Exploring Hadrons through the Microscope of Lattice QCD

David Richards

Jefferson Lab

A history of lattice QCD through no-go theorems

- ~~You can't place a chiral gauge theory on a discretized lattice~~

  Domain-wall Fermions: *D.Kaplan, Phys.Lett.B 288 (1992) 342*
  Overlap Fermions: *R.Narayanan, H.Neuberger, Nucl.Phys.B 443 (1995) 305*

- ~~You can't investigate scattering on a Euclidean lattice~~

  "Luscher's Method": *M.Luscher, Nucl.Phys.B 354 (1991) 531*
  See *David Wilson, Tuesday* and *many parallel talks*

- ~~You can't compute matrix elements of light-cone operators on a Euclidean lattice~~

  LaMET: X.Ji, *Phys.Rev.Lett.* 110 (2013) 262002

Theorems did not fall - we found way to drive around them

Transformed our ability to exploit internal structure of hadrons

HadStruc Collaboration

Robert Edwards, Balint Joo, Jianwei Qiu, David Richards, *Eloy Romero*, Frank Winter, *Nikhil Karthik*

Jefferson Lab

Carl Carlson, Colin Egerer, *Christos Kallidonis*, Tanjib Khan, Christopher Monahan, Kostas Orginos, Raza Sufian

College of William and Mary

Wayne Morris, Anatoly Radyushkin

Old Dominion University

Joe Karpie

Columbia University

Savvas Zafeiropoulos

Aix Marseille Univ, Marseille, France

Yan-Qing Ma

Peking University, Beijing, China

Lattice QCD on a slide

Capability Computing -
Gauge Generation

e.g. Summit at ORNL

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

Euclidean space ⟶
Importance Sampling

Several *V, a, T, m_π*

Capacity Computing -
Observable Calculation

e.g. GPU/KNL clusters at
JLab, BNL, FNAL

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

$$\text{e.g. } C(t) = \sum_{\vec{x}} \langle N(\vec{x}, t) \bar{N}(0) \rangle$$

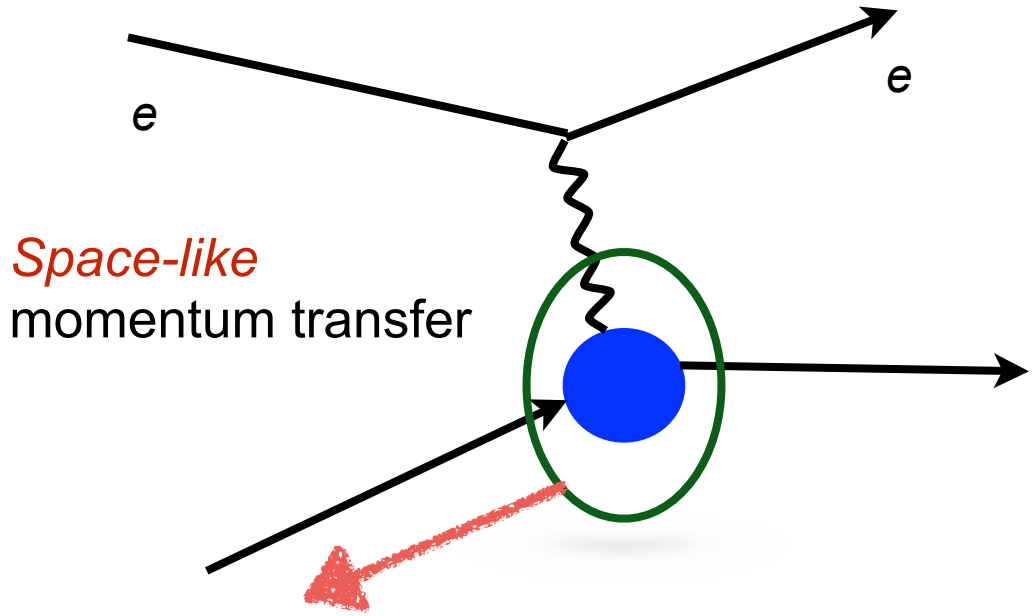
"Desktop" Computing -
Physical Parameters

e.g. Mac at your desk

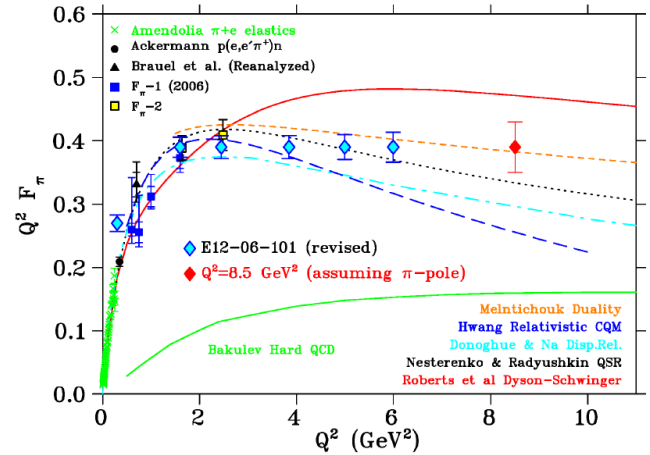
$$C(t) = \sum_n A_n e^{-E_n t}$$

$$M_N(a, m_\pi, V)$$

Paradigm: Pion EM form factor



G. Huber, D.Gaskell, T. Horn
PR12-16-003

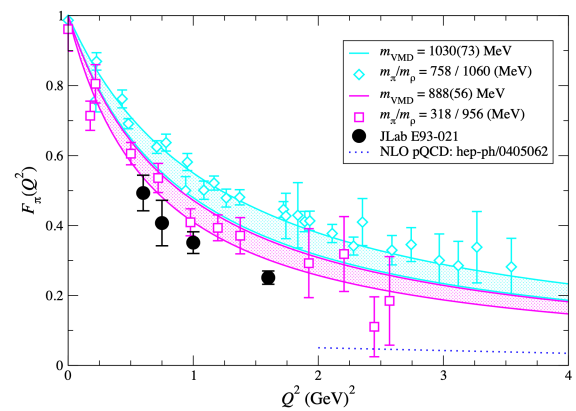


Charge Radius *Partonic DOF*

$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

where
$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$
$$-Q^2 = [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2$$

F.Bonnet *et al.*, PRD 72 (2005) 054506



$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f) V_\mu(\vec{y}, t)\phi^\dagger(0)|\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

No-go Theorem?

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P\rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

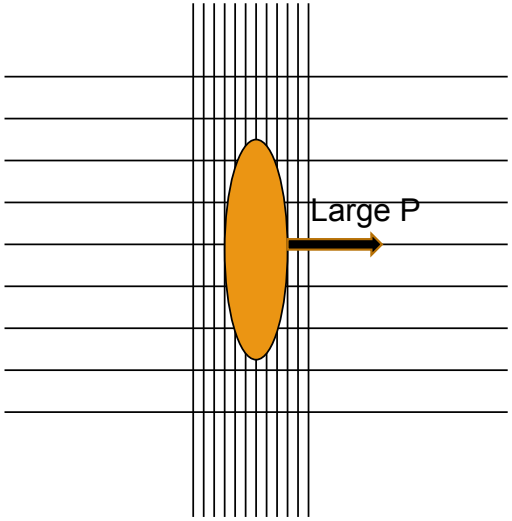
$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi \mid P\rangle \to P_{\mu_1} \ldots P_{\mu_n} a^{(n)}$$

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

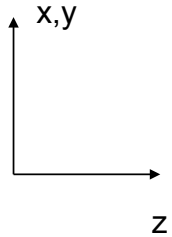
Moment Methods

- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

Solution….



