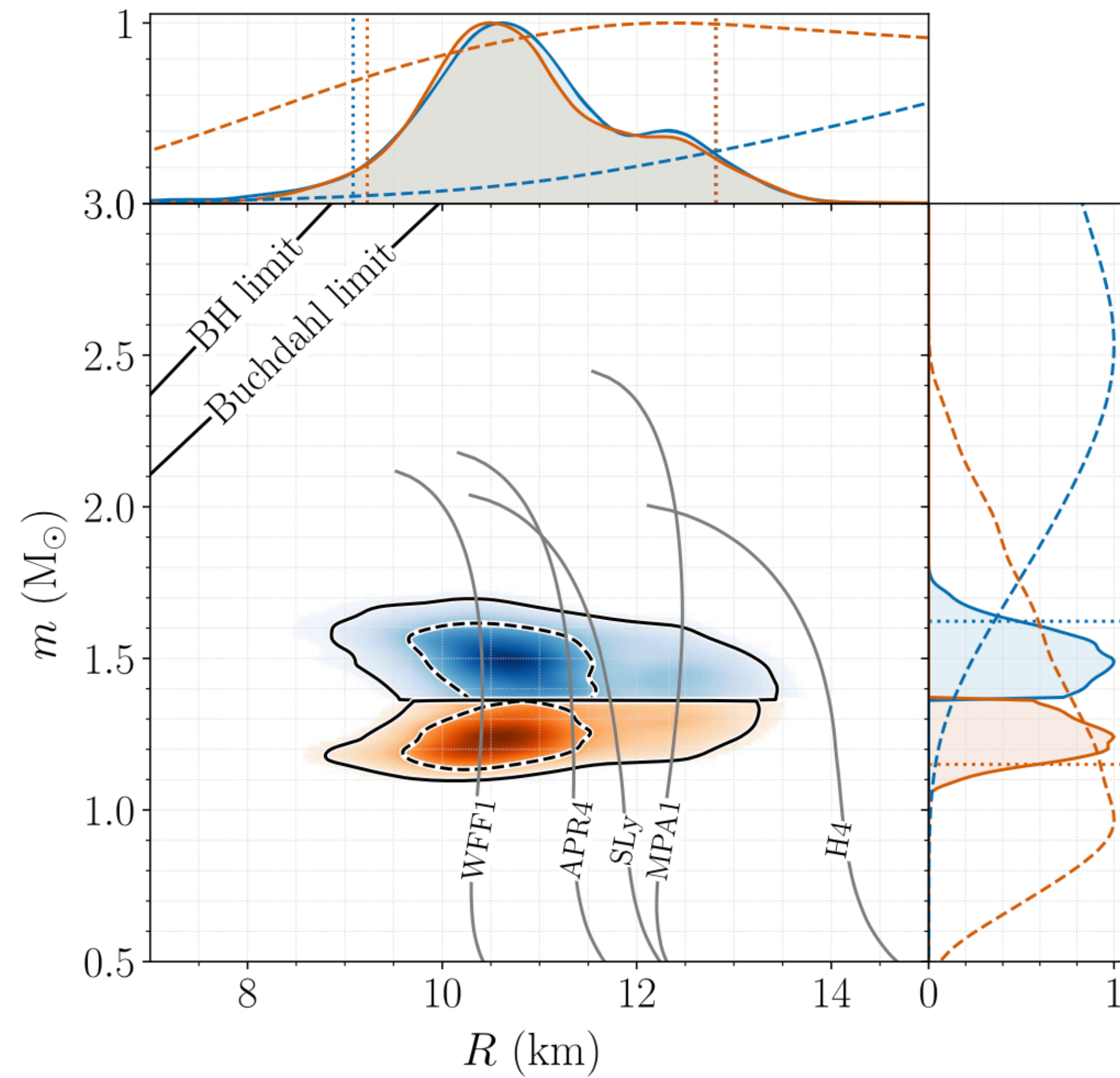


**If you measure Love, you can**  
**(i) infer the radius, and**  
**(ii) you can let the data select between EoS models.**

[see e.g. Yagi & Yunes, Phys. Repts 681 (2017)]



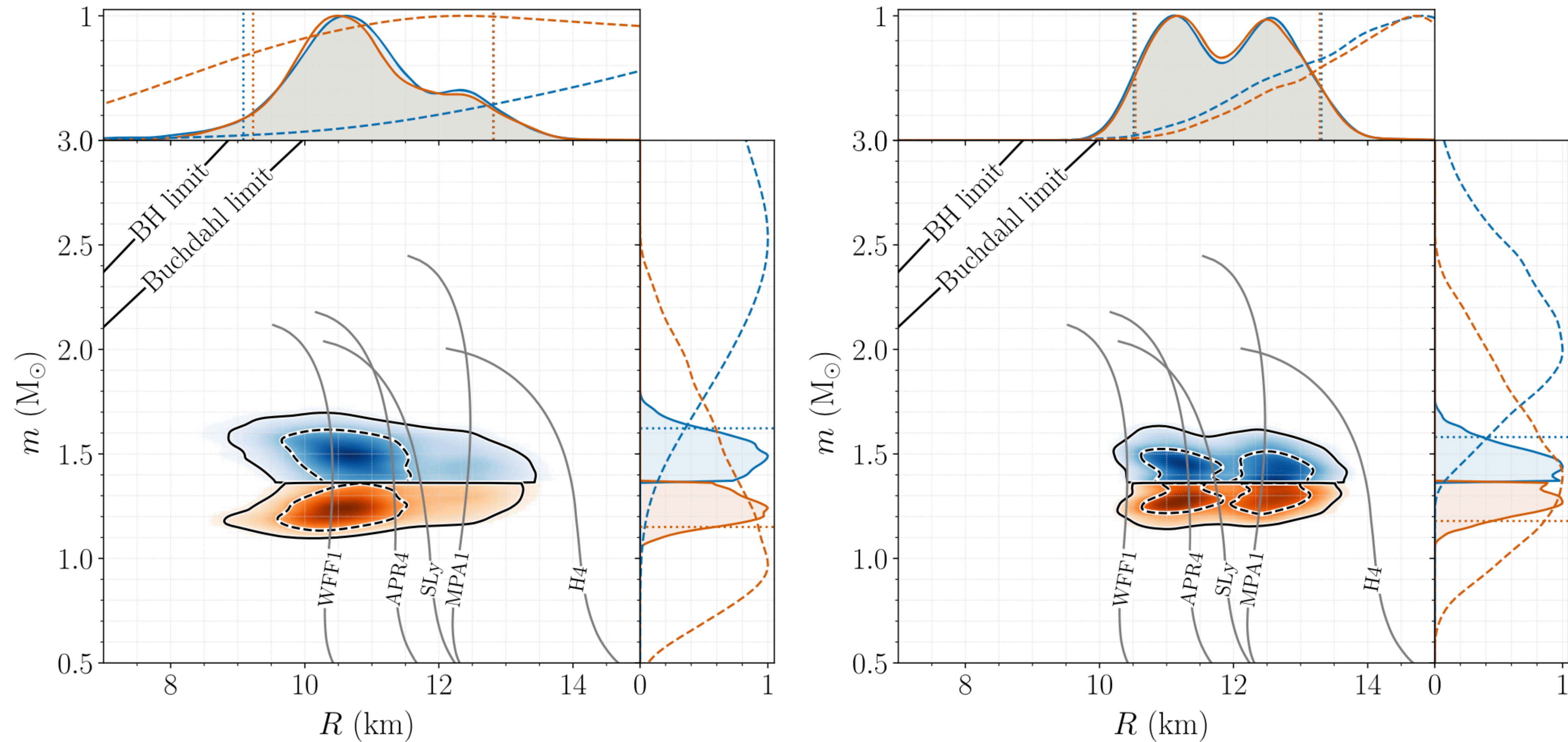
# First GW measurements of Love (and Radius)



[LIGO, PRL 121 ('18)]



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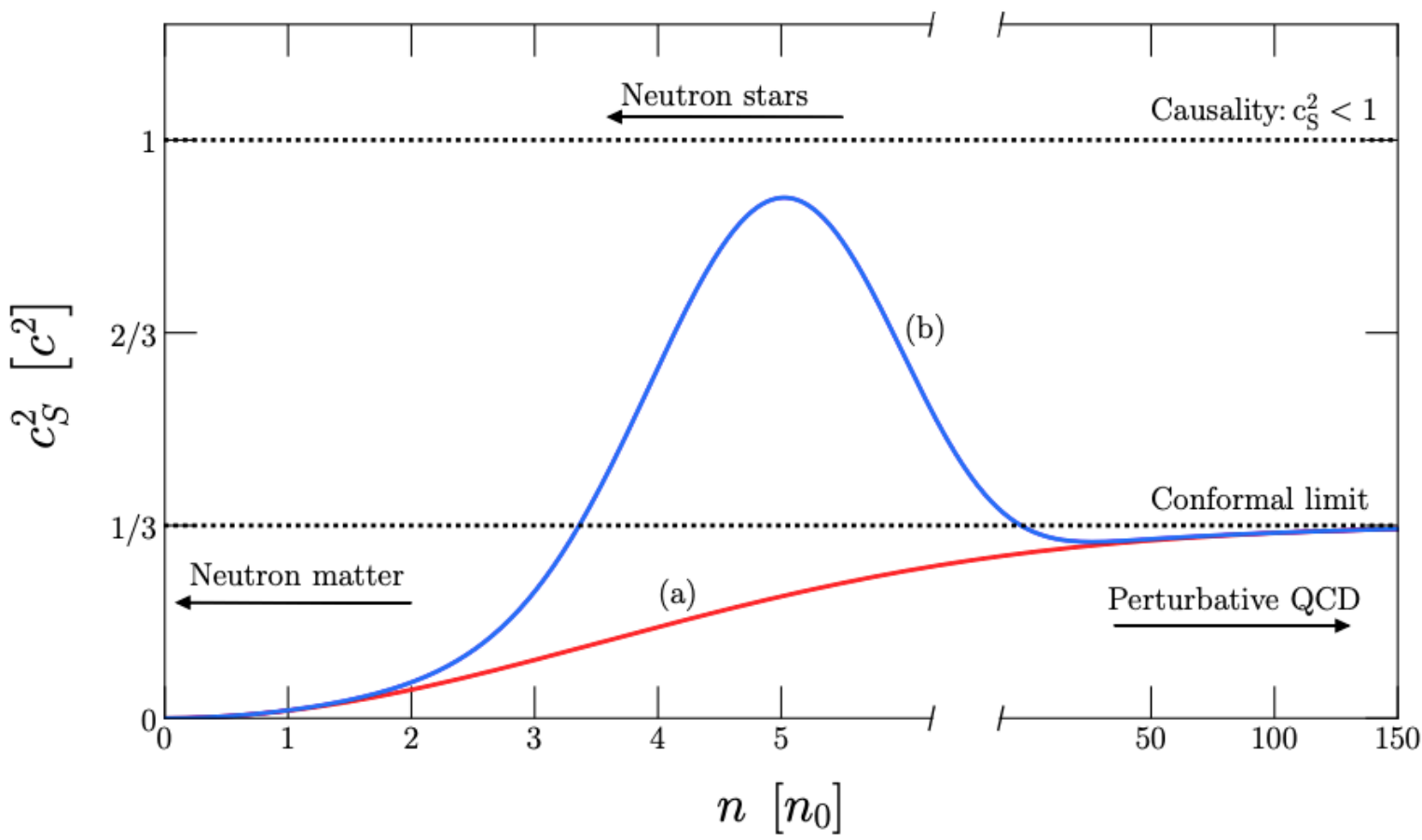


**The GW170817 observation allowed for the first GW constraints on the Love number (and thus the radius)**

[LIGO, PRL 121 ('18)]

# What else can we learn about nuclear physics? Consider $c_s^2$ ...

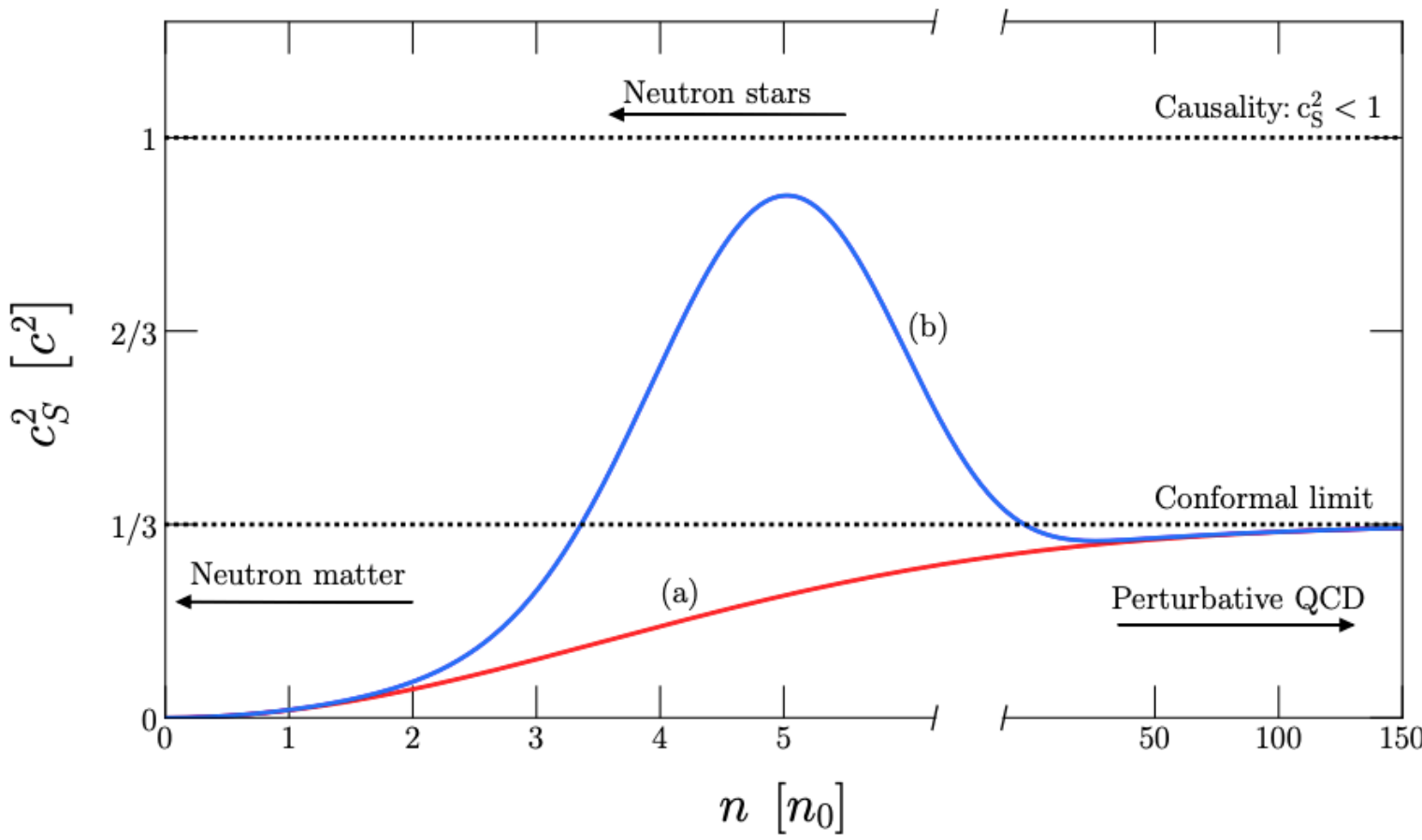
Recent studies indicate  
a steep rise or bump



Bedaque & Steiner, PRL 114, '15; Alford et al, PRD92, '15,  
Ranea-Sandoval, et al, PRC93, Tews, et al, PRC98, '18

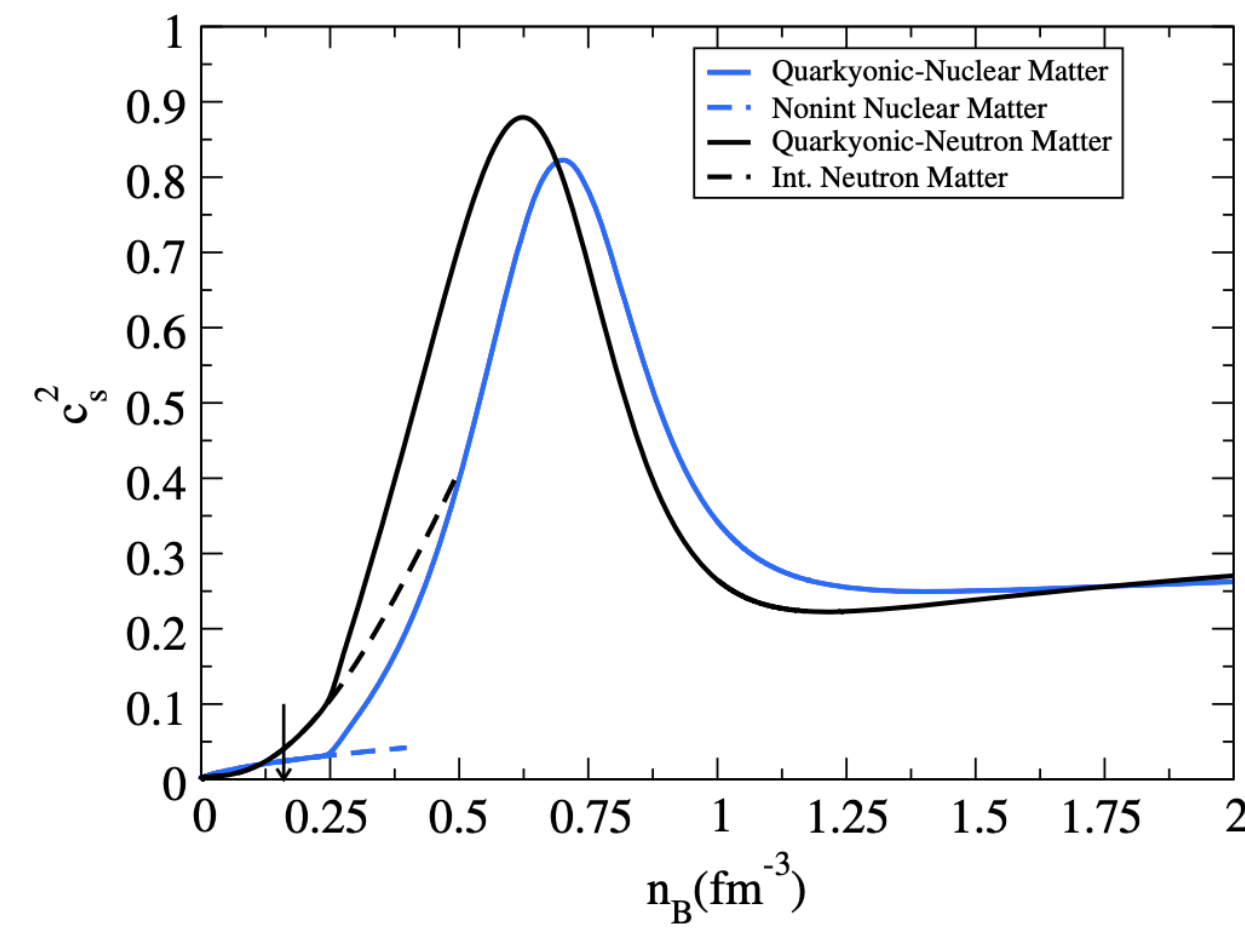
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One physical mechanism:  
Quarkyonic Matter

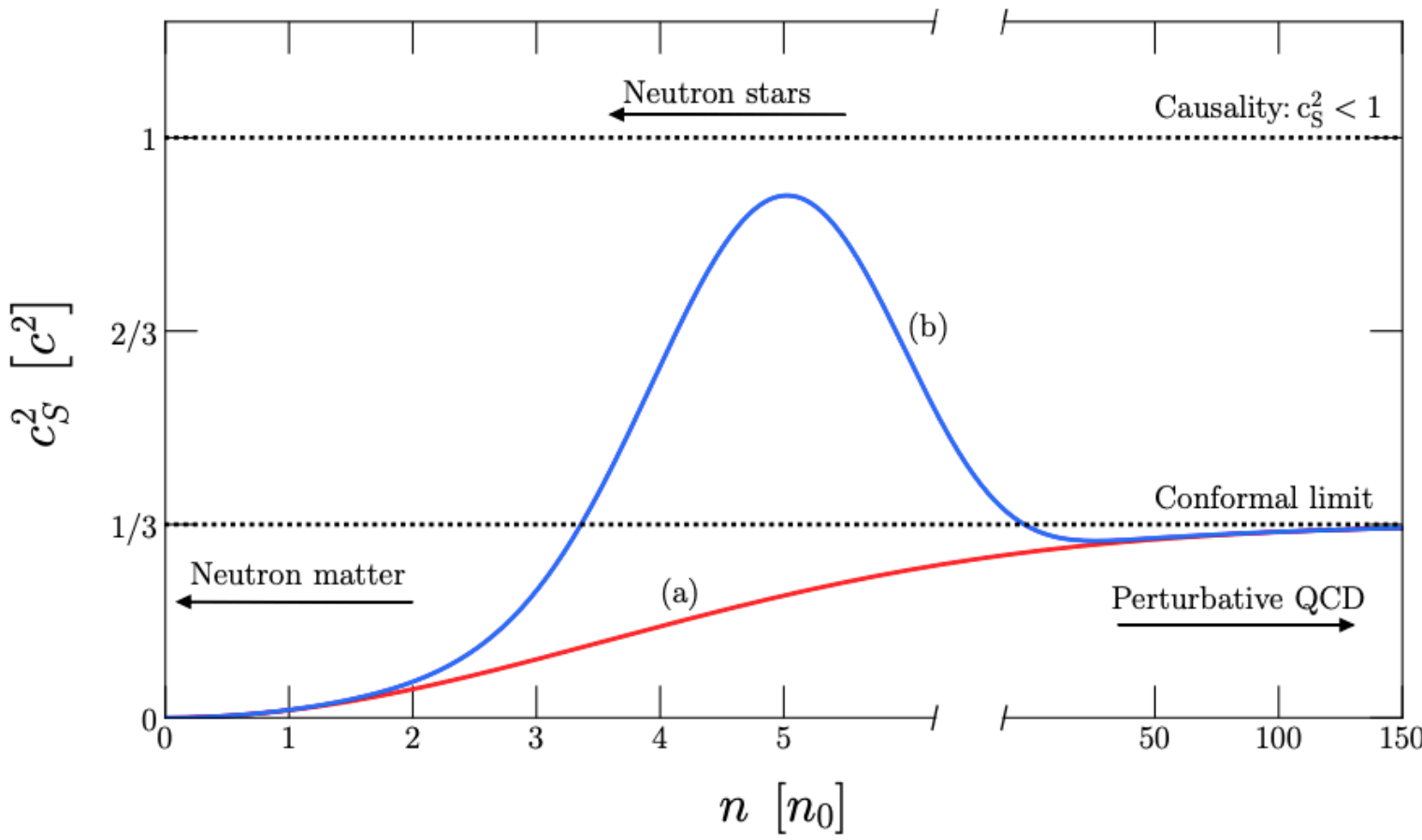


See e.g. Zhao & Lattimer, 2004.08293.



