Non-equilibrium attractor in high temperature QCD plasma

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DA, Aleksi Kurkela, Michael Strickland, PRL. 125, (2020)

9th Workshop of the APS Topical Group on Hadronic Physics (virtual)
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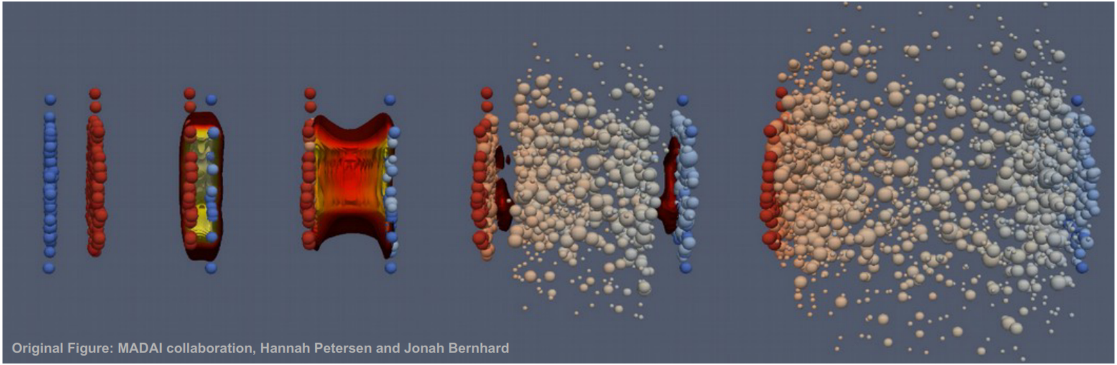
KENT STATE UNIVERSITY

Kinetic transport theory in ultra-relativistic heavy ion collisions



initial state   preequilibrium   Hydrodynamics   Hadronization   Hadronic cascade

QCD kinetic tranport theory

Formalism

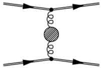
QCD medium at high temperatures: Effective kinetic theory AMY JHEP0301 (2003) 030

$$P^{\mu}\partial_{\mu} f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})]$$

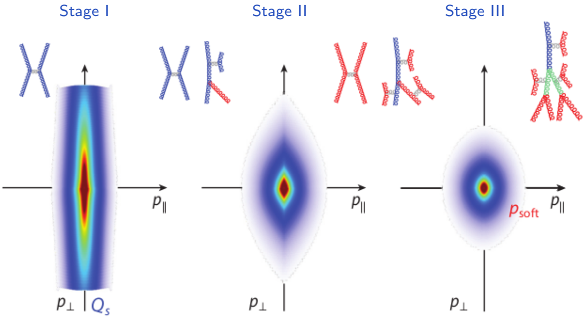
$\underline{0 + 1\mathrm{d}\ \mathrm{Bjorken}}$

At Leading order, transport at different momentum scales in C[f]



regulated by HTL

LPM included



"Bottom up thermalisation"

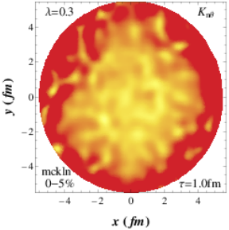
Baier, Mueller, Schiff, and Son (2001); J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019); Arnold, P. Gorda, T. Iqbal, S. JHEP. 2020, 53

Non-equilibrium dynamics

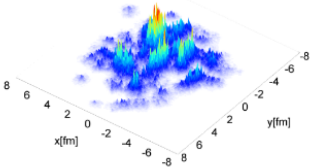
Non-equilibrium effects and Hydrodynamization

Uncertainties in our understanding of the regime applicability of fluid dynamics

$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5$$



Noronha-Hostler,Noronha,Gyulassy PRC 93(2016)



Schenke,Tribedy,Venugopalan PRL. 108,(2012)

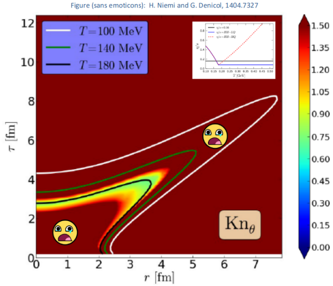
$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} > 1$$

