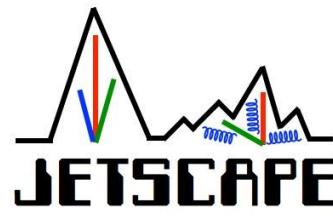


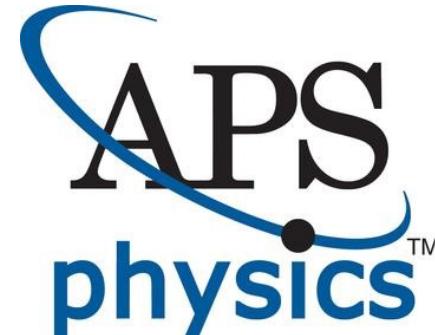
Bayesian constraints on the viscosities of the quark-gluon plasma



Jean-François Paquet (Duke University)

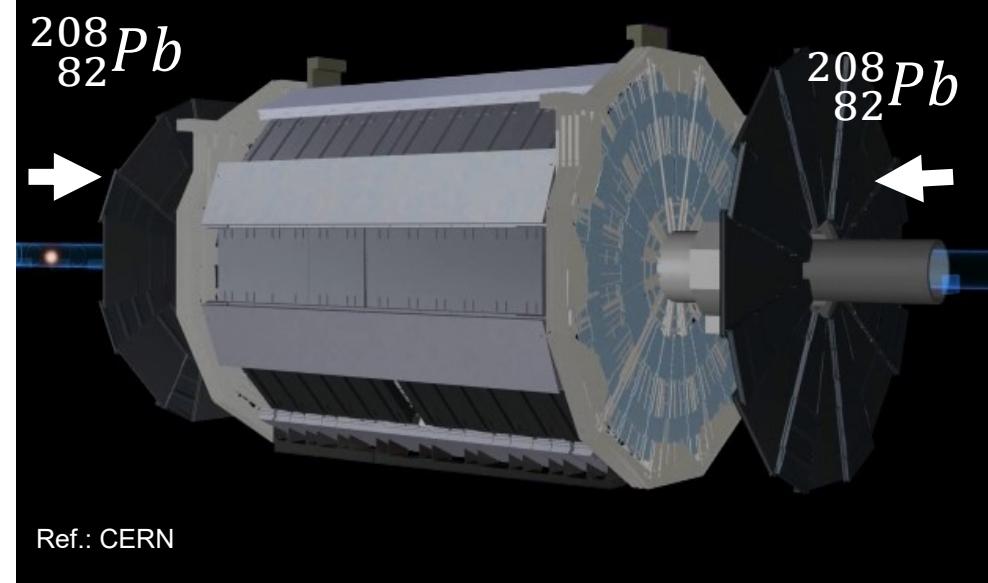
April 14, 2021

9th Workshop of the APS Topical Group on Hadronic Physics

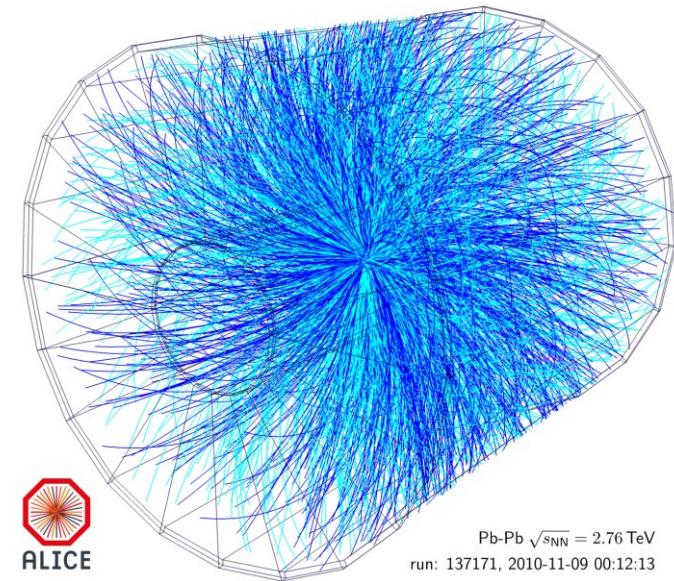


Live from Durham, NC

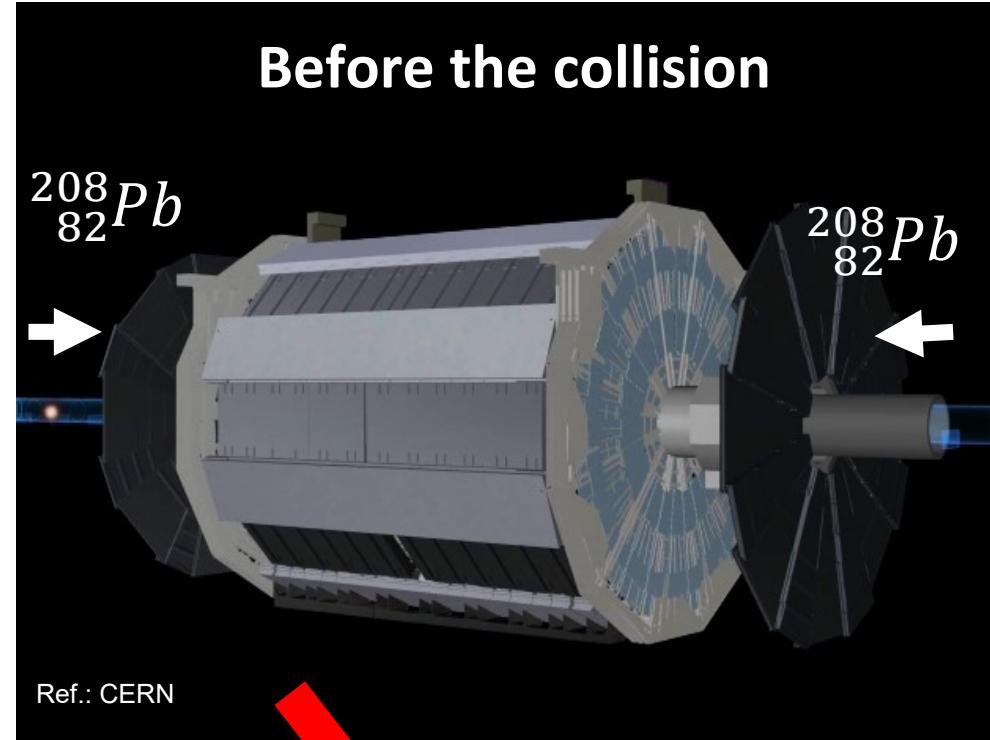
Before the collision



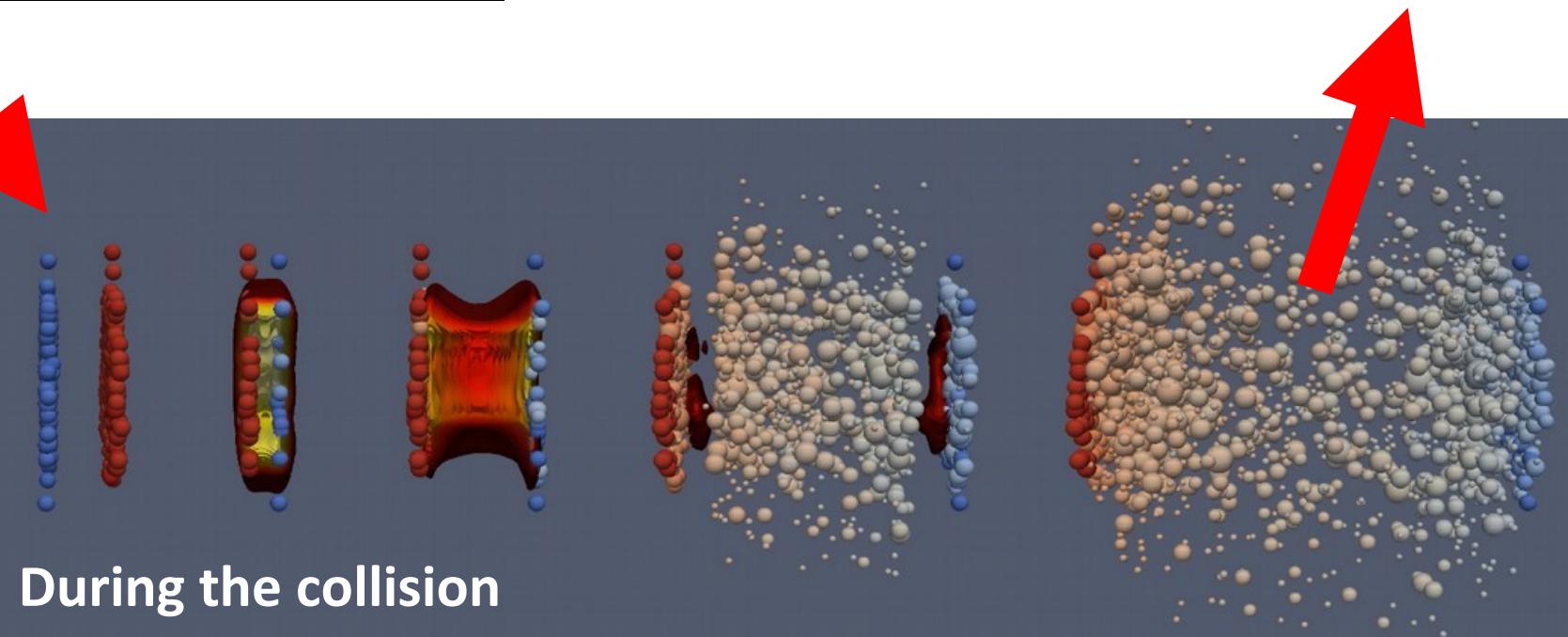
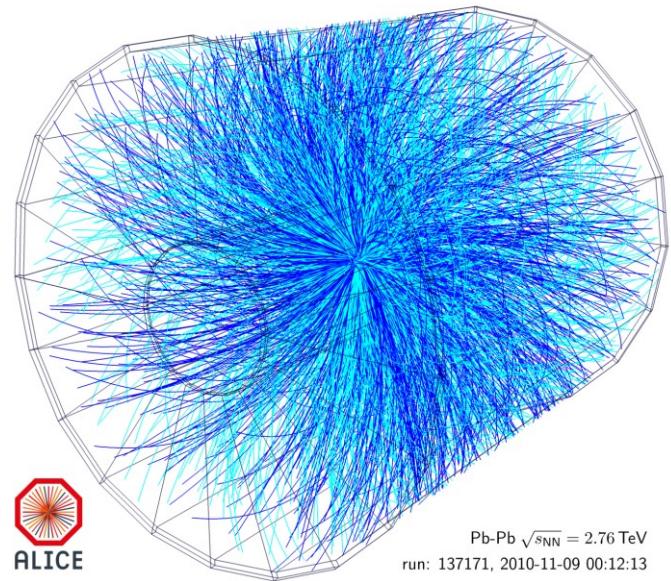
After the collisions