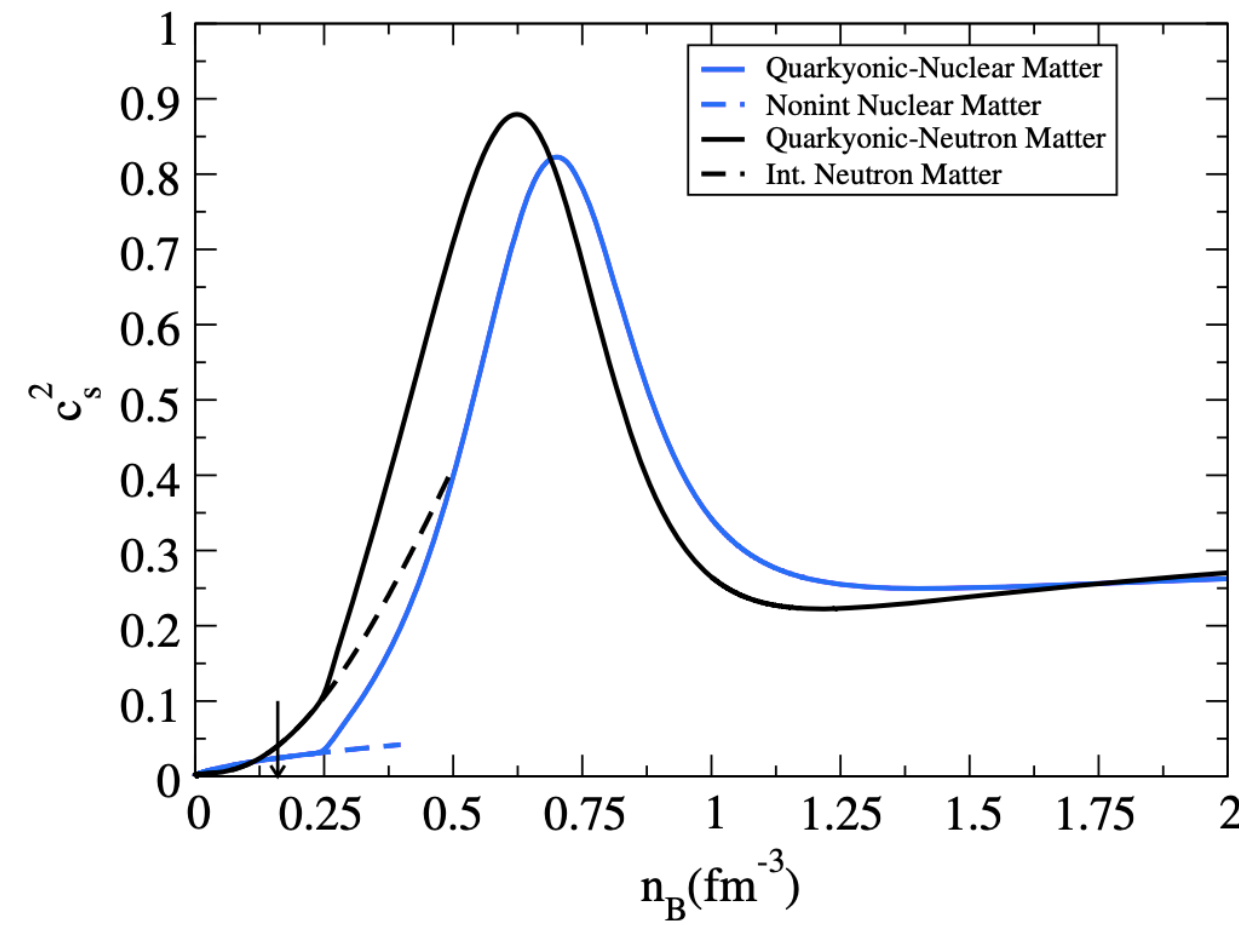
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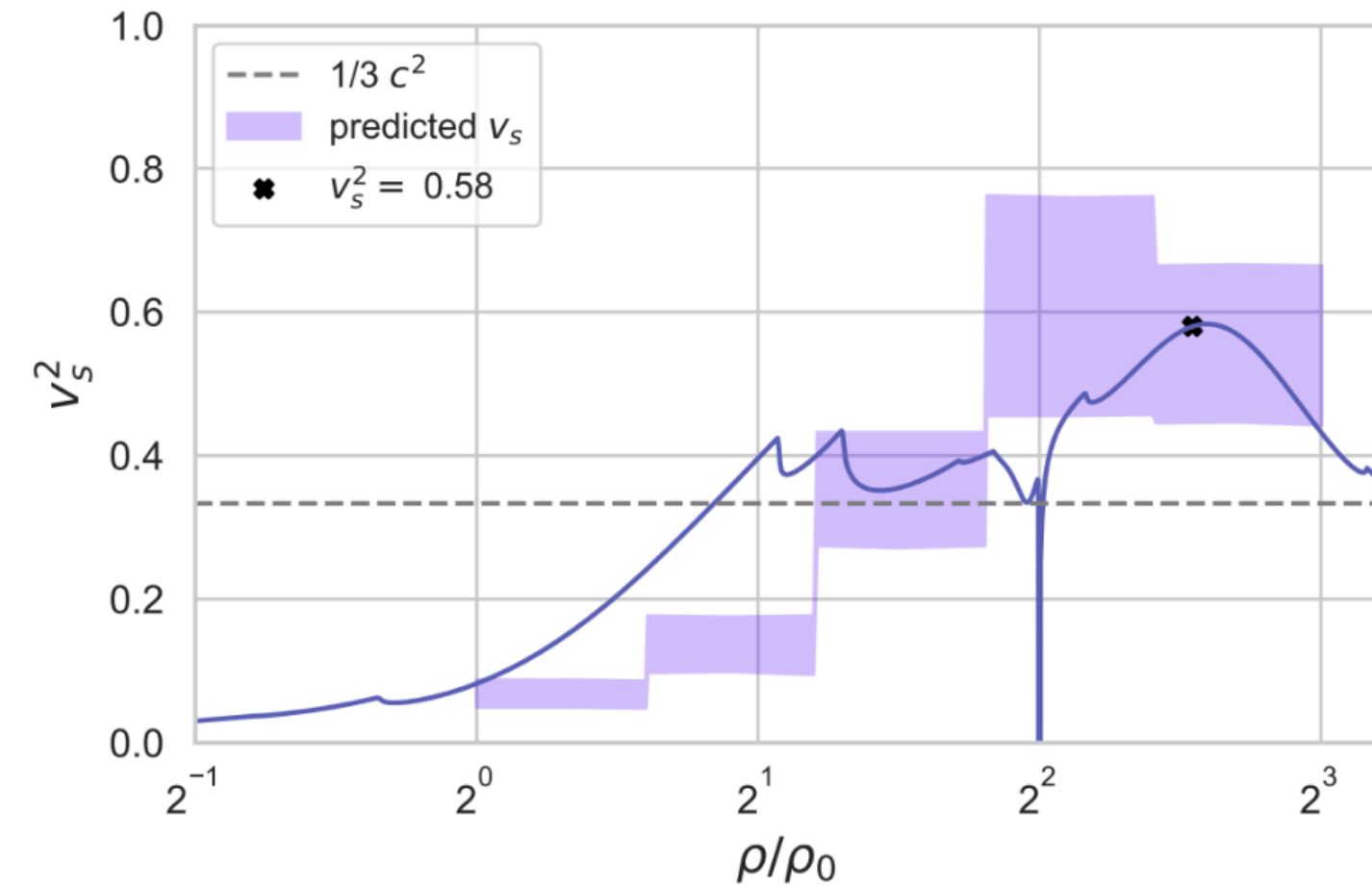
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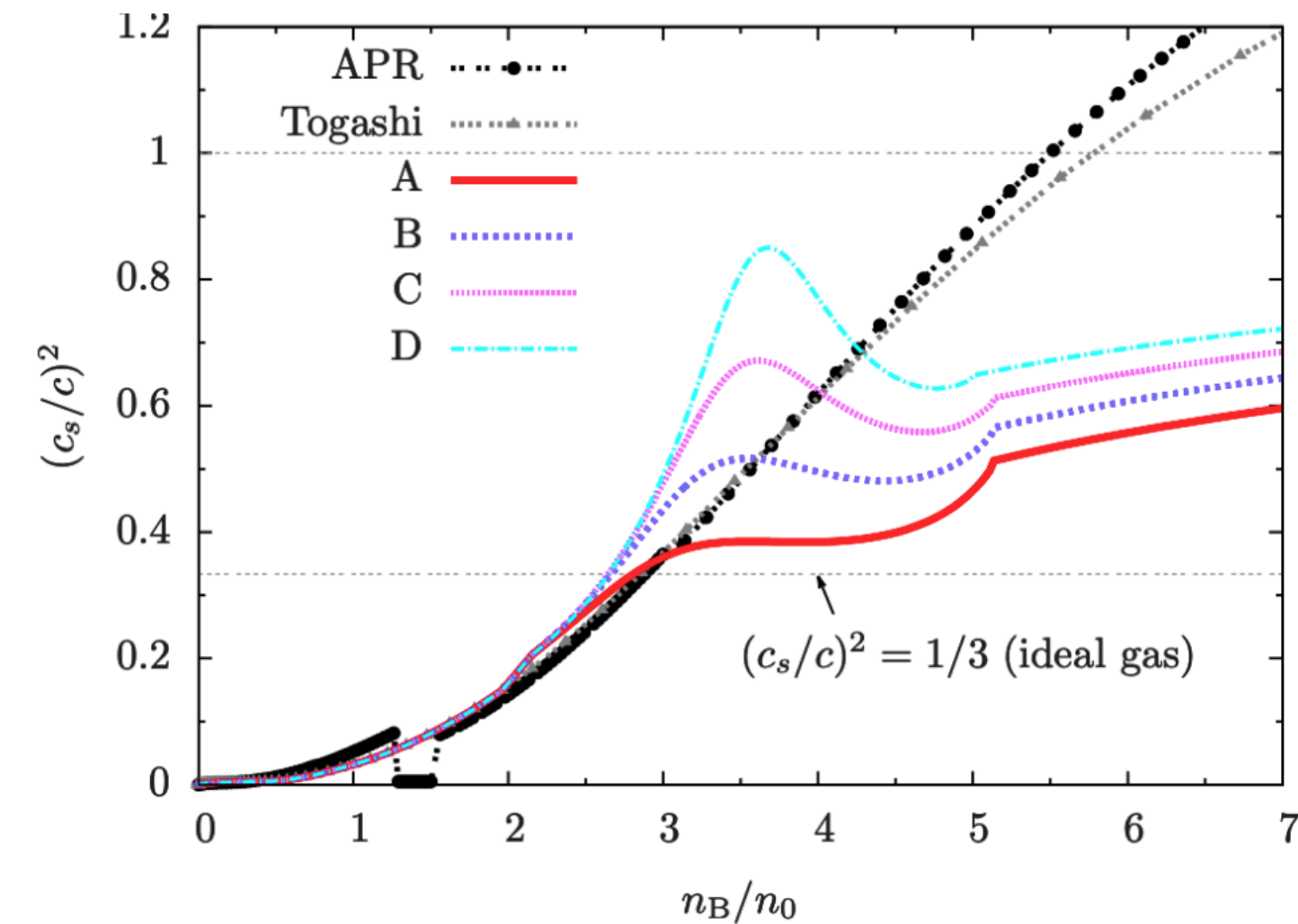
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Other studies indicate  
sharp kinks



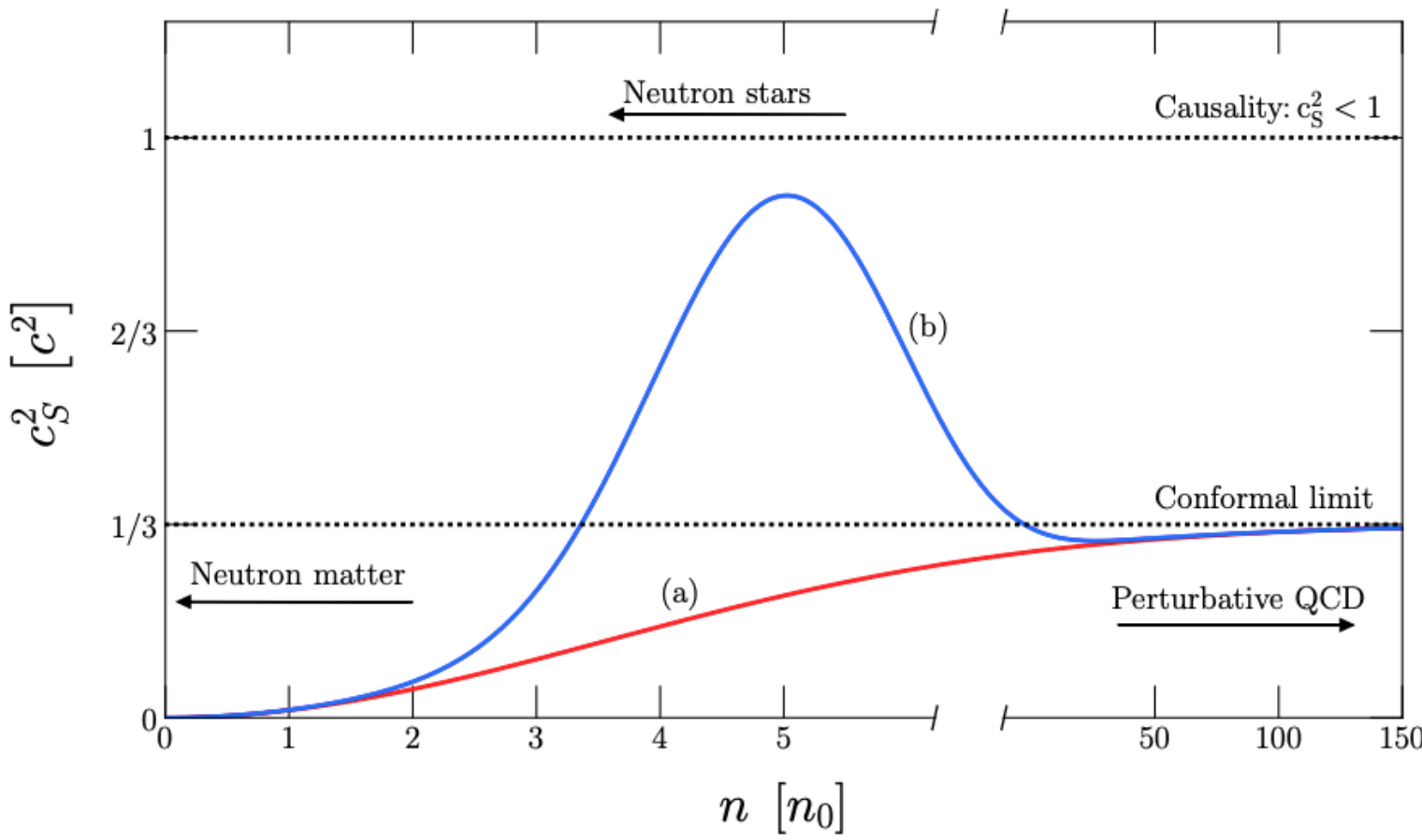
Chiral mean field model  
(hyperons and quarks)  
Dexheimer & Schramm,  
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Jakobus, et al, 2004.07026

QHC19 (crossover to quarks)  
Baym et al, ApJ 885, '19



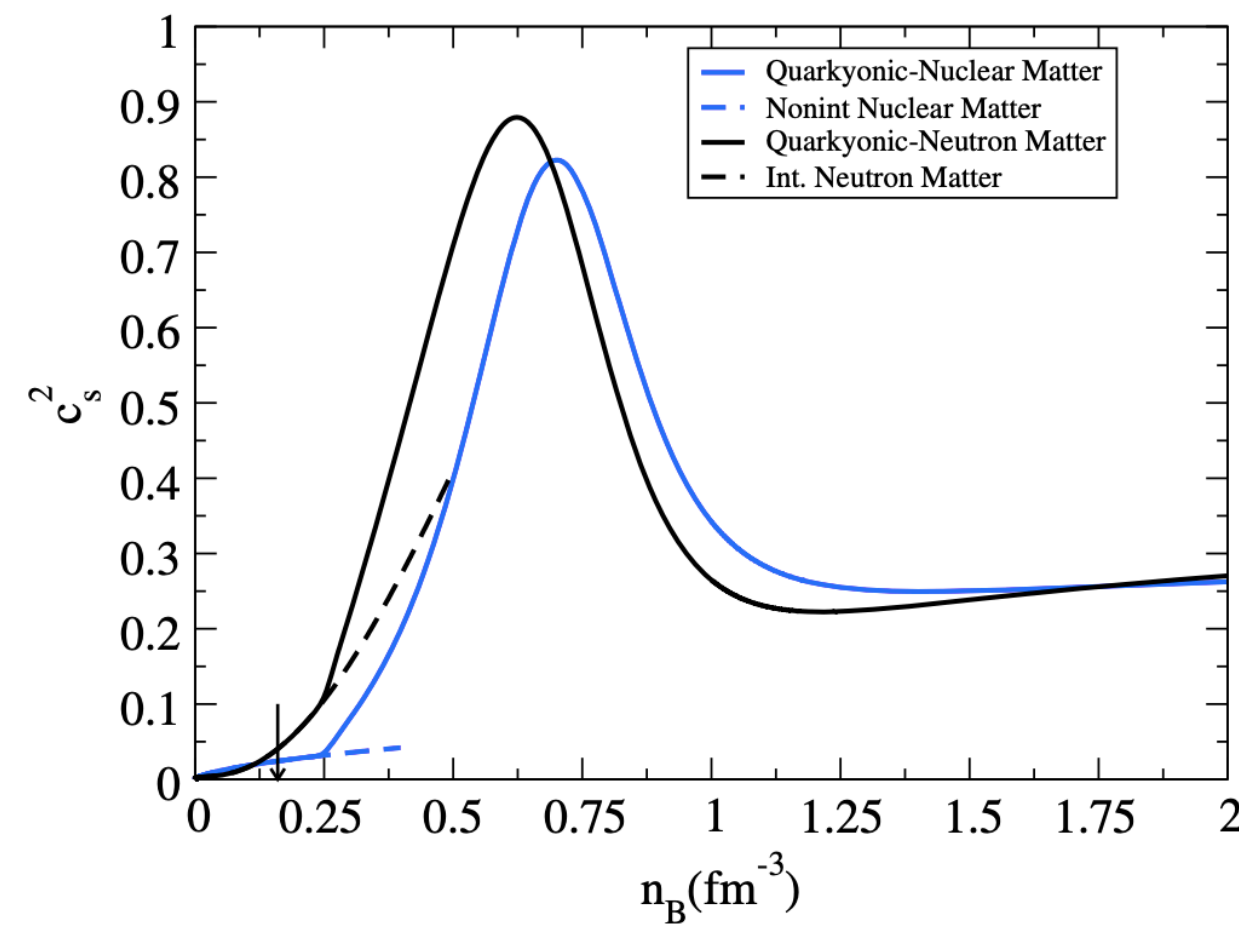
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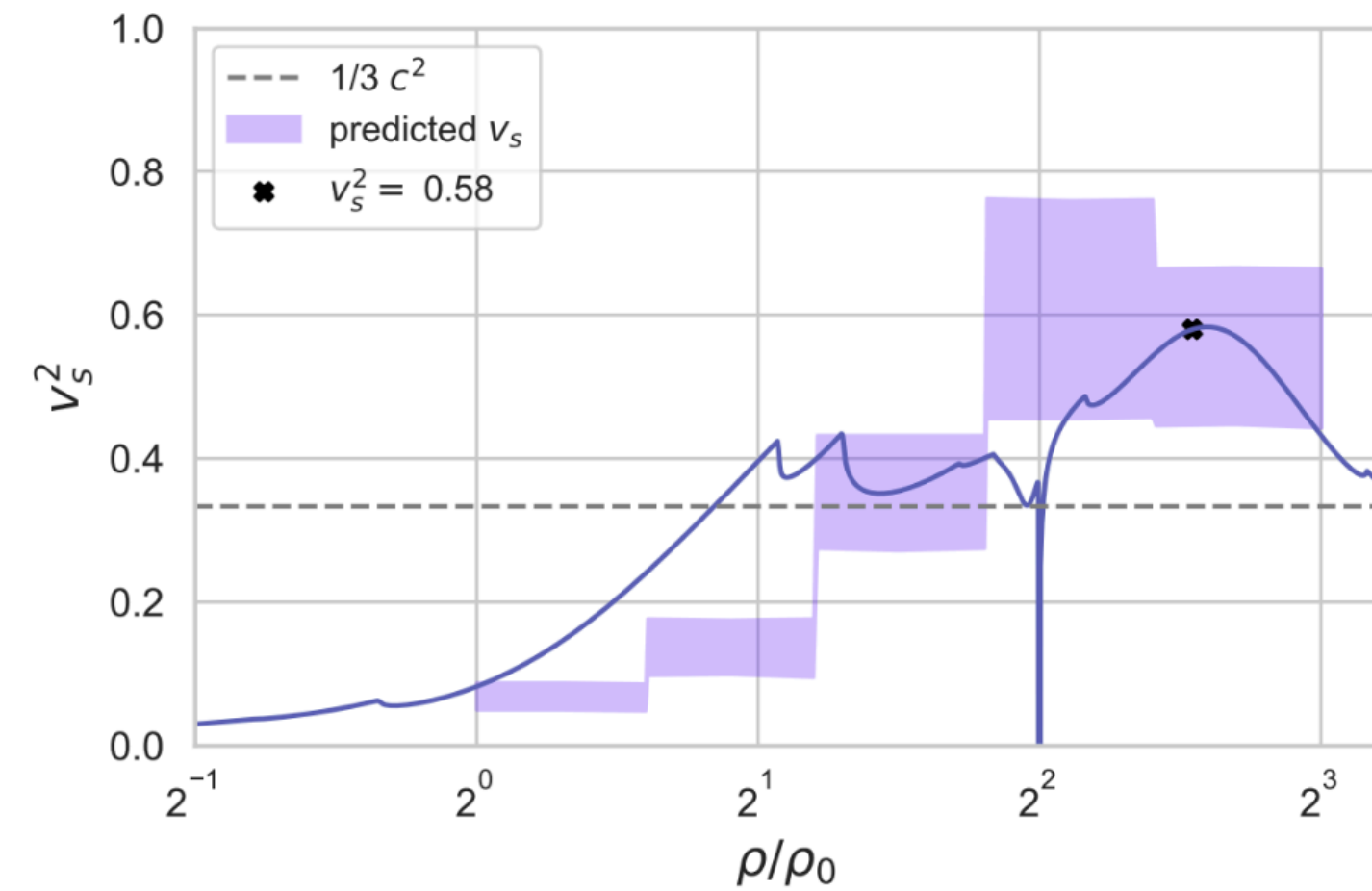
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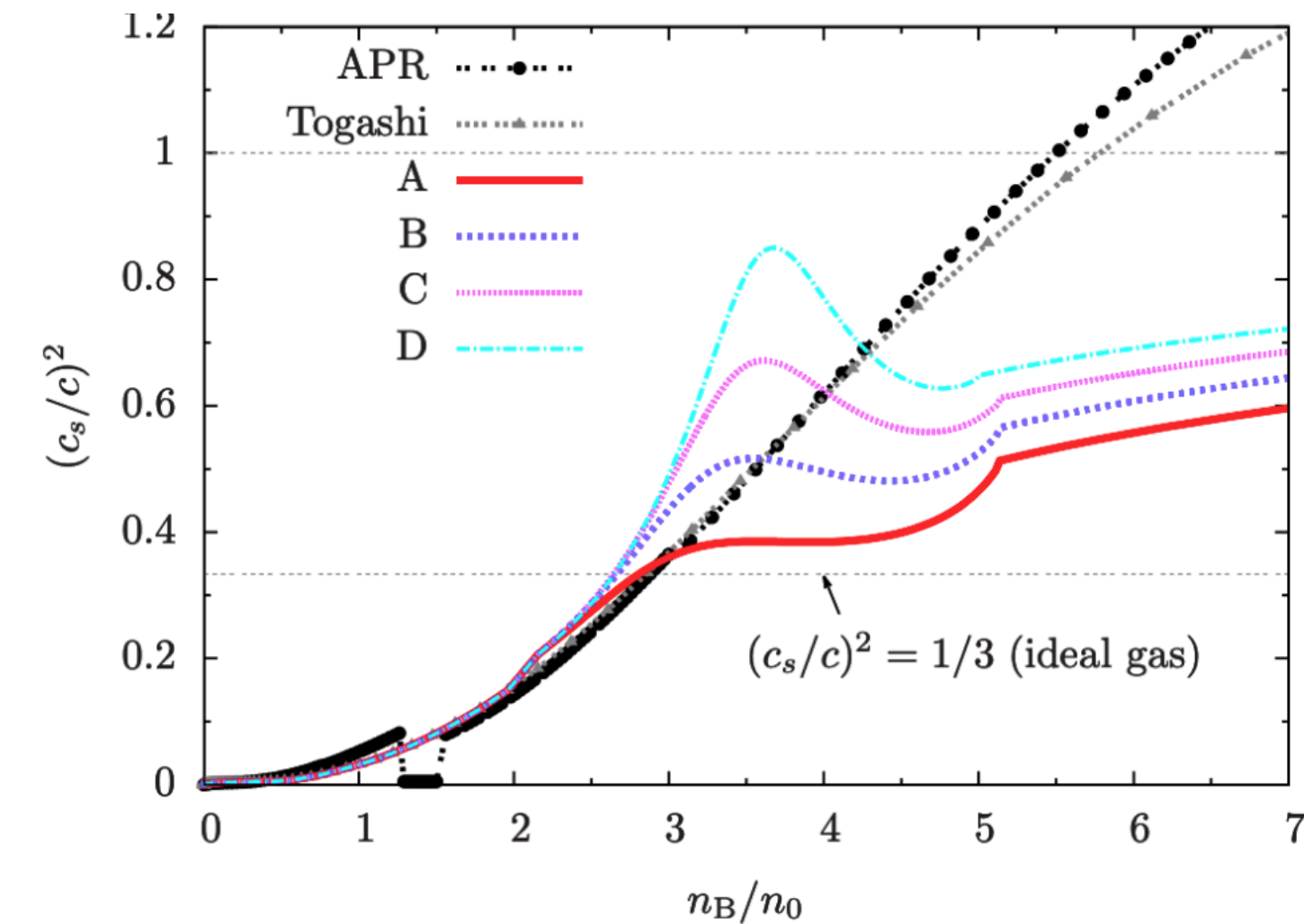
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**A kinky or bumpy speed of sound seems to be somewhat general in  
several nuclear physics models with quarks d.o.f.**



