

# Residual Mean Field Model of Valence Quarks in the Nucleon

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# Introduction:

with Chris Leon and Joseph Maerovitz

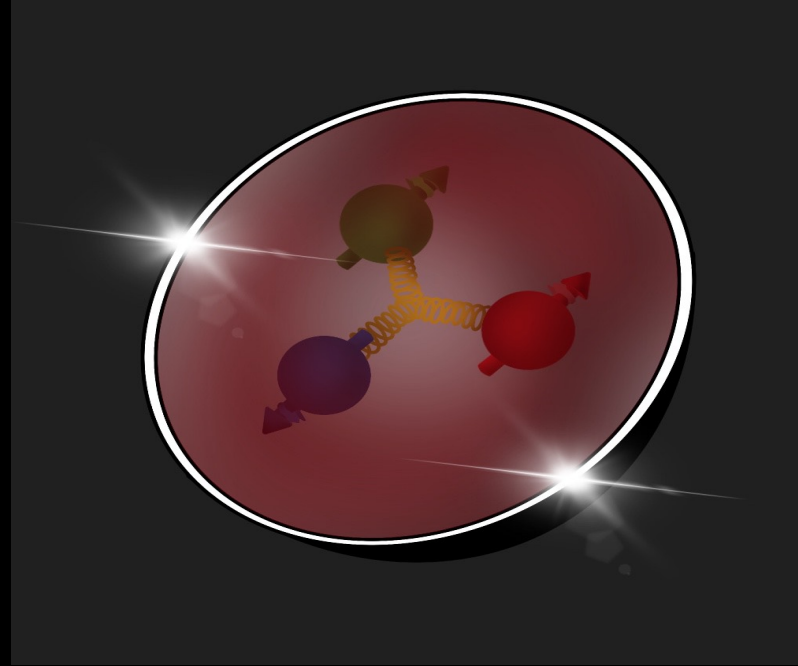
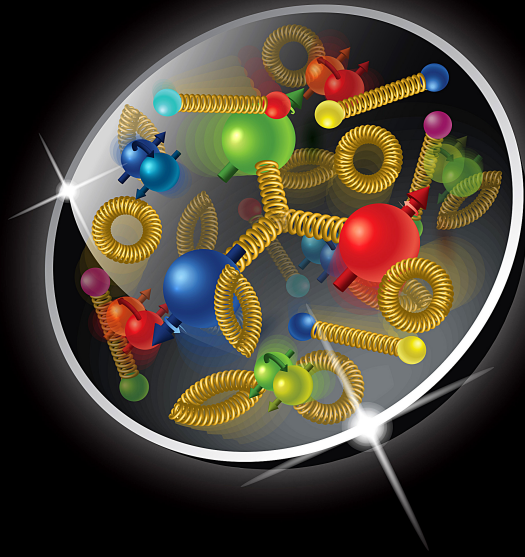
**Valence quarks play a unique role in QCD dynamics of the nucleon**

- Because they define baryonic number of the nucleon:

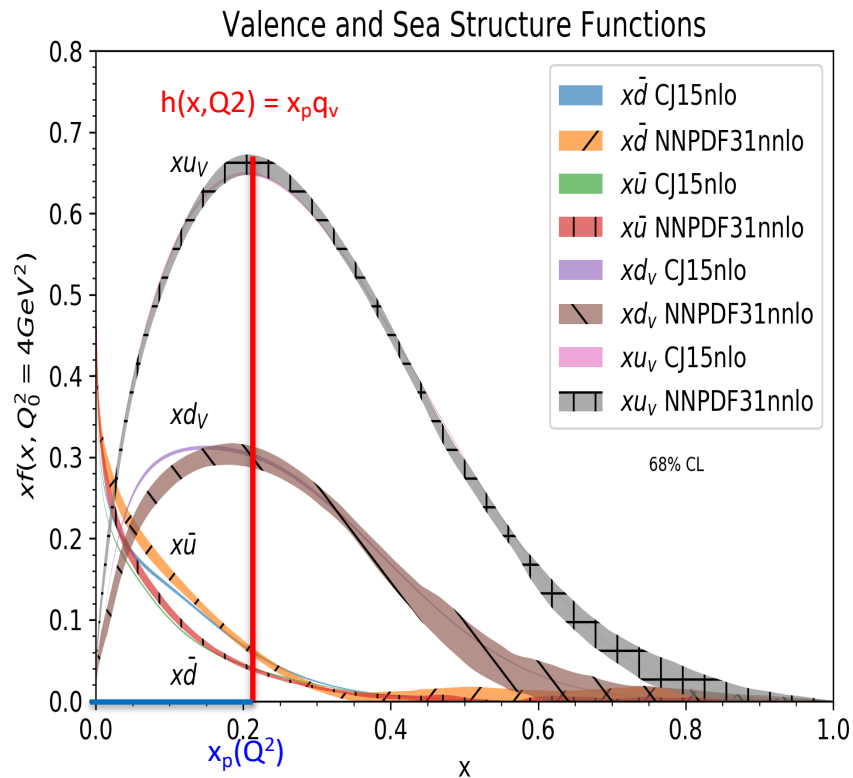
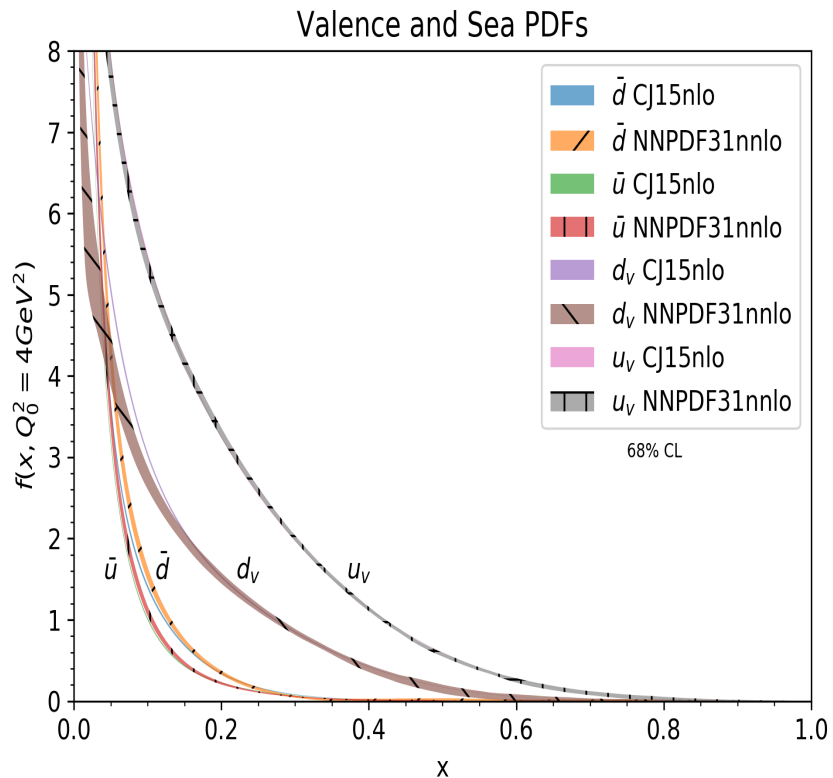
They represent effective “three fermion” system with conserved number

- Because of their conserved number the concept of mean-field interaction can be introduced to discuss their interaction with the nucleon environment
- Quantum mechanically, this becomes a problem of fermions in the strong external field – (see e.g. Migdal Fermions and Bosons in the Strong Field)
- Short range interaction among three valence quarks responsible to the generation of high  $x$  distribution of PDFs

