Quantum Simulation for Lattice Gauge Theories

APS Topical Group on Hadronic Physics
Sacramento, California. April 15th 2021

Natalie Klco
Precisely controlled input
Isolated Interaction with Nature
Precisely designed detection

Clean Observations

Precisely designed detection
Trusted Evolution of System
Precisely controlled input

Extract properties emergent from Inputs

Experiment

Codesign

Computation

(Quasi) Workshop 2: Implementation Strategies for Gauge Theories
Quantum Simulation of Strong Interactions (Quasi) Workshop 1: Theoretical Strategies for Gauge Theories

instructions to be performed by a predictable physical system
Q projection $\rightarrow$ C state creation/annihilation

Quantum Computation of Scattering in Scalar Quantum Field Theories
Stephen P. Jordan (NIST, Wash., D.C. and Caltech), Keith S. M. Lee (Pittsburgh U. and Caltech), John Preskill (Caltech)

Simulating collider physics on quantum computers using effective field theories
Christian W. Bauer* and Benjamin Nachman†
Marat Freytsis‡
arXiv:2102.05044v1

Wilson Line in
Gauge Field

Accelerating Lattice Quantum Field Theory Calculations via Interpolator Optimization Using Noisy Intermediate-Scale Quantum Computing
A. Avkhadiev, P. E. Shanahan, and R. D. Young
Phys. Rev. Lett. 124, 080501 – Published 26 February 2020

Fixed-point quantum circuits for quantum field theories
Natalie Klco and Martin J. Savage
Phys. Rev. A 102, 052422 – Published 30 November 2020

C snapshots $\rightarrow$ Q state prep

Subatomic dynamics from atomic scale interactions

$\alpha$ | Analog Computation via Simile $+$ Digital Discrete Universal Gates
Intersimulatability
A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis
Anthony Ciavarella, Natalie Klco, Martin J. Savage

Ice Picks Required November-March

The structure at the peak is overnight telescope storage, not an outhouse

Don’t Feed the Bears at 2,400 m
Plaquette Operator

Calculate once
Use many
\[ \hat{H} = \frac{\hbar^2}{2a^{d-2}} \sum_{b, \text{links}} |\hat{E}^{(b)}|^2 + \frac{1}{2a^{d-2}g^2} \sum_{\text{plaquettes}} \left[ 6 - \hat{D}(x) - \hat{D}^t(x) \right] \]

Local Hilbert Space

\[ |p, q, T_L, T_L^z, Y_L, T_R, T_R^z, Y_R \rangle \]

\[ T_i = \{0, \frac{1}{2}, \ldots, \frac{1}{2}(p + q)\}, \]

\[ T_i^z = \{-\frac{1}{2}(p + q), -\frac{1}{2}(p + q) + \frac{1}{2}, \ldots, \frac{1}{2}(p + q)\}, \]

\[ Y_i = \{-\frac{1}{3}(q + 2p), -\frac{1}{3}(q + 2p) + \frac{1}{3}, \ldots, \frac{1}{3}(q + 2p)\}. \]

Tensor indices
\[ \begin{array}{c}
T_{a_1 a_2 \ldots a_p} \\
\text{b}_1 \text{b}_2 \ldots \text{b}_q
\end{array} \]

Integrate Local Gauge Space

\[ |p, q \rangle = |R \rangle \]

\[ |0, 0 \rangle = |1 \rangle \]
\[ |1, 0 \rangle = |3 \rangle \]
\[ |0, 1 \rangle = |3 \rangle \]

CGs introduced as gate rotation angles, classically precomputed*

CGs through poly-overhead Operator construction

Bacon, Chuang, Harrow (2006)

De Loera, McAllister (2005)

1D SU(2): Bañuls, Cichy, Cirac, Jansen, Kühn. (2017)

1D\Box SU(2): Klco, Stryker, Savage. (2020)

*Poly-time algorithm for fixed rank
De Loera, McAllister (2005)
Tied together with Vertex CGs

**Sum over local orientations**

Possible Multiplicities in SU(3) e.g., $8 \otimes 8 \otimes 8$

**Local Irreps**

**Pair of link CGs**

**Magnetic Matrix Element**

\[
\langle C_1, R_r, C_4 \mid C_1, R_r, C_4 \rangle = \frac{1}{\dim(Q_2) \dim(Q_4)} \sum_{all} |C_1, a, b; Q_2, c, d; C_2, r, f; |R_4, g, h; |Q_4, i, j; |R_4, k, l; |C_4, m, n; |C_1, p, q)
\]

