Nonperturbative excitations in overoccupied gluon plasmas

with A. Kurkela, T. Lappi, J. Peuron

K. Boguslavski

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Talk mainly based on:

arXiv:2101.02715
PRD 100, 094022 (2019), [arXiv:1907.05892]
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Introduction

The goal is to study Microscopic properties of QCD nonperturbatively

⇒ Spectral functions $\rho(\omega, p)$ of gluons / quarks encode full excitation spectrum!

- Our approach: far from equilibrium
  - Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)
  - Then nonperturbative and perturbative methods available!

- Classical-statistical lattice simulations vs. HTL, kinetic theory
  - class. covariant equ.: $D_\mu F^{\mu\nu} = 0$, HTL: $\Pi_{\mu\nu}^{\text{HTL}}$

- Nonequilibrium application: heavy-ion collisions
Motivation: heavy-ion collisions

Application
Microscopic properties of non-equilibrium QCD

✓ Initial stages in heavy-ion collisions suitable playground
✓ Quark-gluon plasma initially: eff. 2+1D and 3+1D mainly gluonic
  ● Initially color fields approx. boost invariant (Glasma) ⇒ eff. 2+1D
  ● Later: Bottom-up scenario with kinetic theory in anisotropic 3+1D
★ Excitation spectra: Quasiparticles (QP)? When is kinetic theory valid? How does transition 2+1D → 3+1D work microscopically?
★ Why are instabilities suppressed in kinetic evol. of anisotrophic plasmas?
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Considered models and initial conditions

- **SU($N_c$) Yang-Mills theory (simulations: $N_c = 2$)**
  \[ S_{YM}[A] = -\frac{1}{4} \int d^{d+1}x \, F_a^{\mu\nu} F_a^{\mu\nu} \]
  (in gauge-covariant formulation, with links $U_j \approx \exp(ig a_s A_j)$)

- **Models of initial stages (here non-expanding geometry)**
  1. 2+1D
  2. **Glasma-like 2+1D**: add adjoint scalar $\phi$ to model 1
  3. 3+1D: isotropic 3+1D

- **Initial conditions (with $E = \partial_0 A$):** highly occupied
  \[ f(t = 0, p) = \frac{Q}{g^2} \, n_0 \, e^{-\frac{p^2}{2Q^2}} \quad \text{with} \quad f(t, p) \propto \frac{\langle |E_T(t, p)|^2 \rangle}{p} \]

- **Solve classical equations, average over initial ensembles**
  ⇒ classical-statistical lattice simulations
Both 2+1D theories approach a classical self-similar attractor
\[
f(t, p) = (Qt)^\alpha f_s \left((Qt)^\beta p\right)
\]  
(typical nonthermal state, see Backup)

Universal scaling exponents insensitive to details of initial conditions
\[
\beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(parametric kinetic explanation in Backup)}
\]

Hard scale \(\Lambda(t) = \langle p \rangle \sim Q(Qt)^{-\beta}\), soft scale \(m_D(t) \sim Q(Qt)^{\beta}\)

Classical attractor in 3+1D well known, \(\beta = -\frac{1}{7}, \quad \alpha = -\frac{4}{7}\)
Spectral and statistical correlation functions

- Equal-time correlator \( \langle \{ \hat{E}(t), \hat{E}(t) \} \rangle \propto f(t, p) \) is distribution
  \( \Rightarrow \) But what are the relevant gluon excitations?

- Knowledge of spectral function needed (\( \dot{\rho} = \partial_t \rho, \ E = \partial_t A \))

\[ \dot{\rho}(x, x') = \frac{i}{N_c^2 - 1} \left\langle \left[ \hat{E}(x), \hat{A}(x') \right] \right\rangle \]

- Statistical correlator \( \langle EE \rangle \ (\equiv \ddot{F}) \) in general independent of \( \dot{\rho} \)

\[ \langle EE \rangle(x, x') = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x), \hat{E}(x') \right\} \right\rangle \]

- Fourier transf. in \( t - t' \) and \( \vec{x} - \vec{x}' \) to frequency \( \omega \) and momentum \( \vec{p} \)
  Approximation: normally at fixed \( \bar{t} = \frac{1}{2}(t + t') \), we hold \( t \approx \bar{t} \)

- In classical-statistical simulations

\[ \langle EE \rangle(t, t', \vec{p}) = \frac{1}{N_c^2 - 1} \left\langle E(t, \vec{p})E^*(t', \vec{p}) \right\rangle \]

- Gauge at print-out \( t \): temporal \( A_0 = 0 + \text{Coulomb-type} \ \left. \partial^j A_j \right|_t = 0 \)
Perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$; in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...