# Valence quarks in the nucleon at medium to high $x$ : $0.1 < x < 1$



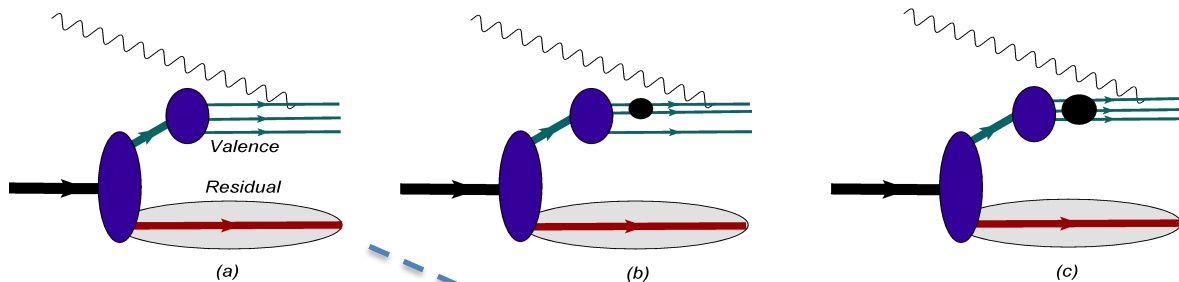
- Treating the **height of the peak  $h(x_p, Q^2)$**  and **position of the peak  $x_p$**  as **physical observables**:



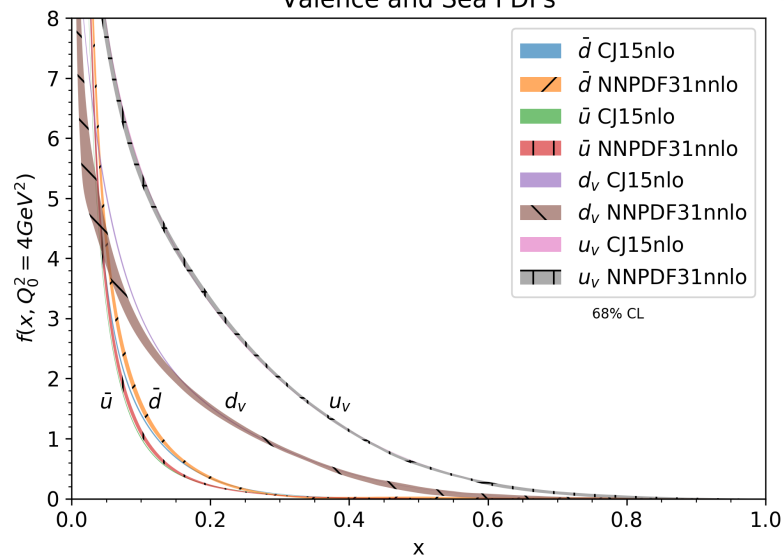


## **"New Approaches"** in modeling valence quark dynamics

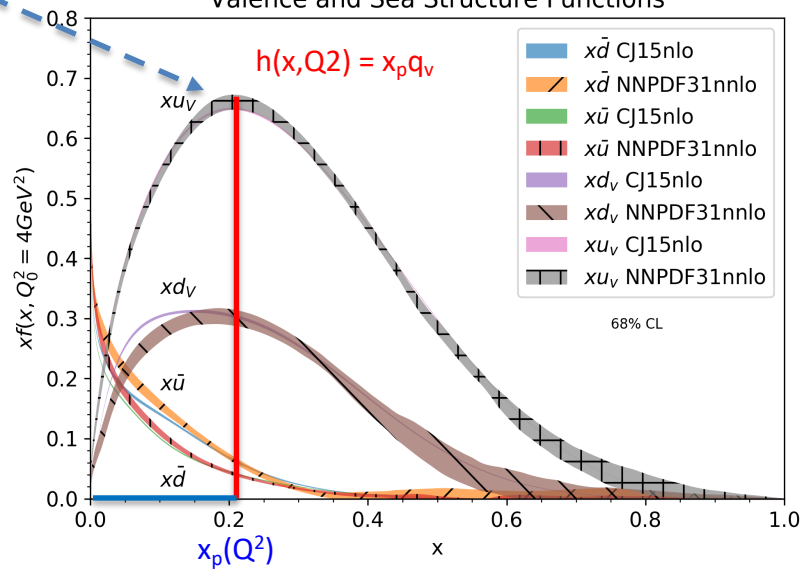
- In general the peaking property of bound Fermi-system is a hallmark for mean-field dynamics
- Our assumption is that the peaking feature of valence quark distributions is due to interaction of valence quarks in the strong mean field generated by "residual nucleon system"
- We introduce concept of "residual nucleon system" as a composite part of the light-front wave function of valence quarks

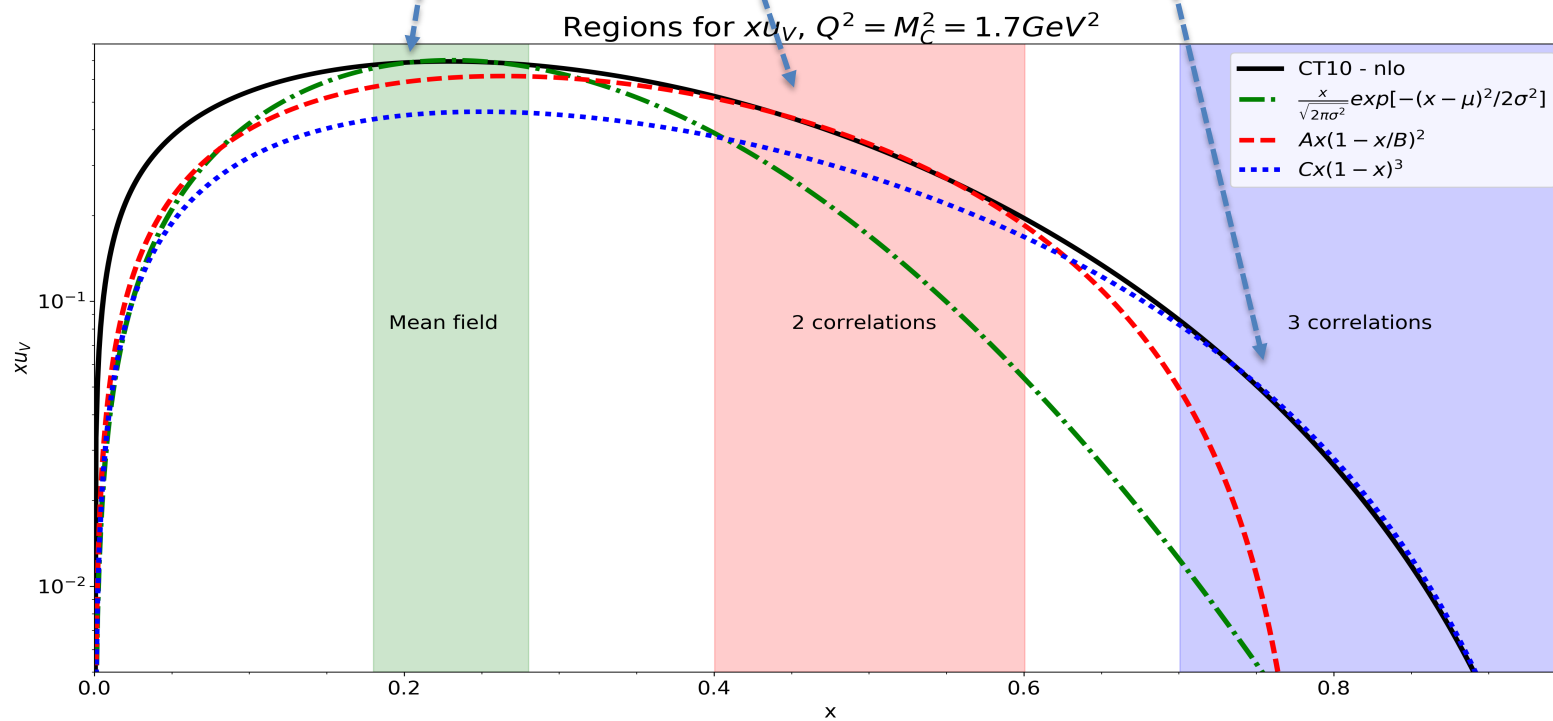
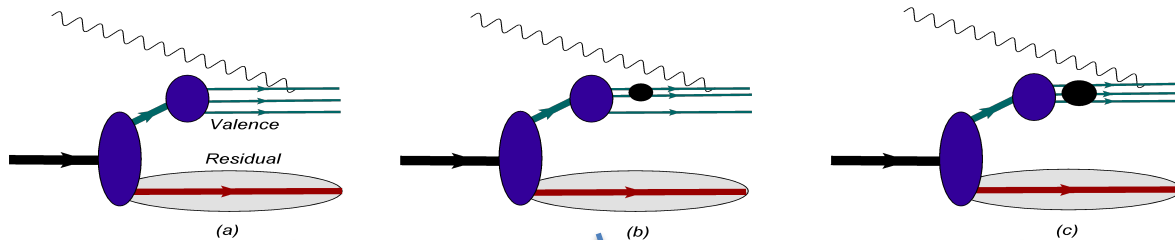


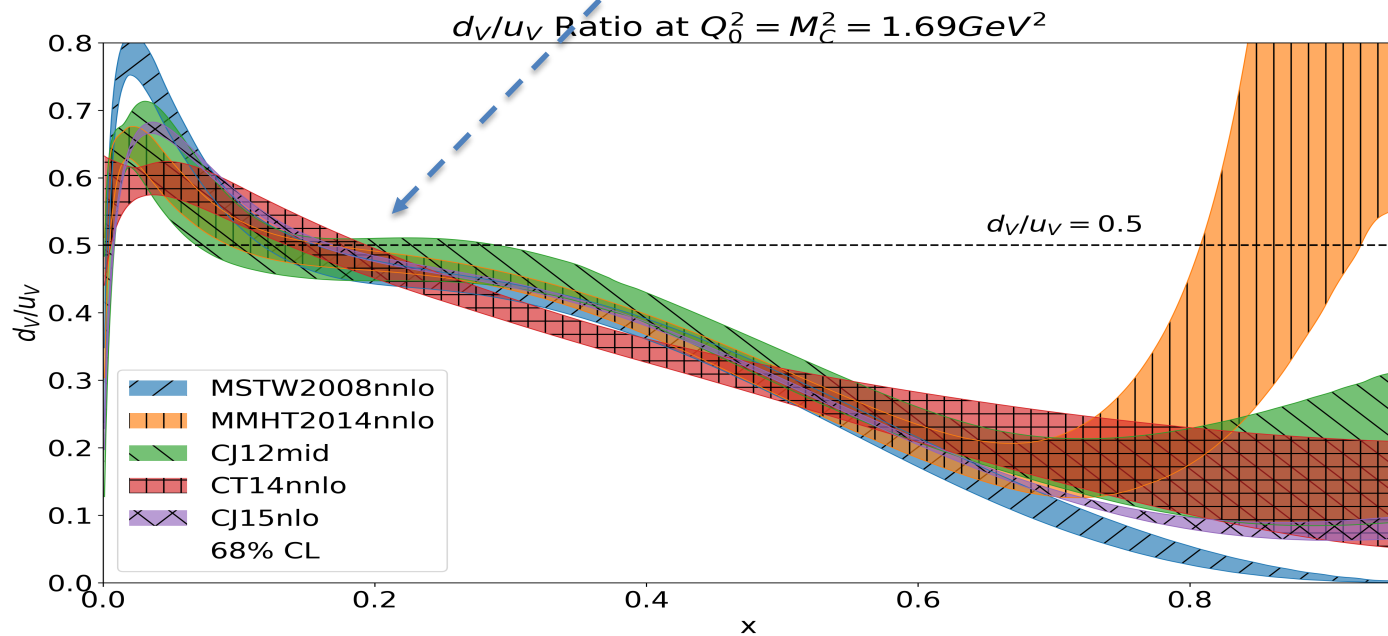
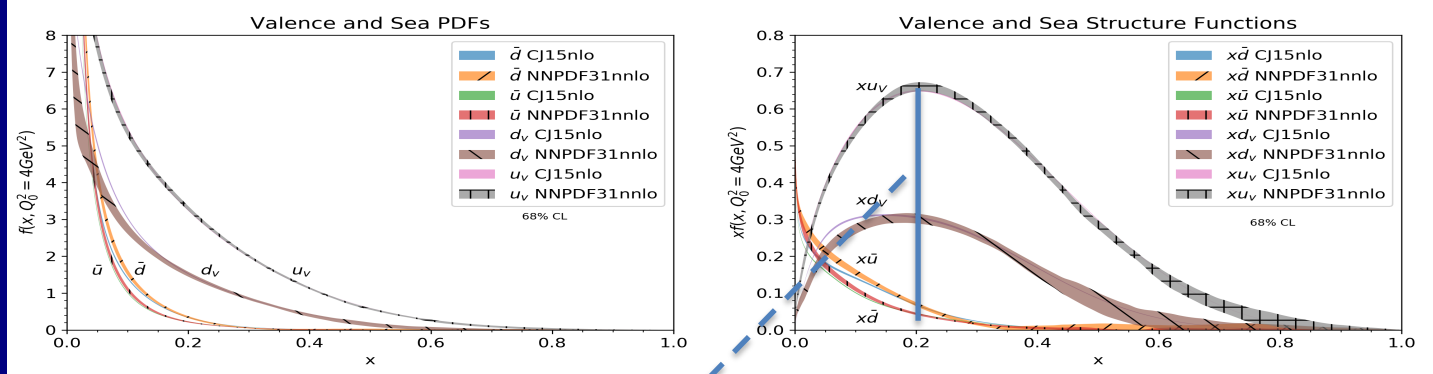
Valence and Sea PDFs

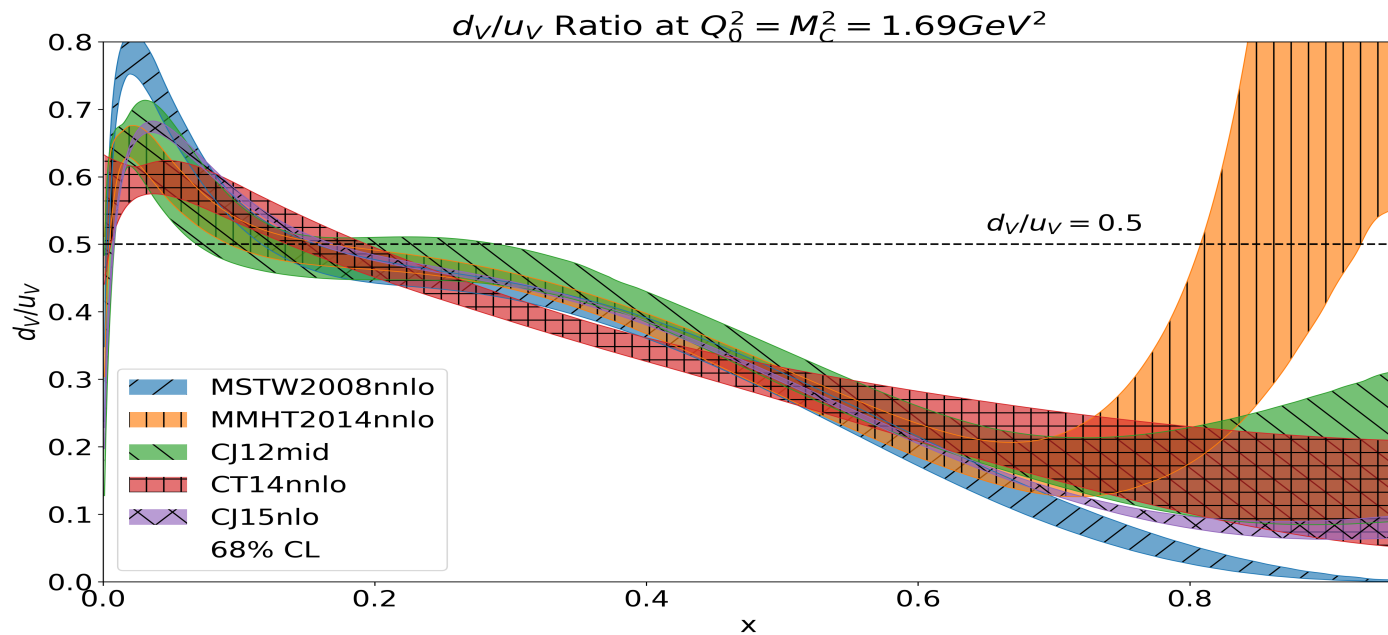
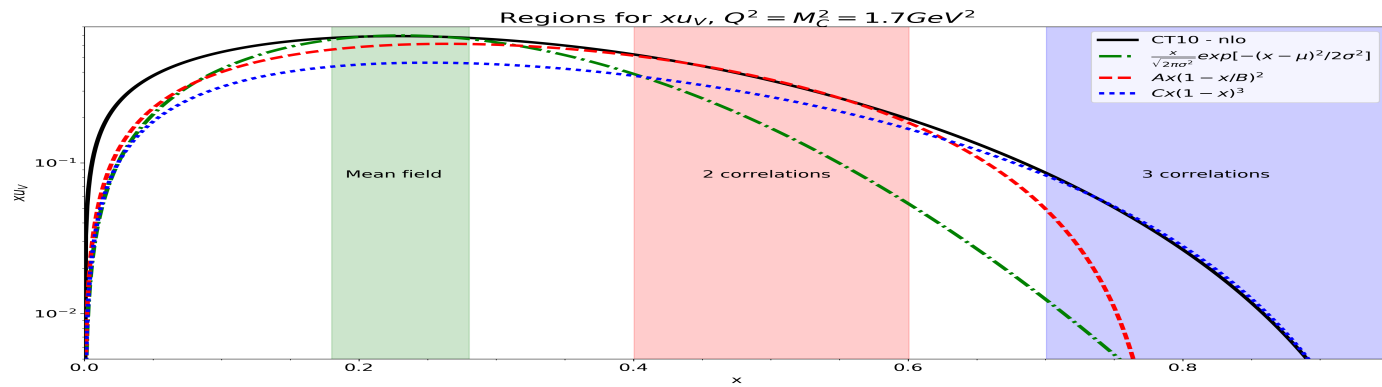


Valence and Sea Structure Functions



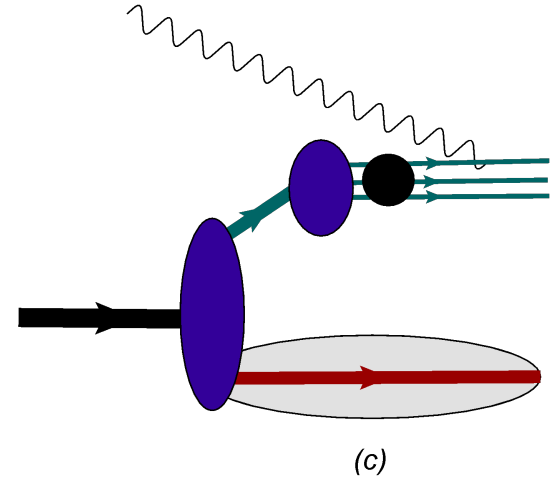
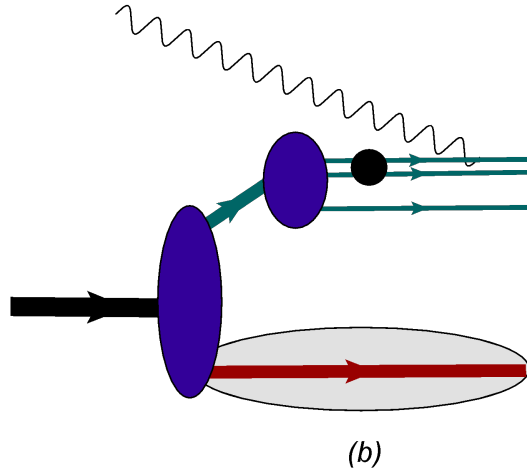
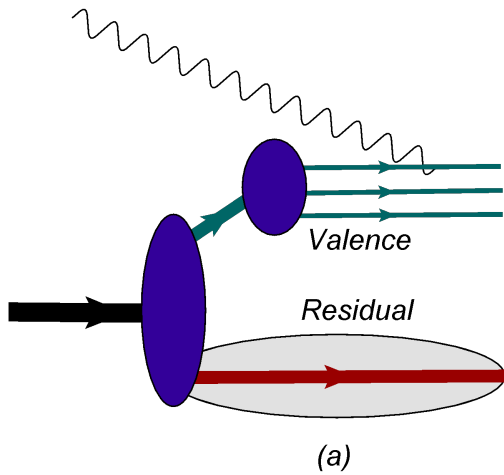






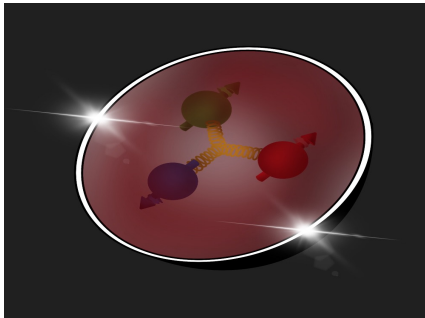
## Main assumptions of the model

**Dynamics:** The main assumption is that the mean field, two- and three- quark short-range correlations define the dynamics of the valence quarks in the range of  $0.1 \leq x \leq 1$ .

