APS GHP Meeting, April 13-16, 2021

Mass radius of the proton

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Based on: DK, arXiv:2102.00110

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BROOKHAVEN NATIONAL LABORATORY

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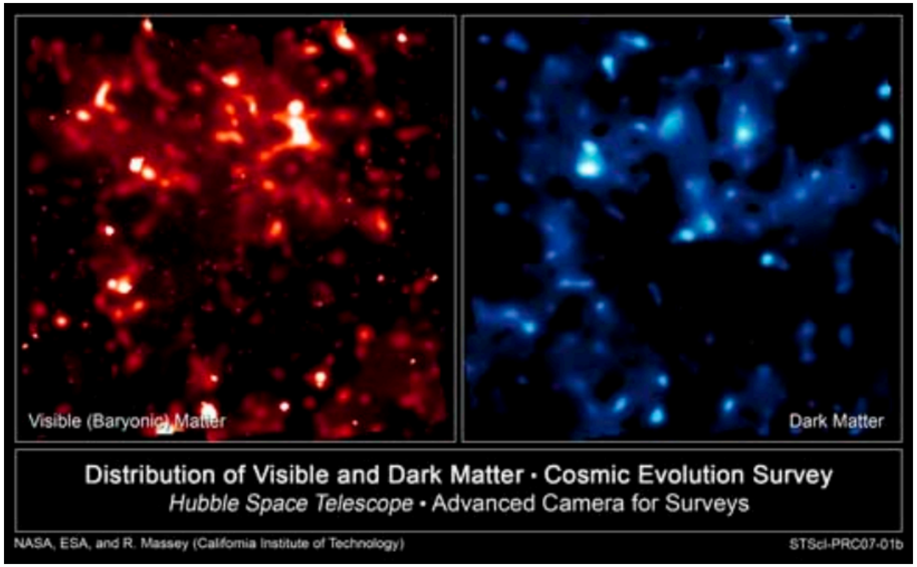
Gravitational formfactors and the mass distribution



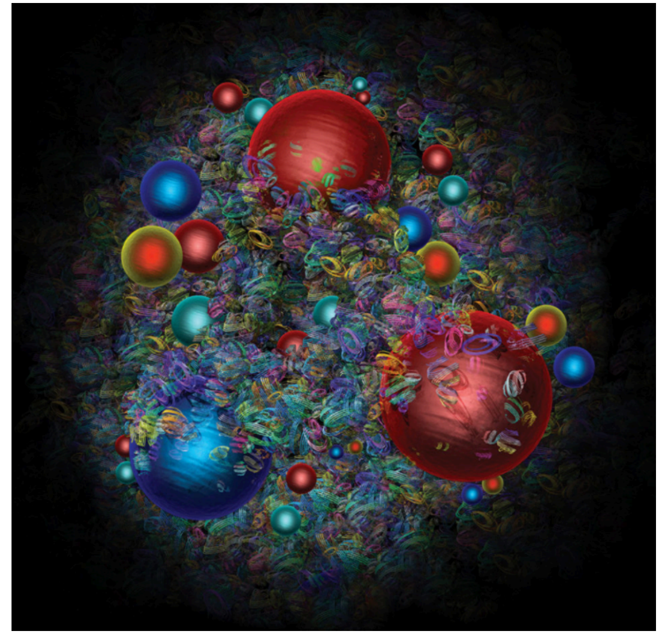


Image: CERN

What is the origin of the proton mass?

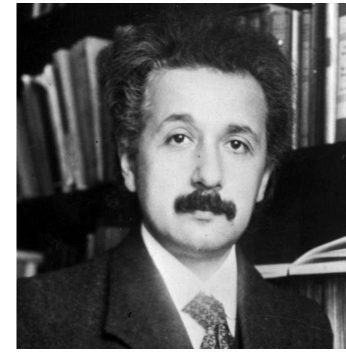
How is the mass distributed inside the proton?

Is it associated with quarks ("visible matter")
or with gluons ("dark matter")?

How can we measure the mass distribution?

The mass distribution in General Relativity


Gunnar Nordstrom
1881-1923


Albert Einstein
1879-1955

Consider Einstein gravity:

Ricci curvature tensor $\rightarrow$ $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \, T_{\mu\nu}$$

Take the trace with metric tensor:

$$-R = 8\pi G \, T \qquad T \equiv T^{\mu}_{\mu}$$

Cf Nordstrom 1912;
Einstein 1913; Einstein-Fokker 1914

Non-relativistic, weak gravitational field limit:

$$g_{00} = 1 + 2\varphi, \qquad T^{\nu}_{\mu} = \mu \; u_{\mu} u^{\nu}, \qquad u_0 = u^0 = 1, \quad u_i = 0.$$

Therefore, in this limit, the distributions of mass and of T coincide:

$$T^{0}_{0} = \mu; \qquad T \equiv T^{\mu}_{\mu} = T^{0}_{0} = \mu$$

[45] Einstein, Albert and Fokker, Adriann, D., "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls", *Annelen der Physik* 44, 1914, pp. 321-328; p. 321.

