Bottomonium suppression in an open quantum system using the quantum trajectories method

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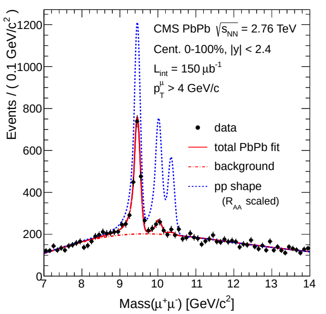
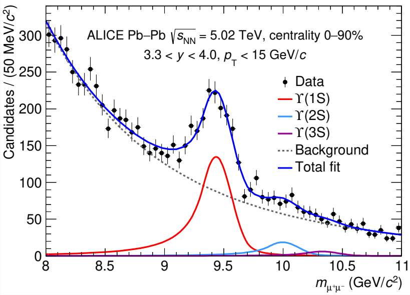
arXiv:2012.01240 and forthcoming

Outline

Bottomonium suppression



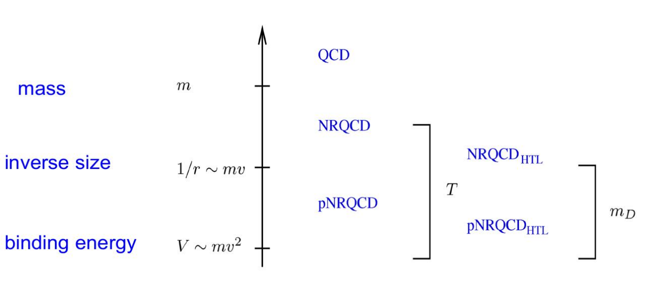
source: CMS (2014)



source: ALICE (2021.05758)

- Evidence of bottomonium suppression in multiple experiments
- Stronger suppression of higher excited states
- Various models are mostly consistent with experiment
- **Systematic, first-principles calculation** is very desirable

Scales governing in-medium quarkonium



- Separation of nonrelativistic scales permits EFT (NRQCD, pNRQCD)
- Intertwined thermal and nonrelativistic scales permit various hierarchies, with or without different types of thermal modification of quarkonium
- Thermal scales at $T \sim$ a few T_c: $\pi T \sim m_D$, strongly-coupled medium!

Quarkonium in effective field theory

- Integrating out scale $M \gg 1/r$ $\rightarrow$ nonrelativistic QCD (NRQCD)
- Integrating out scale $1/r \gg \alpha_s/r$ (multipole expansion!) $\rightarrow$ pNRQCD
- Degrees of freedom are heavy color-singlet and -octet fields + light d.o.f
- Potential nonrelativistic QCD (pNRQCD) Lagrangian[1] at NLO

$$\mathcal{L}_{\mathrm{pNRQCD}} = \mathcal{L}_{\mathrm{light}} + \mathcal{L}_{\mathrm{glue}} + \int d^3r \;\mathrm{tr}\; \left\{ S^\dagger [i\partial_0 - h_s] S + O^\dagger [iD_0 - h_o] O \right\}$$

$$+ V_A \mathrm{tr}\left\{ O^\dagger \boldsymbol{r} \cdot g\boldsymbol{E} S + S^\dagger \boldsymbol{r} \cdot g\boldsymbol{E} O \right\} + \frac{V_B}{2} \mathrm{tr}\left\{ O^\dagger \boldsymbol{r} \cdot g\boldsymbol{E} O + O^\dagger O \boldsymbol{r} \cdot g\boldsymbol{E} \right\} + \dots$$

- Nonlocal Wilson coefficients play the role of potentials

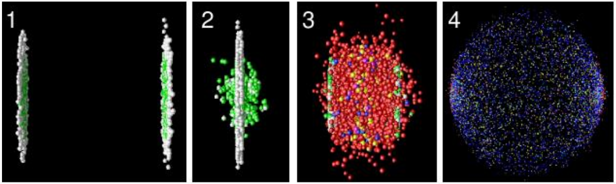
$$h_s = \frac{-\Delta}{2M} + V_s(r), \quad h_o = \frac{-\boldsymbol{D}^2}{2M} + V_o(r), \quad V_A, V_B = 1 + \mathrm{h.o.}$$