"Unreasonable" success of Hydrodynamics

Signs of breakdown

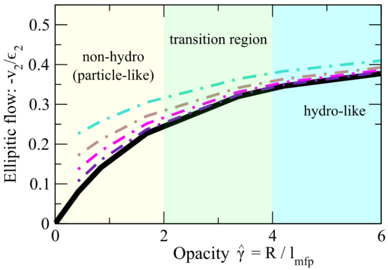
Careful treatment required for the criteria of fluid dynamics

large far from equilibrium effects



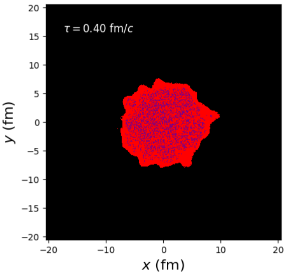
Niemi,Denicol arxiv.1404.7327

non-hydro modes dominate v_2



Kurkela,Wiedemann,Wu, EPJ.C79,(2019)

nonlinear causality violations!!



Plumberg, DA, Dore, Noronha,

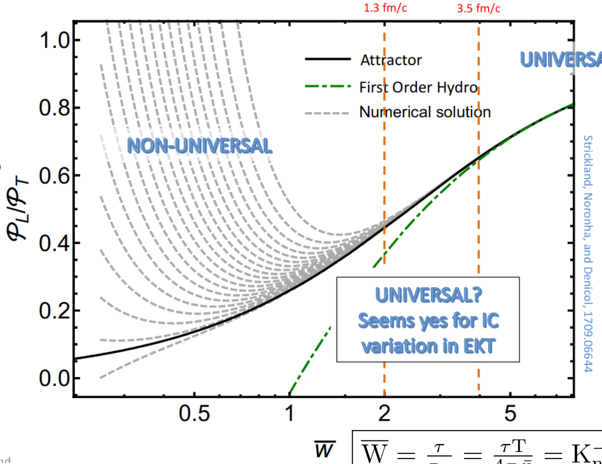
Noronha-Hostler, 2103.15889(2021)

especially true for small systems!.

The attractor: a better way to think of hydrodynamics

How universal are hydrodynamic attractors?

- Hydrodynamics as a universal attractor
  Heller and Spalinski. PRL.115 (2015)
- Non-hydro modes act as regulators to ensure causality
- Memory loss of initial conditions
- Competition between expansion and interaction rate
- Onset of hydrodynamics is set by the decay of non-hydrodynamic modes
- Microscopic model dependent

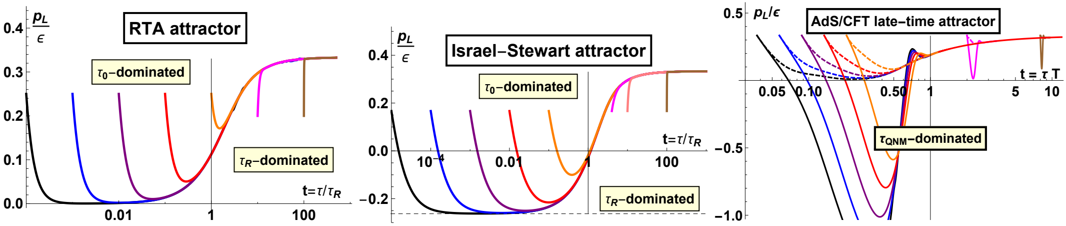


$$\overline{\mathrm{W}} = \frac{\tau}{\tau_R} = \frac{\tau \mathrm{T}}{4\pi\,\bar{\eta}} = \mathrm{K_n}^{-1}$$

Attractors in different microscopic theories

Kurkela,van der Schee,Wiedemann,Wu PRL.124,(2020)

Decay of non-hydro modes depends on the underlying microscopic theory



Strong coupling

Can one access the underlying information?

Non-equilibrium attractor beyond hydrodynamics?

Almaalol,Kurkela, Strickland PRL. 125,(2020)

General moments of the Boltzmann equation

Solving for moments of the Boltzmann equation $\Rightarrow$ reconstruction of $f(x,p)$

- Momentum discretization method: 2D grid $\{x_i, p_j\}$ with 250×2000 grid points (Kurkela and Zhu PRL 115, 182301 (2015))
- A general moment for $\mathrm{N_C} = 3$ in $0+1\mathrm{d}$ Bjorken flow

$$\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)$$

M. Strickland, JHEP2018, 128; 1809.01200.

- Corresponding equilibrium values for Bose distribution,

$$\mathcal{M}^{nm}_{\mathrm{eq}} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}$$

- low moments $\rightarrow$ hydrodynamics degrees of freedom

$$\begin{aligned}\mathcal{M}^{10} &= \quad \text{number density}\\ \mathcal{M}^{20} &= \quad \text{energy density}\\ \mathcal{M}^{01} &= \quad \text{longitudinal pressure}\end{aligned}$$

- Deviations from equilibrium

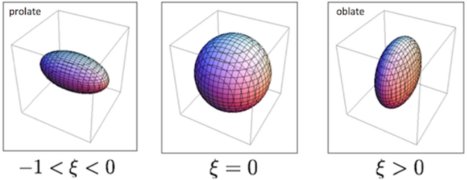
$$\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{nm}_{\mathrm{eq}}(\tau)}$$

.

