Improved α_s determination from hadronic decays of EW bosons at N^3LO accuracy

9th APS GHP Hadronic Physics Workshop
(Sacramento/Virtual), 13–16 April 2021

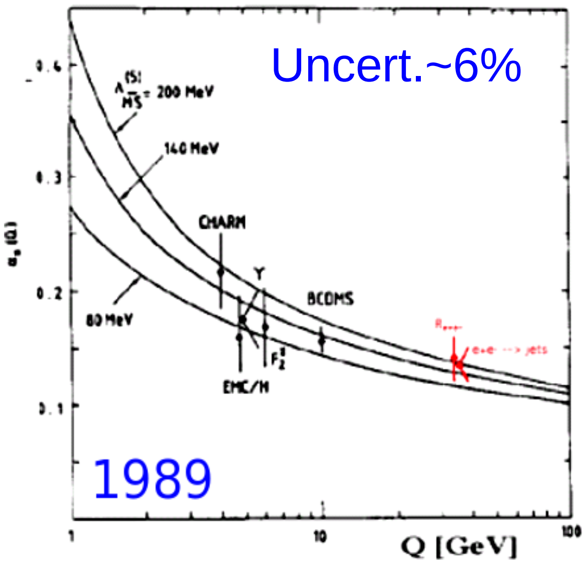
David d'Enterria

CERN

Based mostly on: *D. d'Enterria,V. Jacobsen "Improved strong coupling determinations from hadronic decays of electroweak bosons at N^3LO accuracy",* *https://arxiv.org/abs/2005.04545 [hep-ph]*

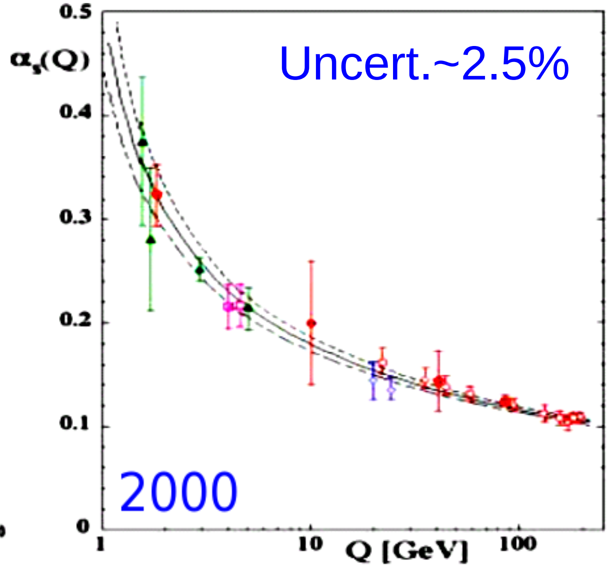
QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q \to 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV



$\alpha_S(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$\alpha_S(M_Z) = 0.1184 \pm 0.0031$ (NNLO)

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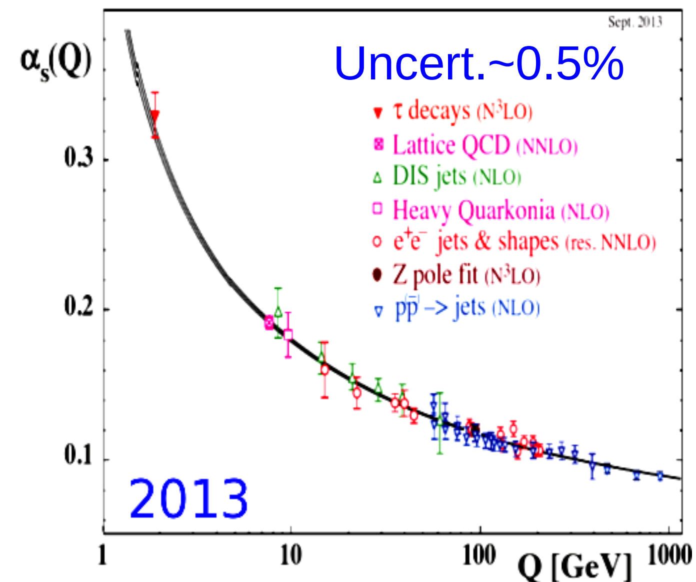

$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (NNLO)

QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q \to 0$ limit.
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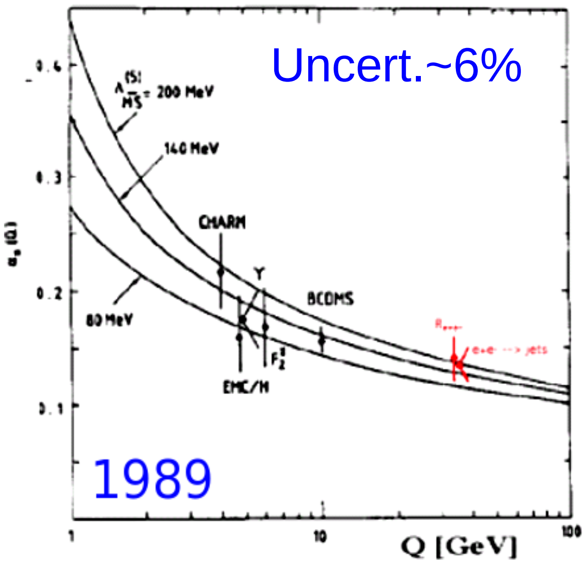

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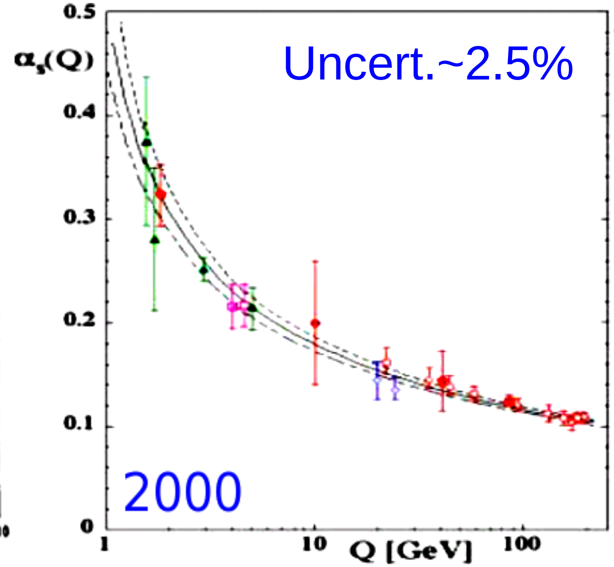

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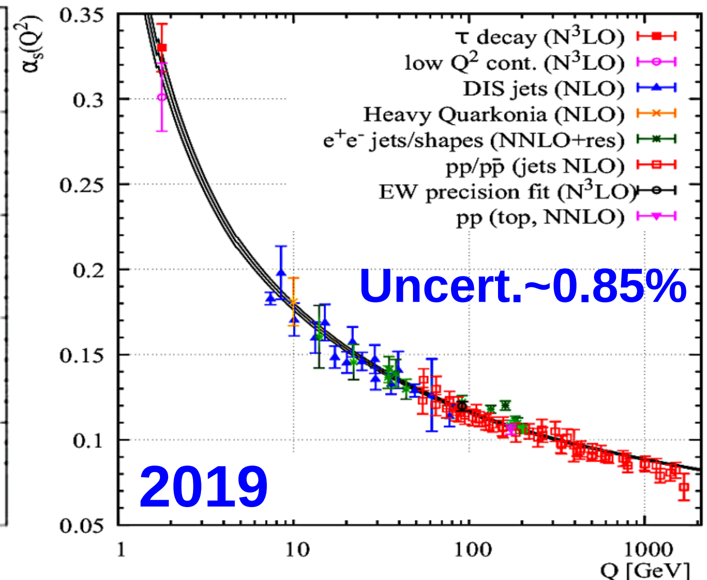


$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010$

- Least precisely known of all interaction couplings !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

Importance of the strong coupling α_s

- Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	σ (pb)	$\delta\alpha_s(\%)$	PDF $+\alpha_s(\%)$	Scale(%)
ggH	49.87	$\pm$ 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	$\pm$ 3.0	$\pm$ 8.9	-9.3 +5.9

Channel	M_H[GeV]	$\delta\alpha_s(\%)$	Δm_b	Δm_c
$H \to c\bar{c}$	126	$\pm$ 7.1	$\pm$ 0.1%	$\pm$ 2.3 %
$H \to gg$	126	$\pm$ 4.1	$\pm$ 0.1%	$\pm$ 0 %

Msbar mass error budget (from threshold scan)

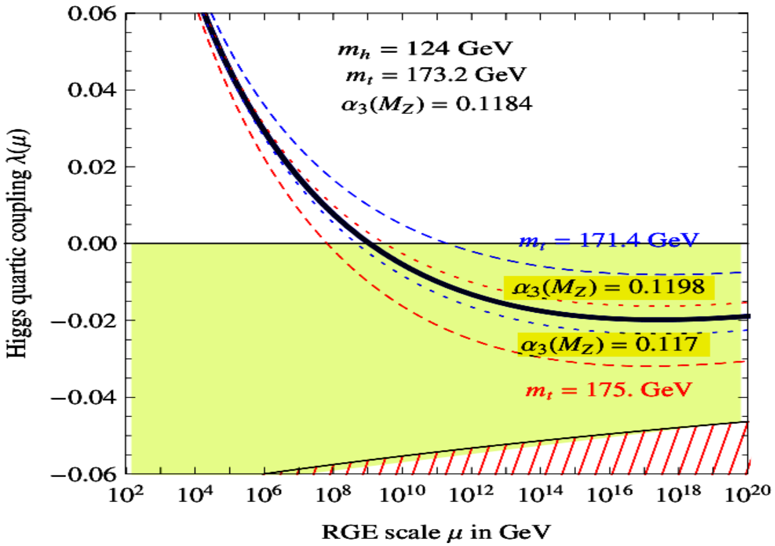
$(\delta M_t^{SD-low})^{exp}$	$(\delta M_t^{SD-low})^{theo}$	$(\delta\overline{m}_t(\overline{m}_t))^{conversion}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

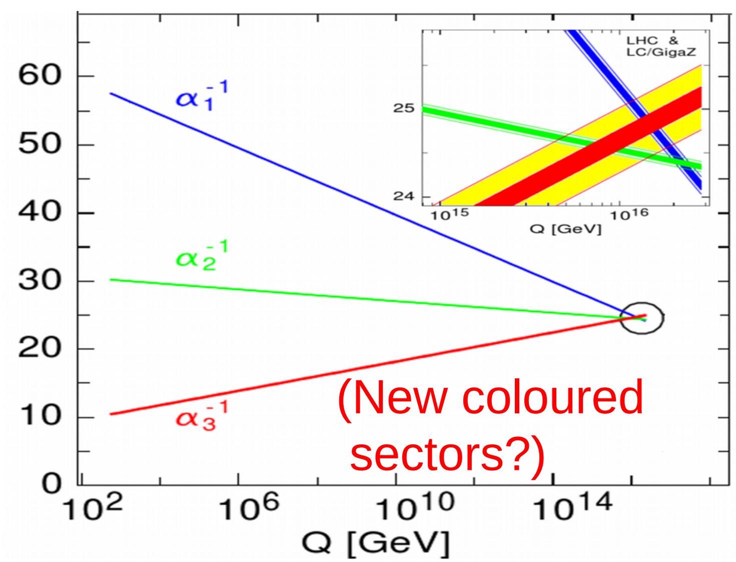
$\Rightarrow$ improvement in α_s crucial $\quad \delta\alpha_s(M_Z) = 0.001$

Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$

Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

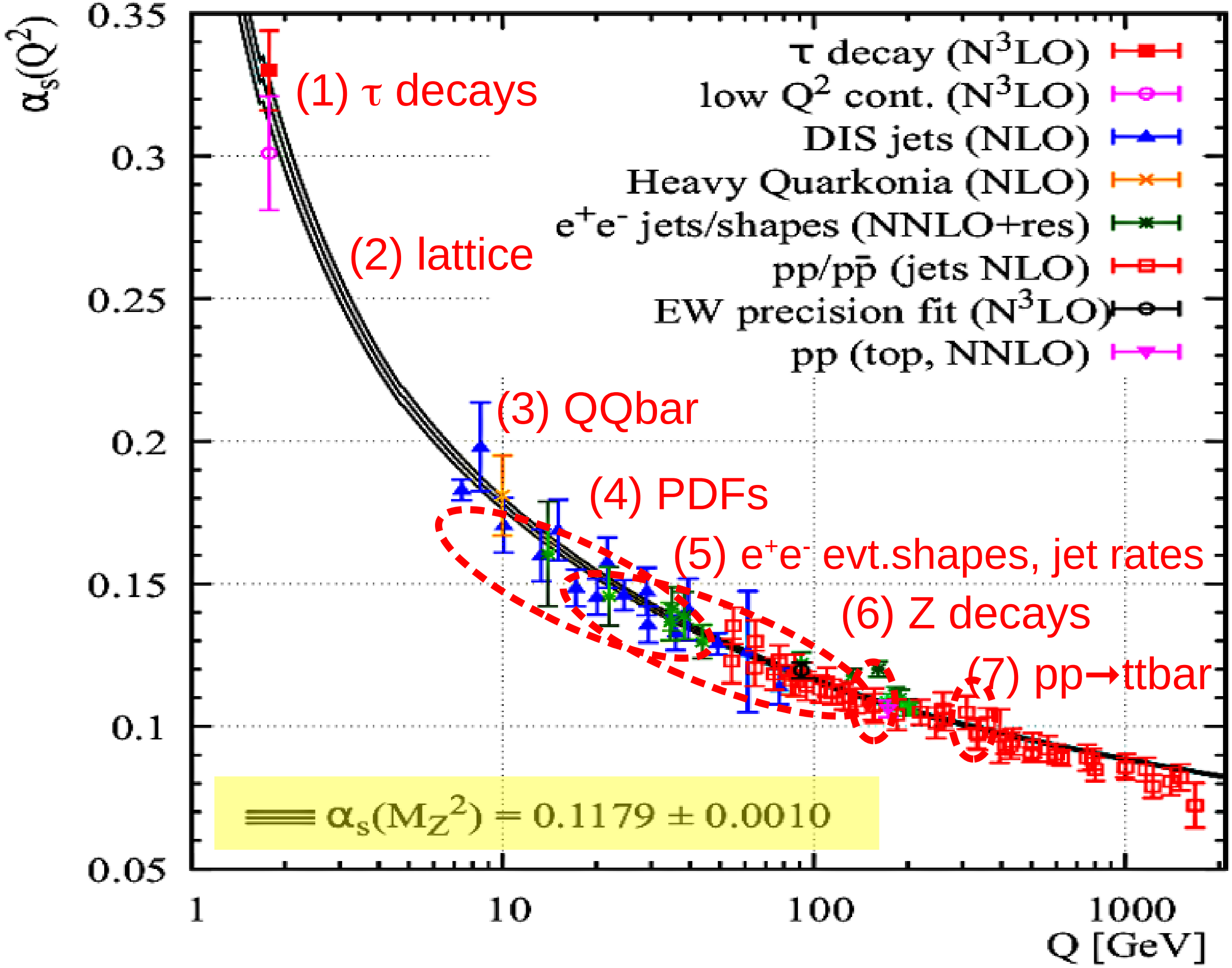
- Impacts physics approaching Planck scale: EW vacuum stability, GUT





World α_s determination (PDG 2019)

- Determined today by comparing 7 experimental observables to pQCD NNLO,N³LO predictions, plus global average at the Z pole scale:



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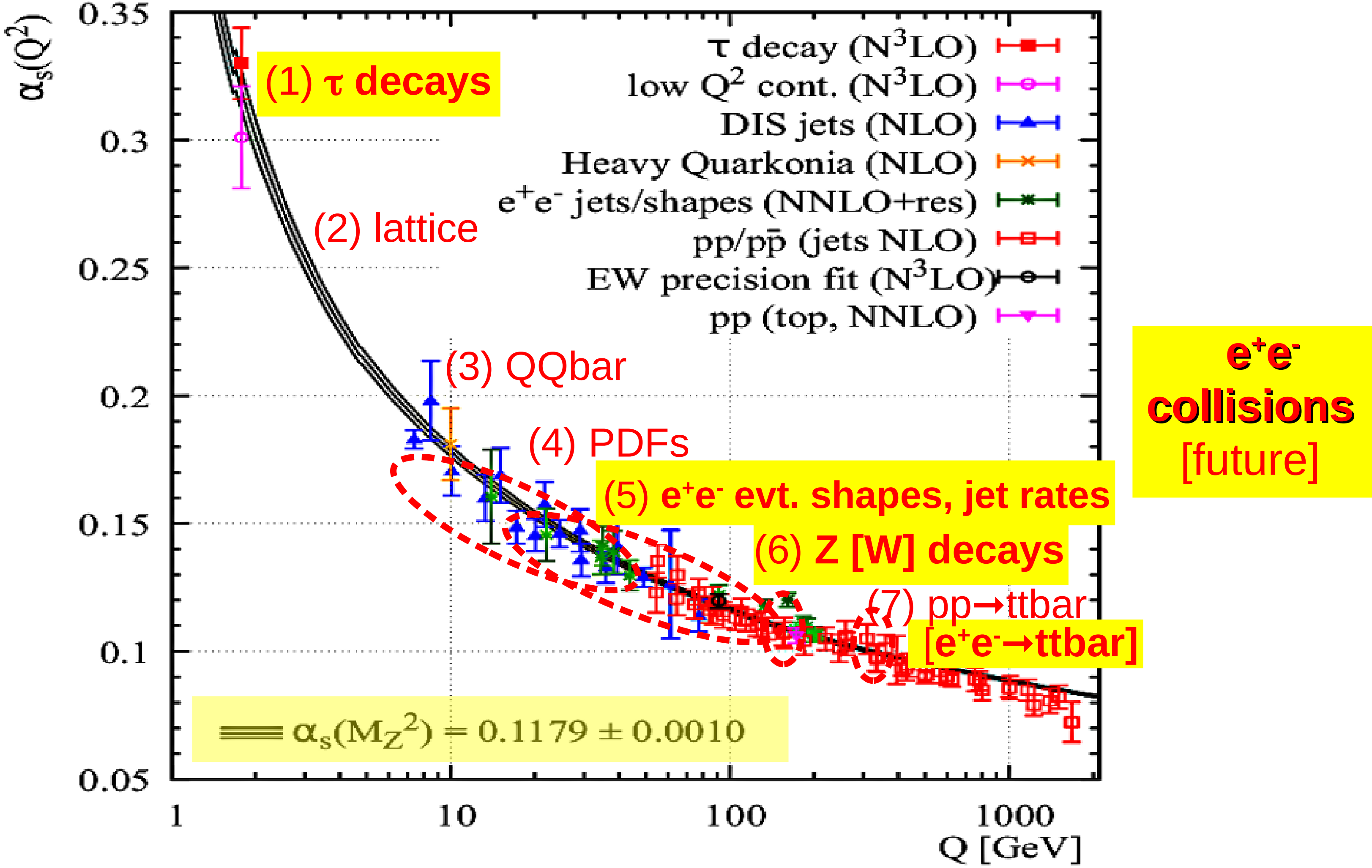
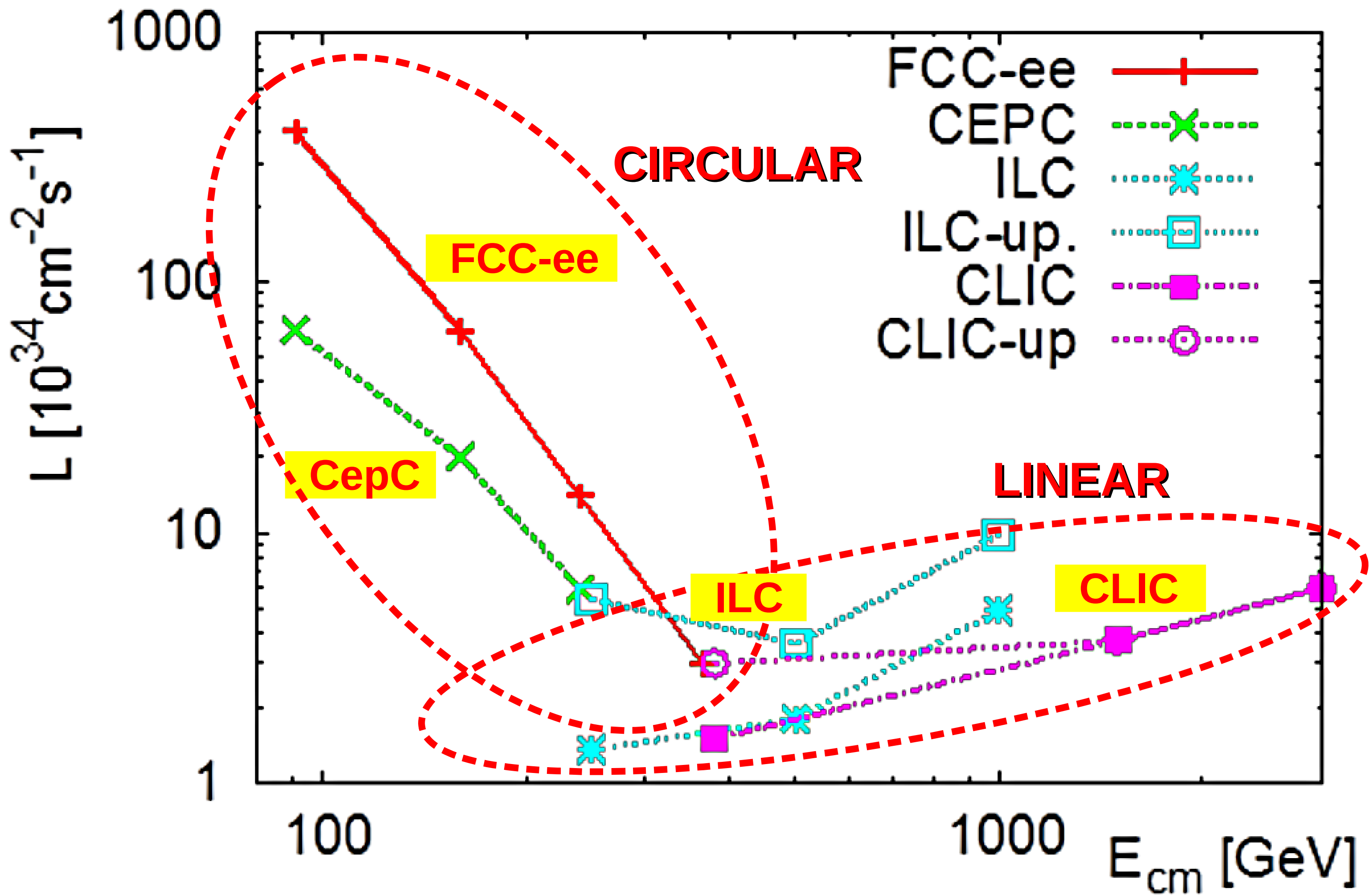


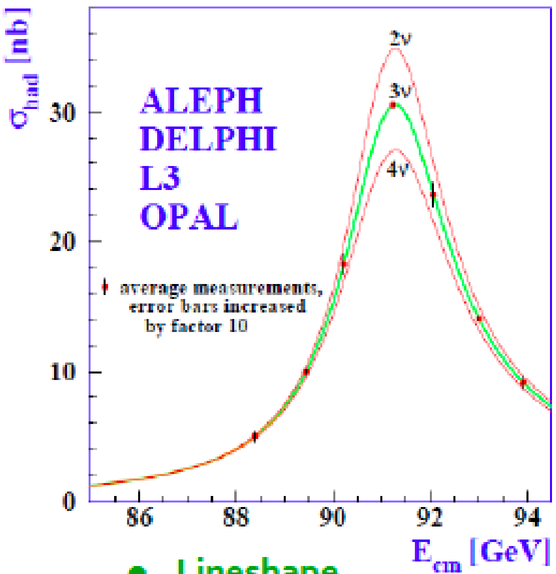
Future e^+e^- colliders under discussion



- FCC-ee features lumis a few times larger than other machines over 90–240 GeV
- Unparalleled Z, W, jets, τ,… data sets: Negligible α_s stat. uncertainties

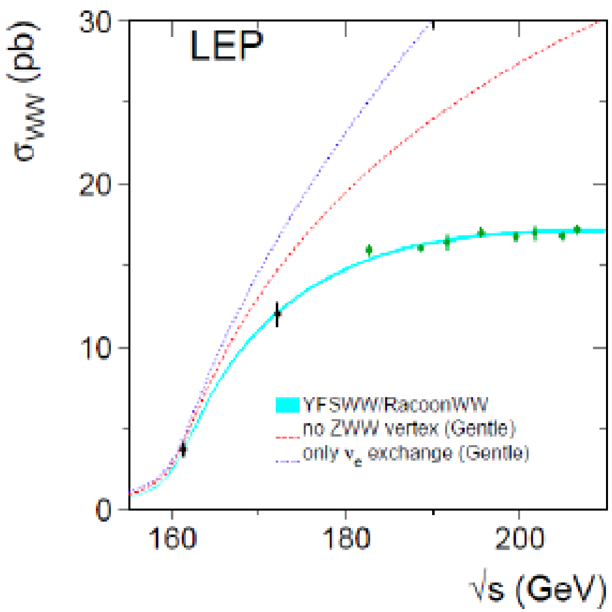
Ultra-precise W, Z, top physics at FCC-ee

$\sqrt{s}$=91 GeV, 10^{12} Z's



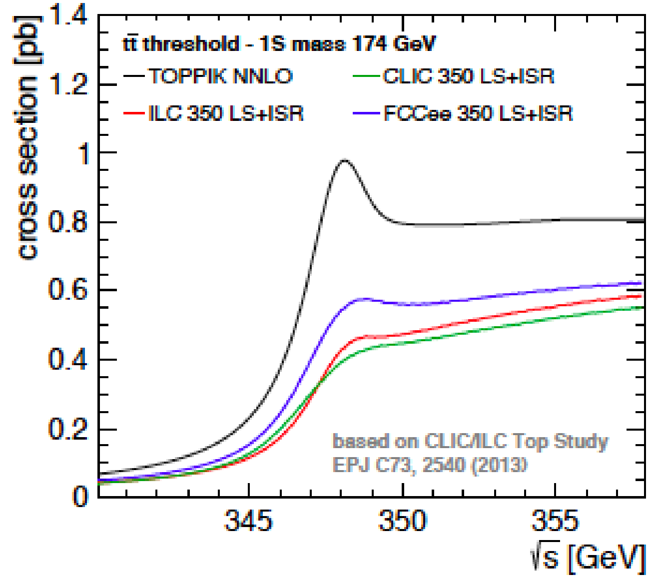
- Lineshape
  - Exquisite E_{beam} (unique!)
  - m_Z, Γ_Z to 10 keV (stat.) 100 keV (syst.)
- Asymmetries
  - $\sin^2\theta_W$ to 5×10^{-6}
- Branching ratios, R_l, R_b
  - $\alpha_s(m_Z)$ to 0.0002
- Predict m_{top}, m_W in SM

$\sqrt{s}$=161 GeV, 10^8 W's



- Threshold scan
  - m_W to 500 keV
- Branching ratios R_l, R_{had}
  - $\alpha_s(m_W)$ to 0.0002
- Radiative returns $e^+e^-\rightarrow\gamma Z$ ($Z\rightarrow\nu\nu$, $\mu^+\mu^-$)
  - N_ν to 0.001

$\sqrt{s}$=350 GeV, 10^6 tops



- Threshold scan + 4D fit
  - m_{top} to 10 MeV (stat.) 40 MeV (th.)
  - λ_{top} to 13%
  - EWK couplings to 1–10%

- Unparalleled Z, W, jets, τ,… data sets: Negligible α_s stat. uncertainties
- Unparalleled syst. uncert.: δE_{cm}(Z,W) ~ 0.1, 0.3 MeV → Very precise $\Gamma_{W,Z}$

Hadronic Z, W decay pseudo-observables

DdE, Jacobsen: arXiv:2005.04545

- Z & W observables theoretically known at N^3LO accuracy:

• The W and Z hadronic widths :

$$\Gamma^{\rm had}_{\rm W,Z}(Q) = \Gamma^{\rm Born}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm EW} + \delta_{\rm mix} + \delta_{\rm np}\right)$$

96.60% 3.78% –0.35% –0.05% → 0

• The ratio of W, Z hadronic-to-leptonic widths :

$$R_{\rm W,Z}(Q) = \frac{\Gamma^{\rm had}_{\rm W,Z}(Q)}{\Gamma^{\rm lep}_{\rm W,Z}(Q)} = R^{\rm EW}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm mix} + \delta_{\rm np}\right)$$

• In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma^{\rm had}_{\rm Z} = \frac{12\pi}{m_{\rm Z}} \cdot \frac{\Gamma^e_{\rm Z}\Gamma^{\rm had}_{\rm Z}}{(\Gamma^{\rm tot}_{\rm Z})^2}$$

TH uncertainties: (α^2,α^3 included for Z):
±0.015–0.03% (Z)
±0.015–0.04% (W)

Parametric uncerts.: (α_s,$m_{Z,W}$; $V_{cs,ud}$):
±0.01–0.03% (Z)
±1.1–1.7% (W)
±0.03% (W, CKM unit)

- Measured at LEP with ±0.1–0.3% (Z), ±0.9–2% (W) exp. uncertainties:

	theory			experiment		
	previous	new (this work)	change	previous [6]	new [20, 21]	change
$\Gamma^{\rm tot}_{\rm Z}$ (MeV)	$2494.2 \pm 0.8_{\rm th}$	$2495.2 \pm 0.6_{\rm par} \pm 0.4_{\rm th}$	+0.04%	2495.2 ± 2.3	2495.5 ± 2.3	+0.012%
$R_{\rm Z}$	$20.733 \pm 0.007_{\rm th}$	$20.750 \pm 0.006_{\rm par} \pm 0.006_{\rm th}$	+0.08%	20.767 ± 0.025	20.7666 ± 0.0247	−0.040%
$\sigma^{\rm had}_{\rm Z}$ (pb)	$41\,490 \pm 6_{\rm th}$	$41\,494 \pm 5_{\rm par} \pm 6_{\rm th}$	+0.01%	$41\,540 \pm 37$	$41\,480.2 \pm 32.5$	−0.144%

W boson observables	GFITTER 2.2 (NNLO)	this work (N^3LO) (exp. CKM)	(CKM unit.)	experiment
$\Gamma^{\rm had}_{\rm W}$ (MeV)	–	$1440.3 \pm 23.9_{\rm par} \pm 0.2_{\rm th}$	$1410.2 \pm 0.8_{\rm par} \pm 0.2_{\rm th}$	1405 ± 29
$\Gamma^{\rm tot}_{\rm W}$ (MeV)	$2091.8 \pm 1.0_{\rm par}$	$2117.9 \pm 23.9_{\rm par} \pm 0.7_{\rm th}$	$2087.9 \pm 1.0_{\rm par} \pm 0.7_{\rm th}$	2085 ± 42
$R_{\rm W}$	–	$2.1256 \pm 0.0353_{\rm par} \pm 0.0008_{\rm th}$	$2.0812 \pm 0.0007_{\rm par} \pm 0.0008_{\rm th}$	2.069 ± 0.019

Recent update of LEP luminosity bias(*) change the Z values by few permil

(*) Voutsinas et al. arXiv:1908.01704, Janot et al. arXiv:1912.02067

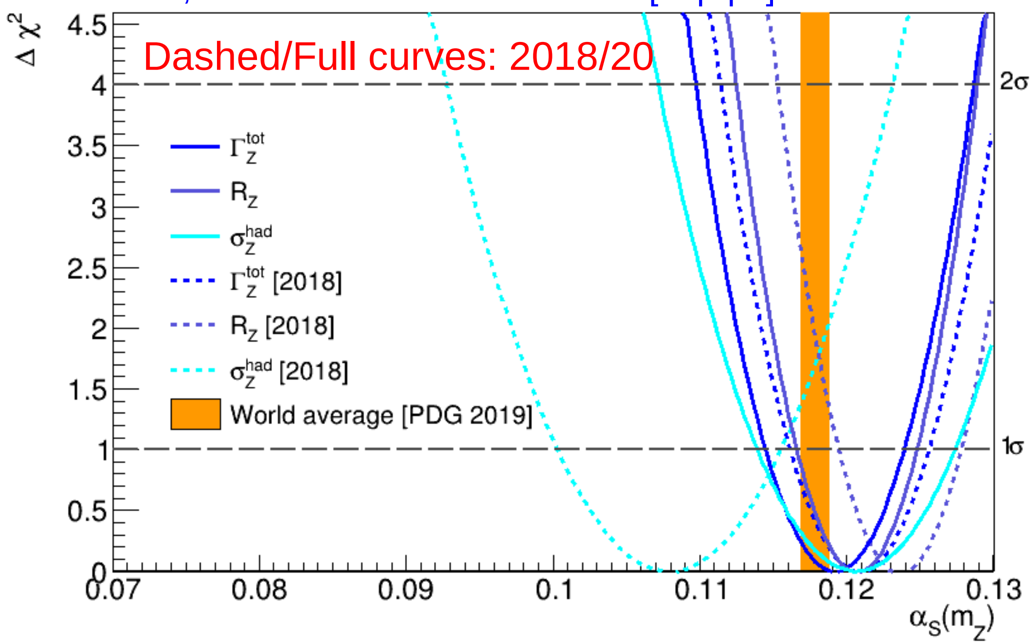
α_s from hadronic Z decays (today)

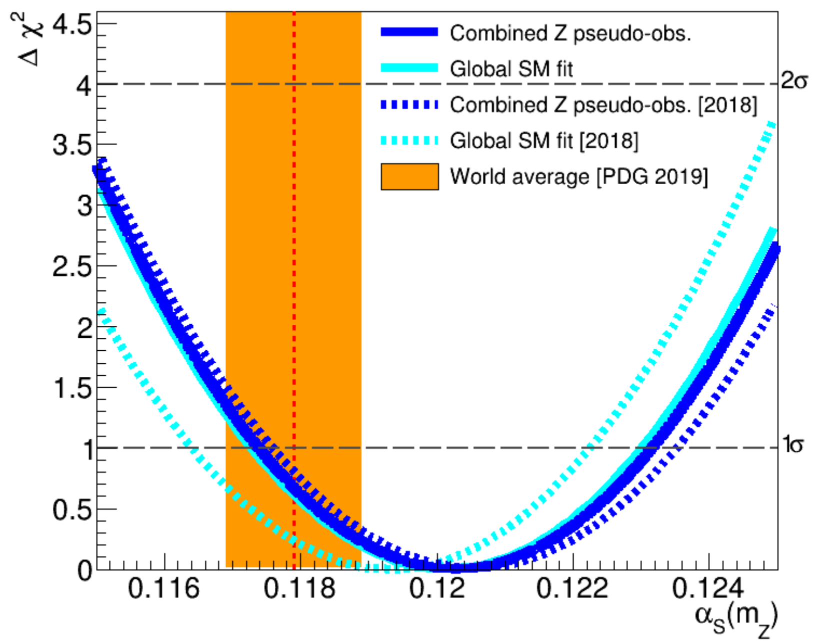
- QCD coupling extracted from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

Z boson observable	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_Z^{tot}	0.1192 ± 0.0047	± 0.0046	± 0.0005	± 0.0008
R_Z	0.1207 ± 0.0041	± 0.0041	± 0.0001	± 0.0009
σ_Z^{had}	0.1206 ± 0.0068	± 0.0067	± 0.0004	± 0.0012
All combined	0.1203 ± 0.0029	± 0.0029	± 0.0002	± 0.0008
Global SM fit	0.1202 ± 0.0028	± 0.0028	± 0.0002	± 0.0008

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]





- LEP lumi-bias updates lead to much better agreement among Γ_Z, R_Z, σ_0 extractions:
- Improved **$\alpha_s(m_z)$ = 0.1203 ± 0.0028 (±2.3%)**

PDG'19: $\alpha_s(m_z)$ = 0.1205 ± 0.0030 (±2.5%)

- EXP/TH updates lead to better agreement with full SM fit:
- **$\alpha_s(m_z)$ = 0.1202 ± 0.0028**

PDG'19: $\alpha_s(m_z)$ = 0.1194 ± 0.0029

α_s from hadronic Z decays (FCC-ee)

- QCD coupling extracted from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

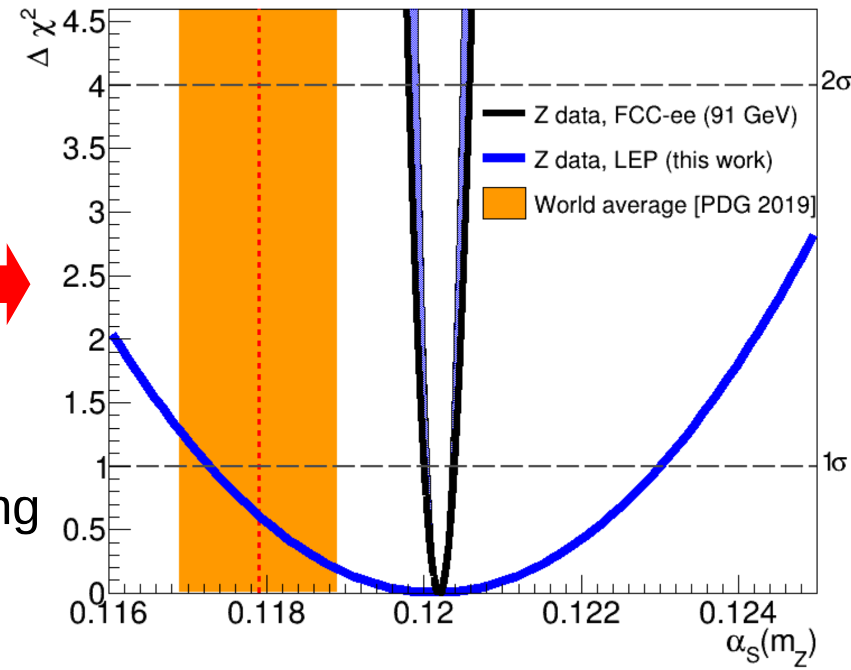
Z boson observable	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
All combined	0.1203 ± 0.0029	± 0.0029	± 0.0002	± 0.0008
Global SM fit	0.1202 ± 0.0028	± 0.0028	± 0.0002	± 0.0008
All combined (FCC-ee)	0.12030 ± 0.00026	± 0.00013	± 0.00005	± 0.00022
Global SM fit (FCC-ee)	0.12020 ± 0.00026	± 0.00013	± 0.00005	± 0.00022

- FCC-ee:
  - Huge Z pole stats. ($\times 10^5$ LEP):
  - Exquisite systematic/parametric precision (stat. uncert. negligible):

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$
$$\Delta\Gamma_Z^{tot} = 0.1 \text{ MeV}, \quad \Gamma_Z^{tot} = 2495.2 \pm 0.1 \text{ MeV}$$
$$\Delta\sigma_Z^{had} = 4.0 \text{ pb}, \quad \sigma_Z^{had} = 41\,494 \pm 4 \text{ pb}$$
$$\Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV}$$
$$\Delta\alpha = 3 \cdot 10^{-5}, \quad \Delta\alpha_{had}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

  - TH uncert. to be reduced by $\times 4$ computing missing α_s^5, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



- 10 times better precision than today: $\delta\alpha_s/\alpha_s \sim \pm 0.2\%$ (exp+th), $\pm 0.1\%$ (exp)
Strong (B)SM consistency test.