- Octet contribution in vacuum $\rightarrow$ heavy $q\bar{q}$ bound into different hadrons
- V_A resp. V_B permit transitions of types singlet↔octet or octet↔octet

[1] Brambilla et al., Nucl. Phys. B566 (2000) 275

Bottomonium as a probe of the medium

VNI Simulations: Geiger, Longacre, Srivastava, nucl-th/9806102



- Few $b\bar{b}$ pairs formed in hard collisions in early stages of HIC
- $b\bar{b}$ pairs propagate through and probe all stages of hot medium
- Eventually hadronize at their respective freezeout temperatures

$$\dot{\rho}_{\text{total}} = -\mathrm{i}\left[H_{\text{total}}, \rho_{\text{total}}\right], \quad \rho_{\text{total}} = \rho_{\text{probe}} \oplus \rho_{\text{medium}}$$

$$H_{\text{total}} = H_{\text{probe}} \otimes \mathrm{Id}_{\text{medium}} + \mathrm{Id}_{\text{probe}} \otimes H_{\text{medium}} + H_{\text{int}}$$

Trace out the medium: $\rho_{\text{probe}} \equiv \mathrm{tr}_{\text{medium}}\left[\rho_{\text{total}}\right]$

Time scales of in-medium bottomonium

- For certain conditions[2] (Markovian, quantum Brownian) bottomonium time evolution has form of Lindblad equation[3]

$$\dot{\rho}_{\text{probe}} = -\mathrm{i}\left[H_{\text{probe}}, \rho_{\text{probe}}\right] + \sum_n \left[C_n \rho_{\text{probe}} C_n^\dagger - \frac{\left\{ C_n^\dagger C_n, \rho_{\text{probe}} \right\}}{2} \right]$$

- Collapse (jump) operators C_n are related to partial widths

$$\Gamma_n \equiv C_n^\dagger C_n, \quad \Gamma = \sum_n \Gamma_n$$

- Non-hermitian effective Hamiltonian + jump term

$$H_{\text{eff}} \equiv H_{\text{probe}} - \frac{\mathrm{i}}{2}\Gamma, \quad \dot{\rho}_{\text{probe}} = -\mathrm{i}\left[H_{\text{eff}}, \rho_{\text{probe}}\right] + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

[2] Brambilla et al., Phys. Rev. D96 (2017) 034021; Phys. Rev. D97 (2018) 074009

[3] Lindblad, Comm. Math. Phys. 48 (1976) 119; V. Gorini et al., J. Math. Phys. 17 (1976) 821

Open Quantum System (OQS) pNRQCD for bottomonium

- Coupled master equations for singlet/octet densities[4]

$$\dot{\rho}_s(t;t) = -\mathrm{i}[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}[\rho_o(t;t), t],$$

$$\dot{\rho}_o(t;t) = -\mathrm{i}[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}[\rho_s(t;t), t] + \Xi_{oo}[\rho_o(t;t), t]$$

- Self-energies $\Sigma_{s,o}$ are related to thermal mass shifts and widths

$$2\mathrm{Re}\left[-\mathrm{i}\Sigma_{s,o}(t)\right] = 2\delta m_{s,o}(t), \quad -2\mathrm{Im}\left[-\mathrm{i}\Sigma_{s,o}(t)\right] = \Gamma_{s,o}(t)$$

- OQSpNRQCD is fully consistent, non-Abelian quantum field theory that conserves the number of heavy quarks at all times

[4] Brambilla et al., Phys. Rev. D97 (2018) 074009

Lindblad equations in OQSpNRQCD

- Scale hierarchy $1/a_0 \gg \pi T \sim m_D \sim g(T)T \gg E \sim \alpha_s(1/a_0)/a_0$ (**strongly-coupled** plasma at **not too low temperatures**)

$$\Sigma_{s,o}(T(t)) = \left\{ \begin{array}{c} 1 \\ \frac{N_c^2-2}{2(N_c^2-1)} \end{array} \right\} \boldsymbol{r}^2 \ (\kappa[T(t)] + \mathrm{i}\gamma[T(t)]) \,,$$

$$\Xi_{ij}(\rho_j, T(t)) = \left\{ \begin{array}{cc} 0 & \frac{1}{N_c^2-1} \\ 1 & \frac{N_c^2-4}{2(N_c^2-1)} \end{array} \right\} \sum_{k=1}^{3} r_k \rho_j r_k \ \kappa[T(t)]$$

Heavy-quark momentum diffusion coefficients

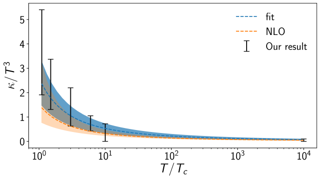
- Heavy-quark momentum diffusion coefficients

$$\kappa(T) \equiv \frac{g^2}{6N_c}\int_0^{+\infty} ds \left\langle \left\{ E^{a,i}(s,\mathbf{0}), E^{a,i}(0,\mathbf{0}) \right\} \right\rangle_T ,$$

$$\gamma(T) \equiv -\mathrm{i}\frac{g^2}{6N_c}\int_0^{+\infty} ds \left\langle \left[E^{a,i}(s,\mathbf{0}), E^{a,i}(0,\mathbf{0}) \right] \right\rangle_T .$$

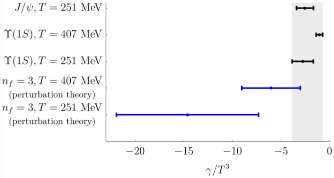
- In principle non-perturbatively computable in lattice QCD.

κ in quenched approximation:



Brambilla et al., Phys.Rev.D 102 (2020) 7, 074503

γ from J/ψ & $\Upsilon(1S)$ mass shifts:



Brambilla et al., Phys.Rev.D 100 (2019) 5, 054025

Solving the Lindblad equation

$$\dot{\rho}_{\mathrm{probe}} = -\mathrm{i}\left[H_{\mathrm{eff}}, \rho_{\mathrm{probe}}\right] + \sum_n C_n \rho_{\mathrm{probe}} C_n^\dagger, \quad H_{\mathrm{eff}} \equiv H_{\mathrm{probe}} - \frac{\mathrm{i}}{2}\Gamma$$

- Directly solving the Lindblad equation is very expensive . . .

- H_{eff} is diagonal both in angular momentum and in color
- Jump term couples sectors with $l' = l \pm 1$ in dipole transitions

$\Rightarrow$ ρ_{probe} is block-diagonal matrix in $l = 0, 1, \ldots, \infty$ and $c = s, o$

- Radial direction discretized: N_r sites, Dirichlet boundary cond.
- Solution of Lindblad equation needs some trunction $l \leq l_{\mathrm{max}}$[5]
- ρ_{probe} has dimensions $[\rho_{\mathrm{probe}}] = N^2 = (N_r \times (l_{\mathrm{max}} + 1) \times 2)^2$
- Large matrix size + inter-block cross-talk – challenging

[5] $l_{\mathrm{max}} = 1$ in: Brambilla et al., Phys. Rev. D97 (2018) 074009

The Monte-Carlo Wave-Function method

$$\dot{\rho}_{\text{probe}} = \underbrace{-\mathrm{i}\left[H_{\text{eff}}, \rho_{\text{probe}}\right]}_{\text{deterministic, non-unitary}} + \underbrace{\sum_n C_n \rho_{\text{probe}} C_n^\dagger}_{\text{stochastic jumps}}, \quad H_{\text{eff}} \equiv H_{\text{probe}} - \frac{\mathrm{i}}{2}\Gamma$$

