Nonperturbative excitations in overoccupied gluon plasmas

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GHP2021,
April 16, 2021

Talk mainly based on:

arXiv:2101.02715
PRD 100, 094022 (2019), [arXiv:1907.05892]
PRD 98, 014006 (2018), [arXiv:1804.01966]

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Introduction

The goal is to study

Microscopic properties of QCD nonperturbatively

$\Rightarrow$ *Spectral functions* $\rho(\omega, p)$ of gluons / quarks encode full excitation spectrum!

- Our approach: *far from equilibrium*
  - ✓ Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)
  - ✓ Then nonperturbative and perturbative methods available!
- Classical-statistical lattice simulations vs. HTL, kinetic theory

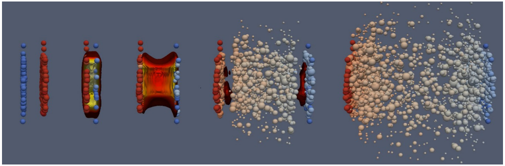
  class. covariant equ.: $D_\mu F^{\mu\nu} = 0$, HTL: $\Pi^{\mathrm{HTL}}_{\mu\nu}$

- Nonequilibrium application: heavy-ion collisions

Motivation: heavy-ion collisions

Application

Microscopic properties of non-equilibrium QCD



MADAI collaboration

✓ Initial stages in heavy-ion collisions suitable playground

✓ Quark-gluon plasma initially: eff. 2+1D and 3+1D mainly gluonic
- Initially color fields approx. boost invariant (Glasma) $\Rightarrow$ eff. 2+1D
- Later: Bottom-up scenario with kinetic theory in anisotropic 3+1D

★ Excitation spectra: Quasiparticles (QP)? When is kinetic theory valid? How does transition 2+1D $\rightarrow$ 3+1D work microscopically?

★ Why are instabilities suppressed in kinetic evol. of anisotr. plasmas?

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Considered models and initial conditions

- $\mathrm{SU}(N_c)$ Yang-Mills theory (simulations: $N_c = 2$)

$$S_{YM}[A] = -\frac{1}{4} \int d^{d+1}x \, F_a^{\mu\nu} F^a_{\mu\nu}$$

(in gauge-covariant formulation, with links $U_j \approx \exp\left(ig\, a_s A_j\right)$)

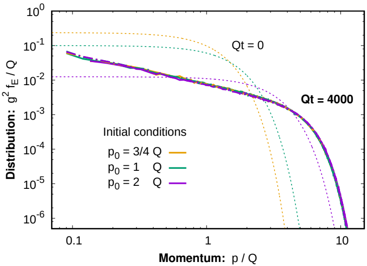
- Models of initial stages (here non-expanding geometry)
  1. 2+1D
  2. Glasma-like 2+1D: add adjoint scalar ϕ to model 1
  3. 3+1D: isotropic 3+1D

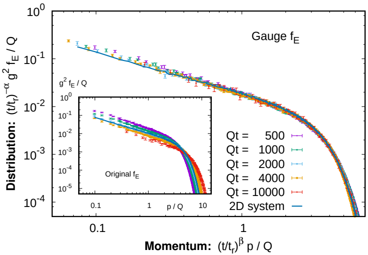
- Initial conditions (with $E = \partial_0 A$): highly occupied

$$f(t=0,p) = \frac{Q}{g^2}\, n_0\, e^{-\frac{p^2}{2Q^2}} \quad \text{with} \quad f(t,p) \propto \frac{\langle |E_T(t,p)|^2 \rangle}{p}$$

- Solve classical equations, average over initial ensembles
  $\Rightarrow$ classical-statistical lattice simulations

Attractor in 2+1D (PRD 100, 094022 (2019))





- Both 2+1D theories approach a classical self-similar attractor

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right) \qquad \text{(typical nonthermal state, see Backup)}$$

- Universal scaling exponents insensitive to details of initial conditions

$$\beta = -\frac{1}{5}, \quad \alpha = 3\beta \qquad \text{(parametric kinetic explanation in Backup)}$$

- Hard scale $\Lambda(t) = \langle p \rangle \sim Q(Qt)^{-\beta}$, soft scale $m_D(t) \sim Q(Qt)^{\beta}$
- Classical attractor in 3+1D well known, $\beta = -\frac{1}{7}$, $\alpha = -\frac{4}{7}$