$\alpha_s(m_Z) = 0.12030 \pm 0.00028$ ($\pm 0.2\%$)

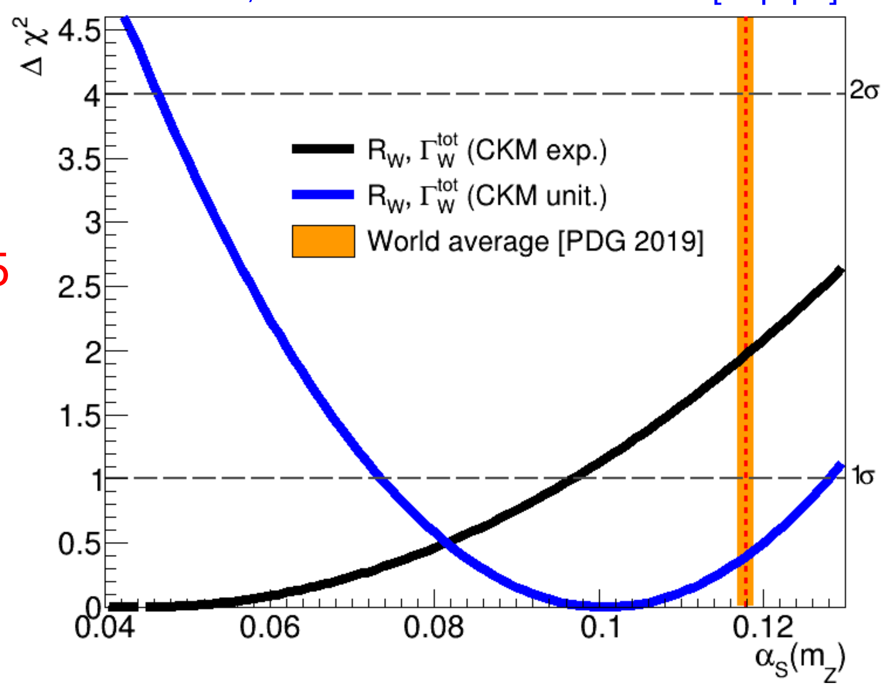
α_s from hadronic W decays (today)

- QCD coupling extracted from new N³LO fit of combined Γ_W, R_W pseudo-observ.:

W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_W^{tot}, R_W (exp. CKM)	0.044 ± 0.052	± 0.024	± 0.047	(± 0.0014)
Γ_W^{tot}, R_W (CKM unit.)	0.101 ± 0.027	± 0.027	(± 0.0002)	(± 0.0016)
Γ_W^{tot}, R_W (FCC-ee, CKM unit.)	0.11790 ± 0.00023	± 0.00012	± 0.00004	± 0.00019

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

- Very imprecise extraction:
  - Large propagated parametric uncert. from poor V_{cs} exp. precision (±2%): QCD coupling unconstrained: 0.04±0.05
  - Imposing CKM unitarity: large exp. uncertainties from Γ_W, R_W (0.9–2%): QCD extracted with ~27% precision
  - Propagated TH uncertainty much smaller today: ~1.5%



$\alpha_s(m_z) = 0.101 \pm 0.027$ (±27%)

α_s from hadronic W decays (FCC-ee)

- QCD coupling extracted from new N^{3}LO fit of combined Γ_W, R_W pseudo-observ.:

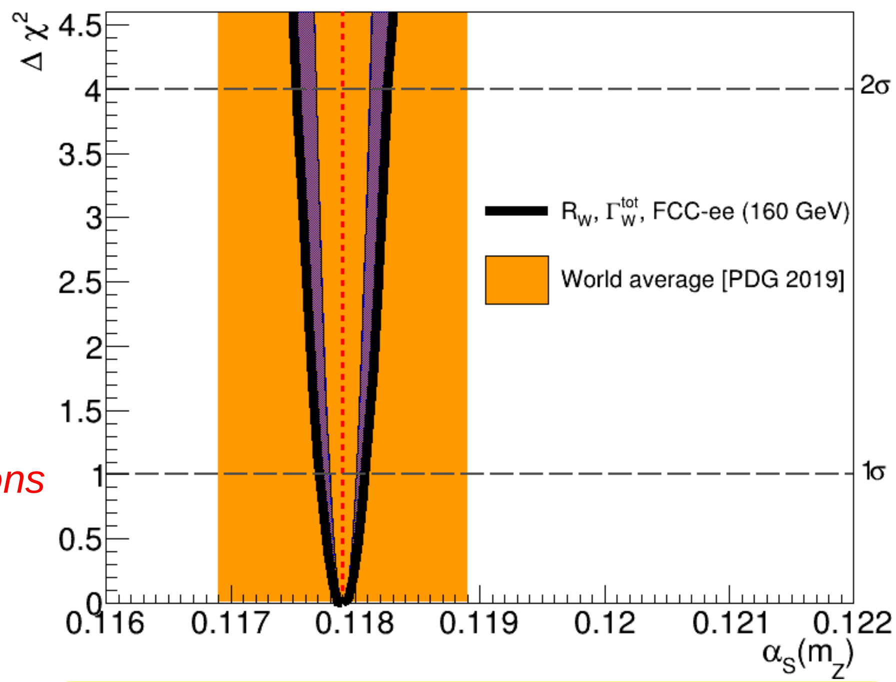
W boson	$\alpha_S(m_Z)$	uncertainties		
observables	extraction	exp.	param.	theor.
Γ_W^{tot}, R_W (exp. CKM)	0.044 ± 0.052	± 0.024	± 0.047	(± 0.0014)
Γ_W^{tot}, R_W (CKM unit.)	0.101 ± 0.027	± 0.027	(± 0.0002)	(± 0.0016)
Γ_W^{tot}, R_W (FCC-ee, CKM unit.)	0.11790 ± 0.00023	± 0.00012	± 0.00004	± 0.00019

- FCC-ee extraction:
  - Huge W pole stats. ($\times 10^4$ LEP-2).
  - Exquisite syst./parametric precision:

$$\Gamma_W^{tot} = 2088.0 \pm 1.2 \text{ MeV}$$
$$R_W = 2.08000 \pm 0.00008$$
$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$
$$|V_{cs}| = 0.97359 \pm 0.00010$$ ← *O(10^{12}) D mesons*

  - TH uncertainty to be reduced by $\times 10$ after computing missing α_s^5, α^2, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

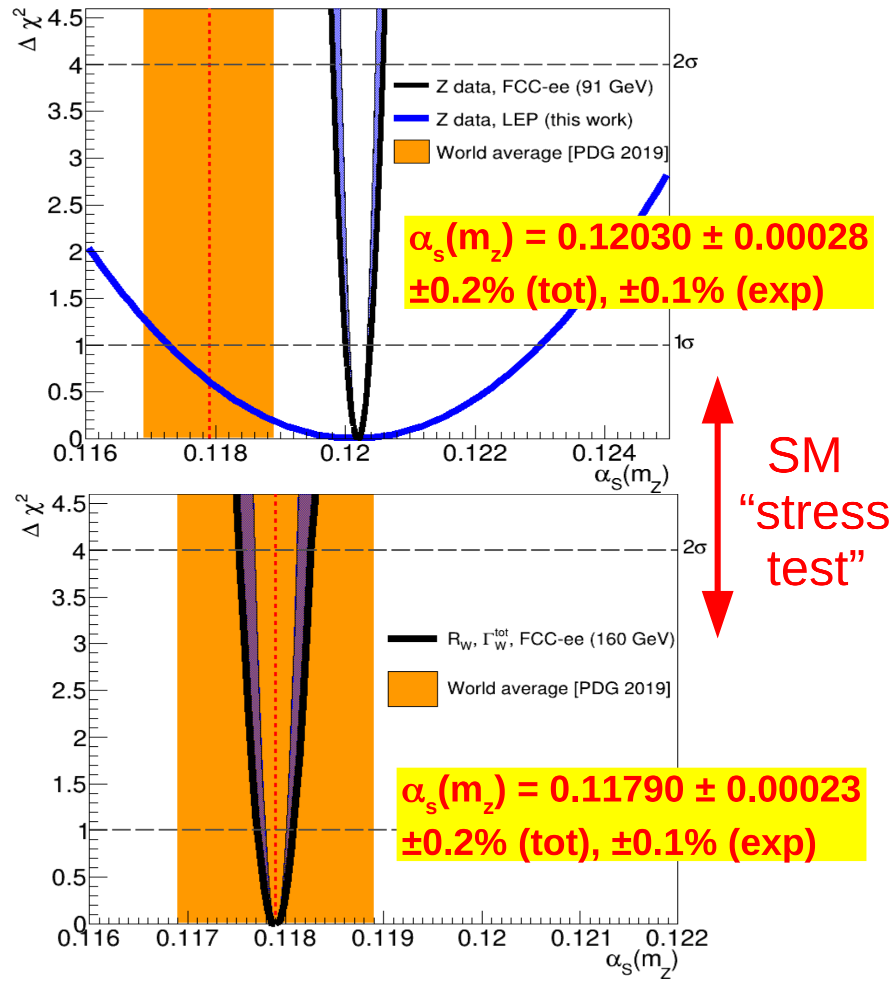
DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



$\alpha_s(m_z) = 0.11790 \pm 0.00023$ ($\pm 0.2\%$)

Summary: α_s from hadronic EW bosons decays

- World-average QCD coupling at N^{2,3}LO today:
  - *Determined from 7 observables with combined ±0.85% uncertainty: Least well-known gauge coupling.*
  - *Impacts all LHC QCD x-sections & decays.*
  - *Role beyond SM: GUT, EWK vacuum stability, New coloured sectors?*
- Uncerts. for e^+e^- extractions today:
  - τ decays: ±1.5% (mostly non-pQCD)
  - Shapes, jets: ±2.6% (mostly non-pQCD)
  - Z pseudo-observ.: ±2.5% (mostly exp.)
- New Z, W extractions:
  - Z boson: New fit with high-order EW corrections + updated LEP data: ±2.3%, ±0.6%, (exp., th.) uncerts.
  - W boson: New N^{3}LO fit to Γ_W, R_W ~47%,~27% (param., exp.) uncerts.
- Future: 0.1% uncertainty possible with a machine like FCC-e^+e^-



Possible detector improvements to bring propagated syst. uncert. on W,Z pseudo-observ. below 0.1%

