MICHAEL PAOLONE
NEW MEXICO STATE UNIVERSITY
FOR THE E05-110 COLLABORATION.

THE COULOMB SUM RULE IN NUCLEI
Inclusive electron scattering cross-section:

\[
\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[ \frac{q^4}{|q|^4} R_L(\omega, |q|) + \left( \frac{q^2}{2|q|^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, |q|) \right]
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\]

- Scattering response due to **charge** properties
- Scattering response due to **magnetic** properties
THE COULOMB SUM RULE IN NUCLEI

COULOMB SUM RULE

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Coulomb Sum Rule definition:

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S_L(|q|) = \int_{\omega^+}^{\omega} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}
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This is measured through inclusive electron scattering on nucleons in nuclei.
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Coulomb Sum Rule definition:

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Scattering response due to charge properties

Scattering response due to magnetic properties

This is measured through inclusive electron scattering on nucleons in nuclei.

This is calculated according to expected structure for unbound nucleons.
Coulomb Sum Rule

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Scattering response due to **charge** properties

Scattering response due to **magnetic** properties

If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.
THE COULOMB SUM RULE IN NUCLEI

COULOMB SUM RULE

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At small $|q|$, $S_L$ will deviate from unity due to long range nuclear effects, Pauli blocking. (directly calculable, well understood).
COULOMB SUM RULE

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At large $|q| >> 2k_f$, $S_L$ should go to 1. Any significant* deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

*Short range correlations will also quench $S_L$, but only by < 10%
COULOMB SUM RULE

- Long standing issue with many years of theoretical interest.
- Even most state-of-the-art models cannot predict existing data.
- New precise data at larger $|q|$ would provide crucial insight and constraints to modern calculations.

$$S_L(|q|) = \int_{\omega^+} |q| \, d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

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EXPERIMENTAL CONSTRAINTS ON INTEGRATION

We consider the scattering on the constituent nucleons in the nucleus, and focus on the quasi-elastic to delta region.

The Coulomb sum rule in nuclei: GHP 2021

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- The lower limit is constrained by avoiding the nuclear elastic contribution.
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- The lower limit is constrained by avoiding the nuclear elastic contribution.

- The upper-limit is first limited by $Q^2 = 0$, and then by accessible experimental phase-space.

\[
S_L(|q|) = \int_{\omega^+} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}
\]

\[
Q^2 = |q|^2 - \omega^2
\]
First group of experiments from Saclay, Bates, and SLAC show a quenching of $S_L$ consistent with medium modified form-factors.

$$S_L(|q|) = \int_{\omega^+} |q| \frac{R_L(\omega, |q|)}{Z \tilde{G}_{Ep}^2(Q^2) + N \tilde{G}_{En}^2(Q^2)}$$

|q_{eff}| is |q| corrected for a nuclei dependent mean coulomb potential.

Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.
PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of $S_L$ consistent with medium modified form-factors.

- Very little data above $|q|$ of 600 MeV/c, where the cleanest signal of medium effects should exist!

- Saclay, Bates limited in beam energy reach up to 800 MeV.

- SLAC limited in kinematic coverage of scattered electron at $|q|$ below 1150 MeV/c.

$|q_{\text{eff}}|$ is $|q|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.
THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

- Located in Newport News, Virginia
- Four main experimental halls
- Recently completed upgrade allows electron beam energies up to 12 GeV
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THE COULOMB SUM RULE IN NUCLEI: GHP 2021
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THE COULOMB SUM RULE INNUCLEI: GHP 2021
**EXPERIMENTAL DESIGN**

- Need $R_L$ → Use Rosenbluth separation!

$$S_L(|q|) = \int_{\omega^+}^{|q|} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

- Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of $R_L$!

- Need data for each angle at a constant $|q|$ over an $\omega$ range starting above the elastic peak up to $|q|$. When running a single arm experiment with fixed beam energy and scattering angle, $|q|$ is NOT constant over your momentum acceptance.

- Need to take data at varying beam energies, and “map-out” $|q|$ and $\omega$ space.

