Quantum Simulation for Lattice Gauge Theories

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Where is the line?

Quantum Computation of Scattering in Scalar Quantum Field Theories

Stephen P. Jordan (NIST, Wash., D.C. and Caltech), <u>Keith S.M. Lee</u> (Pittsburgh U. and Caltech), John Preskill (Caltech) Published in: Quantum Information and Computation 14, 1014-1080 (2014), *Quant.Inf.Comput.* 14 (2014) 1014-1080



Simulating collider physics on quantum computers using effective field theories Christian W. Bauer^{*} and Benjamin Nachman[†] arXiv:2102.05044v1 Marat Freytsis[‡] $\sigma = H \otimes J_1 \otimes \cdots \otimes J_n \otimes S$ $\langle X | T[Y_n Y_{\bar{n}}^{\dagger}] | \Omega \rangle$ Wilson Line in Soft Function **Gauge Field** $\alpha \left| \begin{array}{c} \text{Analog} \\ \text{Computation} \\ \text{via Simile} \end{array} \right\rangle + \left| \begin{array}{c} \text{Digital} \\ \text{Discrete} \\ \text{Universal Gates} \end{array} \right\rangle$ Atomic scale interactions perfections

Accelerating Lattice Quantum Field Theory Calculations via Interpolator Optimization Using Noisy Intermediate-Scale Quantum Computing

A. Avkhadiev, P. E. Shanahan, and R. D. Young Phys. Rev. Lett. **124**, 080501 – Published 26 February 2020

Q projection \rightarrow C state creation/annihilation



Fixed-point quantum circuits for quantum field theories

Natalie Klco and Martin J. Savage Phys. Rev. A **102**, 052422 – Published 30 November 2020

> C snapshots → Q state prep



A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis Anthony Ciavarella, Natalie Klco, Martin J. Savage

The structure at the peak is overnight telescope storage, not an outhouse

Don't Feed the Bears at 2,400 m

Ice Picks Required November-March





Plaquette Operator

Calculate once Use many

S.

. Let S.

. Set

. Her

S.C.

, der

-5,50

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Byrnes and Yamamoto

|0,0
angle = |1
angle

 $|1,0\rangle = |\mathbf{3}\rangle$

 $|0.1\rangle = |\overline{\mathbf{3}}\rangle$

Phys. Rev. A 73 (2006) 022328

$$\hat{H} = \frac{g^2}{2a^{d-2}} \sum_{b,\text{links}} |\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2} \sum_{\text{plaquettes}} \left[6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^{\dagger}(\mathbf{x}) \right]$$

CGs through poly-overhead Operator construction

Local Hilbert Space

$\left|p,q,T_{L},T_{L}^{z},Y_{L},T_{R},T_{R}^{z},Y_{R}\right\rangle$

$$T_{i} = \{0, \frac{1}{2}, \dots, \frac{1}{2}(p+q)\},\$$

$$T_{i}^{z} = \{-\frac{1}{2}(p+q), -\frac{1}{2}(p+q) + \frac{1}{2}, \dots, \frac{1}{2}(p+q)\},\$$

$$Y_{i} = \{-\frac{1}{3}(q+2p), -\frac{1}{3}(q+2p) + \frac{1}{3}, \dots, \frac{1}{3}(p+2q)\},\$$

$$\begin{split} |j,m\rangle_{\rm reg} |\frac{1}{2}, \Delta m\rangle_{\rm reg} \rightarrow \\ \sum_{J=|j-1/2|}^{j+1/2} \langle J,M|j,m;\frac{1}{2},\Delta m\rangle |J,M=m+\Delta m\rangle_{\rm reg} \end{split}$$

Bacon, Chuang, Harrow (2006)

CGs introduced as gate rotation angles, classically precomputed*

rications

Tensor indices $T^{a_1 a_2 \dots a_{\Lambda p}}_{b_1 b_2 \dots b_{\Lambda q}}$

*Poly-time algorithm for fixed rank De Loera, McAllister (2005)



