Reconstructing Pion and Kaon x-dependent PDFs From Lattice Mellin Moments

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Overview

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Motivation

- Pion and kaon structure is important for answering open questions in hadron structure, e.g., SU(3) flavor symmetry breaking caused by heavier strange quark mass
- Accessing x-dependence of PDFs using Lattice QCD (LQCD):
  - Novel methods: quasi-PDFs, pseudo-PDFs, current-current correlators, etc.
  - From Mellin moments:

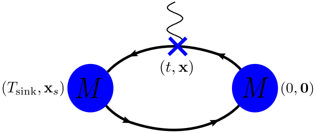
$$\langle x^n \rangle = \int_{-1}^{1} dx\, x^n f(x)$$

- Previously argued that PDF reconstruction is unfeasible using lattice results for the Mellin moments, in particular, the large-x behavior cannot be reliably understood [Detmold *et al.*, arXiv:hep-lat/0108002], [Holt *et al.*, RMP **82**, 2991–3044 (2010)]
- We calculate moments directly from local operators without mixing with lower dimension operators so we attempt a reconstruction with our moment results

Overview

Meson matrix elements

- Moments under study:
  - quark momentum fraction $\langle x \rangle$
  - 2nd Mellin moment $\langle x^2 \rangle$
  - 3rd Mellin moment $\langle x^3 \rangle$



- Matrix elements in the forward limit ($Q^2 = 0$):

$$\langle M(p) | \mathcal{O} | M(p) \rangle$$

- Operators of interest:

$$\mathcal{O}_V^{\{\mu\nu\}} = \overline{q} \gamma^{\{\mu} D^{\nu\}} q$$

$$\mathcal{O}_V^{\{\mu\nu\rho\}} = \overline{q} \gamma^{\{\mu} D^{\nu} D^{\rho\}} q$$

$$\mathcal{O}_V^{\{\mu\nu\rho\tau\}} = \overline{q} \gamma^{\{\mu} D^{\nu} D^{\rho} D^{\tau\}} q$$

Meson decomposition for $\langle x \rangle$, $\langle x^2 \rangle$, and $\langle x^3 \rangle$

- Lattice breaks Euclidean Lorentz group O(4) symmetry to discreet hyber cubic group H(4) $\implies$ mixing among operators
- We only use operators that are free of mixing with lower dimension operators, i.e., all indices are taken different for the 2- and 3-derivative operators
- This leads to decomposition in forward limit for general frame:

$$\Pi^{\{00\}} = \frac{1}{2E}\left(\frac{m^2}{2} - 2E^2\right)\langle x \rangle$$

$$\Pi^{\{0ij\}} = -p_i p_j \langle x^2 \rangle$$

$$\Pi^{\{0ijk\}} = -i p_i p_j p_k \langle x^3 \rangle$$

$$E = \sqrt{m^2 + p^2}$$

$p:$ hadron momentum

- Due to p in kinematic factor, $\langle x^n \rangle$ with $n > 1$ requires boosted frame to calculate $\langle x^n \rangle$
- Since indices i, j, and k are different, we need a boosted frame with at least:
  - $p = (\pm 1, \pm 1, 0)\frac{2\pi}{L}$ for $\langle x^2 \rangle$
  - $p = (\pm 1, \pm 1, \pm 1)\frac{2\pi}{L}$ for $\langle x^3 \rangle$

PDF reconstruction setup

- Standard PDF functional form:

$$q_M^f(x) = Nx^{\alpha}(1-x)^{\beta}(1+\rho\sqrt{x}+\gamma x)$$

- ρ generally assumed to be small, so we neglect $\rho\sqrt{x}$ term
- Normalization factor:

$$\langle 1 \rangle_M = \int_0^1 q_M(x) = 1 \implies N = \frac{1}{B(\alpha+1, \beta+1) + \gamma B(2+\alpha, \beta+1)}$$

- Moment integrals:

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^n (i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^n (i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}$$