The mass distribution in General Relativity

Newtonian limit:

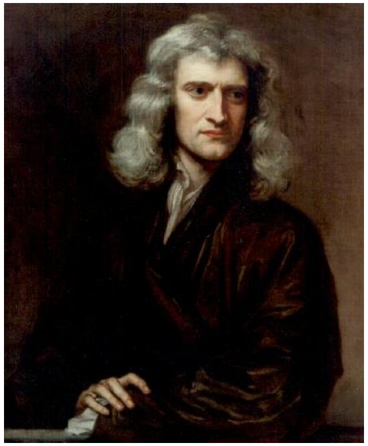
$$R_0^0 = \frac{\partial^2 \varphi}{\partial x^{\mu 2}} \equiv \Delta\varphi,$$

Einstein equation:

$$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T);$$

The only non-vanishing component:

$$R_0^0 = 4\pi G\mu,$$



Isaac Newton
1643-1727

Therefore, the distribution of mass determines the gravitational potential:

$$\Delta\varphi = 4\pi G\mu. \quad \Rightarrow \quad \varphi = -G \int \frac{\mu \, dV}{R} \quad \Rightarrow \quad F_g = -m \, \partial\varphi/\partial R$$

$$M = \int \mu dV$$

$$\boxed{F_g = -G \, \frac{mM}{R^2}.}$$

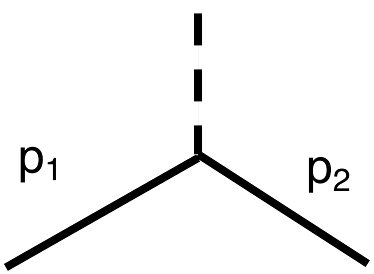
Gravitational formfactors and the mass distribution

The mass distribution is encoded in the gravitational formfactors.

For the spin ½ nucleon, 3 formfactors appear:

H. Pagels '66,
A. Pais, S. Epstein '49

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\; p_{02}} \right)^{1/2} \frac{1}{4M}\, \bar{u}(p_1, s_1) \Big[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + $$
$$ + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$



Energy-momentum conservation:

$$\partial^\mu T_{\mu\nu} = 0$$

$$\Rightarrow \quad q^\mu \langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = 0;$$

Satisfied for on-shell nucleons (use Dirac equation)

$$\textstyle\sum_s \bar{u}(p, s) u(p, s) = (\hat{p} + M)/2M$$

$$p_1^2 = p_2^2 = M^2$$

Gravitational formfactors and the mass distribution

For the spin ½ nucleon, 3 formfactors appear: (no G_1 for spin 0)

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\; p_{02}} \right)^{1/2} \frac{1}{4M}\, \bar{u}(p_1, s_1) \Big[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + \\ + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$

Zero momentum transfer $q \to 0$

$$\langle \mathbf{p} | T_{\mu\nu} | \mathbf{p} \rangle = \left(\frac{M^2}{p_0^2} \right)^{1/2} \bar{u}(p, s) u(p, s)\ \frac{p_\mu p_\nu}{M^2} \left[G_1(0) + G_2(0) \right]$$

(no “stress” G_3)

the Hamiltonian

$$H = \int d^3x\, T_{00}(x)$$

In the rest frame of the nucleon:

$$\langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M$$

$$\Downarrow$$

$$G_1(0) + G_2(0) = M.$$

Formfactor of the trace of the energy-momentum tensor

Let us call it "scalar gravitational formfactor", as it would be a gravitational formfactor in the scalar model of gravity:

Nordstrom 1912
Einstein 1913

$$T \equiv T_\mu^\mu$$

$$\langle \mathbf{p}_1 | T | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\; p_{02}} \right)^{1/2} \bar{u}(p_1, s_1) u(p_2, s_2)\; G(q^2),$$

Scalar gravitational formfactor:

$$G(q^2) = G_1(q^2) + G_2(q^2) \left(1 - \frac{q^2}{4M^2} \right) + G_3(q^2) \frac{3q^2}{4M^2}$$

In the rest frame of the nucleon:

$$\langle \mathbf{p} = 0 | T | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M,$$

$$\Downarrow$$

$$G(0) = M$$

How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$,

the formfactor of T_{00} and the scalar gravitational formfactor coincide if

$$\frac{G_i(0)}{4M} \ll \frac{dG_i}{dt}\Big|_{t=0} \equiv G_i(0)/m_i^2$$

The origin of the difference is frame dependence of T_{00} :

In Breit frame, $\mathbf{p}_2 = \frac{1}{2}\mathbf{q},\ \mathbf{p}_1 = -\frac{1}{2}\mathbf{q}$ the proton is moving with

$$\gamma = E/M = \sqrt{M^2 + (q^2/4)}/M = \sqrt{1 + q^2/(4M^2)},$$

so for $q \equiv |\mathbf{q}| \simeq m_i$ it is Lorentz-contracted with

$$1/\gamma \simeq (1 + m_i^2/(4M^2))^{-1/2}$$

For massive bodies, $m_i << 2M$ – size much larger than the Compton wavelength! In this limit, the formfactors of T_{00} and T coincide.

[the proton: $8M^2 \gg m_s^2$]

See R.L. Jaffe, PRD103(2021) for related discussion

How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$, $\frac{G_i(0)}{4M} \ll \left.\frac{dG_i}{dt}\right|_{t=0}$

the formfactor of T_{00} and the scalar gravitational formfactor coincide, thus the scalar gravitational formfactor can be used to define the **mass radius of the proton**:

$$\langle R_{\mathrm{C}}^2 \rangle = 6 \left.\frac{dG_{\mathrm{EM}}}{dt}\right|_{t=0}. \Rightarrow \boxed{\langle R_{\mathrm{M}}^2 \rangle = \frac{6}{M} \left.\frac{dG}{dt}\right|_{t=0},}$$

In the relativistic region (mass -> energy), it is natural to consider the scalar gravitational formfactor, as T is the Lorentz scalar

The trace of the energy-momentum tensor also plays a special role – it is a generator of dilatations. Its formfactor thus carries information about the Renormalization Group (RG) evolution inside the nucleon.

Scale invariance

Scale transformations (dilatations) are defined by

$$x \to e^{\lambda}\ x$$

the corresponding dilatational current is

$$s^{\mu} = x_{\nu} T^{\mu\nu}$$

Hermann Weyl (1885-1955)

It is conserved (a theory is scale-invariant) if the energy-momentum is traceless:

$$\partial_{\mu} s^{\mu} = T^{\mu}_{\mu} \equiv T$$

Scale invariance

A scale-invariant theory cannot contain massive particles, all particles must be massless

For example, in Maxwell electrodynamics with action

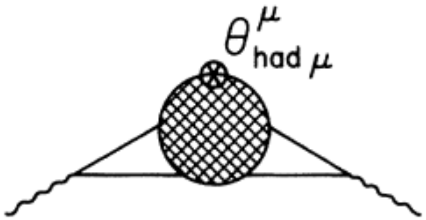
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

the energy-momentum is traceless: (massless photons)

$$T^{\mu}_{\mu} = 0$$

Note: because of this, in scalar gravity (Nordstrom, 1912; Einstein, 1913) there would be no light bending by massive bodies!

Scale anomaly in QCD



The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor:

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g}\, G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h$$

Running coupling -> dimensional transmutation -> mass scale

Gross, Wilczek; Politzer

$$\beta(g) = -b\frac{g^3}{16\pi^2} + \ldots,\ \ b = 9 - \frac{2}{3} n_h,$$

Ellis, Chanowitz; Crewther; Collins, Duncan, Joglecar; …

At small momentum transfer, heavy quarks decouple:

$$\sum_h m_h \bar{Q}_h Q_h \to -\frac{2}{3}\, n_h\, \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \ldots$$

so only light quarks enter the final expression

Shifman, Vainshtein Zakharov '78

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g}\, G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l,$$

The proton mass

At zero momentum transfer, the matrix element of the trace of the energy-momentum tensor defines the mass of the proton:

$$\langle \mathbf{p}=0|T|\mathbf{p}=0\rangle = \langle \mathbf{p}=0|T_{00}|\mathbf{p}=0\rangle = M,$$

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g}\, G^{\mu\nu a}G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l(1+\gamma_{m_l})\bar{q}_l q_l,$$

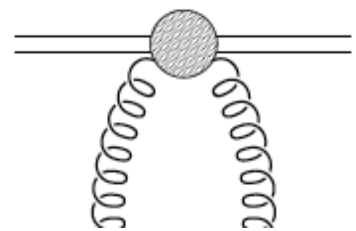
In the chiral limit, the only contribution is from gluons!

How to measure the mass distribution inside the proton?

No dilatons available…
next best thing: a heavy quarkonium

QCD multipole expansion:

Voloshin ‘78; Appelquist, Fischler ‘78; Gottfried ‘78; Peskin ‘79; Novikov, Shifman ‘81; Leutwyler ‘81, …



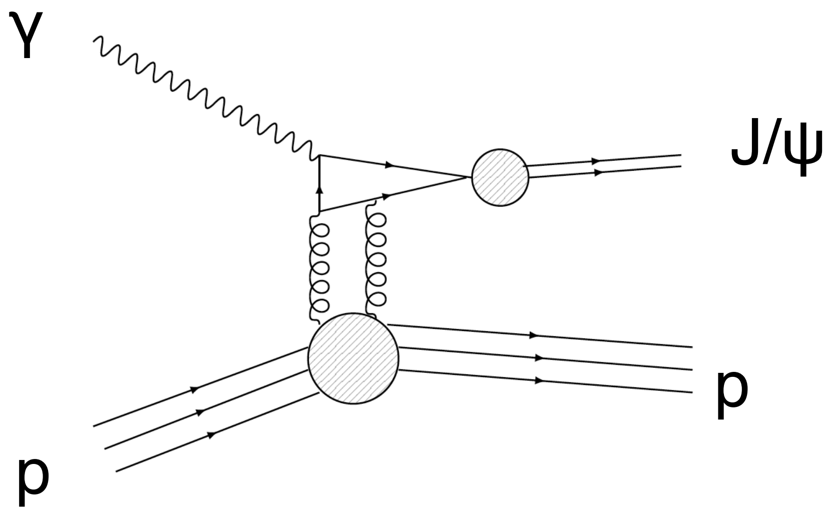
M.B. Voloshin
1953-2020

$$g^2\mathbf{E}^{a2} = \frac{g^2}{2}(\mathbf{E}^{a2} - \mathbf{B}^{a2}) + \frac{g^2}{2}(\mathbf{E}^{a2} + \mathbf{B}^{a2})$$

$$= -\frac{1}{4}g^2 G^a_{\alpha\beta}G^{a\alpha\beta} + g^2(-G^a_{0\alpha}G^{a\alpha}_0 + \frac{1}{4}g_{00}G^a_{\alpha\beta}G^{a\alpha\beta}) = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

$$\theta^\mu_\mu \equiv \frac{\beta(g)}{2g}G^{a\alpha\beta}G^a_{\alpha\beta} = -\frac{bg^2}{32\pi^2}G^{a\alpha\beta}G^a_{\alpha\beta}\ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha}G^{a\alpha}_{\nu} + \frac{1}{4}g_{\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta}$$

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



Near threshold, dominance of

$$g^2\mathbf{E}^{a2} = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

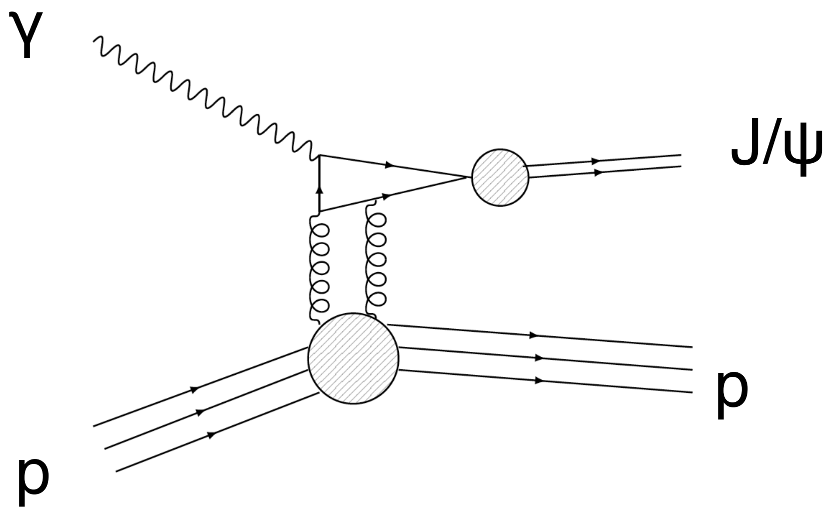
Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK '96; DK, Satz, Syamtomov, Zinovjev '99