- Only one-dimensional in Monte-Carlo wave-function method, sample $\rho_{\text{probe}} = \sum_i |i\rangle\langle i|$ as average of many MC wave-functions

- Each MCWF: evolve non-unitarily with H_{eff} with probability

$$P_{\text{no jump}}(t_1; t_0) = 1 - \int_{t_0}^{t_1} dt \, \langle \psi_{l,c}(t) | \Gamma | \psi_{l,c}(t) \rangle$$

  between t_0 and t_1 until next instantaneous jump occurs at t_1
- Each MCWF: jump $\psi_{l,c}(t_1) \to C_n \psi_{l',c'}(t_1)$ with probability

$$P_{\text{jump},n}(t_1; t_0) = \int_{t_0}^{t_1} dt \, \langle \psi_{l,c}(t) | \Gamma_n | \psi_{l,c}(t) \rangle$$

  as instantaneous jump satisfying selection rules for color and Δl
- Average many **quantum traj.** related to **same physical traj.**

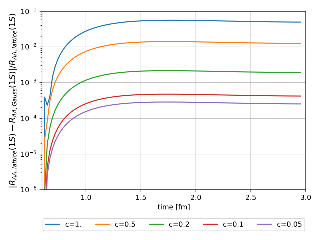
Initial $b\bar{b}$ states

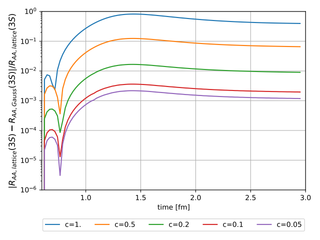
- $b\bar{b}$ production in hard process: delta function in position space

$\rightarrow$ on lattice contaminated by modes $p \sim \pi/a$ near lattice cutoff

- Soften the locality using Gaussian and derivatives @ $c = 0.2a_0$

$$u_l(r, t=0) \propto r^{l+1} e^{-r^2/c^2}$$





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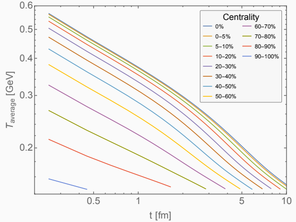
- $c = 0.2a_0$ mimics a delta function for S-wave R_{AA} at the 1% level
- Computational cost increases dramatically for narrower Gaussian

Inequialent physical trajectories

- T dependence via heavy-quark diffusion coefficients $\kappa[T], \gamma[T]$
- Temperature evolution $T(t)$ from anisotropic hydrodynamics[6]

aHydroQP temperature profile:



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Centrality	$\langle b \rangle$ [fm]	$\langle N_{part} \rangle$	T_0^{ct} [GeV]	T_0^{av} [GeV]
0%	0	406.1	0.630	0.565
0-5%	2.32	374.0	0.625	0.561
5-10%	4.25	315.9	0.614	0.550
10-20%	6.01	243.5	0.597	0.533
20-30%	7.78	168.5	0.571	0.504
30-40%	9.21	112.4	0.538	0.470
40-50%	10.45	70.8	0.497	0.430
50-60%	11.55	41.1	0.446	0.381
60-70%	12.56	21.3	0.386	0.325
70-80%	13.49	9.7	0.322	0.267
80-90%	14.38	3.8	0.258	0.214
90-100%	15.66	0.97	0.180	0.157

Centrality bin & collision parameters for a $\sqrt{s_{NN}} = 5.02$ TeV PbPb collision

- Production points sampled according to $N_{b\bar{b}}(x, z, p_T) \propto \frac{N_{bin}(x,y)}{(p_T^2+M^2)^2}$

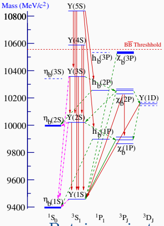
[6] Alqahtani et al., Phys. Rev. C92 (2015) 054910; C95 (2017) 034906; PPNP 101 (2018) 204

Final state feed-down

- Measured vacuum yields related to primordially produced vacuum yields by a feed-down matrix as $\vec{N}_{\text{measured}} = F \cdot \vec{N}_{\text{direct}}$

$$F = \begin{pmatrix} 1 & 0.2645 & 0.0194 & 0.352 & 0.18 & 0.0657 & 0.0038 & 0.1153 & 0.077 \\ 0 & 1 & 0 & 0 & 0 & 0.106 & 0.0138 & 0.181 & 0.089 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0091 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0051 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

source: 2012.01240



source: Patrignani et al., Ann.Rev.Nucl.Part.Sci. 63 (2013) 21-44

- Feed-down matrix treats different $1P$ and $2P$ states as distinct

- $\vec{N}_{QGP} = (N_{1S}, N_{1S}, 3 \times N_{1P}, N_{3S}, 3 \times N_{2P})^T$ obtained as $\vec{N}_{QGP}(i) = \langle N_{\text{binary}}[b] \rangle \times P_{\text{survival}}(i) \times \vec{\sigma}_{\text{direct}}$
- Post feed-down normalize to pp scaled by binary collisions (AA)

Simulation details

- Model: $\alpha_s(1/a_0)$, m_b, $\hat{\kappa} \equiv \kappa/T^3$, $\hat{\gamma} \equiv \gamma/T^3$, $T_F = 250\,\mathrm{MeV}$
- Coupling and mass fixed in terms of vacuum physics

$$\alpha_s(1/a_0) = 0.2071, \qquad m_b = 4.881\,\mathrm{GeV}, \qquad a_0 = 0.146\,\mathrm{fm}.$$

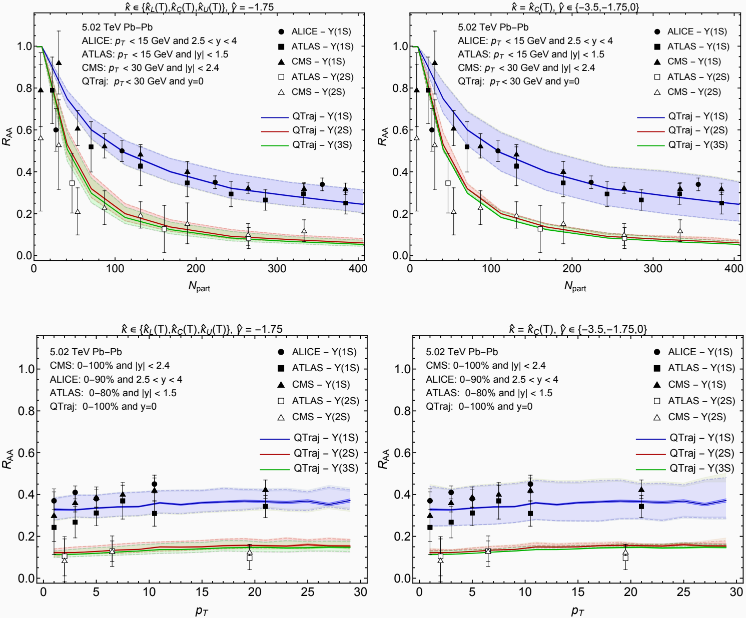
- Fixed with lattice gauge theory input:{dashed, solid, dot-dashed}

$$\hat{\kappa} = 1/(\kappa_0 + \kappa_1 T^{1/2} + \kappa_2 T^1 + \kappa_3 T^{3/2}) \qquad \to \{\kappa_L < \kappa_C < \kappa_U\},$$
$$\hat{\gamma} = -1.75 \pm 1.75 \qquad \to \{\gamma_L < \gamma_C < \gamma_U\}.$$

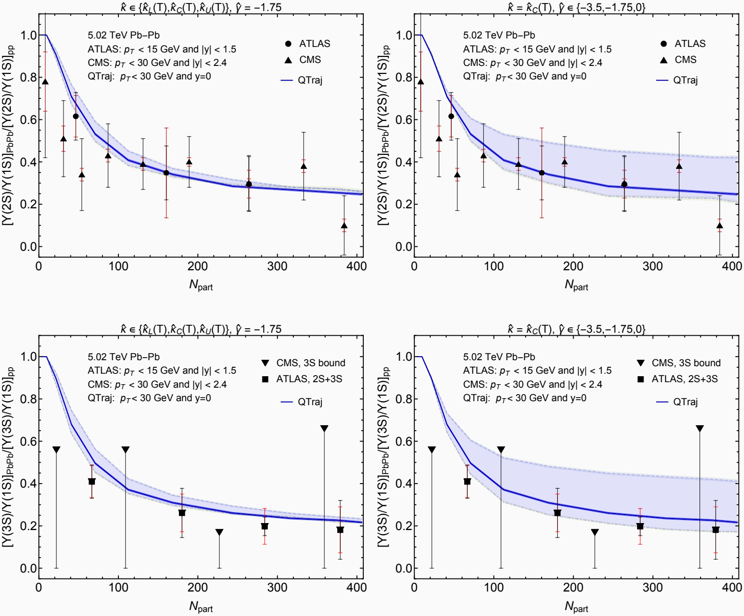
- Also tested constant $\hat{\kappa}$

- $\sim 900\,k$ physical traj. ($45\,M$ q.traj.) for (κ_C, γ_C) pair
- $\sim 700\,k$ physical traj. ($35\,M$ q.traj.) for the other (κ, γ) pairs

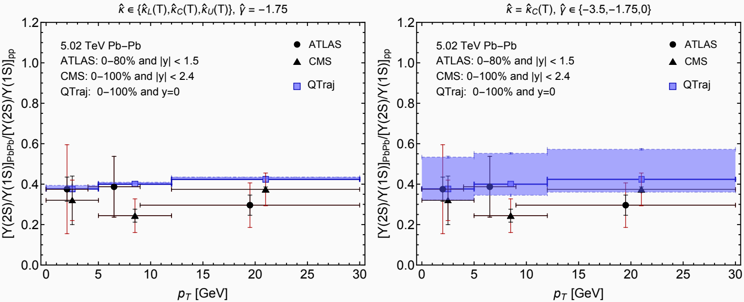
S-wave bottomonium R_{AA}



Double ratios vs $N_{\rm part}$

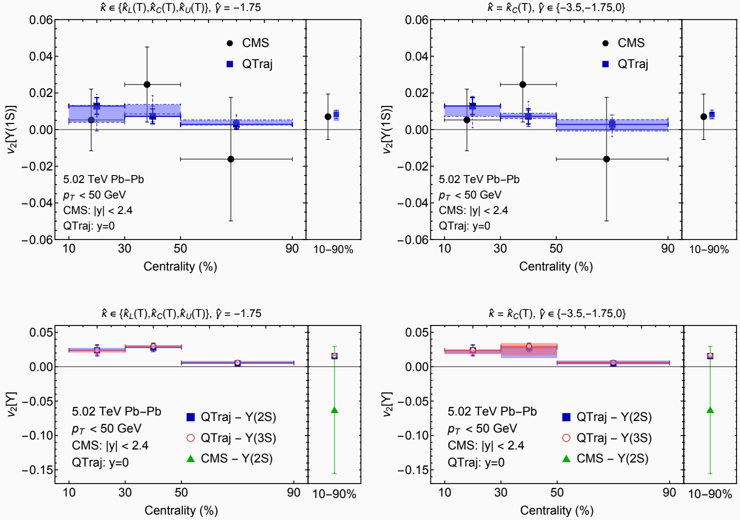


Double ratios vs p_T



γ_L is seems to be disfavored by the double ratio $R_{AA}(\Upsilon(2S))/R_{AA}(\Upsilon(1S))$