Lattice details

- $N_f = 2+1+1$ twisted-clover fermions

Ensemble Parameters					
a [fm]	N_f	m_π [MeV]	m_K [MeV]	volume $L^3 \times T$	L [fm]
0.093	$2+1+1$	260	530	$32^3 \times 64$	3.0

Statistics					
p	p combos.	T_{sink}	confs	src pos.	Total
$(0,0,0)$	1	$12, 14, 16, 18, 20, 24$	122	16	1,920
$(\pm1,\pm1,\pm1)$	8	$12,\ 14, 16, 18$	122	72	70,272

- Boosted frame: $(\pm1,\pm1,\pm1)$ to calculate $\langle x^2 \rangle$ and $\langle x^3 \rangle$

Overview

First three non-trivial moments

- Excited states sizeable (backup slides)
- Find results for 2-state fits including up to $T_{\rm sink} = 2.2$ fm for $\langle x \rangle$ and $T_{\rm sink} = 1.7$ fm for $\langle x^2 \rangle$, $\langle x^3 \rangle$

$$\langle x \rangle_u^{\pi^+} = 0.261(3)(6)$$
$$\langle x \rangle_u^{K^+} = 0.246(2)(2)$$
$$\langle x \rangle_s^{K^+} = 0.317(2)(1)$$

$$\langle x^2 \rangle_u^{\pi^+} = 0.110(7)(12)$$
$$\langle x^2 \rangle_u^{K^+} = 0.096(2)(2)$$
$$\langle x^2 \rangle_s^{K^+} = 0.139(2)(1)$$

$$\langle x^3 \rangle_u^{\pi^+} = 0.024(18)(2)$$
$$\langle x^3 \rangle_u^{K^+} = 0.033(6)(1)$$
$$\langle x^3 \rangle_s^{K^+} = 0.073(5)(2)$$

$$\frac{\langle x^2 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.423(28)(57)$$
$$\frac{\langle x^2 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.391(10)(16)$$
$$\frac{\langle x^2 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.438(8)(11)$$

$$\frac{\langle x^3 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.092(71)(6)$$
$$\frac{\langle x^3 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.135(26)(8)$$
$$\frac{\langle x^3 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.232(16)(1)$$

- $\langle x^2 \rangle / \langle x \rangle \sim 40\%$, $\langle x^3 \rangle / \langle x \rangle \sim 10 - 20\%$
- More details in [Phys. Rev. D **103**, 014508 (2021), arXiv:2010.03495] and [arXiv:2104.02247]

SU(3) flavor symmetry breaking

$$\frac{\langle x\rangle_{\pi}^{u^+}}{\langle x\rangle_{K}^{u^+}} = 1.060(9)(7)$$
$$\frac{\langle x^2\rangle_{\pi}^{u^+}}{\langle x^2\rangle_{K}^{u^+}} = 1.148(57)(106)$$
$$\frac{\langle x^3\rangle_{\pi}^{u^+}}{\langle x^3\rangle_{K}^{u^+}} = 0.717(488)(94)$$

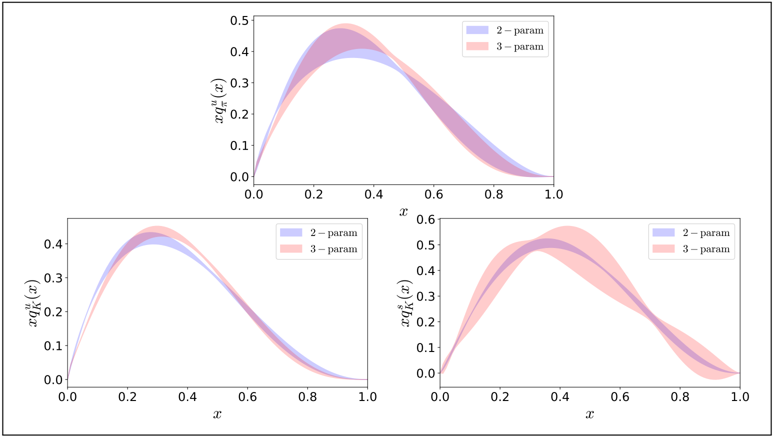
$$\frac{\langle x\rangle_{\pi}^{u^+}}{\langle x\rangle_{K}^{s^+}} = 0.823(8)(10)$$
$$\frac{\langle x^2\rangle_{\pi}^{u^+}}{\langle x^2\rangle_{K}^{s^+}} = 0.795(45)(80)$$
$$\frac{\langle x^3\rangle_{\pi}^{u^+}}{\langle x^3\rangle_{K}^{s^+}} = 0.325(244)(23)$$

- SU(3) symmetry breaking $\sim 5-10\%$ for $\langle x\rangle$
- $\sim 10-20\%$ for $\langle x^2\rangle$
- $\sim 30-50\%$ for $\langle x^3\rangle$
- Symmetry breaking between π and strange part of K is more pronounced in the higher moments

Overview

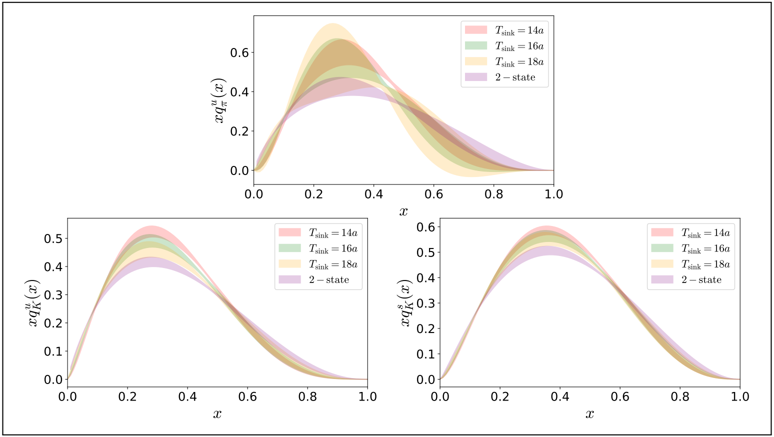
Effect of fit function

- Moments evolved to scale of 5.2 GeV
- 2-parameter fit: α, β
- 3-parameter fit: α, β, γ



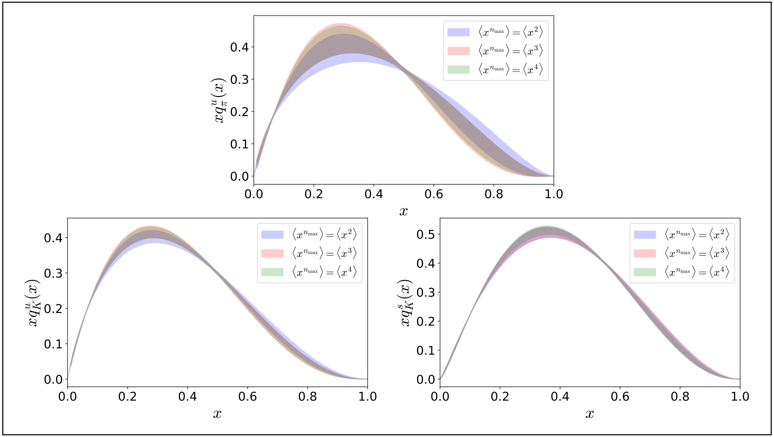
- 3-parameter fit not well behaved
- We find little dependence on the fit function
- We proceed with the 2-parameter fits

Excited-state effects



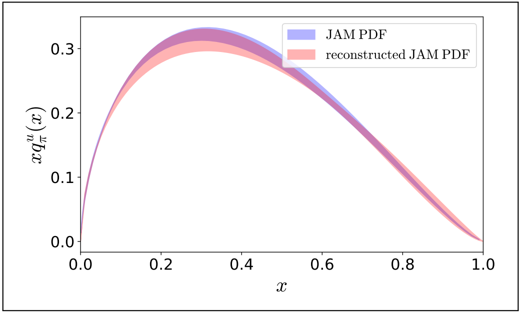
- Excited-state effects appear to raise peak
- We choose the two-state fit as our final estimates