Spectral and statistical correlation functions

- Equal-time correlator $\langle\{\hat{E}(t),\hat{E}(t)\}\rangle \propto f(t,p)$ is distribution
  $\Rightarrow$ But what are the relevant gluon excitations?
- Knowledge of spectral function needed ($\dot{\rho} = \partial_t \rho,\ E = \partial_t A$)

$$\dot{\rho}(x,x') = \frac{i}{N_c^2-1}\left\langle \left[\hat{E}(x), \hat{A}(x')\right]\right\rangle$$

- Statistical correlator $\langle EE\rangle$ ($\equiv \ddot{F}$) in general independent of $\dot{\rho}$

$$\langle EE\rangle(x,x') = \frac{1}{2(N_c^2-1)}\left\langle \left\{\hat{E}(x), \hat{E}(x')\right\}\right\rangle$$

- Fourier transf. in $t-t'$ and $\vec{x}-\vec{x}'$ to frequency ω and momentum $\vec{p}$
  Approximation: normally at fixed $\bar{t} = \frac{1}{2}(t+t')$, we hold $t \approx \bar{t}$
- In classical-statistical simulations

$$\langle EE\rangle(t,t',p) = \frac{1}{N_c^2-1}\left\langle E(t,\vec{p})E^*(t',\vec{p})\right\rangle$$

- Gauge at print-out t: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j\big|_t = 0$

Perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$;
  in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f\, \Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{\rm pol} N_c \int \frac{\mathrm{d}^2p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m^2+p^2}} \sim g^2 f\, \Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$
$\Rightarrow$ Nonperturbative method necessary!

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\rm HTL}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\rm HTL}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

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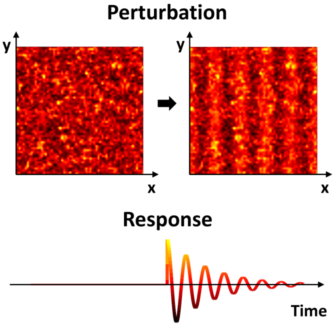
Nonperturbative computation of spectral function ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, A. Kurkela, T. Lappi, J. Peuron, *PRD 98, 014006 (2018)*, Editors' suggestion



- Perturb $A(t,\vec{x}) \mapsto A(t,\vec{x}) + \delta A(t,\vec{x})$
- Class. EOM for A: $D_\mu F^{\mu\nu}[A] = 0$
- Linearized EOM for $\delta A(t,\vec{x})$ (both in gauge-cov. formulation)

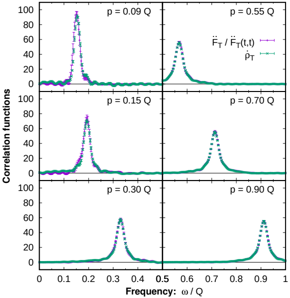
  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $G_R(t,t',p) \propto \langle \delta A(t',\vec{p}) \delta E^*(t,\vec{p}) \rangle$
- $\theta(t-t') \boxed{\rho(t,t',p)} = G_R(t,t',p)$
- $\dot{\rho}(t,\omega,p) \approx \omega \rho(t,\omega,p)$

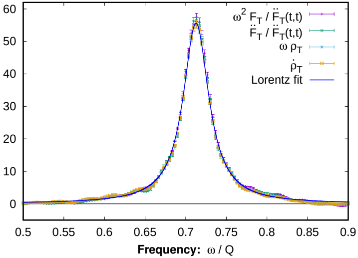
Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020)

First: Isotropic 3+1D gluon plasmas

KB, A. Kurkela, T. Lappi, J. Peuron, PRD 98, 014006 (2018)





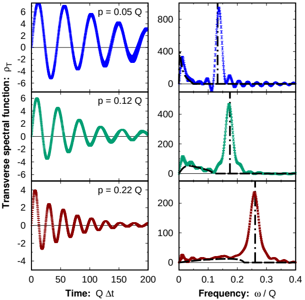
- Narrow Lorentzian quasiparticle peaks for all momenta, even $p \lesssim m_D$
- Generalized fluctuation dissipation relation (FDR) for $\alpha = T, L$