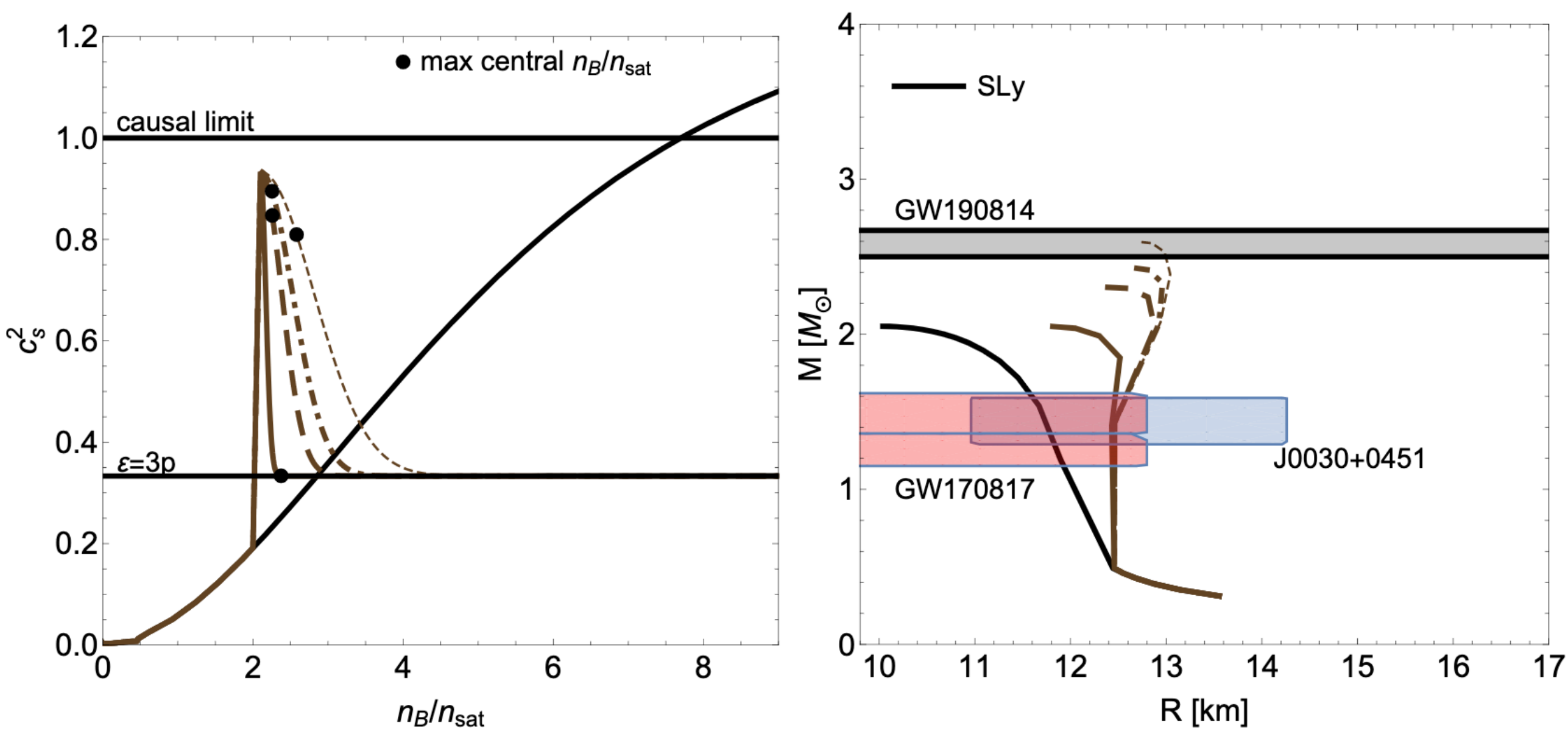


# Kinky and bumpy neutron stars



[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
+ in prep with Dexheimer, Dore]

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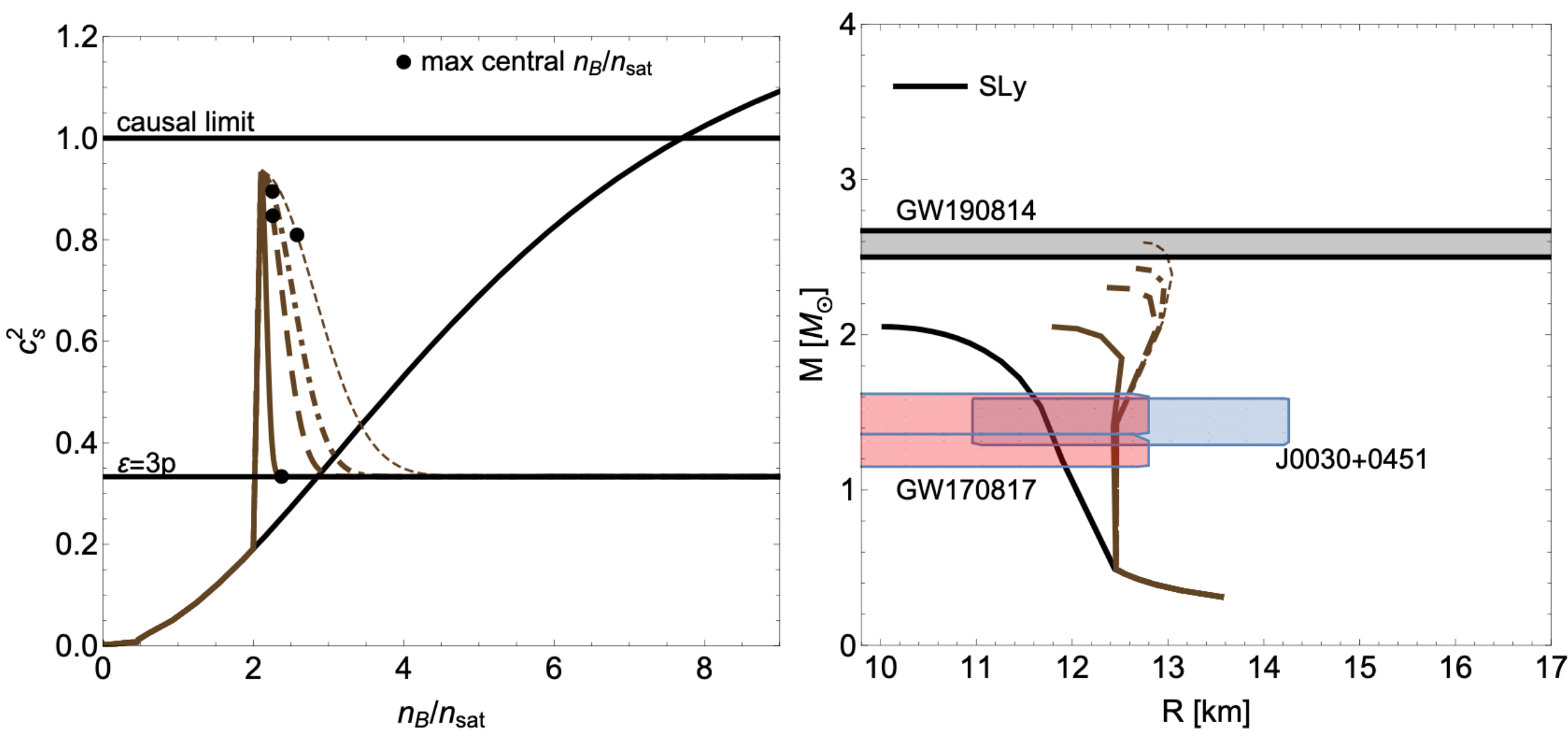
Holding the location of the bump constant,  $\uparrow$  width  $\uparrow M_\odot$



[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
+ in prep with Dexheimer, Dore]



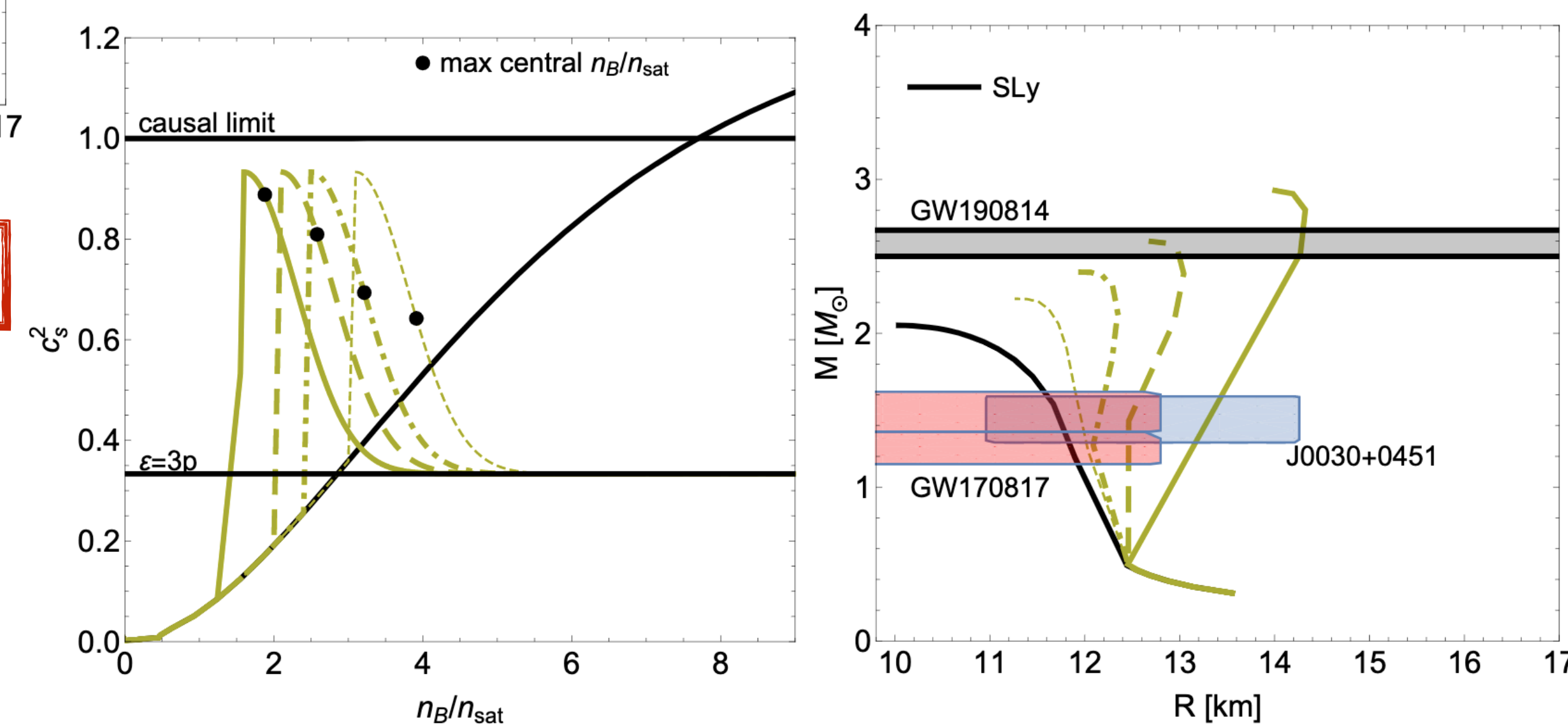
# Kinky and bumpy neutron stars



Holding the location of the bump constant,  $\uparrow$  width  $\uparrow M_\odot$

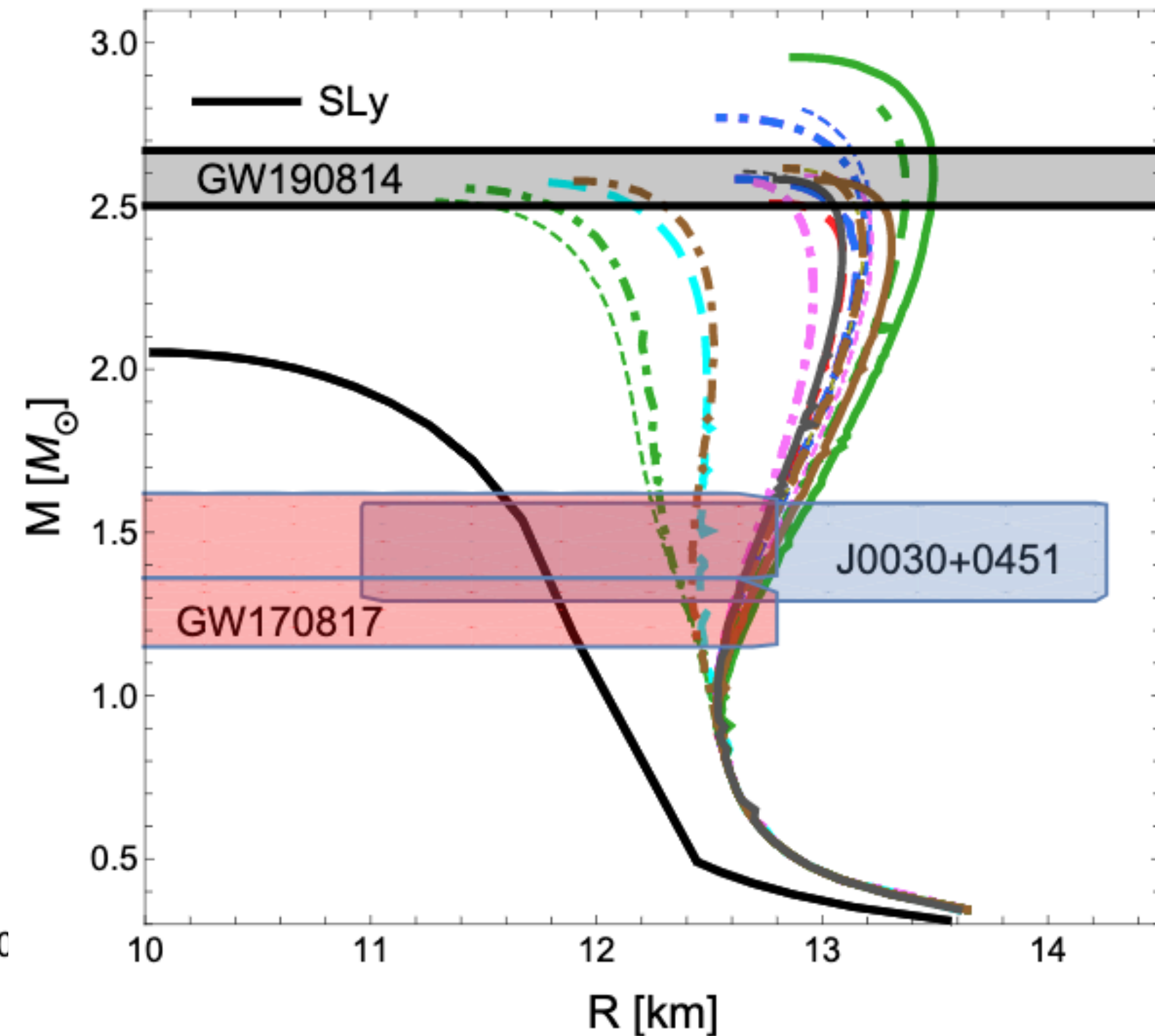
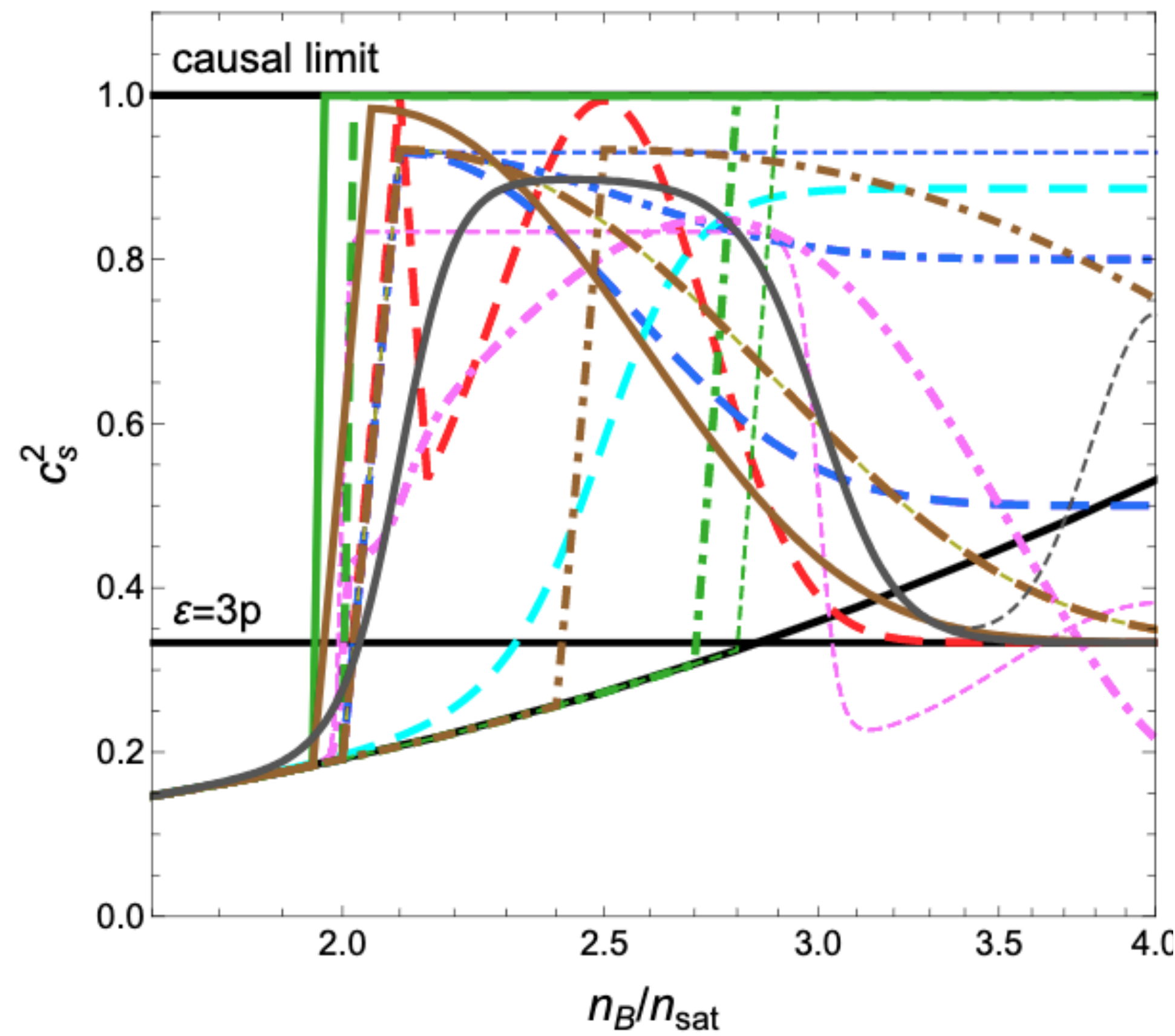


[Tan, Noronha-Hostler, Yunes, PRL 125, '20;  
+ in prep with Dexheimer, Dore]



$\uparrow n_B/n_{\text{sat}}$  for the rise,  $\uparrow$  the radius (and max central density)

# If GW190814 is a NS-BH merger, what does this say about $c_s^2$ ?

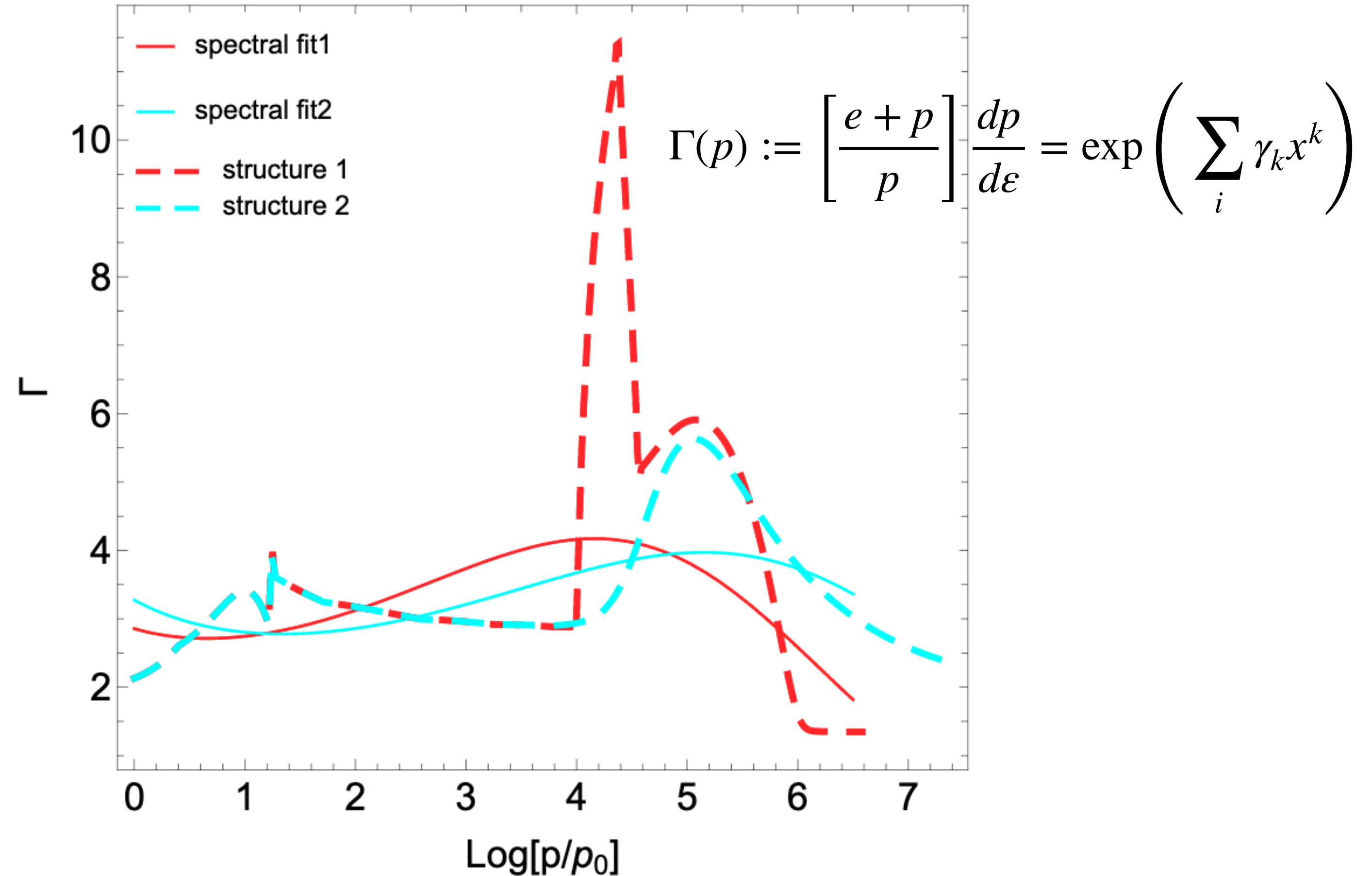
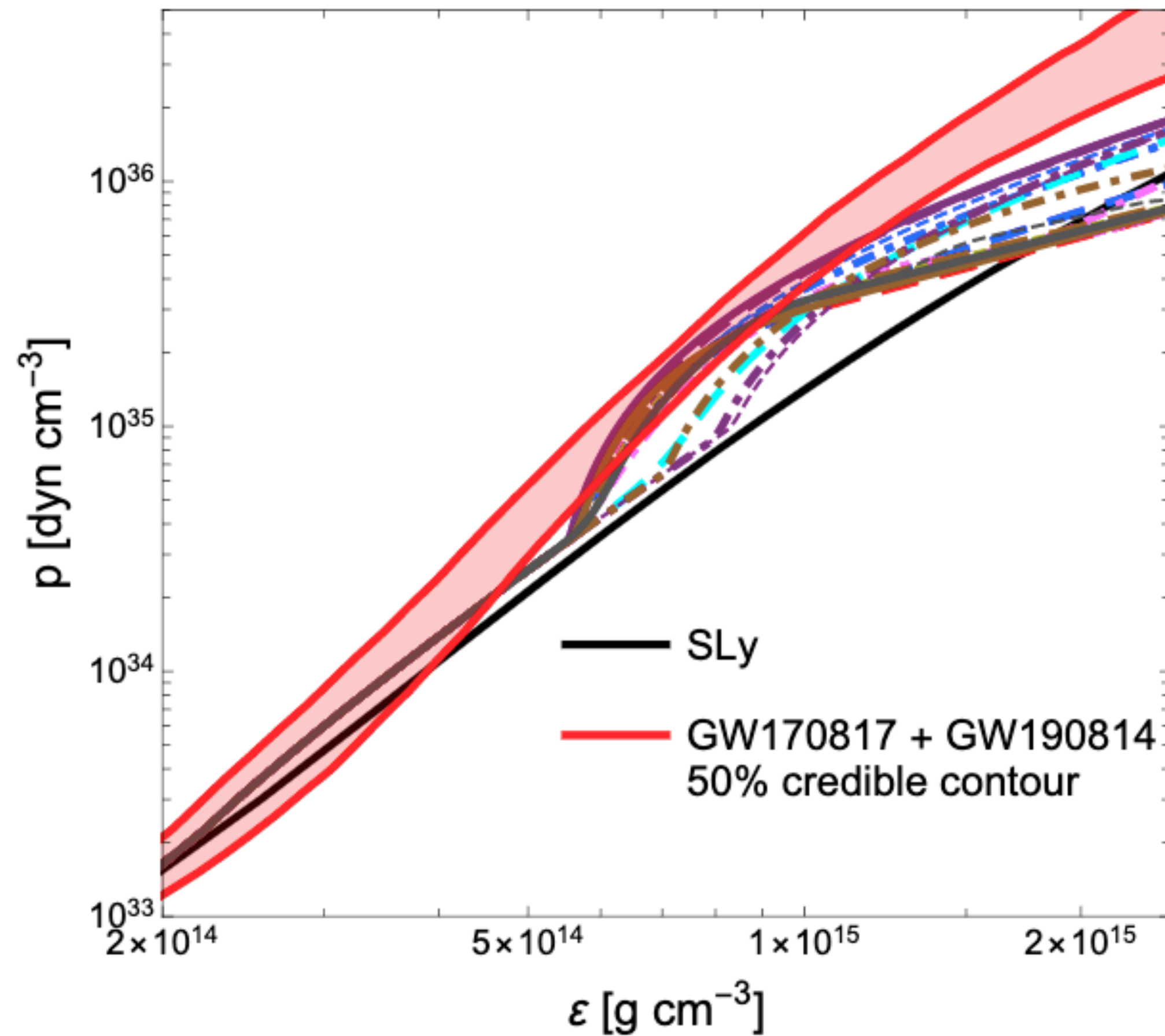