Before the collision

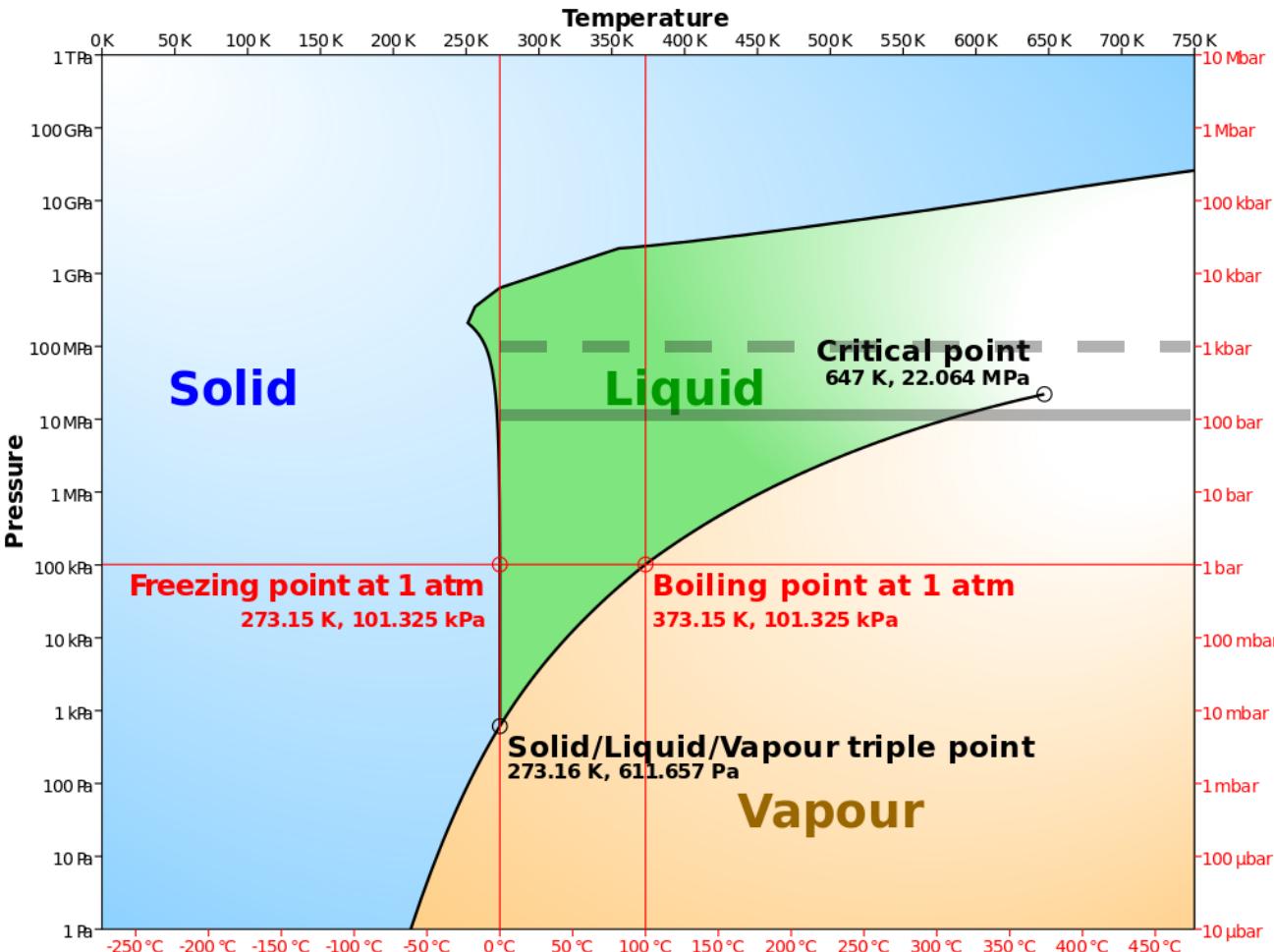


After the collisions



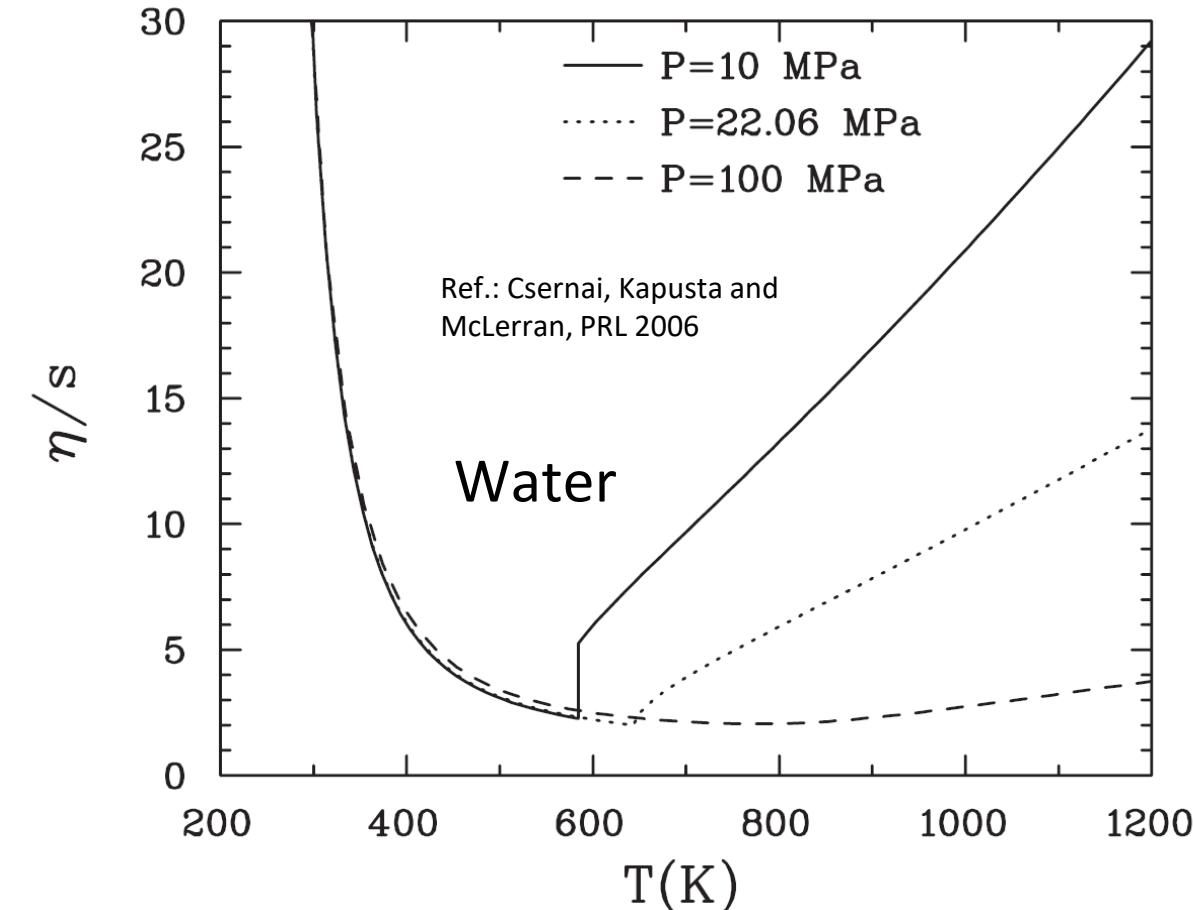
Viscosities as macroscopic properties of fluids: water

Phase diagram



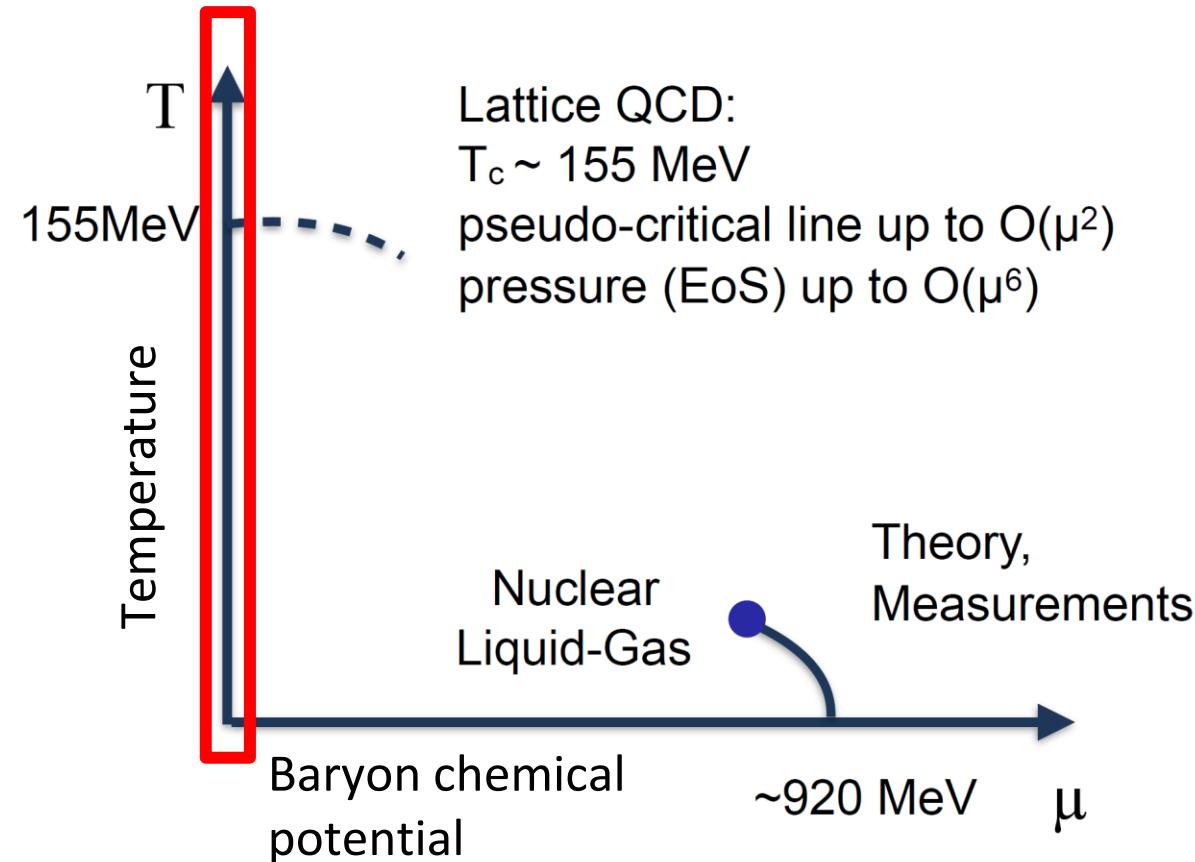
Ref.: https://en.wikipedia.org/wiki/Phase_diagram#/media/File:Phase_diagram_of_water_simplified.svg

Shear viscosity to entropy density ratio
(at different point in the phase diagram)



Viscosities as macroscopic properties of nuclear plasmas

Phase diagram



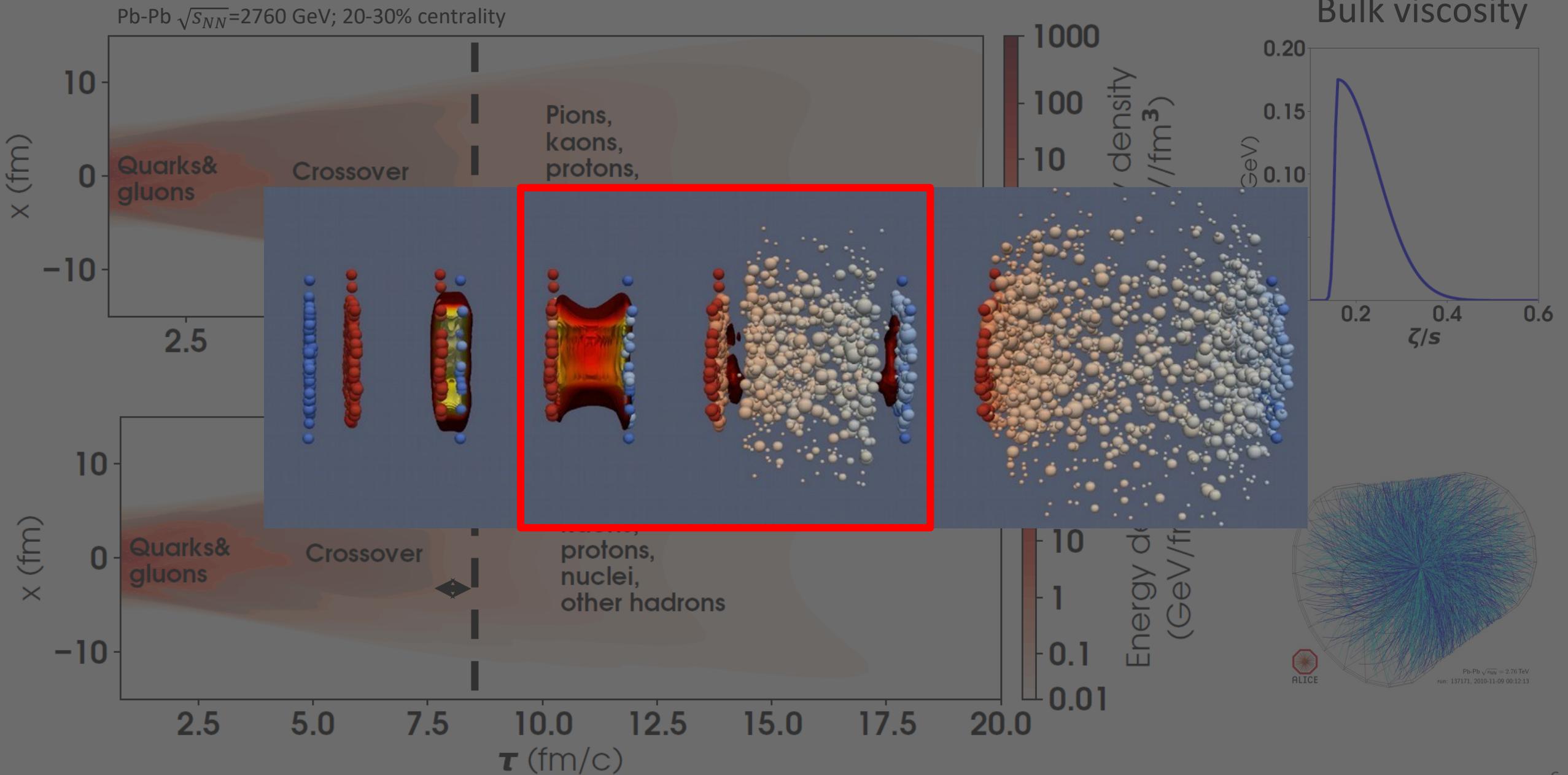
Shear and bulk viscosity to entropy density ratio