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

Pseudo-PDFs

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\quad$ $z = (0, z_-, 0_T)$

Ioffe-Time Distribution

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Lattice "building blocks" that of quasi-PDF approach.

$\Downarrow$ *Lorentz covariant*

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x} \mathcal{P}(x, z^2)$$

$\longleftarrow$ *pseudo-PDF*

$$\Downarrow$$

$$f(x) = \mathcal{P}(x,0) \underset{z_3^2 \to 0}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} \mathcal{M}(\nu, -z_3^2)$$

"Good Lattice Cross Sections"

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Encompasses qPDF/pPDF

Gauge-Invariant Currents

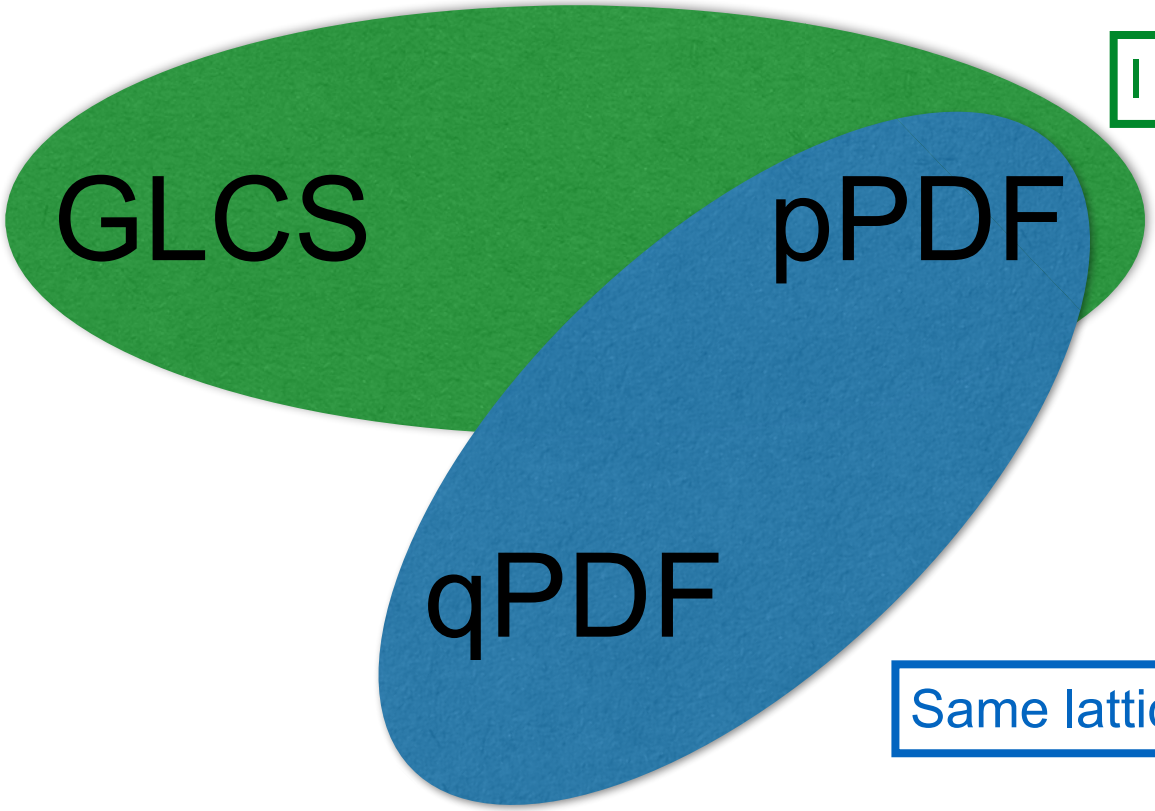
$$\begin{aligned} \mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{aligned}$$

Flavor-changing

+ analogous gluon operators

Analogous matching to light-cone PDFs

I will discuss these



Same lattice building blocks

Pion Valence PDF

JPhys G

Revealing the structure of light pseudoscalar mesons at the Electron-Ion Collider

J Arrington[1], C Ayerbe Gayoso[2], PC Barry[6,21], V Berdnikov[3], D Binosi[4], L Chang[5], M Diefenthaler[6], M Ding[4], R Ent[6], T Frederico[7], Y Furletova[6], TJ Hobbs[6,8,20], T Horn[3,6,*], GM Huber[9], SJD Kay[9], C Keppel[6], H-W Lin[10], C Mezrag[11], R Montgomery[12], IL Pegg[3], K Raya[5,13], P Reimer[14], DG Richards[6], CD Roberts[15,16], J Rodríguez-Quintero[17] D Romanov[6], G Salmè[18], N Sato[6], J Segovia[19], P Stepanov[3], AS Tadepalli[6] and RL Trotta[3]

[1] Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
[2] Mississippi State University, Starkville, MS, USA
[3] Catholic University of America, Washington, DC, USA
[4] European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler Villa Tambosi, Strada delle Tabarelle 286, I-38123 Villazzano (TN) Italy
[5] School of Physics, Nankai University, Tianjin 300071, China
[6] Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA
[7] Instituto Tecnológico de Aeronáutica, 12.228-900 São José dos Campos, Brazil
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[17] Department of Integrated Sciences and Center for Advanced Studies in Physics, Mathematics and Computation, University of Huelva, E-21071 Huelva, Spain

Major experimental initiatives

C.Roberts, D.Richards, T.Horn, L.Chang, arXiv:2102.01765, PPNP

X.Gao, C.Lauer

N.Karthik, Y.Zhao

- Understanding pion goes to heart of origin of mass
- LQCD can study *isolated, unbound pion*
  - *Potential to validate experimental analyses*
- Computationally the most straightforward
  - *But… ..signal-to-noise ratio degrades at high momentum*

Good Lattice Cross Section

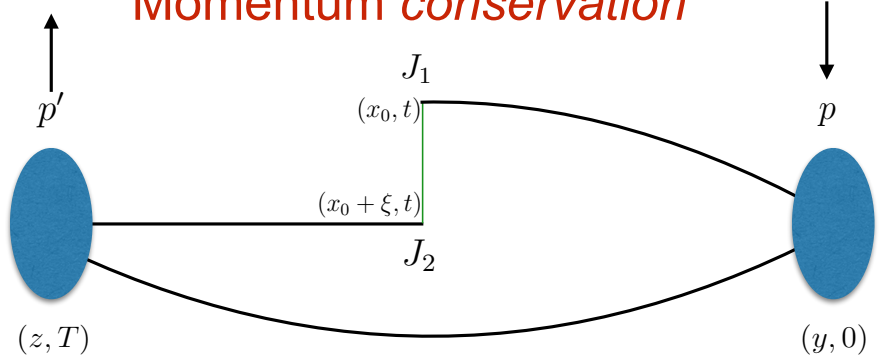
Sufian *et al.*, Phys. Rev. D 99, 074507 (2019); Phys. Rev. D102, 05408 (2020)