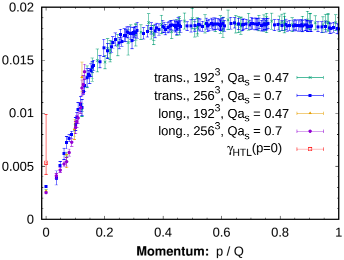
$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t{=}0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t{=}0, p)}$$

- Small width $\gamma_\alpha(p) \ll \omega_\alpha(p)$, decreases faster $\gamma_\alpha(t) \sim (Qt)^{-2/7}\, m_D(t)$

Spectral function in isotr. 3+1D

KB, A. Kurkela, T. Lappi, J. Peuron, PRD 98, 014006 (2018)



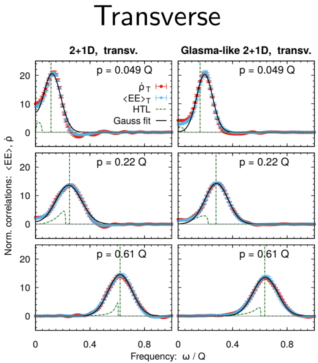


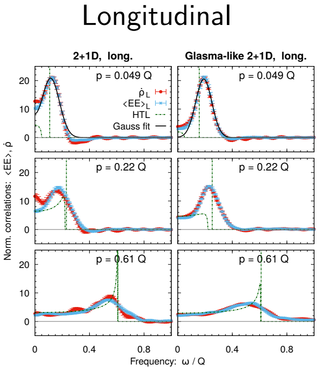
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- $\gamma_{T/L}(p)$ beyond HTL at LO
- $\boxed{\text{First determination}}$ of p dep.
- 'isotropic' $\gamma_T(p) \approx \gamma_L(p)$
- HTL prediction $\gamma(p = 0)$

Braaten, Pisarski, PRD 42, 2156 (1990)

Now: correlations in 2+1D plasmas (2101.02715)

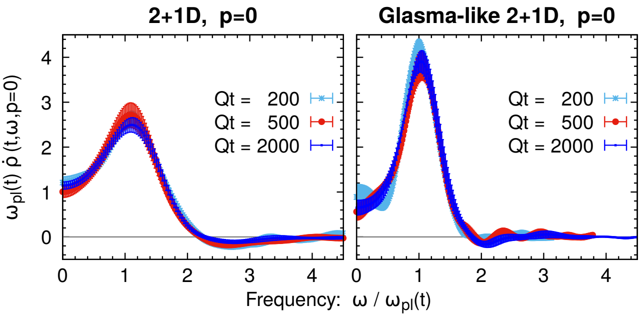




- Generalized FDR observed for $\dot{\rho}$, $\langle EE \rangle$
- Broad peaks $\gamma_\alpha \sim \omega_{\rm pl} \equiv \omega_T(p{=}0) \propto m_D$ [in HTL $2\omega_{\rm pl}^2 = m_D^2$]

  $\Rightarrow$ $\boxed{\text{no quasiparticles for } p \lesssim m_D!}$
- Non-Lorentzian peak shape (Backup)
- HTL curves (green) agree poorly (except for $\omega \ll p$ for long.)

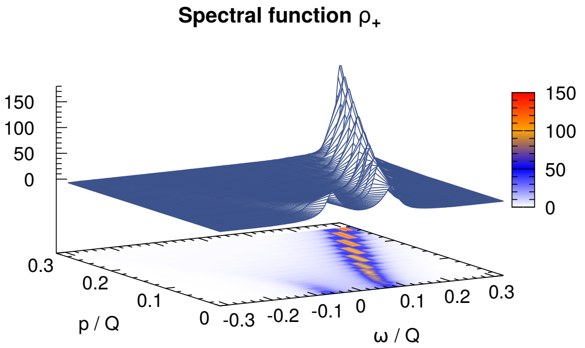
Time dependence of 2+1D gluon plasmas



- $\omega_{\rm pl}\,\dot\rho(t,\omega/\omega_{\rm pl},p/\omega_{\rm pl})$ is time independent (for all p, Backup)
- This implies $\boxed{\gamma_\alpha(t,p) \sim \omega_{\rm pl}(t)} \sim m_D \sim Q(Qt)^{-1/5}$
- Estimates as for 3+1D lead to $Q(Qt)^{-2/5} \Rightarrow$ different mechanism
- Also in classical thermal equilibrium $\gamma \sim \omega_{\rm pl}$ (Backup)
- No quasiparticles at low $p \Rightarrow$ quite general in 2+1D

Teaser: Fermion ρ in 3+1D (mainly) gluon plasma

KB, M. Mace, T. Lappi, S. Schlichting, *in preparation*



Fermions: $\rho(t,\omega,\vec{p}) \approx \gamma^0 \rho_0^V(t,\omega,\vec{p}) + \gamma^j \frac{p_j}{p} \rho_V(t,\omega,\vec{p})\,, \quad \rho_+ = \rho_0^V + \rho_V$

Many similarities with gluons in 3+1D and 2+1D!

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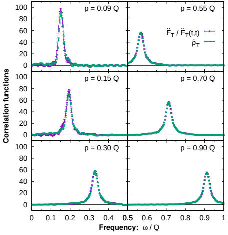
Conclusion

- Equal-time correlators often insufficient to understand microscopics
  $\Rightarrow$ unequal-time correlations required
- Much broader peaks in 2+1D than in 3+1D
  $\Rightarrow$ excitations too short-lived to form quasiparticles for $p \lesssim m_D$
  (an effective kinetic description may be possible for $p \gg m_D$ but requires nonperturbatively determined collision kernel)
- Nonperturbative low-p physics in anisotropic plasmas
  $\Rightarrow$ structure of instabilities in heavy-ion collisions well understood?

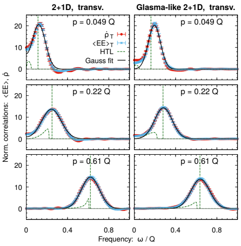
Outlook

- ρ in Bjorken expanding systems (for heavy-ion collisions)
- How does an eff. kinetic theory in 2+1D gauge systems look like?
- Gauge-invariant checks of nonperturbative properties:
  Heavy-quark diffusion KB, A. Kurkela, T. Lappi, J. Peuron, JHEP 09, 077 (2020), transport, ...



3+1D

Thank you for your attention!



2+1D

Backup slides

Classical-statistical lattice simulations

1. Set initial conditions

$$\langle E_T^*(t_0,\vec{p}) E_T(t_0,\vec{q})\rangle \propto p\, f(t_0,p)\, \delta_{jk}(2\pi)^3 \delta(\vec{p}-\vec{q})$$

   with $E_T^j p_j = 0$, initially $\langle E_L^*(t_0,\vec{p}) E_L(t_0,\vec{q})\rangle = 0$, same for A

2. Restore Gauss law (algorithm: G.D. Moore, Nucl. Phys. B 480, 657 (1996))
3. Solve classical field equations on the lattice

$$\begin{aligned} U_j(t+\mathrm{d}t/2,\vec{x}) &= e^{i\mathrm{d}t\, a_s g E_a^j(t,\vec{x})} U_j(t-\mathrm{d}t/2,\vec{x}) \\ gE_a^i(t+\mathrm{d}t,\vec{x}) &= gE_a^i(t,\vec{x}) - \frac{\mathrm{d}t}{a_s^3}\sum_{j\neq i}\left[U_{ij}\left(t-\frac{\mathrm{d}t}{2},\vec{x}\right) + U_{i(-j)}\left(t-\frac{\mathrm{d}t}{2},\vec{x}\right)\right]_{\mathrm{ah}} \end{aligned}$$

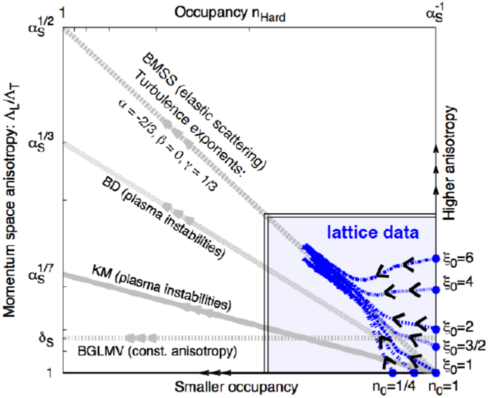
4. Evolve each initial configuration $\{U(t_0,\vec{x}), E(t_0,\vec{x})\}$ until $t > t_0$
5. Compute observable $O[U,E]$ that depends on the fields

$$O(t) = \frac{1}{\#k}\sum_k O[U(t),E(t)]$$

Classical attractors: Distinguishing kinetic descriptions

Real-time lattice simulations

Berges, KB, Schlichting, Venugopalan,
PRD 89, 114007 (2014), PRD 89, 074011 (2014)