$$\eta/s(T, \mu_B = 0) = ?$$

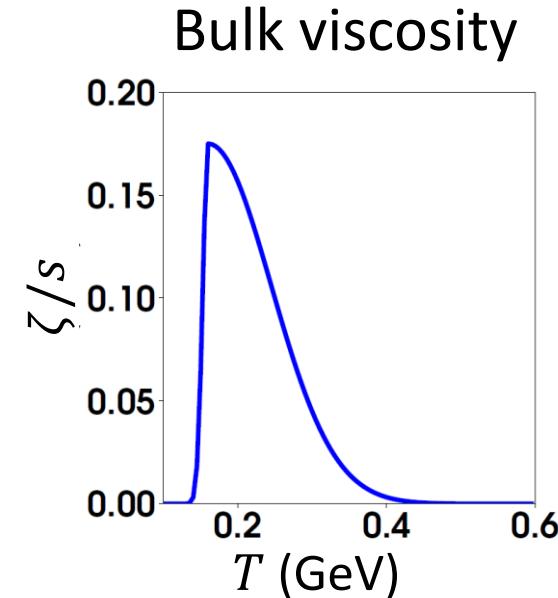
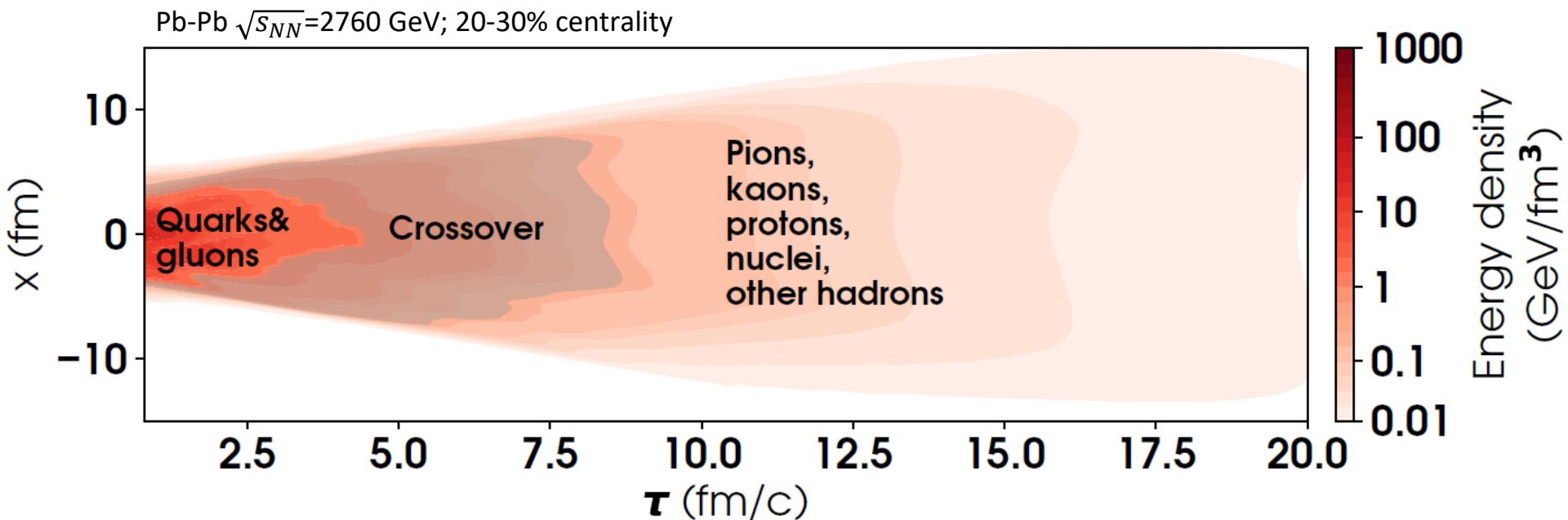
$$\zeta/s(T, \mu_B = 0) = ?$$

Ref.: Volker Koch - Search for the Critical End Point
[Student day - Quark Matter 2019]

Viscosities affect the evolution of the plasma



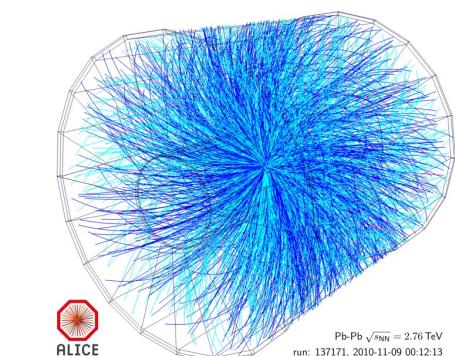
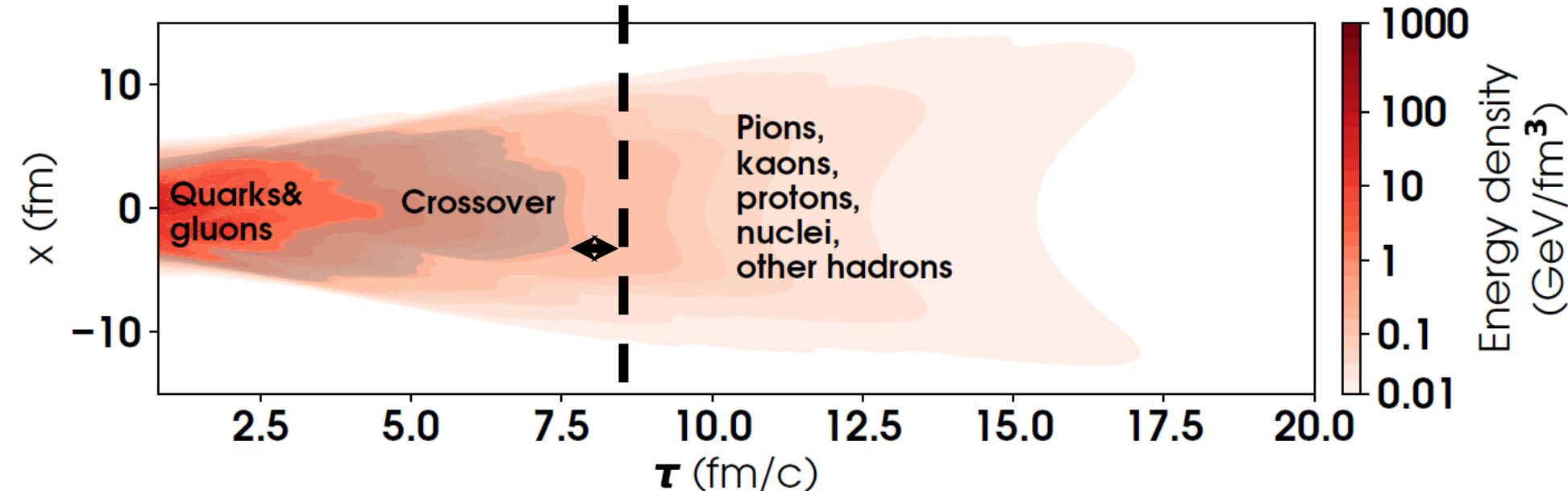
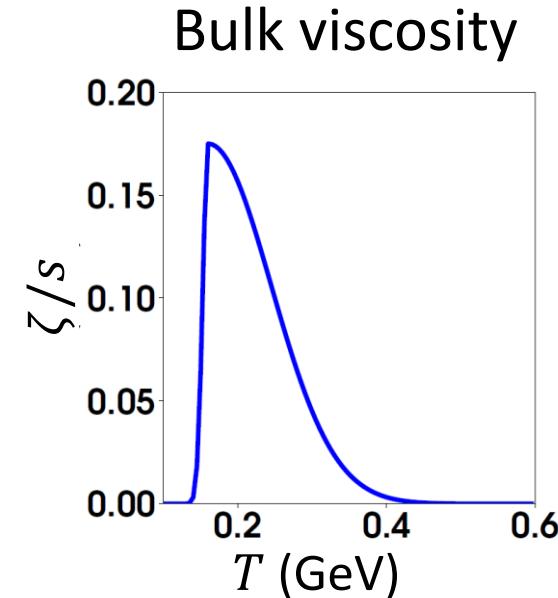
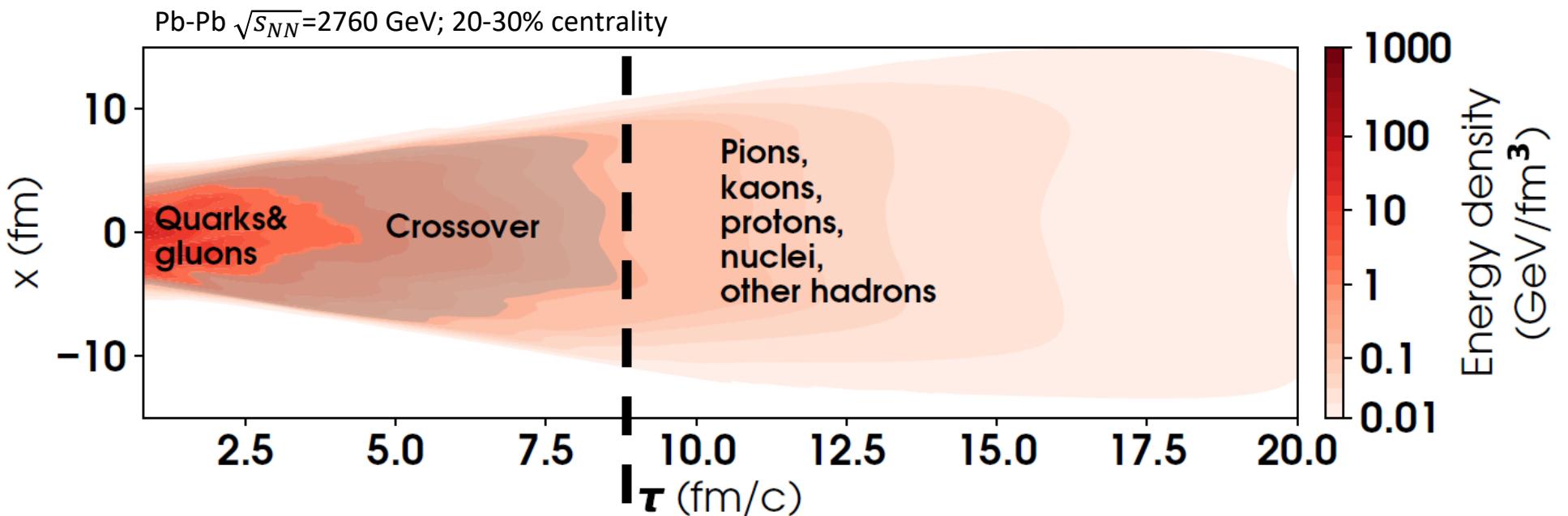
Hydrodynamic evolution of the plasma



Ref.: Gale, JFP, Schenke and
Shen (2021) NPA;
[+in preparation]

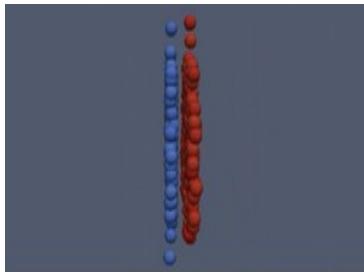
Viscosities affect the evolution of the plasma

Ref.: Gale, JFP, Schenke and Shen
(2021) NPA; [+in preparation]



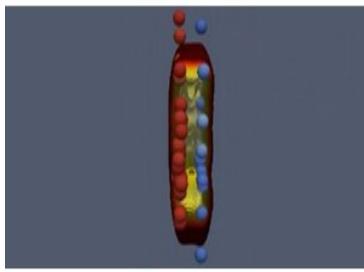
Viscosities and other model parameters

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Collaboration,
arXiv:2011.01430,
arXiv:2010.03928



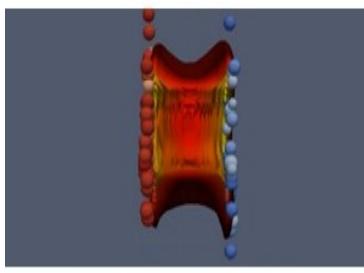
$\tau = "0^+"$: Nuclei collide

- Trento ansatz used to parametrize the energy deposition
- 5 parameters: (i-iii) nucleon width, fluctuation & minimum distance, (iv) transparency parameter, (v) normalization



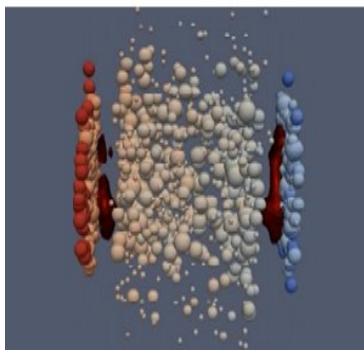
$\tau \sim 0.1 \text{ fm}$: “Pre-equilibrium phase”