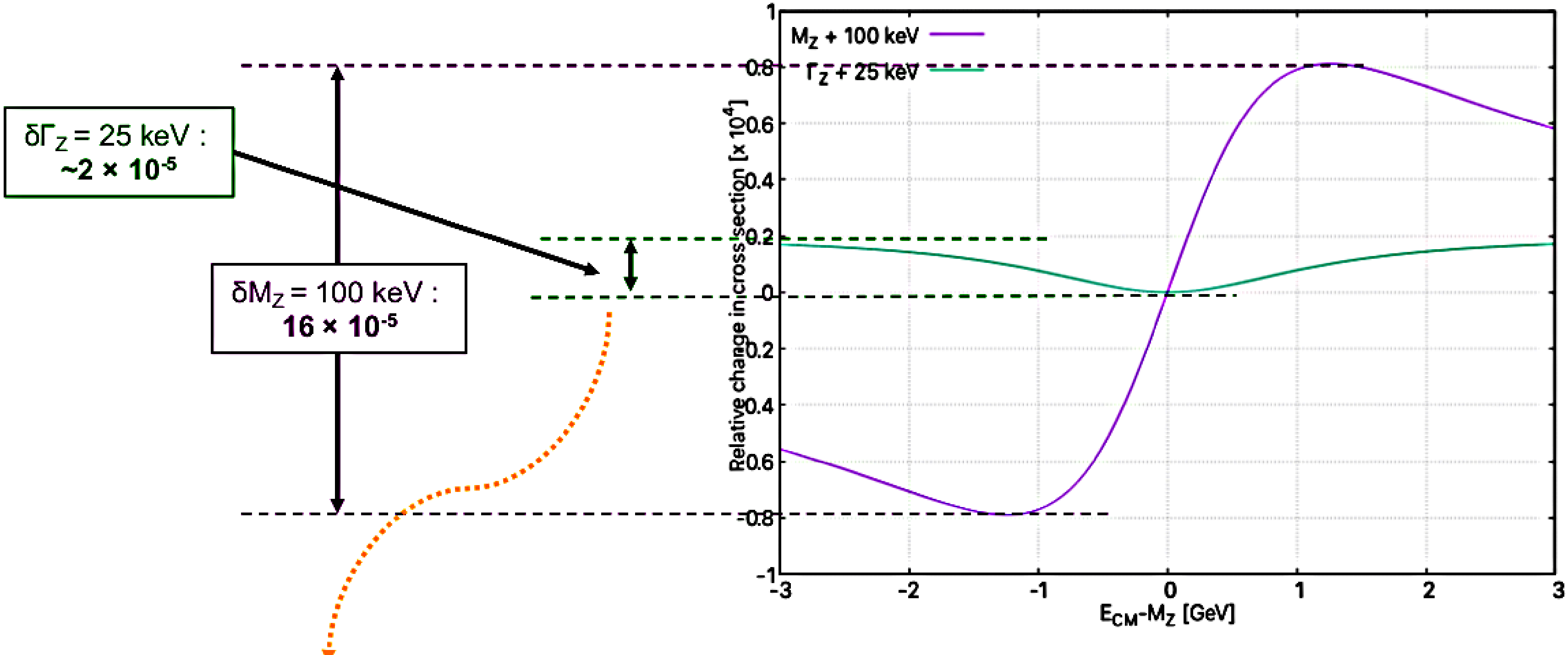
Backup slides

FCC-ee (91 GeV) syst. uncertainties

- FCC-ee goal: Via Z line-shape scan, determine Z parameters to precisions:

$\delta M_Z = 100$ keV ; $\delta\Gamma_Z = 25$ keV

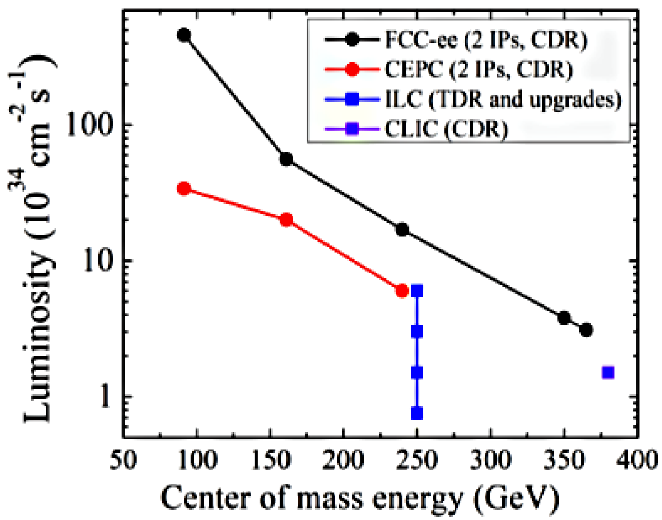
  - Plot shows relative change in cross section across Z resonance for parameter variation of this size

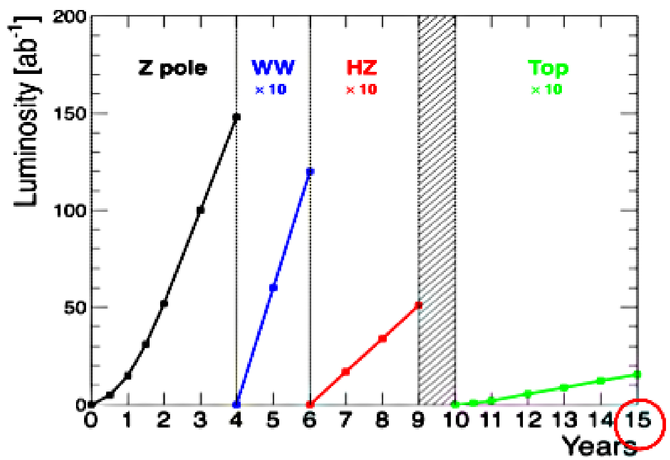


- Z width measurement most demanding: Need **relative** normalisation to about 10^{-5}
  - Need statistics of order 10^{10}
  - Need careful control of energy dependent effects

FCC-ee Luminosity, Operation, Data samples

Largest luminosities in the 88 – 365 GeV energy range





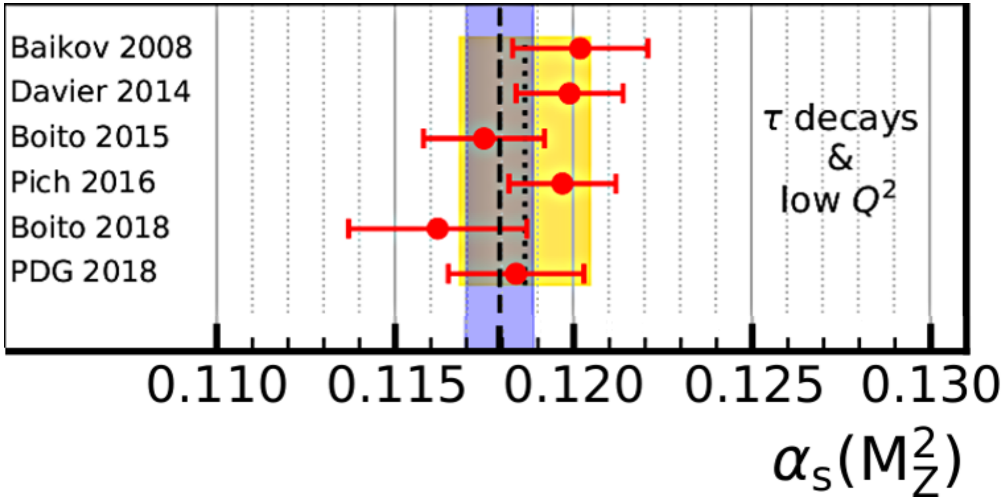
Event statistics	√s precision
5×10^{12} $e^+e^- \to Z$	100 keV
10^8 $e^+e^- \to W^+W^-$	300 keV
10^6 $e^+e^- \to HZ$	1 MeV
10^6 $e^+e^- \to tt$	2 MeV

Working point	Z, years 1-2	Z, later	WW	HZ	tt threshold...	... and above
√s (GeV)	88, 91, 94		157, 163	240	340 – 350	365
Lumi/IP (10^{34} cm^{-2}s^{-1})	100	200	25	7	0.8	1.4
Lumi/year (2 IP)	24 ab^{-1}	48 ab^{-1}	6 ab^{-1}	1.7 ab^{-1}	0.2 ab^{-1}	0.34 ab^{-1}
Physics goal	150 ab^{-1}		10 ab^{-1}	5 ab^{-1}	0.2 ab^{-1}	1.5 ab^{-1}
Run time (year)	2	2	2	3	1	4

α_s from hadronic τ-lepton decays

- Computed at N^{3}LO: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np})$
- Experimentally: $R_{\tau,exp}$ = 3.4697 ± 0.0080 (±0.23%)
- Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections $(\Lambda/m_\tau)^2$ ~2%, yield different results.

  Uncertainty slightly increased: 2013 (±1.3%) → 2019 (±1.5%)



$\alpha_s(M_Z^2) = 0.1187 \pm 0.0018$ **(±1.5%)**

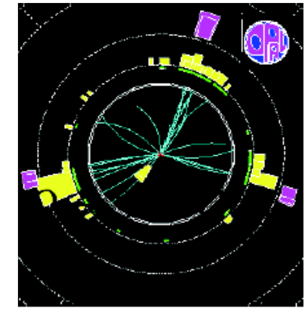
- Future :
  - TH: Better understanding of FOPT vs CIPT differences.
  - Better spectral functions needed (high stats & better precision): B-factories (BELLE-II)?
  - High-stats: $\mathcal{O}(10^{11})$ from Z→ττ at FCC-ee(90) : **$\delta\alpha_s/\alpha_s$ << 1%**

α_s from e^+e^- event shapes & jet rates (today)

- Computed at N[2,3]LO+N[(2)]LL accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  n-jet x-sections

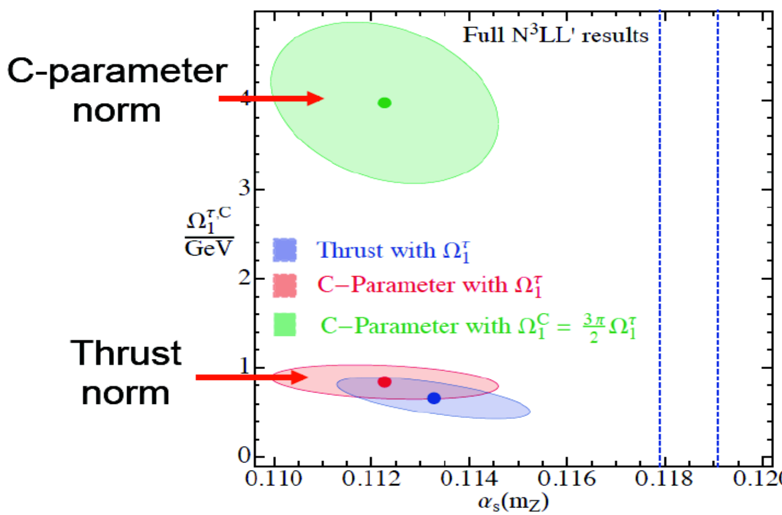
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

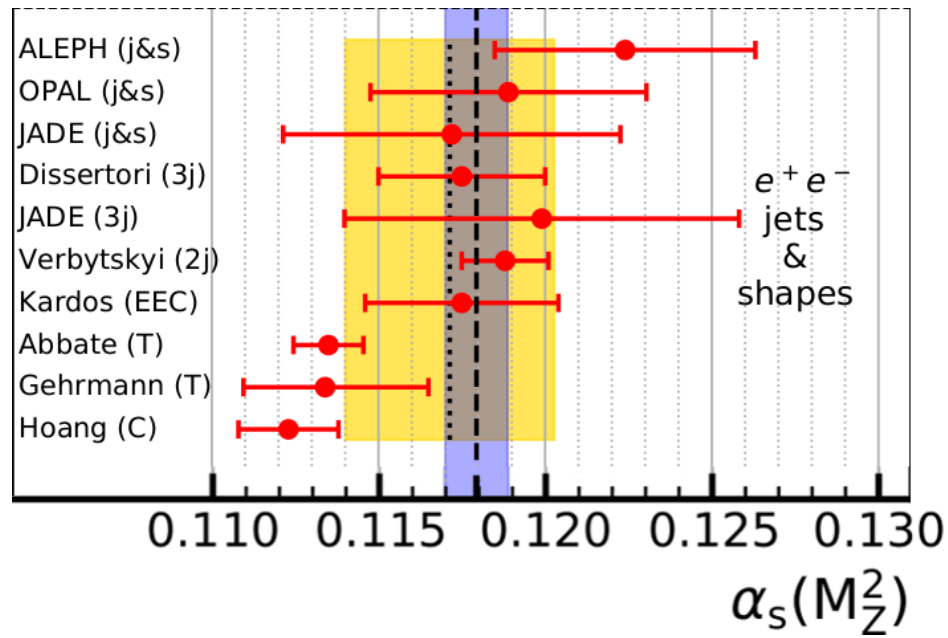
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:





- Wide span of TH extractions...