**THE COULOMB SUM RULE IN NUCLEI**

12C

$|q| = 650$ MeV/c

$\omega = 300$ MeV

Slope = $\frac{Q^4}{4|q|^4} R_L$

Intercept = $\frac{Q^2}{2|q|^2} R_T$
EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c

CSR calculated at constant $|q|$ !!

$$S_L(|q|) = \int_{\omega^+}^{q} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$
EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c
- Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$.

q vs. $\omega$ coverage for 15 degree Iron data
If one wants to measure from 100 to 600 MeV \( \omega \) at constant \(|q| = 650\) MeV/c

- Take data at different beam energies, and interpolate to determine cross-section at constant \(|q|\).
- \(|q|\) can be selected between 550 and 1000 MeV/c

Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.
EXPERIMENTAL DESIGN

If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c

▸ Take data at different beam energies, and interpolate to determine cross-section at constant |q|.

|q| can be selected between 550 and 1000 MeV/c

- $E_{\text{beam}} = 1.26 \text{ GeV}$
- $E_{\text{beam}} = 1.65 \text{ GeV}$
- $E_{\text{beam}} = 2.15 \text{ GeV}$
- $E_{\text{beam}} = 2.45 \text{ GeV}$
- $E_{\text{beam}} = 2.85 \text{ GeV}$
- $E_{\text{beam}} = 3.68 \text{ GeV}$

Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.

q vs. ω coverage for 15 degree Iron data
**EXPERIMENTAL SPECIFICS**

- **E05-110:**
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: \(^{4}\text{He}, ^{12}\text{C}, ^{56}\text{Fe}, ^{208}\text{Pb}\).
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**THE COULOMB SUM RULE IN NUCLEI**

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All these spectra again for the RHRS
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And again (LHRS & RHRS) for $^4$He, $^{56}$Fe, and $^{208}$Pb
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All these spectra again for the RHRS
And again (LHRS & RHRS) for $^4$He, $^{56}$Fe, and $^{208}$Pb
Aiming for 1% to 2% precision on XS
EXPERIMENTAL CHALLENGES

- Data collection for many different settings!
  - Special calibration data needed to be strategically scheduled.
    - Elastic runs and optics runs could not be reasonably done for all settings, so improved algorithms needed to be developed to apply calibrations to all run-settings.
  - Very low momentum settings push the limits of the HRS spectrometers.
    - Negligible effects in high momentum settings must be considered with care.
      - Hysteresis effects and magnetic field read-back on the central momentum of the spectrometer
      - Backgrounds from magnet-rescattering.
    - Extra care needed for calculations of multiple-scattering and radiation loss.
  - Accurate interpolation routines needed to be developed for both radiative corrections and general analysis.
  - Systematic uncertainty analysis of each spectrometer arm independently.
ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS

- Blue histograms are reconstructed data.
- Red histograms are monte-carlo:
  - Event sample generated from expected XS calculations (Fourier-Bessel fit to world data)
  - Radiative effects (internal, external, vertex) are handled, including exact bremsstrahlung distributions.
- Resolution effects are applied by calculating the expected material effects of tracks passing through the VDC chamber materials.
12C elastic XS at 1260 MeV, 15 degrees

< 1% deviation from calculation
ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS

$^{12}$C elastic XS at 400 MeV, 35 degrees

< 1 % deviation from calculation
Extractions of excited elastic states based on fit of transition form-factors to world data.

Functional form follows an analytic, global, and model-independent analysis introduced recently* (mostly in the study of the $0^+_2$ "Hoyle" state)

\[
F(q) = \frac{1}{Z} e^{-\frac{1}{2}(bq)^2} \sum_{n=1}^{n_{\text{max}}} c_n (bq)^{2n}
\]

EXCITED ELASTIC STATES

Data
Sim

Beam Energy: 400 MeV
Angle: 26 degs
Target: $^{12}$C

Beam Energy: 1260 MeV
Angle: 15 degs
Target: $^{12}$C

Excited states calculation allows for:

A) More accurate elastic + excited tail subtraction

B) Subtraction and access to lower energy transfer

Higher States (not calculated)
Continuum to Quasi-elastic estimation
Elastic Peak

COMPARISON TO WORLD DATA

- By using our available $\omega / |q|$ space over 4 angles, we can interpolate to any $\omega / |q|$, and use Rosenbluth fits to calculate at a given angle.

Iron Comparisons

- **56Fe**
  - Beam-E: 3300 MeV
  - Angle: 15 deg
  - World Data: Chen 1990
  - Hall-A CSR interpolated

- **56Fe**
  - Beam-E: 2020 MeV
  - Angle: 20 deg
  - World Data: Day 1998
  - Hall-A CSR interpolated

- **56Fe**
  - Beam-E: 440 MeV
  - Angle: 90 deg
  - World Data: Meziani 1984
  - Hall-A CSR interpolated
By using our available $\omega/|q|$ space over 4 angles, we can interpolate to any $\omega/|q|$, and use Rosenbluth fits to calculate at a given angle.
COMPARISON TO WORLD DATA

- By using our available $\omega / |q|$ space over 4 angles, we can interpolate to any $\omega / |q|$, and use Rosenbluth fits to calculate at a given angle.

**Carbon Comparisons**

- Preliminary
  - $^{12}\text{C}$
  - Beam-E: 961 MeV
  - Angle: 38 deg
  - World Data: Sealock 1989
  - Hall-A CSR interpolated

- Preliminary
  - $^{12}\text{C}$
  - Beam-E: 2020 MeV
  - Angle: 15 deg
  - World Data: Day 1998
  - Hall-A CSR interpolated

- Preliminary
  - $^{12}\text{C}$
  - Beam-E: 2222 MeV
  - Angle: 15.54 deg
  - World Data: Dai 2018
  - Hall-A CSR interpolated

THE COULOMB SUM RULE IN NUCLEI: GHP 2021
SOME RESULTS

- Three of the four targets ($^{12}$C, $^{56}$Fe) have been analyzed extensively.
  - $^{208}$Pb and $^4$He need extended target analysis (preliminary results exist for $^4$He, but are not shown here)
  - $^{208}$Pb needs analytic calculations for the Coulomb Correction, plus proper LH2 subtraction
- Large $\omega$ is the region where sensitivity to systematics is most pronounced. This is especially true in the region above the quasi-elastic peak (the "dip" and $\Delta$ region where $R_T$ dominates).
- This area needs higher order corrections to reduce systematic uncertainty. Some of these studies are ongoing.
We have some coverage at the lowest end of $|q|$ to compare results at $|q| = 550$ MeV/c with prior calculations.

Note: This is on the edge of our available phase space. Most slopes are calculated with only 2 or 3 angles.
COMPARISONS TO EXISTING SUM-RULE CALCULATIONS

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- For Iron:
COMPARISONS TO EXISTING SUM-RULE CALCULATIONS

- We have some coverage at the lowest end of \(|q|\) to compare results at \(|q| = 550\) MeV/c with prior calculations.
  - Note: This is on the edge of our available phase space. Most slopes are calculated with only 2 or 3 angles.
- For Carbon: we have very nice agreement with Barreau, et. al.
- For Iron: we also have nice agreement with Meziani, et. al.
We can look at various larger $|q|$ values, here I will discuss $|q| = 650$ MeV/c and $|q| = 800$ MeV/c.
We can look at various larger $|q|$ values, here I will discuss $|q| = 650$ MeV/c and $|q| = 800$ MeV/c.

The highest omega portion of each spectra has uncertainties dominated by the linearity of the Rosenbluth fit.

Systematic effects in this region are still being investigated.
The Coulomb Sum Rule can be used to directly compare the integrated electric response of nucleons bound in a nucleus to the expected incoherent sum of electric structure of unbound nucleons:

- In this way, we get a model independent test of medium-modification of nucleon structure with a purely electromagnetic probe.

Early experiments reported a quenching of the CSR in heavy nuclei, since then theoretical interest has remained high.

Experiment E05-110 was run at Jefferson Lab to investigate a critical region of $|q|$ where no world data existed.

Latest results show an agreement with earlier calculations at $|q| = 550$ MeV/c.

The longitudinal response for larger $|q|$ have been measured, but the especially sensitive large omega region of the response is still being investigated with hopes of correcting systematic effects and reducing the uncertainty of the total sum.
THANK YOU!!!


PhD Students  Spokespersons  Run Coordinators  Special Contributors  PEOPLE

Hall-A collaboration