Integrate Local Gauge Space

 $|p,q
angle = |\mathbf{R}
angle$

1D SU(2): Bañuls, Cichy, Cirac, Jansen, Kühn. (2017) 1D_□ SU(2): Klco, Stryker, Savage. (2020)



 $\bigvee \dim(\mathbf{R}'_{t})\dim(\mathbf{Q}'_{\ell})\dim(\mathbf{Q}_{\ell})\dim(\mathbf{Q}_{\ell})\dim(\mathbf{Q}'_{\ell})^{3}\dim(\mathbf{Q}'_{\ell})^{3} \longrightarrow \mathcal{O}$ $\sum \langle \mathbf{C}_{1}, b, \overline{\mathbf{R}}_{t}, g | \overline{\mathbf{Q}}_{\ell}, d \rangle_{\Gamma_{1}} \langle \mathbf{R}_{t}, g, \overline{\mathbf{3}}, \delta | \mathbf{R}'_{t}, g' \rangle_{\Gamma_{2}} \langle \mathbf{Q}'_{\ell}, d' | \mathbf{Q}_{\ell}, d, \overline{\mathbf{3}}, \delta \rangle_{\Gamma_{3}} \langle \overline{\mathbf{Q}}'_{\ell}, d' | \mathbf{C}_{1}, b, \overline{\mathbf{R}}'_{t}, g' \rangle_{\Gamma_{4}}$ $\sum \langle \mathbf{R}_{t}, h, \overline{\mathbf{C}}_{3}, m | \overline{\mathbf{Q}}_{r}, j \rangle_{\Gamma_{5}} \langle \mathbf{Q}'_{r}, j' | \mathbf{Q}_{r}, j, \mathbf{3}, \gamma \rangle_{\Gamma_{6}} \langle \mathbf{R}'_{t}, h' | \mathbf{R}_{t}, h, \overline{\mathbf{3}}, \gamma \rangle_{\Gamma_{7}} \langle \overline{\mathbf{Q}}'_{r}, j' | \mathbf{R}'_{t}, h', \overline{\mathbf{C}}_{3}, m \rangle_{\Gamma_{8}}$ $\sum \langle \mathbf{C}_{2}, f, \overline{\mathbf{R}}_{b}, k | \mathbf{Q}_{\ell}, c \rangle_{\Gamma_{9}} \langle \mathbf{R}_{b}, k, \mathbf{3}, \alpha | \mathbf{R}'_{b}, k' \rangle_{\Gamma_{10}} \langle \mathbf{Q}_{\ell}, c, \overline{\mathbf{3}}, \alpha | \mathbf{Q}'_{\ell}, c' \rangle_{\Gamma_{11}} \langle \mathbf{Q}'_{\ell}, c' | \mathbf{C}_{2}, f, \overline{\mathbf{R}}'_{b}, k' \rangle_{\Gamma_{12}}$ $\sum \langle \langle \mathbf{R}_{b}, \ell, \overline{\mathbf{C}}_{4}, p | \mathbf{Q}_{r}, i \rangle_{\Gamma_{13}} \langle \mathbf{R}'_{b}, \ell' | \mathbf{R}_{b}, \ell, \mathbf{3}, \beta \rangle_{\Gamma_{14}} \langle \mathbf{Q}_{r}, i, \mathbf{3}, \beta | \mathbf{Q}'_{r}, i' \rangle_{\Gamma_{15}} \langle \mathbf{Q}'_{r}, i' | \mathbf{R}'_{b}, \ell', \overline{\mathbf{C}}_{4}, p \rangle_{\Gamma_{12}}$

Identify Vertex Factors









Controls incorporate Vertex CGs into operator Gauge-Violating states in Hilbert Space!





 $(p,q)\otimes(1,0)=(p+1,q)\oplus(p-1,q+1)\oplus(p,q-1)$

Flexibility for Codesign with existing and developing hardware e.g., SRF cavities, Ions, Superconducting Circuits, ...

e.g., SKF Cavilies, Ions, Superconducting Circuits,





Global Basis

$$|\Psi\rangle = \sum c\left(\vec{R}\right)|\vec{R}\rangle$$

 Classical Preprocessing of Hilbert Space.

