UNIVERSITY OF MARYLAND

New digitization strategies for relativistic quantum field theories

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based on Phys. Rev. A 103, 042410

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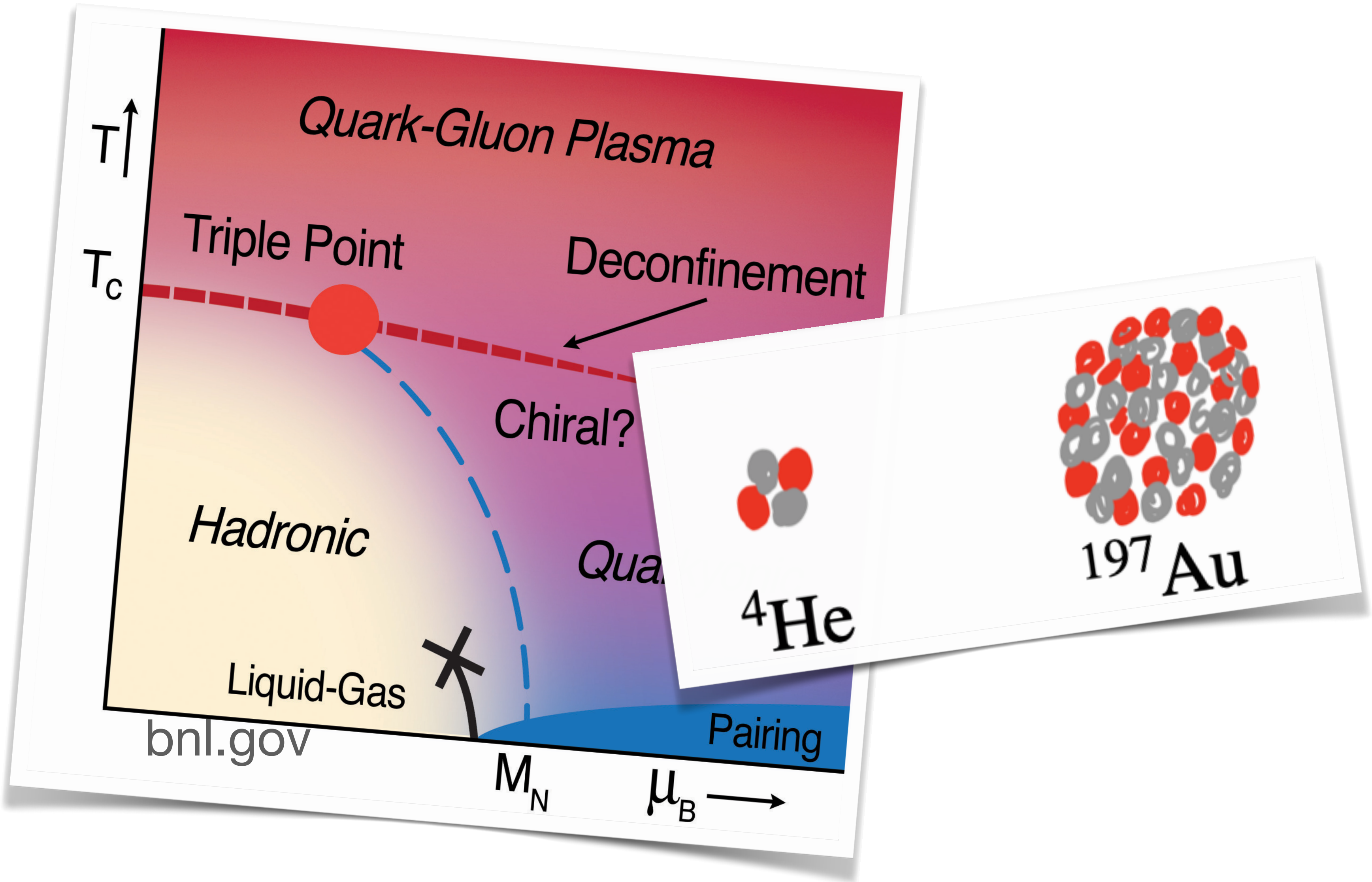
Motivation

Quantum ~~Computing~~ *Computing quantum many-body systems is ~~exponentially~~ hard (most of the time)*

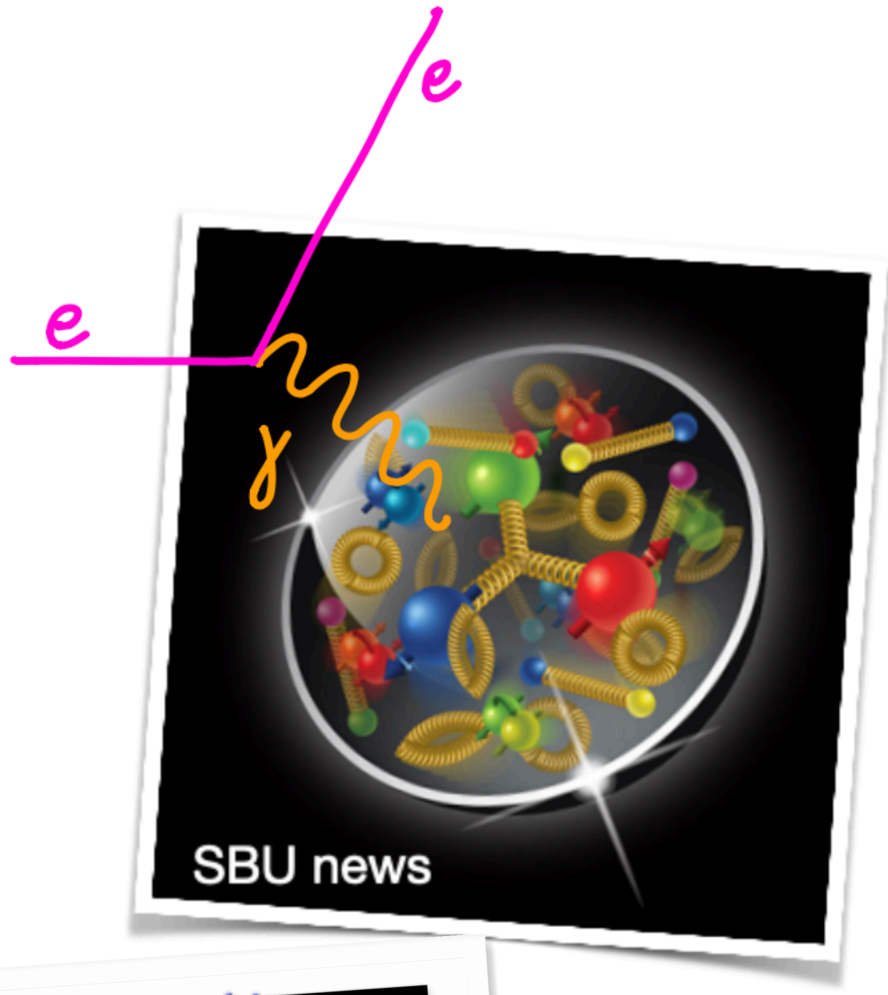
Static problems

Thermal or ground state properties, bound states etc.