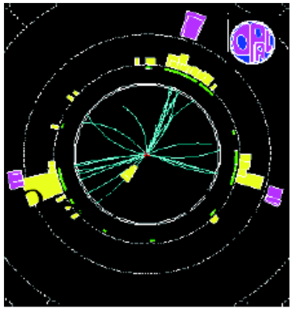
$$\alpha_s(M_Z^2) = 0.1171 \pm 0.0031$$ **(±2.6%)**

α_s from e^+e^- event shapes & jet rates (FCC-ee)

- Computed at $N^{2,3}LO+N^{(2)}LL$ accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections

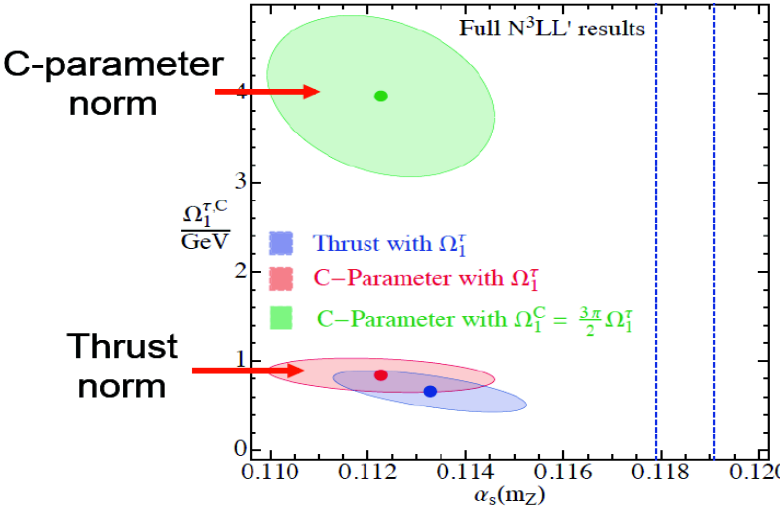
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



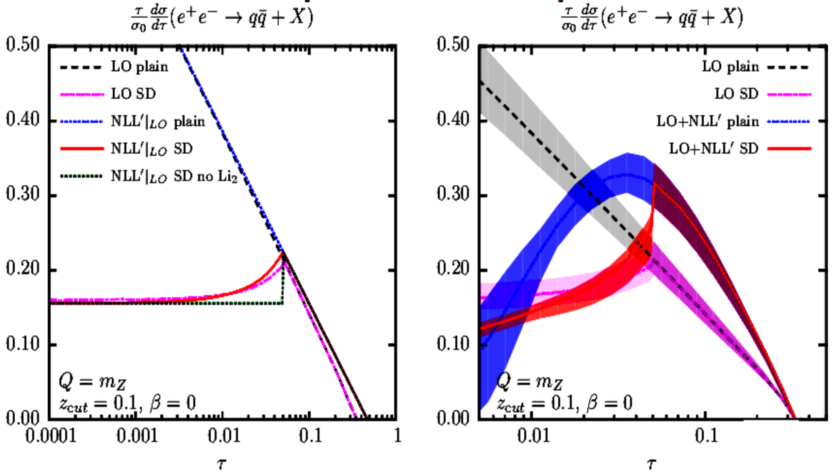
OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:



- Modern jet substructure techniques: "Soft drop" can help reduce non-pQCD corrections for thrust:

[Baron, Marzani, VT; '18]



- Future: **$\delta\alpha_s/\alpha_s < 1\%$**

– FCC-e^+e^- : Lower-√s (ISR) for shapes, higher-√s for jet rates

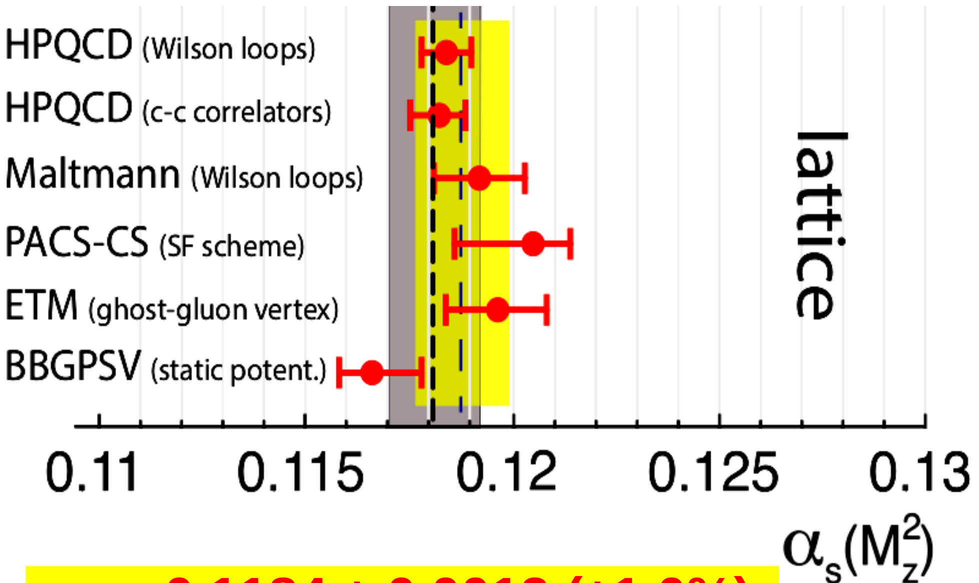
– TH: Improved ($N^{2,3}LL$) resummation for rates, hadronization for shapes

α_s from lattice QCD

- Comparison of short-distance quantities (Wilson loops, q static potential, vacuum polariz.,...) computed at NNLO in pQCD, to lattice QCD "data":

$$K^{\rm NP} = K^{\rm PT} = \sum_{i=0}^{n} c_i \alpha_s^i$$

[FLAG Collab. http://itpwiki.unibe.ch/flag]



- Currently, it's extraction with smallest uncertainties: ±1% (lattice spacing & statistics).

  Extracted value depends on observables:

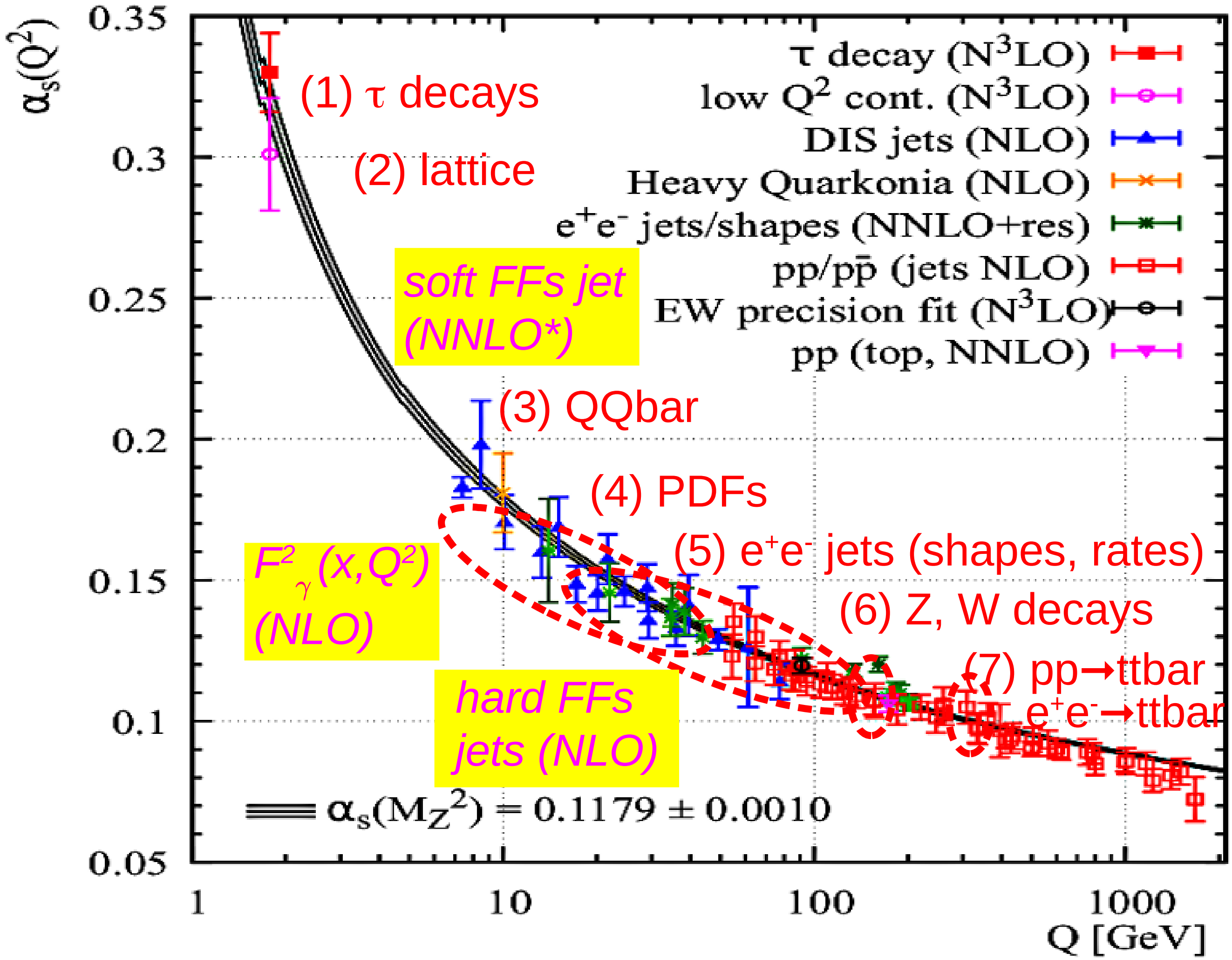
  Uncertainty increased: 2013 (±0.4%) → 2017 (±1.0%)

α_s = 0.1184 ± 0.0012 (±1.0%)

- Future prospects:
  - Uncertainty in α_s could be halved with (much) better numerical data.
  - Reaching ±0.1% requires 4th-loop perturbation theory (~10 years?)