- Free-streaming
- Free-streaming time is parameterized



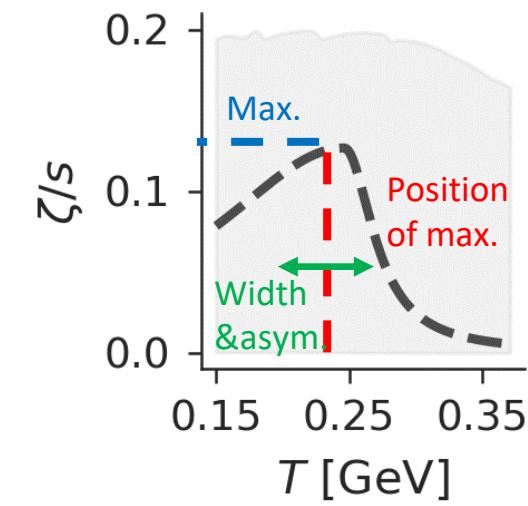
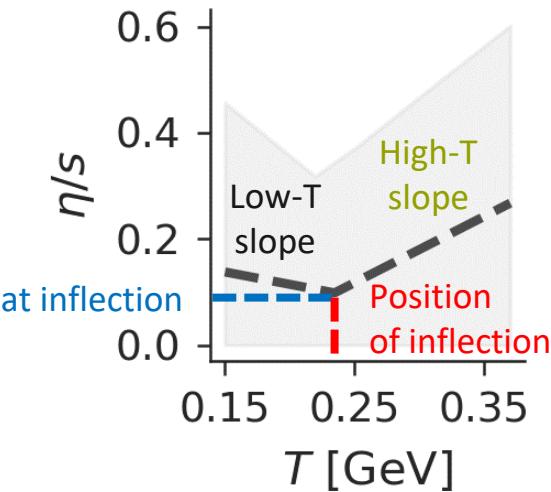
$\tau \sim 1 \text{ fm}$: Beginning of “hydrodynamic phase”

- 2+1D relativistic viscous hydrodynamics [MUSIC]
- Equation of state: hadron resonance gas + lattice QCD [HotQCD]
- Shear and bulk viscosity: $\frac{\eta}{s}(T)$ and $\frac{\zeta}{s}(T)$ parametrized
- Shear relaxation time normalization

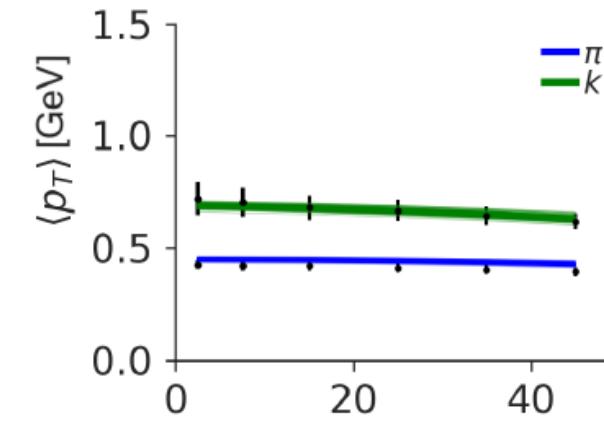
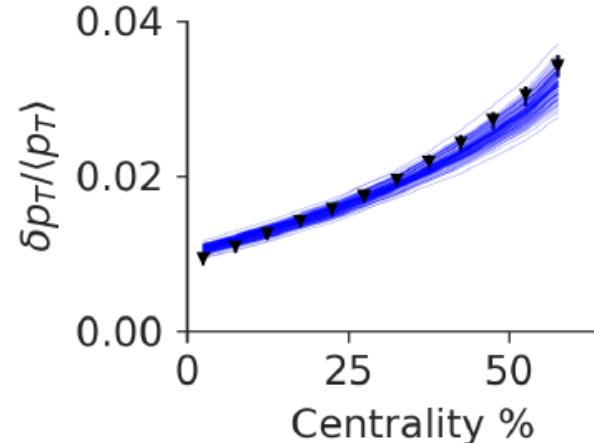
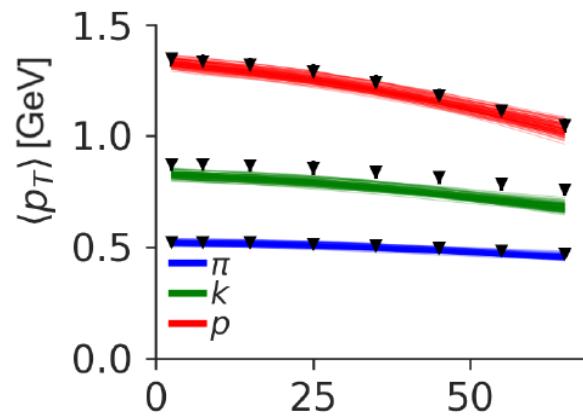
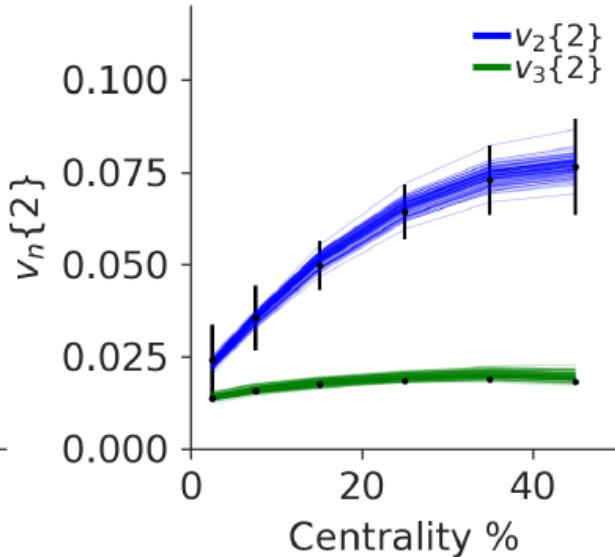
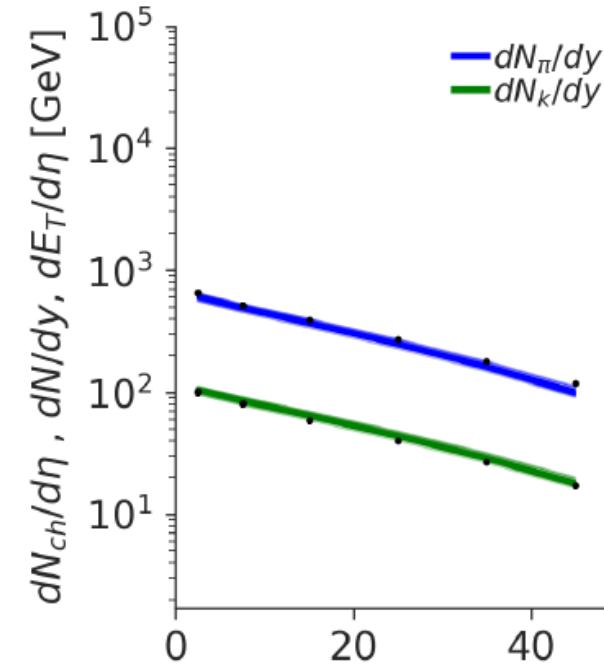
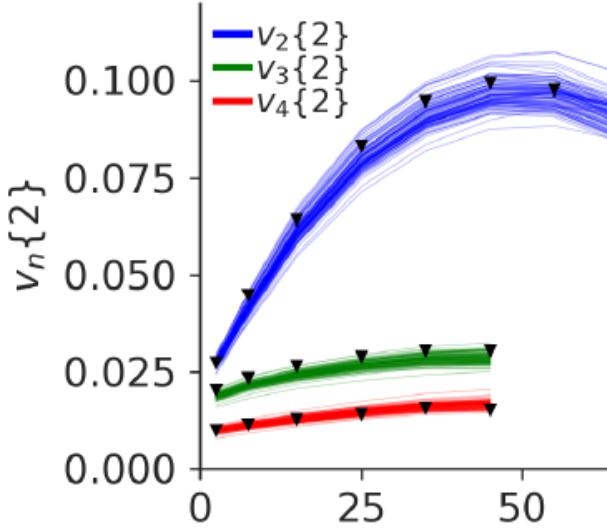
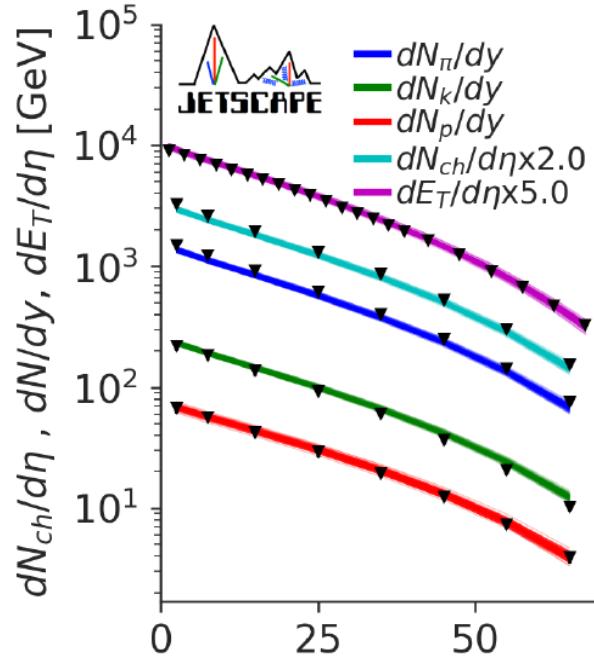


$\tau \sim 10 \text{ fm}$: End of “hydrodynamic phase”

- Fluid converted to hadrons [iS3D]: Cooper-Frye at temperature T_{sw}
- Viscous corrections in Cooper-Frye: 4 different models
- Hadronic interactions with SMASH hadronic transport



Calibrating on measurements



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arXiv:2010.03928

Bayes constraints on model parameters

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Mean value of
measurements

$$\text{posterior}(p|D) \propto \text{prior}(p) \times \exp\left(-\frac{1}{2} \left(D - \text{Model}(p)\right)^T \text{Cov}^{-1} \left(D - \text{Model}(p)\right)\right)$$

↓
↑

Prediction of model for
given set of parameters
 $(\eta/s, \zeta/s, \text{initial}$
 $\text{conditions}, \dots)$

Bayes constraints on model parameters

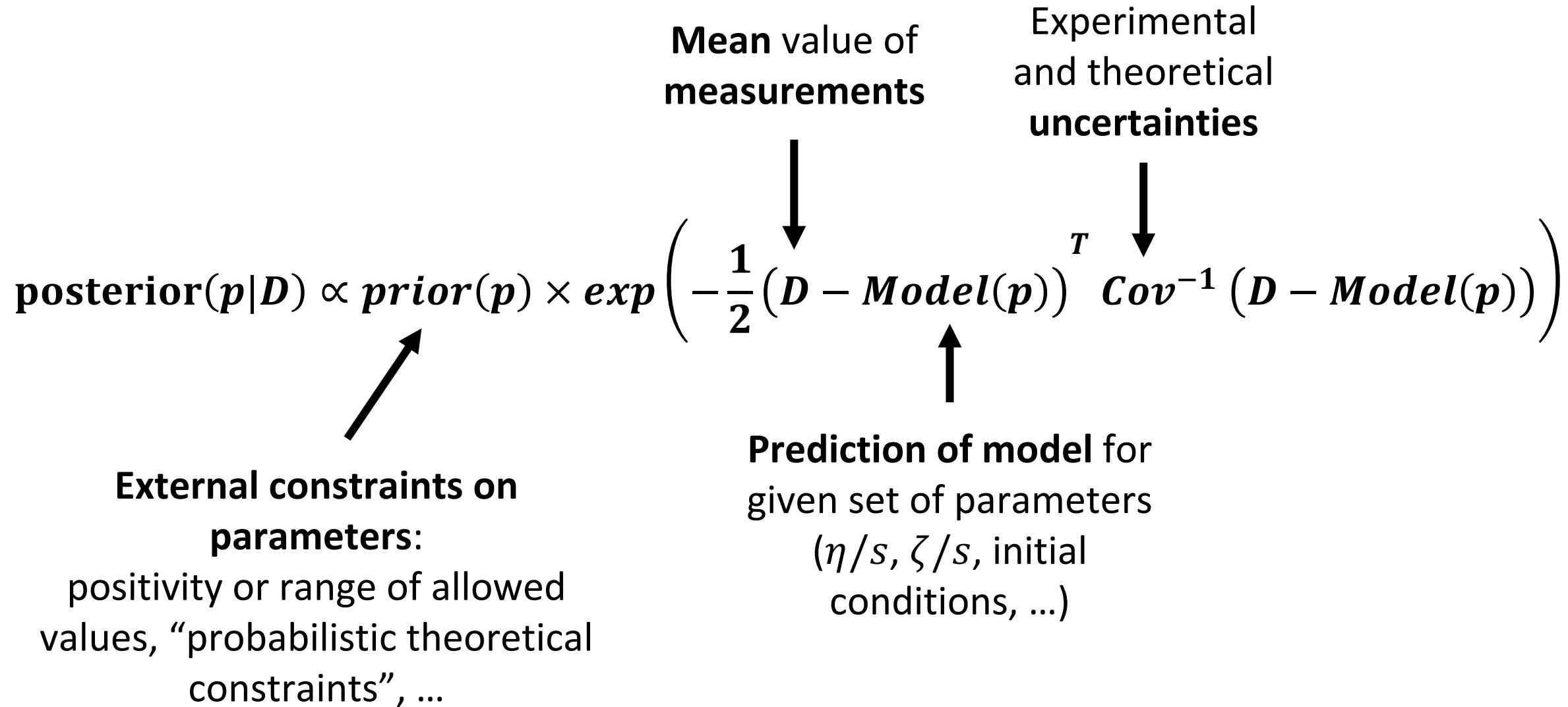
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Collaboration,
arXiv:2011.01430,
arXiv:2010.03928

$$\text{posterior}(p|D) \propto \text{prior}(p) \times \exp\left(-\frac{1}{2} (D - \text{Model}(p))^T \text{Cov}^{-1} (D - \text{Model}(p))\right)$$

↓ ↓
Mean value of measurements **Experimental and theoretical uncertainties**
↑
Prediction of model for given set of parameters
(η/s , ζ/s , initial conditions, ...)

Bayes constraints on model parameters

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arXiv:2010.03928



Bayes constraints on model parameters

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arXiv:2010.03928

Probabilistic constraints
on parameters

$$\text{posterior}(p|D) \propto \text{prior}(p) \times \exp\left(-\frac{1}{2}(D - \text{Model}(p))^T \text{Cov}^{-1}(D - \text{Model}(p))\right)$$

External constraints on
parameters:

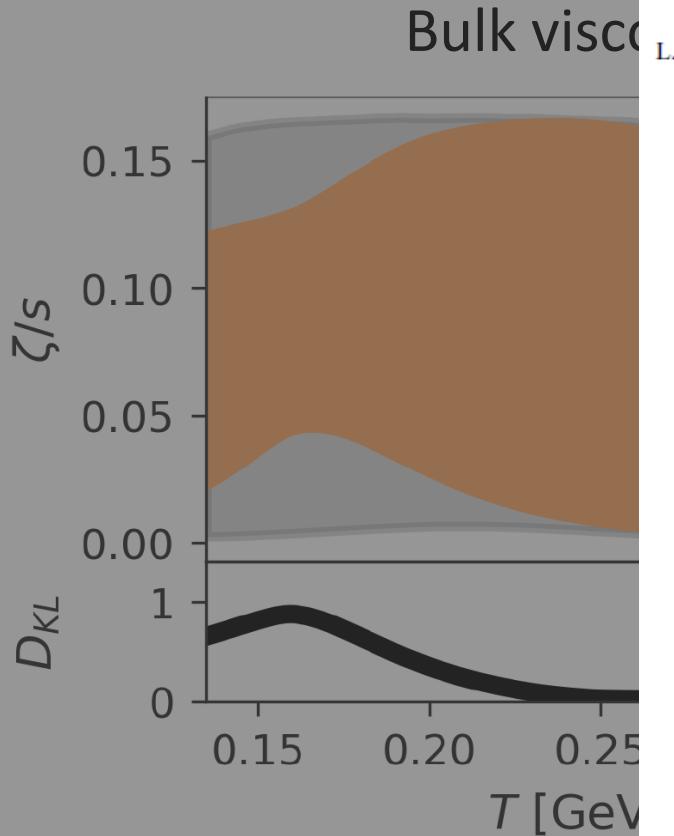
positivity or range of allowed
values, “probabilistic theoretical
constraints”, ...

Mean value of
measurements

Experimental
and theoretical
uncertainties

Prediction of model for
given set of parameters
 $(\eta/s, \zeta/s, \text{initial}$
 $\text{conditions}, \dots)$

Latest constraints



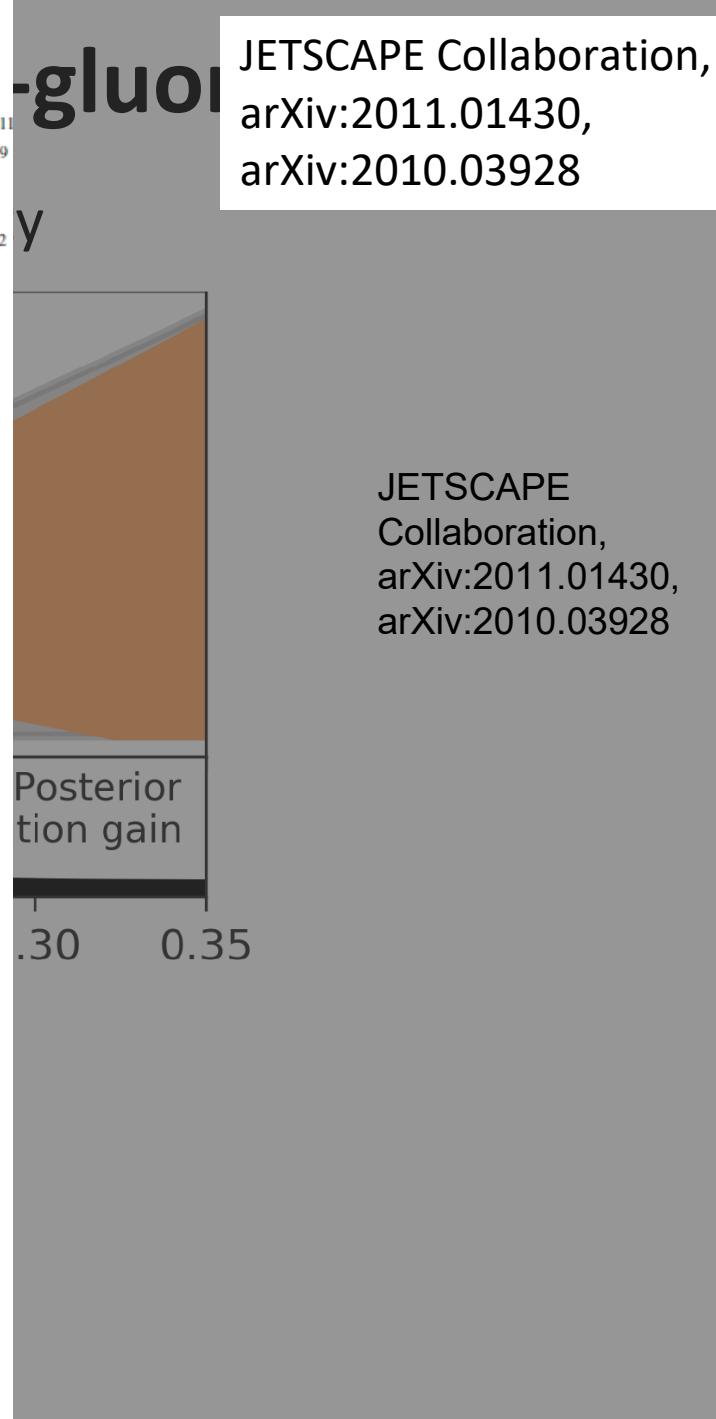
- Result of Bayesian inference
- State-of-the art multi-system hydrodynamics

Multi-system Bayesian constraints on the transport coefficients of QCD matter

D. Everett,¹ W. Ke,^{2,3} J.-F. Paquet,⁴ G. Vujanovic,⁵ S. A. Bass,⁴ L. Du,¹ C. Gale,⁶ M. Heffernan,⁶ U. Heinz,¹ D. Liyanage,¹ M. Luzum,⁷ A. Majumder,⁵ M. McNeils,¹ C. Shen,^{5,8} Y. Xu,⁴ A. Angerami,⁹ S. Cao,⁵ Y. Chen,^{10,11} J. Coleman,¹² L. Cunqueiro,^{13,14} T. Dai,⁴ R. Ehlers,^{13,14} H. Elfner,^{15,16,17} W. Fan,⁴ R. J. Fries,^{18,19} F. Garza,^{18,19} Y. He,²⁰ B. V. Jacak,^{2,3} P. M. Jacobs,^{2,3} S. Jeon,⁶ B. Kim,^{18,19} M. Kordell II,^{18,19} A. Kumar,⁵ S. Mak,¹² J. Mulligan,^{2,3} C. Nattrass,¹³ D. Oliinychenko,³ C. Park,⁶ J. H. Putschke,⁵ G. Roland,^{10,11} B. Schenke,²¹ L. Schwiebert,²² A. Silva,¹³ C. Sirimanna,⁵ R. A. Soltz,^{5,9} Y. Tachibana,⁵ X.-N. Wang,^{20,2,3} and R. L. Wolpert¹²