Other approaches to threshold photoproduction:

Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19
Recent: Ji, 2102.07830; Gao, Ji, Liu, 2103.11506; Sun, Tong, Yuan, 2103.12047…

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



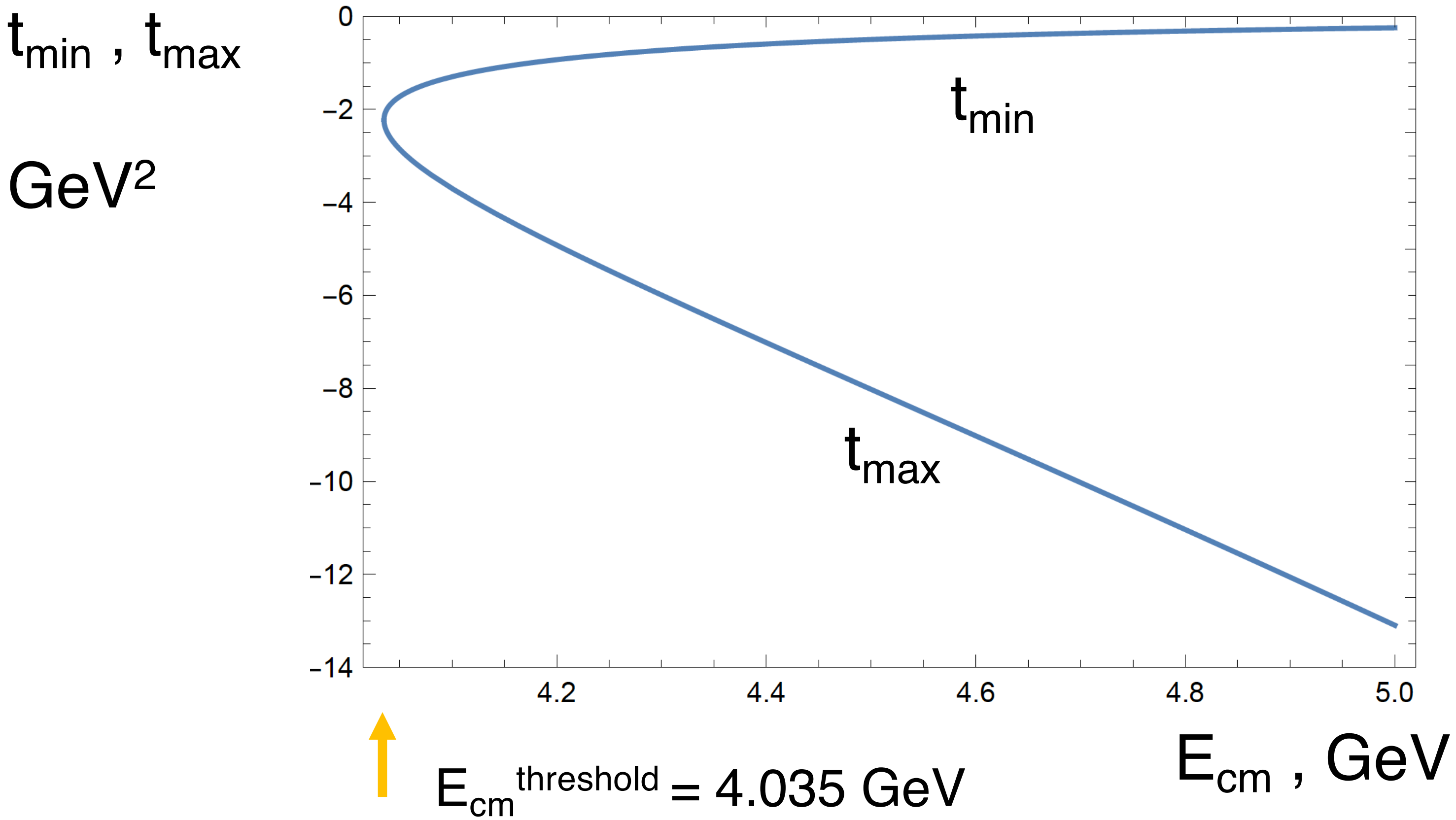
Large minimum momentum transfer at threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23\ \mathrm{GeV}^2 \simeq -(1.5\ \mathrm{GeV})^2$$

–> VDM questionable.

but, scanning the energy range near the threshold,
we measure the scalar gravitational formfactor –
can extract the proton mass distribution!

