Attractor in pre-equilibrium quark-gluon plasma and initial distribution

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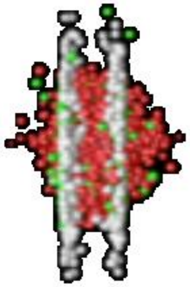
XD, SS, arXiv: 2012.09068, 2012.09079
XD, YH, SS, in preparation

9th Workshop of the APS Topical Group on Hadronic Physics 2021
April 16, 2021
Sacramento, CA [Online]

DFG Deutsche Forschungsgemeinschaft German Research Foundation
UNIVERSITÄT BIELEFELD
CRC-TR 211 Strong-interaction matter under extreme conditions

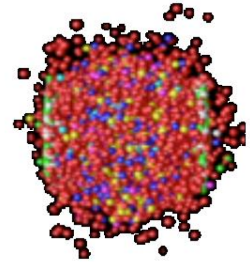
Attractor from QCD effective kinetic theory



Initial Collision
Off-Thermal
Gluon Saturation

Attractor

Pre-Equilibrium QGP
Thermalization
Chemical Equilibration



Hydrodynamic
Thermal
Gluon/Quarks

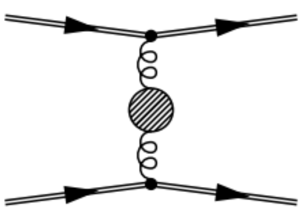
Effective Kinetic Theory (Arnold, Moore, Yaffe) at LO

AMY,JHEP01 (2003) 030
AMY,JHEP0206(2002)030
Kurkela, Mazeliauskas,PRD99 (2019) 054018

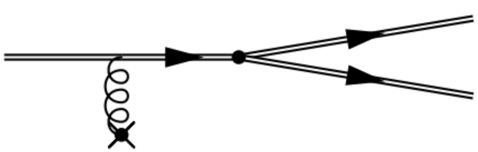
$$\left[\frac{\partial}{\partial\tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right] f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

Boltzmann equation for massless gluon and 3 light quarks/anti-quarks $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

Including $2 \leftrightarrow 2$ elastic processes and $1 \leftrightarrow 2$ inelastic processes



$2 \leftrightarrow 2$: Color screening by Debye mass fit to HTL calculation



$1 \leftrightarrow 2$: Collinear radiation including LPM effect via effective vertex resummation

Pressure attractor

System initially highly anisotropic with CGC inspired gluon dist. & finite baryon/charge density

Universal scaling variable:

$$\tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

1st-order hydrodynamics near equilibrium

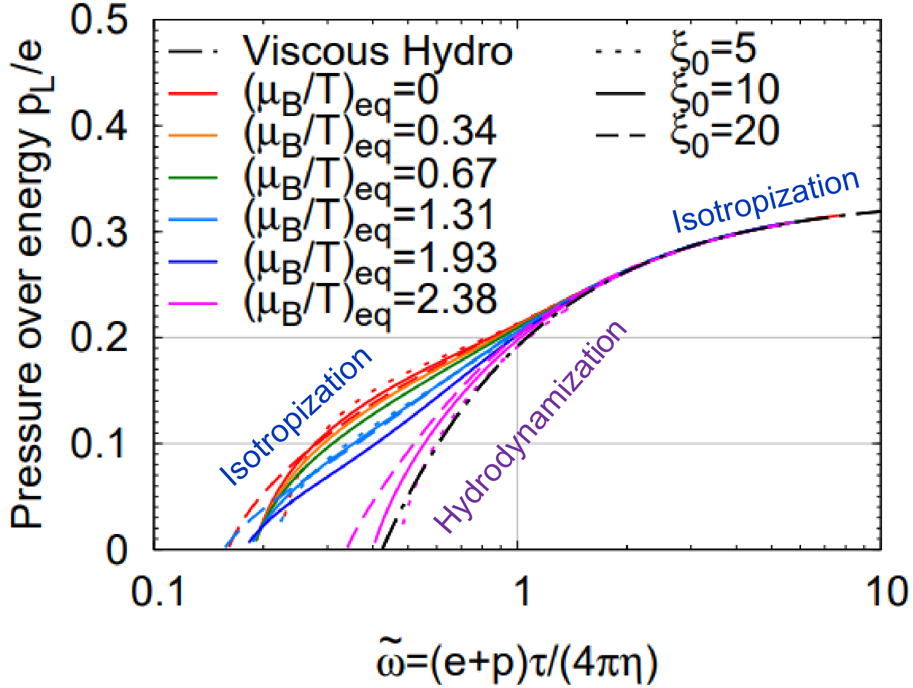
$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi}\left(\frac{\eta T_{\text{eff}}}{e+p}\right)\frac{4\pi}{\tau T_{\text{eff}}} \Rightarrow \frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}}$$