- In 3+1D $m_D^2 = 4N_c \int \frac{d^3p}{(2\pi)^3} \frac{g^2f(t,p)}{p} \sim g^2 f \Lambda^2 \Rightarrow$ HTL applicable

- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{pol} N_c \int \frac{d^2p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m^2 + p^2}} \sim g^2 f \Lambda \ln \left( \frac{\Lambda}{m_D} \right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$  
$\Rightarrow$ Nonperturbative method necessary!

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{HTL}(\omega, p) \sim \delta(\omega - \omega^{HTL}_\alpha(p))$
- All expressions depend only on $m_D$, computed consistently in HTL
Nonperturbative computation of spectral function \( \rho \)

Classical-statistical \( SU(N_c) \) simulations + linear response theory


- Perturb \( A(t, \vec{x}) \mapsto A(t, \vec{x}) + \delta A(t, \vec{x}) \)
- Class. EOM for \( A \): \( D_\mu F^{\mu\nu}[A] = 0 \)
- Linearized EOM for \( \delta A(t, \vec{x}) \)
  (both in gauge-cov. formulation)

Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*

- \( G_R(t, t', p) \propto \langle \delta A(t', \vec{p}) \delta E^*(t, \vec{p}) \rangle \)
- \( \theta(t - t') \rho(t, t', p) = G_R(t, t', p) \)
- \( \dot{\rho}(t, \omega, p) \approx \omega \rho(t, \omega, p) \)

Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020)
First: Isotropic 3+1D gluon plasmas


- **Narrow Lorentzian quasiparticle peaks** for all momenta, even \( p \lesssim m_D \)
- **Generalized fluctuation dissipation relation** (FDR) for \( \alpha = T, L \)

\[
\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t=0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t=0, p)}
\]

- **Small width** \( \gamma_\alpha(p) \ll \omega_\alpha(p) \), decreases faster \( \gamma_\alpha(t) \sim (Qt)^{-2/7} m_D(t) \)
Spectral function in isotr. 3+1D


- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

$\gamma_{T/L}(p)$ beyond HTL at LO
- First determination of $p$ dep.
- ‘isotropic’ $\gamma_T(p) \approx \gamma_L(p)$
- HTL prediction $\gamma(p = 0)$

Braaten, Pisarski, PRD 42, 2156 (1990)
Now: correlations in 2+1D plasmas (2101.02715)

- Generalized FDR observed for $\dot{\rho}, \langle EE \rangle$
- Broad peaks $\gamma_\alpha \sim \omega_{pl} \equiv \omega_T(p=0) \propto m_D$ [in HTL $2\omega_{pl}^2 = m_D^2$]
  $\Rightarrow$ no quasiparticles for $p \lesssim m_D$!
- Non-Lorentzian peak shape (Backup)
- HTL curves (green) agree poorly (except for $\omega \ll p$ for long.)
\( \omega_{\text{pl}} \dot{\rho}(t, \omega/\omega_{\text{pl}}, p/\omega_{\text{pl}}) \) is time independent (for all \( p \), Backup)

This implies \( \gamma_\alpha(t, p) \sim \omega_{\text{pl}}(t) \sim m_D \sim Q(Qt)^{-1/5} \)

Estimates as for 3+1D lead to \( Q(Qt)^{-2/5} \Rightarrow \) different mechanism

Also in classical thermal equilibrium \( \gamma \sim \omega_{\text{pl}} \) (Backup)

No quasiparticles at low \( p \) \( \Rightarrow \) quite general in 2+1D
Teaser: Fermion $\rho$ in 3+1D (mainly) gluon plasma

KB, M. Mace, T. Lappi, S. Schlichting, *in preparation*

Fermions:
$$\rho(t, \omega, \vec{p}) \approx \gamma^0 \rho^V_0(t, \omega, \vec{p}) + \gamma^j \frac{p_j}{p} \rho^V(t, \omega, \vec{p}), \quad \rho_+ = \rho^V_0 + \rho^V$$

Many similarities with gluons in 3+1D and 2+1D!
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Conclusion

- Equal-time correlators often insufficient to understand microscopics
  \(\Rightarrow\) unequal-time correlations required

- Much broader peaks in 2+1D than in 3+1D
  \(\Rightarrow\) excitations too short-lived to form quasiparticles for \(p \lesssim m_D\)
  (an effective kinetic description may be possible for \(p \gg m_D\) but requires nonperturbatively determined collision kernel)

- Nonperturbative low-\(p\) physics in anisotropic plasmas
  \(\Rightarrow\) structure of instabilities in heavy-ion collisions well understood?