Sequential-Source Approach

Momentum *conservation*

Momentum projection

Momentum projection



Process, i.e. current, dependent

$$\frac{1}{2}[\sigma^{\mu\nu}_{V,A}(\xi,p) + \sigma^{\mu\nu}_{A,V}(\xi,p)]$$

$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\nu,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)T_2(\nu,\xi^2)$$

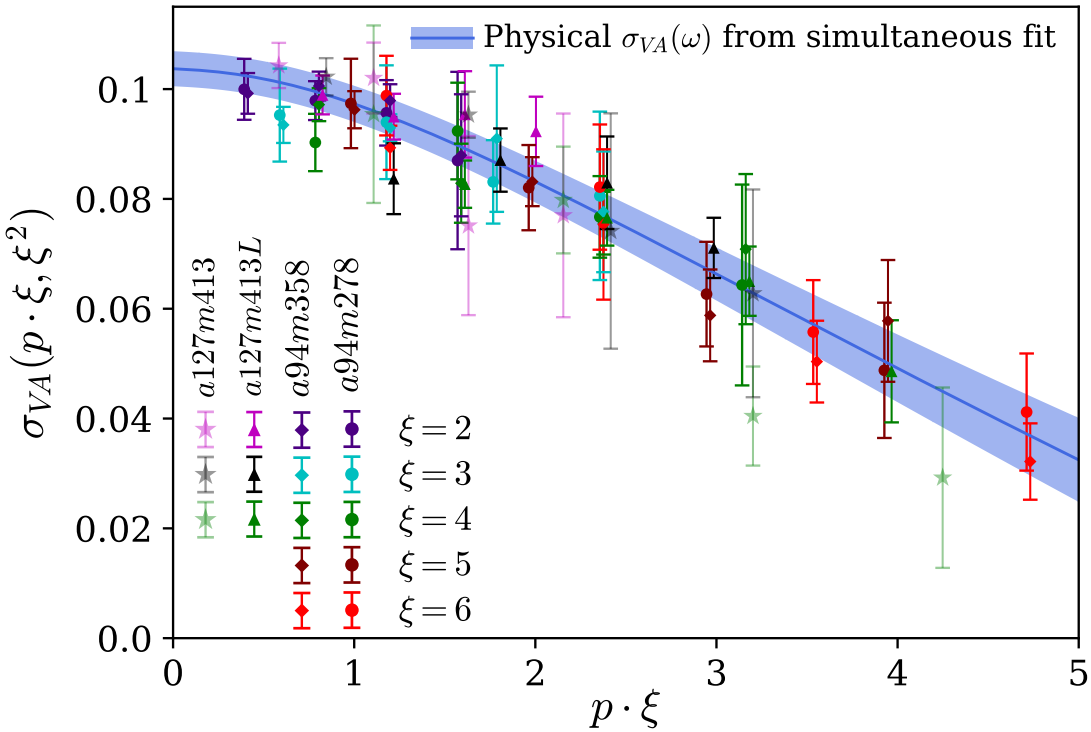
Perturbative kernel:

N.B. We're inconsistent $\omega \leftrightarrow \nu$!

$$\tilde{\sigma}^{q(1)}_{VA}(\tilde{\omega},q^2) = \int_0^1 \frac{dx}{x}\,\tilde{K}^{(1)}(x\tilde{\omega},q^2,\mu^2) f^{(0)}_{q_v/q}(x,\mu^2) + \int_0^1 \frac{dx}{x}\,\tilde{K}^{(0)}(x\tilde{\omega},q^2,\mu^2) f^{(1)}_{q_v/q}(x,\mu^2).$$

Y-Q Ma

Lattice Cross Sections



"Z-expansion fit"

$$\sigma_{VA}(\omega,\xi^2) = \sum_{k=0}^{k_{\max}=4} \lambda_k \tau^k + b_1 m_\pi + b_2 a + b_3 \xi^2 + b_4 a^2 p^2 + b_5 e^{-m_\pi(L-\xi)}$$

$$\tau = \frac{\sqrt{\omega_{\rm cut}+\omega} - \sqrt{\omega_{\rm cut}}}{\sqrt{\omega_{\rm cut}+\omega} + \sqrt{\omega_{\rm cut}}}$$

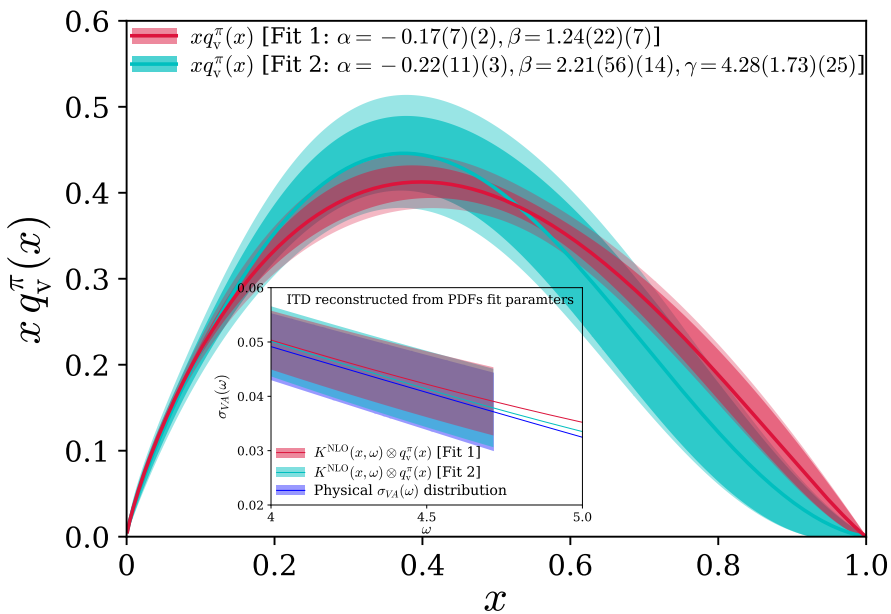
Inverse problem: extract PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

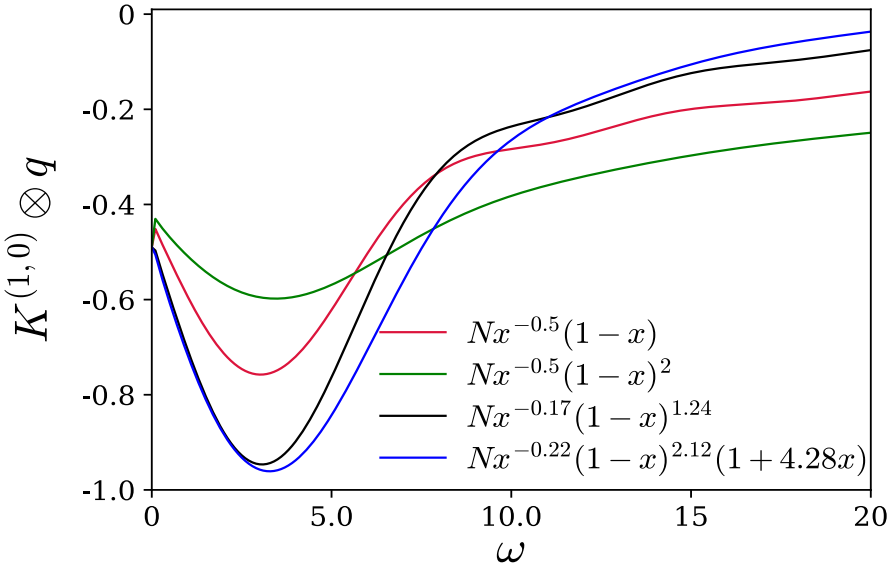
Calculate on Lattice Extract PDF? Calculate in PQCD

Similar challenge to global fitting community!



$$q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1, \beta+1) + \gamma B(\alpha+2, \beta+1)}$$

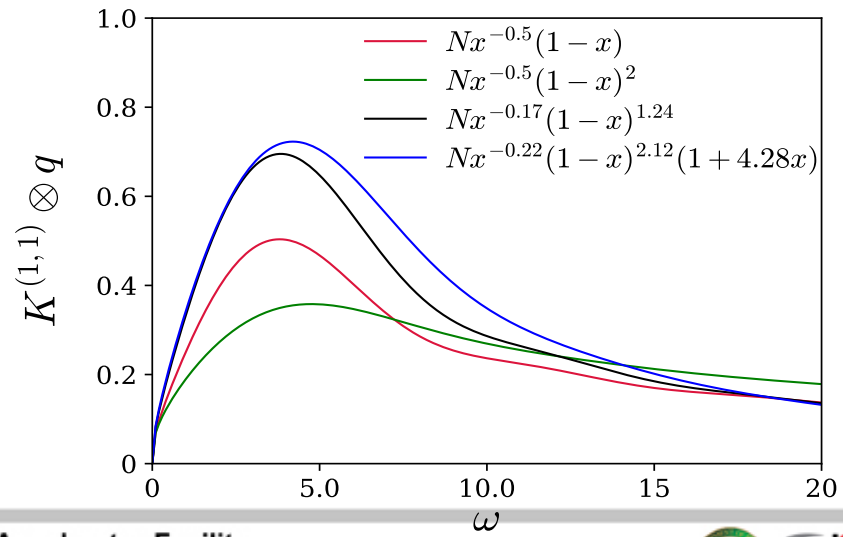
NLO term well-controlled

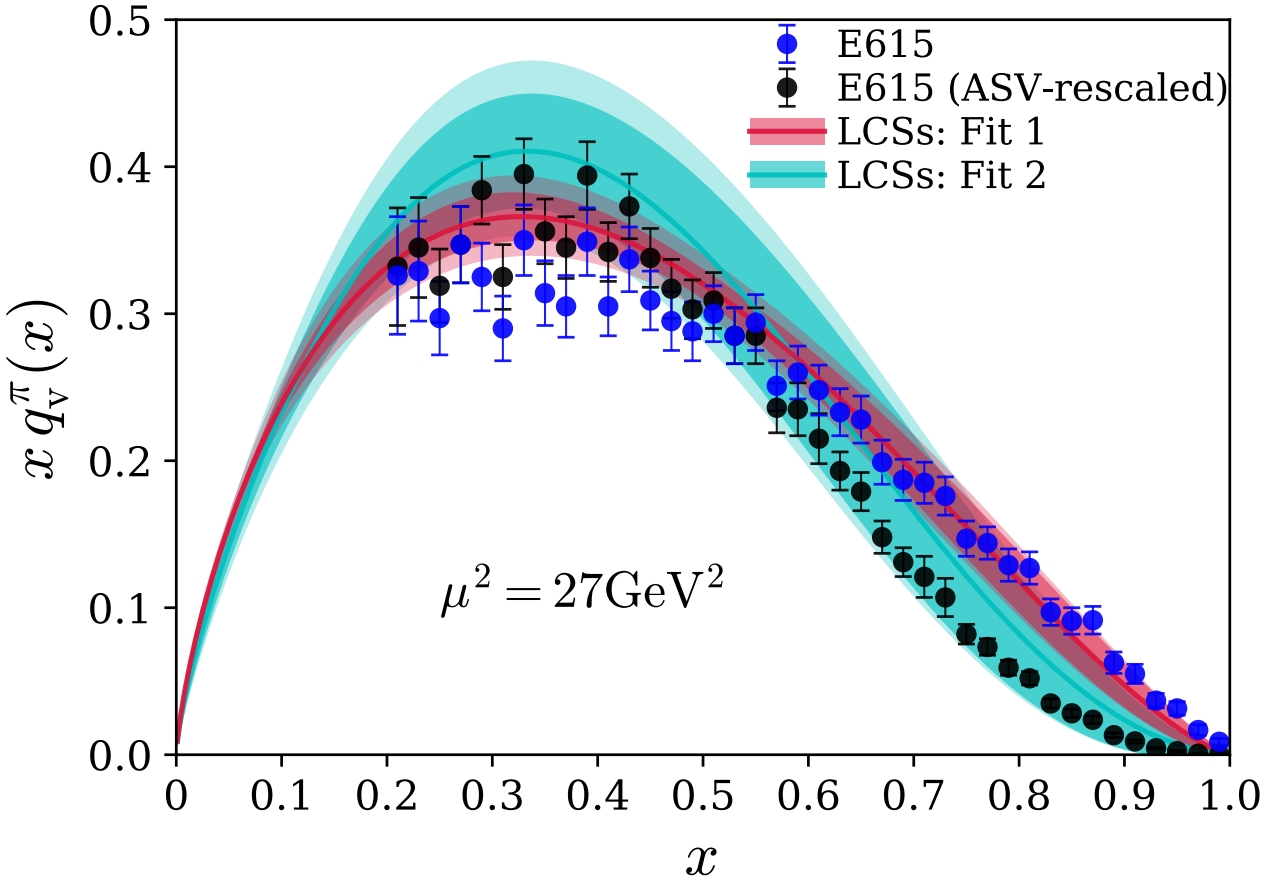


Pion Valence Quark Distribution at Large x from Lattice QCD

Raza Sabbir Sufian,[1] Colin Egerer,[2] Joseph Karpie,[3] Robert G. Edwards,[1] Bálint Joó,[1] Yan-Qing Ma,[4,5,6] Kostas Orginos,[1,2] Jian-Wei Qiu,[1] and David G. Richards[1]

Sufian *et al.*, Phys. Rev. D102, 05408 (2020)



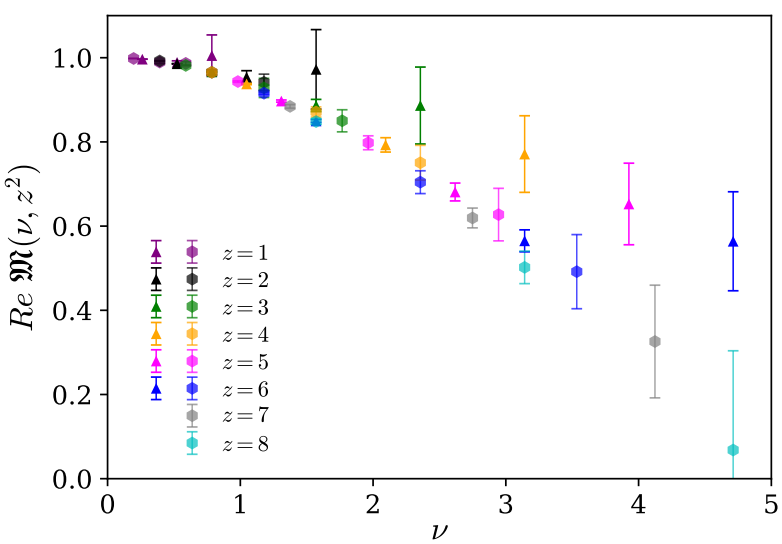


Determine large-x behavior → *need for finer resolution and reach in Ioffe time.*

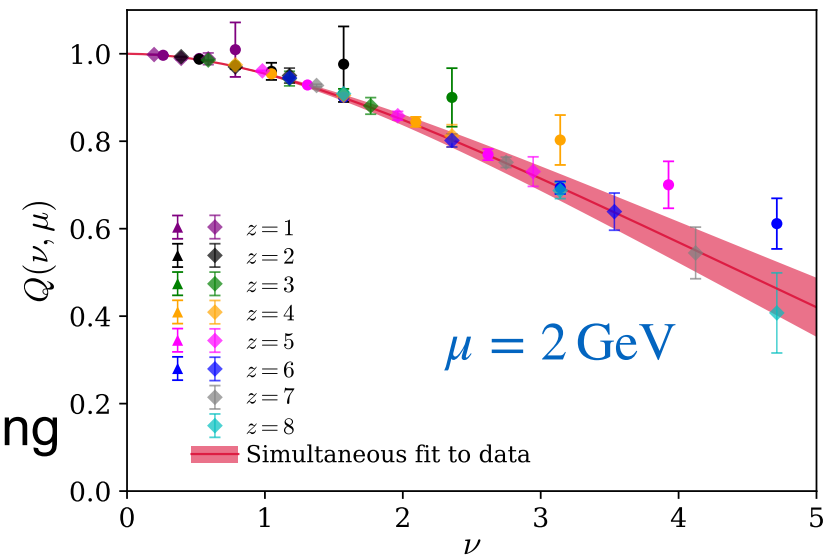
Pseudo-PDF Approach

ID	a (fm)	m_π (MeV)	β	am_l	am_s	$L^3 \times N_t$	$N_{\rm cfg}$
$a127m415$	0.127(2)	415(23)	6.1	-0.280	-0.245	$24^3 \times 64$	2147
$a127m415L$	0.127(2)	415(23)	6.1	-0.280	-0.245	$32^3 \times 96$	2560

Same ensemble as LCS

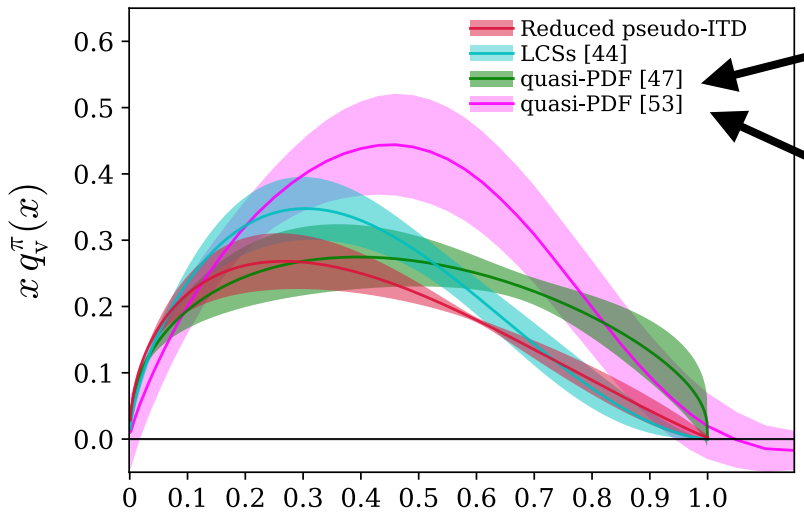


After Matching



B.Joó et al., Phys. Rev. D 100, 114512 (2019).

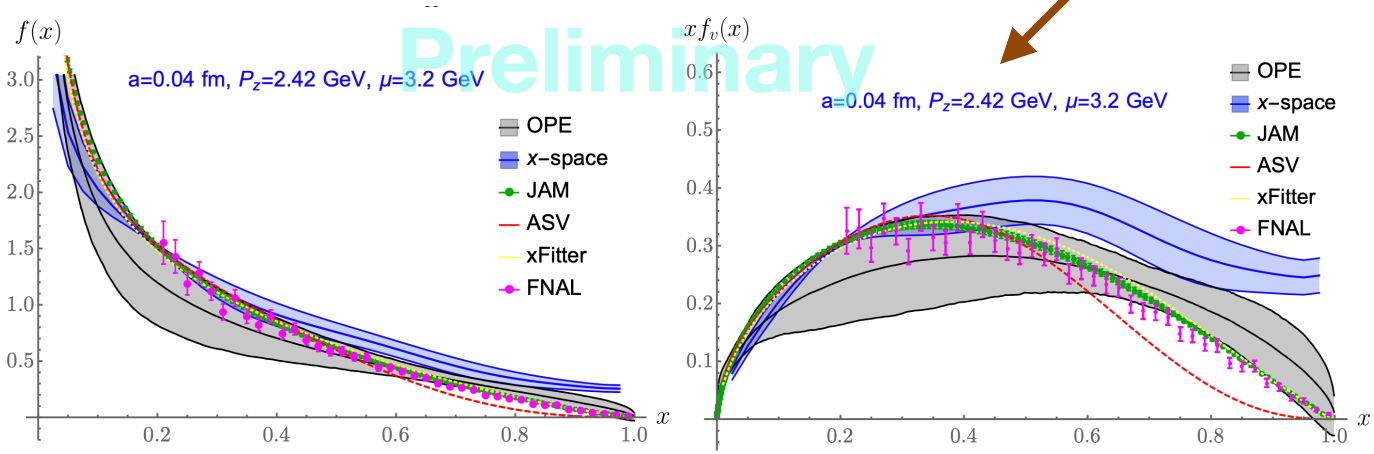
Pion pPDF



T.Izubuchi *et al.*, Phys. Rev. D 100, 034516

J-H Zhang *et al.*, Phys. Rev. D **100**, 034505

Y.Zhao, Tues



Better agreement with experimental fits for 0.1<x<0.5 compared to our previous analysis using OPE in coordinate space.