Scalability Unknown

 Project into Local and Global Symmetry Sectors
 Economical use of Hilbert space
 Errors incapable of violating symmetry

• Non-local distribution of lattice-local information



Jan F. Haase^{1,2}, Luca Dellantonio^{1,2}, Alessio Celi^{3,4}, Danny Paulson^{1,2}, Angus Kan^{1,2}, Karl Jansen⁵, and Christine A. Muschik^{1,2,6}

Color-Parity Symmetry



$$\hat{H} = \frac{g^2}{2} \sum_{b,\text{links}} |\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2g^2} \left(6 - \hat{\Box} - \hat{\Box}^{\dagger} \right)$$

6

$$\left|R^{+}\right\rangle = \frac{1}{\sqrt{2}} \left[\left|R\right\rangle + \left|\bar{R}\right\rangle \right]$$



Dynamical Peak Benchmarks

Theory-Experiment tradeoff



Trotter Order vs Steps







Magnetic time evolution expanded in <u>Gauge Invariant</u> product of operators organized by control sectors, \vec{C}

(No Trotter errors between control sectors)

2-Level unitaries

 $\hat{\Box} + \hat{\Box}^{\dagger}$

$$\begin{bmatrix} \mathbf{3} & \mathbf{\overline{3}} \\ \hat{\Box} + \hat{\Box}^{\dagger} \\ \mathbf{\overline{3}} & \mathbf{3} \end{bmatrix} = \exp\left[-i\alpha\left(\frac{1}{3}\mathcal{X}_{12}\mathcal{X}_{01}\mathcal{X}_{12}\mathcal{X}_{01} + \frac{1}{3\sqrt{3}}\mathcal{X}_{02}\mathcal{X}_{12}\mathcal{X}_{01}\mathcal{X}_{02} + \frac{1}{3\sqrt{3}}\mathcal{X}_{01}\mathcal{X}_{02}\mathcal{X}_{02}\mathcal{X}_{12}\right)\right]$$

Gauge Invariant Matrix Element



• Each unitary operator is associated with a physical plaquette transition

 Coefficients determined from matrix elements between gauge-invariant states



Colors: local qudit basis $(3^6 = 729 \text{ dim}, 27 \text{ cGivens})$



{1, 3, 3bar}

Two PBC Plaquettes

Scaling

Physical 4-pt vertices 10⁷ Order 3 10⁵ 4 5 iduals 6 1000 7

1000

5

 $\Lambda_p = \Lambda_a$

2 4

10

Polynomial Order

4-pt \leq control sectors

2

10

 $\sim \Lambda^8$

8 9

10

20



Gauss's law constraint does not reduce the asymptotic polynomial scaling.



Potential non-zero matrix element per physical state constant: $3^4 = 81$ Limited by Gauss' Law Givens rotations ~ Λ^{16} Retain exponential color space convergence? EFT for inclusion of "hot" links?

Field V Quantum d.o.f.

Quantum Simulation of Strong Interactions (QuaSI) Workshop 1 : Theoretical Strategies for Gauge Theories Position space lattice Momentum Mode Lattice Loop, String, Hadron Excitations Eigenbasis of Field Operator Gauge Field Integration in (1+1) Hybrid/Analog Representation Orbifold Lattice

Irreducible Representations Local Free-Field Eigenstates **Group Space Decimation** Link Models/Qubit Regularization **Magnetic Basis Discrete Subgroups** Mesh Digitization **Light-Front Formulations**

Initialization Ground States Connectivity gates Local wave packets Time Evolution Measurement Procedure Detector Design **Entanglement Structures**

Summary

Multiplet basis integrated over the local gauge space. Qubit requirements:

qubits ~
$$2L^D \log_2(\Lambda_p + 1)$$

L = 10, D = 3, $\Lambda = 1 \rightarrow 2,000$ logical qubits 1,000 ops/plaquette

Implementation of Global basis

Gauge Invariant Time Evolution organized by local link qudit structure

Flexibility for Codesign with existing and developing hardware



(p,q), **O** mix:

