Advances in analyses of three-body decays

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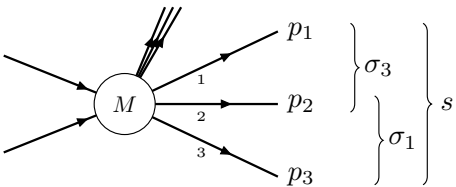
The plan for the talk

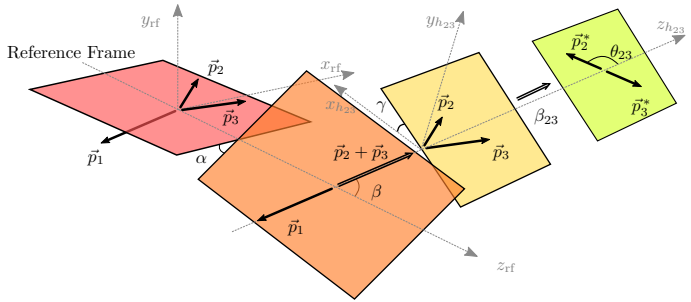
1. Search for CPV in $\Xi_b^- \to pK^-K^-$
2. Lineshape analysis for $\Lambda_b^{**0} \to \Lambda_b^0 \pi^+ \pi^-$

Three-body problem in HEP



$$M = M(\alpha, \beta, \gamma, s, m_{12}^2, m_{23}^2)$$

- s is a total invariant mass. **3b dynamics**
- $\sigma_3 \equiv m_{12}^2 \times \sigma_1 \equiv m_{23}$. **Dalitz plot**
- (α, β, γ) decay plane orientation. **Euler**

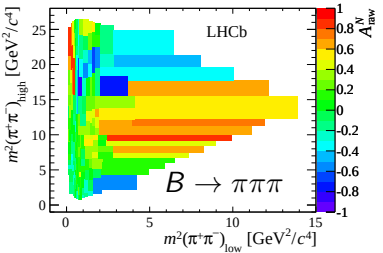


Amplitude analysis of $\Xi_b^- \to pK^-K^-$

[LHCb-PAPER-2020-017 (NEW! in preparation)]

CP violation in hadronic decays

CPV is well seen in $B \to hhh$:



[PRD 90, 112004 (2014)]

Appearance of CPV effects:

$$B \to f: \quad A = \sum_i |A_i| e^{i(\delta_i + \phi_i)}$$

$$\overline{B} \to \overline{f}: \quad \overline{A} = \sum_i |A_i| e^{i(\delta_i - \phi_i)},$$

- Strong phase (δ) does not change under CP,
- Weak phase (ϕ) flip the sign.

$$\mathcal{A}_{\rm CP} = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2}$$

$$\to \sum_i |A_i||A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

NOTHING in baryonic decays

In Λ_b^0 decay:

- pK^-, $p\pi^-$, $K^0_S p\pi^-$, $\Lambda K^- K^+$
- $p\pi^-\pi^-\pi^+$, $p\pi^- K^- K^+$, $pK^-\pi^+\pi^+$, $pK^- K^+ K^+$

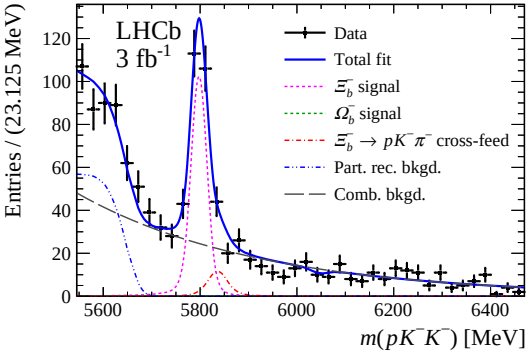
In Ξ_b^- decay:

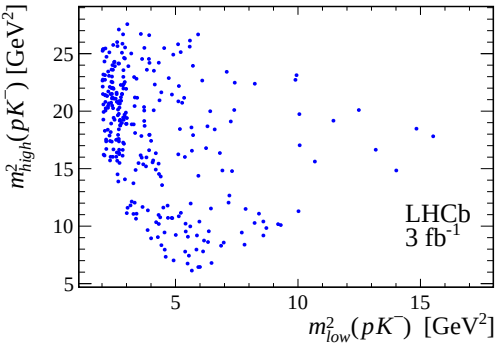
- $pK^-\pi^+\pi^+$, $pK^-\pi^+ K^+$

? $\boxed{pK^-K^-}$

Observation of $\Xi_b \to pK^-K^-$ decay

[LHCb-PAPER-2020-017]



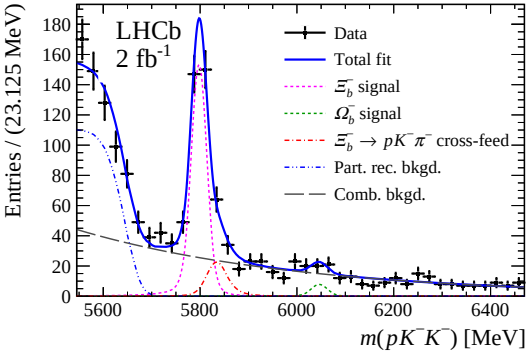


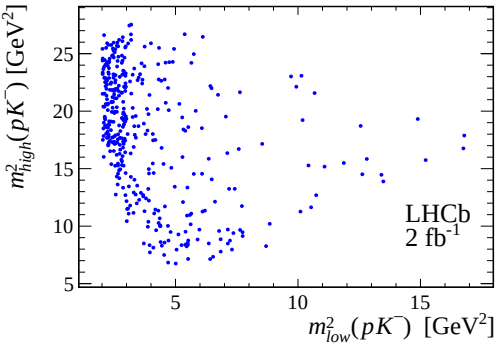
- Run I + a part of Run II $\Rightarrow 5\,\text{fb}^{-1}$.
- 460 signal candidates,
  - + combinatorial background,
  - + $\Xi_b^- \to pK^-K^-$ misidentification.

- K^-K^- symmetry makes Dalitz plot (double entry) symmetric
- Only half is analysed (single entry)

Observation of $\Xi_b \to pK^-K^-$ decay

[LHCb-PAPER-2020-017]





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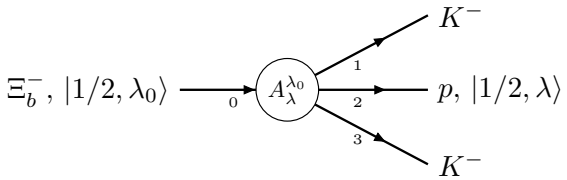
Ξ_b^- decay amplitude



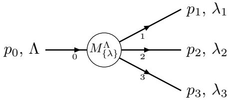
- Baryon decay (half-integer spin)
  $\Rightarrow 4\pi$-fold azimuthal symmetry of the decay amplitude
- Two identical kaons in the final state
  $\Rightarrow$ Symmetry of the decay amplitude.
- Intermediate Σ, Λ states decaying to pK^-

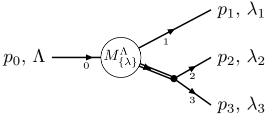
Conventional helicity approach

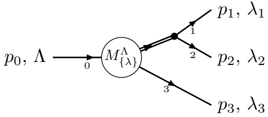
Complicated cases: particles with spin in isobar model [Hansen (1974)], [Herndon(1975)]

$$M^{\Lambda}_{\{\lambda\}} = M^{\Lambda}_{1,\{\lambda\}} + M^{\Lambda}_{2,\{\lambda\}} + M^{\Lambda}_{3,\{\lambda\}}$$