Effects of number of moments in fit



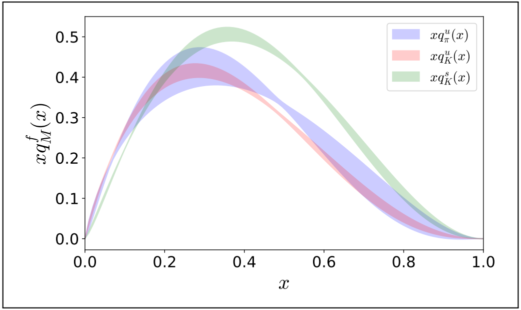
- $\langle x^{n_{\max}}\rangle = \langle x^4\rangle$: add constraint from
  - phenomenological result $\langle x^4\rangle_\pi^u = 0.027(2)$
  - model calculations $\langle x^4\rangle_K^s = 0.029^{+0.005}_{-0.004}$, $\langle x^4\rangle_K^u = 0.021^{+0.003}_{-0.003}$
- We choose $n_{\max} = 3$ as our final estimates

Can PDF be accurately reconstructed from 3 moments?



- Calculate moments from JAM global fit [P. C. Barry et. al. (JAM collaboration), arXiv:1804.01965]
- Reconstruct PDF from 1st 3 JAM moments
- Reconstructed PDF has larger errors, agrees well with actual JAM PDF
- Reconstructed $n = 4$ moment: $\langle x^4 \rangle_\pi^u = 0.026(2)$
- Actual JAM $n = 4$ moment: $\langle x^4 \rangle_\pi^u = 0.027(2)$

SU(3) flavor symmetry breaking



- Up quark equally prevalent in pion as in kaon for most regions of x
- Small difference between $xq^u_\pi(x)$ and $xq^u_K(x)$ around $x \approx 0.5$
- Distribution of strange quark in kaon is greater than up quark in pion for $x \approx 0.3$–0.8
- Peaks at

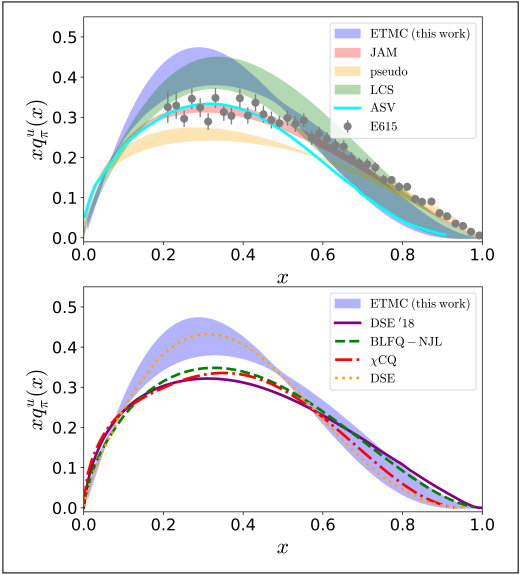
$$xq^u_\pi(x = 0.30) = 0.43(5)$$
$$xq^u_\pi(x = 0.28) = 0.42(2)$$
$$xq^u_\pi(x = 0.36) = 0.51(2)$$

Mellin moments from reconstructed PDFs

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
q_π^u	0.230(3)(7)	0.087(5)(8)	0.041(5)(9)
q_K^u	0.217(2)(5)	0.079(2)(1)	0.036(2)(2)
q_K^s	0.279(1)(5)	0.115(2)(6)	0.058(2)(2)
q_M^f	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.023(5)(6)	0.014(4)(5)	0.009(3)(3)
q_K^u	0.019(1)(2)	0.011(1)(2)	0.007(1)(1)
q_K^s	0.033(2)(2)	0.021(1)(2)	0.014(1)(2)