Kinetic descriptions

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

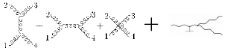
- Numerics consistent with 1st stage of BMSS scenario

  Baier, Mueller, Schiff, Son, *PLB 502, 51 (2001)*

- ⇨ Effective kinetic theory (EKT)

  Arnold, Moore, Yaffe, *JHEP 0301, 030 (2003)*

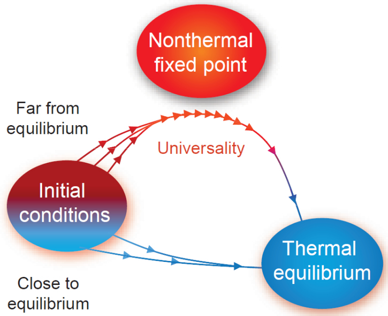
$$\frac{df}{d\tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} =$$



- Used as standard description now

Universal classical attractors: nonthermal fixed points

- Rich nonequilibrium dynamics in gauge and scalar systems
- Share similar universal features:



Nonthermal fixed point (NTFP)

★ Large initial occupancy
$\Rightarrow$ may approach attractor

★ System 'forgets' initial conditions

★ Self-similar dynamics

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

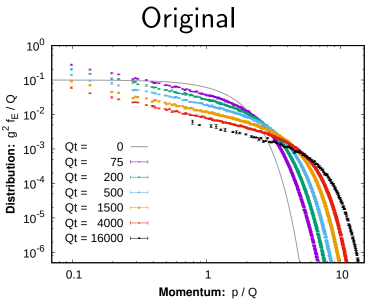
★ *Universal* α, β, $f_s(p)$

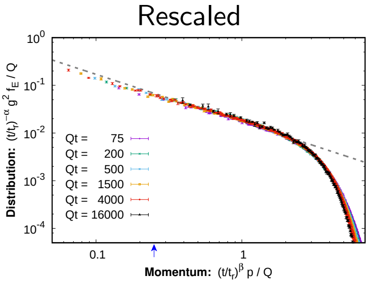
NTFP: Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

Universality: Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

Experimental observations: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)

Self-similarity of 2+1D theory (PRD 100, 094022 (2019))





- Self-similar evolution

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal scaling exponents

$$\beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(energy conserv.)}$$

Perturbative explanation of scaling exponents (PRD 100, 094022 (2019))

- Soft scale (Debye mass) from HTL

$$m_D^2 \approx d_{\rm pol} N_{\rm c} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{p} \sim g^2 f\,\Lambda\,\ln(\Lambda/m_D)$$

(Log from soft-soft interactions $\Rightarrow$ breakdown of HTL at m_D in 2+1D)

- Scaling exponents from kinetic arguments:

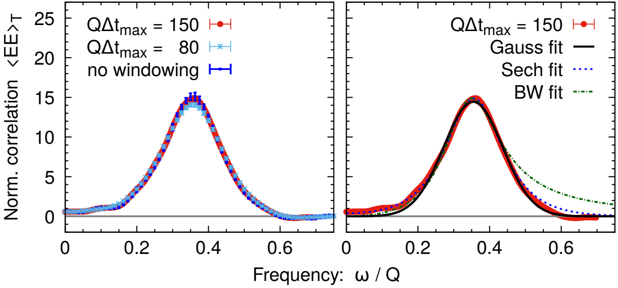
Elastic scattering rate: $$\frac{\mathrm{d}\Gamma}{\mathrm{d}q_\perp} \sim \frac{g^4}{(q_\perp^2 + m_D^2)^2} \int \mathrm{d}^2 p\, f(1+f)$$

Momentum diffusion: $$\hat{q} \sim \int \mathrm{d}q_\perp \frac{\mathrm{d}\Gamma}{\mathrm{d}q_\perp} q_\perp^2 \sim \frac{\Lambda^2\,(g^2 f)^2}{m_D}$$