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

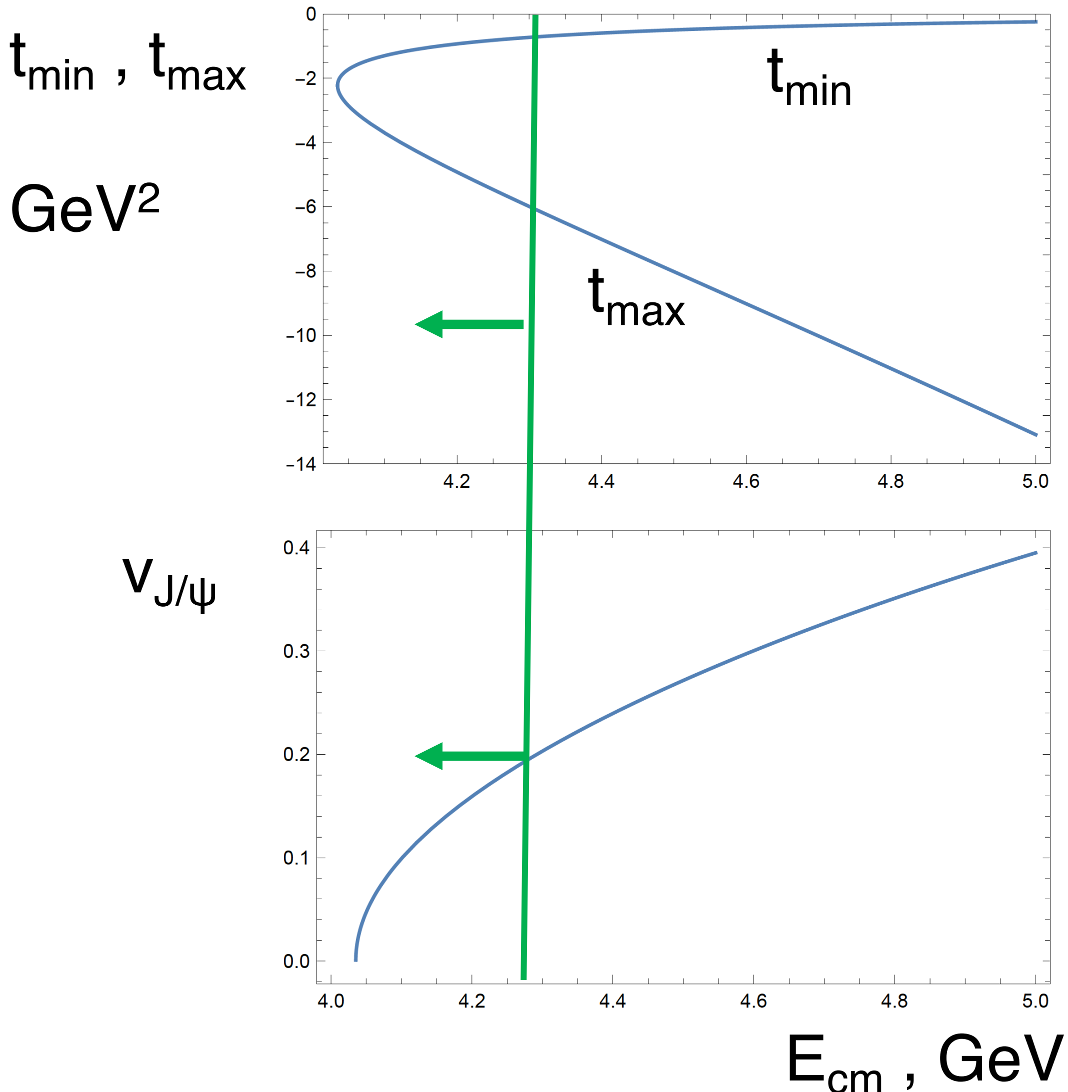


Can measure scalar gravitational formfactor in a broad kinematical domain!

At threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23\ \mathrm{GeV}^2 \simeq -(1.5\ \mathrm{GeV})^2$$

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



The scalar operator dominates for small velocity of heavy quarkonium;

Limiting $v_{J/\psi} < 0.2$, (corrections ~ $v_{J/\psi}^2$) the optimal kinematical region is:

$E_{cm} < 4.25$ GeV
$E_\gamma < 9.2$ GeV

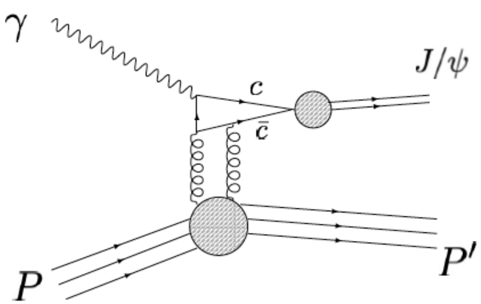
$-t < 6$ GeV^2

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

The amplitude:

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ 2M\ \langle P'|g^2\mathbf{E}^{a2}|P\rangle,$$

$$Qe = 2e/3$$



$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ \frac{16\pi^2 M}{b}\ \langle P'|T|P\rangle$$

Differential cross section:

$$\frac{d\sigma_{\gamma P\to\psi P}}{dt} = \frac{1}{64\pi s}\ \frac{1}{|\mathbf{p}_{\gamma cm}|^2}\ |\mathcal{M}_{\gamma P\to\psi P}(t)|^2$$

$$\sigma_{\gamma P\to\psi P}(s) = \int_{t_{min}}^{t_{max}} dt\ \frac{d\sigma_{\gamma P\to\psi P}}{dt},$$