**Large enough  $M_\odot$  and match of R constraints  
requires step rise in  $c_s^2$  between  $n_B/n_{\text{sat}} \sim 2 - 3$**

[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
+ in prep with Dexheimer, Dore]



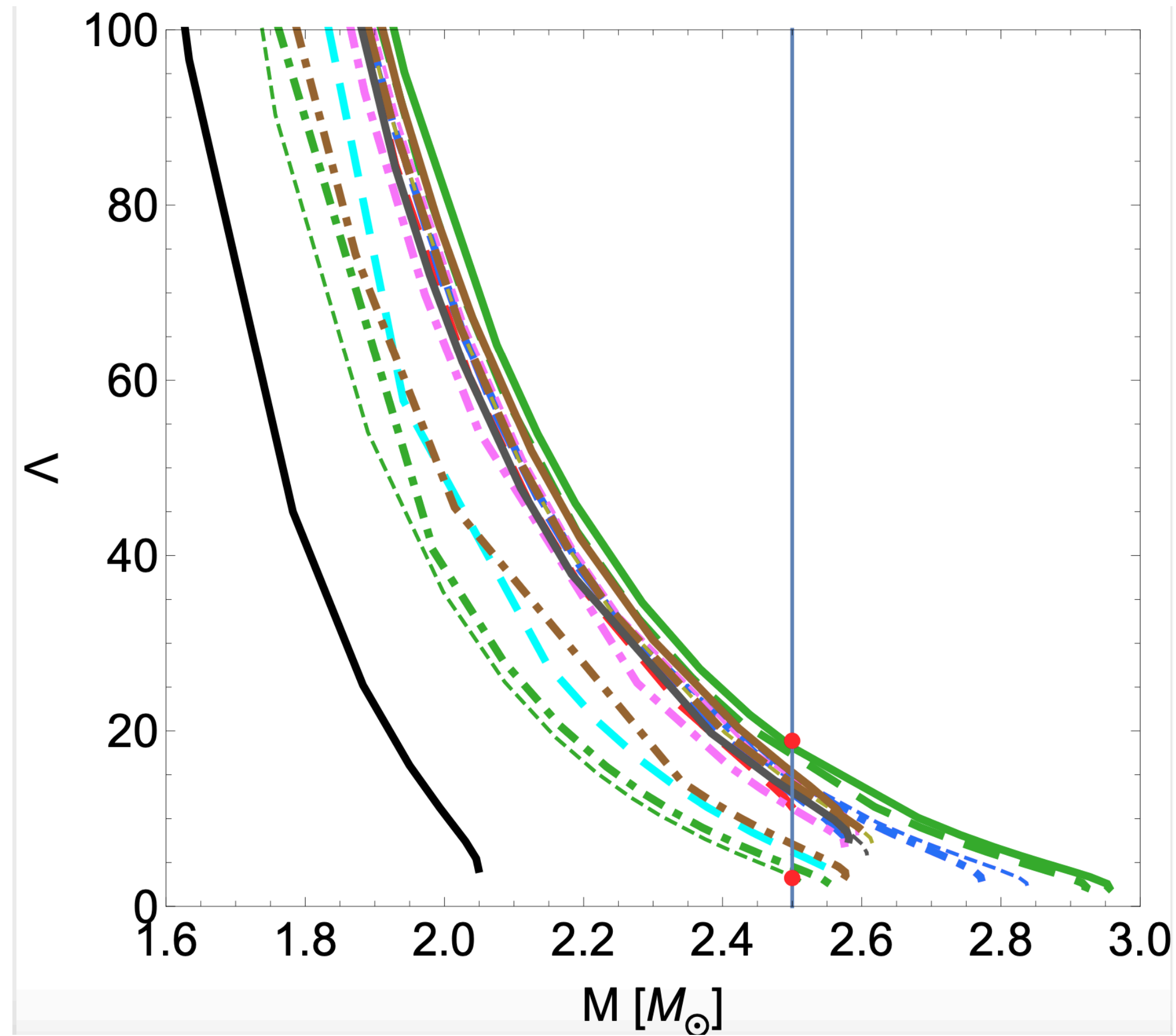
# Isn't this in conflict with LIGO's observations?



**The spectral representation cannot capture bumps/kinks/jumps in the EOS, can push the M-R curve out-of-bounds!**

[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
+ in prep with Dexheimer, Dore]

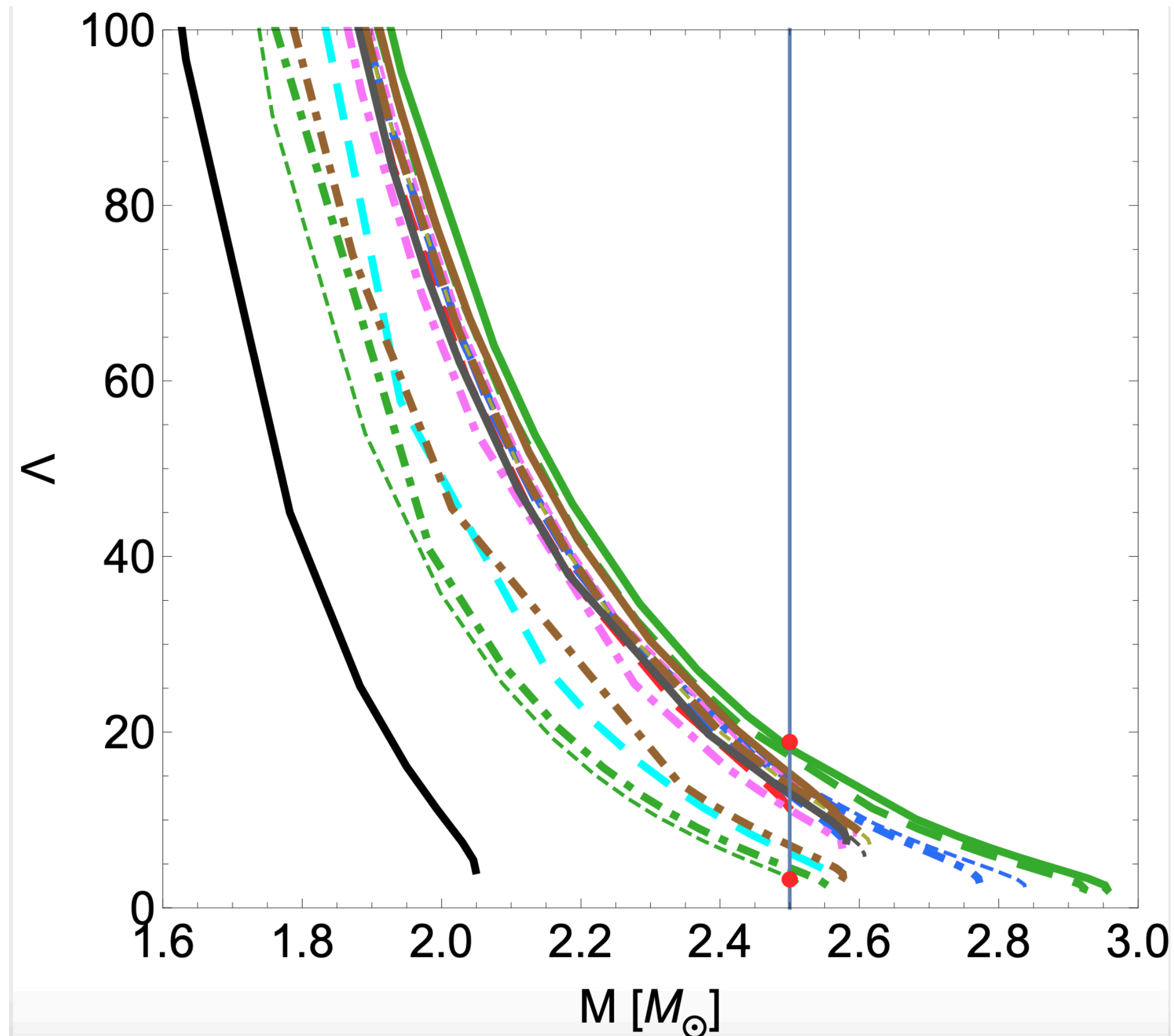
# How can we be sure that it's a neutron star and not a black hole?



[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
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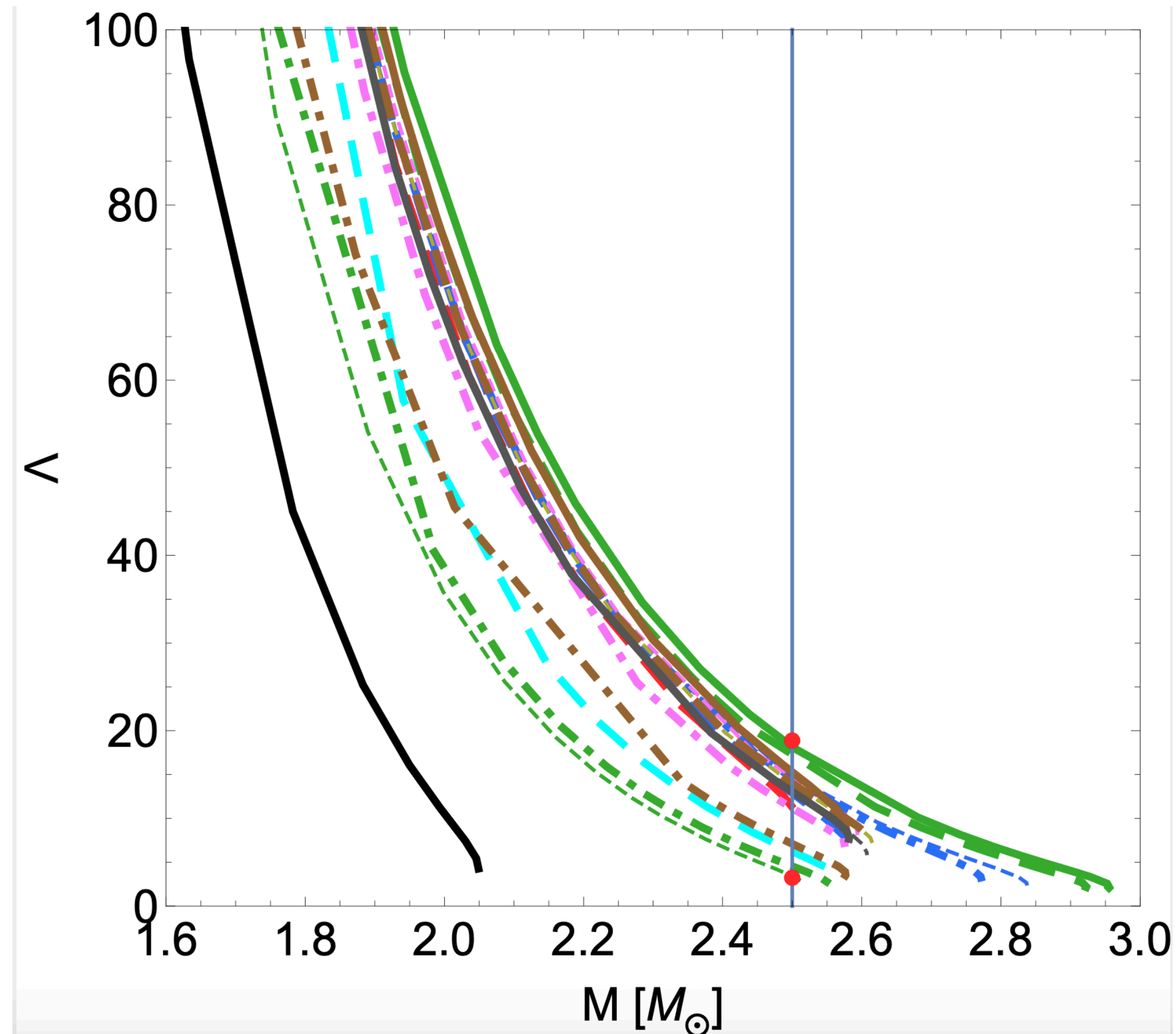
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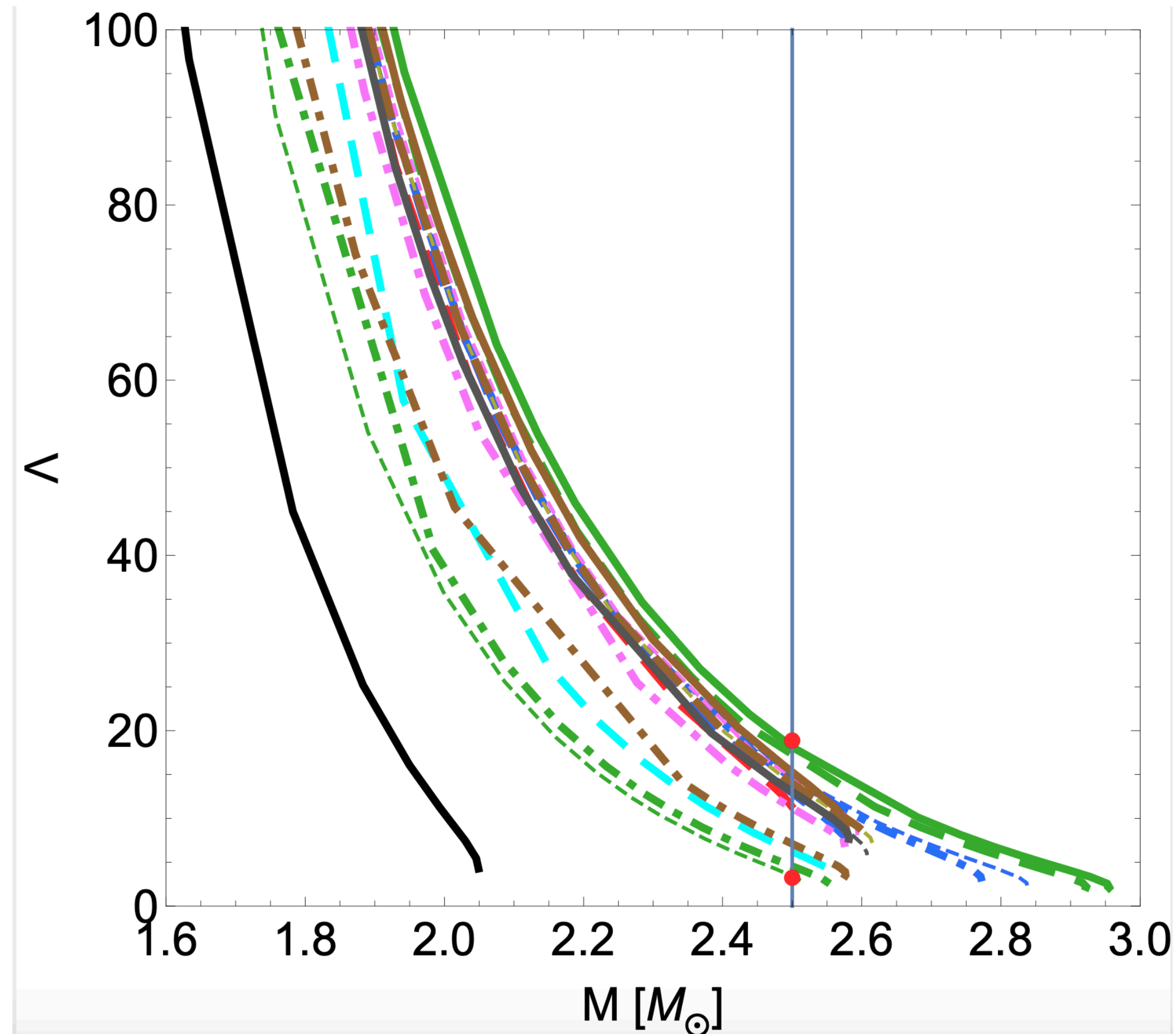
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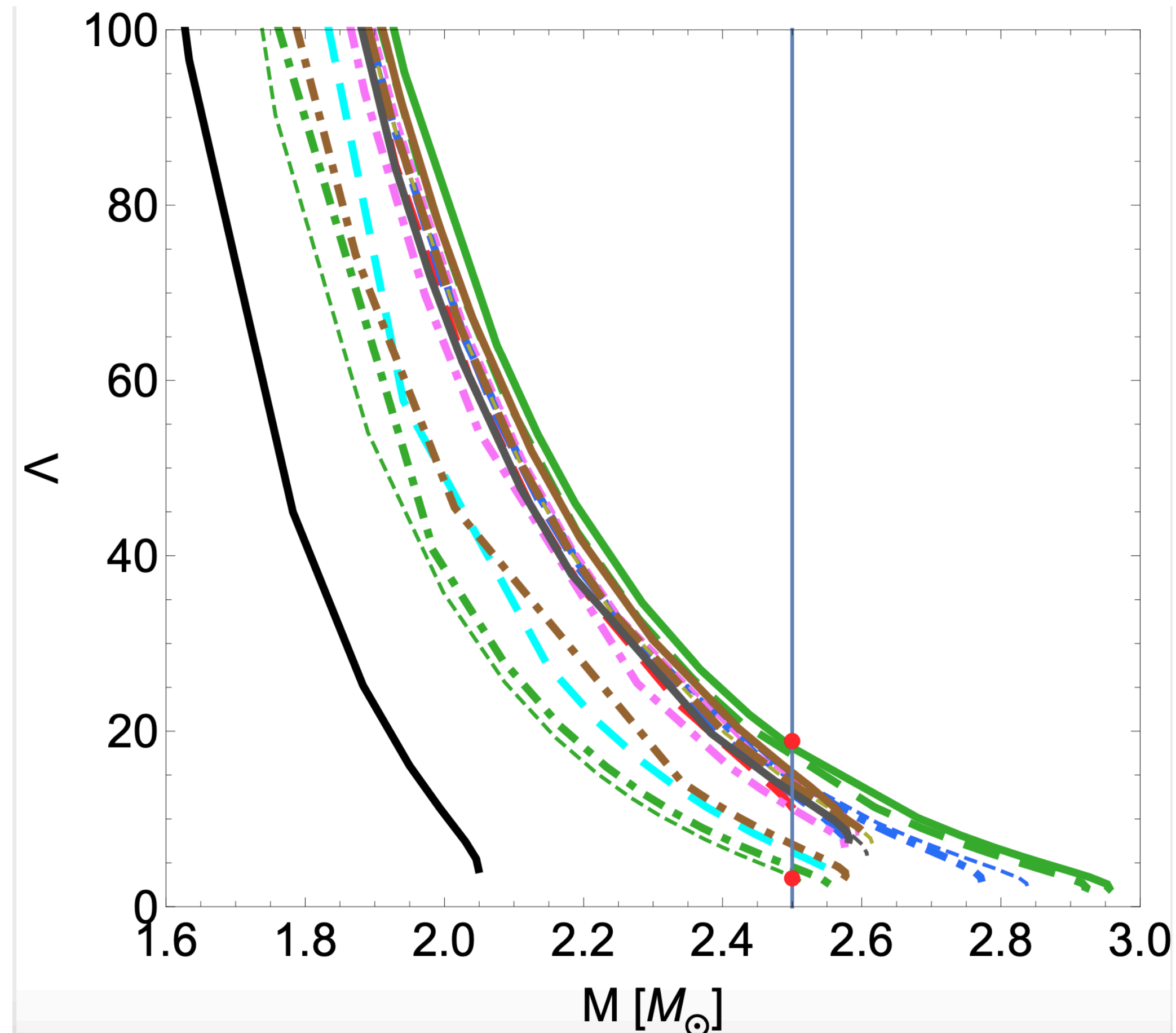


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[**Tan**, Noronha-Hostler, Yunes, PRL 125, '20;  
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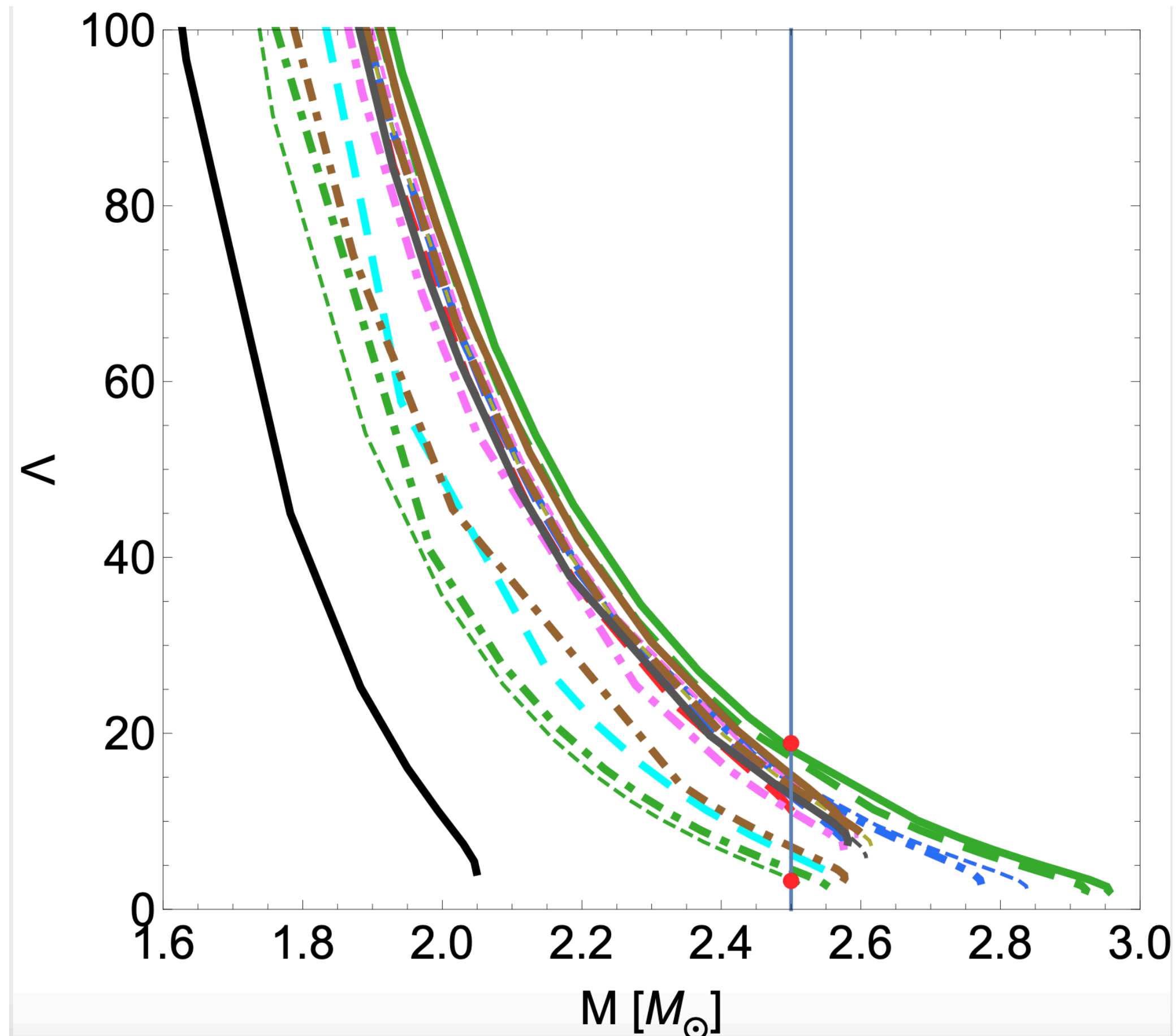
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**Needs measurements of  $\Lambda \sim 3-20$ , current detectors can measure  $\Lambda \sim 100 - 400$**





# Summary and Outlook

- Large maximum mass of a neutron star is possible when incorporating dramatic change in the degrees of freedom (quarks???)