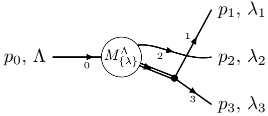
$$\underbrace{M^{\Lambda}_{\{\lambda\}}} \quad = \underbrace{H_1 D(\phi_1, \theta_1, 0)\, D(\phi_{23}, \theta_{23}, 0) W_1(\dots)} + \underbrace{H_3 D(\phi_3, \theta_3, 0)\, D(\phi_{12}, \theta_{12}, 0) W_3(\dots)}$$







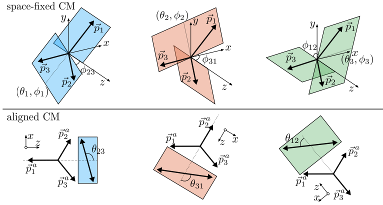
$$+ \underbrace{H_2 D(\phi_2, \theta_2, 0)\, D(\phi_{31}, \theta_{31}, 0) W_2(\dots)}$$



- A special set of angles for every decay chain
- Consistently of quantization direction – **Wigner rotations**

$$W_i(\dots) = D^{j_1}(\tilde{\phi}^i_1, \tilde{\theta}^i_1, 0)\, D^{j_2}(\tilde{\phi}^i_2, \tilde{\theta}^i_2, 0)\, D^{j_3}(\tilde{\phi}^i_3, \tilde{\theta}^i_3, 0)$$

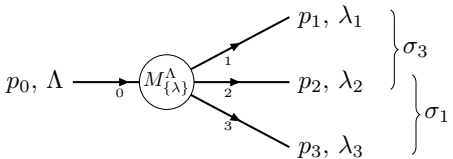
- Fails for non-integer spin



The Dalitz-Plot decomposition

[MM et al.(JPAC), PRD 101, 034033 (2020)]

Reformulation of the helicity approach



$$= \sum_{\nu} \underbrace{D^{J*}_{\Lambda\nu}(\phi_1, \theta_1, \phi_{23})}_{\text{Decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(\{\sigma\})}_{\text{Dalitz-plot function}}$$

Model-independent factorization of the overall rotation:

- Exploits properties of the Lorentz group (orientation – just three Euler angles)
- Dalitz-plot function depends entirely on 2 variables, $\{\sigma\} \equiv \{\sigma_1, \sigma_2, \sigma_3\}$
- No azimuthal phase factors in $O^{\nu}_{\{\lambda\}}$.

Gives significant benefits to

- Pentaquark analysis, Λ_b/Λ_c polarionation measurements, Baryonic decay chains,...

Dalitz-Plot function

[MM et al.(JPAC), arXiv:1910.04566]

Master formula $0 \to 1\,2\,3$ decay with arbitrary spins

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s \, d^{J}_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k) \, d^{s}_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij}) \times d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^1_{k(0)}) \, d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^2_{k(0)}) \, d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^3_{k(0)}),$$

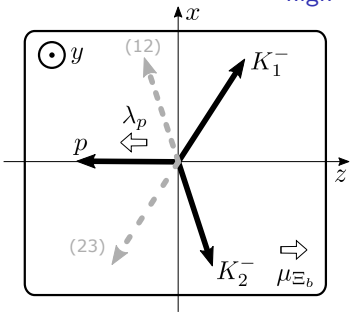
- Three decay chains, $(ij)k \in \{(12)3, (23)1, (31)2\}$.
- $\theta_{ij} = \theta_{ij}(\{\sigma\})$ is an isobar decay angle
- $\hat{\theta}_{k(1)} = \hat{\theta}_{k(1)}(\{\sigma\})$ is the particle-0 Wigner angle
- $\zeta^i_{k(0)} = \zeta^i_{k(0)}(\{\sigma\})$ is the particle-i Wigner angle

Applied in LHCb

- P_c in $\Lambda_b^0 \to J/\psi p K^-$
- P_{cs} in $\Xi_b^- \to J/\psi \Lambda^0 K^-$
- $\Lambda_c^+/\Xi_c^+ \to p K^- \pi^+$
- Some analyses in progress

Symmetrization: $\underbrace{(K^-p)}_{\text{high}}K^- + K^-\underbrace{(pK^-)}_{\text{low}}$