NUCLEON STRUCTURE

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0,z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu,0)}\right) / \left(\frac{\mathcal{M}(0,z^2)}{\mathcal{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu, \mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$$\Downarrow$$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

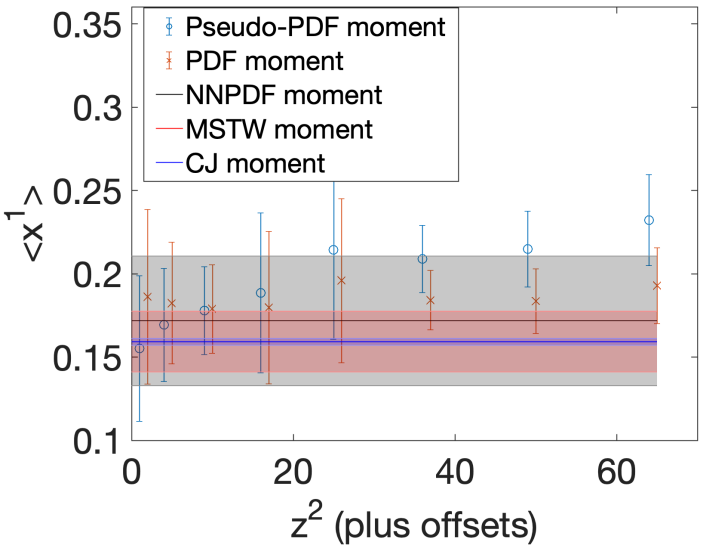
$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

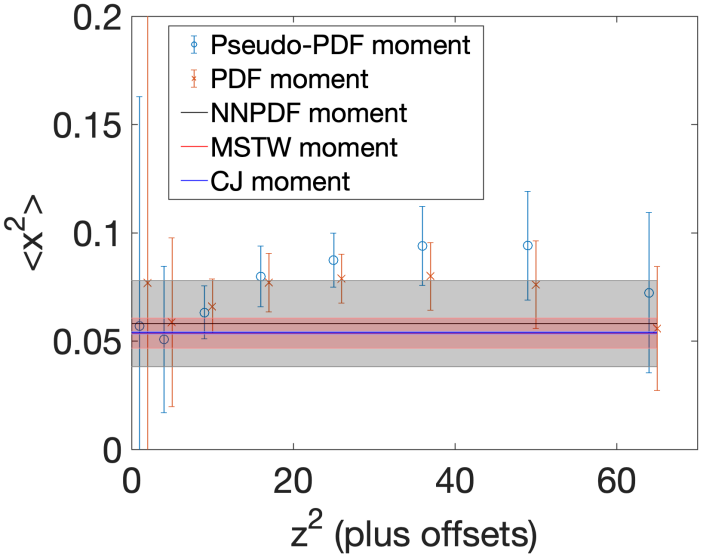
Need data for all v, *or additional physics input*

Inverse problem

Moments of PDFs

Can extract moments - which does not require tackling the inverse problem





Different systematics from computation through local operators

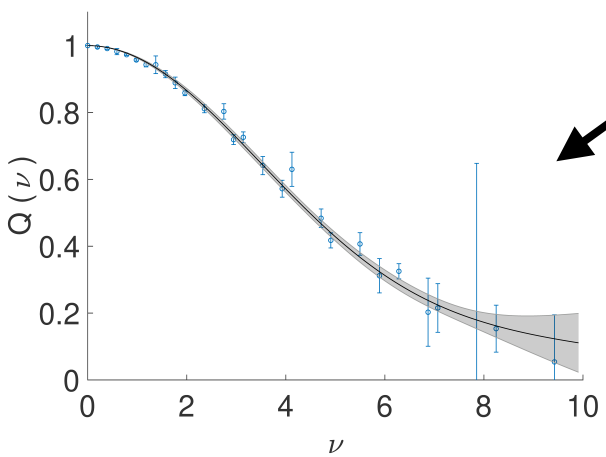
Ioffe-Time Distribution to PDF

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

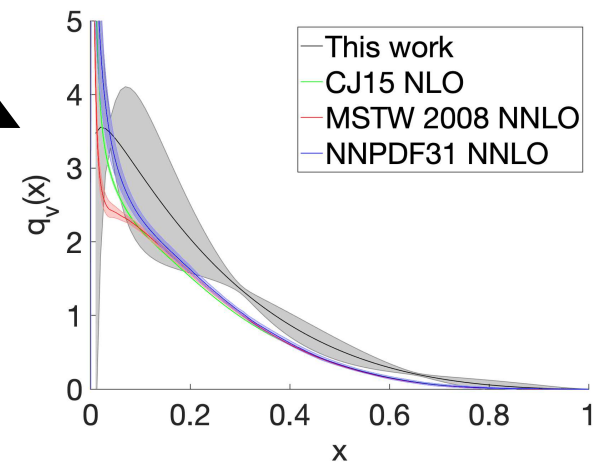
$$f(x) = x^a(1-x)^b P(x)$$

MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$

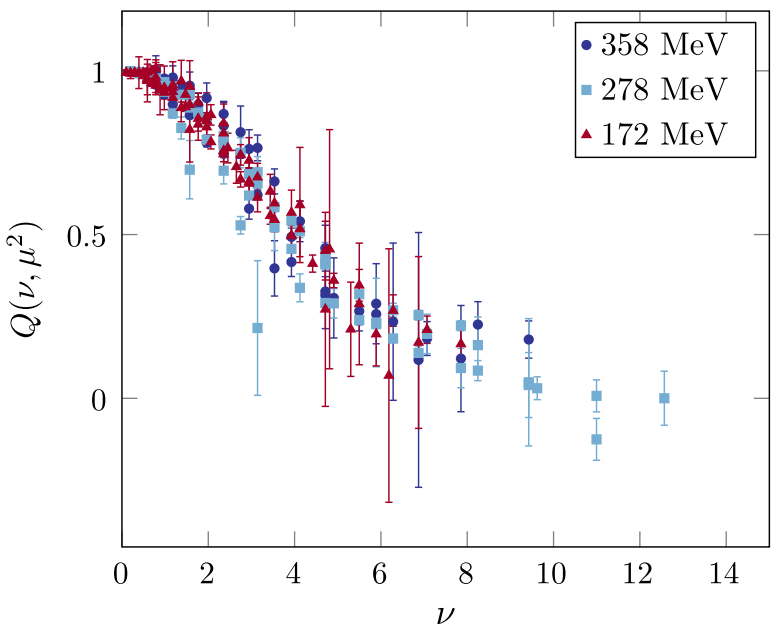


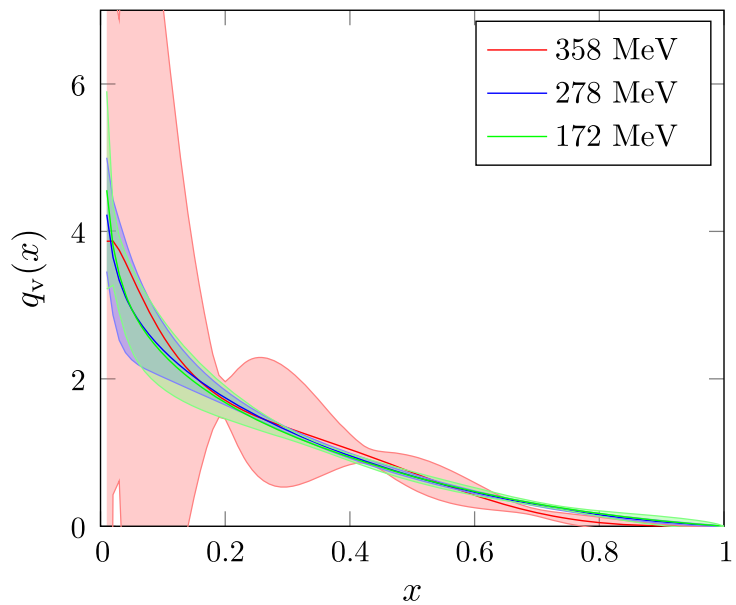
a127m415L



PDFs at Physical Quark Masses

ID	a(fm)	M_π(MeV)	β	$c_{\rm SW}$	am_l	am_s	$L^3 \times T$	$N_{\rm cfg}$
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175