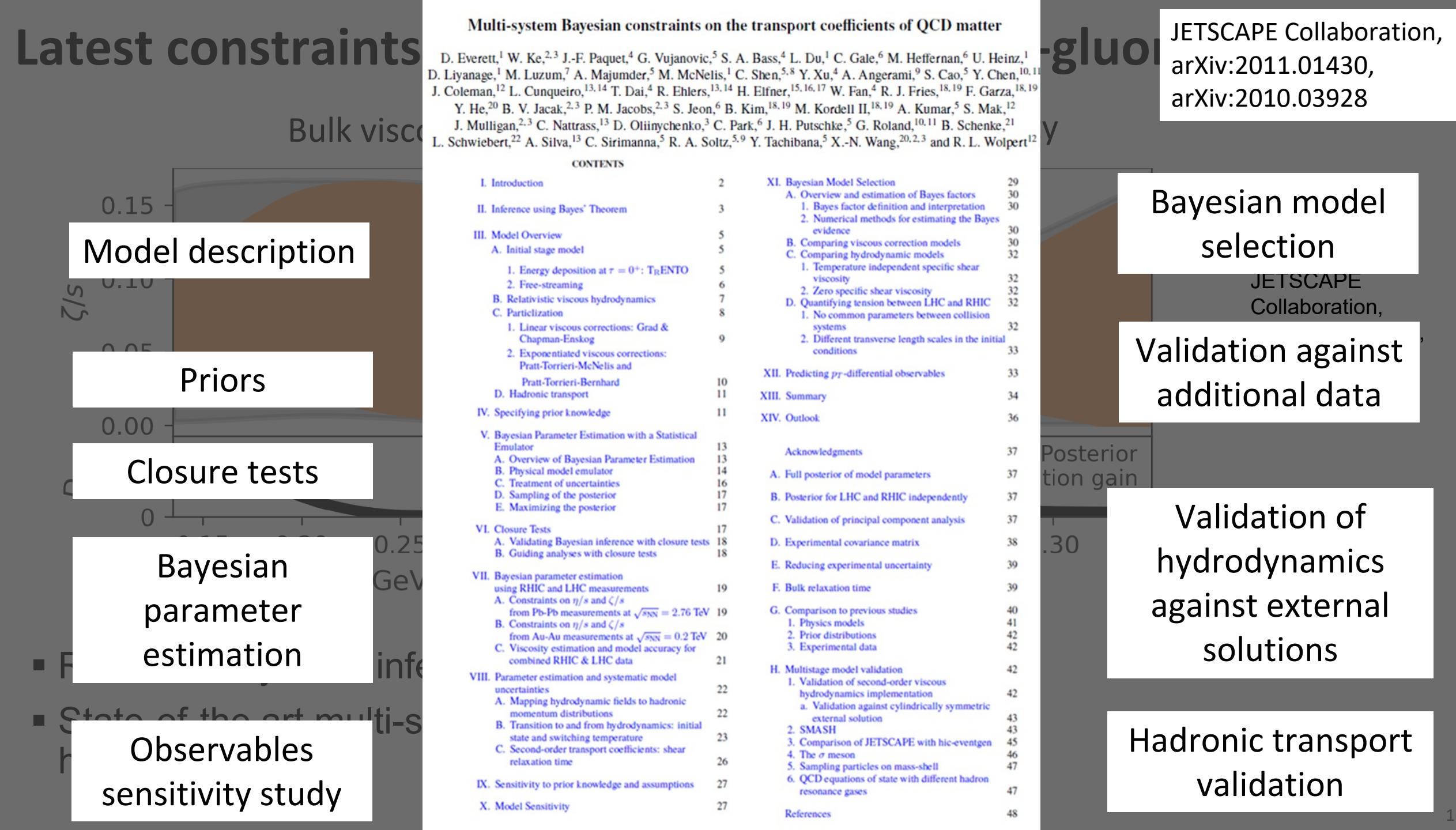
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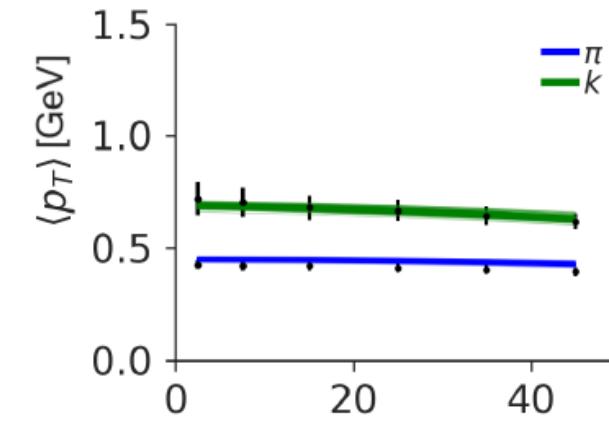
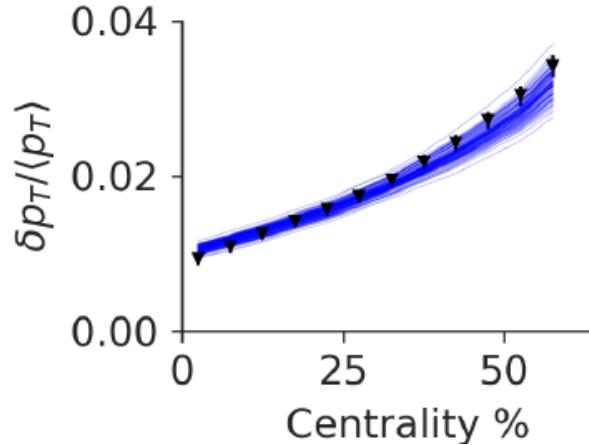
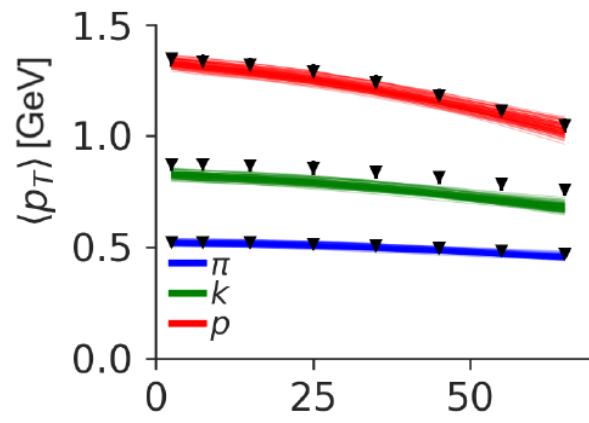
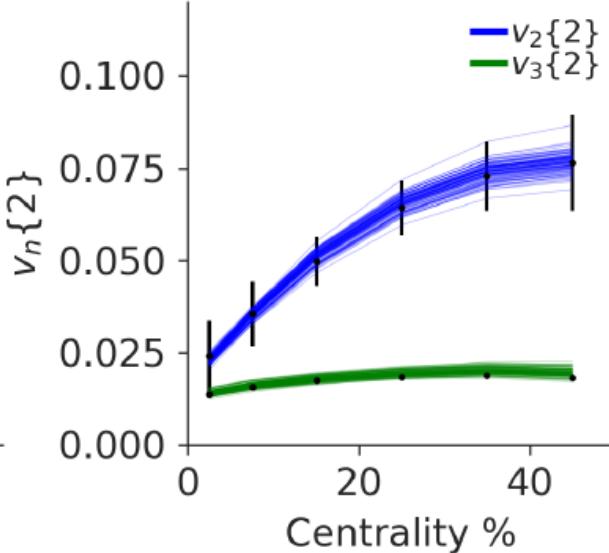
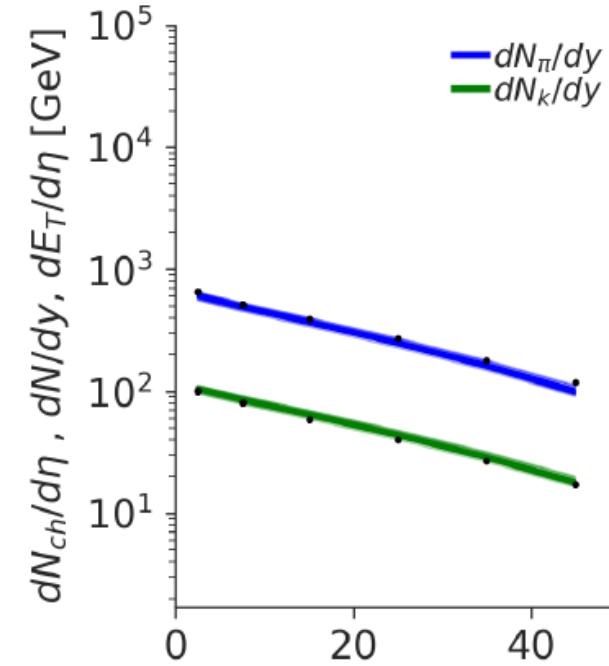
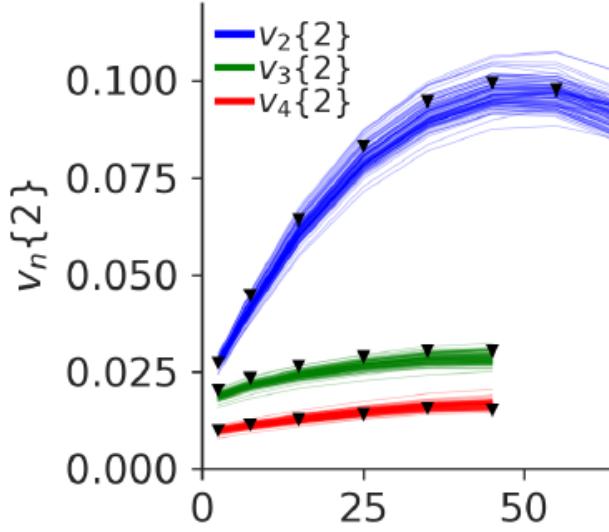
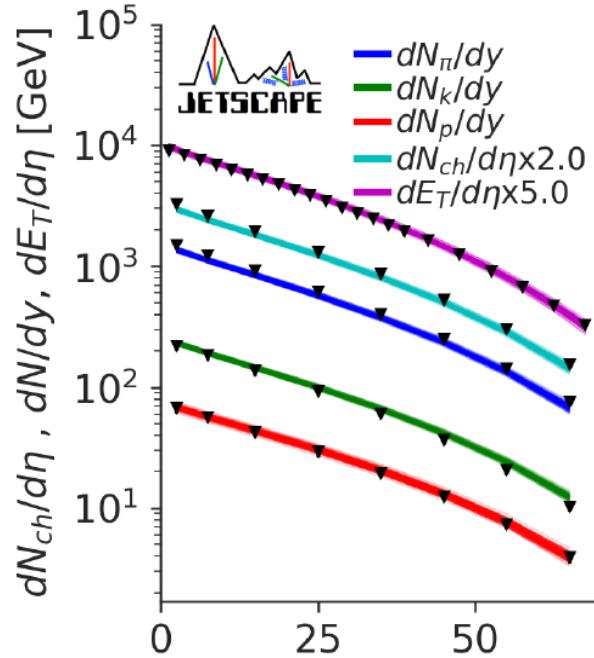


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arXiv:2010.03928

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arXiv:2010.03928



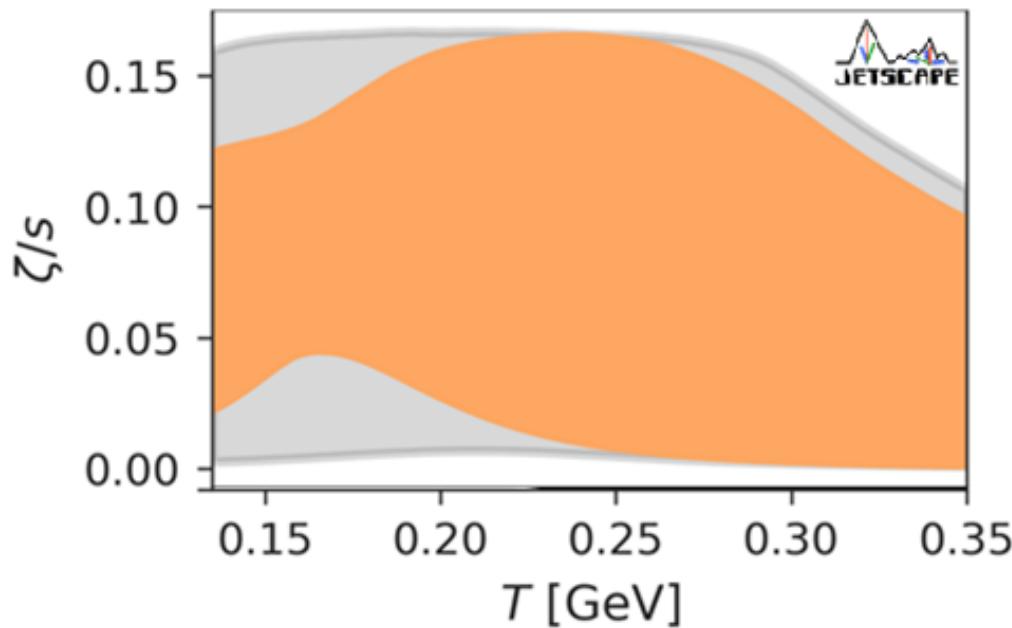
Agreement with measurements



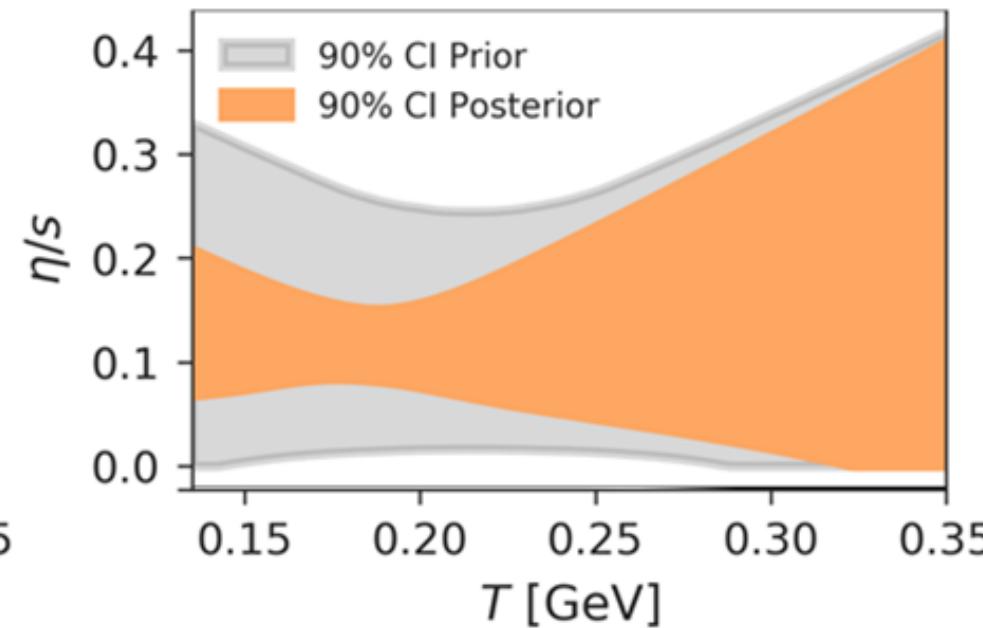
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Latest constraints on the viscosities of quark-gluon plasma

Bulk viscosity



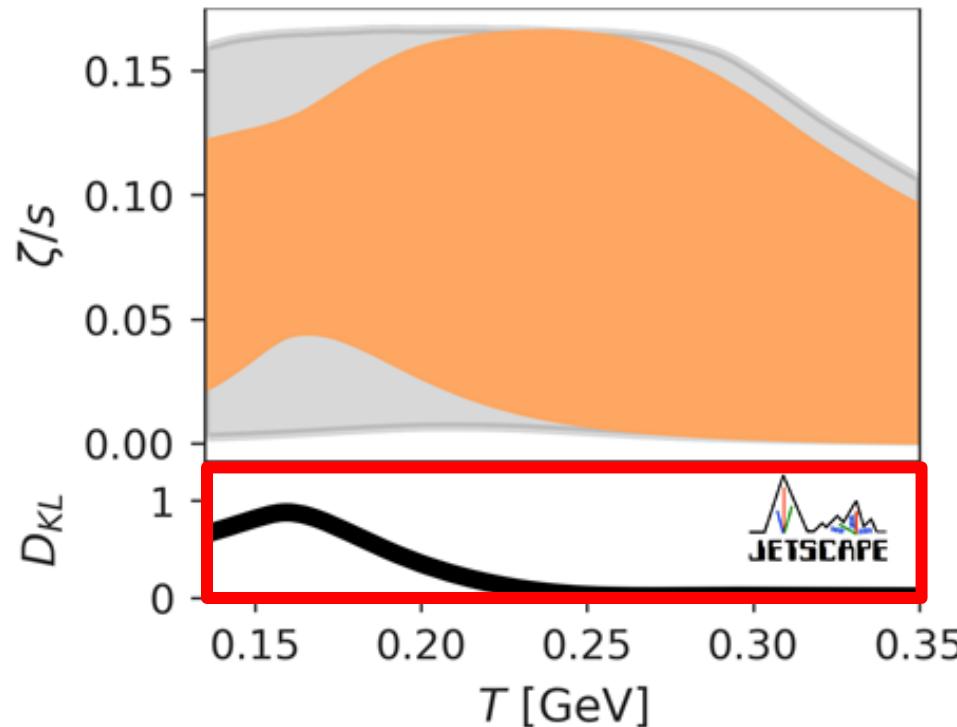
Shear viscosity



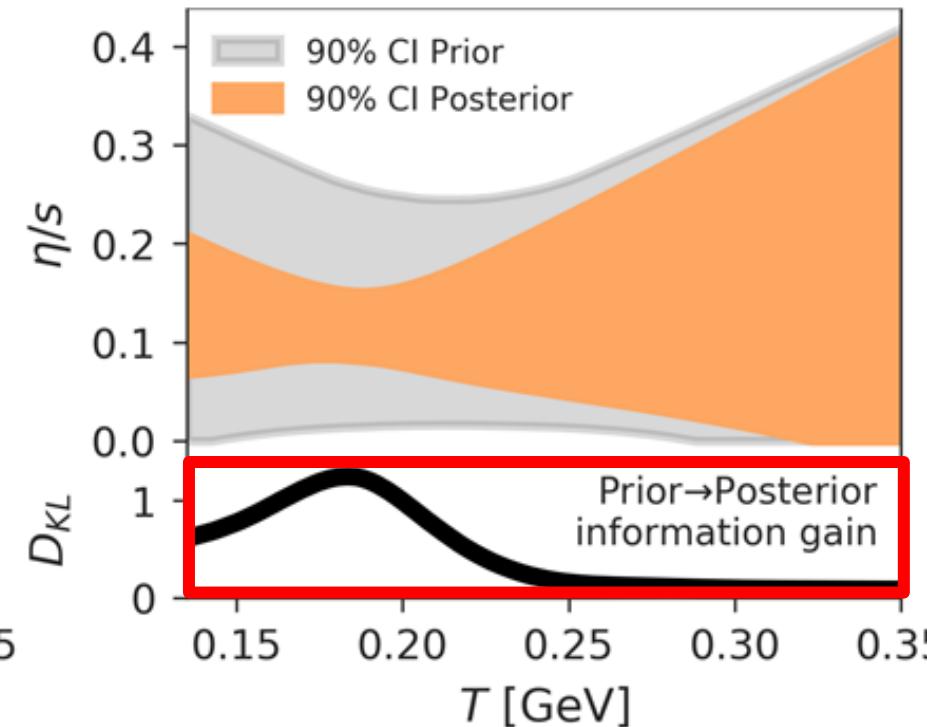
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How to understand Bayesian constraints

Bulk viscosity



Shear viscosity

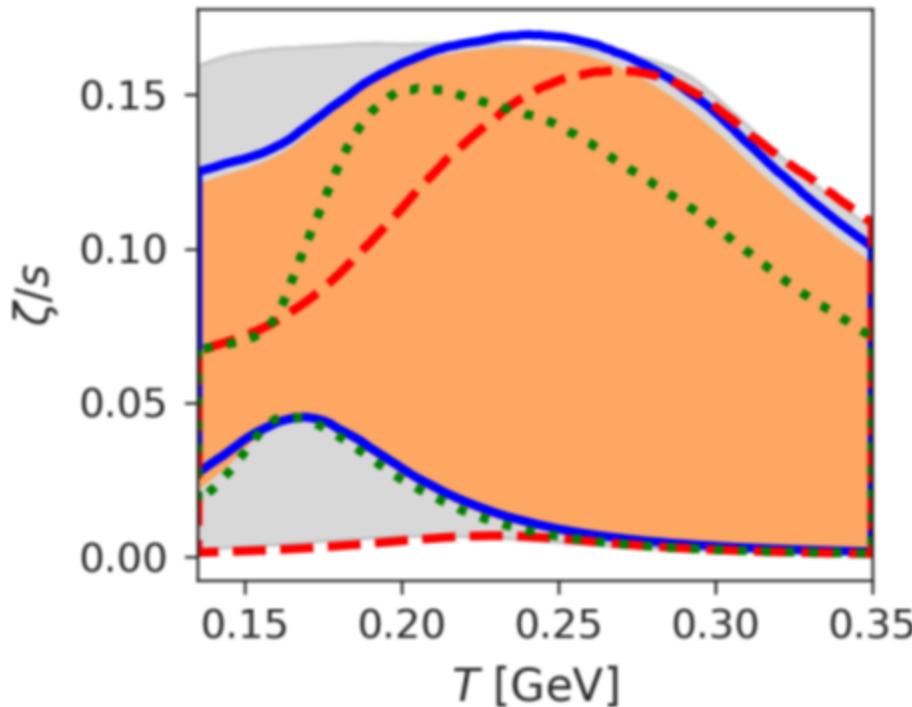


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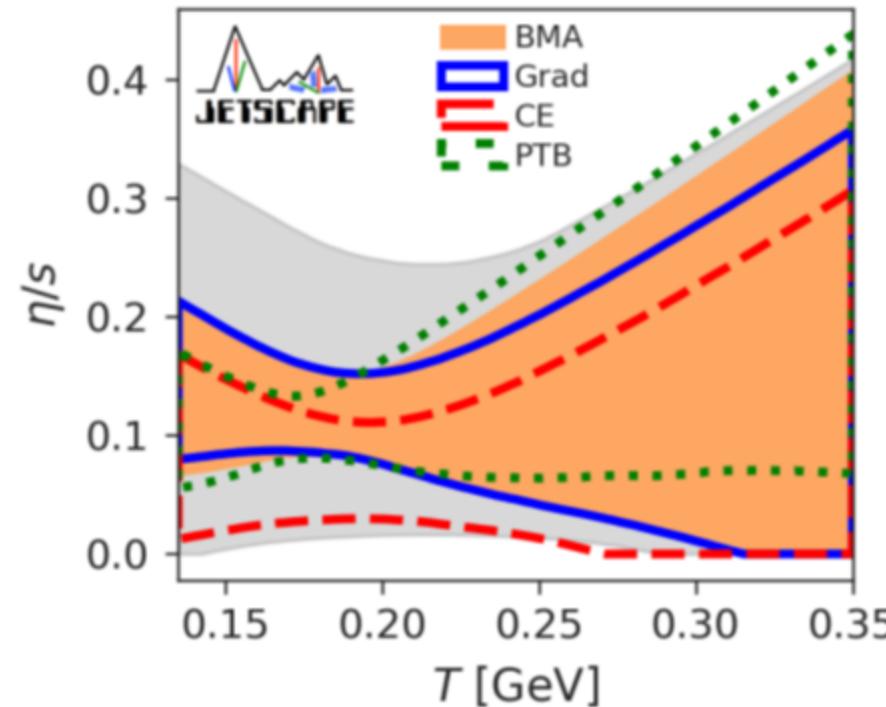
- Compare with priors
- Kullback–Leibler divergence D_{KL} can quantify prior-posterior differences

The importance of including theoretical uncertainties

Bulk viscosity



Shear viscosity

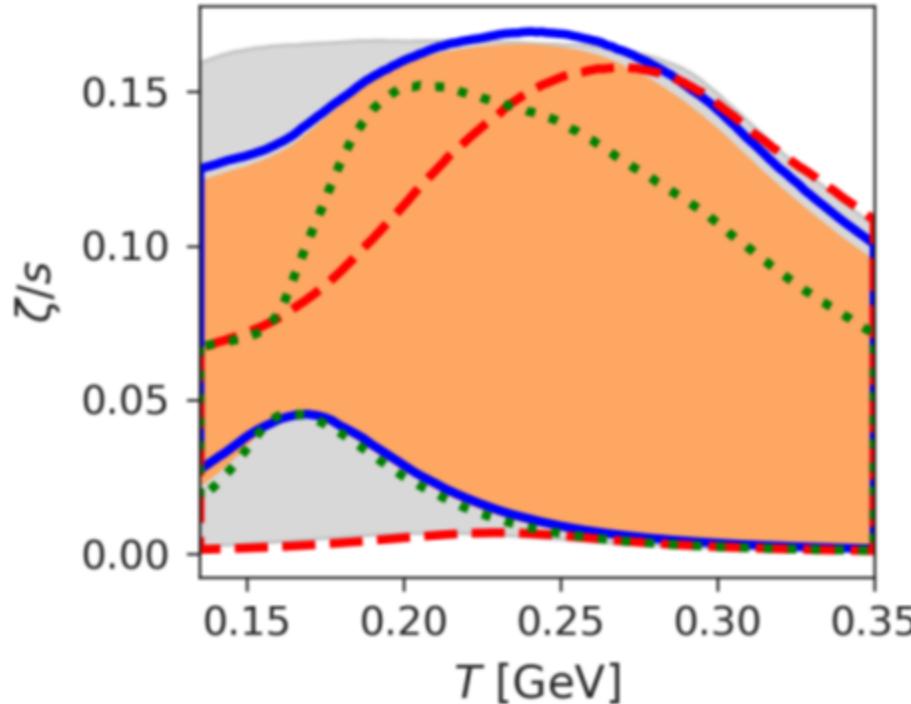


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[arXiv:2011.01430](https://arxiv.org/abs/2011.01430),
[arXiv:2010.03928](https://arxiv.org/abs/2010.03928)

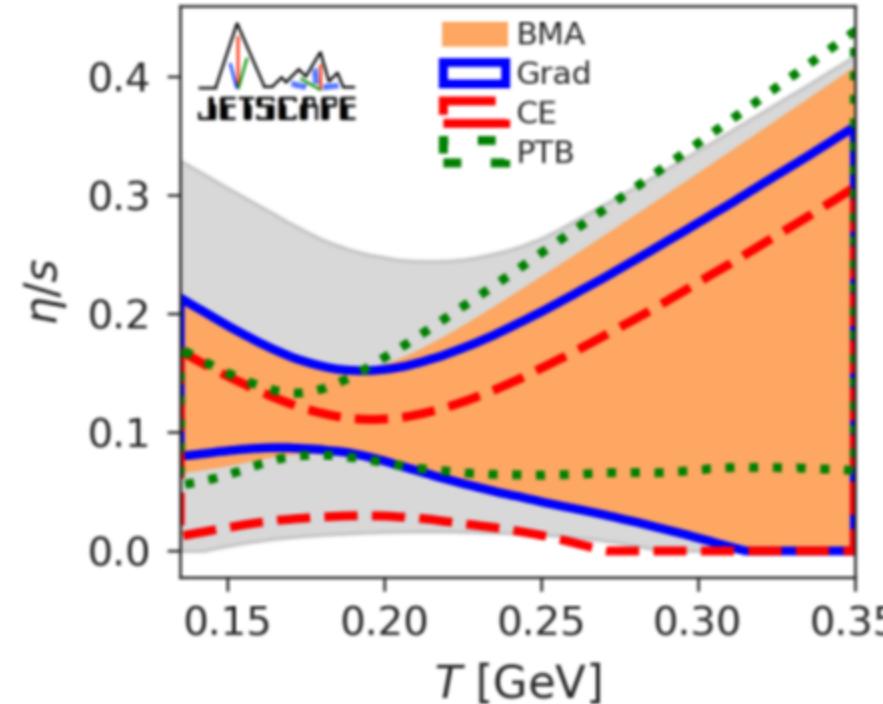
- Different line colours are different “particilization model”
(particilization = transition from hydrodynamics to hadronic transport)