**Identify Vertex Factors**

\[
\langle C_1, R_r, C_4 \mid C_1, R_r, C_4 \rangle = \frac{1}{\dim(Q_2) \dim(Q_4)} \sum_{all} |C_1, a, b| Q_2, c, d; C_2, r, f; |R_4, g, h; |Q_4, i, j; |R_4, k, l; |C_4, m, n; |C_1, p, q)
\]

**Link Operator**

\[
\hat{U}_{\alpha, \beta}^{\gamma} = \sum_{\Omega} \sum_{\Omega'} \sqrt{\frac{\dim(R')}{\dim(R)}} |R', a', b'\rangle \langle R, a, r, \alpha | R', a' \rangle_{\Gamma_1} (R', b' | R, b, \beta \rangle_{\Gamma_2}
\]
Classical Integration of Local Gauge Space

\[ \langle \chi (C_1, R_t, C_3) | \mathcal{D} | \chi (C_1, R_t, C_3) \rangle = \]

\[ \{ j_r, j_a, q_r \} \{ j_r, j_a, q_r \} \{ j_f, j_a, q_f \} \{ j_r, j_a, q_r \} \{ j_f, j_a, q_f \} \{ j_f, j_a, q_f \} \]

\[ | j, m, m' \rangle \rightarrow | j \rangle \]

6j vertex factors

\[ | R, T_L, T_R^Z, Y_L, T_R^Z, Y_R \rangle \rightarrow | R \rangle \]

“9R” vertex factors

\[ \{ A, B, C \} = \sum | D, y', B, z | E, q \rangle | \{ A, y, B, z | C, q \} | \{ A, y, B, z | D, y' \} | \{ C, q, z | E, q \} | \{ D, y' | E, q \} \]
Plaquette Operator Structure

1D:

- 4 control registers

2D:

- 8 control registers

D:

- 4D control registers
  Consolidated into 4 virtual link insertions
**Vertex CGs**

\[ |\psi_{3pt}\rangle \sim \sum_{b,g,d,\Gamma} \langle C_1, b, R_4, g | Q_f, d \rangle \Gamma | C_1, a, b | Q_i, c, d \rangle | R_4, g, h \rangle \]

**Absent in unstructured Qubit Hilbert Space**

\[ \delta \text{ contraction} \]

\[ \varepsilon \text{ contraction} \]

**Link CGs**

\[ 3 \bigoplus 6 \bigoplus 3 \]

\[ 3 \bigoplus 6 \bigoplus 3 \]

**Controls incorporate Vertex CGs into operator Gauge-Violating states in Hilbert Space!**
Irrep basis states, not qubits
\[(p, q) \otimes (1, 0) = (p + 1, q) \oplus (p - 1, q + 1) \oplus (p, q - 1)\]
Flexibility for Codesign with existing and developing hardware

e.g., SRF cavities, Ions, Superconducting Circuits, ...

\[(p, q) \otimes (1, 0) = (p + 1, q) \oplus (p - 1, q + 1) \oplus (p, q - 1)\]
Global Basis

\[ |\Psi\rangle = \sum c(\vec{R}) |\vec{R}\rangle \]

- Classical Preprocessing of Hilbert Space.
  Scalability Unknown

- Project into Local and Global Symmetry Sectors
  Economical use of Hilbert space
  Errors incapable of violating symmetry

- Non-local distribution of lattice-local information
One plaquette

Hilbert space ~ Link Hilbert space

\[ |R\rangle = \frac{1}{\text{dim}(R)^2} \sum_{\alpha, \beta, \gamma, \delta} |R, \alpha, \beta\rangle_1 |R, \beta, \gamma\rangle_2 |R, \gamma, \delta\rangle_3 |R, \delta, \alpha\rangle_4 \]

SU(2)

\[ \hat{H} = \frac{g^2}{2} \sum_{\text{all links}} |E|^2 + \frac{1}{2} g^2 \left( 4 - \mathbf{\nabla} \cdot \mathbf{\nabla} \right) \]

\[ \nabla_j^2 \psi(j) + \left( g^2 E - \frac{1}{2} g^4 j(j + 1) \right) \psi(j) = 0 \]

\[ \psi \rightarrow e^{-\frac{g^2}{2\sqrt{2}} (j + \frac{1}{2})^2} \]

SU(3)

Wavefunction

Observables

Gaussian Convergence in “color” space
Decomposition for Dynamics

Pauli 2-Level unitary

Binary and Gray Encoding

Polynomial Time Evolution

$\hat{H} = \frac{g^2}{2} \sum_{b, \text{links}} |\vec{E}(b)|^2 + \frac{1}{2g^2} \left( 6 - \hat{\Delta} - \hat{\Delta}^\dagger \right)$

Distinct Sectors

Trivial vacuum +

$|\mathcal{R}^+\rangle = \frac{1}{\sqrt{2}} \left[ |\mathcal{R}\rangle + |\overline{\mathcal{R}}\rangle \right]$
Confidence Intervals:

- CNOT extrapolation
- Measurement calibration matrix inverse
- Majority voting (aux. qubits)

Global Basis: \( |R_{max}| = 6 \)

- 38 CNOTS
- 20 CNOTS

CP projection
Dynamical Peak Benchmarks

Theory-Experiment tradeoff

Trotter Order vs Steps
**Gauge Variant Completion**

phys ↔ phys  
phys ↔ unphys  
unphys ↔ unphys  

![Checkmark]  

Free!