const.

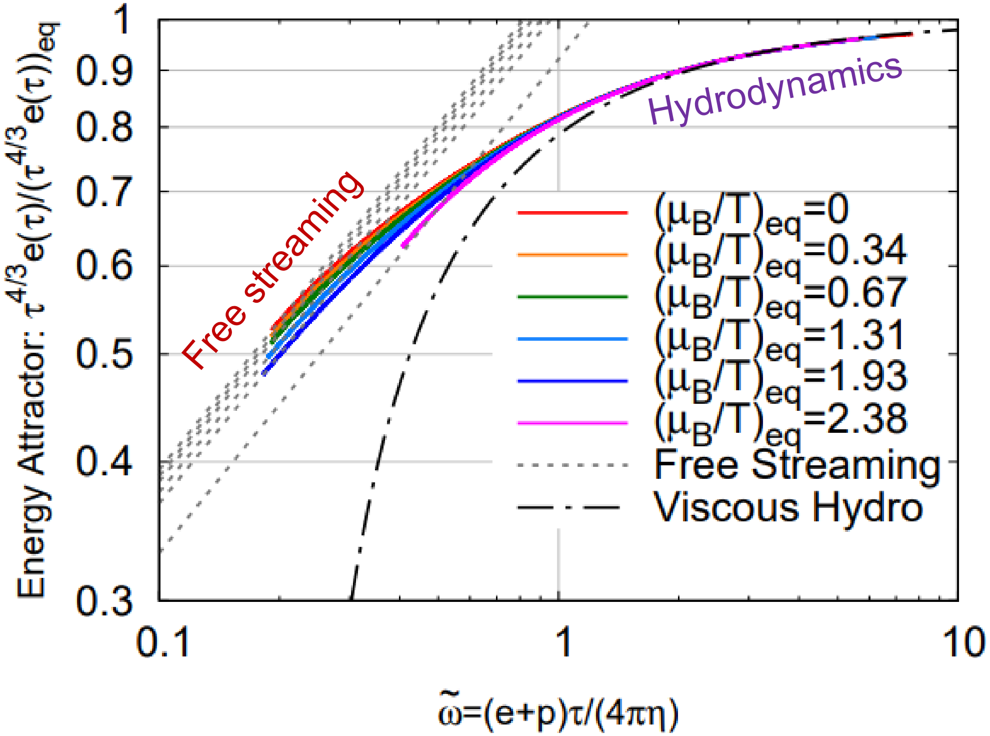
Isotropization longer than hydrodynamization

Non-equilibrium attractors from kinetic theory:
Effective constitutive relations (far-from equilibrium)

$$\frac{p_L}{e} = f(\tilde{\omega})$$



Energy attractor



Non-equilibrium energy attractor

$$\mathcal{E}\left(\tilde{\omega}=\left(\frac{e+p}{\eta T_{\text{eff}}}\right)\frac{\tau T_{\text{eff}}}{4\pi}\right)=\frac{\tau^{\frac{4}{3}}e}{\left(\tau^{\frac{4}{3}}e\right)_{\text{eq}}}$$

Asymptotes:

$$\mathcal{E}(\tilde{\omega}\gg 1)\simeq 1-\frac{2}{3\pi\tilde{\omega}}$$ Hydrodynamics

$$\mathcal{E}(\tilde{\omega}\ll 1)\simeq C_\infty^{-1}\tilde{\omega}^{\frac{4}{9}}$$ Free streaming

Universal pre-equilibrium description connects initial state to hydrodynamics

$$\left(\tau^{\frac{4}{3}}e\right)_{\tilde{\omega}}=\left(4\pi\frac{\eta T_{\text{eff}}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2\nu_{\text{eff}}}{30}\right)^{\frac{1}{9}}(e\tau)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}}=(\tau\Delta n_f)_0$$

Two-way:

Provide input for hydrodynamics

Learn the past (pre-eq, initial, ..)

Giacalone, Mazeliauskas, Schlichting PRL123(2019)26

Learn the past: Pre-equilibrium trajectory

In equilibrium:

Entropy: $$(s\tau)_{\mathrm{eq}} = \frac{\left(e+p-\sum_f \mu_f \Delta n_f\right)\tau}{T}$$

Net Baryon Number: $$\Delta n_B = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

Fixed from experimental/lattice data

Charged particle multiplicity: $$\frac{dN_{ch}}{d\eta} \simeq \frac{N_{ch}}{JS}(\tau s)_{\mathrm{eq}} S_T \simeq 0.12(\tau s)_{\mathrm{eq}} S_T$$

Entropy per baryon: $$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B}\right)_{\mathrm{eq}}$$

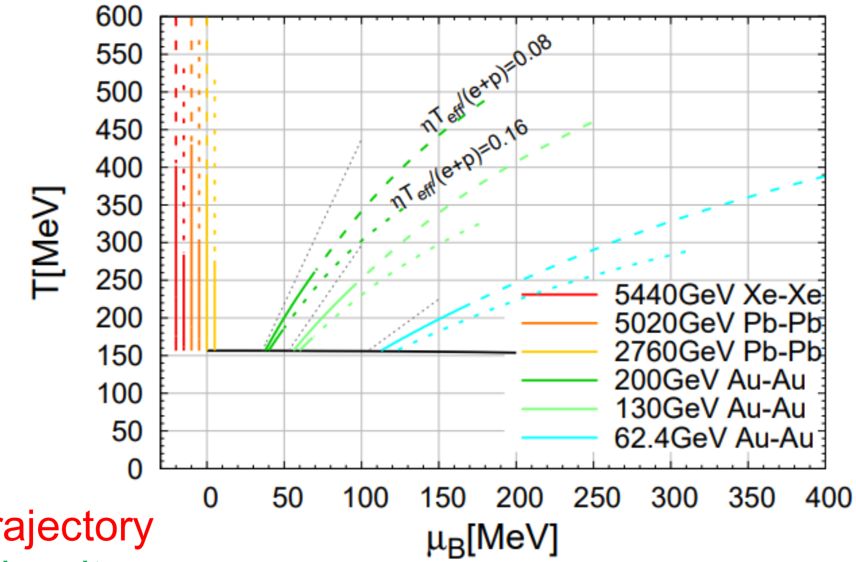
From non-equilibrium attractor:

$$\left(\tau^{\frac{4}{3}} e\right)_{\tilde{\omega}} = \left(4\pi \frac{\eta T_{\mathrm{eff}}}{e+p}\right)^{\frac{4}{9}} \left(\frac{\pi^2 \nu_{\mathrm{eff}}}{30}\right)^{\frac{1}{9}} (e\tau)_0^{\frac{8}{9}} C_\infty \mathcal{E}(\tilde{\omega})$$

$$(\tau \Delta n_f)_{\tilde{\omega}} = (\tau \Delta n_f)_0$$

From Landau matching:
Time-dependent T and μ

Non-equilibrium trajectory
to higher baryon density



Learn the past: Initial energy deposition

Push the attractor curve further to initial time, further constrain the initial energy density

$$\frac{dN_{\rm ch}}{d\eta} \approx \frac{1}{J}\frac{4}{3}C_\infty^{\frac{3}{4}}\left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{\frac{1}{3}}\frac{N_{\rm ch}}{S}\int d^2\mathbf{b}\,\boxed{[(e\tau)_0]^{2/3}}$$