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**“Assumptions are made and most assumptions are wrong.”**

**Thank You**

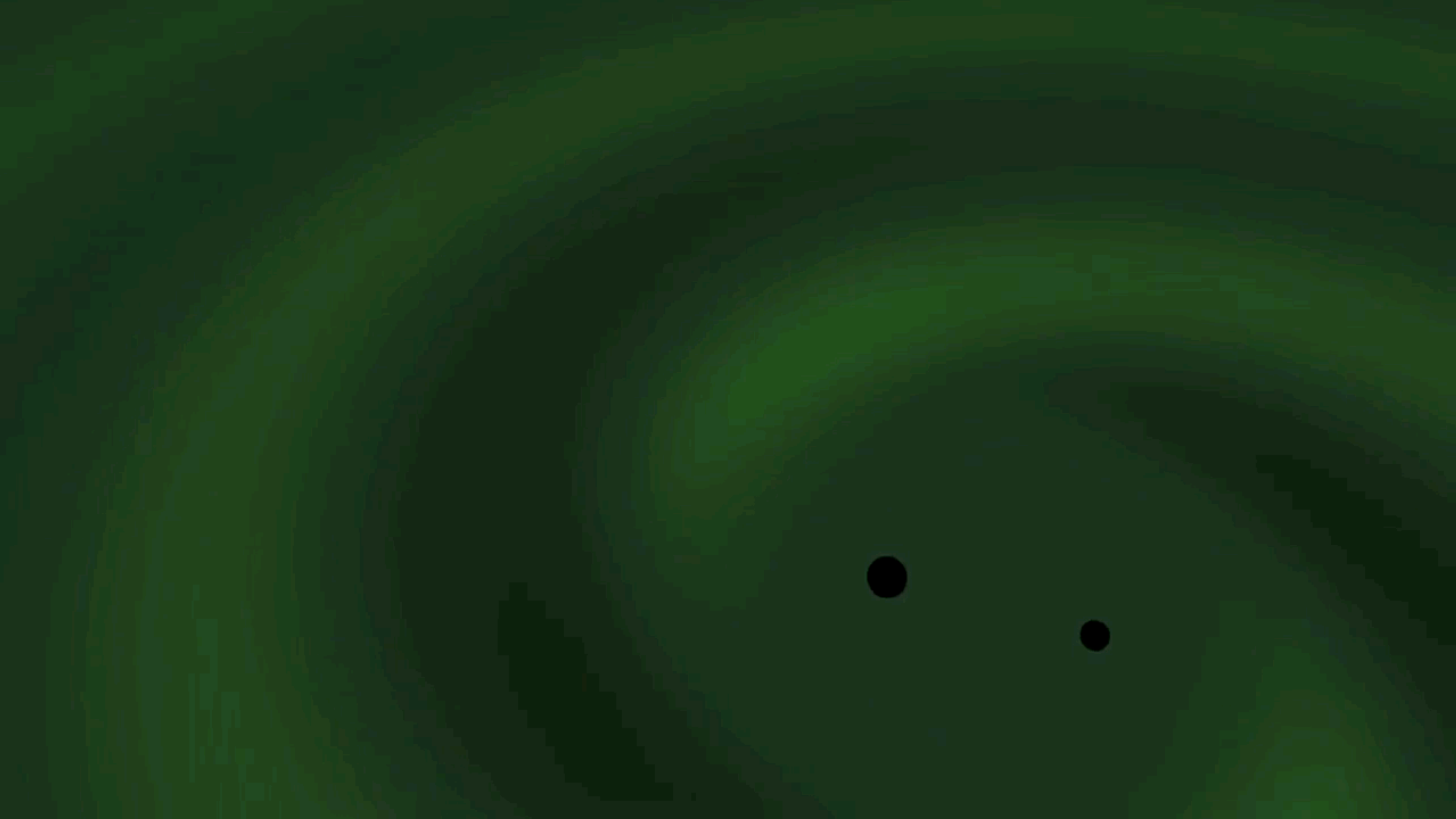


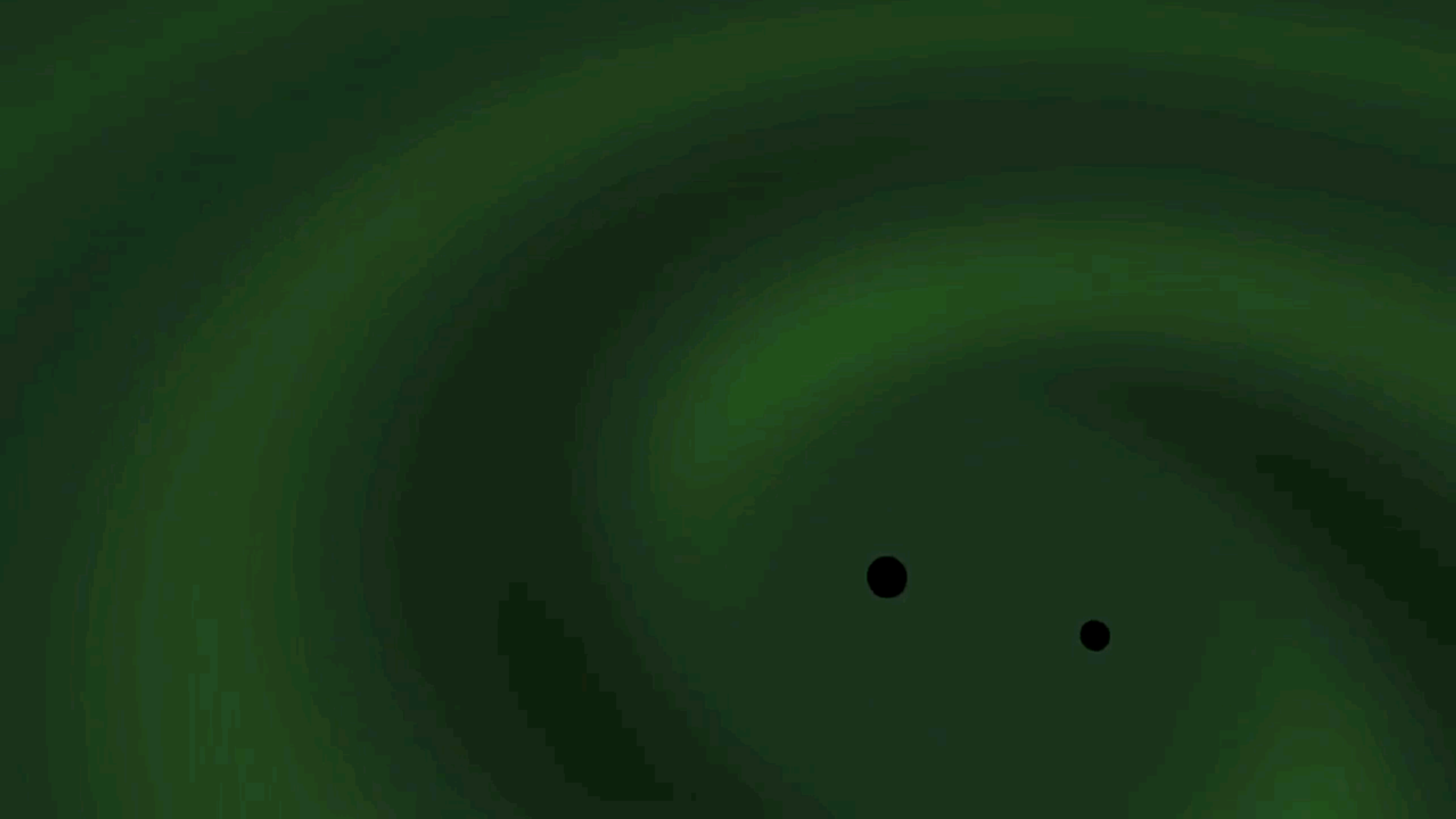
# Why start with speed of sound: $c_s^2$ ?

Connection to the susceptibilities

$$\chi_2 = \frac{d^2 P}{d\mu_B^2} \text{ at } T=0:$$
$$c_s^2 = n_B / (\mu_B \chi_2)$$

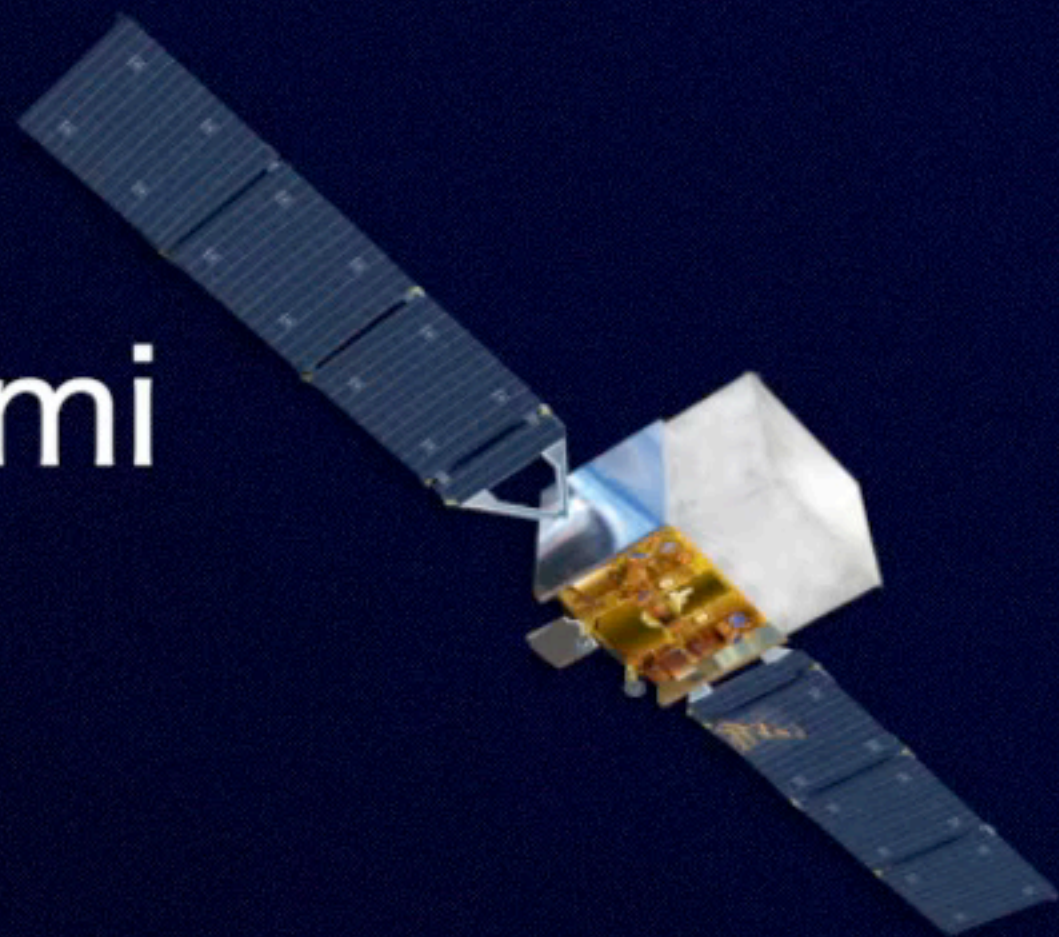
McLerran & Reddy, Phys. Rev. Lett.122, 122701 (2019)







Fermi



Gamma rays, 50 to 300 keV

GRB 170817A

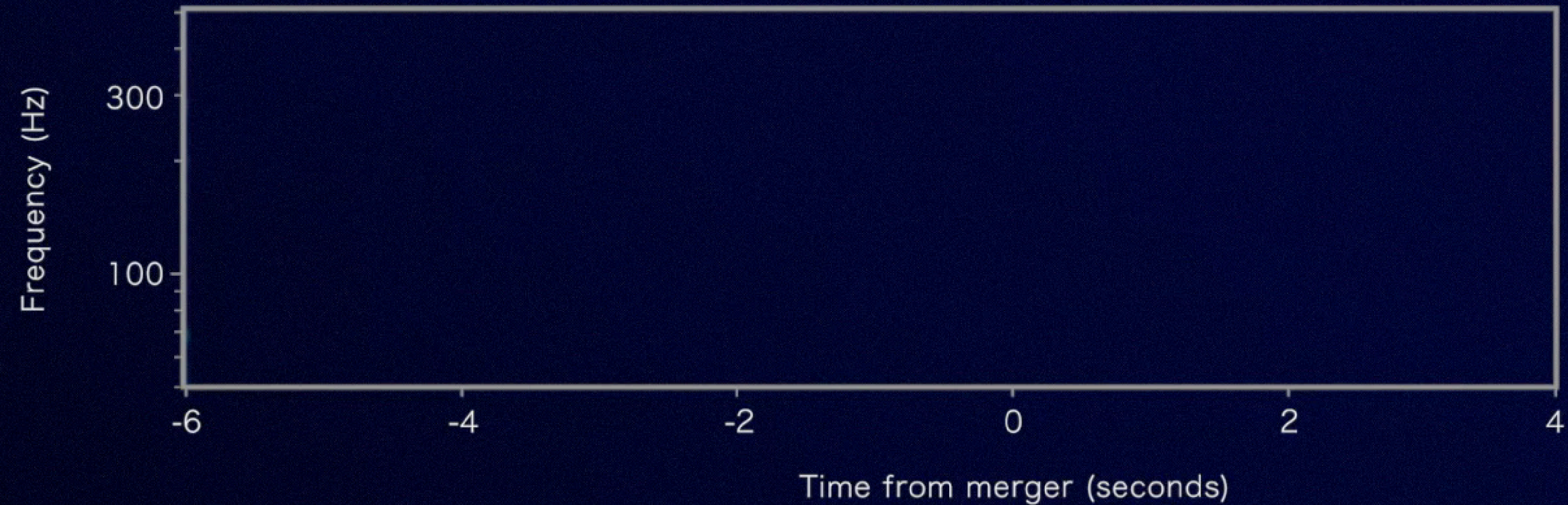


LIGO



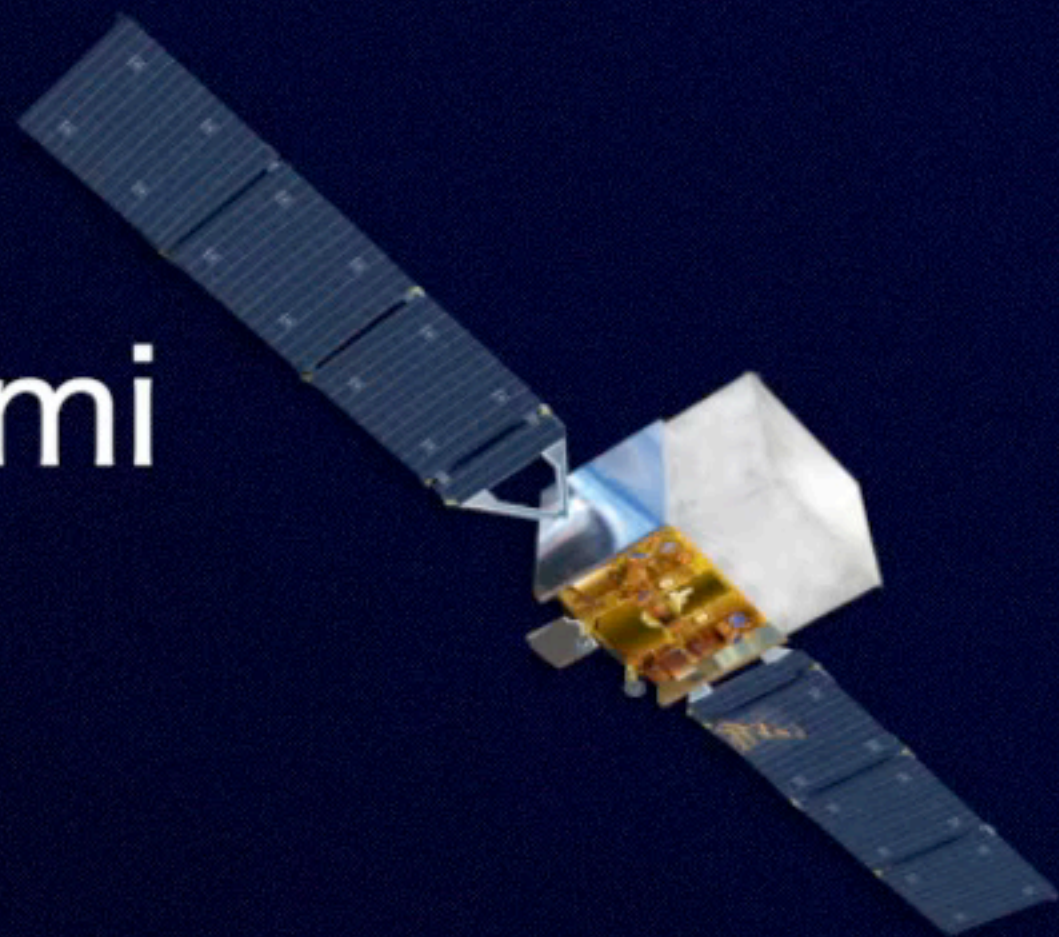
Gravitational-wave strain

GW170817





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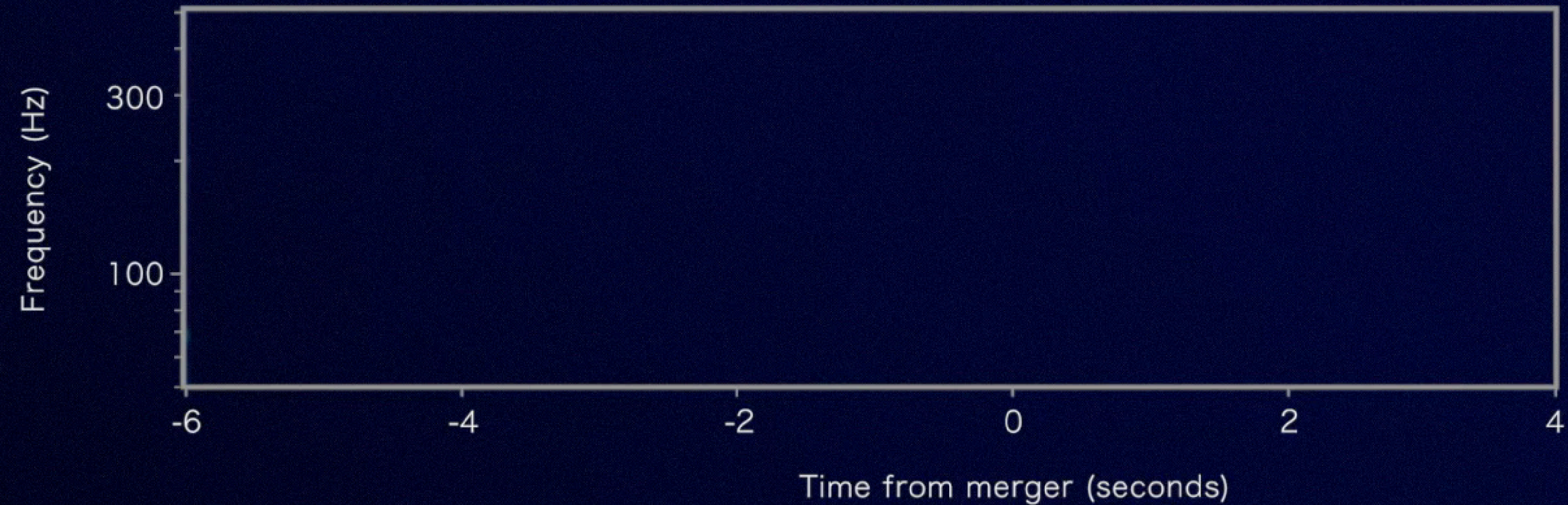


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$$\tilde{h}(f) = \mathcal{A}(f) e^{i\psi_{\text{pp}}(f) + i\psi_{\text{tidal}}(f)}$$

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$$\Lambda = g(m_1, m_2) \lambda_1 + h(m_1, m_2) \lambda_2$$



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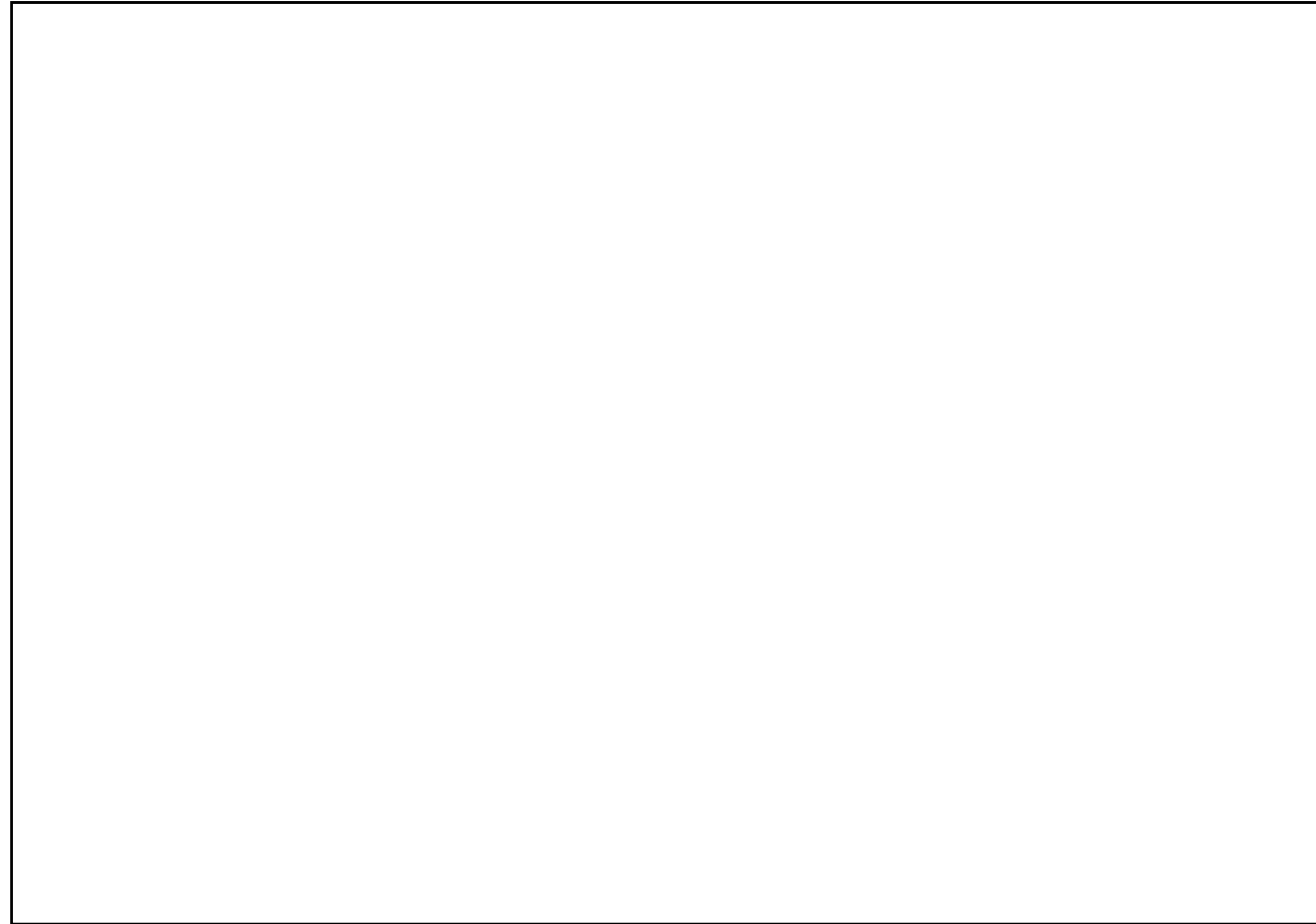
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**If you have measured  $(m_1, \lambda_1)$  and  $(m_2, \lambda_2)$ , then  $\lambda_1 = \lambda_1(C_1)$  and  $\lambda_2 = \lambda_2(C_2)$  relations give you  $(m_1, R_1)$  and  $(m_2, R_2)$  !**