Bottomonium elliptic flow v_2 vs centrality



Summary & Outlook

- QTraj project aims at predicting bottomium suppression via **ab-initio OQSpNRQCD + aHydro** MC lattice simulations[7]
- Systematic expansion with a consistent power counting scheme
- Effect of **dissociation & recombination** (i.e. quantum jumps) is fully quantifiable
- **Off-diagonal overlaps** studied as well → statistically irrelevant

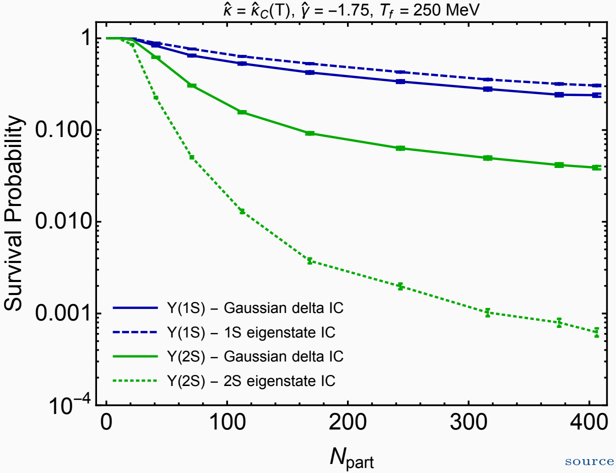
- Good agreement for $N_{\text{part}} \gtrsim 100$ (like all models)
- Cold nuclear matter effects are expected to lower R_{AA} at low T

- Parameters motivated by vacuum physics or lattice gauge theory
- Could constrain **transport coefficients** with more precise data
- Double ratios quite insensitive to $\hat{\kappa}$; $\gamma \approx 0$ favored by the data

[7] Session L04: Heavy Quarkonia as a Probe of Hot and Cold QCD Media: Current and Future (Sunday, 04/18/2021): N. Brambilla (3:45 pm) & M. Strickland (4:21 pm)

Thank you for listening!

Initial state dependence



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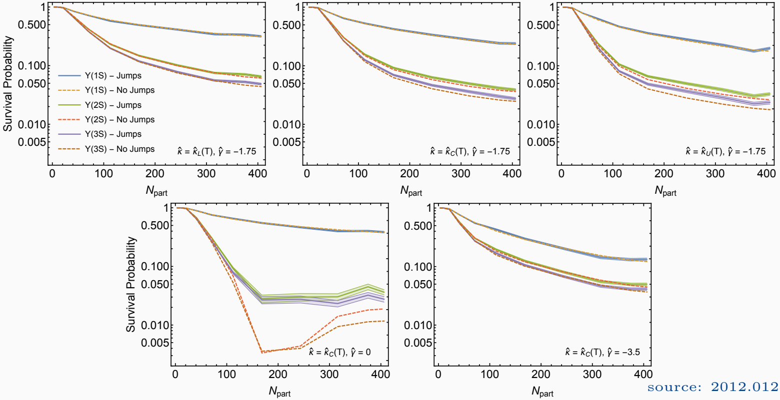
- Gaussian vs bound state initial condition: with Gaussian less suppression of excited states, more suppression of ground state

Jump vs no-jump evolution



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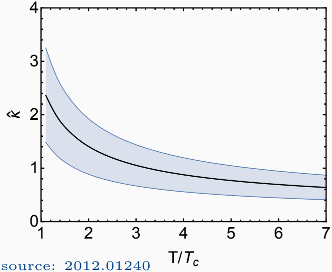
- With jumps vs without jumps: excited state survival probabilities differ substantially only for $\hat{\gamma} \approx 0$ and $N_{part} \approx 200$