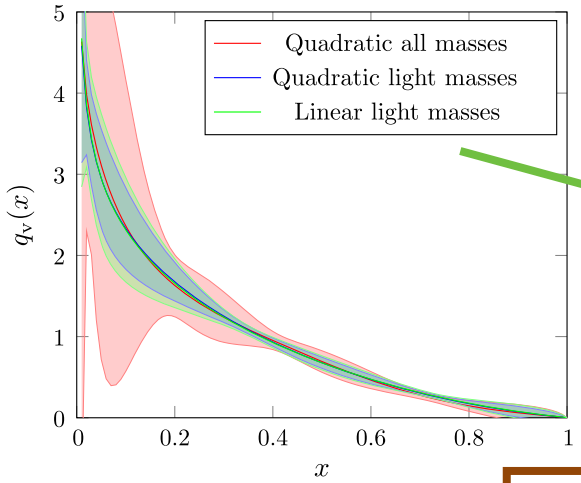
B.Joo *et al.*, arXiv:2004.01687, PRL (in press)

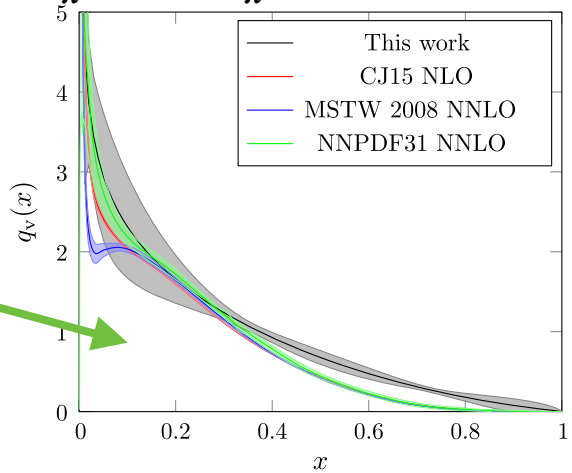
Physical pion

$$q_v(x, \mu^2, m_\pi) = q_v(x, \mu^2, m_0) + a\Delta m_\pi + b\Delta m_\pi^2$$

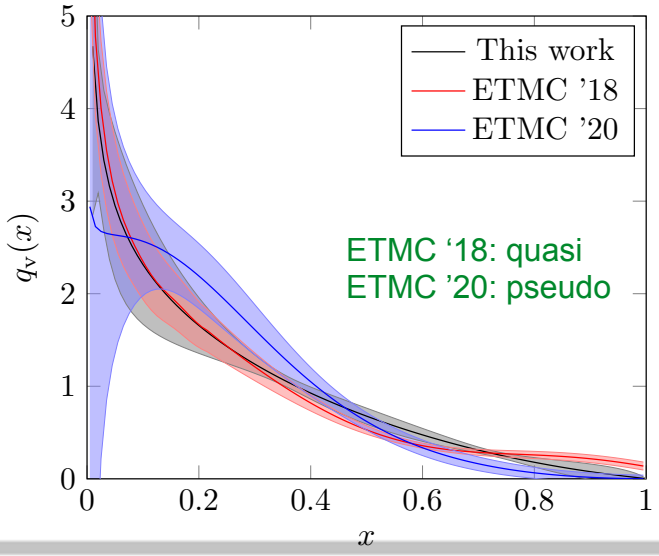
PDFs at Physical Mass

$$q_v(x,\mu^2,m_\pi) = q_v(x,\mu^2,m_0) + a\Delta m_\pi + b\Delta m_\pi^2$$





Colin Egerer, Tues

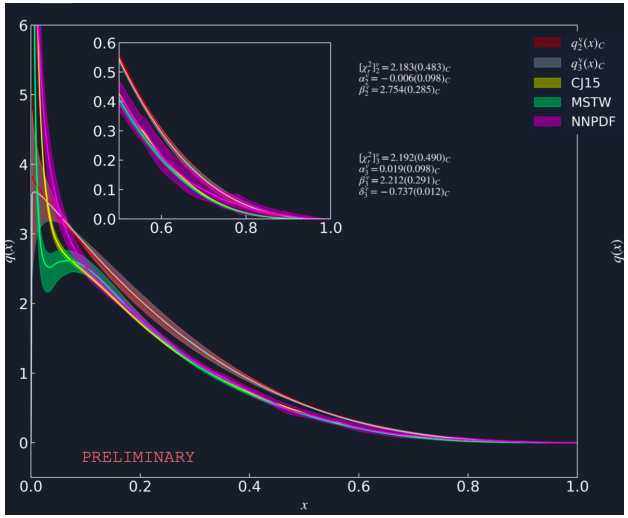


C.Egerer et al., PRD103, 034502 (2021)

