Hadron resonances from lattice QCD

David Wilson

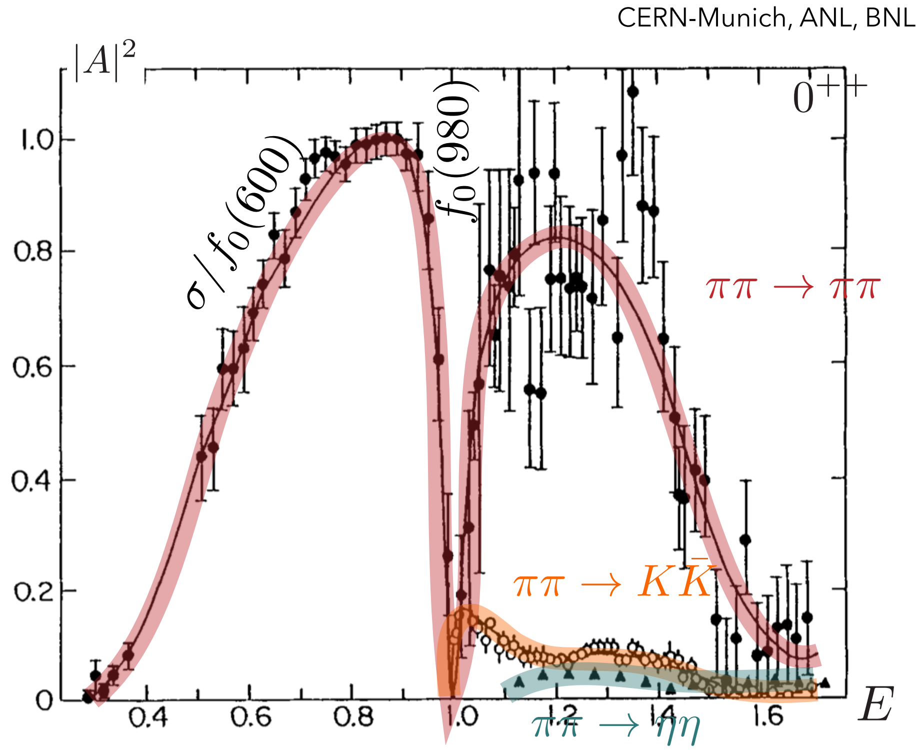
hadspec.org

GHP Meeting
Tuesday 13th April 2021

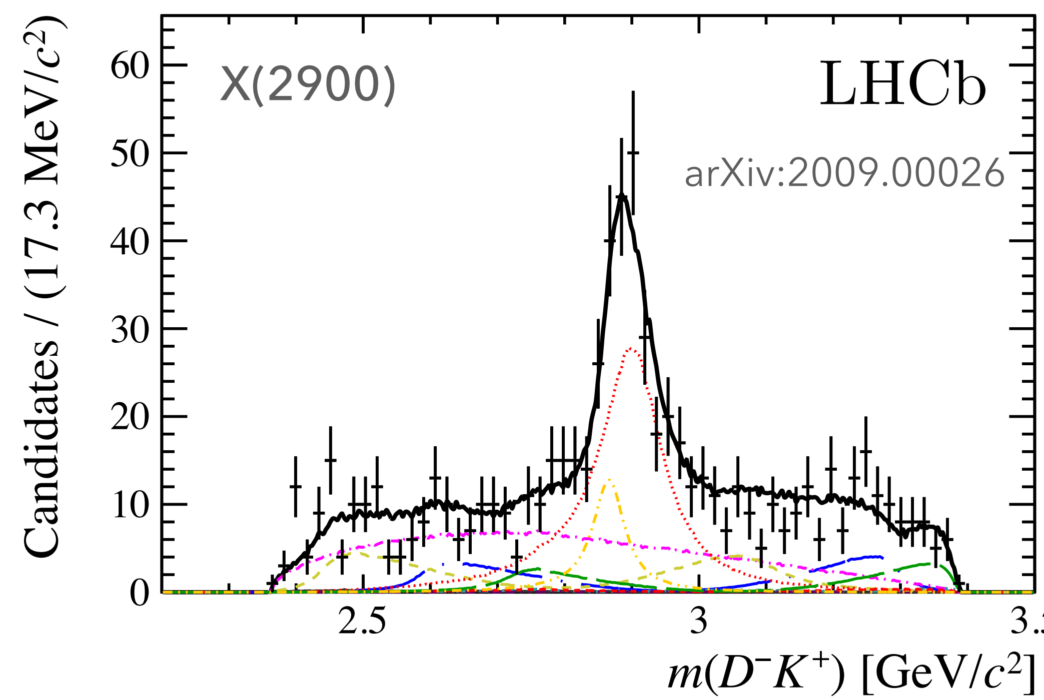
UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