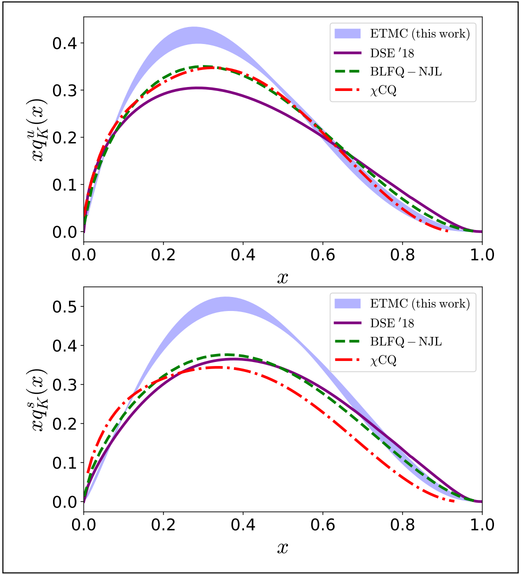
- Calculate by integrating over reconstructed PDFs
- Uncertainties under control even for higher moments
- Our $\langle x^4 \rangle_\pi^u$ in agreement with moment from JAM PDF $\langle x^4 \rangle_\pi^u = 0.027(2)$ [P. C. Barry et. al. (JAM collaboration), arXiv:1804.01965]

Comparison to other studies, pion



- pseudo, LCS: lattice results using non-local operators
- Qualitative comparison, studies have different systematic uncertainties which are not all quantified

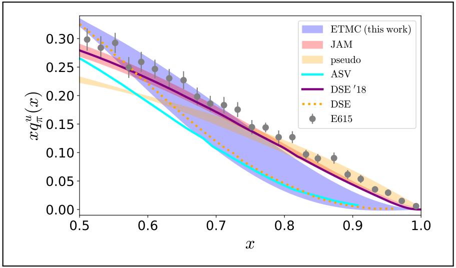
Comparison to other studies, kaon



- Good agreement at high- and low-x, most tension in intermediate-x region
- Qualitative comparison

Large-x behavior for pion

- There is some tension between studies of the high-x behavior of the pion PDF
- Un-quantified systematics
- Original analysis of Fermilab E615 (gray circles) experiment finds $\sim (1-x)^1$ $(\beta = 1)$



- More recent analysis of the same data (solid cyan line) finds $\sim (1-x)^2$ $(\beta = 2)$
- This study: $\beta = 2.23(65)$
- Our results favor $(1-x)^2$ large-x behavior, in agreement with ASV and DSE
- Our kaon results also favor $(1-x)^2$ large-x behavior

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Summary

- Calculated first three non-trivial Mellin moments of PDFs
- Pioneering study has shown for the first time that PDFs can be reconstructed using the first three moments
- Higher order Mellin moments not included in fit can be calculated from reconstructed PDFs with well-controlled uncertainties

Summary

- Calculated first three non-trivial Mellin moments of PDFs
- Pioneering study has shown for the first time that PDFs can be reconstructed using the first three moments
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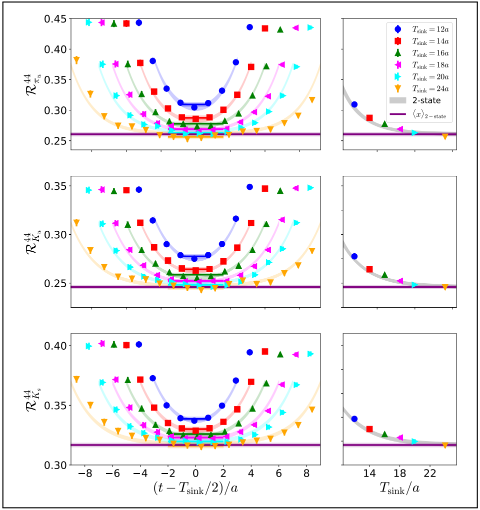
Thank you

Backup Slides



- Plateau: $\frac{C^{3\rm pt}_{\mathcal{O}}(t, T_{\rm sink})}{c_0 e^{-E_0 T_{\rm sink}}}$

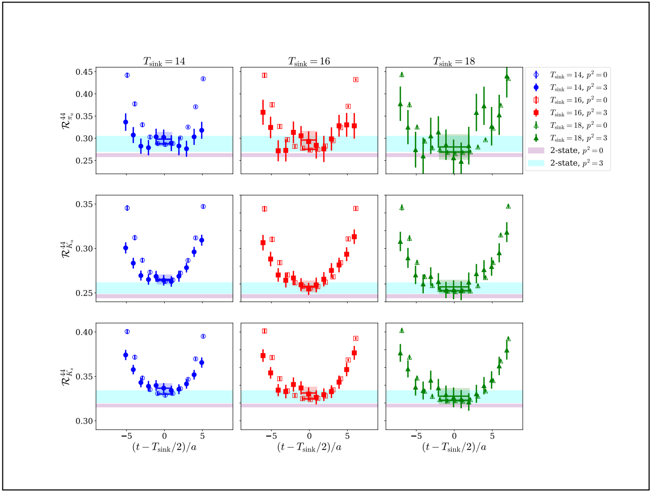
  from two-state fit
- Two-state fit consistent with plateau for $T_{sink} \geq 18a$ (1.6 fm)
- $\overline{MS}(2GeV)$

$$\langle x \rangle_u^{\pi^+} = 0.261(3)_{\rm stat}(6)_{\rm syst}$$
$$\langle x \rangle_u^{K^+} = 0.246(2)_{\rm stat}(2)_{\rm syst}$$
$$\langle x \rangle_s^{K^+} = 0.317(2)_{\rm stat}(1)_{\rm syst}$$

- Phenomonological results: $2\langle x \rangle_u^{\pi^+} = 0.48(1)$

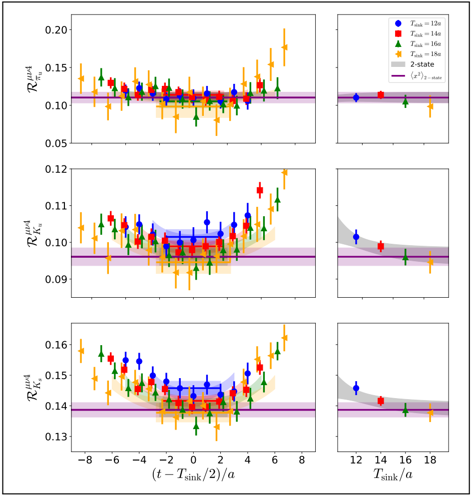
  [Barry et. al., arXiv:1804:01965]
- Compatible with other lattice calculations at similar m_π. Comparisons in [Alexandrou et. al., arXiv:2010.03495]
- $\langle x \rangle_u^{K^+} < \langle x \rangle_u^{\pi^+} < \langle x \rangle_s^{K^+}$

First Mellin moment $\langle x \rangle$, momentum frame comparison



- Serves to test signal of $\langle x \rangle$ in boosted frame
- Useful for selecting $T_{\rm sink}$ to optimize computer resources
- Agreement between two frames in plateau and two-state fit
- More details in [Alexandrou et. al., arXiv:2010.03495]

Second Mellin moment $\langle x^2 \rangle$



$$\langle x^2 \rangle_u^{\pi^+} = 0.110(7)(12)$$
$$\langle x^2 \rangle_u^{K^+} = 0.096(2)(2)$$
$$\langle x^2 \rangle_s^{K^+} = 0.139(2)(1)$$

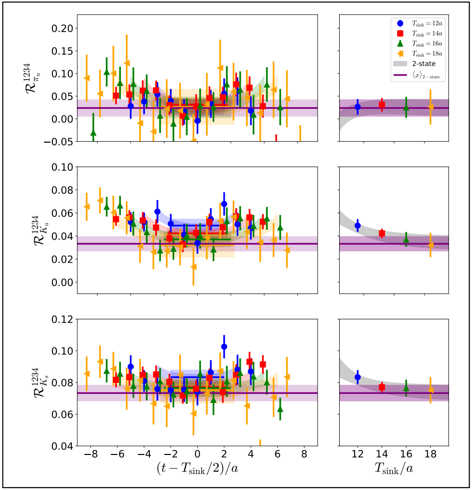
- Phenomonological results: $2\langle x^2 \rangle_u^{\pi^+} = 0.210(5)$

  [Barry et. al., arXiv:1804:01965]