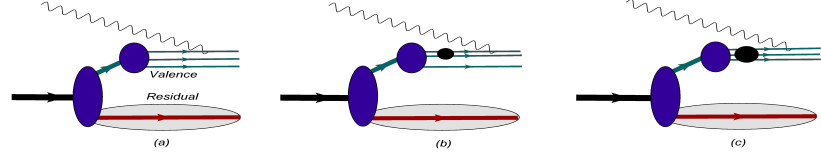


## Valence Quarks in the Nucleon:

- The model assumes an existence of almost massless valence three-quark cluster  $V$  in the nucleon.
- The cluster is compact with the transverse separation between any  $qq$ ,  $b_{qq} \lesssim 0.3$  Fm.
- Valence quark system defines the baryonic number but not necessarily the total isospin of the nucleon. It can have total isospin,  $I_V = 1/2$  or  $3/2$  each of them corresponding to the different excitations or masses of the residual nucleon system.  
*(For the lowest mass of the recoil system one expects the  $3q$  system to have the same isospin and its projection that the considered nucleon has.)*



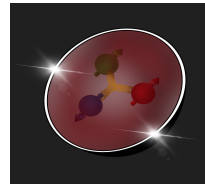
## Residual Structure:



- Introducing residual structure of the nucleon with the spectrum of mass,  $m_R$  - (*spectral function formalism in the description of the nucleon structure*)
- The model assumes a certain universality of the residual structure,  $R$ , entering in all three mechanisms of generation of valence quark distribution.
- This universality is reflected in the fact that one can fix its main properties within mean field and apply it in the calculation of 2q- and 3q- correlation contributions.
- The mass spectrum of the residual system is continuous and effectively depends on whether **u**- or **d**- valence quarks are probed.

$$m_R(u/d) = \alpha_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = \frac{1}{2}) + \beta_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = -\frac{1}{2}) + \gamma(u/d) \sum_{I_V^3 = -\frac{3}{2}}^{\frac{3}{2}} m_R(I_V = \frac{3}{2}, I_V^3) + \dots$$

For proton:  $m_R(u) < m_R(d)$

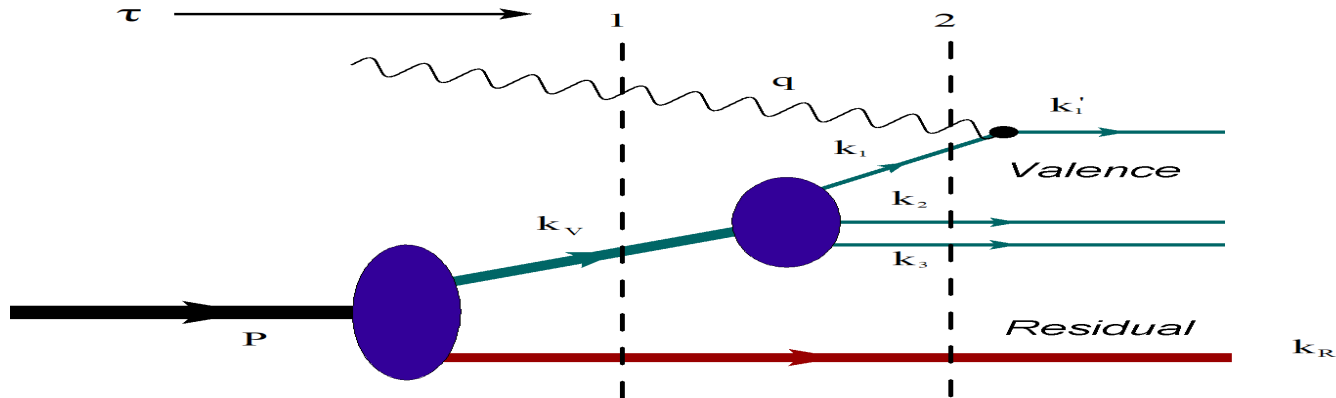


- QCD evolution will increase:  $m_R(Q^2)$



## Mean-Field Model of Valence Quark Distributions

- The valence 3q system occupies a region of 0.6Fm and is described by mutually coupled three-dimensional harmonic oscillators, thus satisfying confinement condition.
- Valence quarks are almost massless with the invariant energy of 3q system contributing to the nucleon mass.
- The residual system generates the mean field and occupies a volume less or equal to the nucleon volume.



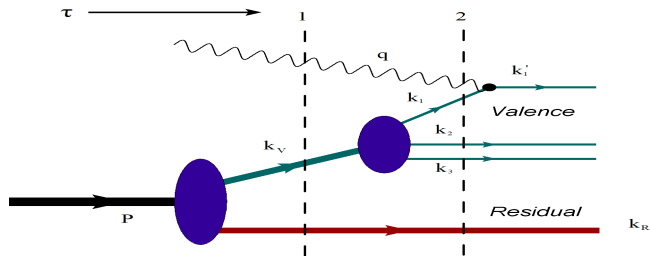
## Reference Frame, Kinematics and Structure Function

$$p_N^\mu = (p_N^+, \frac{m_N^2}{p_N^+}, \mathbf{0}_\perp), \quad q^\mu = (0, \frac{2p \cdot q}{p_N^+}, \mathbf{q}_\perp), \quad Q^2 = -q^2 = |\mathbf{q}_\perp|^2,$$

$$p_N^+ \gg m_N, k_i^-, k_{i,\perp},$$

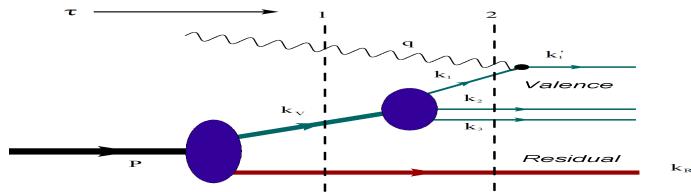
$$F_2(x, Q^2) \equiv \sum_i e_i^2 x f_i(x, Q^2), = \frac{MQ^2}{2x(p_N^+)^2} W_N^{++}$$

$$W_N^{\mu\nu} = \frac{1}{4\pi M} \int \sum_X \sum_{s_X} J^{\mu,\dagger}(p_X, s_X, p_N, s_N) J^\nu(p_X, s_X, p_N, s_N) (2\pi)^4 \delta^4(q + p_N - p_X) \delta(p_X^2 - M_X^2) \frac{d^4 p_X}{(2\pi)^3} \frac{1}{2(2\pi)}$$



$$A^\mu = \sum_{h_V, h_1} \frac{1}{k_V^+} \frac{1}{k_1^+} \frac{\bar{u}(k'_1, h'_1) (ie_1 \gamma^\mu) u(k_1, h_1)}{\mathcal{D}_1} \frac{\prod_{i=1}^3 \bar{u}(k_i, h_i) \Gamma^{V \rightarrow 3q} \chi_V \bar{\chi}_V \bar{\chi}_R \Gamma^{B \rightarrow V R} u(p_N, h_N)}{\mathcal{D}_2}$$

# Calculation of the scattering amplitude



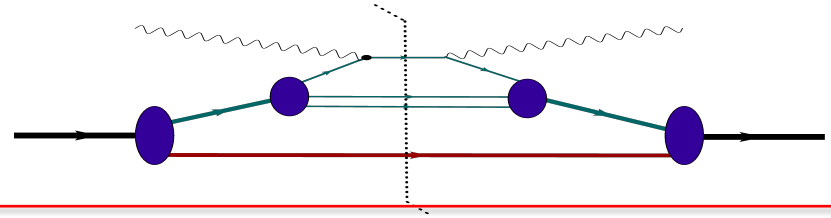
$$A^\mu = \sum_{h_1, h_V} \frac{1}{x_V} \frac{1}{\beta_1} \frac{\bar{u}(k'_1, h'_1)(ie_1\gamma^\mu)u(k_1, h_1)}{m_V^2 - \sum_{i=1}^3 \frac{k_{i,\perp}^2 + m_i^2}{\beta_i}} \frac{\prod_{i=1}^3 \bar{u}(k_i, h_i)\Gamma^{V \rightarrow 3q}\chi_V\bar{\chi}_V\bar{\chi}_R\Gamma^{B \rightarrow VR}u(p_N, h_N)}{M^2 - \frac{k_{V,\perp}^2 + m_V^2}{x_V} - \frac{k_{R,\perp}^2 + m_R^2}{x_R}}$$

$$\begin{aligned} \psi_{VR}(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp}) &= \frac{\chi_V\bar{\chi}_R\Gamma^{B \rightarrow VR}u(p_N, h_N)}{m_N^2 - \frac{k_{V,\perp}^2 + m_V^2}{x_V} - \frac{k_{R,\perp}^2 + m_R^2}{x_R}} \\ \psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3) &= \frac{\prod_{i=1}^3 \bar{u}(k_i, h_i)\Gamma^{V \rightarrow 3q}\chi_V}{m_V^2 - \sum_{i=1}^3 \frac{k_{i,\perp}^2 + m_i^2}{\beta_i}} \end{aligned}$$

where  $\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3$  denotes the LC momenta and helicities of the three valence quarks in the wave function.