Temperature dependence of $\hat{\kappa}$



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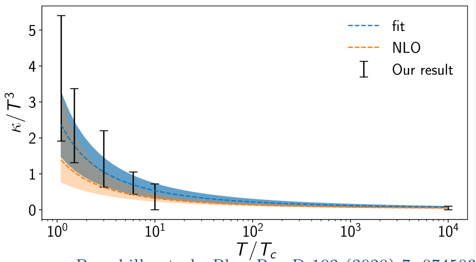


source: Brambilla et al., Phys.Rev.D 102 (2020) 7, 074503

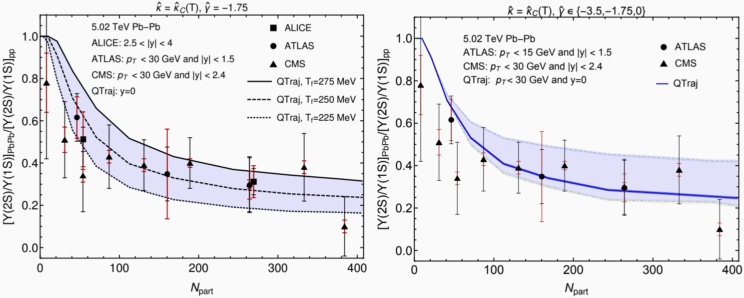
$$\hat{\kappa}(T) = 1/(\kappa_0 + \kappa_1 T^{1/2} + \kappa_2 T^1 + \kappa_3 T^{3/2})$$

–	κ_L	κ_C	κ_U
κ_0	-0.8071156107961191	-0.5188965324059919	-0.3819693126876343
κ_1	1.5560083173857689	0.98030718982413288	0.71513037386309615
κ_2	-0.14261556412444807	-0.083409768363912536	-0.058542116432588807
κ_3	0.0071217431979965582	0.0039726666253277474	0.0027170443410800557

- Tests with $\hat{\kappa} = 1$ are covered by uncertainty bands of $\hat{\kappa}$ variation
- Past value[8] of $\hat{\kappa} = 2.6$ lead to substantially more suppression

[8] Brambilla et al., Phys. Rev. D96 (2017) 034021; Phys. Rev. D97 (2018) 074009

Dependence on T_f



- Varied $T_f = 250\,\text{MeV}$ by $\pm 10\%$; effect mildly dependent on N_{part}
- Magnitude of the effect is comparable to variation of $\hat{\kappa}$ and $\hat{\gamma}$

Details of the feed-down

- Use experimental, rapidity-averaged ($|y| \leq 2.4$) cross-sections
- Fold di-muon branching ratios B with S-wave di-muon yields[9]

$$B^{d\sigma}/_{dy}(S) \approx (1.44, 0.37, 0.15)^T \,\text{nb}, \quad B \approx (2.5, 1.9, 2.2)^T\,\%$$

- We extrapolate ratios of (mP)- and (nS) yields[10] to $\sqrt{s} = 5\,\text{TeV}$
- Theory expectation ${}^{\sigma[\chi_{b2}]}/_{\sigma[\chi_{b1}]} \approx 1.176$ (for $1P$ and $2P$)
- Theory-based estimate $\sigma[\chi_{b0}] \approx {}^{(\sigma[\chi_{b1}] + \sigma[\chi_{b2}])}/_{8}$ (for $1P$ and $2P$)

$$\vec{\sigma}_{\text{exp}} = (57.6, 19.3, 3.72, 13.69, 16.1, 6.8, 3.27, 12.0, 14.15)^T \,\text{nb}$$

for $\vec{N} = (\Upsilon(1S), \Upsilon(2S), \chi_{b0}(1P), \chi_{b1}(1P), \chi_{b2}(1P), \Upsilon(3S), \chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P))^T$

For state i and (centrality) class c: $R^i_{AA}(c) = \frac{F\cdot S(c)\cdot\vec{\sigma})\text{direct})^i}{\sigma^i_{\text{exp}}}$

[9] Sirunyan et al. [CMS], Phys. Lett. B 790 (2019) 270
[10] Aaij et al. [LHCb], Eur. Phys. J. C74 (2014) 3092