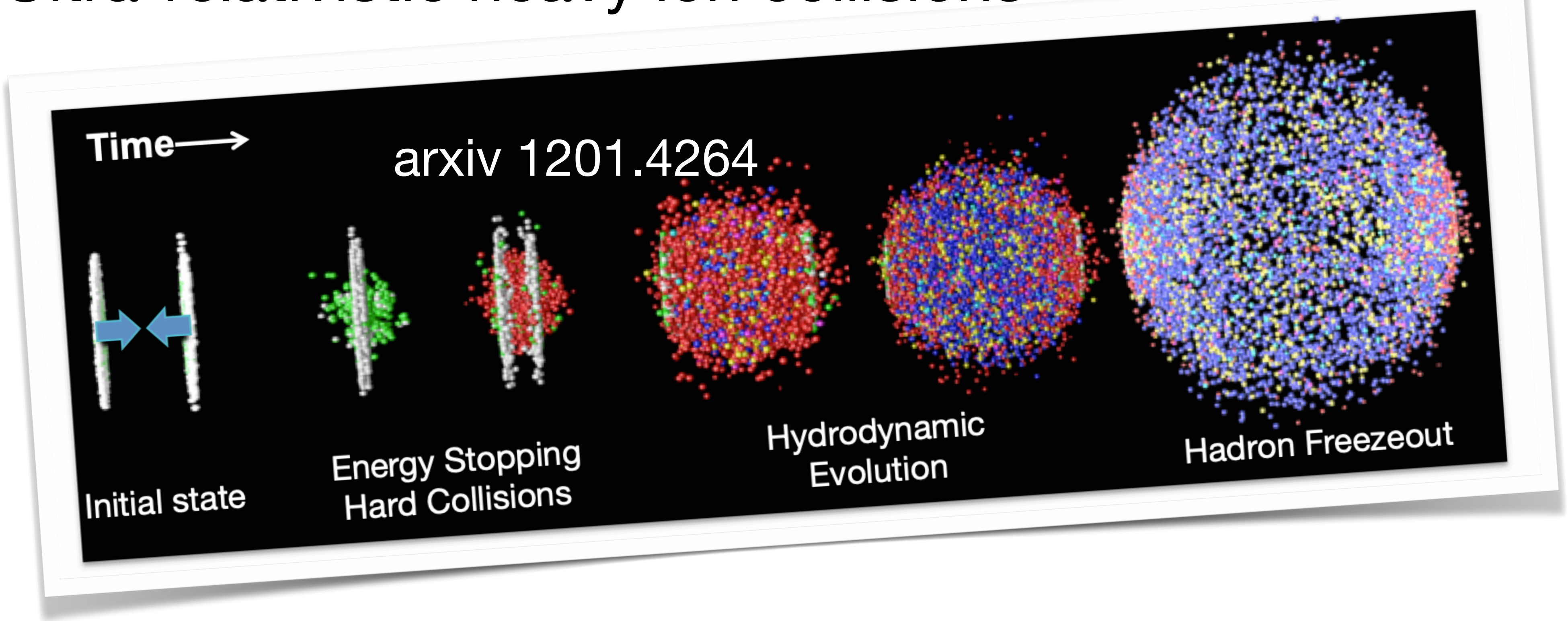


Dynamical problems

Scattering at the Electron-Ion Collider



Ultra-relativistic heavy ion collisions



Motivation

Desired Model

$$H = \sum_n \left[\frac{\pi_n^2}{2} + \frac{1}{2}(\nabla\phi_n)^2 + \frac{m^2}{2}\phi_n^2 + \frac{\lambda}{4!}\phi_n^4\right]$$

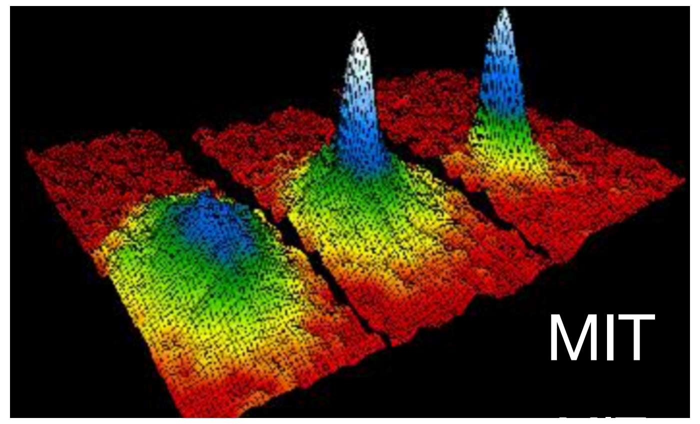
(real scalar field theory d+1 dim.)

$\phi_n|\phi\rangle_n = \bar{\phi}_n|\phi\rangle_n$ or $\pi_n|\pi\rangle_n = \bar{\pi}_n|\pi\rangle_n$

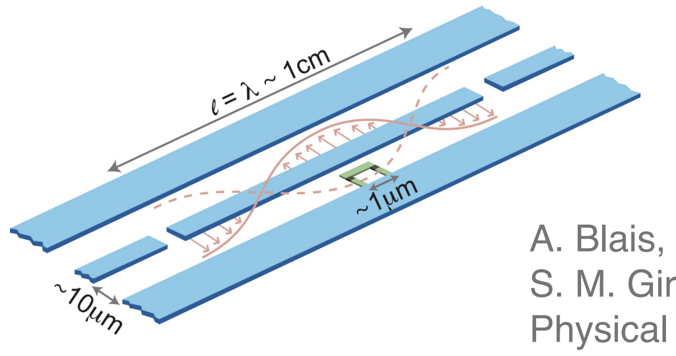
available Hilbert spaces ("hardware")

Digital (sc-qubit, ions …)

Analog (atoms, molecules, …)



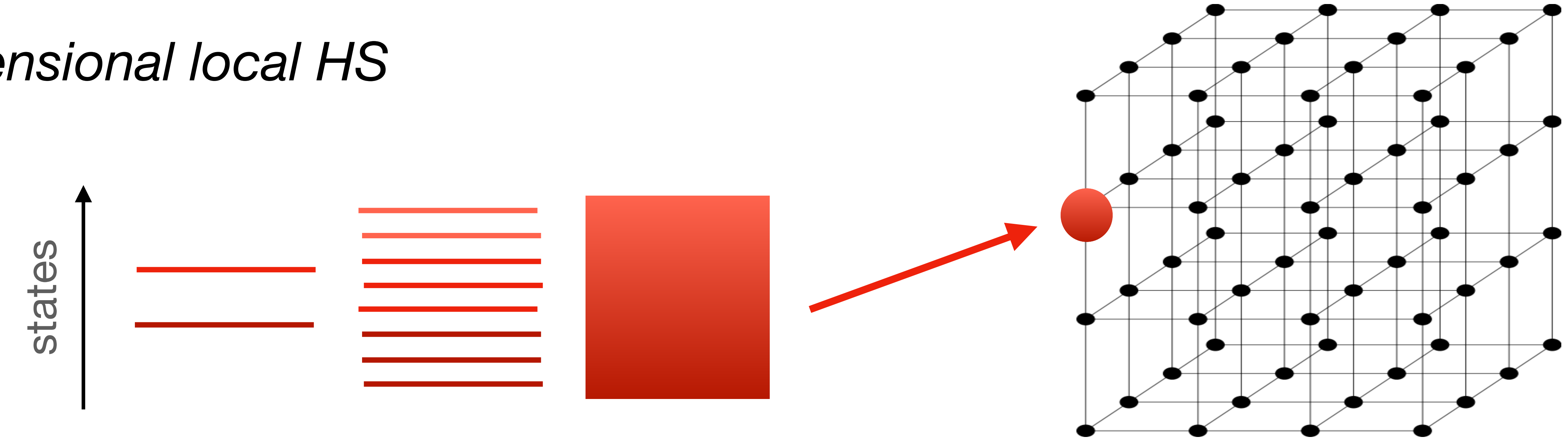
Digital-Analog (cQED, optical…)



A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Physical Review A **69**, 062320 (2004).

For bosonic theories spectrum continuous, infinite dimensional local HS

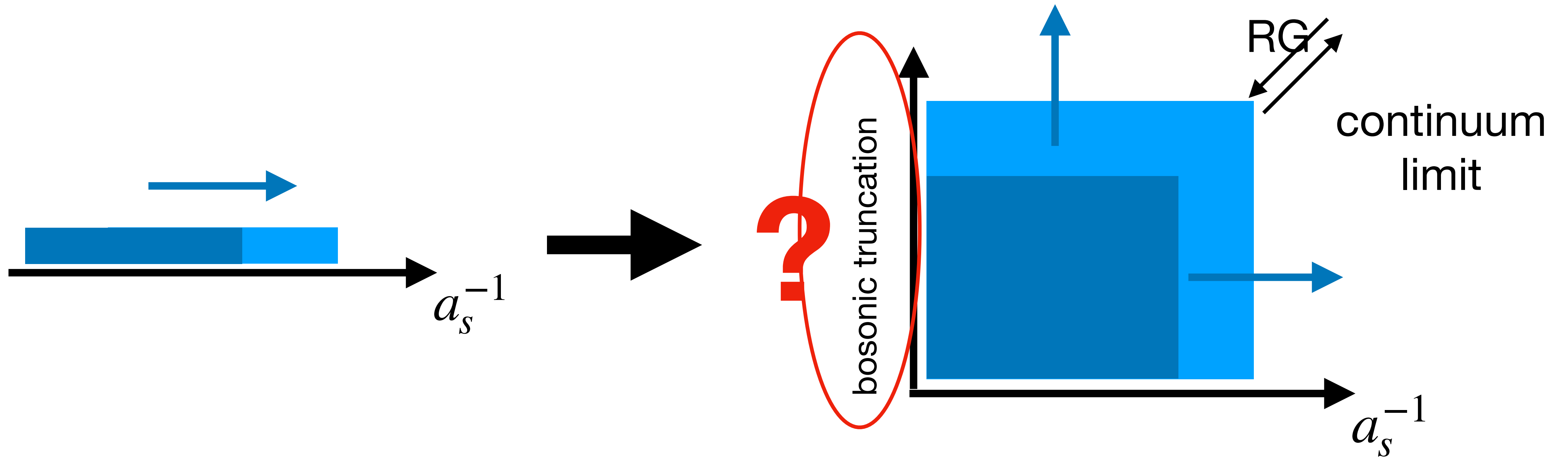
$\phi_x|\phi\rangle_x = \bar{\phi}_x|\phi\rangle_x$ or $\pi_x|\pi\rangle_x = \bar{\pi}_x|\pi\rangle_x$



Here will focus mostly on digital implementations

Digitization Strategies

- Challenge in NISQ era:
  how to do best with limited resources?



("Strategy 1"): Field Based Digitization

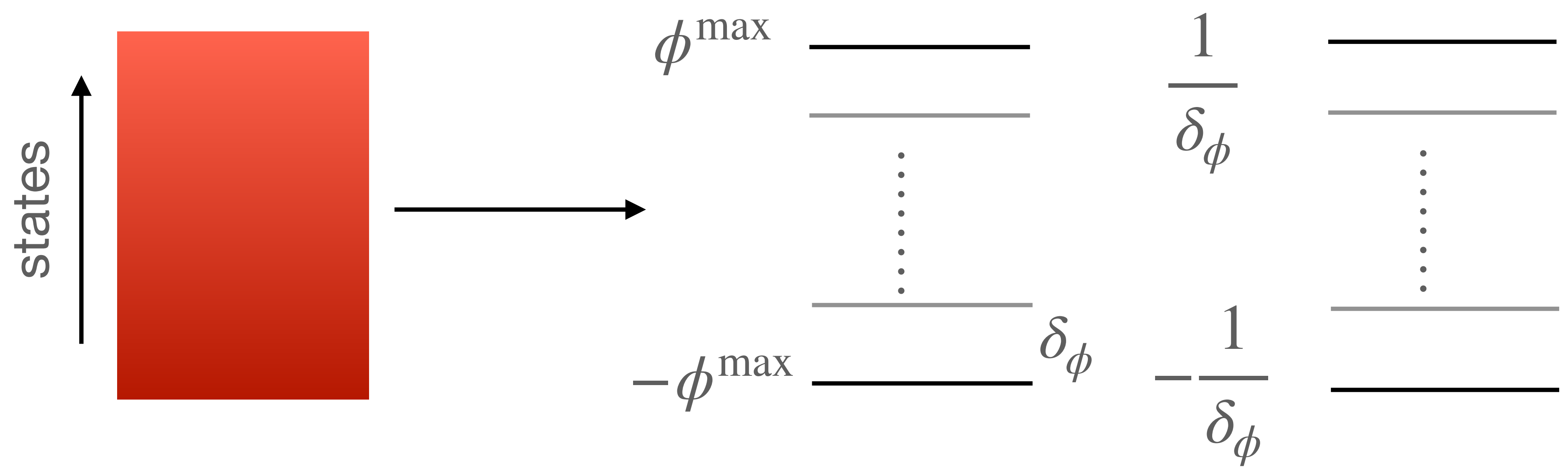
Jordan, Lee & Preskill, Science 336, 1130 (2012) arXiv:1112.4833; Klco, Savage PRA 98, 052335 (2019)

$\phi_x|\phi\rangle_x = \bar{\phi}_x|\phi\rangle_x \qquad \bar{\phi} \in [-\phi^{\max}, \phi^{\max}]$

$\bar{\phi} = -\bar{\phi}^{\max} + \beta_\phi \delta_\phi \quad \beta \in 0,1,\ldots\phi^{\max}/\delta_\phi$ (integer)

$|\phi\rangle \rightarrow |\beta_\phi\rangle = |0\rangle, |1\rangle, |2\rangle, \ldots$

$\pi_x|\pi\rangle_x = \bar{\pi}_x|\pi\rangle_x \qquad \bar{\pi} \in [-\pi^{\max}, \pi^{\max}] \qquad \pi^{\max} = \frac{1}{a_s^d \delta_\phi}$



- Binary representation $|0\rangle = |\ldots000\rangle,\ |1\rangle = |\ldots001\rangle,\ |2\rangle = |\ldots010\rangle$

number of qubits

$O(V)$

~ few thousand to ten thousand for simplest problem

("Strategy 2"): Harmonic Oscillator Based Digitization

$$\phi_x = \frac{1}{V^{1/2}} \sum_q \frac{1}{(2\omega_q)^{1/2}} [a_q + a_{-q}^\dagger] e^{2\pi i x\cdot q/N} \qquad \pi_x = \frac{-i}{V^{1/2}} \sum_q \left(\frac{\omega_q}{2}\right)^{1/2} [a_q - a_{-q}^\dagger] e^{2\pi i x\cdot q/N}$$

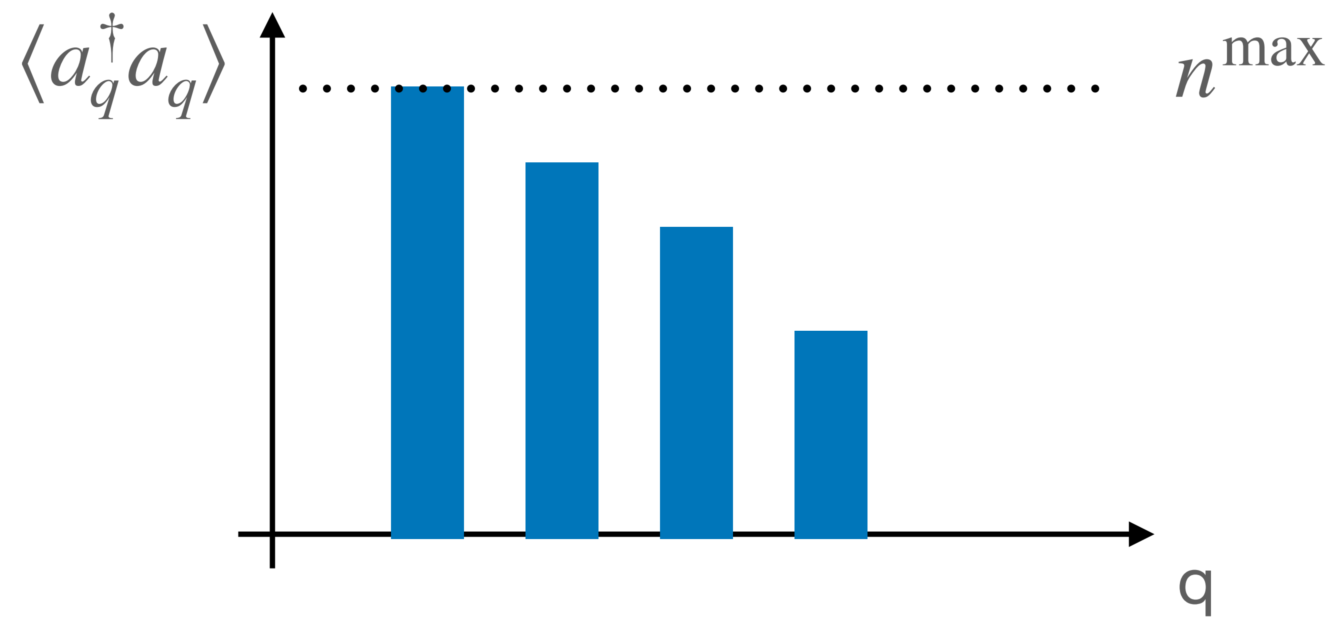
Yeter-Aydeniz, Siopsis PRD(7, 036004 (2018) Klco, Savage PRA 98, 052335 (2019) Yeter-Aydeniz, Siopsis, Dumitrescu, McCaskey, Bennink, Poser PRA 99, 032306, truncation of HO, see Macridin, Spentzouris, Amundson, Harnik PRL 121, 110504 (2018)), PRA 98, 042312 (2018),

- Position space vs. momentum space

$$a_x^\dagger = \frac{\phi_x - i\pi_x}{2} \qquad a_x = \frac{\phi_x + i\pi_x}{2}$$

- Truncation $n_q, n_x < n^{\max}$, binary representation

$|n=0\rangle = |\ldots000\rangle, |n=1\rangle = |\ldots001\rangle, |n=2\rangle = |\ldots010\rangle, \ldots$



Digitization Strategies

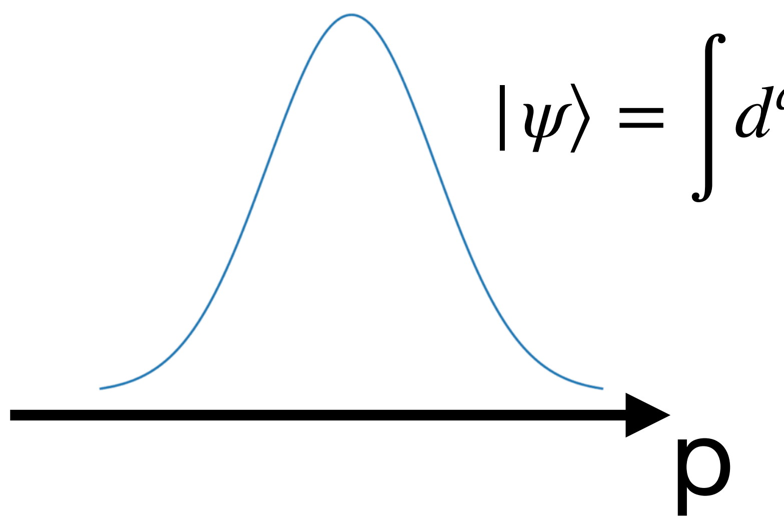
(“Strategy 3”): Particle basis

PRA 103, 042410, arXiv:2012.00020 [hep-th]

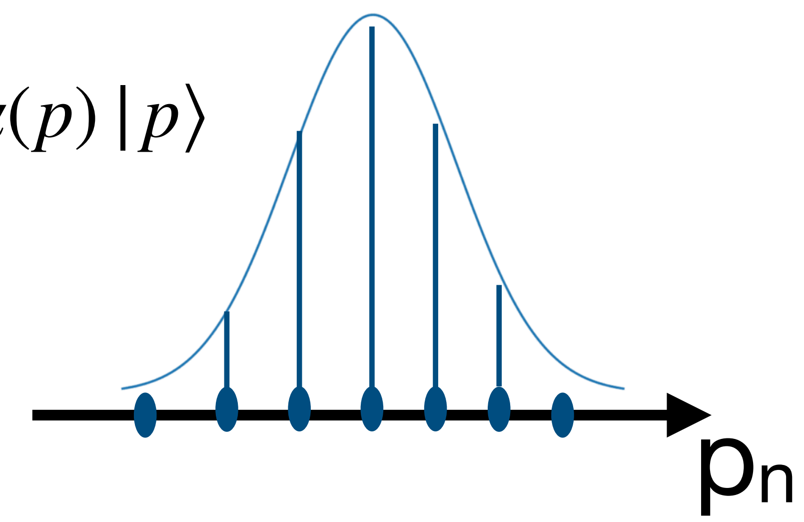
Joao Barata (Santiago→BNL),
NM (UMD), Andrey Tarasov (Ohio State),
Raju Venugopalan (BNL)

- Non-relativistic system: $|\mathbf{p}\rangle = |0\rangle, |1\rangle, |2\rangle, \ldots$

$$|\mathbf{x}\rangle = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}} |\mathbf{p}\rangle$$



$$|\psi\rangle = \int d^d p\, \alpha(p)\, |p\rangle$$



- binary encoding: $|n\rangle = |0\rangle, |1\rangle, \ldots |L-1\rangle = |0\ldots01\rangle, |0\ldots10\rangle, \ldots, |1\ldots11\rangle$ using $d\log(L) = \log(V)$ qubits

In relativistic QFT more complicated!

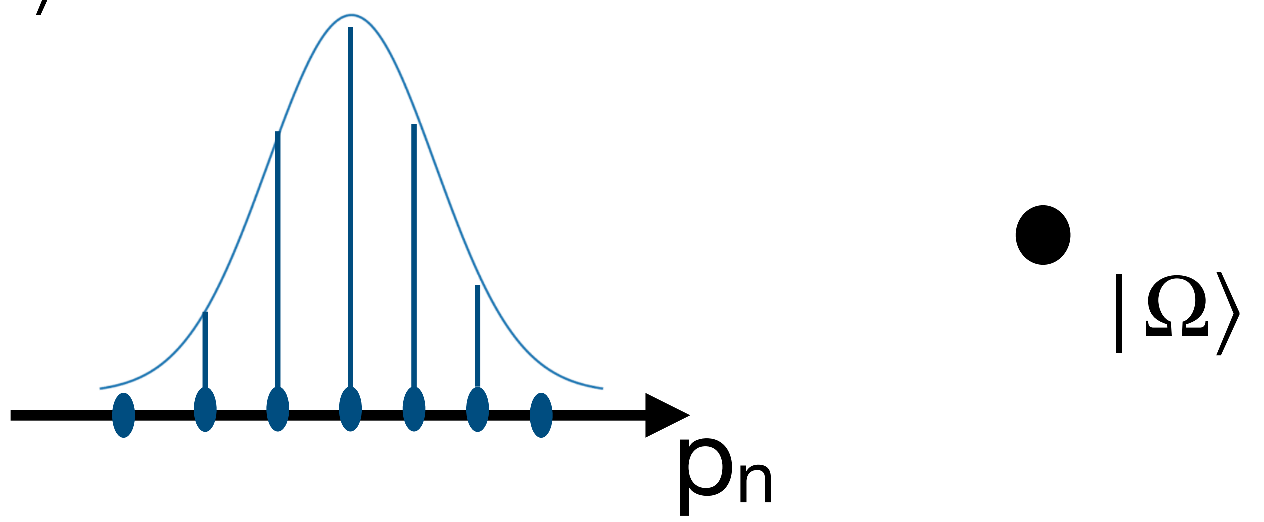
Particle register

$\mathcal{H} \equiv \mathrm{span}(|\mathbf{p}\rangle, |\Omega\rangle)$ $\quad |\mathbf{p}\rangle = |1011\ldots;1\rangle$ $\quad |\Omega\rangle = |0000\ldots;0\rangle$

i) particle ‘exists’ i.e. three-momentum eigenstate $|\mathbf{p}\rangle$

ii) particle ‘does not exist’ $|\Omega\rangle$,

iii) particle can ’exist’ and ‘not exist’, i.e. be off-shell $|\psi\rangle = \alpha_0|\Omega\rangle + \int d^d\mathbf{p}\, \alpha_{\mathbf{p}} |\mathbf{p}\rangle$



Relation to many-body states and quantum field theory

- $|\mathrm{vac}\rangle = \bigotimes_{i=0}^{M-1} |\Omega\rangle$ is the Fock vacuum of the theory
- $|p, p'\rangle = \mathcal{N}\{|p\rangle|p'\rangle|\Omega\rangle\ldots + |p'\rangle|p\rangle|\Omega\rangle + \ldots\} \equiv |n_p = 1, n_{p'} = 1\rangle$ are the Fock states

… actually a hard core boson representation

$$a_p^\dagger = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} a_p^{(i)\dagger} \qquad a_p^\dagger |\mathrm{vac}\rangle = |p\rangle \qquad [a_p, a_{p'}^\dagger] = \delta_{p,p'} + O\left(\frac{n}{M}\right)$$

If $n/V < 1$, compact $\log(V)$ versus V

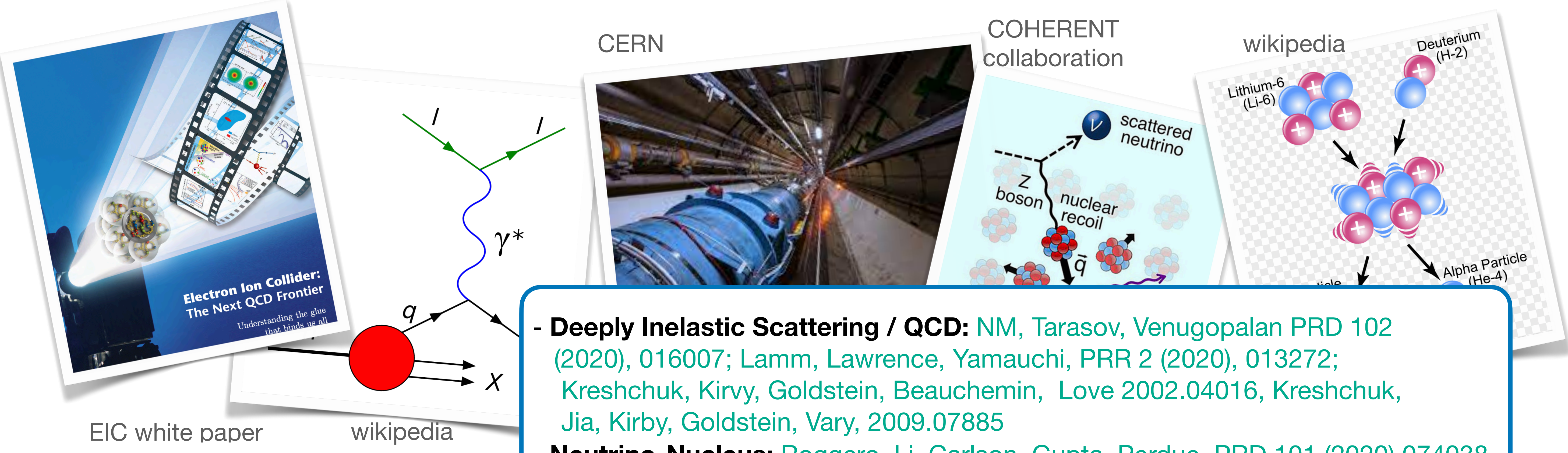
If $n/V \sim 1$, similar $V\log(V)$

If $n/V > 1$, worse, still polynomial

note dimensionless operators/units +different normalization as “textbook” single particle states $(2\omega_p)^{1/2} a_p^\dagger |\mathrm{vac}\rangle = |p\rangle$

Algorithms: Scattering problem

- Interesting because reveals dynamical information of quantum system
  (the only "tomography tool" for high energy experimentalists)



- **Deeply Inelastic Scattering / QCD:** NM, Tarasov, Venugopalan PRD 102 (2020), 016007; Lamm, Lawrence, Yamauchi, PRR 2 (2020), 013272; Kreshchuk, Kirvy, Goldstein, Beauchemin, Love 2002.04016, Kreshchuk, Jia, Kirby, Goldstein, Vary, 2009.07885
- **Neutrino-Nucleus:** Roggero, Li, Carlson, Gupta, Perdue, PRD 101 (2020) 074038
- **Particle decays:** Ciavarella, PAR 102 (2020), 094505
- **Universal QC:** Bao, Hayden, Salton, Thomas, NJP 17 (2015), 093028, Jordan, Lee Preskill Quantum 2, 44 (2018)

• S-matrix

$$S = U(\infty, -\infty) \qquad S_{\beta\alpha} = \langle \psi_\beta^{\text{free}} | U_{\infty,-\infty} | \psi_\alpha^{\text{free}} \rangle \qquad U_{t,t'} = \exp\{-iH(t-t')\}$$

• Computational perspectives

(a) Heisenberg picture, quantum variational algorithms Yeter-Aydeniz, Sipsis, Pooser 2008.08763

$$|\psi_\alpha^{\text{in/out}}\rangle = (V - VG_0V)^{-1}V|\phi_\alpha\rangle \qquad H = H_0 + V \qquad G_0 \equiv (E_\alpha - H_0 \pm i\epsilon)^{-1}$$

(b) Finite volume energies vs. scattering amplitudes (from euclidean and minkowski correlation functions)

Luescher Comm. MathPhys 105, 153 (1986), NPB 354, 531 (1991), Rummukainen, Gottlieb NPB450, 397 (1995), Davoudi, Savage, PRD84, 114502 (2011), Hansen, Sharpe PRD86, 016007 (2012), Brinceno, Davoudi, PRD88 094507 & Lu PRD88, 034502, … , Briceno, Guerrero, Hansen, Sturzu PRD 103, 014506 (2021)

(c) Schroedinger picture, real time evolution Jordan, Lee & Preskill, Science 336, 1130 (2012)

Jordan, Lee & Preskill, Science 336, 1130 (2012)

Algorithms: Scattering problem

Schroedinger picture

(a) prepare single particle states of non-interacting theory

$|\psi(-\infty)\rangle = |\phi(-\infty)\rangle$

(b) evolve into single particle states of interacting theory

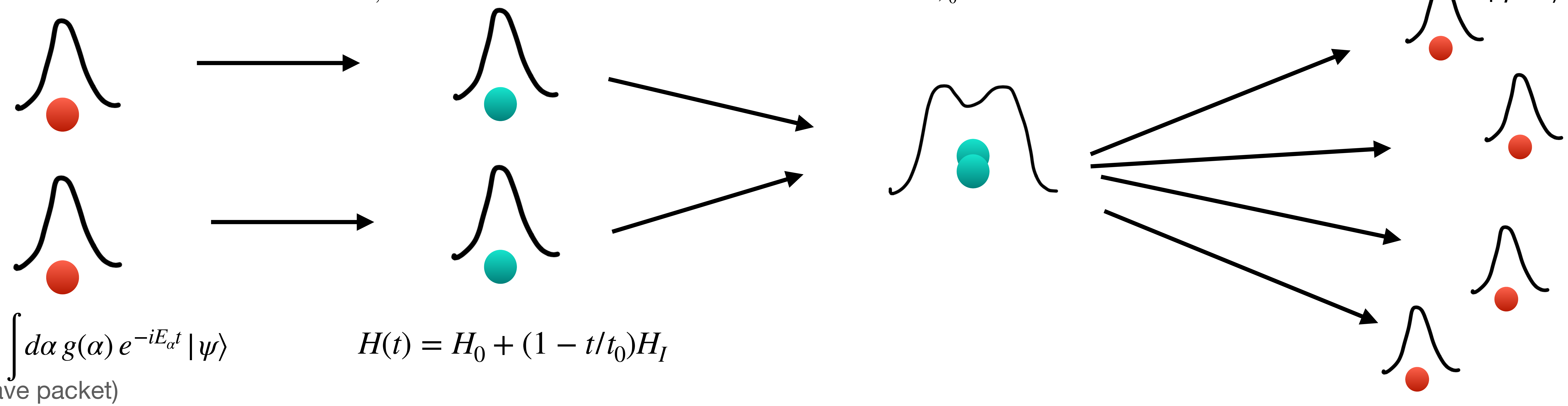
$U_{t,-\infty}|\psi(-\infty)\rangle$

(c) wave packets overlap / interact

$U_{t,t_0}|\psi(t_0)\rangle$

(d) measure outgoing particle states or energy density

$|\psi^{\mathrm{out}}\rangle$



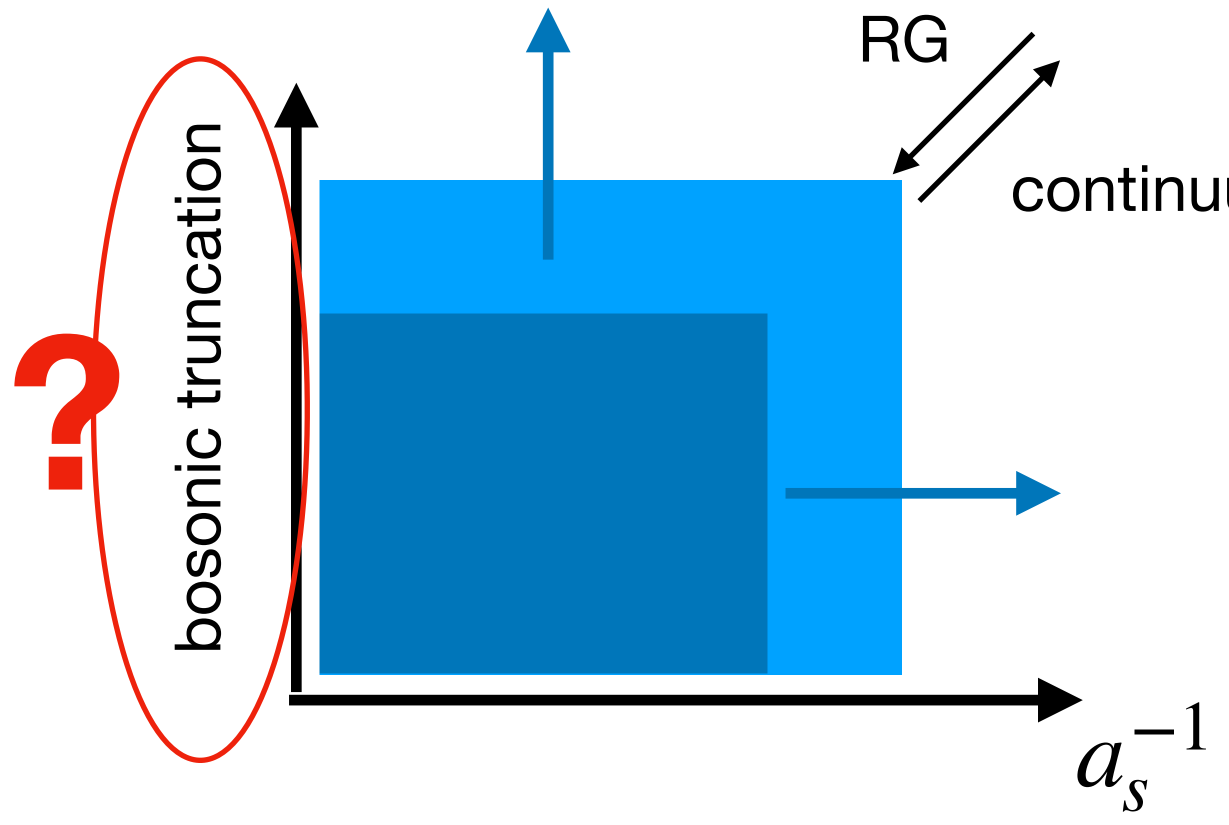
$|\psi(t)\rangle = \int d\alpha\, g(\alpha)\, e^{-iE_\alpha t}|\psi\rangle$

(wave packet)

$H(t) = H_0 + (1 - t/t_0)H_I$

(e) Renormalization

Relate to continuum scattering cross sections



Time Evolution Algorithm - Single Particle basis

- Trotter scheme $$U = \prod_t U(\delta t) \qquad U(\delta t) = e^{-iH_0\delta t}e^{-iH_I\delta t} + O(\delta t^2)$$
- Split up Hamiltonian

$$H = \sum_n \left[\frac{\pi_n^2}{2} + \frac{1}{2}(\nabla\phi_n)^2 + \frac{m^2}{2}\phi_n^2 + \frac{\lambda}{4!}\phi_n^4\right]$$

[reminder:

$$a_x^\dagger = \frac{\phi_x - i\pi_x}{2} \qquad a_x = \frac{\phi_x + i\pi_x}{2}$$

are Fock operators in position space]

$$H_0 = \sum_x \left[\frac{\pi_x^2}{2} + \frac{1}{2}(\nabla\phi_x)^2 + \frac{m^2}{2}\phi_x^2\right] = \sum_p \omega_p a_p^\dagger a_p$$

(position space) (momentum space)

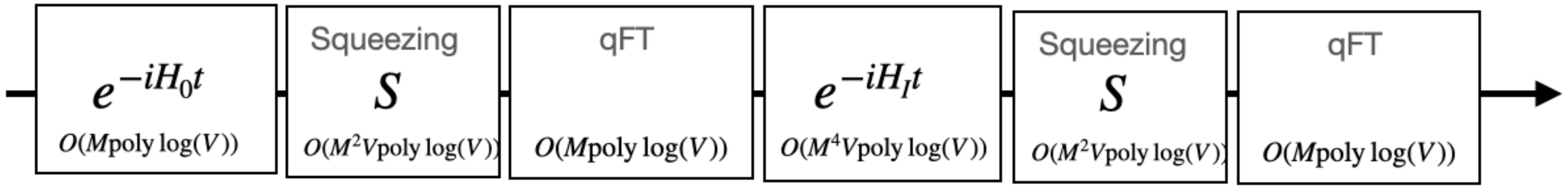
- Diagonal in momentum space
- Complicated in position space

$$H_I = \sum_x \frac{\lambda}{4!}\phi_x^4 = \sum_{q_1q_2q_3q_4} \frac{\lambda}{4!}\phi_{q_1}\phi_{q_2}\phi_{q_3}\phi_{q_4}\delta(q_1+q_2+q_3+q_4)$$

(position space) (momentum space)

- Very complicated in momentum space
- Local in position space

$U(\delta t) =$



Time Evolution Algorithm - Single Particle basis

Squeezing transformation?

Squeezing
S
$O(M^2 V \text{poly} \log(V))$

$$\phi_x = \frac{1}{V^{1/2}} \sum_q \frac{1}{(2\omega_q)^{1/2}} [a_q + a_{-q}^\dagger] e^{2\pi i x \cdot q/N}$$

$$\pi_x = \frac{-i}{V^{1/2}} \sum_q \left(\frac{\omega_q}{2}\right)^{1/2} [a_q - a_{-q}^\dagger] e^{2\pi i x \cdot q/N}$$

- Fock operators in position space $\quad a_x \equiv \frac{1}{\sqrt{2}}(\phi_x + i\pi_x) \qquad a_x^\dagger \equiv \frac{1}{\sqrt{2}}(\phi_x - i\pi_x)$
- Fourier conjugates $\quad a_x \equiv \frac{1}{\sqrt{V}} \sum_q A_q e^{2\pi i q x/N}$
- but $A_q \neq a_q$, instead $\quad A_q \equiv \frac{1}{2}\left[\frac{1}{\sqrt{\omega_q}} + \sqrt{\omega_q}\right] a_q + \frac{1}{2}\left[\frac{1}{\sqrt{\omega_q}} - \sqrt{\omega_q}\right] a_{-q}^\dagger = S_q a_q S_q^\dagger$

Two-mode squeezing

$$S_q \equiv \exp\{-z_q(a_q^\dagger a_{-q}^\dagger - a_{-q} a_q)\}$$
$$z_q \equiv \log(\omega_q)/2$$

- Particle/Fock states in position vs momentum space

$$|\mathbf{x}\rangle \neq \int \frac{d^d\mathbf{p}}{(2\pi)^d} e^{i\mathbf{p}\mathbf{x}} |\mathbf{p}\rangle$$

but actually

$$|\mathbf{x}, x^0\rangle = \int \frac{d^4p}{((2\pi)^4)} \delta(p^2 - m^2)\theta(p^0) e^{ipx} |\mathbf{p}, p^0\rangle$$

- Circuits

		Elementary gate operations	Ancilla qubits
Time Evolution	Free part U_0	$O(M \text{poly} \log(\mathcal{V})\, t)$	$O(\log(\mathcal{V})/d)$
	Squeezing transform S	$O(M^2 \mathcal{V} \text{poly} \log(\mathcal{V})\, t)$	0
	quantum Fourier transform	$O(M \text{poly} \log(\mathcal{V})\, t)$	0
	Interaction part U_I	$O(M^4 \mathcal{V} \text{poly} \log(\mathcal{V})\, t)$	$O(\log(\mathcal{V})/d)$
	Total	$O(M^4 \mathcal{V} \text{poly} \log(\mathcal{V})\, t)$	$O(\log(\mathcal{V})/d)$

Barata, NM, Tarasov, Venugopalan , **PRA 103, 042410, arXiv:2012.00020 [hep-th]**

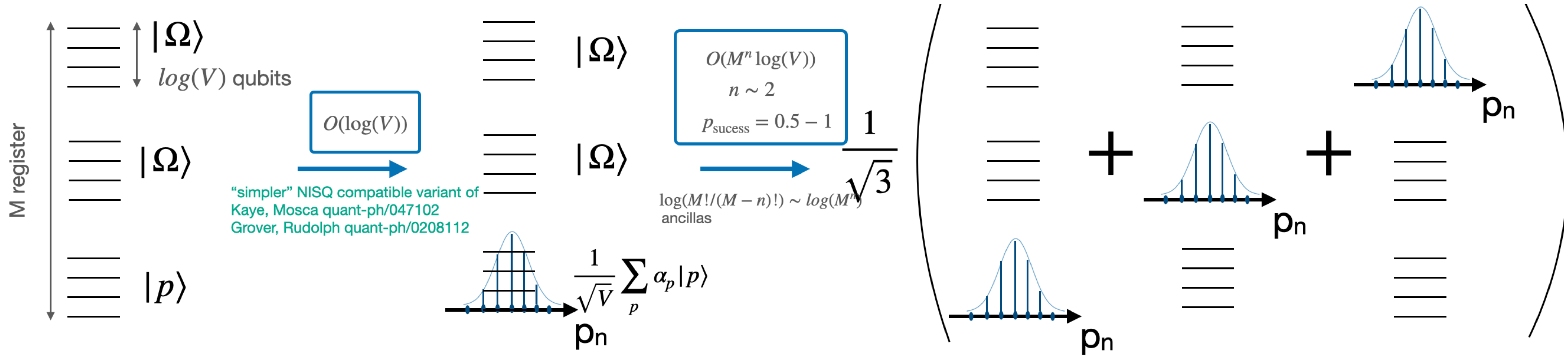
Jordan, Lee & Preskill, Science 336, 1130 (2012)
Barata, NM, Tarasov, Venugopalan **PRA 103, 042410, arXiv:2012.00020**