[LHCb-PAPER-2020-017]



$$A_{\lambda_0,\lambda} = \sum_\nu D^{1/2}_{\lambda_0,\nu}(\alpha,\beta,\gamma) O^\nu_\lambda(m^2_{\text{high}}, m^2_{\text{low}})$$

The helicity amplitude with explicit permutation symmetry:

$$O^\nu_\lambda = T^\nu_\lambda(m^2_{\text{high}}, m^2_{\text{low}}) \quad \boxed{+} \quad (-1)^{\nu+\lambda} T^\nu_\lambda(m^2_{\text{low}}, m^2_{\text{high}})$$

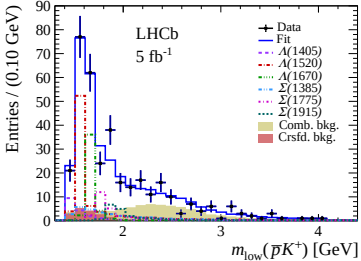
T^ν_λ is the resonance decay amplitude $\Xi_b^- \to R^0(\to K^-p)K^-$.

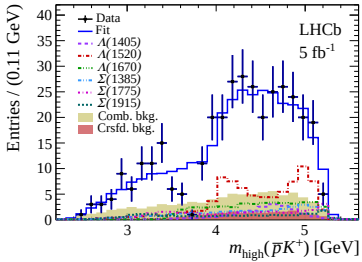
$(-1)^{\nu+\lambda}$: permutation of kaons $K_1^- \leftrightarrow K_3^-$ flips the x-z plane $\Rightarrow R_z(\pi)$

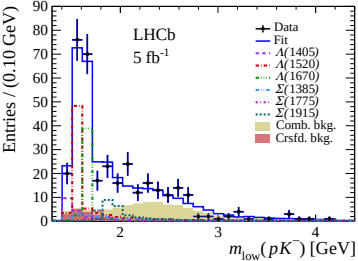
Clearly $\boxed{+}$ in covatiant: $A = \bar{u}(p,\lambda)\left[\frac{\not{p}_{R_1}+m_{\text{high}}}{2m^2_{\text{high}}} + \frac{\not{p}_{R_3}+m_{\text{low}}}{2m^2_{\text{low}}}\right] u(p_b,\nu) \Rightarrow$ check!

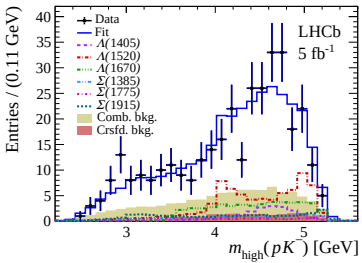
Fit to $\Xi_b^- \to pK^-K^-$. CP test!

[LHCb-PAPER-2020-017]







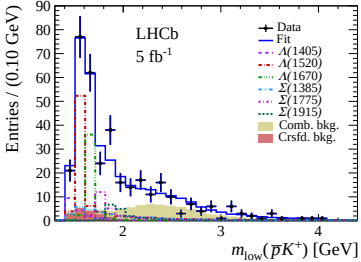


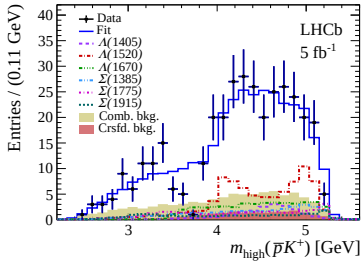
Fractions:

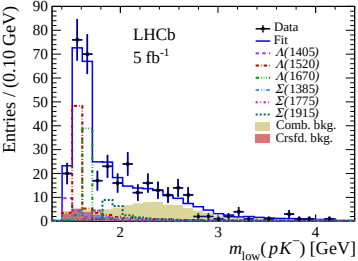
$\Sigma(1385) \to 0.26(11)(17)$
$\Lambda(1405) \to 0.19(6)(7)$
$\Lambda(1520) \to 0.76(9)(8)$
$\Lambda(1670) \to 0.45(7)(13)$
$\Sigma(1775) \to 0.22(8)(9)$
$\Sigma(1915) \to 0.26(9)(21)$
errors: (stat.)(syst.)