Giacalone, Mazeliauskas, Schlichting
PRL123(2019)26

with equation of state e≈3p, realistic area S_T from MC-Glauber model

Constraining initial distribution from initial energy density:
A simple energy deposition model: k_T-factorized Color Glass Condensate (CGC) form

$$(e\tau)_0 = \int d^2\mathbf{P}\ |\mathbf{P}|\,\frac{\alpha_s N_c}{\pi^4\mathbf{P}^2(N_c^2-1)}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\ \Phi_A(x_A,\mathbf{b}+\mathbf{b}_0/2,\mathbf{k})\Phi_B(x_B,\mathbf{b}-\mathbf{b}_0/2,\mathbf{P}-\mathbf{k})$$

Lappi, Schlichting
PRD97(2018)3,034034

with Golec-Biernat Wusthoff (GBW) gluon distribution

$$\phi^{(1)}_{(U)}(x,\mathbf{k}) = 4\pi^2\frac{(N_c^2-1)}{g^2N_c}\frac{\mathbf{k}^2}{Q_s^2(x,\mathbf{b})}\exp\left\{-\frac{\mathbf{k}^2}{Q_s^2(x,\mathbf{b})}\right\}$$

Golec-Biernat, Wusthoff
PRD59(1998)014017

and can be analytically related to the adjoint saturation scale Q_A, Q_B of nucleus A and B

$$(e\tau)_0 = \frac{(N_c^2-1)}{4g^2N_c\sqrt{\pi}}\frac{Q_A^2Q_B^2}{(Q_A^2+Q_B^2)^{5/2}}\left[2Q_A^4+7Q_A^2Q_B^2+2Q_B^4\right]$$

Parameterization of saturation scales

The saturation scales Q_A, Q_B might be parameterized as

Golec-Biernat, Sapeta
JHEP1803(2018)102

$$Q^2_{s,A}(x,\mathbf{b}) \simeq Q^2_{s,0}\left(\frac{Q^2_{s,0}\sigma_0 T_A(\mathbf{b})}{x_0^2 s_{NN}}\right)^{-\lambda/(2+\lambda)} \sigma_0 T_A(\mathbf{b})$$

with thickness function T_A from MC-Glauber

►Now we have model calculating observables from parameters

Physical Modelings

$\frac{dN_{ch}}{d\eta}$ attractor $(e\tau)_0$ CGC $Q_{s,A/B}$ parameterization $Q_{s,0}$, λ, $\left(\frac{C_\infty}{0.95}\right)^{\frac{3}{4}} \left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}}$

observables energy density saturation scale parameters

Statistical Constraints

► We want to construct the initial distribution by constraining parameters from observables
Find probability of parameters using Bayesian inference updating the likelihood function from model simulation

$$P(\mathbf{x}|\mathbf{y}_{\text{exp}}) = \frac{P(\mathbf{y}_{\text{exp}}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{y}_{\text{exp}})}$$

Bayesian inference of model parameters

AuAu,
PbPb,
XeXe,
…

$y_{exp} = \frac{dN_{ch}}{d\eta}$

② Numerically intensity calculations
likelihood function
Markov Chain Monte Carlo (MCMC)

① Numerically intensity calculations
physical modeling
Gaussian process (GP) emulator

Physical Modelings

$y_{th} = \frac{dN_{ch}}{d\eta}$

Norm

X= $Q_{s,0} \quad \lambda \quad \left(\frac{C_\infty}{0.95}\right)^{\frac{3}{4}} \left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}}$