$$A^\mu = \sum_{h_1, h_V} \bar{u}(k_1, h_1)(ie_1\gamma^\mu)u(k_1, h_1) \frac{\psi_{VR}(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})}{x_V} \frac{\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3)}{\beta_1}$$

# Calculation of Structure Function



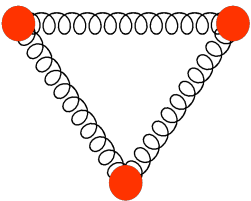
$$W_N^{\mu\nu}(x, Q^2) = \frac{1}{4\pi M_N} \sum_{\{h_i, \tau_i\}} \int \delta(1 - \sum_{i=1}^3 x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^3 \frac{dx_i}{x_i} 16\pi^3 \delta^{(2)}(\sum_{i=1}^3 \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2 \mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^3 \frac{d^2 \mathbf{k}_{i,\perp}}{16\pi^3} A^{\mu\dagger} A^\nu$$

$$F_2(x, Q^2) \equiv \sum_i e_i^2 x \underline{f_i(x, Q^2)}, = \frac{MQ^2}{2x(p_N^+)^2} W_N^{++}$$

$$\underline{f_q(x_B)} = \sum_{h_i} \int \delta(1 - \sum_{i=1}^3 x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^3 \frac{dx_i}{x_i} [d^2 \mathbf{k}_\perp] 16\pi^3 \delta^{(2)}(\sum_{i=1}^3 \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2 \mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^3 \frac{d^2 \mathbf{k}_{i,\perp}}{16\pi^3} \\ \times \delta(x_1 - x_B) |\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3)|^2 |\psi_V(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})|^2.$$

# Modeling Wave Functions

## Wave function of 3q valence system: Relativistic coupled Harmonic Oscillator



$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp \left[ -\frac{B_V}{2} (k_{12,cm}^2 + k_{23,cm}^2 + k_{31,cm}^2) \right] \sqrt{x_2 x_3}, \tag{1}$$

where  $A_V$  and  $B_V$  are parameters and  $x_i, k_{i,\perp}$ , ( $i \neq j = 1, 2, 3$ ) are LC momentum fractions and transverse momenta of each valence quark in the reference considered frame. The  $k_{ij,cm}^2$ s, ( $i \neq j = 1, 2, 3$ ) in the exponent of the wave function represent relative three momenta in the CM system of  $i, j$  pairs defined as follows:

$$k_{ij,cm}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{4s_{ij}}, \tag{2}$$

where the invariant energy of the  $i, j$  pair is:

$$s_{ij} = (k_i + k_j)^+ (k_i + k_j)^- - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2 = (x_i + x_j) \left( \frac{k_{i,\perp}^2 + m_i^2}{x_i} + \frac{k_{j,\perp}^2 + m_j^2}{x_j} \right) - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2. \tag{3}$$

$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp \left[ -\frac{B_V}{8} \left( \sum_{i=1}^3 x_V \frac{\tilde{k}_{i,\perp}^2 + m^2}{x_i} - 9m^2 \right) \right]$$

$$\tilde{k}_{i,\perp} = k_{i,\perp} - \frac{x_i}{x_V} \mathbf{k}_{V,\perp}, \quad (i = 1, 2, 3)$$

$$\sum_{i=1}^3 \tilde{k}_{i,\perp} = 0.$$

## Modeling Wave Functions

Wave function of V-R system: Model in a Gaussian form

$$\psi_R(x_R, \mathbf{p}_{R,\perp}) = \sqrt{16\pi^3 m_N} A_R e^{-B_R p_R^2} \sqrt{x_R}$$

$x_R$  – light-cone momentum fraction of the recoil system

$p_R$  – relative momentum between CMs of V and R system

- Considering a non-relativistic approximation for recoil system  $p_R \ll m_R$

$$p_{R,z} \approx (x_R m_N - m_R).$$

$$f_q(x_B) = \sum_{h_i} \int \delta(1 - \sum_{i=1}^3 x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^3 \frac{dx_i}{x_i} [d^2 \mathbf{k}_\perp] 16\pi^3 \delta^{(2)}(\sum_{i=1}^3 \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2 \mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^3 \frac{d^2 \mathbf{k}_{i,\perp}}{16\pi^3} \\ \times \delta(x_1 - x_B) |\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3)|^2 |\psi_V(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})|^2.$$

$$f_q(x_B, Q^2) = \mathcal{N} \int_0^{1-x_B} dx_2 \int_0^{1-x_B-x_2} dx_3 \exp \left[ -\frac{B_V x_V}{4} \sum_{i=1}^3 \frac{m_i^2}{x_i} - B_R M_N^2 (x_V - (1 - \frac{M_R}{M_N}))^2 \right] \\ \times \frac{x_2 x_3}{x_V^3} \left( 1 - e^{-a_{cm} Q_{cm}^{max2}} \right) \left( 1 - e^{-a_{rel} Q_{rel}^{max2}} \right) \left( 1 - e^{-B_R Q^2} \right)$$

where  $a_{cm} = \frac{B_V x_V}{4} \frac{x_V}{x_3(x_1+x_2)}$  and  $a_{rel} = \frac{B_V x_V}{4} \frac{x_1+x_2}{x_1 x_2}$ .

considering large  $Q^2$  and  $m_q \rightarrow 0$  limit

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$$f_q(x_B, Q^2) = \frac{\mathcal{N}}{6} \int_{x_B}^1 dx_V \exp \left[ -B_R m_N^2 \left( x_V - \left( 1 - \frac{m_R}{m_N} \right) \right)^2 \right] \frac{(x_V - x_B)^3}{x_V^3}$$

$$\text{where } \mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2} e^{\frac{9}{4} B_V m_q^2}$$

## Qualitative Features of the Model

- Evaluating the integral at the maximum of exponent:  $f_q(x_B, Q^2) \sim \left( 1 - x_B - \frac{m_R}{m_N} \right)^3$

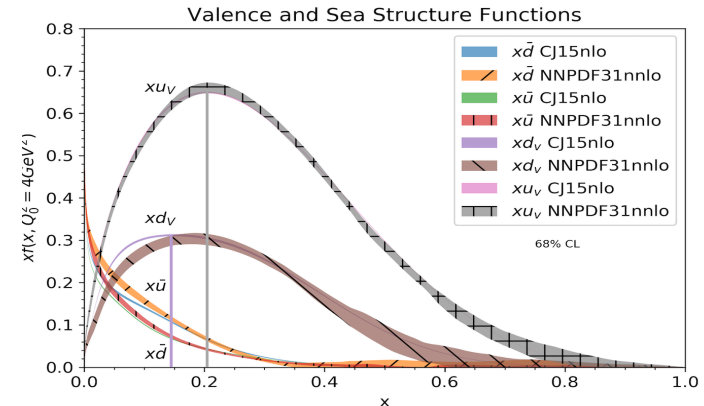
In this case  $h(x_B, t) = x_B f_q(x_B, Q^2) \sim x_B \left( 1 - x_B - \frac{m_R}{m_N} \right)^3$

which peaks at  $x_p \approx \frac{1}{4} \left( 1 - \frac{m_R}{m_N} \right)$ .

At moderate  $Q^2$  ( $M_c^2$ ) characteristic  $x_p \sim 0.2$  resulting in  $m_R \sim m_\pi$ .

- In the model:  $m_R(u) < m_R(d)$ :

Explains  $x_p^d < x_p^u$





$$f_q(x_B, Q^2) = \frac{\mathcal{N}}{6} \int_{x_B}^1 dx_V \exp \left[ -B_R m_N^2 \left( x_V - \left( 1 - \frac{m_R}{m_N} \right) \right)^2 \right] \frac{(x_V - x_B)^3}{x_V^3}$$

$$\text{where } \mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2} e^{\frac{9}{4} B_V m_q^2}$$

## Qualitative Features of the Model

- One can also evaluate the analytic behavior of  $f_q(x_B, Q^2)$  at  $x_B \rightarrow 1$ :

For this we substitute  $x_B = 1 - \epsilon$  and in the  $\epsilon \rightarrow 0$  limit evaluate the integral which results in

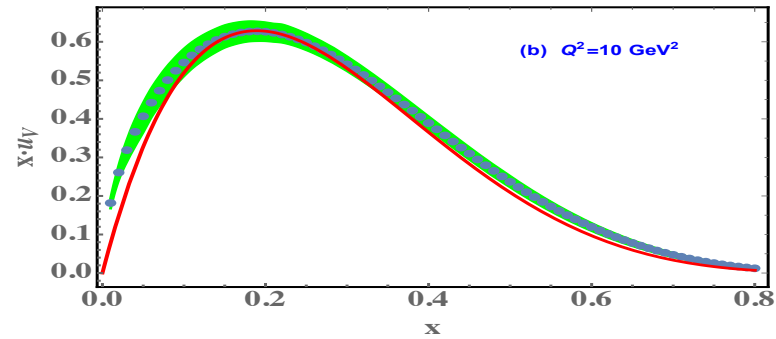
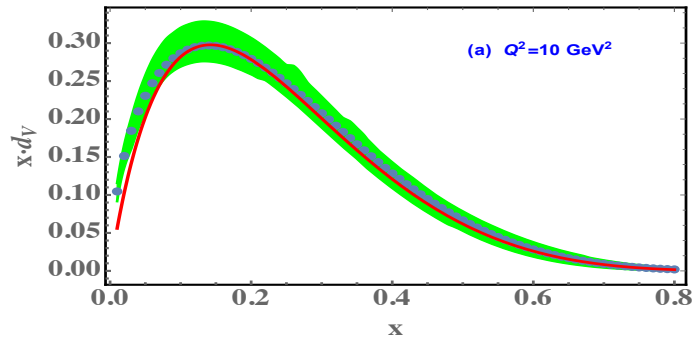
$$f_q(x_B, Q^2) |_{x_B \rightarrow 1} = \frac{\mathcal{N}}{24} e^{-B_R m_R^2} \cdot (1 - x_B)^4.$$

this should be compared with  $\sim (1 - x)^3$  behavior following from pQCD

**Numerical Estimates:** choosing the parameters of the model

the model has five parameters  $A_V$ ,  $B_V$ ,  $A_R$ ,  $B_R$  and  $m_R$ .

- For valence quarks we assume that characteristic separations in the 3q system in the impact parameter space is  $\langle b_{i,j}^2 \rangle \sim (0.3\text{Fm})^2$ . This allows us to evaluate  $B_V = 4\langle b_{i,j}^2 \rangle \frac{x_i}{x_V} \approx \frac{4}{3}\langle b_{i,j}^2 \rangle$ .
- We assume that this parameter does not change with the QCD evolution.
- For the recoil system, because of the use of we can relate  $A_R = \left(\frac{B_R}{\pi}\right)^{\frac{3}{4}}$ .
- We expect the parameter  $B_R$ , which characterizes the size of the residual system to depend on the residual mass and as a result to be  $Q^2$  dependent.
- the parameter  $A_V$  is fixed through the normalization factor,  $\mathcal{N}$  using:  $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2}$
- the remaining parameters  $\mathcal{N}$ ,  $m_R$  and  $B_R$  are evaluated by fitting to empirical PDFs

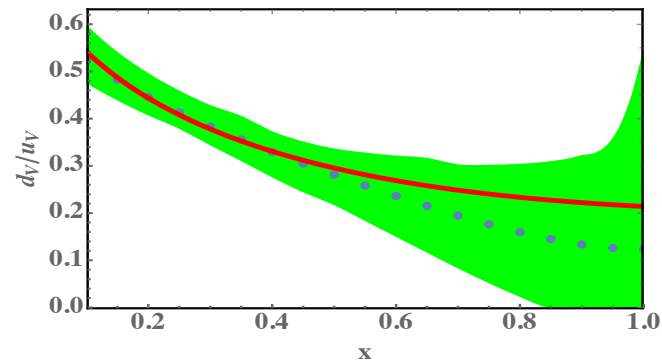


Once parameters are fixed we can evaluate the strength of the missing high momentum component

- For the normalization one obtains for d-quark  $N_d = 0.8$  and for u-quark  $N_u = 1.54$ . These results also indicates that one expects that Regge mechanism at  $x < x_p$  and  $qq$ -correlations to contribute  $\sim 20\%$  and  $\sim 23\%$  of total normalizations for d- and u- quarks respectively.

- The evaluation of the momentum sum rule ( $P_q = \int xq_V(x)dx$ ) yields  $P_d = 0.1$  and  $P_u = 0.246$  compared to  $P_d = 0.108$  and  $P_u = 0.264$  of CTNN distribution. Because of lesser contribution from the Regge mechanism in this case these estimates evaluate better the contribution from qq correlations. Here for qq-correlations one obtains for the d-quark,  $\sim 8\%$  and for the u-quark is  $\sim 11\%$ .

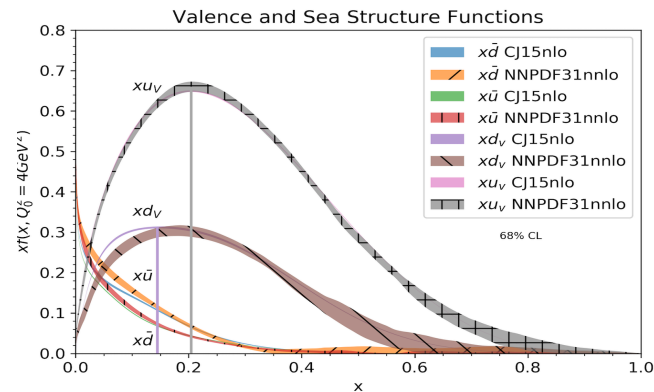
## Predication of d/u ratio at $x_B \rightarrow 1$