- Ratio $\langle x^2 \rangle / \langle x \rangle$ is an indication of how quickly the PDFs lose support at large x

$$\frac{\langle x^2 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.423(28)(57)$$
$$\frac{\langle x^2 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.391(10)(16)$$
$$\frac{\langle x^2 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.438(8)(11)$$

Third Mellin moment $\langle x^3 \rangle$



$$\langle x^3 \rangle_u^{\pi^+} = 0.024(18)(2)$$
$$\langle x^3 \rangle_u^{K^+} = 0.033(6)(1)$$
$$\langle x^3 \rangle_s^{K^+} = 0.073(5)(2)$$

- π near zero due to high uncertainties
- Clear signal for both flavors of K
- $\langle x^3 \rangle < \langle x^2 \rangle < \langle x \rangle$

$$\frac{\langle x^3 \rangle_u^{\pi^+}}{\langle x \rangle_u^{\pi^+}} = 0.092(71)(6)$$
$$\frac{\langle x^3 \rangle_u^{K^+}}{\langle x \rangle_u^{K^+}} = 0.135(26)(8)$$
$$\frac{\langle x^3 \rangle_s^{K^+}}{\langle x \rangle_s^{K^+}} = 0.232(16)(1)$$

Effect of fit function

fit type	α_π^u	β_π^u	γ_π^u
2-parameter	-0.04(20)	2.23(65)	0
3-parameter	-0.54(22)	2.76(64)	22.17(17.87)
fit type	α_K^u	β_K^u	γ_K^u
2-parameter	-0.05(7)	2.42(24)	0
3-parameter	-0.56(72)	3.01(23)	25.11(5.23)
fit type	α_K^s	β_K^s	γ_K^s
2-parameter	0.21(8)	2.13(20)	0
3-parameter	0.18(95)	2.051(3.46)	0.347(16.10)

Vector form factors

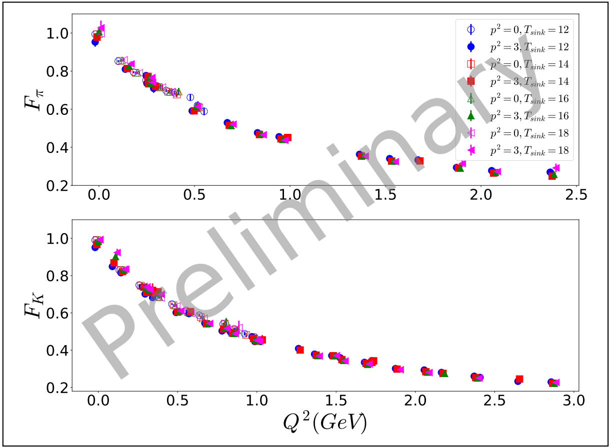
- GPD Mellin moments
- Functions of momentum transfer squared Q^2
- Operators of interest:

$$\mathcal{O}_V^{\mu} = \overline{q}\gamma^{\mu} q$$

- $F_K(Q^2) = q_u F_K^u(Q^2) + q_s F_K^s(Q^2),\ q_u = 2/3\epsilon,\ q_s = -1/3\epsilon$

$$R(\vec{q} = \vec{p}\,' - \vec{p}; t, T_{\rm sink}) = \frac{C^{3pt}(\vec{p}\,', \vec{p}; T_{\rm sink}, t)}{C^{2pt}(\vec{p}\,'; t)} \sqrt{\frac{C^{2pt}(\vec{p}; T_{\rm sink} - t) C^{2pt}(\vec{p}\,'; t) C^{2pt}(\vec{p}\,'; T_{\rm sink})}{C^{2pt}(\vec{p}\,'; T_{\rm sink} - t) C^{2pt}(\vec{p}; t) C^{2pt}(\vec{p}; T_{\rm sink})}}$$

Vector form factors, momentum frame comparison



- Boosted frame gives us access to denser range of $Q^2 = \vec{q}^2 - (E_f - E_i)^2$
- Access to higher Q^2 in the boosted frame because some require two-point functions at low momentum in the ratio
- Good agreement between frames
- Intend to look at SU(3) flavor symmetry breaking