observable

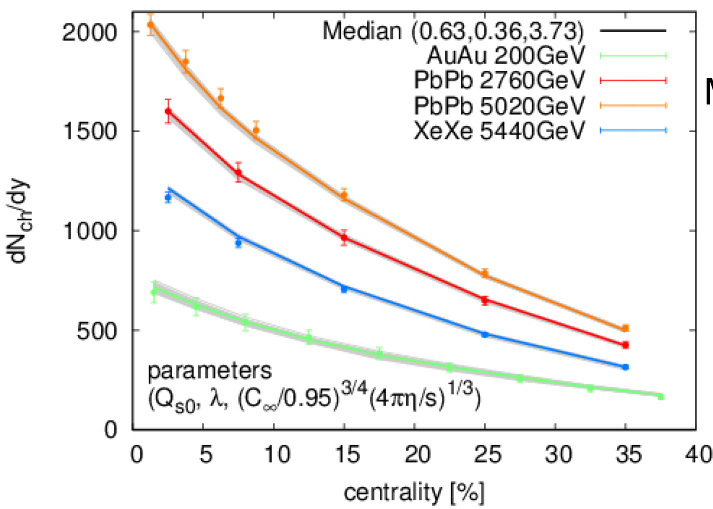
parameters

Statistical Constraints

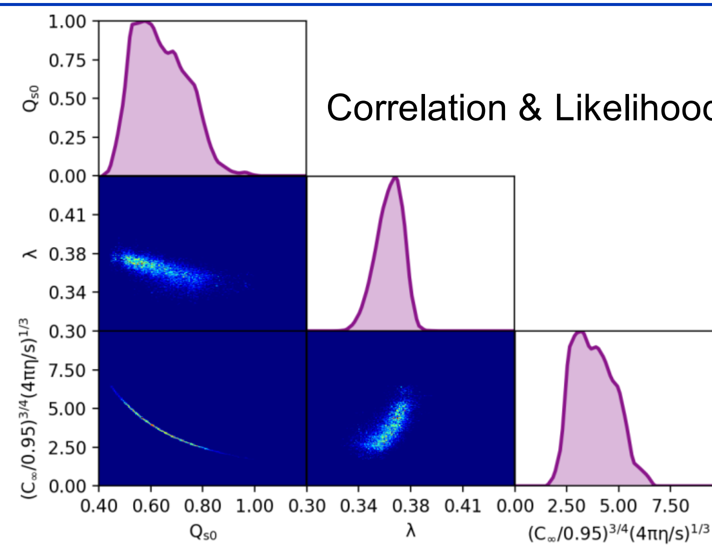
$$P(\mathbf{y}_{\rm exp}|\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^n \det\Sigma}} \exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{y}_{\rm exp})^T \Sigma^{-1}(\mathbf{y}-\mathbf{y}_{\rm exp})\right)$$

$$\Sigma = {\rm diag}(\sigma^2_{\rm exp}) + {\rm diag}(\sigma^2_{\rm th})$$

③ $$P(\mathbf{x}|\mathbf{y}_{\rm exp}) = \frac{P(\mathbf{y}_{\rm exp}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{y}_{\rm exp})}$$



Model/Data comparison



Correlation & Likelihood

Conclusions

⇒ Learn the pre-equilibrium stage:

- Non-equilibrium effective constitutive relation from QCD kinetic theory
  - Before hydrodynamic regime
- Trajectory of pre-equilibrium QGP in T-μ diagram
  - Before hydrodynamic regime
- Applicable time and temperature for hydrodynamics
  - By fixing scales from experiments/lQCD

$$\tau \simeq 1.3\ \mathrm{fm}/c \left(\frac{4\pi\eta/s}{2}\right)^{\frac{3}{2}} \left(\frac{dN_{ch}/d\eta}{1942}\right)^{-\frac{1}{2}} \left(\frac{S_\perp}{138\mathrm{fm}^2}\right)^{\frac{1}{2}}$$

$$T \simeq 300\mathrm{MeV} \left(\frac{4\pi\eta/s}{2}\right)^{-\frac{1}{2}} \left(\frac{dN_{ch}/d\eta}{1942}\right)^{\frac{1}{2}} \left(\frac{S_\perp}{138\mathrm{fm}^2}\right)^{-\frac{1}{2}}$$

⇒ Learn the initial condition:

- Model construction

  Pre-equilibrium attractor + k_T-factorization equipped with GBW model
- Statistical learning

  Bayesian inference of model parameters with GP+MCMC numerically economic simulation

⇒ More applications:

- Appear today on arXiv: Dilepton production: MC, XD, JYO, SS, MW: 2104.07622