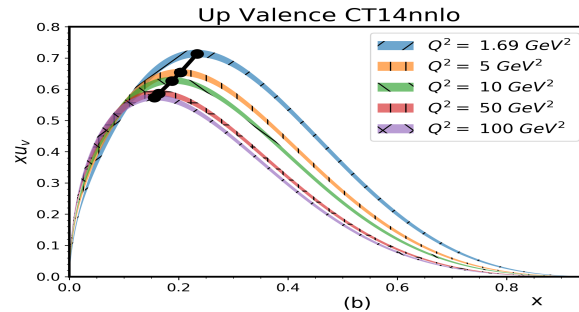
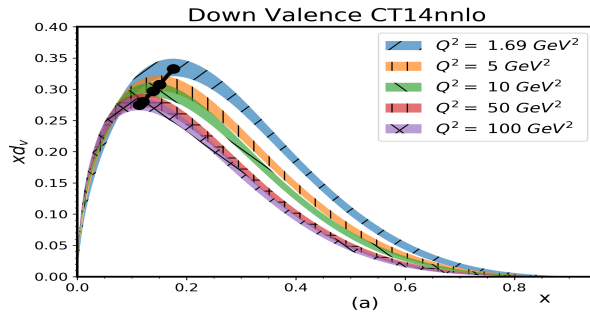
$$\frac{d_V}{u_V} \Big|_{x \rightarrow 1} \approx 0.21.$$

The reason of d/u fall-off is  $m_R(u) < m_R(d)$

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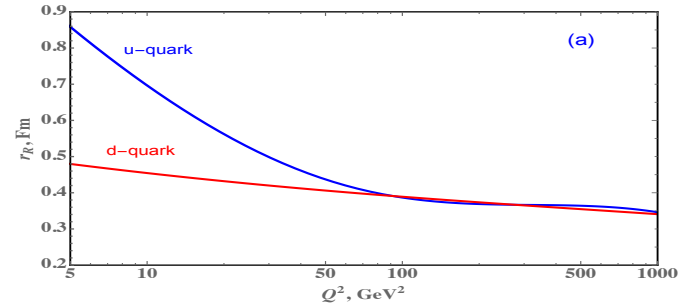
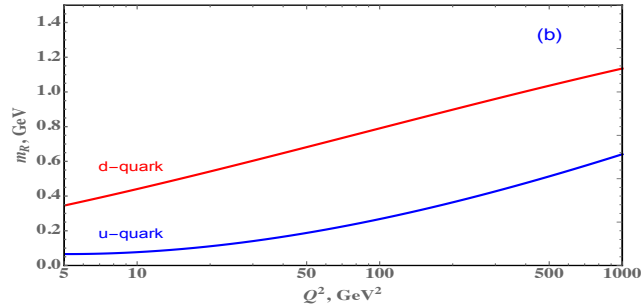
## Q<sup>2</sup> dependence of parameters: N, B<sub>R</sub> and m<sub>R</sub>



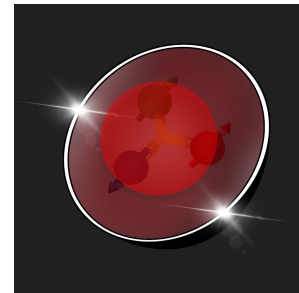
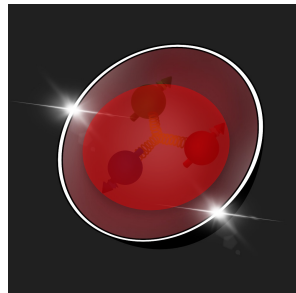
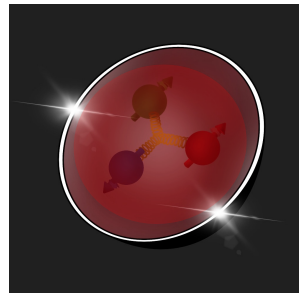
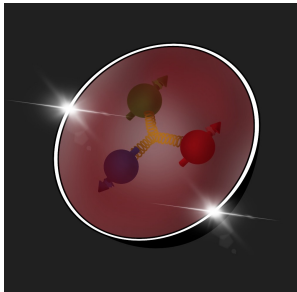
$$\begin{aligned}
 \mathcal{N}(t) &= N_0 + N_1 t + N_2 t^2 + N_3 t^3, \\
 B_R(t) &= B_{R0} + B_{R1} t + B_{R2} t^2 + B_{R3} t^3, \\
 m_R(t) &= m_{R0} + m_{R1} t + m_{R2} t^2 + m_{R3} t^3
 \end{aligned}
 \quad \text{where } t = \log \frac{Q^2}{1 \text{ GeV}^2}.$$

$$r_R(t) \approx \sqrt{\frac{3}{2} B_R(t)} \cdot 0.197327 \text{ Fm.}$$

## Q2 dependence of parameters: N, BR and mR



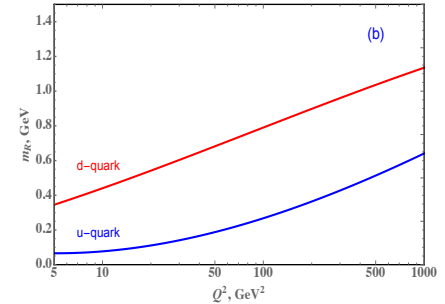
- Mass of the residual system increases as  $\log Q^2$
- Gluon radiation of valence quarks confined in 3q system
- Mass gap is almost constant starting  $Q^2 > 20$  GeV<sup>2</sup>



All these observations generate predictions that can be checked at EIC

- Mass of the residual system increases as  $\log Q^2$

Invariant mass of gluonic jets in target fragmentation region in  $ep \rightarrow e' \text{ jets}$  will increase with  $\log Q^2$  at  $x=0.2$  and the increase rate can be calculated

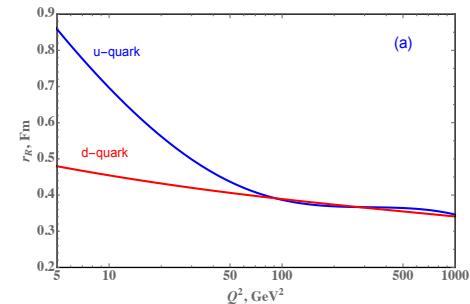


- Mass gap is almost constant starting  $Q^2 > 20 \text{ GeV}^2$

Multiplicity of target fragmented pions depends whether valence u or d quarks was struck by virtual photon

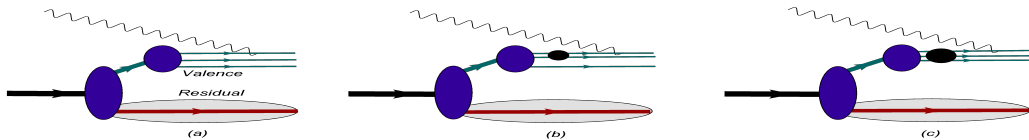
- Gluon radiation of valence quarks confined in 3q system

Angular correlation of gluon radiation in target fragmentation region with probed  $x$ .



## Conclusion and some outlook

- We developed model of valence  $3q+$  recoil nucleon system in which explicitly included wave function of the recoil nucleon with mass spectrum
- Fitting to phenomenological valence quark PDFs allowed to fix parameters of both  $3q$  mean field and residual nucleon system's wave function
- Estimated the overall possible contribution from  $qq$  and  $qqq$  correlations
- These wave functions will allow us to calculate many other observables, such as form-factors, hard real and virtual Compton scattering, GPDs TMDs .... sensitive to  $x=0.2$  region of valence quarks
- The project currently underway is to include the  $qq$  to  $qqq$  correlations that can contribute in generation of high  $x$  tail of PDFs



- Refinement and improvement of the model



# Quarks in Nuclei