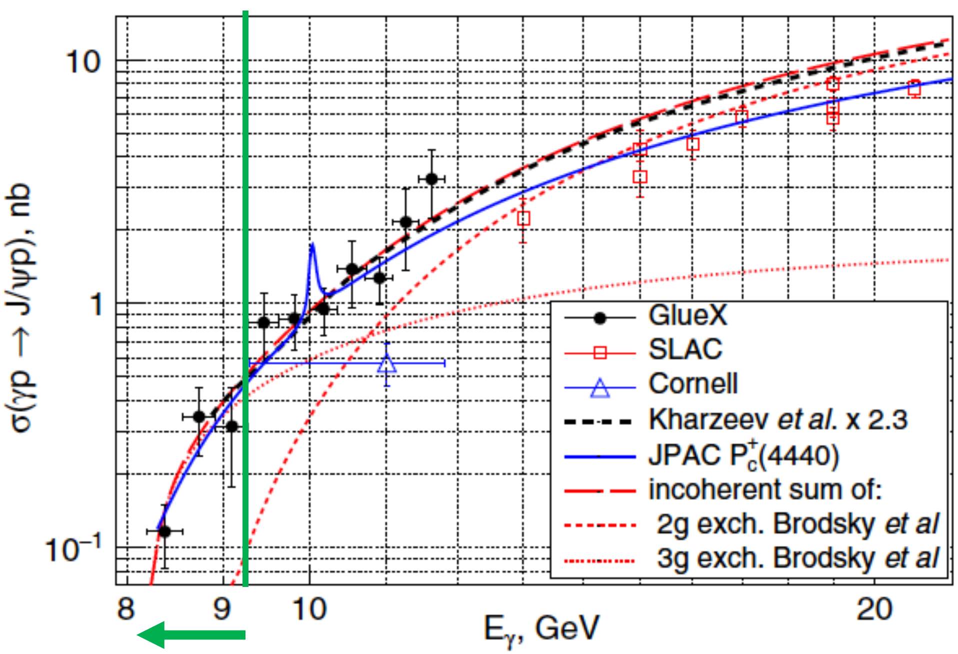
PHYSICAL REVIEW LETTERS **123**, 072001 (2019)

Editors' Suggestion

First Measurement of Near-Threshold J/ψ Exclusive Photoproduction off the Proton

A. Ali,[10] M. Amaryan,[22] E. G. Anassontzis,[2] A. Austregesilo,[3] M. Baalouch,[22] F. Barbosa,[14] J. Barlow,[7] A. Barnes,[3] E. Barriga,[7] T. D. Beattie,[23] V. V. Berdnikov,[17] T. Black,[20] W. Boeglin,[6] M. Boer,[4] W. J. Briscoe,[8] T. Britton,[14] W. K. Brooks,[24] B. E. Cannon,[7] N. Cao,[11] E. Chudakov,[14] S. Cole,[1] O. Cortes,[8] V. Crede,[7] M. M. Dalton,[14] T. Daniels,[20] A. Deur,[14] S. Dobbs,[7] A. Dolgolenko,[13] R. Dotel,[6] M. Dugger,[1] R. Dzhygadlo,[10] H. Egiyan,[14] A. Ernst,[7] P. Eugenio,[7] C. Fanelli,[16] S. Fegan,[8] A. M. Foda,[23] J. Foote,[12] J. Frye,[12] S. Furletov,[14] L. Gan,[20] A. Gasparian,[19] V. Gauzshtein,[25,26] N. Gevorgyan,[27] C. Gleason,[12] K. Goetzen,[10] A. Goncalves,[7] V. S. Goryachev,[13] L. Guo,[6] H. Hakobyan,[24] A. Hamdi,[10] S. Han,[29] J. Hardin,[16] G. M. Huber,[23] A. Hurley,[28] D. G. Ireland,[9] M. M. Ito,[14] N. S. Jarvis,[3] R. T. Jones,[5] V. Kakoyan,[27] G. Kalicy,[4] M. Kamel,[6] C. Kourkoumelis,[2] S. Kuleshov,[24] I. Kuznetsov,[25,26] I. Larin,[15] D. Lawrence,[14] D. I. Lersch,[7] H. Li,[3] W. Li,[28] B. Liu,[11] K. Livingston,[9] G. J. Lolos,[23] V. Lyubovitskij,[25,26] D. Mack,[14] H. Marukyan,[27] V. Matveev,[13] M. McCaughan,[14] M. McCracken,[3] W. McGinley,[3] J. McIntyre,[5] C. A. Meyer,[3] R. Miskimen,[15] R. E. Mitchell,[12] F. Mokaya,[5] F. Nerling,[10] L. Ng,[7] A. I. Ostrovidov,[7] Z. Papandreou,[23] M. Patsyuk,[16] P. Pauli,[9] R. Pedroni,[19] L. Pentchev,[14,*] K. J. Peters,[10] W. Phelps,[8] E. Pooser,[14] N. Qin,[21] J. Reinhold,[6] B. G. Ritchie,[1] L. Robison,[21] D. Romanov,[17] C. Romero,[24] C. Salgado,[18] A. M. Schertz,[28] R. A. Schumacher,[3] J. Schwiening,[10] K. K. Seth,[21] X. Shen,[11] M. R. Shepherd,[12] E. S. Smith,[14] D. I. Sober,[4] A. Somov,[14] S. Somov,[17] O. Soto,[24] J. R. Stevens,[28] I. I. Strakovsky,[8] K. Suresh,[23] V. Tarasov,[13] S. Taylor,[14] A. Teymurazyan,[23] A. Thiel,[9] G. Vasileiadis,[2] D. Werthmüller,[9] T. Whitlatch,[14] N. Wickramaarachchi,[22] M. Williams,[16] T. Xiao,[21] Y. Yang,[16] J. Zarling,[12] Z. Zhang,[29] G. Zhao,[11] Q. Zhou,[11] X. Zhou,[29] and B. Zihlmann[14]

(GlueX Collaboration)



Need to focus on the threshold region!

$E_{cm} < 4.25$ GeV

$E_{\gamma} < 9.2$ GeV

Threshold photoproduction of quarkonium: the effect of the scalar gravitational formfactor

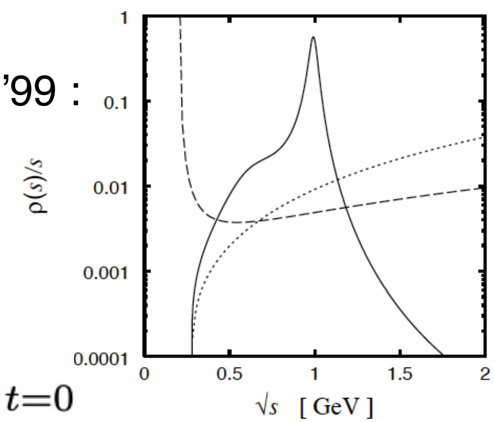
The scalar gravitational formfactor can be constrained theoretically by using:

i) dispersion relations;

ii) low-energy theorems of broken scale invariance;

iii) experimental data on π π phase shifts and scalar mesons

See e.g. Fujii, DK'99 :



However, as a first step, can try a simple dipole formfactor of the type used for electromagnetic formfactor:

$$G(t) = \frac{M}{(1 - t/\mathrm{m}_s^2)}$$

radius

$$\langle R_{\mathrm{m}}^2 \rangle = \frac{6}{M} \frac{dG}{dt}\Big|_{t=0}$$

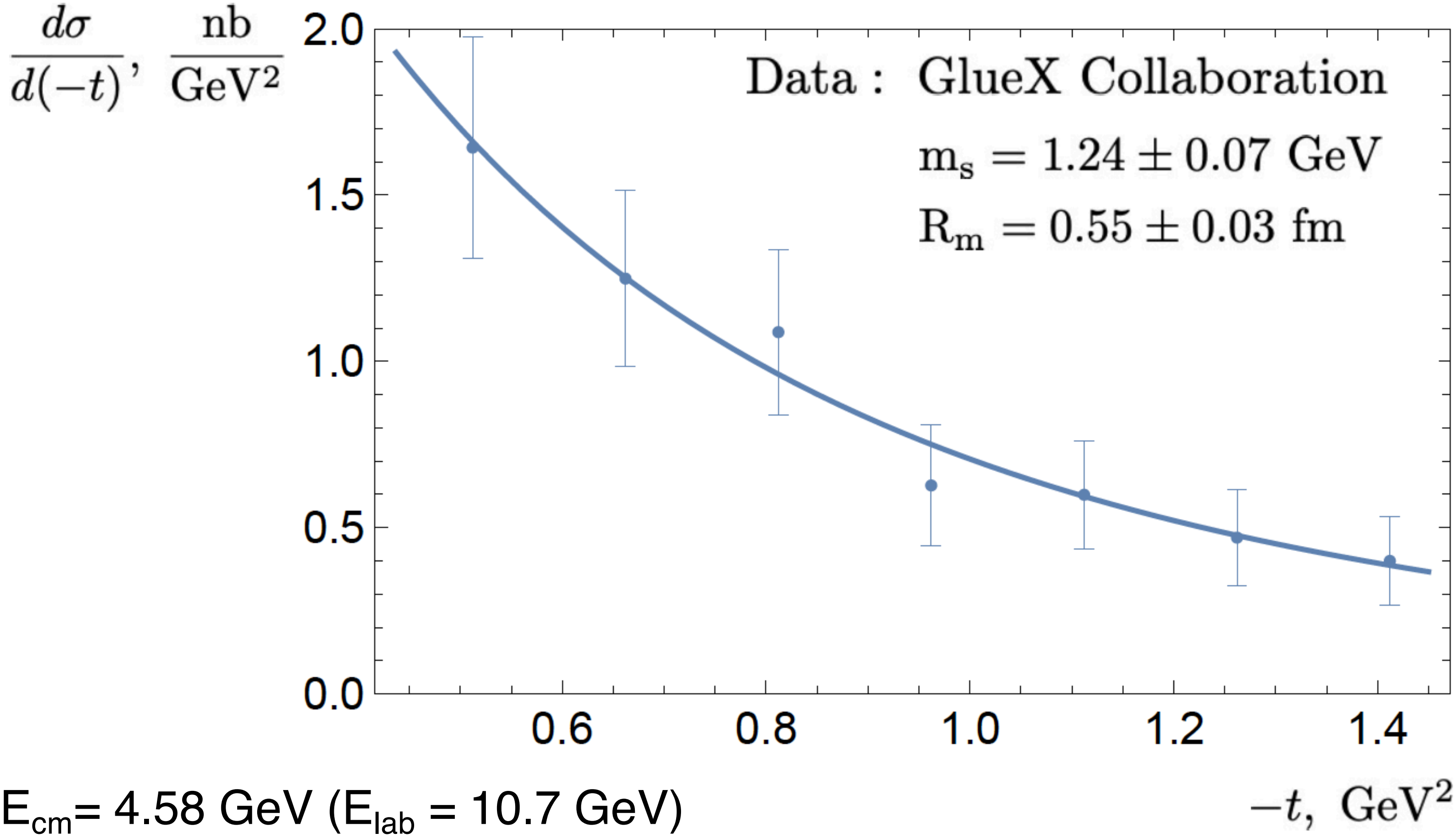
Dipole formfactor was also used for 2-gluon coupling in perturbative models

See e.g. Frankfurt, Strikman '02

but: T formfactor cannot be computed perturbatively

Differential cross section

DK, arXiv:2102.00110



E_{cm}= 4.58 GeV (E_{lab} = 10.7 GeV)

$$|c_2|^2 = 0.043 \pm 0.006 \ \mathrm{fm}^4$$

Lattice QCD, P. Shanahan, W. Detmold PRD'19:

$$m_s = 1.13 \pm 0.06 \ \ \mathrm{GeV}$$ (Traceless gluon operator)

$$c_2 \sim \pi r_{c\bar{c}}^2 \qquad r_{c\bar{c}} \simeq 0.1 \ \mathrm{fm}$$

The proton mass radius

The r.m.s. "proton mass radius" from GlueX data:

DK, arXiv:2102.00110

$$R_{\mathrm{m}} \equiv \sqrt{\langle R_{\mathrm{m}}^2 \rangle} = 0.55 \pm 0.03 \ \mathrm{fm}$$

Compare to the proton charge radius:

$$\bar{\mathrm{R}}_{\mathrm{c}} \equiv \sqrt{R_c^2} = 0.8409 \pm 0.0004 \quad \mathrm{fm}$$

See J.Bernauer, EPJ 234 (2020) for review

A more compact mass distribution? Need more data!

