Chromoelectric Distribution Function of Nuclear Matter Probed by Quarkonium

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arXiv: 2009.02408
arXiv: 2102.01736

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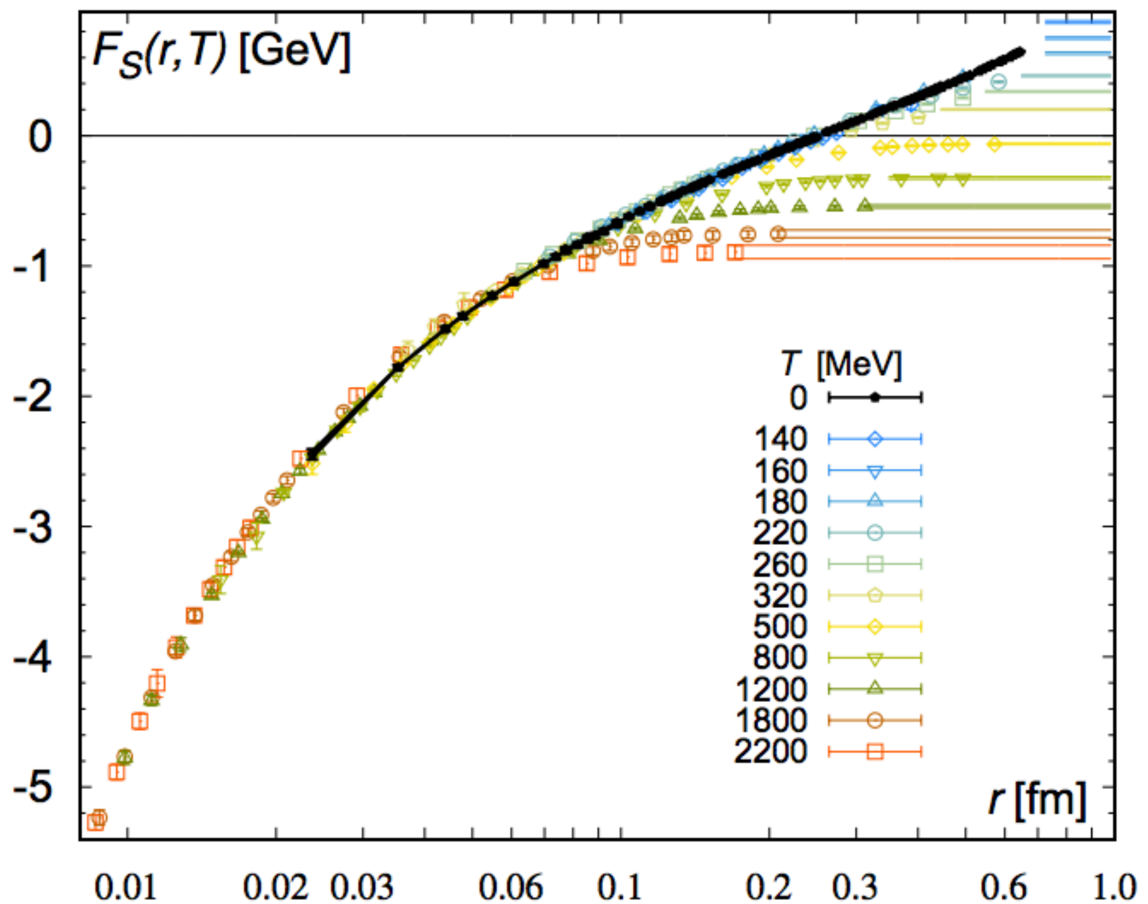
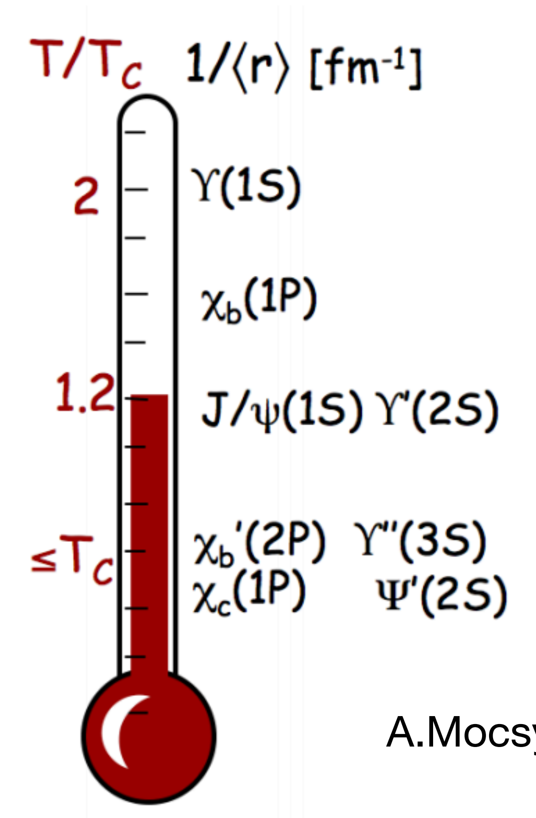
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