From broadening $\Lambda^2 \sim \hat{q}\,t$ follows $\Lambda \sim Q(Qt)^{1/5}$

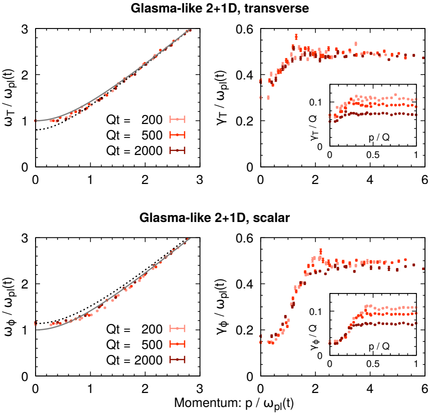
- In 2+1D $q_\perp \sim m_D$ crucial! If $q_\perp \sim \Lambda$, then $\Lambda \sim Q(Qt)^{1/7}$ instead!
- Kinetic estimates work $\Rightarrow$ eff. kinetic descr. may exist for $p \gg m_D$

Shape of the excitation peaks in 2+1D (arXiv:2101.02715)



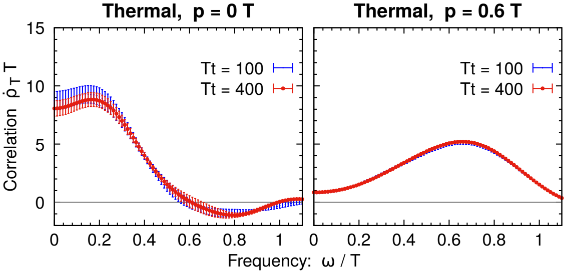
- *Left:* Different ways of computing the Fourier transform are consistent
- *Right:* Peak has non-Lorentzian shape (not Breit-Wigner)

Dispersion relations, damping rates in Glasma-like 2+1D (arXiv:2101.02715)



- *Left:* Dispersions $\omega_\alpha(t,p)/\omega_{\rm pl}(t)$
- *Right:* Peak width $\gamma_\alpha(t,p)/\omega_{\rm pl}(t)$
- As functions of $p/\omega_{\rm pl}(t)$ time independent $\Rightarrow \gamma(t,p) \sim \omega_{\rm pl}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same t dependence

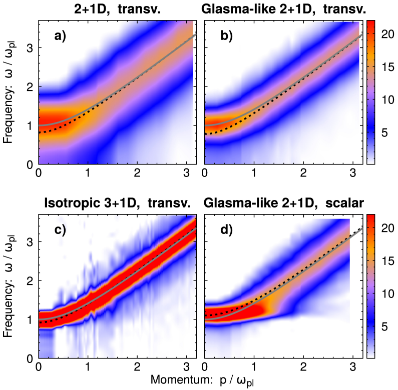
ρ in 2+1D classical thermal equilibrium

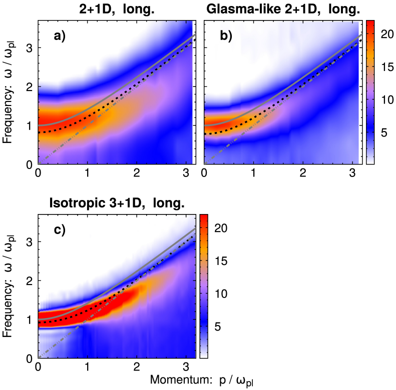


- Qualitatively similar behavior as far from equilibrium
  - ✓ Broad gluonic excitations with $\gamma(p) \sim \omega_{\rm pl}$
  - ✓ HTL provides poor description
  - ✓ For $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low p
- Interpretion:

  These qualitative features seem generic in 2+1D gauge theories

Summary of $\langle EE \rangle_\alpha(t{=}const, \omega, p)$





Black dashed: $\omega_\alpha^{\mathrm{HTL}}(p)$, gray: $\sqrt{\omega_{\mathrm{pl}}^2 + p^2}$, with $\omega_{\mathrm{pl}} \propto m_D$

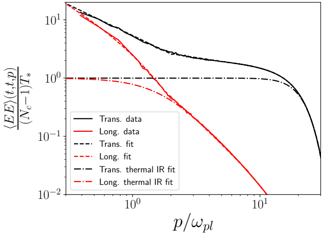
Heavy quark diffusion: IR gluon excess in isotropic 3+1D