Other α_s extractions (not *yet* in world average)

- There are few other classes of e^+e^- observables, computed today at lower accuracy (NLO, NNLO*), that can be used to extract the QCD coupling:



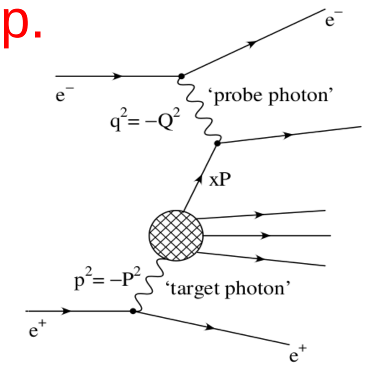
α_s from photon QCD structure function (NLO)

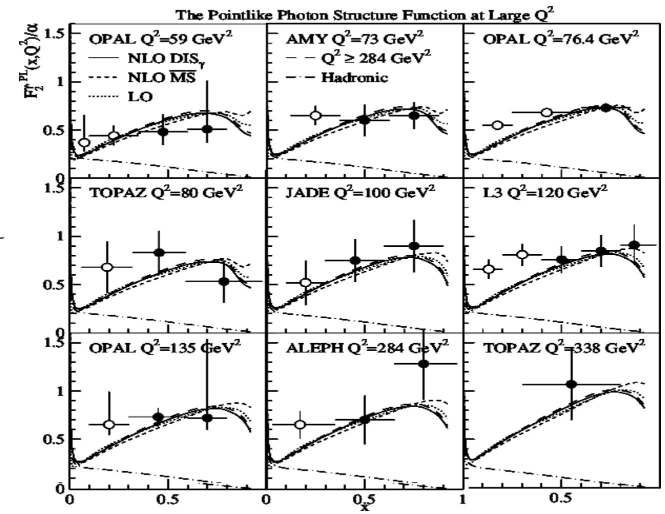
- Computed at NNLO: $$\int_0^1 dx F_2^\gamma(x,Q^2,P^2) = \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{\frac{4\pi}{\alpha_s(Q^2)}c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi}c_{NNLO} + \mathcal{O}(\alpha_s^2)\right\}$$
- Poor $F^2_\gamma(x,Q^2)$ experimental measurements:
- Extraction (NLO) with large exp. uncertainties today:

α_s (m$_z$) = 0.1198 ± 0.0054

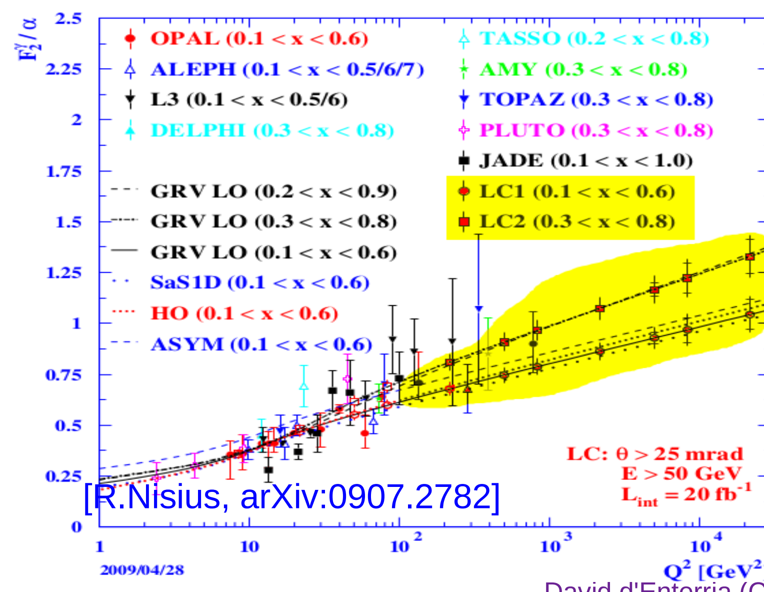
(±4.5%)

[M.Klasen et al. PRL89 (2002)122004]





- Future prospects:
  - Fit with NNLO F^2_γ evolution (ongoing)
  - Better data badly needed: Belle-II ?
  - Dedicated simul. studies at ILC exist:
  - Huge $\gamma\gamma$ (EPA) stats at FCC-ee will lead to: **$\delta\alpha_s/\alpha_s$ < 1%**

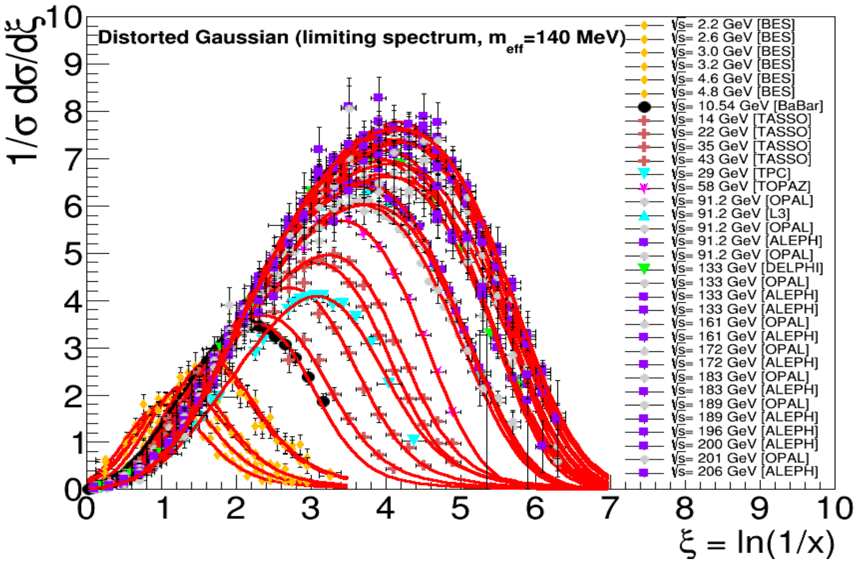


[R.Nisius, arXiv:0907.2782]

α_s extractions from jet fragmentation (NLO,NNLO*)

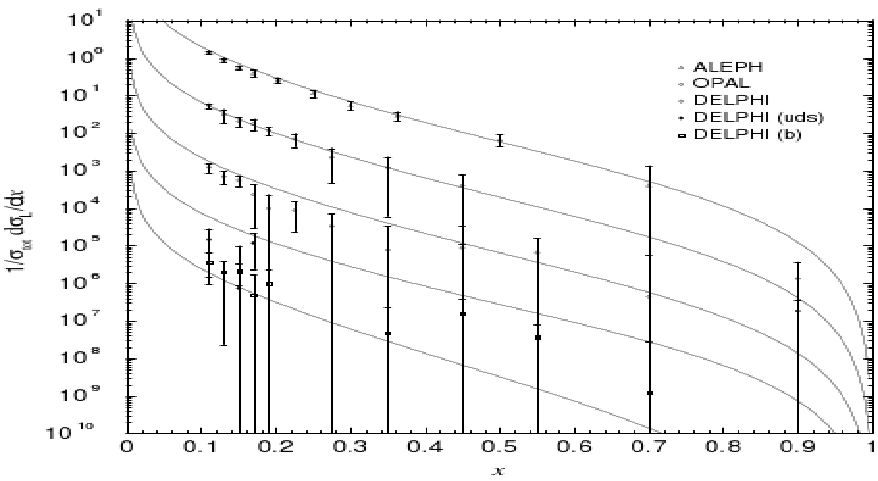
- Soft parton-to-hadron FFs (NNLO*+NNLL):

[D.d'E.,R.Perez-Ramos, arXiv:1505.02624]



- Hard parton-to-hadron FFs (NLO):

$\alpha_s(m_Z) = 0.1176 \pm 0.0055$ (±4.7%)

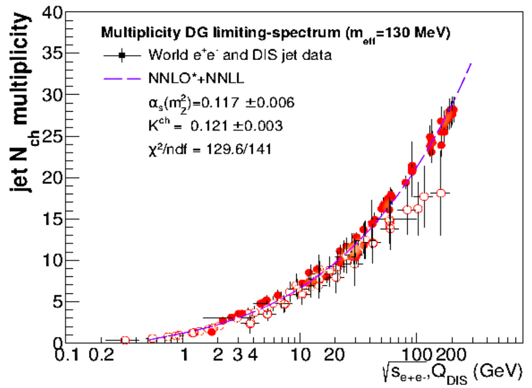


[AKK, B. Kniehl et al.,NPB 803(2008)42]

Combined fit of the jet-energy evolution of the FF moments (multiplicity, peak, width,…) with α_s as single free parameter:

$\alpha_s(m_Z) = 0.1205 \pm 0.0022$ (±2%)

(full-NNLO corrections missing)



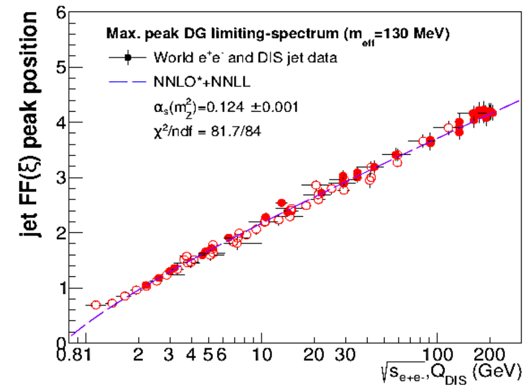


Figure 3: Energy evolution of the charged-hadron multiplicity (left) and of the FF peak position (right) measured in e^+e^- and DIS data fitted to the NNLO*+NNLL predictions. The obtained $\mathcal{K}_{ch}$ normalization constant, individual NNLO* $\alpha_s(m_Z)$ values, and the goodness-of-fit per degree-of-freedom χ^2/ndf.