1804.10600, A.Bazavov, N.Brambilla
P.Petreczky, A.Vairo, J.H.Weber



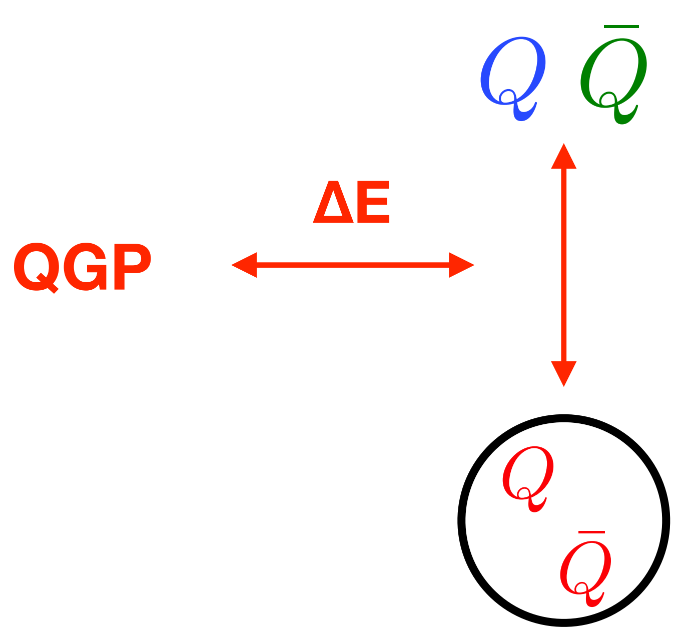
A.Mocsy, 0811.0337

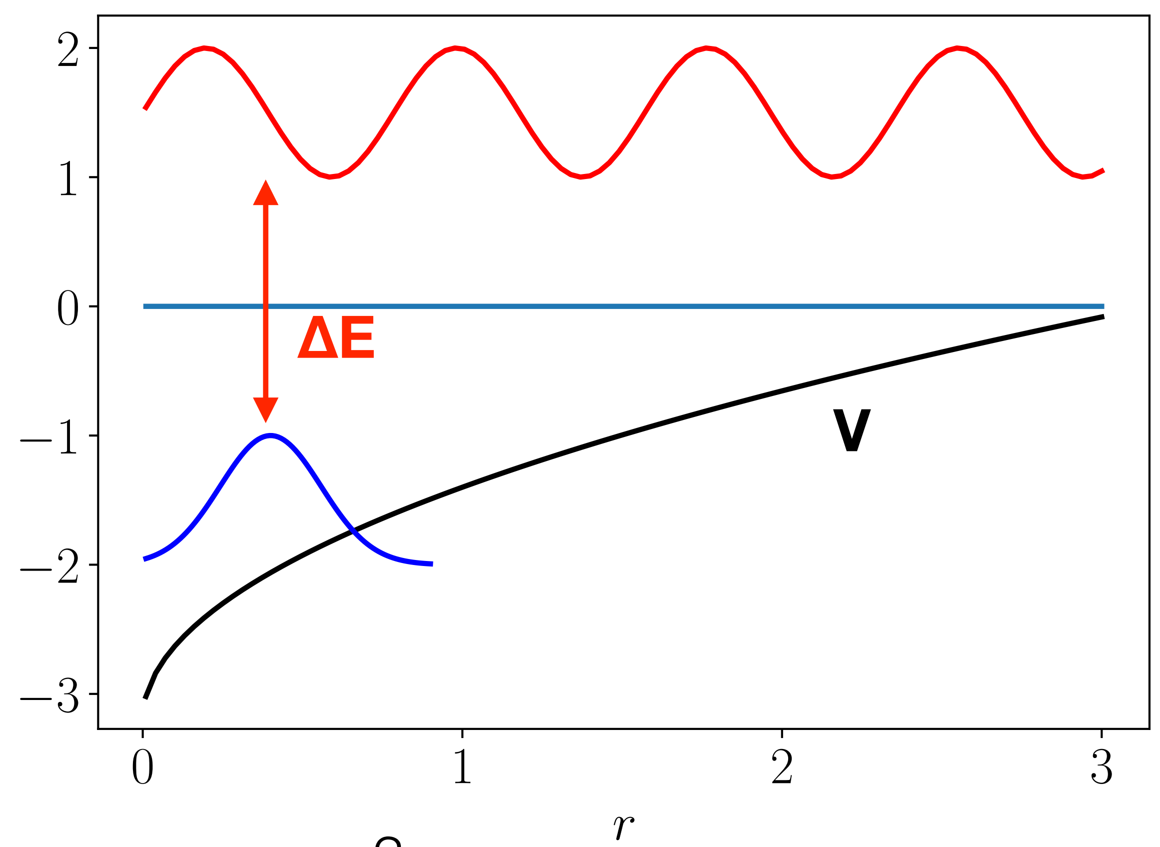
Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: related to imaginary potential, **dissociation** induced by dynamical process, lead to suppression even when T(QGP) < melting T

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for phenomenology and theory consistency





Simple physics picture of thermometer does not work

What QGP properties are we probing by measuring quarkonium?

This talk:

In certain limit, we are probing chromoelectric distribution functions of QGP/nuclear medium

Leading-power, all-order construction, gauge invariant

Two tools: open quantum systems, effective field theory

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$\rho(t=0) = \rho_S \otimes \rho_E$

Subsystem & environment

(Heavy quark pairs & QGP)

Unitary evolution →

Time reversible

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

Subsystem & environment

↓ **Trace out (integrate out) environment** ↓

$\rho_S(t=0)$

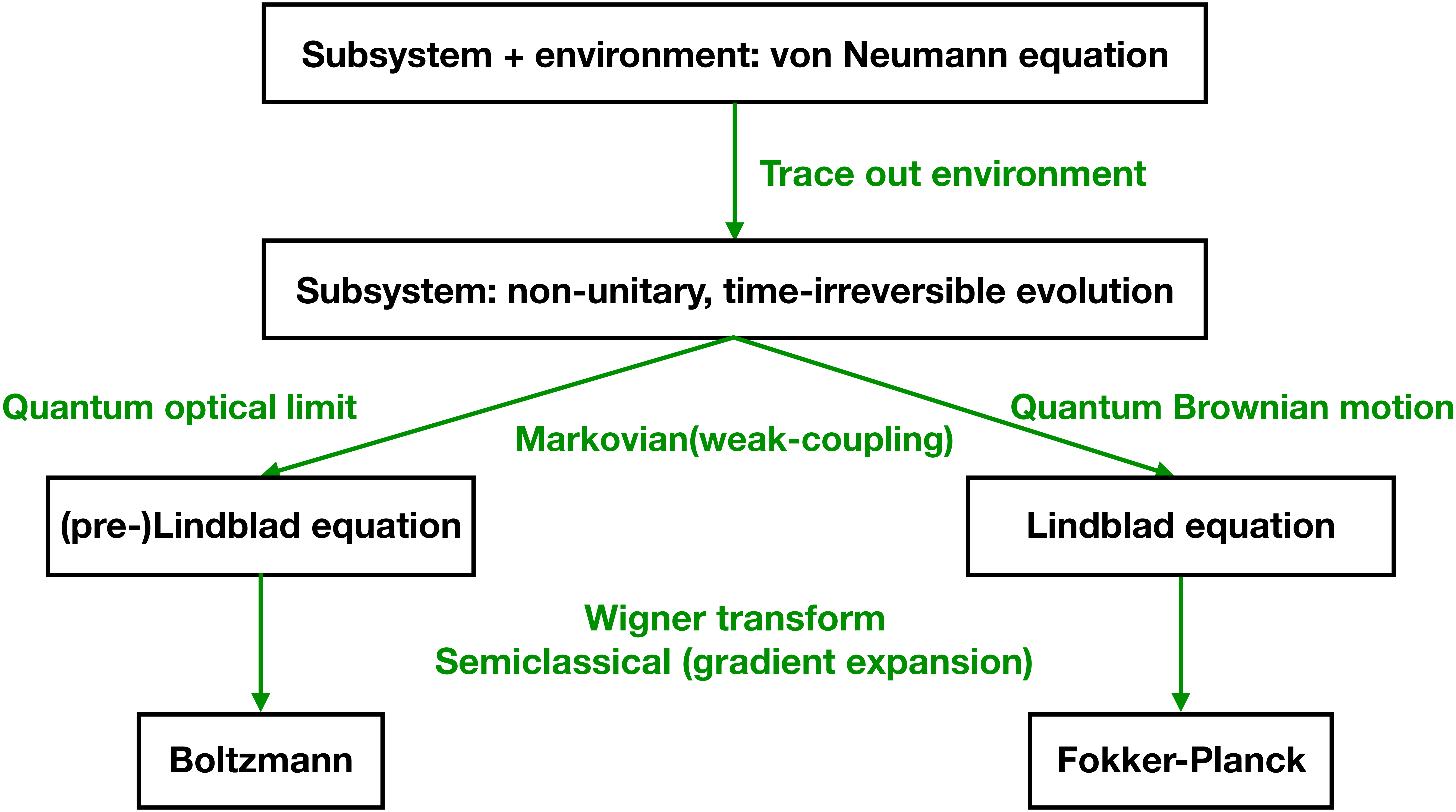
Subsystem

Non-unitary →

Time irreversible

$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

From Open Quantum System to Semiclassical Transport



Wigner transform $$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1 \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1 \right\rangle$$

Separation of Scales and pNRQCD

Separation of scales

$$M \gg Mv \gg Mv^2,\ T,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

QCD

Hard M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise
hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Soft Mv — perturbative / non-perturbative matching

potential NRQCD

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Ultrasoft $Mv^2 \sim 500\ \mathrm{MeV}$, T

Hard $p_h \sim M(1,1,1,1)$

Soft $p_s \sim Mv(1,1,1,1)$

Ultrasoft $p_{us} \sim Mv^2(1,1,1,1)$

Case 1: $Mv \gg T \gg Mv^2$

Lindblad equation in limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S,\, \rho_S(t)\right] + \frac{D(\omega=0,\boldsymbol{R}=0)}{N_c^2-1}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\left\{L_{\alpha i}^\dagger L_{\alpha i},\, \rho_S(t)\right\}\right)$$

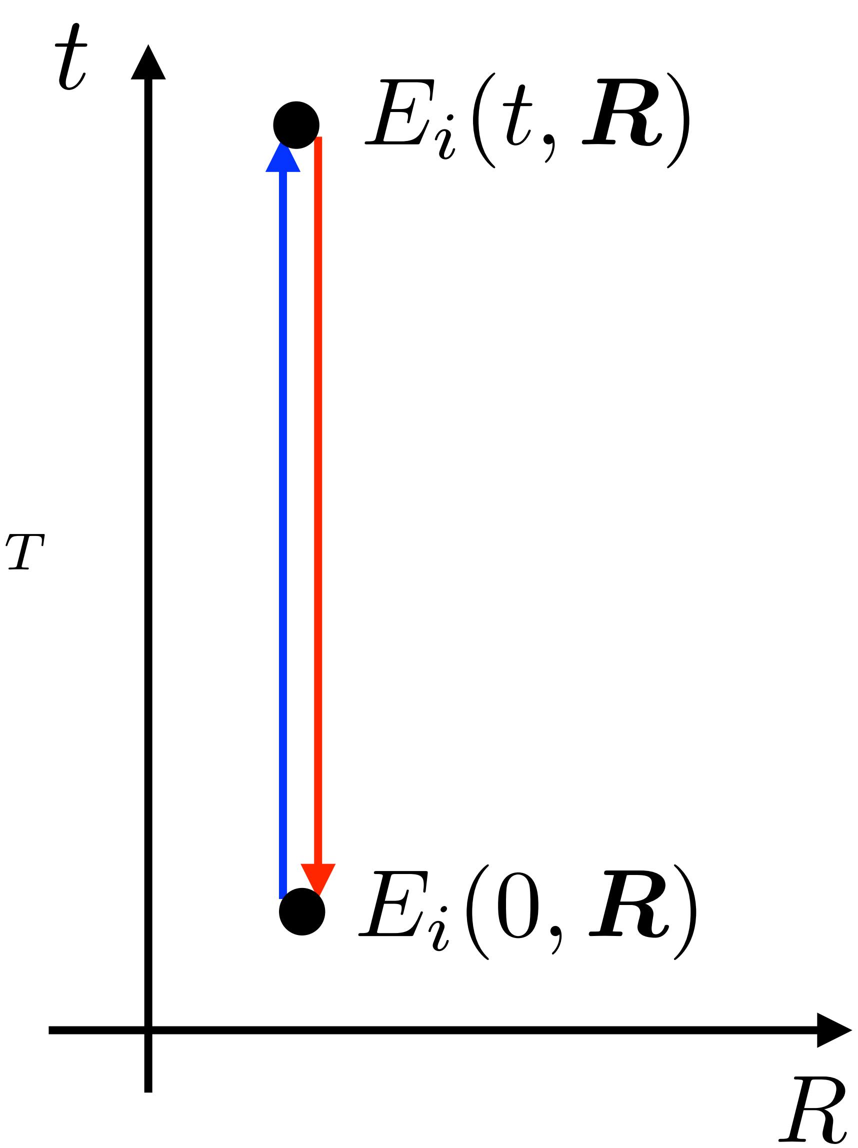
$$\Delta H_S = \frac{\Sigma(\omega=0,\boldsymbol{R}=0)}{2(N_c^2-1)} r^2 \begin{pmatrix} C_F & 0 \\ 0 & \frac{N_c^2-2}{4N_c} \end{pmatrix}$$

N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber arXiv:2012.01240

Evolution determined by transport coefficients

$$D(\omega=0,\boldsymbol{R}=0) = \int dt\, \langle E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$

$$\Sigma(\omega=0,\boldsymbol{R}=0) = \mathrm{Im}\int dt\, \langle \mathcal{T} E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$



D is just the heavy quark diffusion coefficient

Why HQ diffusion coefficient affects quarkonium?

$T \gg Mv^2$ **binding energy effect is subleading**

Case 2: $Mv \gg Mv^2 \gtrsim T$

Quantum optical and semiclassical limits: Boltzmann equation

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

Dissociation term

$$\mathcal{C}^{-}_{nl} = \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{\mathrm{cm}}}{(2\pi)^3} \frac{d^3 p_{\mathrm{rel}}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\mathrm{cm}} + \boldsymbol{q}) \delta(E_{nl} - E_p + q^0)$$
$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\mathrm{rel}}} \rangle \langle \Psi_{\boldsymbol{p}_{\mathrm{rel}}} | r_{i_2} | \psi_{nl} \rangle D_{i_1 i_2}(q^0, \boldsymbol{q}) f_{nl}(\boldsymbol{x}, \boldsymbol{k})$$

Chromoelectric structure function of QGP

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = \int dt\, d^3R\, e^{iq^0(t_1 - t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1 - \boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1) \mathcal{W} E_{i_2}(t_2, \boldsymbol{R}_2) \rangle_T$$

More general than the previous case:

Binding energy effect matters here: different quarkonium states respond differently

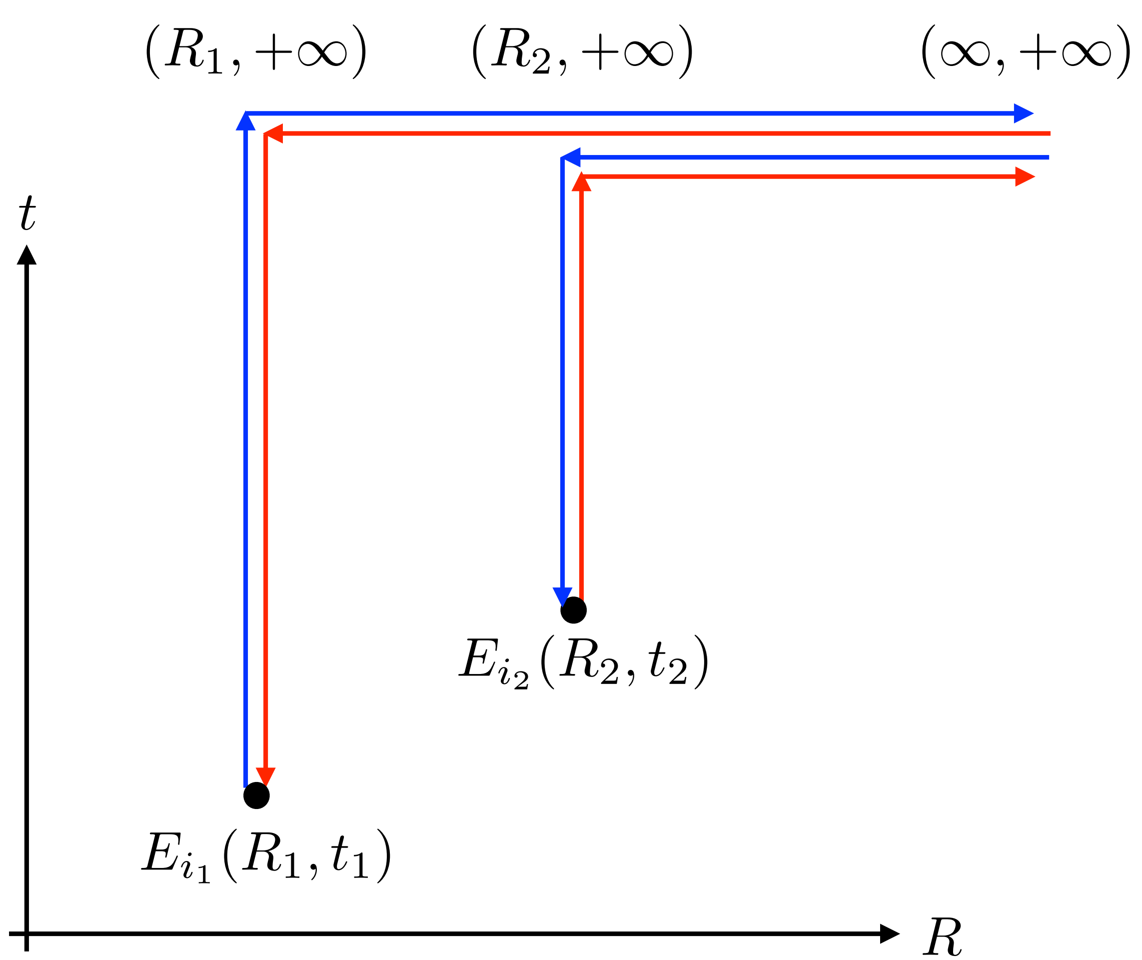
Finite momentum transfer, momentum dependence

Chromoelectric Distribution Function of QGP

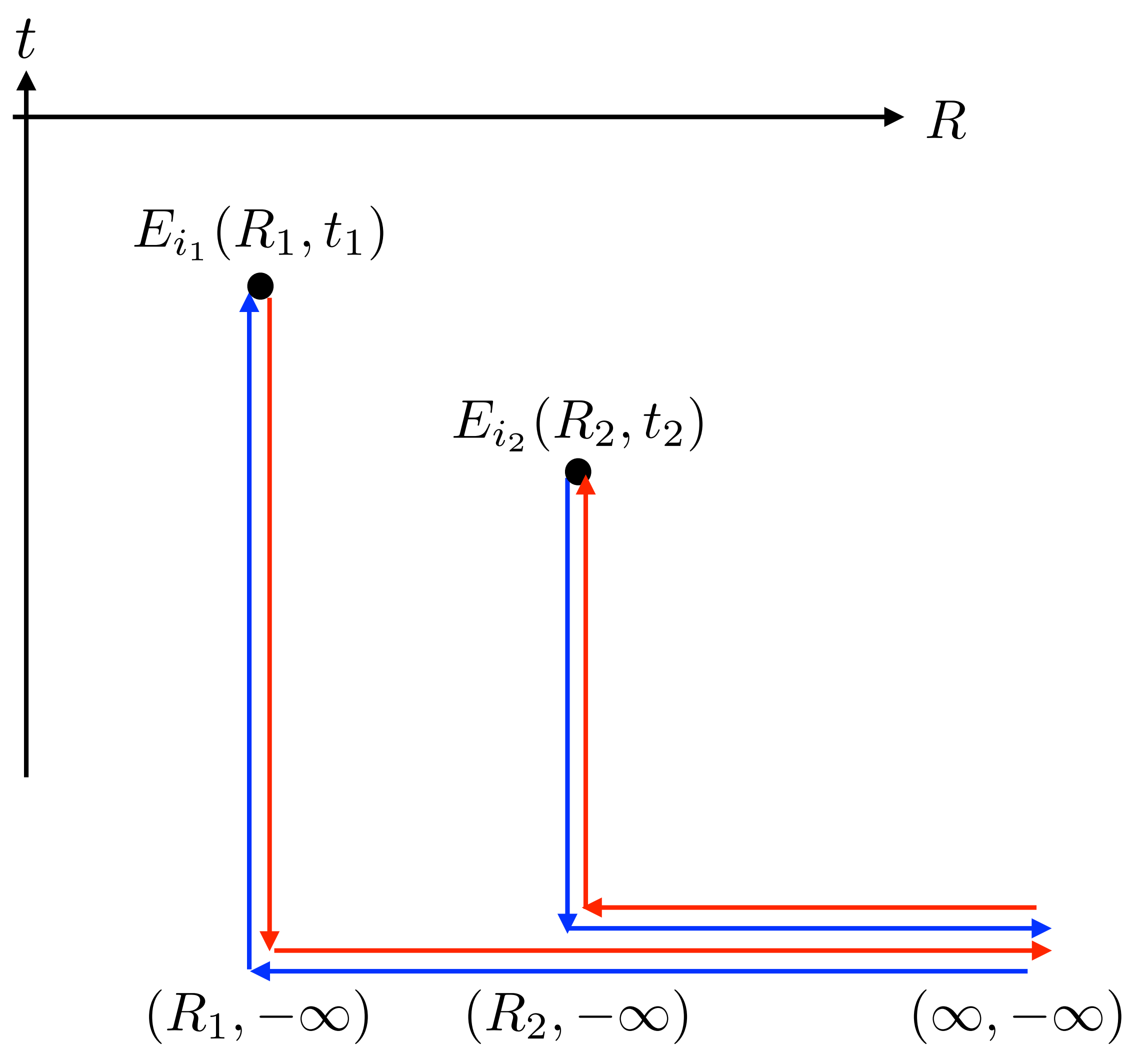
Staple shaped Wilson lines

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = \int dt\, d^3R\, e^{iq^0(t_1-t_2)-i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1)\mathcal{W}E_{i_2}(t_2, \boldsymbol{R}_2)\rangle_T$$

For dissociation: final-state interaction



For recombination: initial-state interaction



Inclusive v.s. Differential Reaction Rates

Take dissociation rate as example

$$R^-_{nl} = \sum_{i_1,i_2} \int \frac{d^3p_{\rm cm}}{(2\pi)^3} \frac{d^3p_{\rm rel}}{(2\pi)^3} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(E_{nl} - E_p + q^0) d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\rm rel}) D_{i_1 i_2}(q^0, \boldsymbol{q})$$

$$d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\rm rel}) = \frac{T_F}{N_c} \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \langle \Psi_{\boldsymbol{p}_{\rm rel}} | r_{i_2} | \psi_{nl} \rangle$$

Inclusive rate

$$R^-_{nl} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3} \bar{d}^{nl}(\boldsymbol{p}_{\rm rel}) D\Big(\frac{p^2_{\rm rel}}{M} - E_{nl}, \boldsymbol{R} = 0\Big)$$

$$D(q^0, \boldsymbol{R} = 0) = \int dt\, e^{iq^0 t} \langle E_i(t, \boldsymbol{R}) \mathcal{W}_{[t,0]} E_i(0, \boldsymbol{R}) \rangle_T$$

Momentum independent distribution

Zero frequency limit = HQ diffusion coefficient, appear in quantum Brownian motion

Differential rate

$$(2\pi)^3 \frac{dR^-_{nl}}{d^3p_{\rm cm}} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3} \bar{d}^{nl}(\boldsymbol{p}_{\rm rel}) D\Big(\frac{p^2_{\rm rel}}{M} - E_{nl}, \boldsymbol{p}_{\rm cm} - \boldsymbol{k}\Big)$$

Momentum dependent distribution

Similar to PDF v.s. TMDPDF, though different in time axis

Summary

- What are we probing by measuring quarkonium?
- Open quantum + EFT: leading-power, all-order construction
- Reaction rates depend on chromoelectric distribution function in hierarchy $Mv \gg Mv^2 \gtrsim T$ which all quarkonia go through
  - Inclusive rates depend on momentum independent one, straight-line Wilson line structure, affect inclusive RAA
  - Differential rates depend on momentum dependent one, staple-shape Wilson line structure, affect pT differential RAA, v2
- Easily generalized to cold nuclear matter by replacing environment density matrix

Backup: Case 1: $Mv \gg T \gg Mv^2$

Lindblad equation in the limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S,\, \rho_S(t)\right] + \frac{D(\omega=0,\boldsymbol{R}=0)}{N_c^2-1}\Big(L_{\alpha i}\rho_S(t)L^\dagger_{\alpha i} - \frac{1}{2}\big\{L^\dagger_{\alpha i}L_{\alpha i},\, \rho_S(t)\big\}\Big)$$

$$\Delta H_S = \frac{\Sigma(\omega=0,\boldsymbol{R}=0)}{2(N_c^2-1)} r^2 \begin{pmatrix} C_F & 0 \\ 0 & \frac{N_c^2-2}{4N_c} \end{pmatrix}$$

$$L_{1i} = \sqrt{C_F}\Big(r_i + \frac{1}{2MT}\nabla_i - \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\Big)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L_{2i} = \sqrt{\frac{T_F}{N_c}}\Big(r_i + \frac{1}{2MT}\nabla_i + \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\Big)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$L_{3i} = \sqrt{\frac{N_c^2-4}{4N_c}}\Big(r_i + \frac{1}{2MT}\nabla_i\Big)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Backup: Leading Power

- **Nonrelativistic & multipole expansions: v & r**

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, \mathrm{Tr}\Big(\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger (iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots \Big)$$

Dipole interaction

- **Boltzmann equation at leading-power in v & r, leading-order in g**

Dissociation and recombination rates depend on QGP via

XY, T.Mehen 1811.07027

$$\mathrm{Tr}_E\big(\rho_E E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\big) = \langle E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\rangle_T$$

Not gauge invariant !

Backup: All-Order Construction: Sum A0 Interactions

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\,{\rm Tr}\Big({\rm S}^\dagger(i\partial_0 - H_s){\rm S} + \boxed{{\rm O}^\dagger(iD_0 - H_o){\rm O}} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E},{\rm O}\} + \cdots\Big)$$

Octet—A0 interaction not suppressed by v or r

Need sum A0 to all orders at leading power

Field redefinition:

$$\begin{aligned} {\rm O}(\boldsymbol{R},\boldsymbol{r},t) &= \mathcal{W}_{[(\boldsymbol{R},t),(\boldsymbol{R},t_0)]}\widetilde{\rm O}(\boldsymbol{R},\boldsymbol{r},t) \\ \widetilde{E}_i(\boldsymbol{R},t) &= \mathcal{W}_{[(\boldsymbol{R},t_0),(\boldsymbol{R},t)]}E_i(\boldsymbol{R},t) \\ \mathcal{W}_{[(\boldsymbol{R},t_f),(\boldsymbol{R},t_i)]} &= \mathcal{P}\exp\left(ig\int_{t_i}^{t_f} ds\mathcal{A}_0(\boldsymbol{R},s)\right) \end{aligned}$$

New form of dipole interaction:

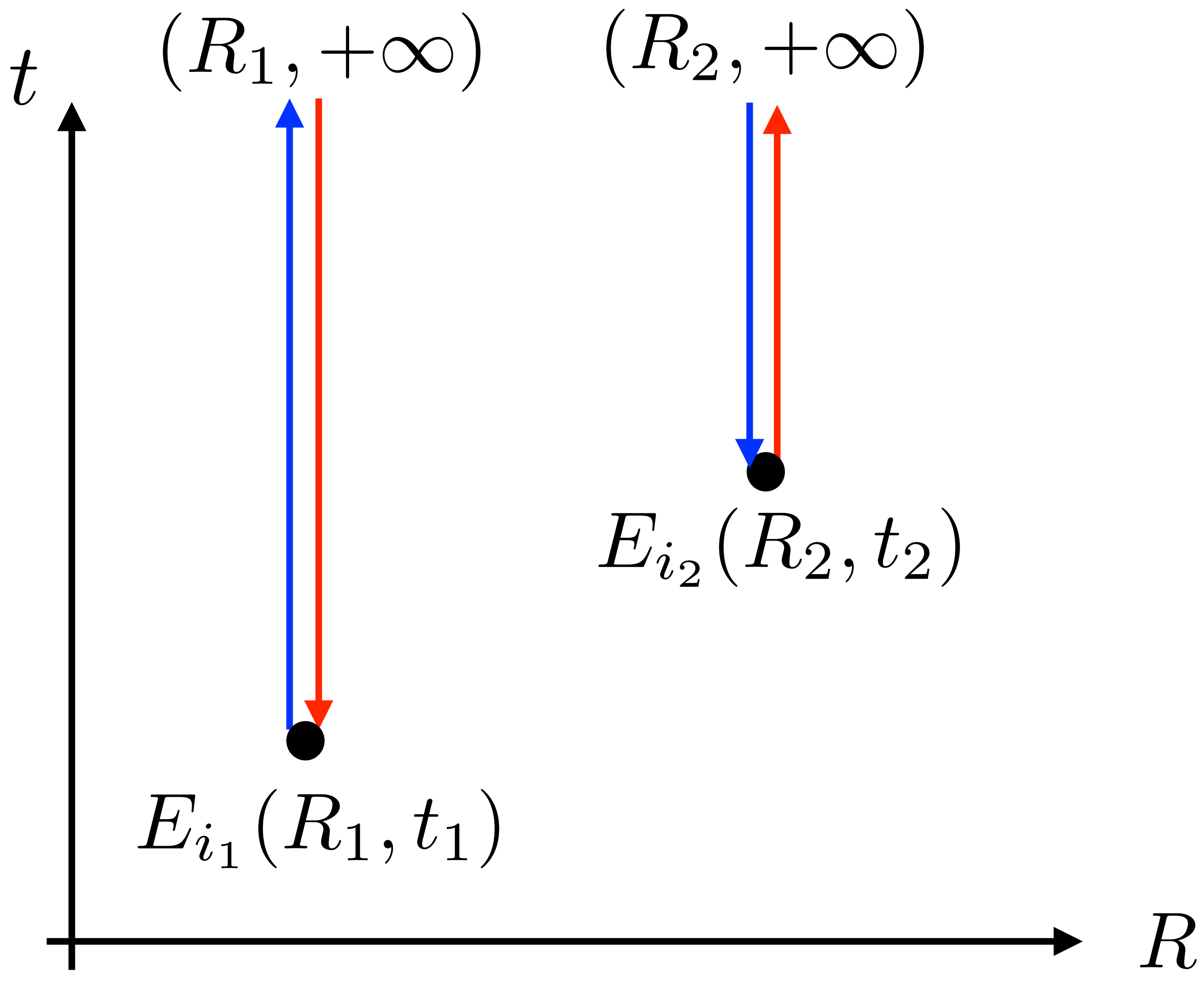
$$g\int d^3r{\rm Tr}\Big(\widetilde{\rm O}^\dagger r_i\widetilde{E}_i{\rm S} + {\rm S}^\dagger r_i\widetilde{E}_i^\dagger\widetilde{\rm O}\Big)$$

Backup: Chromoelectric Distribution Function of QGP

$$g_{i_1 i_2}^{E++}(t_1, t_2, \boldsymbol{R}_1, \boldsymbol{R}_2) = \left\langle E_{i_1}(\boldsymbol{R}_1, t_1) \mathcal{W}_{[(\boldsymbol{R}_1, t_1),(\boldsymbol{R}_1, +\infty)]} \mathcal{W}_{[(\boldsymbol{R}_2, +\infty),(\boldsymbol{R}_2, t_2)]} E_{i_2}(\boldsymbol{R}_2, t_2) \right\rangle_T$$

Wilsons not connected at infinite time!

For gauge invariance, need spatial gauge link



Backup: Wilson Lines at Infinite Time

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r\,\text{Tr}\Big(\text{S}^\dagger(i\partial_0 - H_s)\text{S} + \text{O}^\dagger(iD_0 - H_o)\text{O} + V_A(\text{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\text{S} + \text{h.c.}) + \frac{V_B}{2}\text{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \text{O}\} + \cdots\Big)$$

Coulomb interaction between octet heavy quark pair included in potential

But Coulomb between octet center-of-mass motion and medium not considered

For Coulomb modes $p_c^\mu \sim A_c^\mu \sim M(v^2, v, v, v)$

$$\int \mathrm{d}^3r\,\text{Tr}\Big(\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(iD_0 + \boxed{\frac{\boldsymbol{D}_{\boldsymbol{R}}^2}{4M}} + \frac{\nabla_{\boldsymbol{r}}^2}{M} - V_o(\boldsymbol{r}) + \cdots\Big)\text{O}(\boldsymbol{R},\boldsymbol{r},t)\Big)$$