Outlook

- \(\rho\) in Bjorken expanding systems (for heavy-ion collisions)

- How does an eff. kinetic theory in 2+1D gauge systems look like?

- Gauge-invariant checks of nonperturbative properties:
  Heavy-quark diffusion KB, A. Kurkela, T. Lappi, J. Peuron, JHEP 09, 077 (2020), transport, ...


Thank you for your attention!

3+1D

2+1D
Backup slides
Set initial conditions

\[ \langle E^*_T(t_0, \vec{p}) E_T(t_0, \vec{q}) \rangle \propto p f(t_0, p) \delta_{jk} (2\pi)^3 \delta(\vec{p} - \vec{q}) \]

with \( E^j_T p_j = 0 \), initially \( \langle E^*_L(t_0, \vec{p}) E_L(t_0, \vec{q}) \rangle = 0 \), same for \( A \)


Solve classical field equations on the lattice

\[
U_j(t + dt/2, \vec{x}) = e^{idt asgE^j_a(t, \vec{x})} U_j(t - dt/2, \vec{x}) \\
gE^i_a(t + dt, \vec{x}) = gE^i_a(t, \vec{x}) - \frac{dt}{a^3} \sum_{j \neq i} \left[ U_{ij} \left(t - \frac{dt}{2}, \vec{x}\right) + U_{i(-j)} \left(t - \frac{dt}{2}, \vec{x}\right) \right]_{ah}
\]

Evolve each initial configuration \( \{ U(t_0, \vec{x}), E(t_0, \vec{x}) \} \) until \( t > t_0 \)

Compute observable \( O[U, E] \) that depends on the fields

\[
O(t) = \frac{1}{\#k} \sum_k O[U(t), E(t)]
\]
Suppression of instabilities in kinetic evol. in heavy-ion coll.

Classical attractors: Distinguishing kinetic descriptions

Real-time lattice simulations

Berges, KB, Schlichting, Venugopalan, PRD 89, 114007 (2014), PRD 89, 074011 (2014)

Kinetic descriptions

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

• Numerics consistent with 1\textsuperscript{st} stage of BMSS scenario

Baier, Mueller, Schiff, Son, PLB 502, 51 (2001)

• Effective kinetic theory (EKT)


\[ \frac{df}{d\tau} - \frac{p_z}{\tau} \frac{\partial f}{\partial p_z} = \ldots \]

• Used as standard description now
Universal classical attractors: nonthermal fixed points

- Rich nonequilibrium dynamics in gauge and scalar systems
- Share similar universal features:
  - **Nonthermal fixed point (NTFP)**
    - Large initial occupancy
      \[ \Rightarrow \text{may approach attractor} \]
    - System ‘forgets’ initial conditions
    - Self-similar dynamics
      \[ f(t, p) = t^\alpha f_s(t^{\beta} p) \]
  - Universal \( \alpha, \beta, f_s(p) \)

**Nonthermal fixed point (NTFP):** Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

**Universality:** Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

**Experimental observations:** Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)
Self-similarity of 2+1D theory (PRD 100, 094022 (2019))

- Self-similar evolution

\[ f(t, p) = (Qt)^\alpha f_s \left( (Qt)^\beta p \right) \]

- Universal scaling exponents

\[ \beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(energy conserv.)} \]
Perturbative explanation of scaling exponents (PRD 100, 094022 (2019))

- Soft scale (Debye mass) from HTL

\[ m_D^2 \approx d_{\text{pol}} N_c \int \frac{d^2 p}{(2\pi)^2} \frac{g^2 f(t, p)}{p} \sim g^2 f \Lambda \ln(\Lambda/m_D) \]

(Log from soft-soft interactions \( \Rightarrow \) breakdown of HTL at \( m_D \) in 2+1D)

- Scaling exponents from kinetic arguments:

  Elastic scattering rate:
  \[
  \frac{d\Gamma}{dq_\perp} \sim \frac{g^4}{(q_\perp^2 + m_D^2)^2} \int d^2 p f(1 + f)
  \]

  Momentum diffusion:
  \[
  \hat{q} \sim \int dq_\perp \frac{d\Gamma}{dq_\perp} q_\perp^2 \sim \frac{\Lambda^2 (g^2 f)^2}{m_D}
  \]

  From broadening \( \Lambda^2 \sim \hat{q} t \) follows \( \Lambda \sim Q(Qt)^{1/5} \)

- In 2+1D \( q_\perp \sim m_D \) crucial! If \( q_\perp \sim \Lambda \), then \( \Lambda \sim Q(Qt)^{1/7} \) instead!