# The bafeness of compactly objects



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$\bar{I}$

$$\bar{I} = \frac{I}{M^3}$$

# The baldness of compact objects

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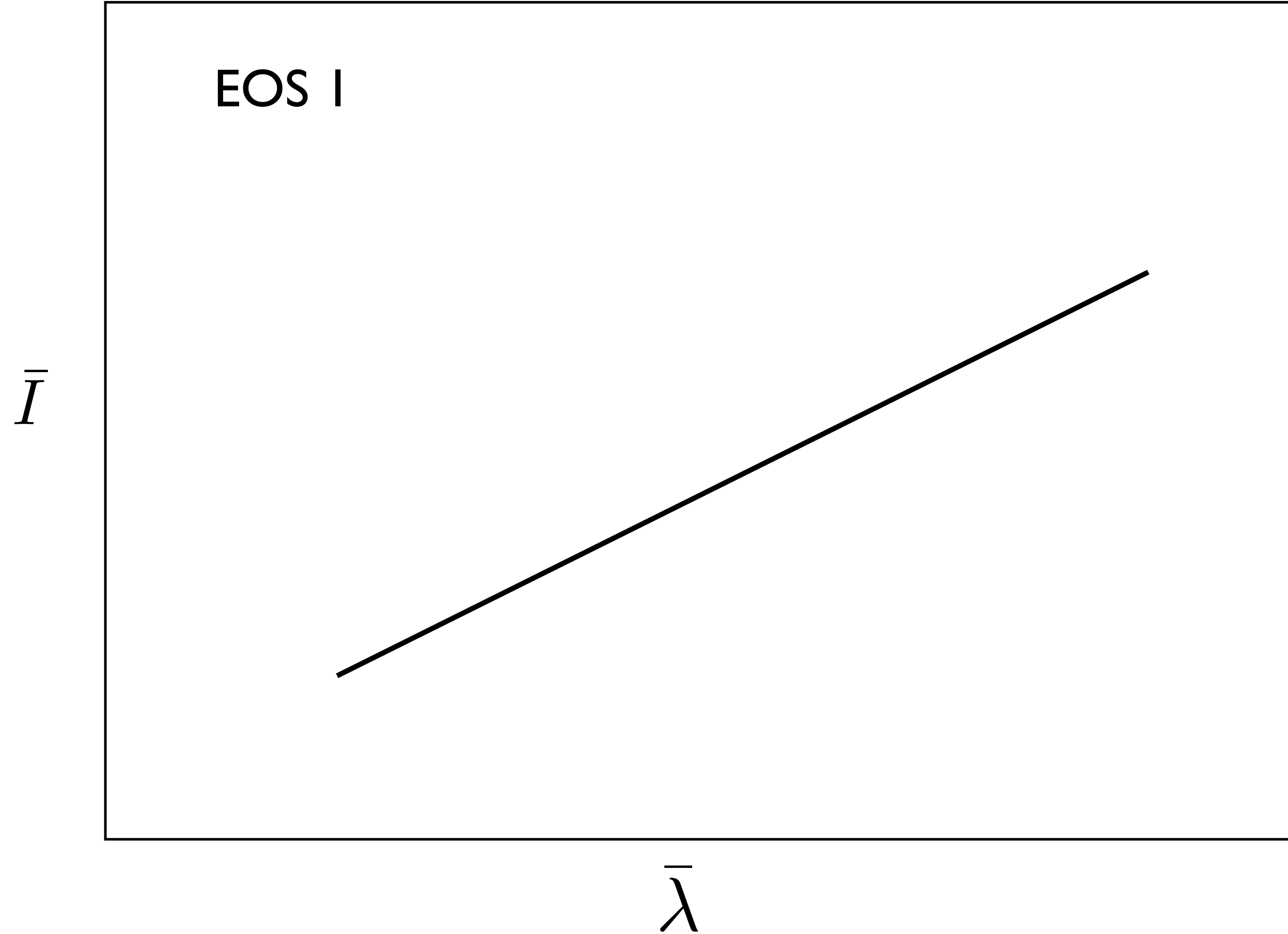
$\bar{\lambda}$

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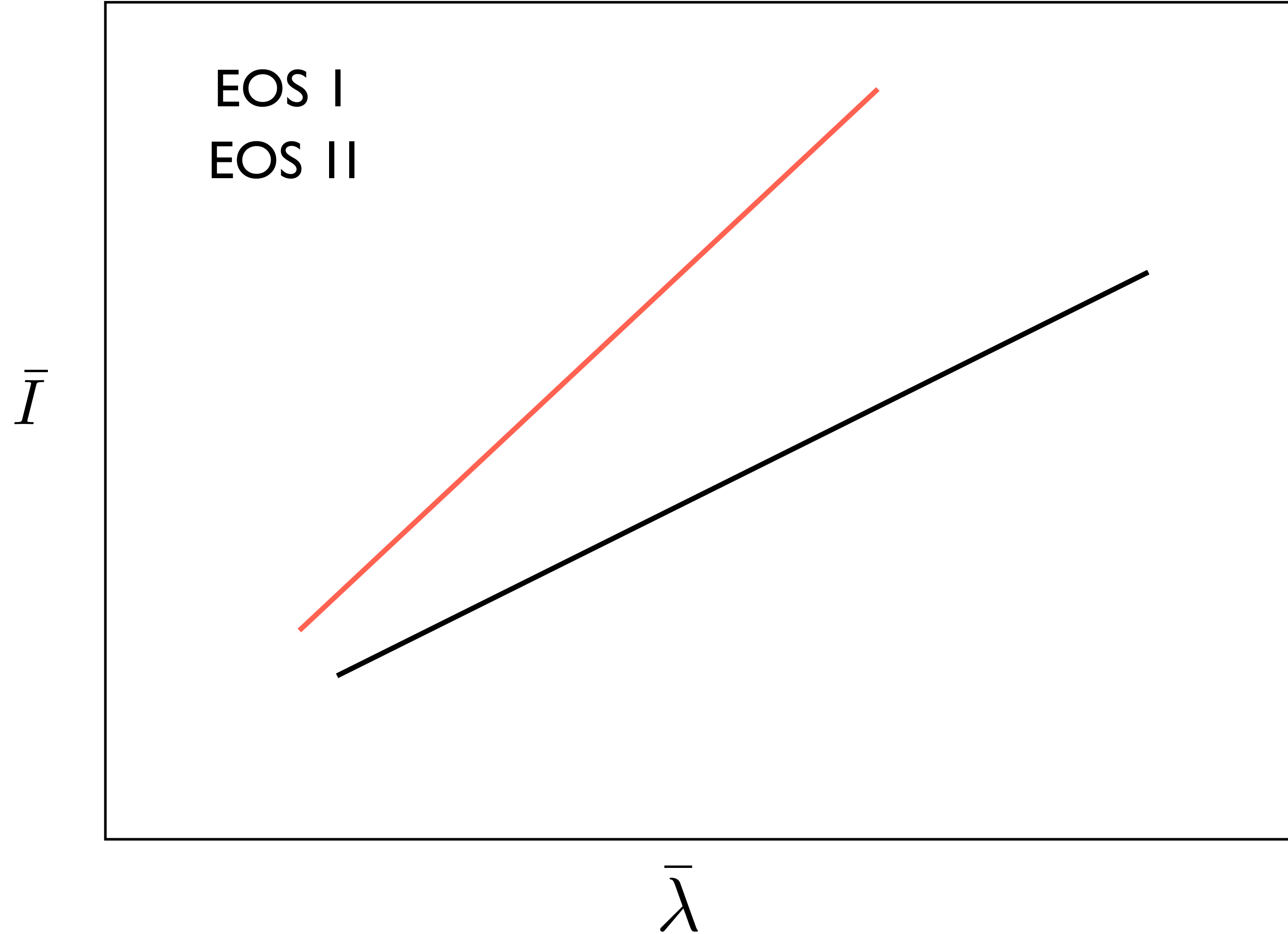
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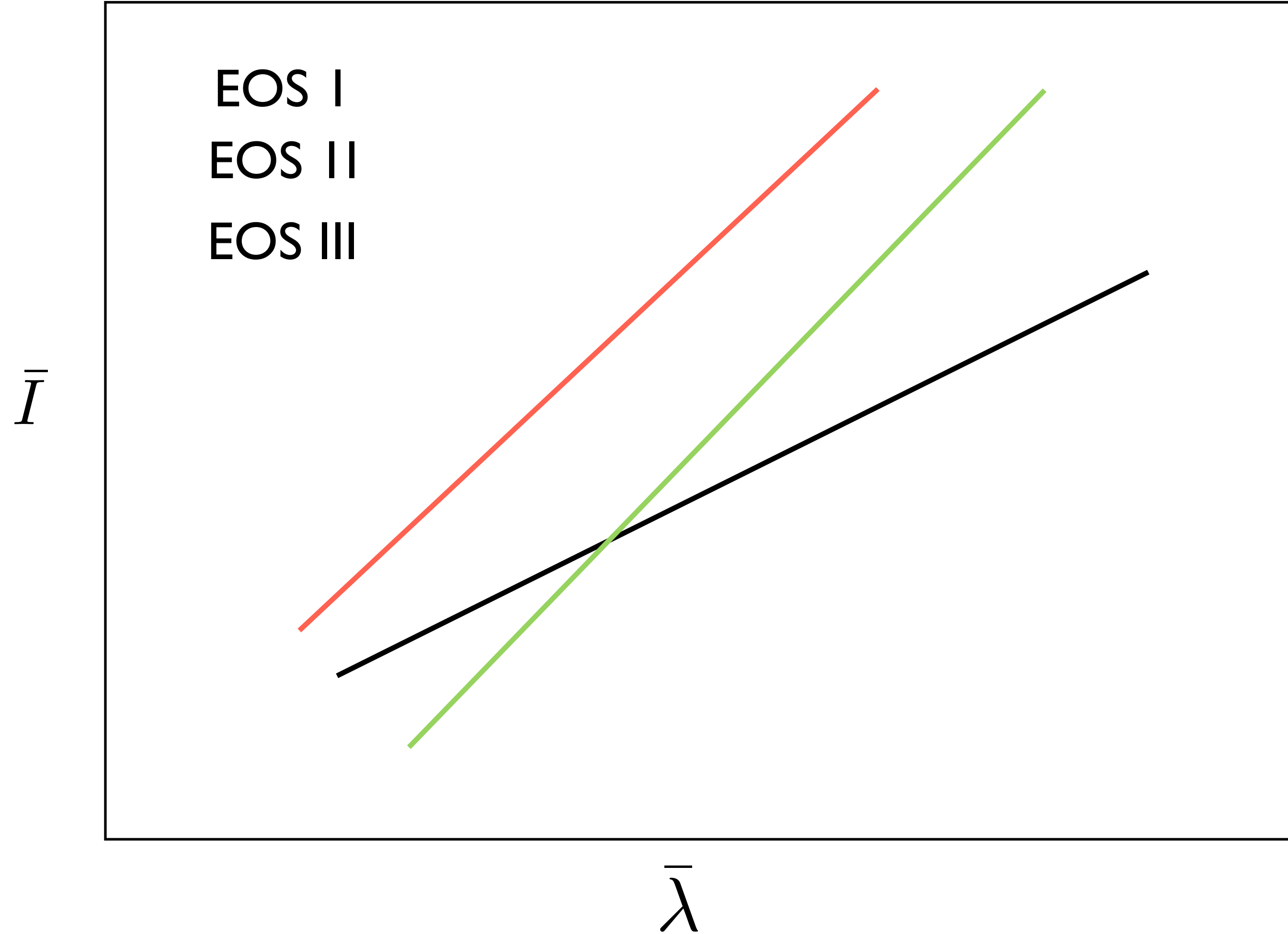


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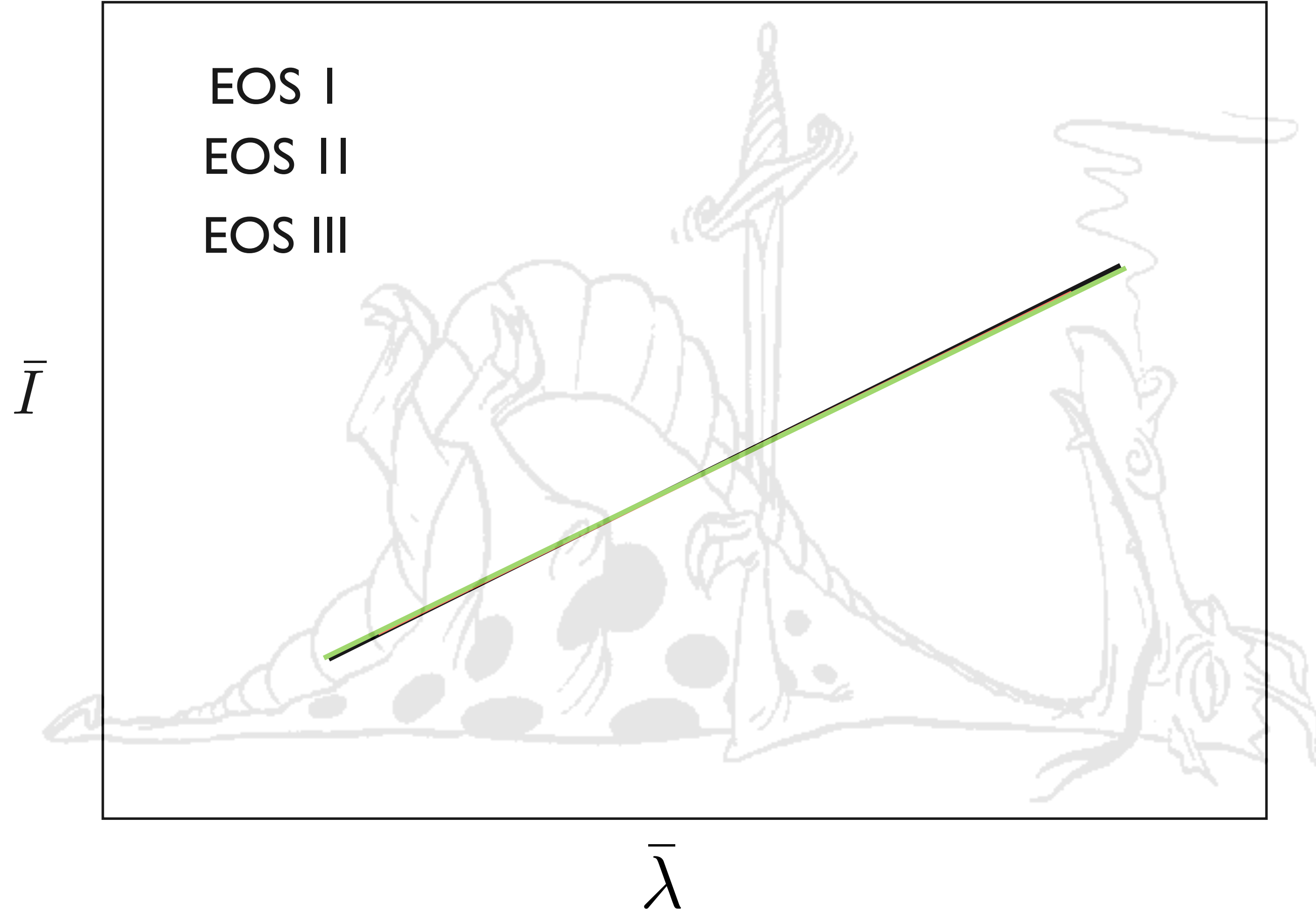
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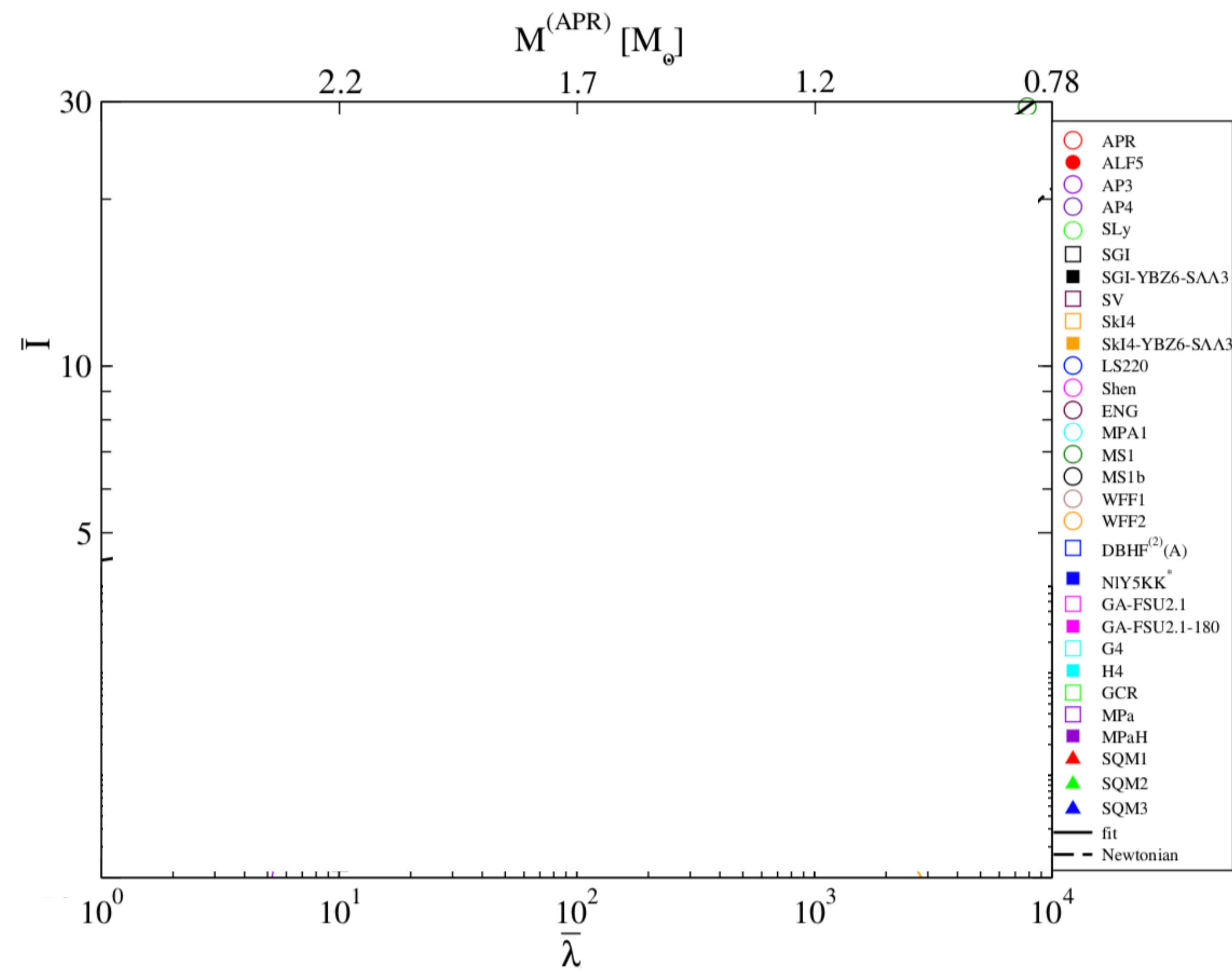


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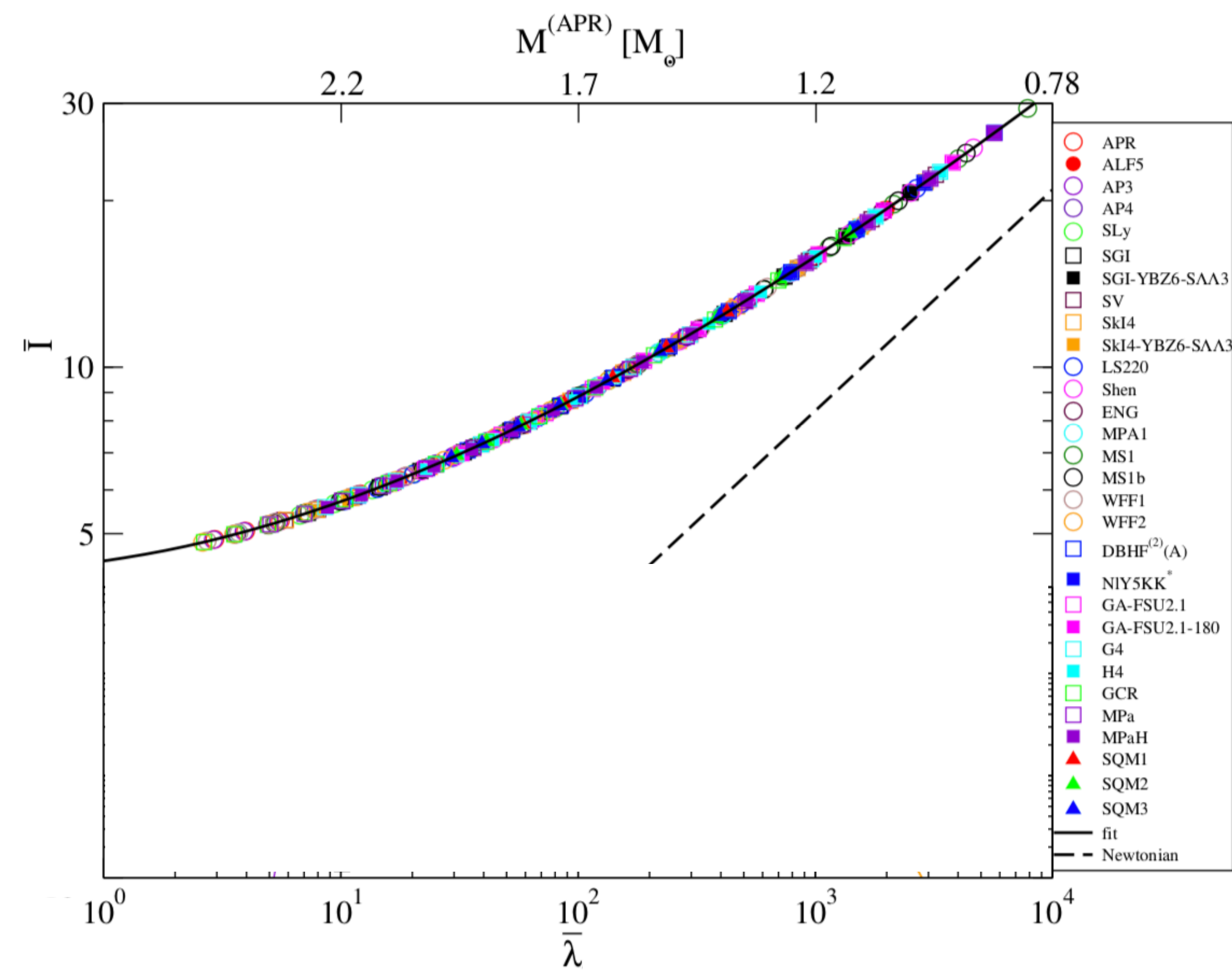


# I-Love-Q relations



[Yagi & Yunes, Science 341 ('13), Yagi & Yunes, PRD 88 ('13)]