C.m. kinetic term same order as D0, so leading power in v

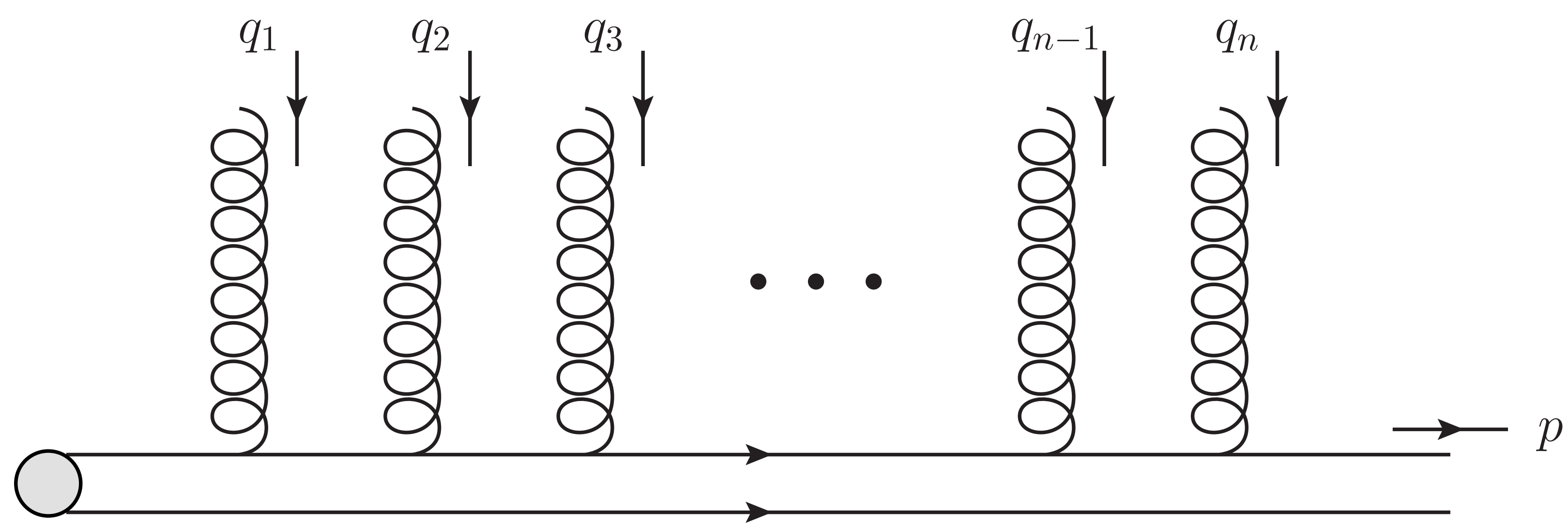
Write out c.m. kinetic term

$$\int \mathrm{d}^3r\,\text{Tr}\Big(\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\frac{\nabla_{\boldsymbol{R}}^2}{4M}\text{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{ig}{4M}\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(\boldsymbol{A}(\boldsymbol{R},t)\cdot\nabla_{\boldsymbol{R}}$$

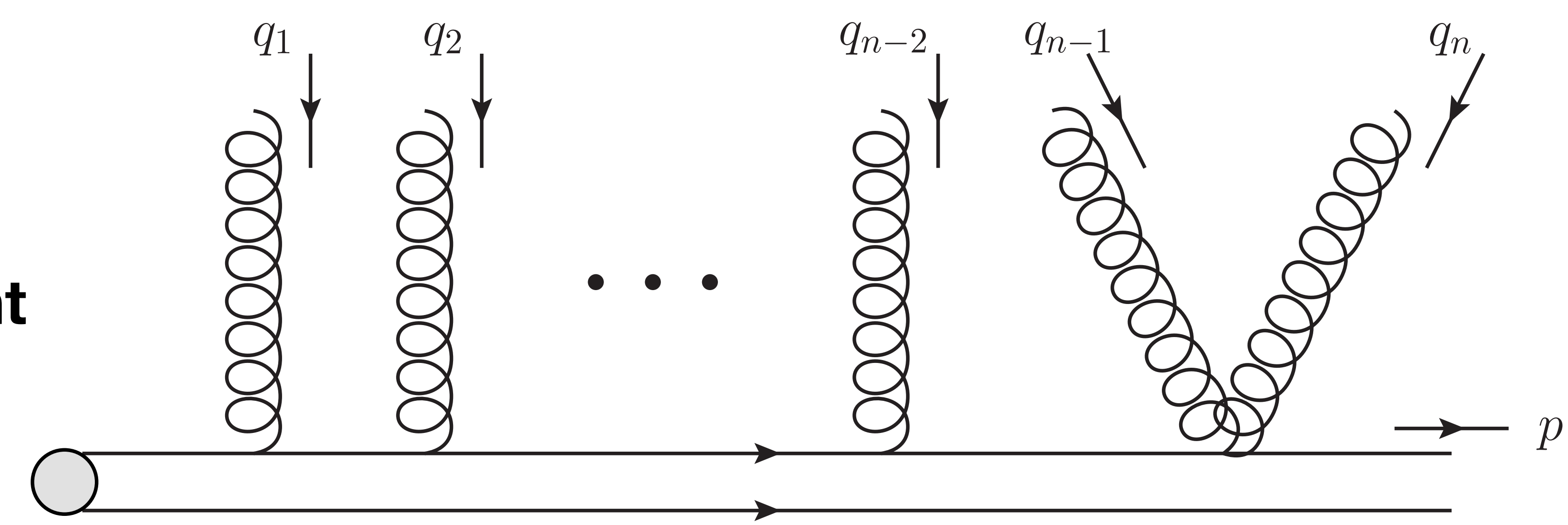
$$+ \nabla_{\boldsymbol{R}}\cdot\boldsymbol{A}(\boldsymbol{R},t)\Big)\text{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{g^2}{4M}\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{A}^2(\boldsymbol{R},t)\text{O}(\boldsymbol{R},\boldsymbol{r},t)\Big)\,.$$

Backup: Wilson Lines at Infinite Time: Resum Coulomb

Single Coulomb attachment



Double Coulomb attachment



Calculations have some similarity with A.V. Belitsky, X. Ji, F. Yuan hep-ph/0208038