- Kinetic estimates work \( \Rightarrow \) eff. kinetic descr. may exist for \( p \gg m_D \)
Shape of the excitation peaks in 2+1D (arXiv:2101.02715)

- **Left**: Different ways of computing the Fourier transform are consistent
- **Right**: Peak has non-Lorentzian shape (not Breit-Wigner)
Dispersion relations, damping rates in Glasma-like 2+1D (arXiv:2101.02715)

- **Left:** Dispersions $\omega_\alpha(t, p)/\omega_{pl}(t)$
- **Right:** Peak width $\gamma_\alpha(t, p)/\omega_{pl}(t)$
- As functions of $p/\omega_{pl}(t)$ time independent $\Rightarrow \gamma(t, p) \sim \omega_{pl}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same $t$ dependence
**Qualitatively similar behavior as far from equilibrium**

- Broad gluonic excitations with $\gamma(p) \sim \omega_{pl}$
- HTL provides poor description
- For $\omega \rightarrow 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low $p$

**Interpretation:**

These qualitative features seem generic in 2+1D gauge theories
Summary of $\langle EE \rangle_\alpha(t=\text{const}, \omega, p)$

Black dashed: $\omega^\text{HTL}_\alpha(p)$, gray: $\sqrt{\omega^2_{\text{pl}} + p^2}$, with $\omega_{\text{pl}} \propto m_D$
What are the consequences of the nonperturbative properties of correlation functions?

E.g., consider correlations in 3+1D at self-similar attractor

Excess of infrared gluons

- $\langle EE \rangle_{T/L}(t, t, p)$ shown
- Dashed: HTL expectations
- Excess of gluons w.r.t. HTL for $p \lesssim m_D \sim \omega_{pl}$
Heavy quark diffusion: far from equilibrium in 3+1D

- Correlations like $\langle EE \rangle_T(t, t', p)$ are not gauge-invariant
- We used $A_0 = 0$ and $\vec{\nabla} \vec{A} = 0 \big|_t$ gauges
- Are effects visible in gauge-invariant observables?

Example

Heavy quark in QGP

- Quark experiences ‘kicks’ from the medium
  \[ \dot{p}_i(t) = F_i(t) \]
- Gauge-inv. force-force correlator leads to momentum broadening
  \[
  \langle \dot{p}_i(t) \dot{p}_i(t') \rangle = g^2 \frac{\text{Tr} \langle E_i(t, \vec{x}) U_0(t, t', \vec{x}) E_i(t', \vec{x}) U_0(t', t, \vec{x}) \rangle}{\text{Tr} \mathbb{1}}
  \]
  \[
  = \frac{g^2}{2N_c} \langle E^a_i(t, \vec{x}) E^a_i(t', \vec{x}) \rangle \equiv \frac{g^2}{2N_c} \langle EE \rangle(t, t')
  \]
Models to understand evolution of $\kappa(t, \Delta t)$

SR: ‘Spectral reconstruction’

$$3\kappa(t, \Delta t) = \frac{d}{d\Delta t} \langle p^2(t, \Delta t) \rangle$$

$$\approx \frac{g^2}{N_c} \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega}$$

$$\times \left[ 2\langle EE \rangle_T(t, t, p) \frac{\dot{\rho}_T(t, \omega, p)}{\dot{\rho}_T(t, t, p)} + \langle EE \rangle_L(t, t, p) \frac{\dot{\rho}_L(t, \omega, p)}{\dot{\rho}_L(t, t, p)} \right]$$

- use extracted equal-time and spectral functions in computation
- use $\langle EE \rangle_T/L(t, t, p)$ with IR excess (‘data’) & without (‘thermal HTL’)
- in $\dot{\rho}_T/L(t, \omega, p)$ Landau ($\omega < p$) and q.p. terms can be distinguished
Heavy-quark diffusion: IR gluon excess observable

\[ \kappa(t, \Delta t) \approx \frac{g^2}{3N_c} \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \times \left[ 2 \langle EE \rangle_T(t, t, p) \frac{\dot{\rho}_T(t, \omega, p)}{\dot{\rho}_T(t, t, p)} + \langle EE \rangle_L(t, t, p) \frac{\dot{\rho}_L(t, \omega, p)}{\dot{\rho}_L(t, t, p)} \right] \]

Total \( \kappa(t, \Delta t) \equiv \sum_i \kappa_i(t, \Delta t) \)

- Nonperturbative effects of \( \langle EE \rangle_\alpha(t, t, p) \) and \( \dot{\rho}_\alpha(t, \omega, p) \) visible!
- Oscillations with \( \omega_{pl} \) due to QP excitations, sign of IR excess
- Heavy quarks, quarkonia, jets may reveal IR dynamics of QGP