State Preparation and measurement

- Vacuum and single particle states are computational basis states

$$\mathscr{H} \equiv \text{span}(|\mathbf{p}\rangle, |\Omega\rangle) \qquad |\mathbf{p}\rangle = |1011\ldots;1\rangle \qquad |\Omega\rangle = |0000\ldots;0\rangle$$

- **Initial state preparation:** Wave packet superposition and Bose symmetrization



- **Measurement:** computational basis = particle number basis

$$|\psi(t)\rangle = \sum_{\ell} \alpha_\ell |\psi_\ell\rangle = \sum_{\text{basis states}} \left[\frac{\alpha_{q,q'\ldots}}{N_{q,q'}^{1/2}}(t)\,|q, q', \Omega\ldots + \text{sym}\right]$$

will collapse onto particle number state upon measurement

- If particle (parton) number not meaningful (such as in QCD):
Phase Estimation Algorithm (*) circuits are the same as for $e^{-iH_0\delta t}$, $e^{-iH_I\delta t}$

Barata, NM, Tarasov, Venugopalan **PRA 103, 042410, arXiv:2012.00020**

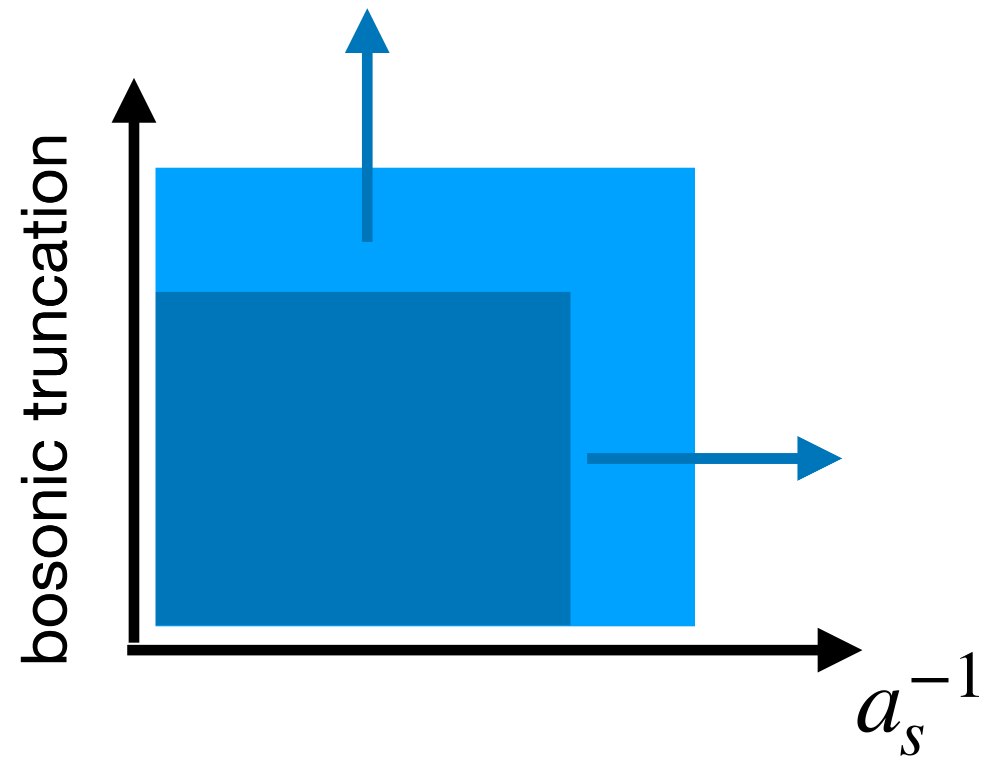
Non-perturbative Renormalization

Renormalization scheme should match (the precision of) the actual computation.
*Thus in principle **the same** computational complexity.*

- Good understanding for static quantities, but not so much for dynamics

- Hamiltonian operator renormalization

$$H = \begin{pmatrix} H_{\ell\ell} & H_{\ell h} \\ H_{h\ell} & H_{hh} \end{pmatrix} \qquad H^{\text{eff}} = P_\ell (THT^\dagger) P_\ell \qquad T = e^{i\eta}$$



- Perturbation theory: Schrieffer Wolf transformation Perry & Wilson NPB 403, 587 (1993), Wilson & Glaczek PRD 48, 5863 (1993)
  Non-perturbative: Similarity RG Wegner Ann. d. Phys. 506, 77 (1994) , Bogner, Furnstahl, Schwenk, Prog. Part. & Nucl. Phys 65, 94 (2010)

In practice
(oversimplified)

1. Quantum compute dimensionless **static low energy quantity**, e.g. ratio of low lying excitations for some a_s, M. Adjust M, λ to reproduce physical value.
2. Repeat at different a_s, M along $a_s \to 0, M \to \infty$, adjust M, λ (line of constant physics)
3. Once $\lambda^{\text{ren}}, m^{\text{ren}}$ known for many a_s, M, use as input for scattering experiment + operator renormalization.
4. Continuum extrapolation of scattering results.

Summary

- Many approaches to digitization of scalar field theory and scattering algorithms

- Few are systematically explored

- Single particle digitization may be very efficient for dilute systems, not for all systems

- Initial State preparation and measurement of scattering cross sections very simple

Summary

	Field Based (JLP)	Particle Based		Harmonic Oscillator based (*)	
				position space	momentum space
State rep.	$O(V\log(V))$	$O(\log(V))$ $> O(V\log(V))$	best case worst case (still polynomial)	$O(V\log(V))$	$O(V\log(V))$
initial state prep	additional Trotter complexity $O(V \times \text{const})$	easy		?? (**)	easy
time evolution	$O(V)$	$O(V)$ polynomial + continiuum dispersion	best case worst case	it depends, see Klco, Savage PRA 98, 052335 (2019)	$O(V^4)$ + continiuum dispersion
measurement	additional Trotter complexity $O(V^2 \times \text{const})$	easy, integrated CS for free		?? (**)	easy
verification	classical-statistical (at weak coupling) (large ϕ^{max})	lattice perturbation theory or kinetic theory (at weak coupling)		?? (**)	lattice perturbation theory or kinetic theory (at weak coupling)

(*) Note: no strict separation. Connected by unitary transformations

(**) Reflects my ignorance, not necessarily that of others.