Initial distribution $-\frac{d\mathbf{f_p}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{\exp}[\mathbf{f_p}].$

- Thermal Romatschke-Strickland

$$\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)$$



anisotropy parameter $(-1 < \xi_0 < \infty)$

Λ_0 is set by Landau matching

Romatschke,Strickland,PRD68, (2003)

- Non-thermal CGC

$$\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda}\frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}} \exp^{-\frac{2}{3}\left(\mathbf{p}^2 + \xi_0 \hat{p}_z^2\right)/\tilde{\Lambda}_0^2}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T}\rangle_0 \approx 1.8\,\mathrm{Q_s}$

A is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation

A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)

T. Lappi, Phys. Lett.B703, 325-330 (2011)

Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

Non-equilibrium evolution becomes insensitive to initial conditions at very early times

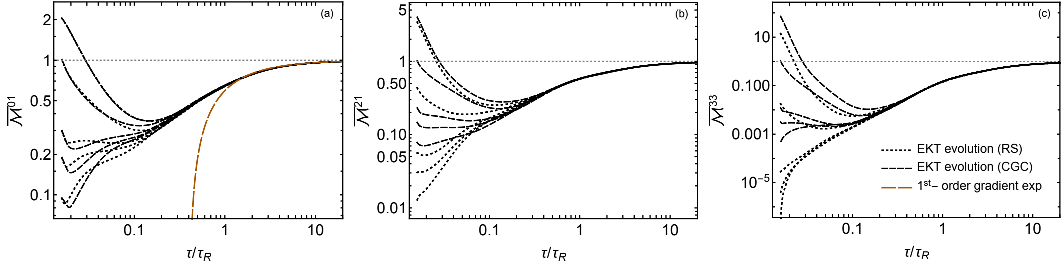
$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Forward attractor

Pressure anisotropy

$\mathcal{M}^{01}[f] =< p^{-1}p_z^2 >$ $\mathcal{M}^{21}[f] =< p^{1}p_z^2 >$ $\mathcal{M}^{33}[f] =< p^{2}p_z^6 >$



Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
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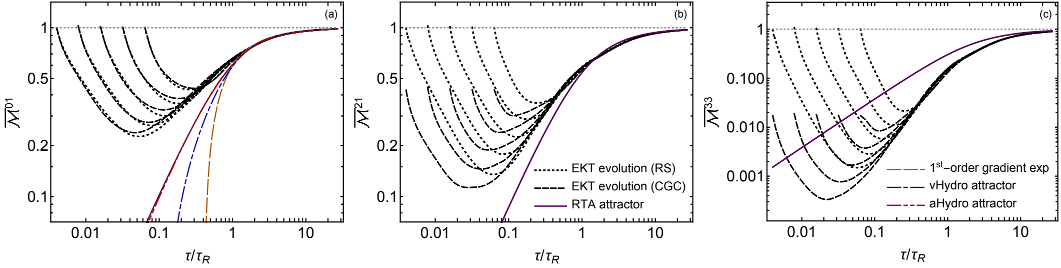
5	14.1 fm/c
10	38.5 fm/c

- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

Pressure anisotropy

$\mathcal{M}^{01}[f] = < p^{-1} p_z^2 >$ $\mathcal{M}^{21}[f] = < p^{1} p_z^2 >$ $\mathcal{M}^{33}[f] = < p^{2} p_z^6 >$



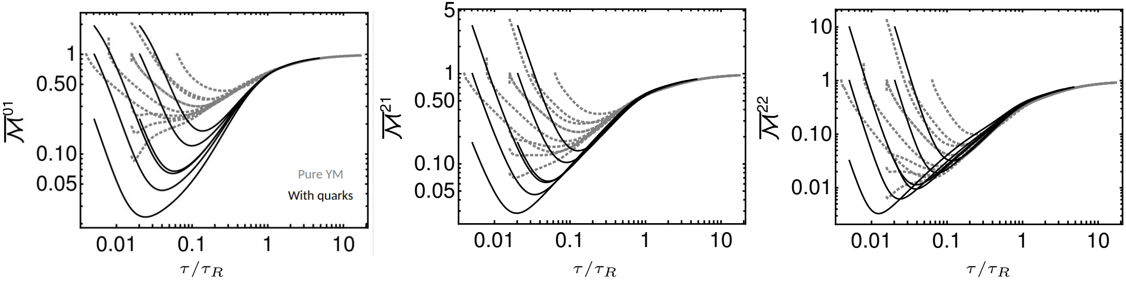
Chemical equilibration?

Quarks?

DA, Mazeliauskas, Strickland. forthcoming

Inclusion of quarks increases anisotropy

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $\mathrm{gg} \to \mathrm{q\bar{q}}$ and splitting $\mathrm{g} \to \mathrm{q\bar{q}}$



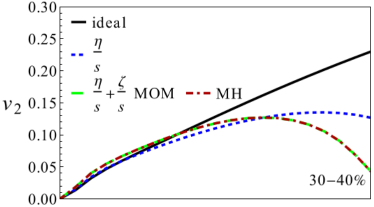
- An attractor exists for all moments in QCD with $N_f = 3$ quarks

Non-equilibrium effects at Freeze out

Almaalol,Kurkela, Strickland PRL. 125,(2020)

freezeout

(Noronha-Hostler, Noronha, Grassi, PRC 90 (2014))



- δf corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$

$$E\frac{d^3N_s}{d^3p} = \frac{\nu_s}{(2\pi)^3}\int_\sigma (f_s(\tilde{p}) + \delta f)$$

- δf can be computed for a particular form of $C[f]$ (Dusling, Moore, Teaney PRC 81, (2008))

The *quadratic ansatz* $(\alpha = 0)$

$$\frac{\delta f_{(i)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{3\overline{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

The *LPM ansatz* $(\alpha = 0.5)$

$$\frac{\delta f_{(ii)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{16\overline{\Pi}}{21\sqrt{\pi}\,T^{3/2}}\left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}}\right)$$

The *aHydro freeze-out ansatz*

$$f(p) = f_{\rm Bose}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

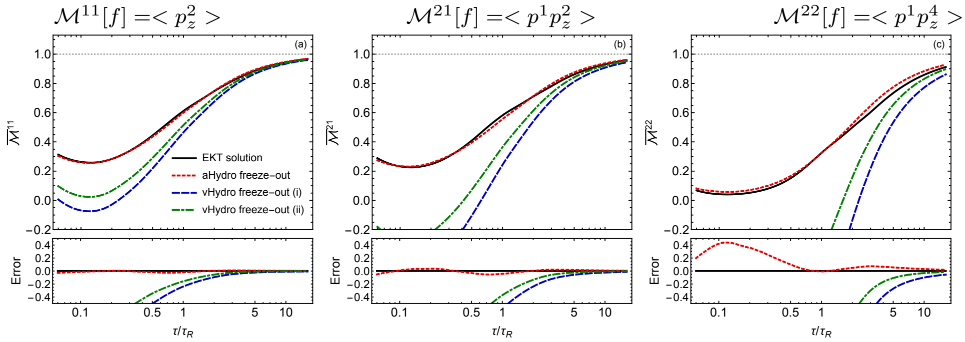
$$\overline{\mathcal{M}}^{nm}_{\rm aHydro}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}$$

Insights into the freezeout perscription

(Almaalol,Kurkela,Strickland PRL 125, (2020))

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times



For earlier implementation: (Pratt,Torrieri PRC 82(2010) (Weller, Romatchke PLB 774 (2017)

Conclusions

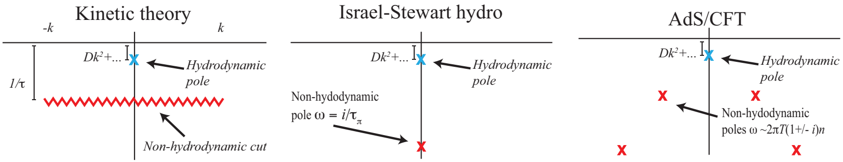
- A non-equilibrium attractor for the phase space distribution of QCD at high temperature EKT
- Ahydro distribution for further improvements in th FO
- Inclusion of quarks: chemical equilibration at high moments ?
  (DA, A. Mazeliauskas, and M.Strickland, forthcoming)
- Conformal and $0 + 1\mathrm{d} \Rightarrow$ transverse dynamcis + non-conformal?

Thank you for your attention!

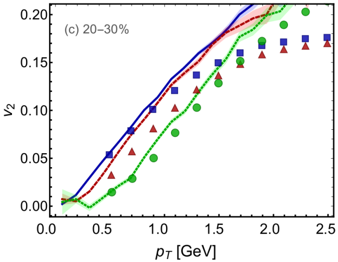
Hydrodynamics

Fluid dynamics is an effective theory of long wavelength modes

$$G_R^{\mu\nu,\alpha\beta}(x;t) = \langle [T^{\mu\nu}(x,t),\, T^{\alpha\beta}(0,0)] \rangle$$



Successfully describes low momentum regime of hadronic spectra and flow variables



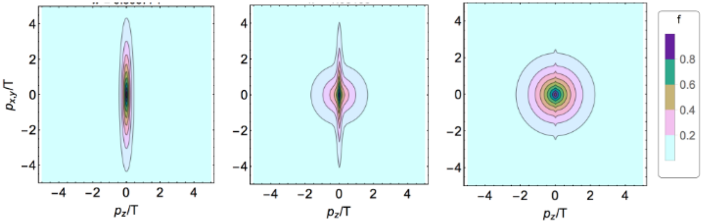
add reference,(2019)

Relaxation time approximation: RTA

J.L. Anderson, H.R. Witting Physica, 74(1974)

- Approach to equilibrium set by an equilibration rate.

$$-\frac{df_g(\mathbf{p})}{d\tau} + \frac{p_z}{\tau}\partial_{p_z} f_g(\mathbf{p}) = \frac{p^\mu u_\mu}{\tau_R}\left(f_g(\mathbf{p}) - f_g^{eq}(\mathbf{p})\right)$$



M. Strickland. JHEP.2018,128 (2018)

Popular approach $\Rightarrow$ direct affect on transport coefficients calculations

From kinetic theory to hydrodynamics

Kinetic based hydrodynamics equations

Moment integral operator

$$\hat{\mathcal{O}}_n\, g = \mathcal{O}^{\mu_1\mu_2\cdots\mu_n}[g] \equiv \int dP\, p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n}\, g(p) n^{th}$$

$$p^\mu \partial_\mu f_p = C[f_p]$$

$$\partial_\mu I^{\mu\nu_1\nu_2\cdots\nu_n} = \mathcal{C}^{\nu_1\nu_2\cdots\nu_n}$$

$$I^{\mu\nu_1\nu_2\cdots\nu_n} \equiv \int dP\, p^\mu p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n}\, f$$

$$\mathcal{C}^{\nu_1\nu_2\cdots\nu_n} \equiv \int dP\, p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n}\, C[f]$$

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C} = 0$$

$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu = 0$$

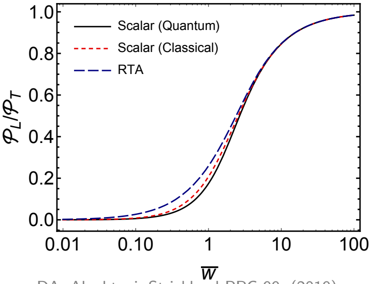
Phenomenological implications?

$$\mathcal{C}_{RTA} = \frac{u^\mu . p^\mu}{\tau_{\rm R}} \left[f_{\rm eq}(p/T) - f_p \right]$$

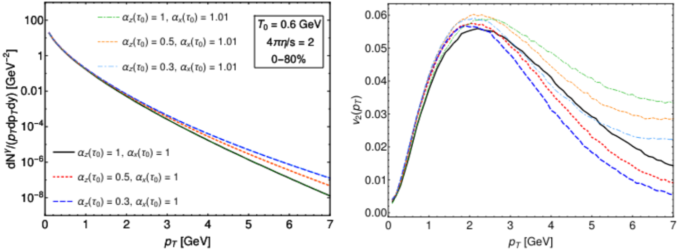
$$\partial_\mu n^\mu = \frac{1}{\tau_{\rm R}} \left[n_{\rm eq} - n \right]$$
$$\partial_\mu T^{\mu\nu} = \frac{1}{\tau_{\rm R}} \left[\epsilon_{\rm eq} - \epsilon \right]$$

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C}$$
$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu$$



DA, Alqahtani, Strickland PRC 99, (2019)



Kasmaei, Strickland PRD 102(2019)