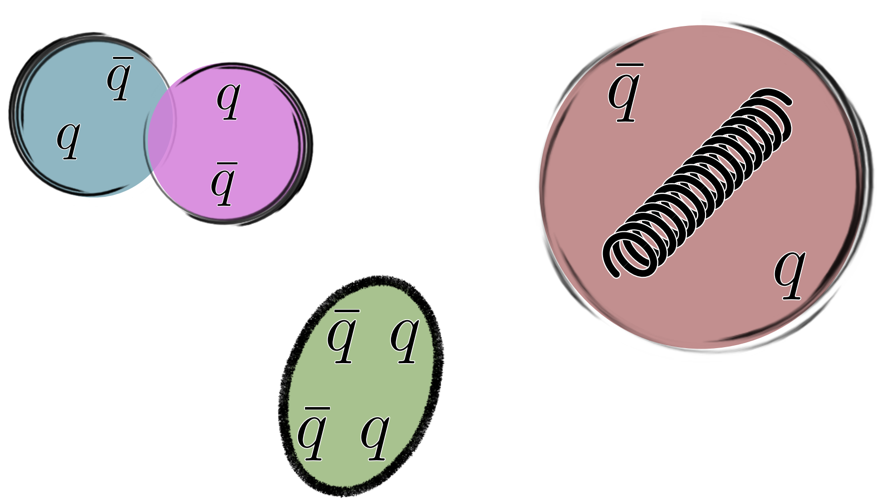
spectroscopy from first-principles is a hard problem



many puzzles - new and old



the quark model is a good guide for low-lying states



models can be useful, but what does QCD say?

provides a rigorous approach to hadron spectroscopy

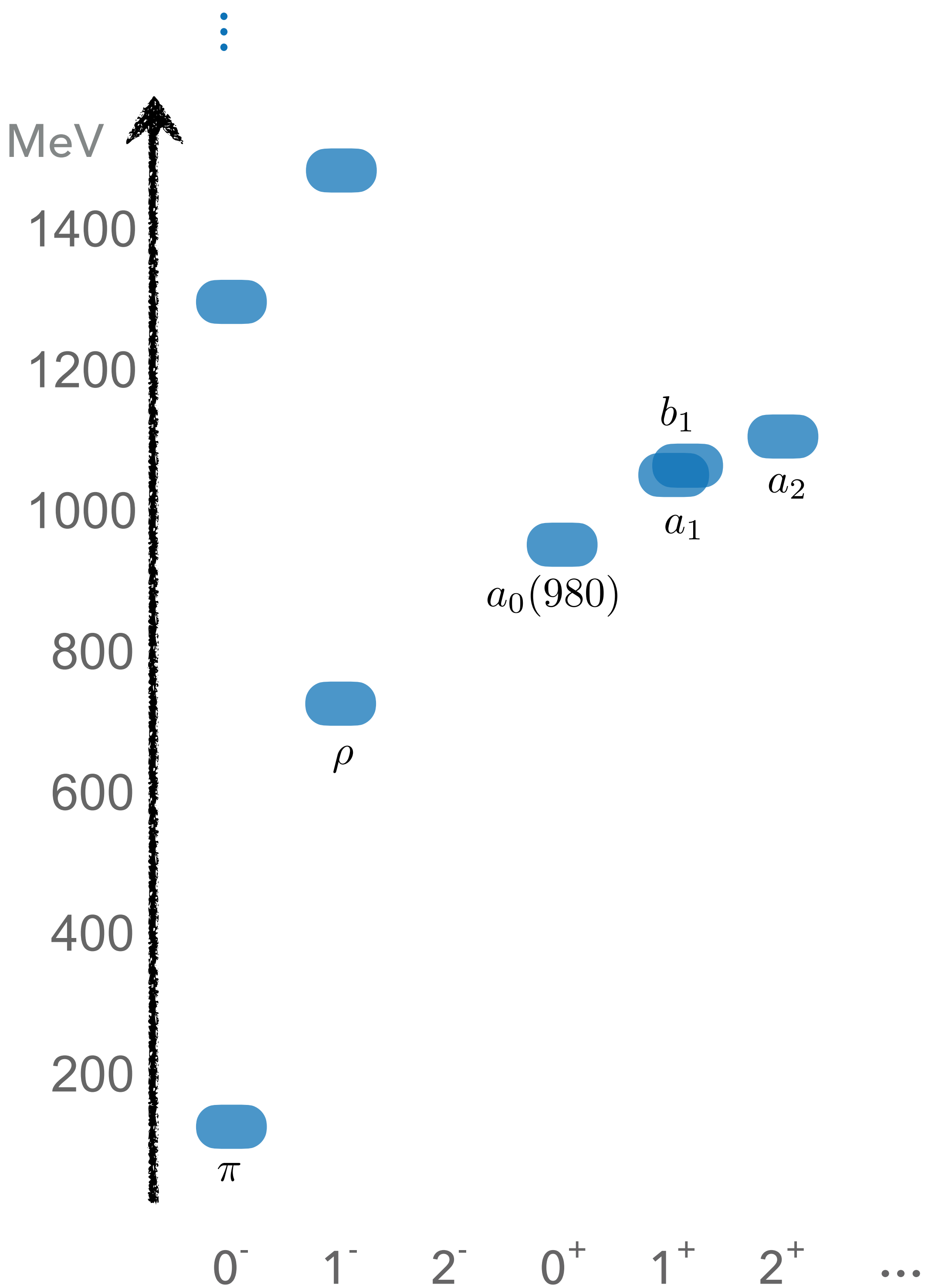
- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude
- minimal inputs (quark masses)