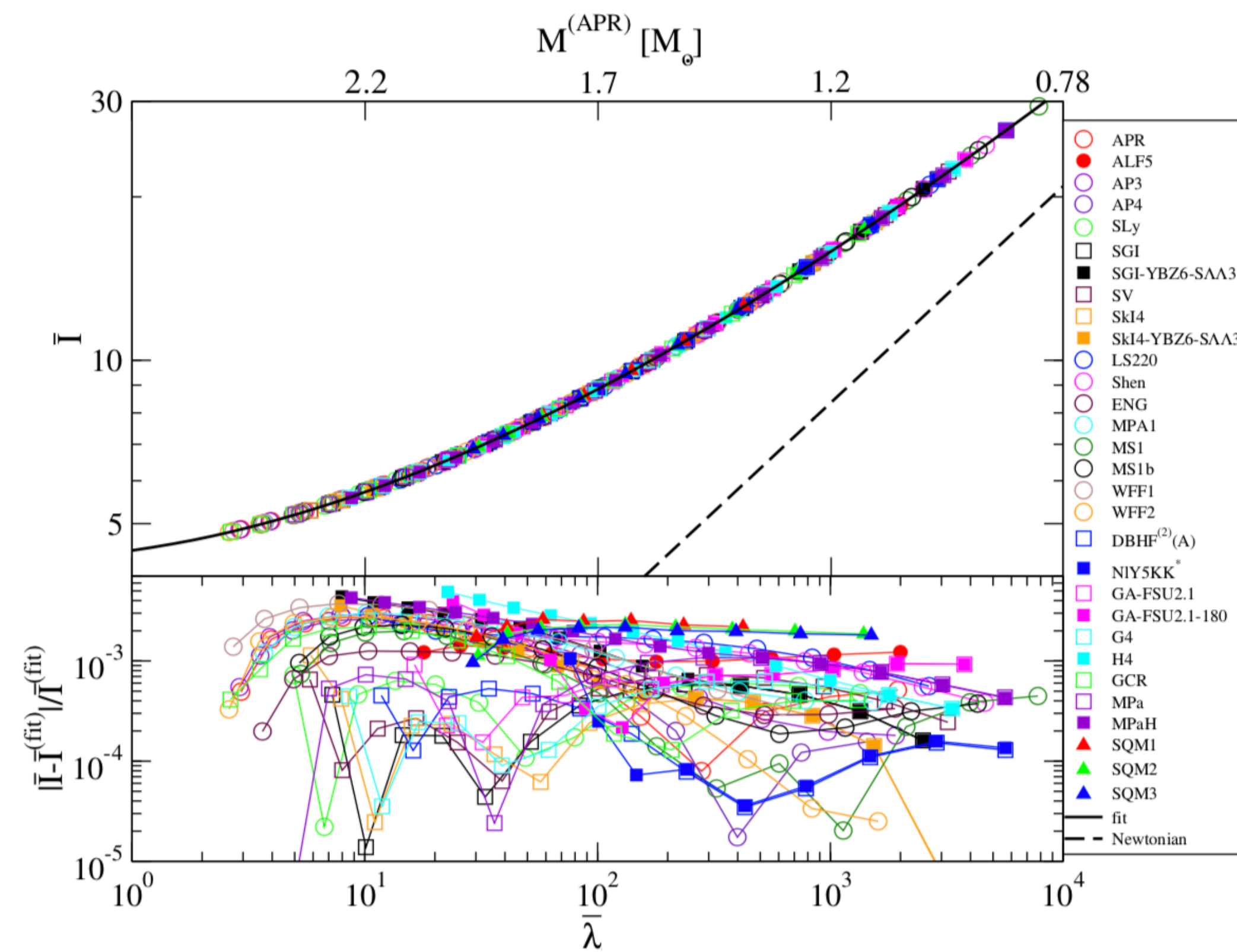
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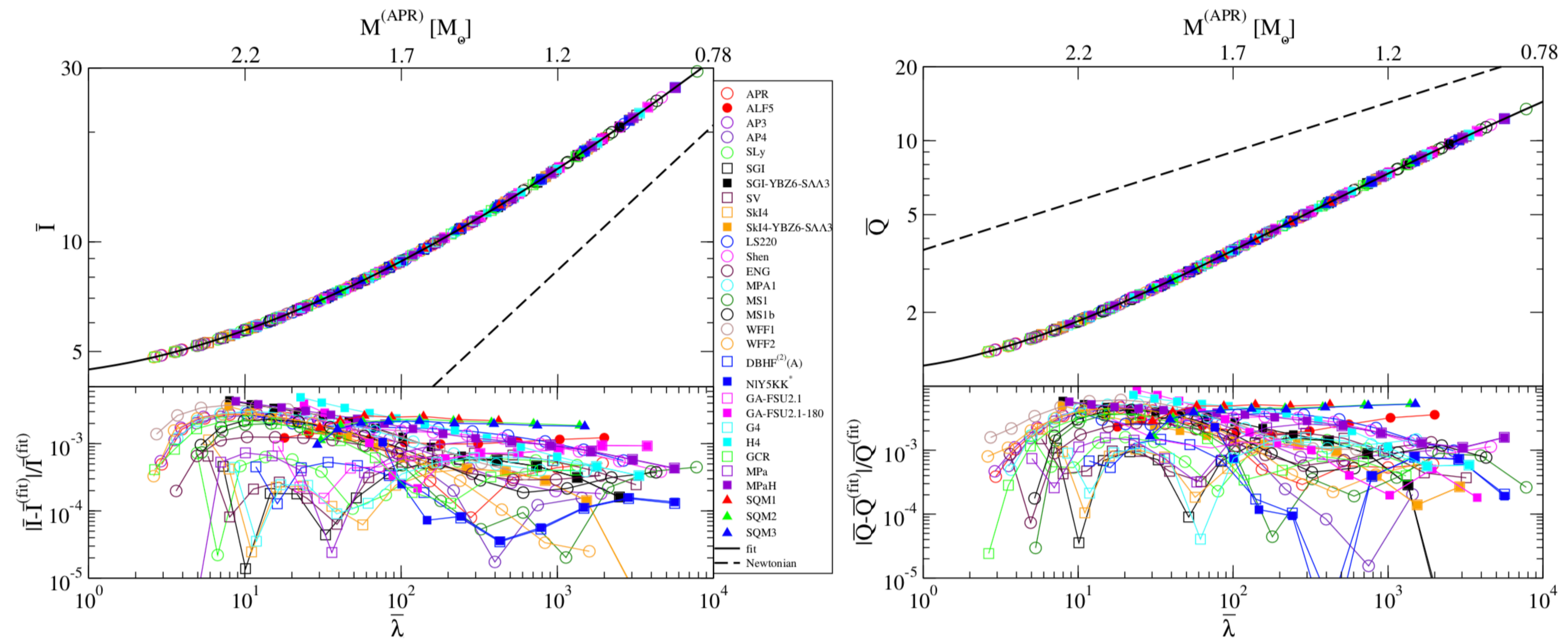


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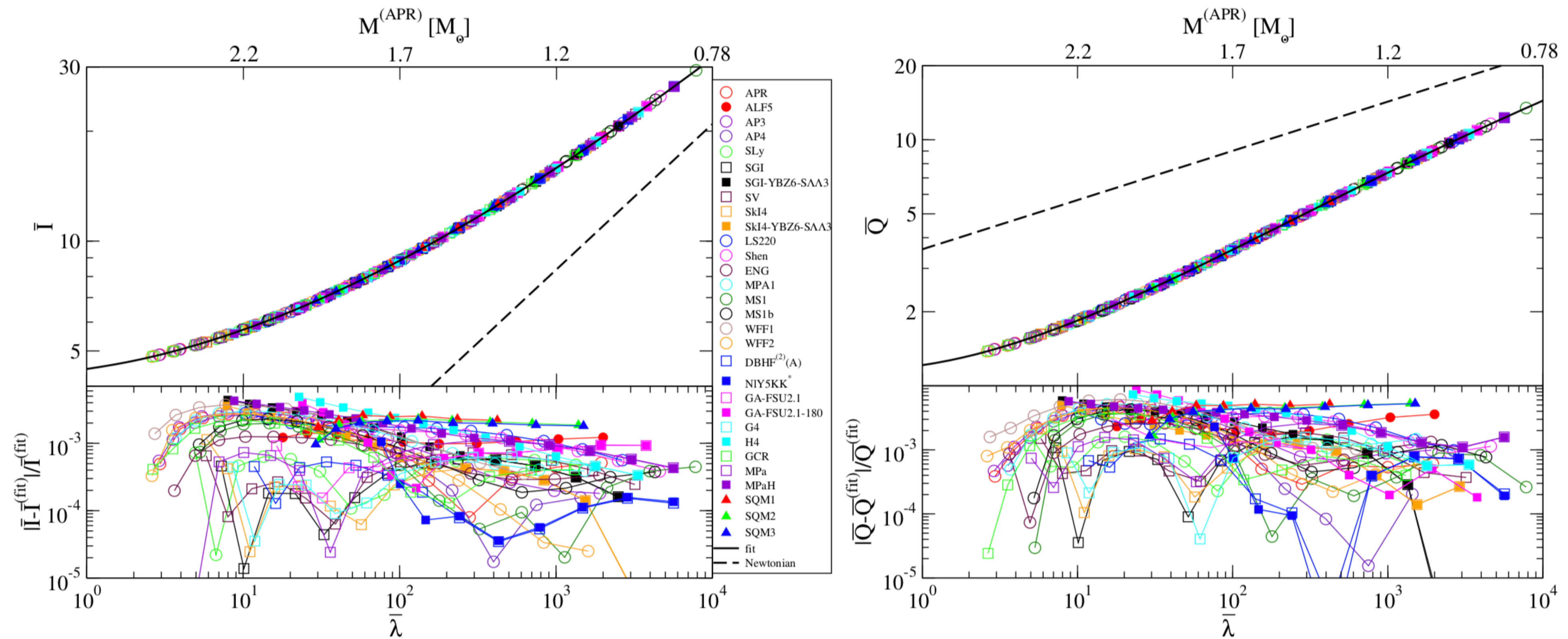
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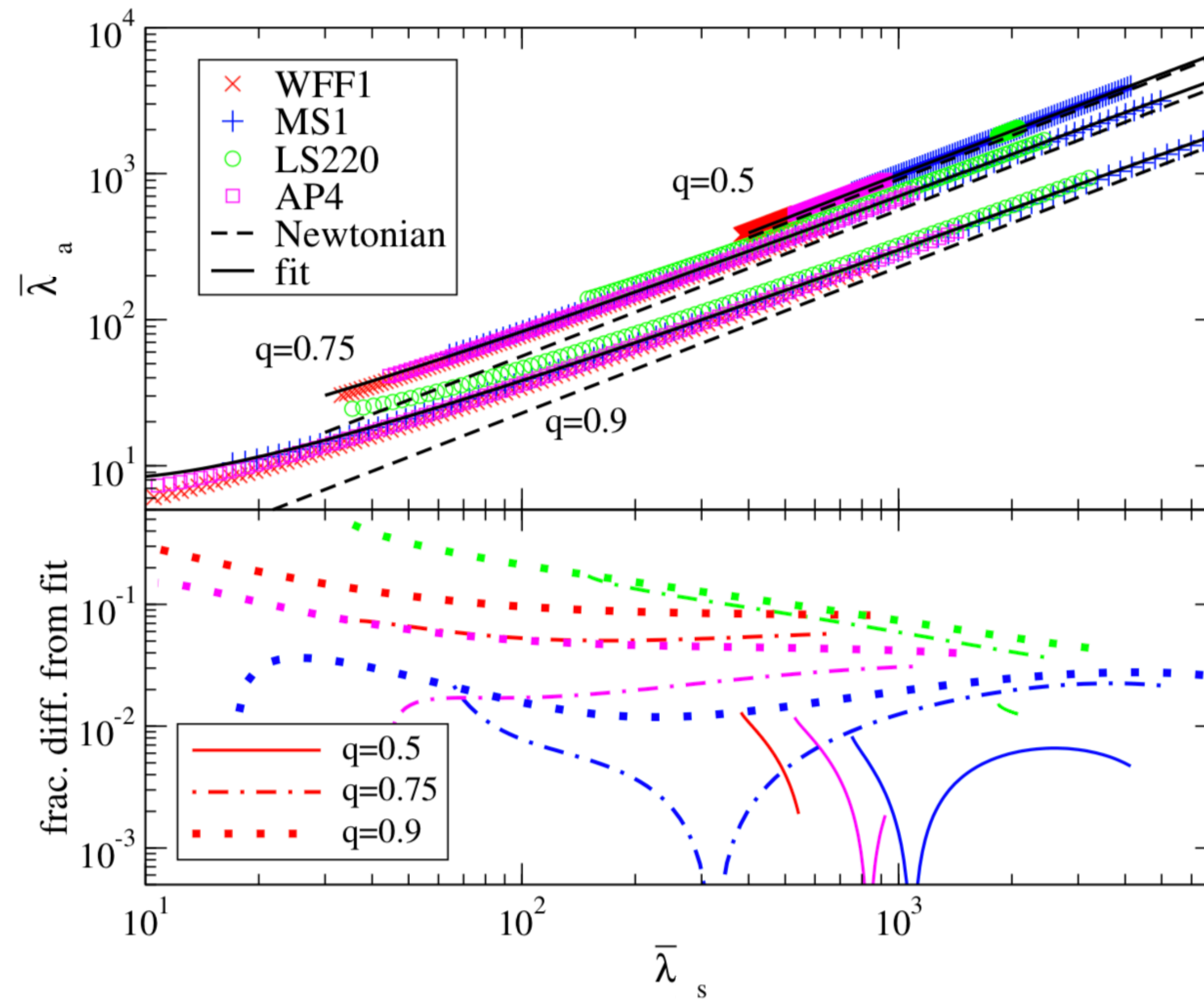
[Yagi & Yunes, Science 341 ('13), Yagi & Yunes, PRD 88 ('13)]

**The moment of inertia, quadrupole moment and Love number satisfy (approx Universal), EoS-insensitive relations!**



# Binary Love relations

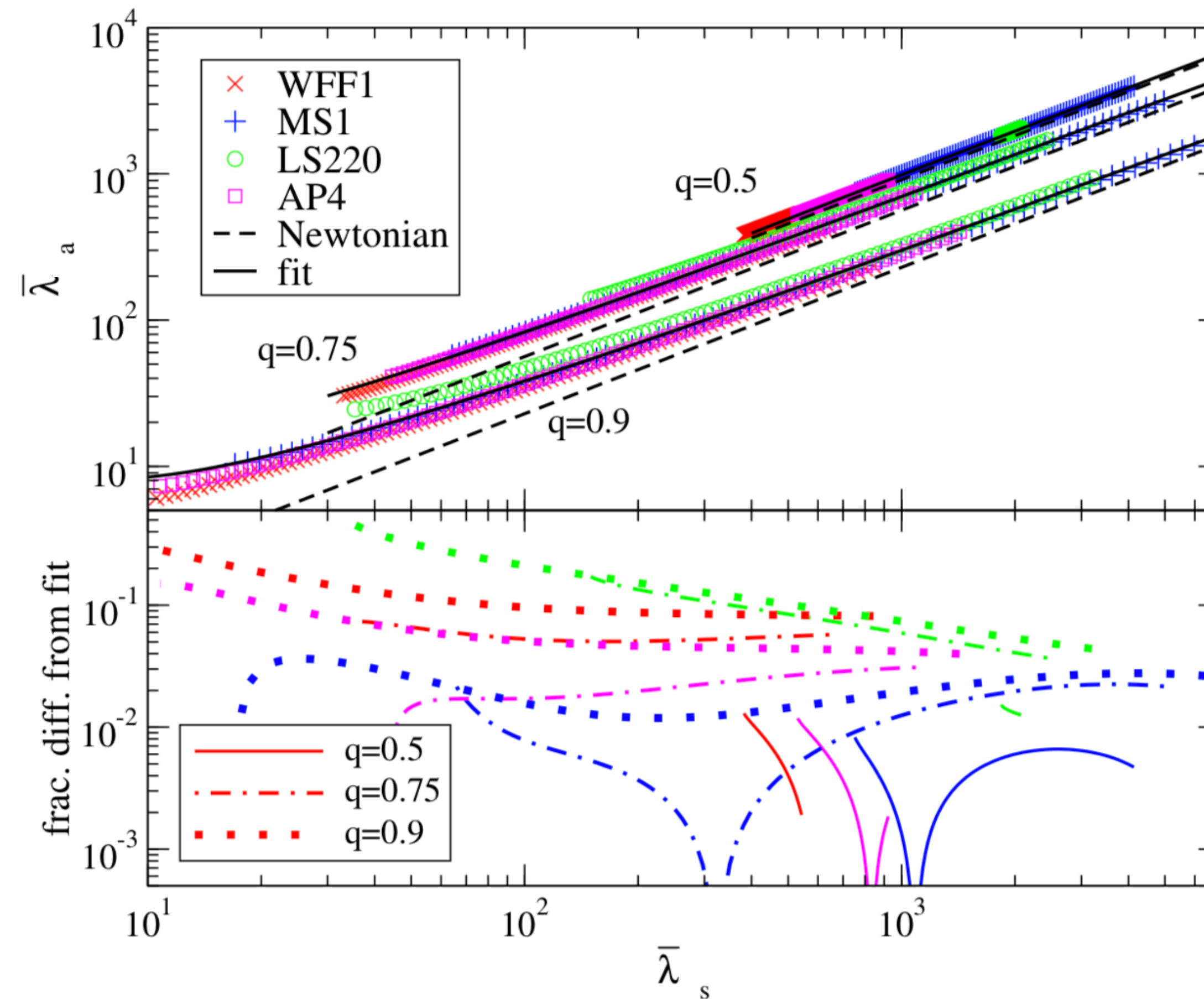
$$\bar{\lambda}_{s,a} = \frac{1}{2} (\bar{\lambda}_1 \pm \bar{\lambda}_2)$$



[Yagi & Yunes, CQG Letters 33 ('16)]

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[Yagi & Yunes, CQG Letters 33 ('16)]

**The tidal Love numbers satisfy (approx Universal), EoS-insensitive relations (that only depend on the mass ratio)!**

# Improvements in extraction of EoS



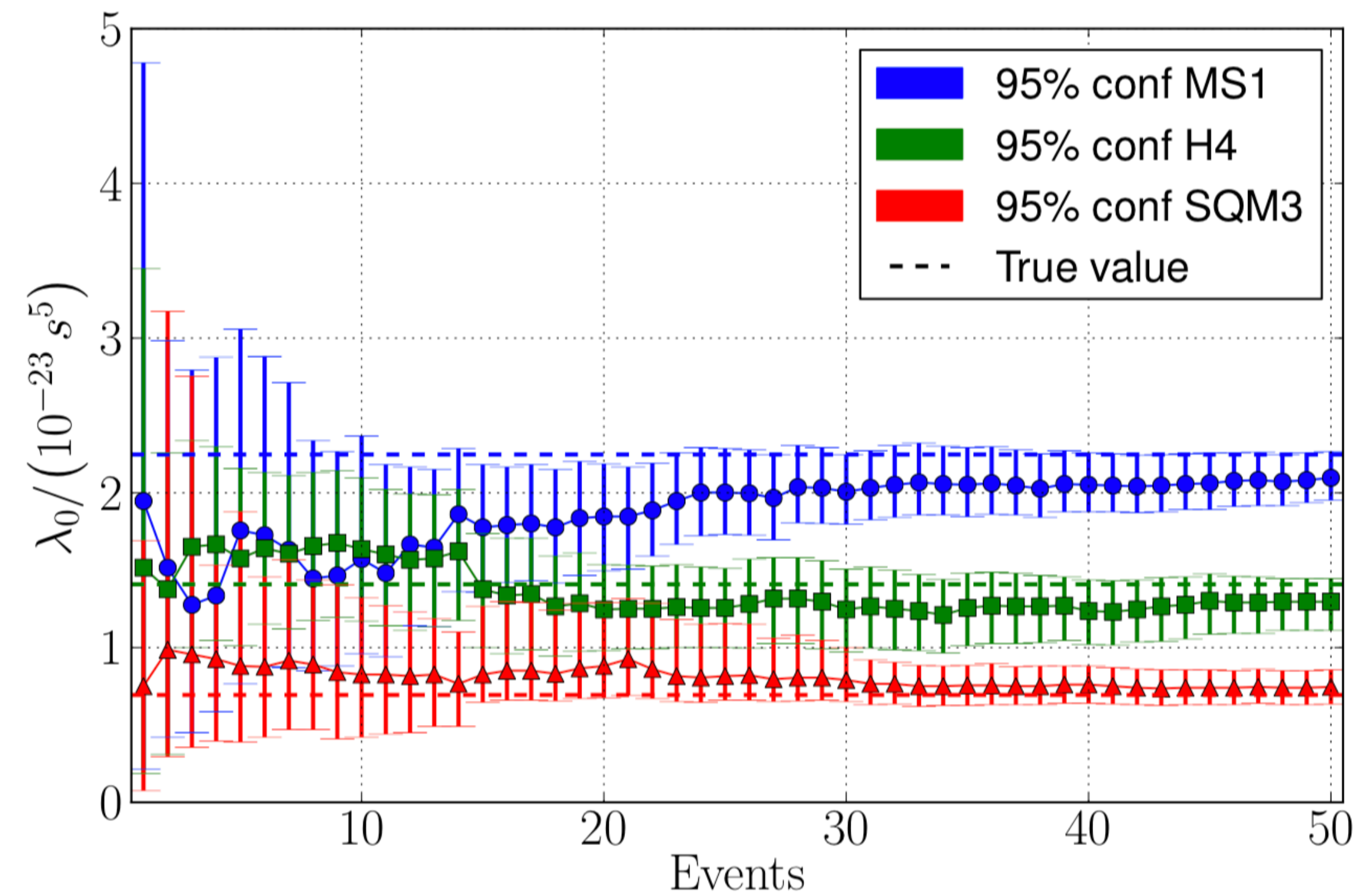
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Via stacking  
(with aLIGO at design sensitivity, 2021-2023)

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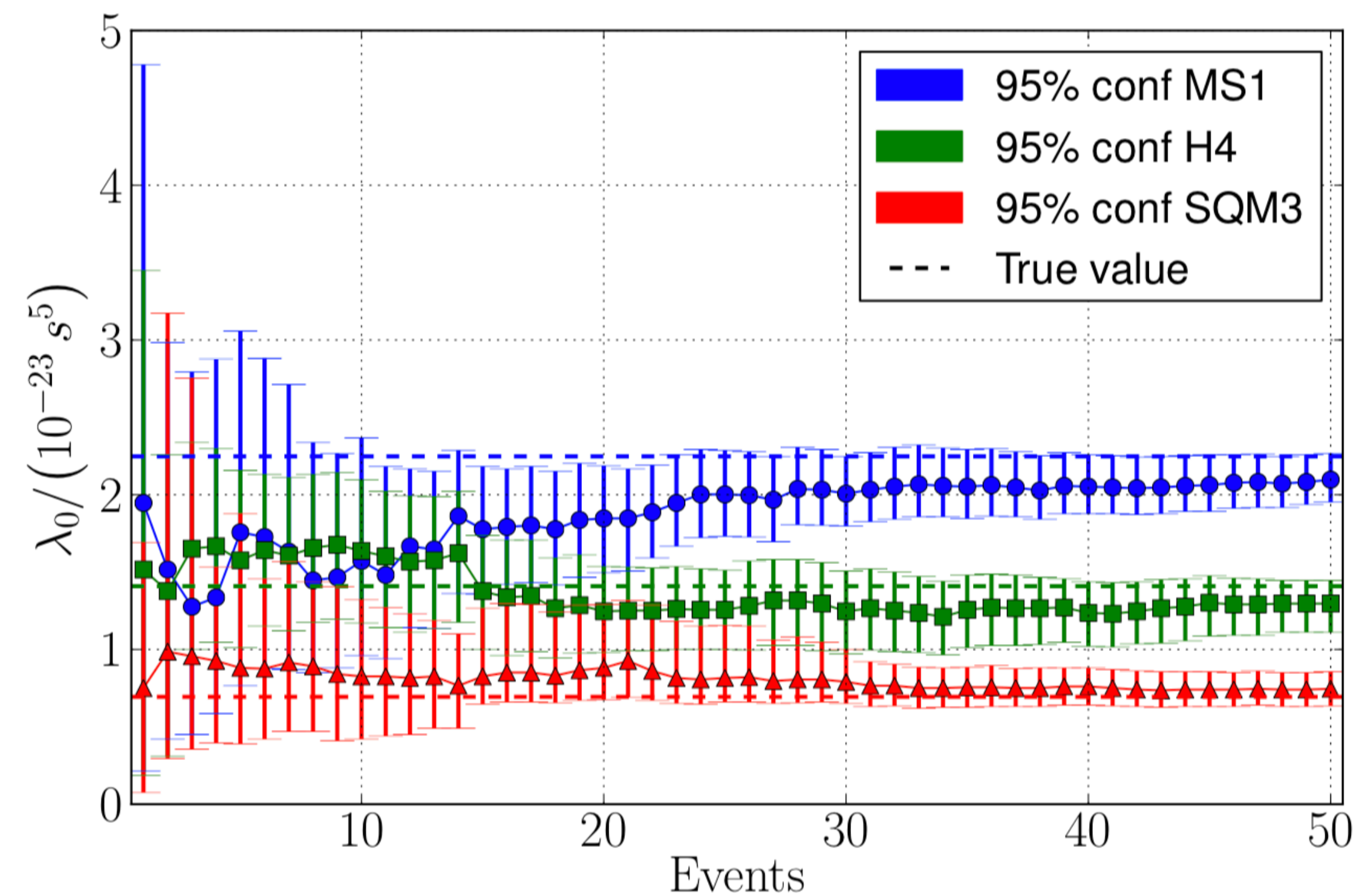
(with aLIGO at design sensitivity, 2021-2023)