The importance of including theoretical uncertainties

Bulk viscosity



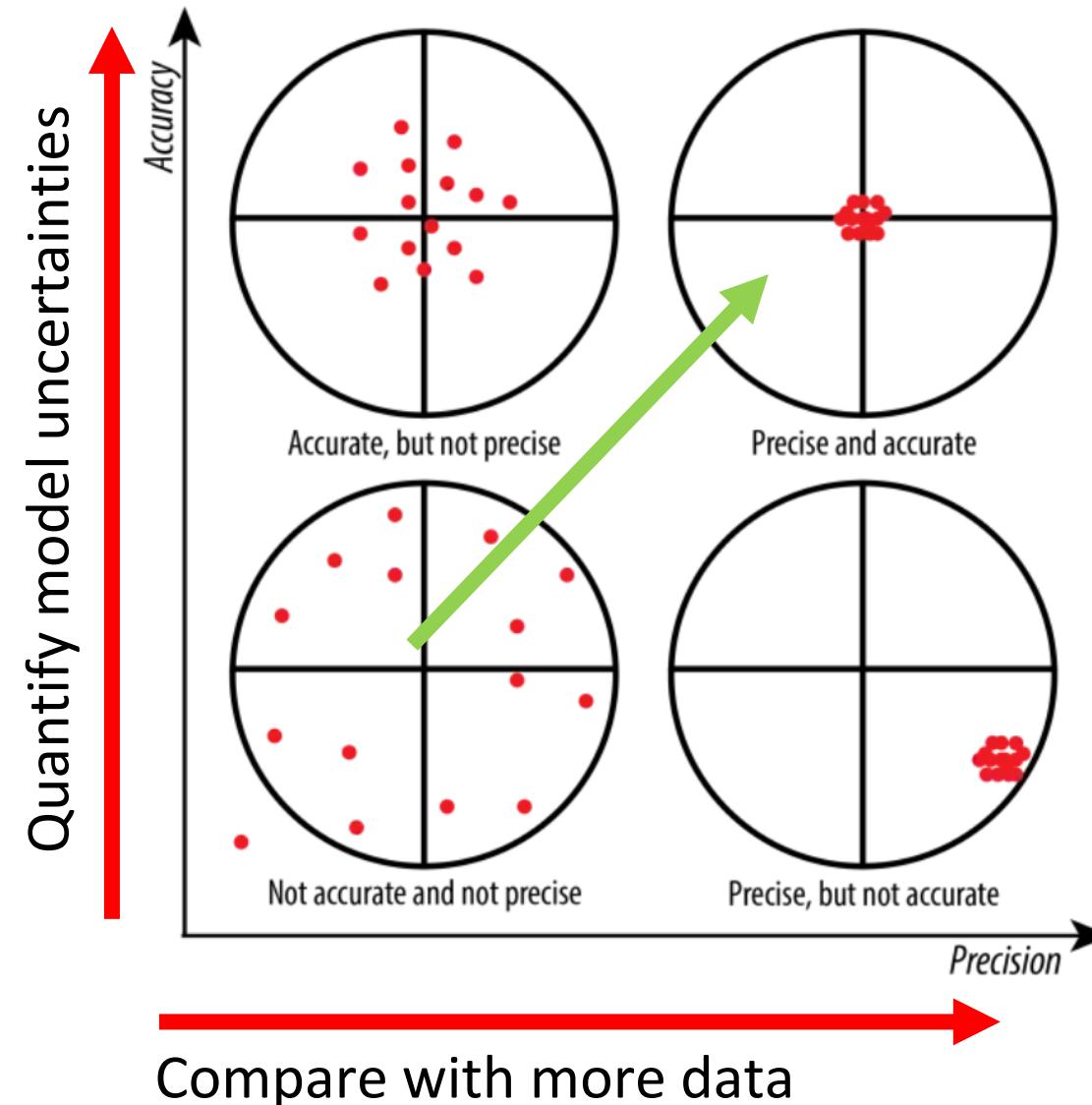
Shear viscosity



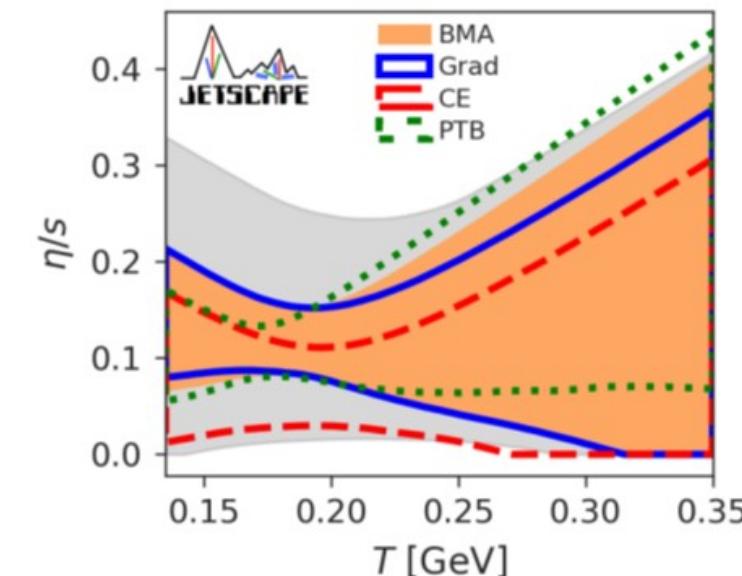
JETSCAPE
Collaboration,
[arXiv:2011.01430](https://arxiv.org/abs/2011.01430),
[arXiv:2010.03928](https://arxiv.org/abs/2010.03928)

- Different line colours are different “particilization model”
(particilization = transition from hydrodynamics to hadronic transport)
- Shaded average is “Bayesian model average”:
average weighted by degree of agreement with data (“Bayes factor”)

Precision vs accuracy

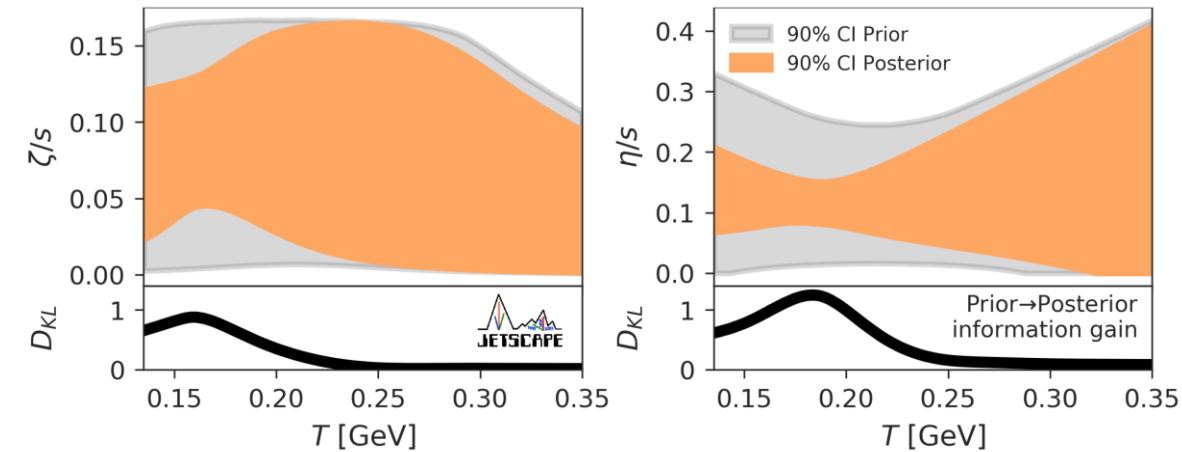


- Important not to confuse narrow constraints with accurate ones
- Relaxing model assumptions generally lead to weaker but more accurate constraints
- More data + inclusion/reduction of model uncertainties = Better constraints on viscosities



Summary: Bayesian constraints on $\eta/s(T)$ and $\zeta/s(T)$ of QCD

- “State-of-the-art” constraints on the QCD viscosities from recent JETSCAPE analysis
- Systematic constraints
- Importance of prior-posterior comparisons
- Including model uncerts increases accuracy
- Precision \neq accuracy



Outlook:

- Additional data can constrain viscosity at higher temperature; model dependence? [Nijs, Van Der Schee, Gürsoy, Snellings, 2010.15134, 2010.15130]
- Additional source of theoretical uncertainties must be accounted for
⇒ **Initial conditions**; Also, equation of state [Auvinen et al (2020) PRC]
- Experimental uncertainties: correlations between uncertainties [2102.11337]

arXiv:2011.01430 is a systematic framework that can include future improvements

Acknowledgements

Bayesian inference results are from the JETSCAPE Collaboration, with **Derek Everett** (OSU) & **Weiyao Ke** (UC Berkeley & LBNL) driving the effort.

See:

<https://inspirehep.net/literature/1827929>

<https://inspirehep.net/literature/1821941>



Cornell University

High Energy Physics - Phenomenology

arXiv:2011.01430 (hep-ph)

[Submitted on 3 Nov 2020 (v1), last revised 6 Nov 2020 (this version, v2)]

Multi-system Bayesian constraints on the transport coefficients of QCD matter



Cornell University



High Energy Physics - Phenomenology

arXiv:2010.03928 (hep-ph)

[Submitted on 8 Oct 2020]

Phenomenological constraints on the transport properties of QCD matter with data-driven model averaging

This work was supported by:



J.-F.P. was supported by the U.S. Department of Energy (DOE) under award number DE-FG02-05ER41367 and by the National Science Foundation (NSF) under award number ACI-1550300

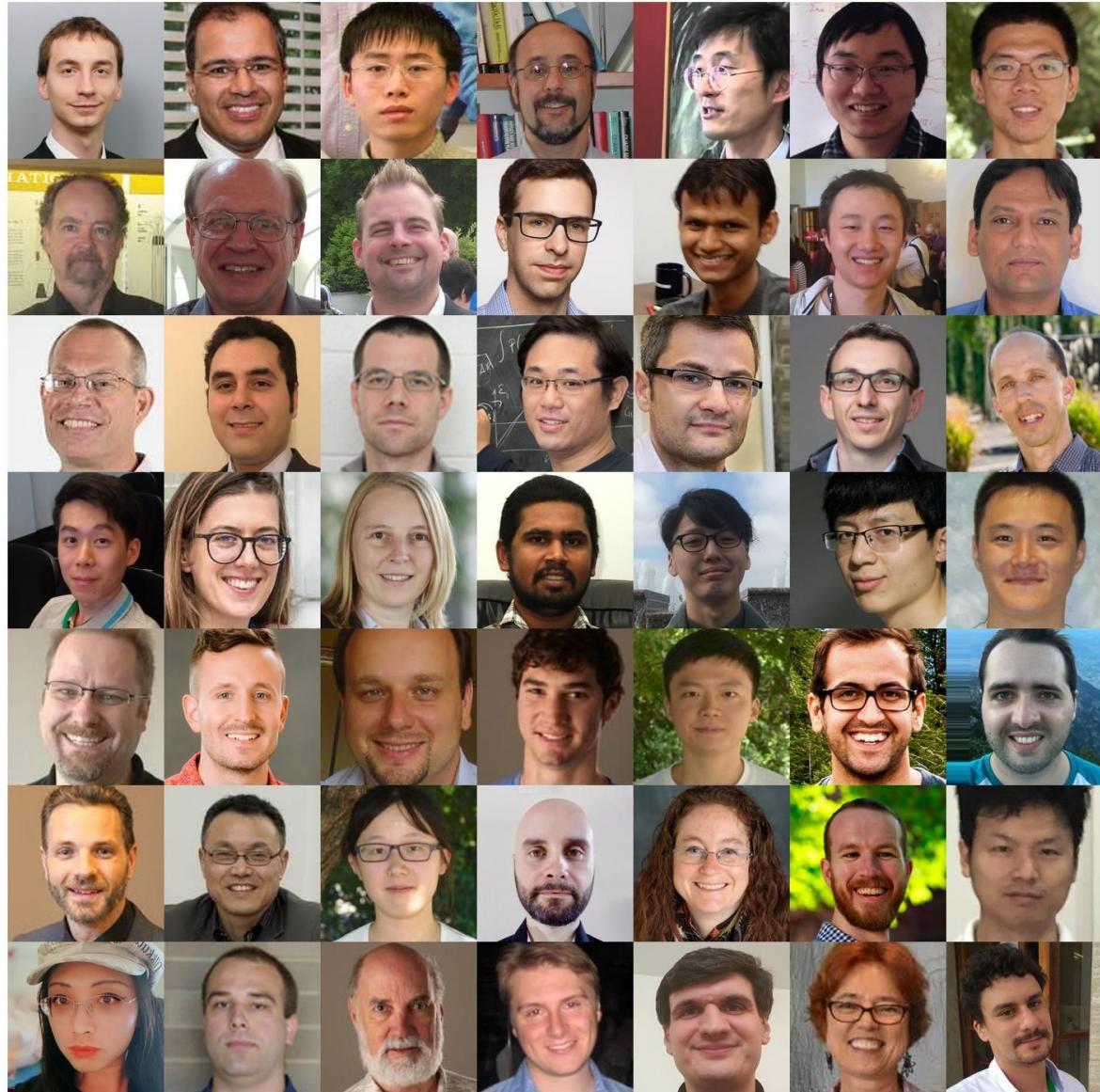


XSEDE

Extreme Science and Engineering Discovery Environment

Computational resources from XSEDE & TACC

The JETSCAPE Collaboration



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Questions?

References for multistage model

JETSCAPE
Collaboration,
arXiv:2011.01430,
arXiv:2010.03928

$\tau = "0^+"$: Nuclei collide

- Trento ansatz used to parametrize the energy deposition

Ref.: Moreland, Bernhard, Bass (2015) PRC92,011901

$\tau \sim 0.1 \text{ fm}$: “Pre-equilibrium phase”

- Free-streaming

Ref.: Everett (2018), <https://github.com/derekeverett/freestream-milne>

$\tau \sim 1 \text{ fm}$: Beginning of “hydrodynamic phase”

- 2+1D relativistic viscous hydrodynamics [MUSIC]
- Equation of state: hadron resonance gas + lattice QCD [HotQCD Collaboration (2014) PRD90,094503]
- Shear and bulk viscosity

MUSIC ref.: Schenke, Jeon, Gale (2010) PRC82,014903; (2011) PRL106,042301; Paquet, Shen, Denicol, Luzum, Schenke, Jeon, Gale (2016) PRC93,044906

Hadron resonance gas + lattice combination: https://github.com/j-f-paquet/eos_maker

$\tau \sim 10 \text{ fm}$: End of “hydrodynamic phase”

- Fluid converted to hadrons [iS3D]
- Hadronic interactions with SMASH hadronic transport

iS3D ref.: McNelis, Everett, Golden & Heinz, in preparation; <https://github.com/derekeverett/iS3D>

SMASH ref.: Weil, Steinberg, Staudenmaier, Pang, Oliinychenko, Mohs, Kretz, Kehrenberg, Goldschmidt, Bäuchle, Auvinen, Attems, Petersen (2016) PRC94, 054905
<https://smash-transport.github.io/>

Agreement with measurements

$$\frac{y_{model} - y_{exp}}{\sigma_{exp}}$$