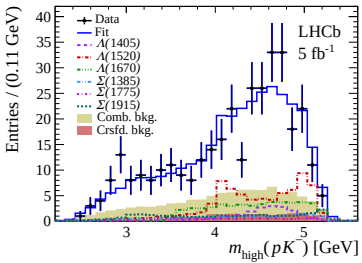
Fit to $\Xi_b^- \to pK^-K^-$. CP test!

[LHCb-PAPER-2020-017]









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errors: (stat.)(syst.)

Asymmetries $A_{\rm CP} \times 10^{-2}$:

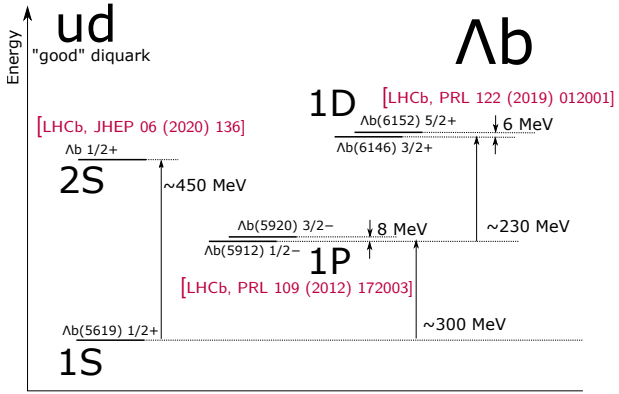
$\Sigma(1385) \to -27(34)(73)$
$\Lambda(1405) \to -1(24)(32)$
$\Lambda(1520) \to -5(9)(8)$
$\Lambda(1670) \to +3(14)(10)$
$\Sigma(1775) \to -47(26)(14)$
$\Sigma(1915) \to +11(26)(22)$
errors: (stat.)(syst.)

Three-body dynamics in decays of Λ_b^{0**}

[LHCb, JHEP 06 (2020) 136]

Excited Λ_b resonances

Spectroscopy of the good diquark

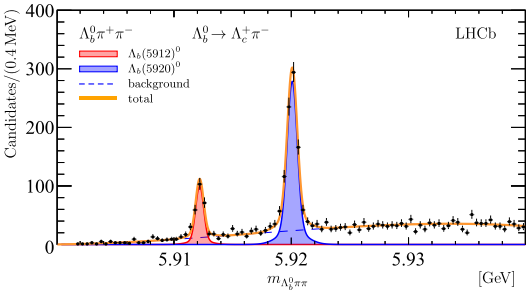


Diquark picture, $Q(qq)$ with (qq) having $J^P = 0^+$, works very well for observed-so-far states.

$\Lambda_b^0\pi^+\pi^-$ spectrum

[LHCb, JHEP 06 (2020) 136]

Low-mass part of the spectrum



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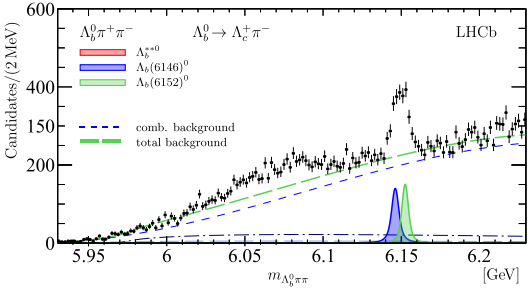


The broad structure:

- An elastic resonance in a system $\Lambda_b^0\pi^+\pi^-$
- Three coupled quasi-stable channels: $\Lambda_b^0 f_0$, $\Sigma_b^{\pm}\pi^{\mp}$, and $\Sigma_b^{*\pm}\pi^{\mp}$

$\Rightarrow$ an excellent opportunity to explore the three-body unitarity

Three-body unitarity constraint

Unitairity equation for the reduced amplitude

$$\mathcal{T}(\sigma',s,\sigma)-\mathcal{T}^\dagger(\sigma',s,\sigma)=$$

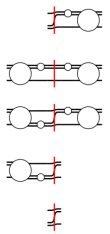
$$2i\,\frac{1}{\lambda_s^{1/2}(\sigma')}\int_{\sigma^-(\sigma',s)}^{\sigma^+(\sigma',s)}\mathrm{d}\sigma_3'\,t(\sigma_3')\mathcal{T}(\sigma_3',s,\sigma)$$

$$+\frac{i}{3}\int_4^{(\sqrt{s}-1)^2}\frac{\mathrm{d}\sigma''}{2\pi}\mathcal{T}^\dagger(\sigma',s,\sigma'')\,t(\sigma'')t^\dagger(\sigma'')\,\rho(\sigma'')\rho_s(\sigma'')\,\mathcal{T}(\sigma'',s,\sigma)$$

$$+2i\iint_{\phi(\sigma_2'',\sigma_3'',s)>0}\frac{\mathrm{d}\sigma_2''\mathrm{d}\sigma_3''}{2\pi s}\,\mathcal{T}\,(\sigma',s,\sigma_2'')t\,(\sigma_2'')t(\sigma_3'')\mathcal{T}(\sigma_3'',s,\sigma)$$

$$+\frac{2i}{3}\,\frac{1}{\lambda_s^{1/2}(\sigma)}\int_{\sigma^-(\sigma,s)}^{\sigma^+(\sigma,s)}\mathrm{d}\sigma_2\,\mathcal{T}^\dagger(\sigma',s,\sigma_2)t^\dagger(\sigma_2)$$

$$+6i\,\frac{2\pi s}{\lambda_s^{1/2}(\sigma')\lambda_s^{1/2}(\sigma)}\,\theta(\phi(\sigma',\sigma,s)).$$



Related works:

- [Mai et al., EPJA 53 (2017) 9, 177]
- [Jackura et al.(JPAC), EPJC 79 (2019) 1, 56]
- [MM et al.(JPAC), JHEP 08 (2019) 080]

Three-main aspects:

- Two-body rescattering effects
  influence of subchannel resonances to each-other lineshape
- Genuine three-to-three dinamics, generated states (the ladder)
- Lineshape of the regular states decaying to three particles

Unitairy resonance kernel

[MM et al.(JPAC), JHEP 08 (2019) 080]

side reminder

- A two-body resonance amplitude

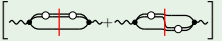
$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - im\Gamma(s)} = \frac{g^2}{m^2 - s - ig^2\Phi_2(s)/2}$$

$\Phi_2(s)$ is a two-body phase-space

A resonance in three-body system

An approximate-three-body unitarity

$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2 \left[\quad + \quad \right]}$$



contains effect of the subchannel-resonances **interference**

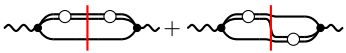
The quasi-two-body approximation

$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2 \left[\quad \right]}$$

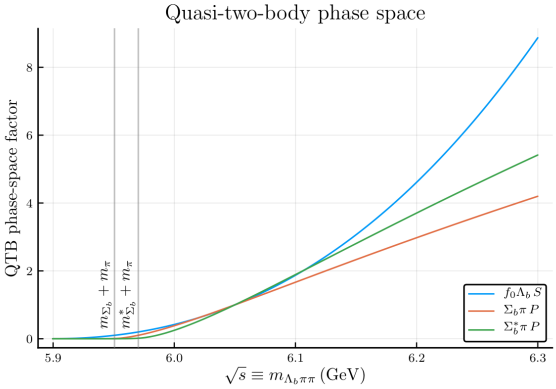
naively accounts for the subchannel-resonance decay

+ dispersion integral to ensure analytic structure (important close to threshold)