New calculation using "distillation" at high p

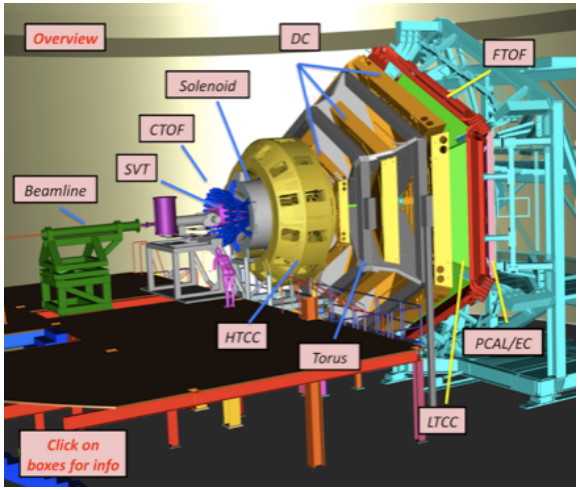
↓

Paper in preparation



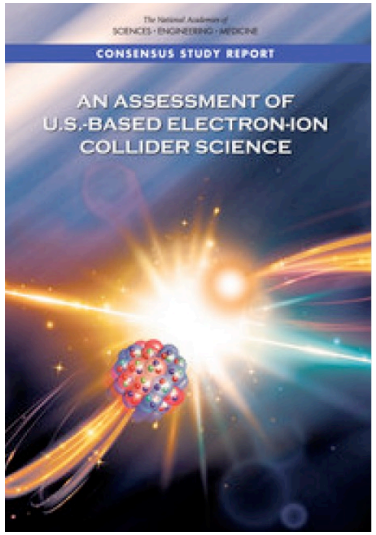
Opportunities and Challenges

A New Opportunity in Hadron Structure



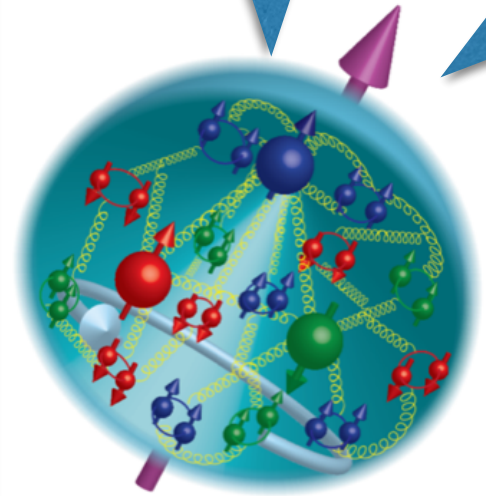
JLab@12GeV

Lattice QCD



Future Electron-Ion Collider

CENTER *for* NUCLEAR FEMTOGRAPHY



3D Image of nucleon and nuclei at the femtoscale

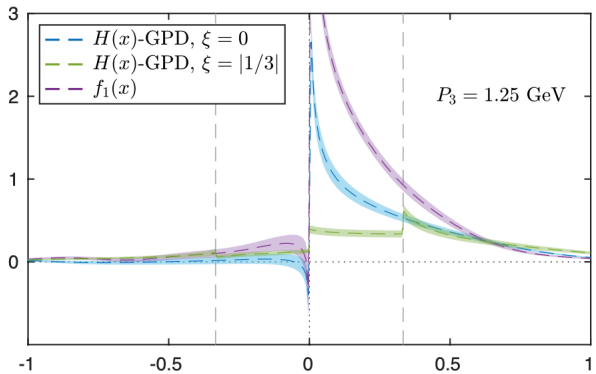
Hadron Femtography

- Three-dimensional imaging of hadrons

M.Constantinou, Tues

Generalized Parton Distributions

Transverse Momentum Dependent dist



C.Alexandrou *et al.*, PRL125, 262001 (2020)

M.Constantinou, Tues

A.Radyushkin, PRD100, 116011 (2019)

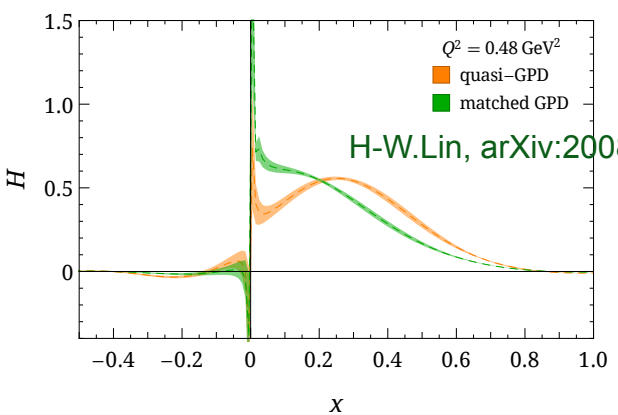
pseudo-GITD

pseudo-GPD

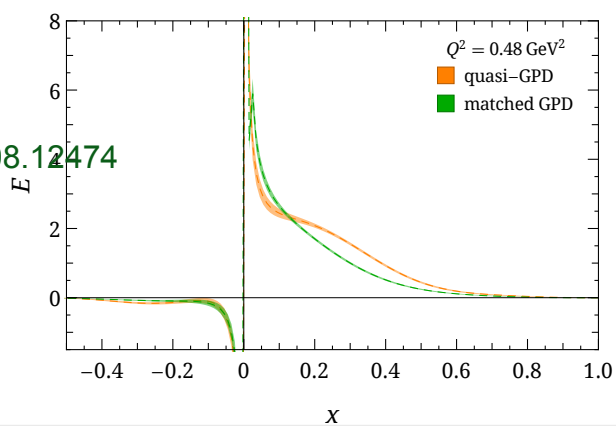
$$\mathcal{M}(\nu, \xi, t; z^2) = e^{i\xi\nu} \int_{-1}^{1} dx\, e^{ix\nu} \mathcal{H}(x, \xi, t, z^2)$$

$$\nu = (\nu_1 + \nu_2)/2 \qquad \xi = \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2}$$

A.Rajan



H-W.Lin, arXiv:2008.12474



C.Egerer, Tues

Lattice is complementary to experiment and essential!

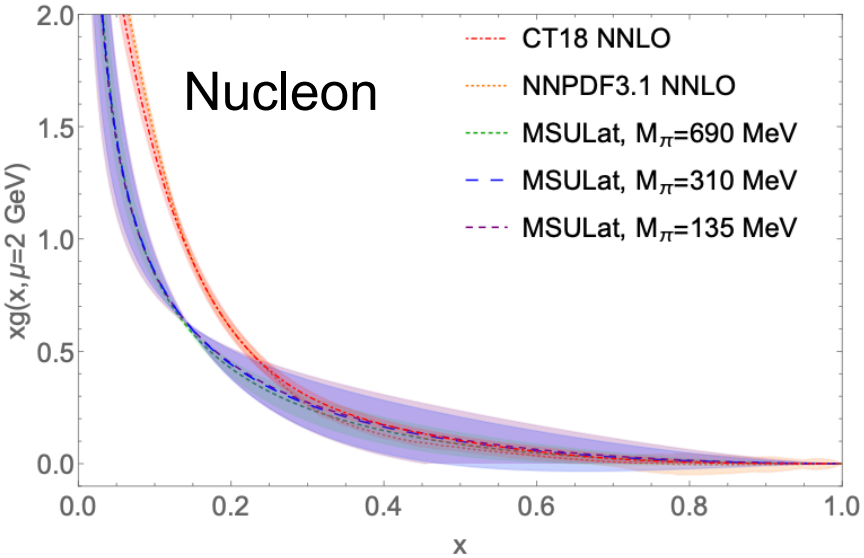
Next Frontier: Flavor Singlet

Glue that binds as all…

The 2015 LONG RANGE PLAN for NUCLEAR SCIENCE

Z.Fan et al., arXiv:2008.15113

D.Pefkou



Gluons in pseudo-PDF approach

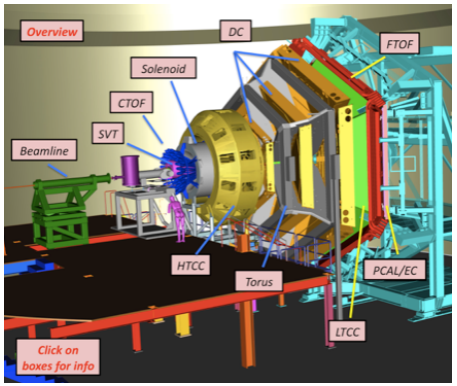
Wayne Morris, Tues

Calculations far more demanding: *signal-to-noise ratio + flavor-singlet quark.*

C.Egerer et al., Phys. Rev. D 103, 034502 (2021)

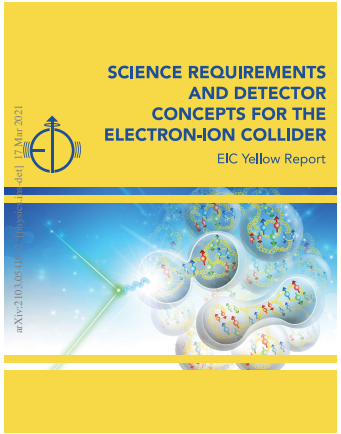
Outlook

- The breadth of physics that lattice QCD can address has grown to encompass most of the key physics of GHP
- Increasing trend: lattice QCD working with phenomenological analysis: JAM, NNPDF,…
- We are entering an exciting time:



EXASCALE COMPUTING PROJECT

+ quantum computing, machine learning!



GHP is an essential forum and voice!