SU(2): “human neural network” for low Λ “color” truncation  
Magnetic time evolution expanded in **Gauge Invariant** product of operators organized by control sectors, $\vec{C}$

(No Trotter errors between control sectors)

$$\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} = \exp \left[ -i\alpha \left( \frac{1}{3} x_{12} x_{01} x_{12} x_{01} + \frac{1}{3\sqrt{3}} x_{02} x_{12} x_{01} x_{02} + \frac{1}{3\sqrt{3}} x_{01} x_{02} x_{02} x_{12} \right) \right]$$

2-Level unitaries

Gauge Invariant Matrix Element
Trotterization with Givens
Preserves Gauge Invariance and Color Parity

\[
\exp \left[ -i \alpha \left( \frac{3}{3} ( \hat{\mathbf{1}} + \hat{\mathbf{1}}^\dagger) \right) \right] \rightarrow \exp \left[ -i \frac{\alpha}{3} x_{12} x_{02} x_{12} x_{01} \right] \exp \left[ -i \frac{\alpha}{9} x_{02} x_{01} x_{01} x_{02} \right] \exp \left[ -i \frac{\alpha}{3} x_{01} x_{12} x_{02} x_{12} \right] \otimes \Lambda_1 \otimes \Lambda_2 \otimes \Lambda_1 \otimes \Lambda_2
\]

- Each unitary operator is associated with a physical plaquette transition
- Coefficients determined from matrix elements between gauge-invariant states

Black: global ++ basis
(4 dim)

Colors: local qudit basis
\(3^6 = 729\) dim, 27 cGivens

Two PBC Plaquettes \(\{1, 3, 3\text{bar}\}\)
Gauss’s law constraint does not reduce the asymptotic polynomial scaling.

<table>
<thead>
<tr>
<th>$\Lambda_p = \Lambda_q$</th>
<th>dimensions</th>
<th>physical states</th>
<th>matrix elements</th>
<th>elements/states</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(1, 3)$</td>
<td>81</td>
<td>81</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>$(1, 3, 8)$</td>
<td>529</td>
<td>1,018</td>
<td>1.92</td>
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<tr>
<td>2</td>
<td>$(1, 3, 8, 6)$</td>
<td>5,937</td>
<td>19,594</td>
<td>3.30</td>
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<td>419,316</td>
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<td>2</td>
<td>$(1, 3, 8, 6, 15, 27)$</td>
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<tr>
<td>3</td>
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<td>4,001,111</td>
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<tr>
<td>3</td>
<td>$(1, 3, 8, 6, 15, 27, 10, 24)$</td>
<td>2,008,297</td>
<td>24,648,819</td>
<td>12.27</td>
</tr>
</tbody>
</table>

• Potential non-zero matrix element per physical state constant: $3^4 = 81$
• Limited by Gauss’ Law
• Givens rotations $\sim \Lambda^{16}$

Retain exponential color space convergence? EFT for inclusion of “hot” links?
Field

Quantum d.o.f.

Position space lattice
Momentum Mode Lattice
Loop, String, Hadron Excitations
Eigenbasis of Field Operator
Gauge Field Integration in (1+1)
Hybrid/Analog Representation
Orbifold Lattice

Irreducible Representations
Local Free-Field Eigenstates
Group Space Decimation
Link Models/Qubit Regularization
Magnetic Basis
Discrete Subgroups
Mesh Digitization
Light-Front Formulations

Initialization
Ground States
Local wave packets

Time Evolution
Measurement Procedure
Detector Design

connectivity
gates
Coherence times

Entanglement Structures**
• Multiplet basis integrated over the local gauge space. Qubit requirements:

\[ \# \text{qubits} \sim 2L^D \log_2(\Lambda_p + 1) \]

\[ L = 10, \quad D = 3, \quad \Lambda = 1 \quad \rightarrow \quad 2,000 \text{ logical qubits} \]
\[ 1,000 \text{ ops/plaquette} \]

• Implementation of Global basis

• Gauge Invariant Time Evolution organized by local link qudit structure

• Flexibility for Codesign with existing and developing hardware

(p,q):

\((p,q), \bigcirc \text{mix:} \)