Appliction to $\Lambda_b^0\pi^+\pi^-$. Self-energy function

$\rho(s) =$  $+$

$$= \begin{pmatrix} g_{\Lambda_b^0 f_0} \\ g_{\Sigma_b^\pm \pi^\mp} \\ g_{\Sigma_b^{*\pm} \pi^\mp} \end{pmatrix}^\dagger \begin{pmatrix} \Lambda_b^0 f_0 | \Lambda_b^0 f_0 & \Lambda_b^0 f_0 | \Sigma_b^\pm \pi^\mp & \Lambda_b^0 f_0 | \Sigma_b^{*\pm} \pi^\mp \\ \Sigma_b^\pm \pi^\mp | \Lambda_b^0 f_0 & \Sigma_b^\pm \pi^\mp | \Sigma_b^\pm \pi^\mp & \Sigma_b^\pm \pi^\mp | \Sigma_b^{*\pm} \pi^\mp \\ \Sigma_b^{*\pm} \pi^\mp | \Lambda_b^0 f_0 & \Sigma_b^{*\pm} \pi^\mp | \Sigma_b^\pm \pi^\mp & \Sigma_b^{*\pm} \pi^\mp | \Sigma_b^{*\pm} \pi^\mp \end{pmatrix} \begin{pmatrix} g_{\Lambda_b^0 f_0} \\ g_{\Sigma_b^\pm \pi^\mp} \\ g_{\Sigma_b^{*\pm} \pi^\mp} \end{pmatrix}$$



$$\sim \sqrt{\quad} \begin{pmatrix} 1 & z & z_* \\ \cdot & 1+y & x \\ \cdot & \cdot & 1+y_* \end{pmatrix} \sqrt{\quad}$$

with $x(m^2) = -0.014 + 0.004i$
$z(m^2) = -0.05 - 0.12i$,
$z_*(m^2) = -0.01 - 0.07i$,
$y(m^2) = 0.003$, and
$y_*(m^2) = -0.02$

$\Rightarrow$ **rescattering effects are negligible**

Fit to the data

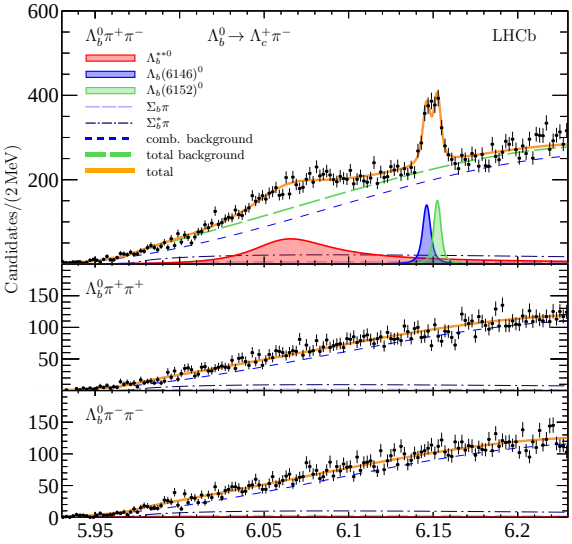
[LHCb, JHEP 06 (2020) 136]

Parameters of the new $\Lambda_b^0(6072)$:

$$m = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2\,\mathrm{MeV}$$
$$\Gamma = 72 \pm 11 \pm 2\,\mathrm{MeV}$$

Advance parametrization impacts the width measurement with respect to the naive BW

Framework works $\Rightarrow$ move to complex examples $X \to D\bar{D}\pi(\gamma)$



Summary

Amplitude analysis is a critical tool for the further progress

Three parts of the three-body problem:

- 3b dynamics – **3b unitarity**
- Angular analysis of the decay – **Dalitz-Plot decomposition**
- Two-body dynamics on the Dalitz – **rescattering** corrections (not covered in this talk)

Wide range of applications of the advanced amplitudes in LHCb. Just two examples:

- Search for CP violation in baryonic decays
- Lineshape analysis for Λ_b^0 spectroscopy