VALUE (fm)		*DOCUMENT ID*		*TECN*	*COMMENT*
0.8409 ± 0.0004		**OUR AVERAGE**			
0.833 ±0.010	1	BEZGINOV	2019	LASR	2S-2P transition in H
0.831 ±0.007 ±0.012	2	XIONG	2019	SPEC	$e\,p \to e p$ form factor
0.84087 ±0.00026 ±0.00029		ANTOGNINI	2013	LASR	μp -atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.877 ±0.013	3	FLEURBAEY	2018	LASR	1S-3S transition in H
0.8335 ±0.0095	4	BEYER	2017	LASR	2S-4P transition in H
0.8751 ±0.0061		MOHR	2016	RVUE	2014 CODATA value
0.895 ±0.014 ±0.014	5	LEE	2015	SPEC	Just 2010 Mainz data
0.916 ±0.024		LEE	2015	SPEC	World data, no Mainz
0.8775 ±0.0051		MOHR	2012	RVUE	2010 CODATA, ep data
0.875 ±0.008 ±0.006		ZHAN	2011	SPEC	Recoil polarimetry
0.879 ±0.005 ±0.006		BERNAUER	2010	SPEC	$e\,p \to e p$ form factor

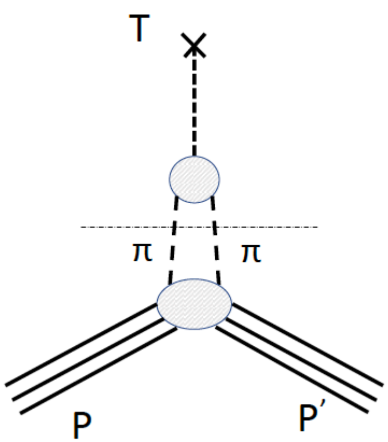
2020 Review of Particle Physics.
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

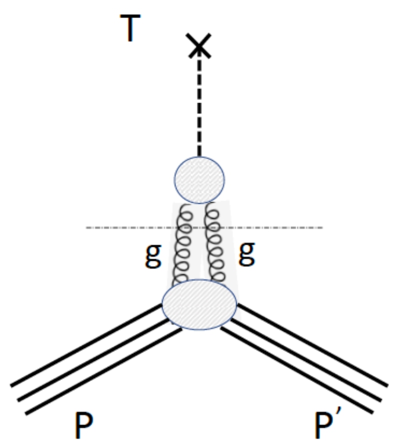
Some day:
p MASS RADIUS in PDG?

Theoretical uncertainties

- Higher dimensional operators (suppressed by $1/m_c$)
- Chiral limit (we omitted the scalar quark operator)
- Gluon operators with derivatives (~ 5% close to threshold)
- t-dependence of short-distance coefficient c_2 ($\sim t/4m_c^2$)
- Dipole parameterization of formfactor

Why is proton mass radius smaller than the charge radius?





Spectral representation –

EM formfactor: M_ρ = 0.77 GeV

Scalar gravitational formfactor: scalar glueball M = 1.5 GeV

But: scalar gluon current mixes with the scalar quark current – σ(500) is lighter than the ρ!

The real reason (?) – decoupling of Goldstone bosons:

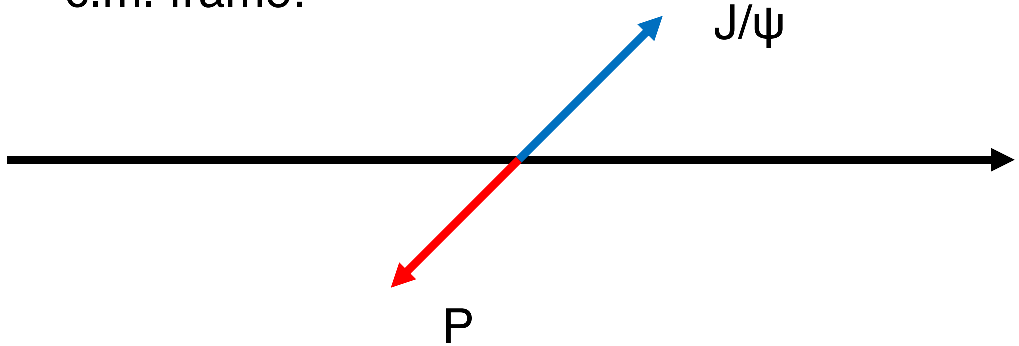
$$\langle 0|T|\pi^+\pi^-\rangle = q^2$$

DK, arXiv:2102.00110

Future measurements

- GlueX has 10 times more data (~ May 2021)
- Future: SoLID@Jlab (~ 2028), EIC (including Υ !)
- Also: ultra-peripheral collisions at RHIC?

c.m. frame:



For a fixed invariant mass (cms energy), measure the angular distribution – differential cross section of photoproduction

Summary

- The proton mass to large extent originates from quantum anomalies

- The threshold photoproduction of J/ψ probes the mass distribution inside the proton; current data and a simple dipole model favor

$$R_{\rm m} \equiv \sqrt{\langle R_{\rm m}^2 \rangle} = 0.55 \pm 0.03\ {\rm fm}$$

- We need a quantitative theory of the scalar gravitational formfactor and precise data at $E_{cm} < 4.3$ GeV to understand the mass distribution inside the proton, and the origin of the proton mass!