The following is based on

KB, A. Kurkela, T. Lappi, J. Peuron, *JHEP 09, 077 (2020), [arXiv:2005.02418]*

- What are the consequences of the nonperturbative properties of correlation functions?
- E.g., consider correlations in 3+1D at self-similar attractor

Excess of infrared gluons



- $\langle EE\rangle_{T/L}(t,t,p)$ shown
- Dashed: HTL expectations
- Excess of gluons w.r.t. HTL for $p \lesssim m_D \sim \omega_{\rm pl}$

Heavy quark diffusion: far from equilibrium in 3+1D

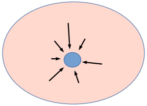
- Correlations like $\langle EE \rangle_T(t, t', p)$ are not gauge-invariant
- We used $A_0 = 0$ and $\vec{\nabla}\vec{A} = 0|_t$ gauges
- Are effects visible in gauge-invariant observables?

Example

Heavy quark in QGP

- Quark experiences 'kicks' from the medium

$$\dot{p}_i(t) = \mathcal{F}_i(t)$$



- Gauge-inv. force-force correlator leads to momentum broadening

$$\langle \dot{p}_i(t)\dot{p}_i(t')\rangle = g^2 \frac{\mathrm{Tr}\langle E_i(t,\vec{x})U_0(t,t',\vec{x})E_i(t',\vec{x})U_0(t',t,\vec{x})\rangle}{\mathrm{Tr}\,\mathbb{1}}$$

$$= \frac{g^2}{2N_c}\langle E_i^a(t,\vec{x})E_i^a(t',\vec{x})\rangle \equiv \frac{g^2}{2N_c}\langle EE\rangle(t,t')$$

Models to understand evolution of $\kappa(t, \Delta t)$

SR: 'Spectral reconstruction'

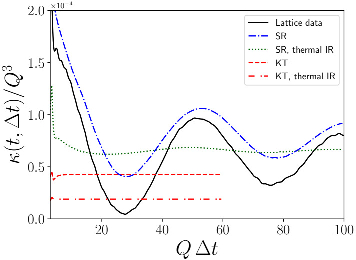
$$
\begin{aligned}
3\kappa\,(t, \Delta t) &= \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t, \Delta t)\rangle \\
&\approx \frac{g^2}{N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \frac{\sin(\omega\,\Delta t)}{\omega} \\
&\quad \times \left[2\langle EE\rangle_T(t, t, p) \frac{\dot\rho_T(t, \omega, p)}{\dot\rho_T(t, t, p)} + \langle EE\rangle_L(t, t, p) \frac{\dot\rho_L(t, \omega, p)}{\dot\rho_L(t, t, p)} \right]
\end{aligned}
$$

- use extracted equal-time and spectral functions in computation
- use $\langle EE\rangle_{T/L}(t, t, p)$ with IR excess ('data') & without ('thermal HTL')
- in $\dot\rho_{T/L}(t, \omega, p)$ Landau ($\omega < p$) and q.p. terms can be distinguished

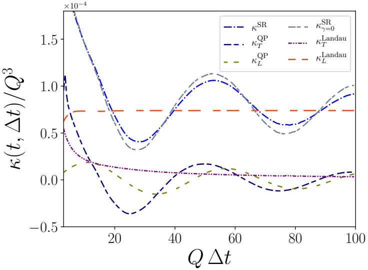
Heavy-quark diffusion: IR gluon excess observable

$$\kappa\,(t,\Delta t) \approx \frac{g^2}{3N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \frac{\sin(\omega\,\Delta t)}{\omega}$$
$$\times \left[2\langle EE\rangle_T(t,t,p)\,\frac{\dot\rho_T(t,\omega,p)}{\dot\rho_T(t,t,p)} + \langle EE\rangle_L(t,t,p)\,\frac{\dot\rho_L(t,\omega,p)}{\dot\rho_L(t,t,p)} \right]$$

Total $\kappa\,(t,\Delta t) \equiv \sum_i \kappa_i\,(t,\Delta t)$



Components $\kappa_i\,(t,\Delta t)$



- Nonperturbative effects of $\langle EE\rangle_\alpha(t,t,p)$ and $\dot\rho_\alpha(t,\omega,p)$ visible!
- Oscillations with ω_{pl} due to QP excitations, sign of IR excess
- Heavy quarks, quarkonia, jets may reveal IR dynamics of QGP