[Agathos et al, PRD 92 ('05)]

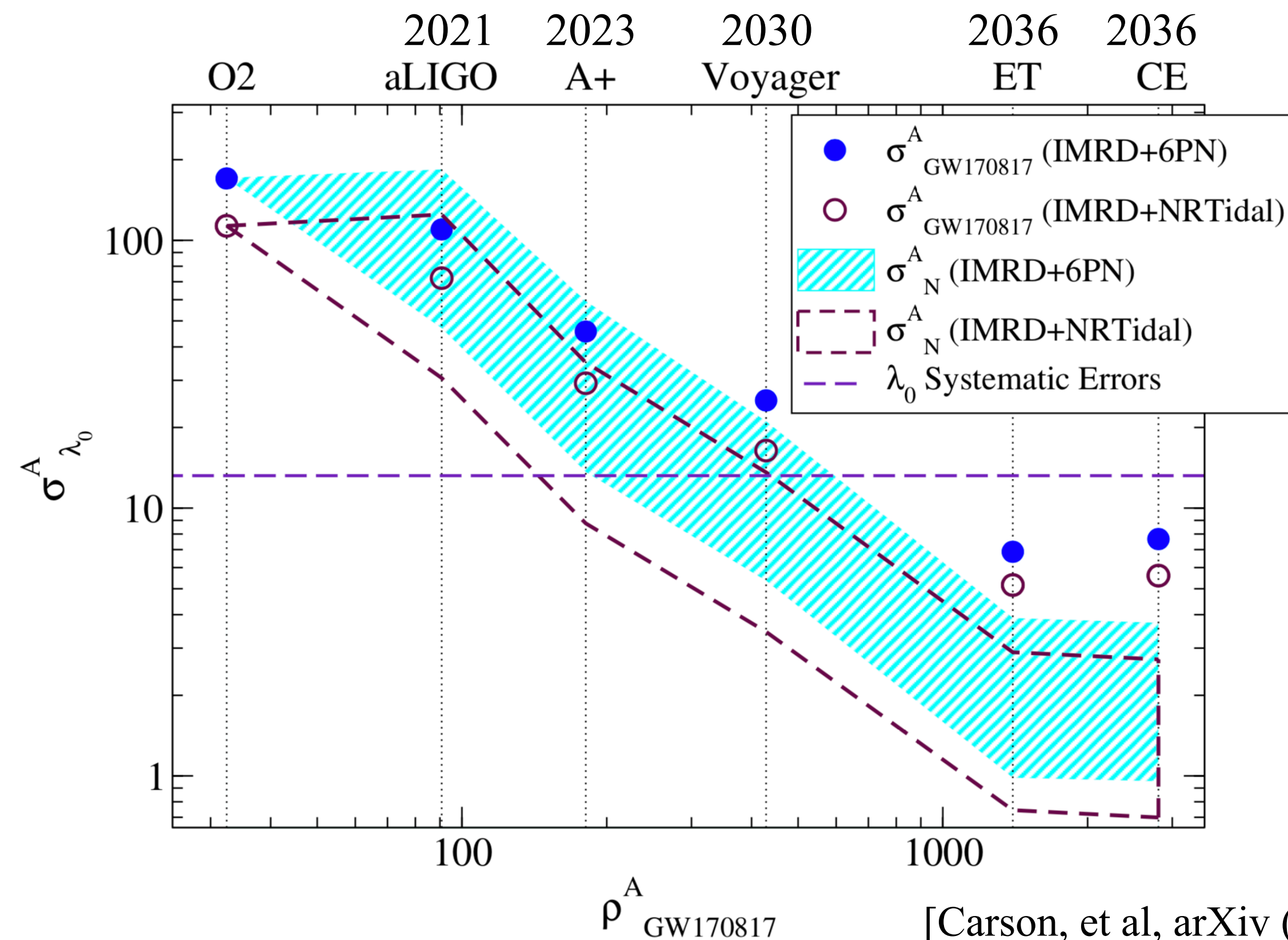
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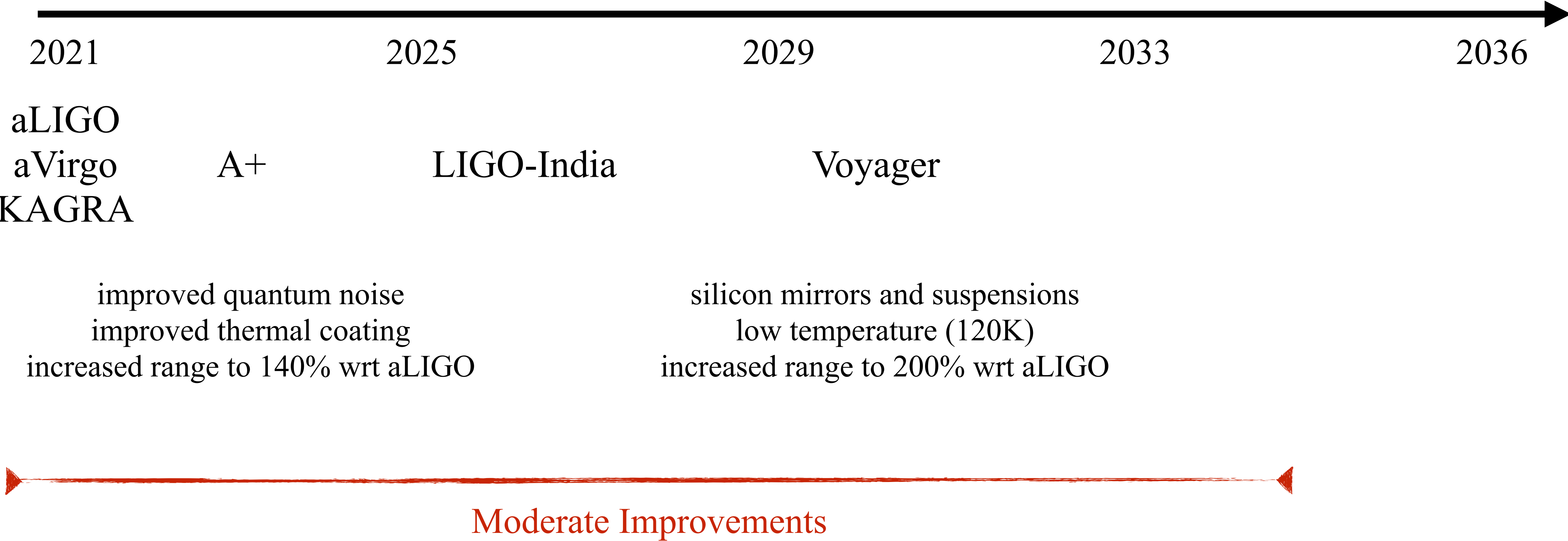
Single and future observations with 3G detectors  
( $\lambda_0=150$ , GW170817)



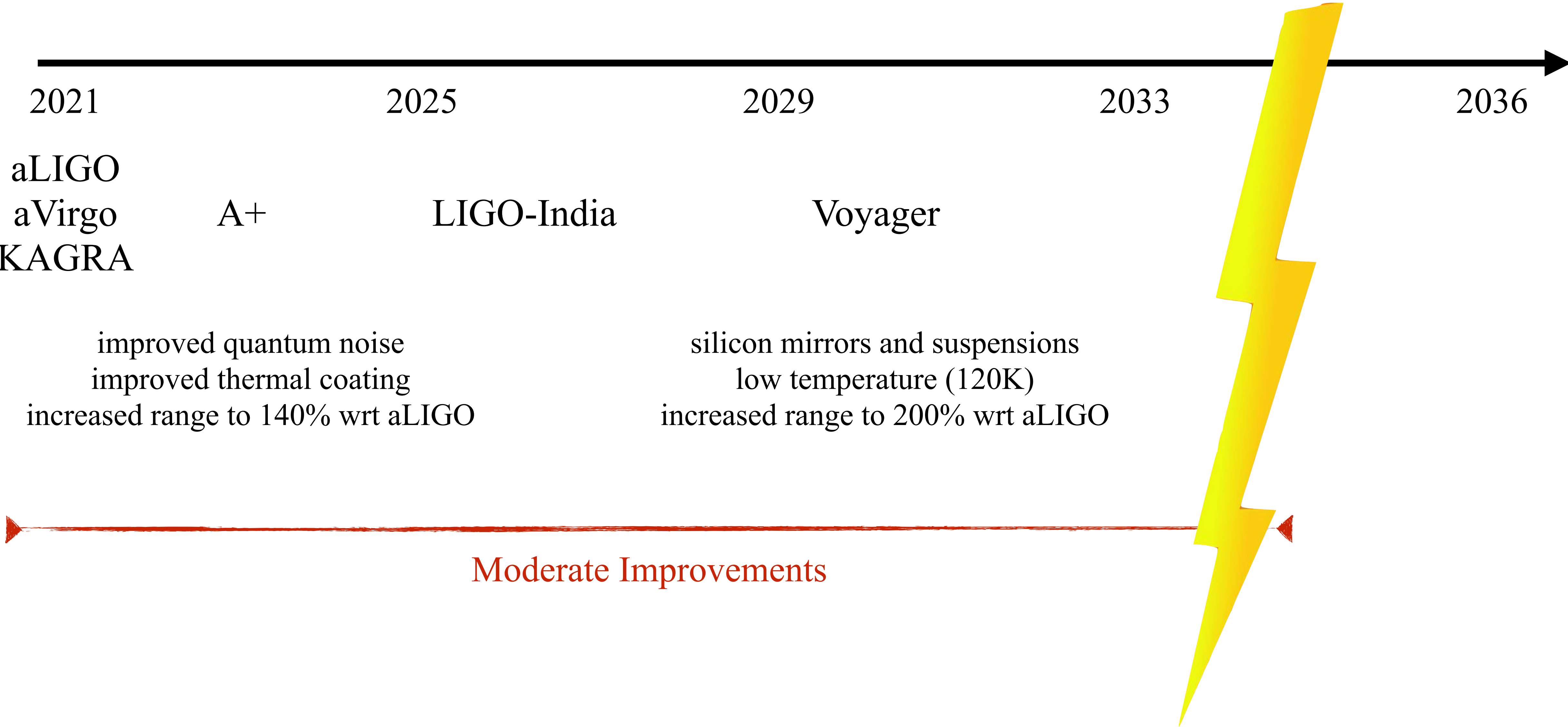
[Carson, et al, arXiv ('19)]



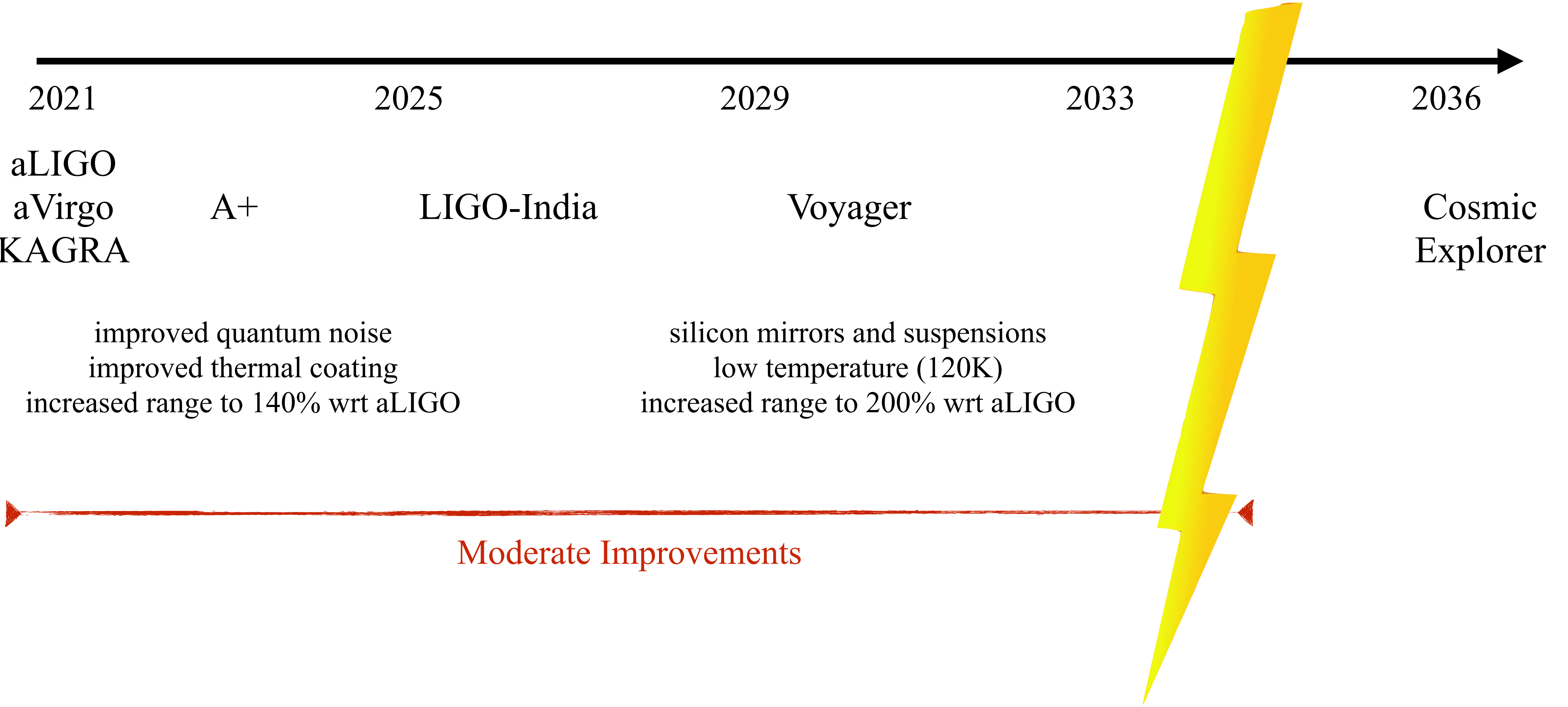
# Beyond 2G detectors



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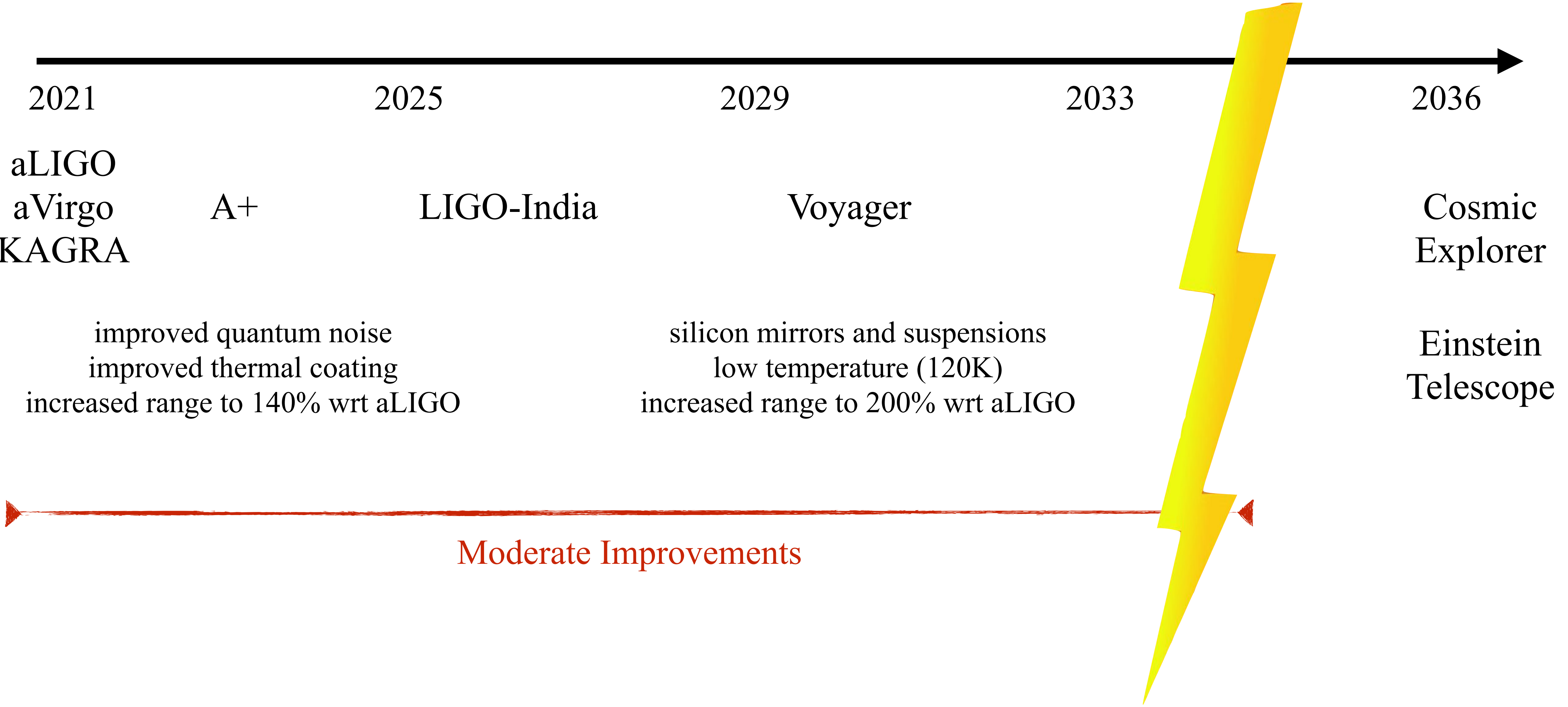


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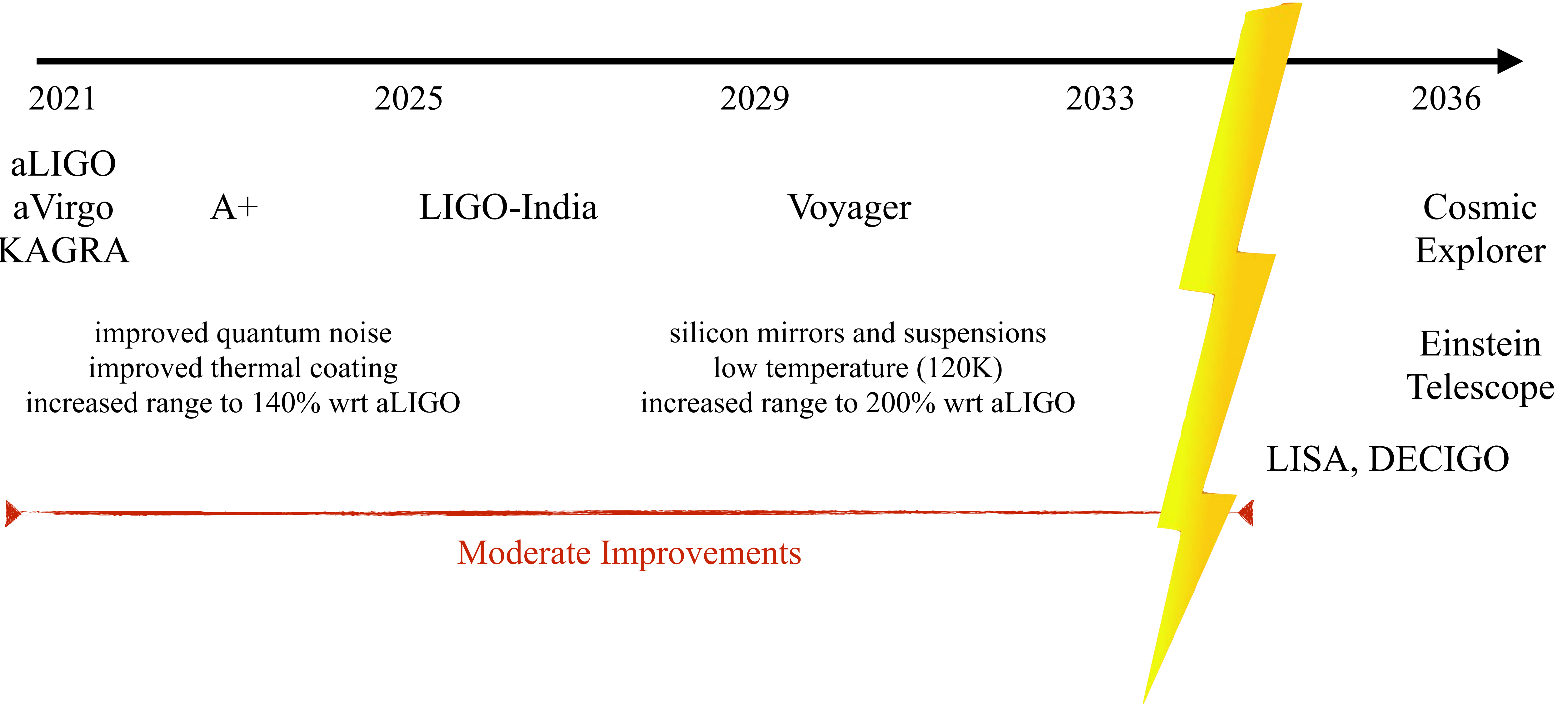




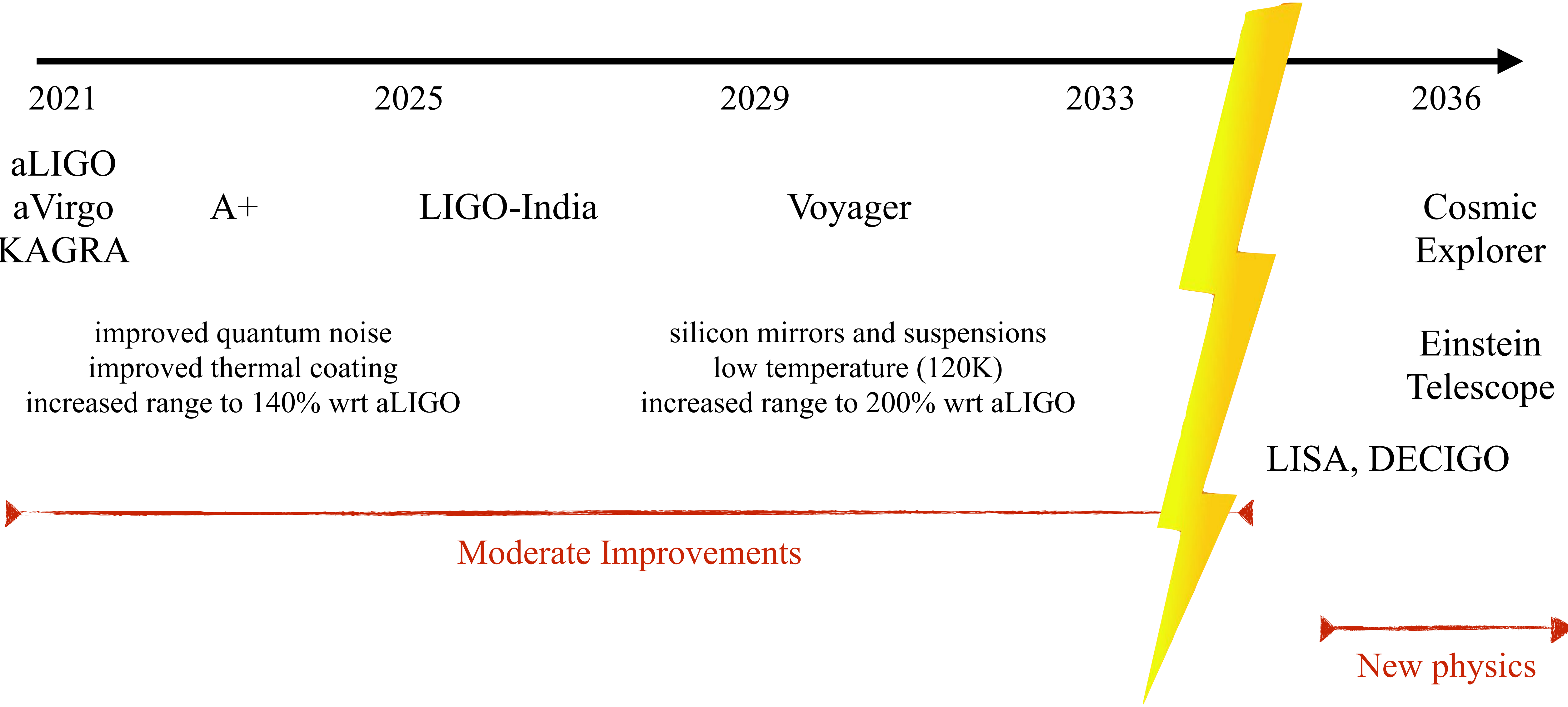
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# 3G ground-based detectors



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