tremendous progress in recent years
but not yet ready for precision comparisons

- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- small effects not considered in general:

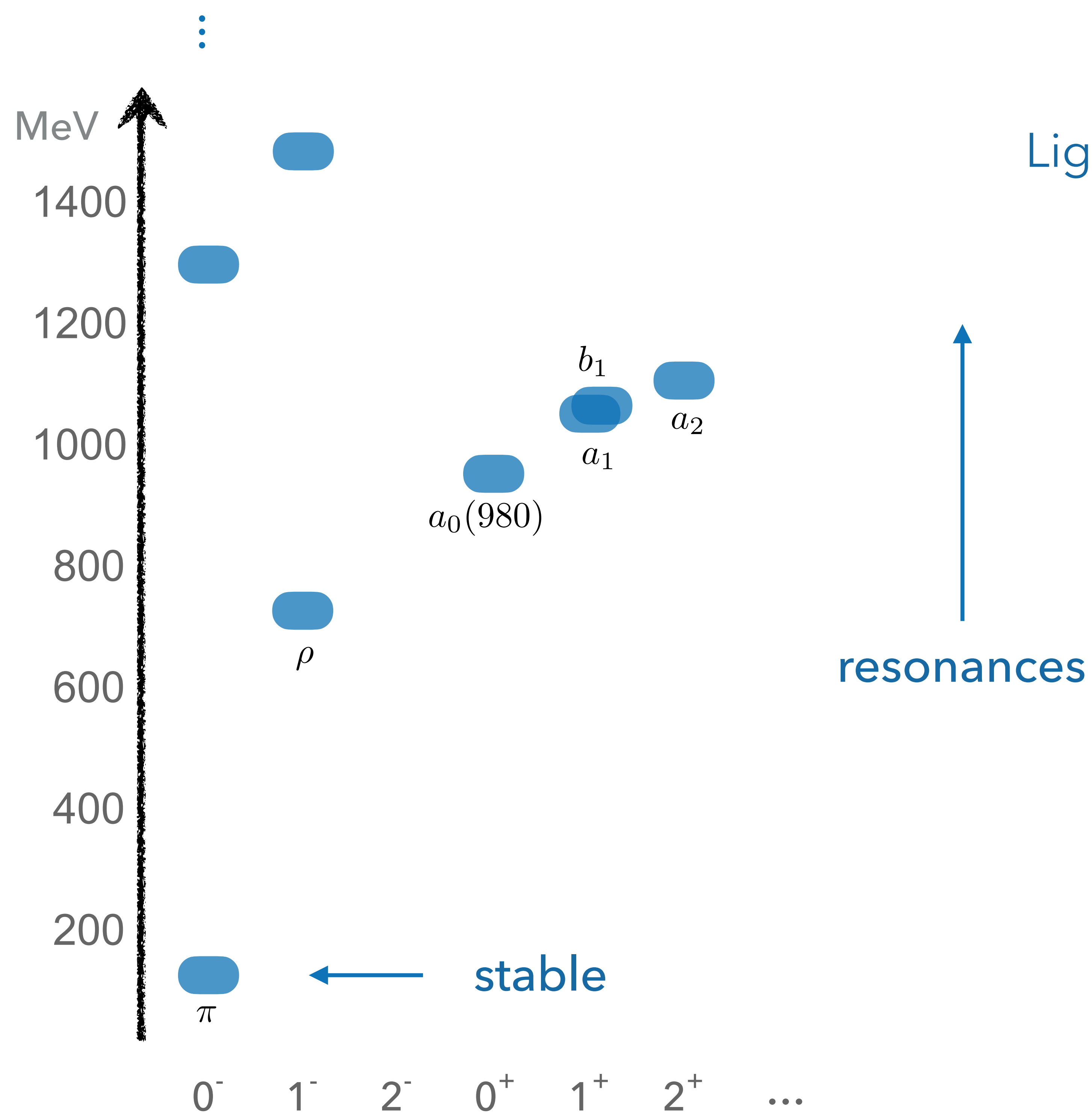
  finite lattice spacing, isospin breaking, EM interactions

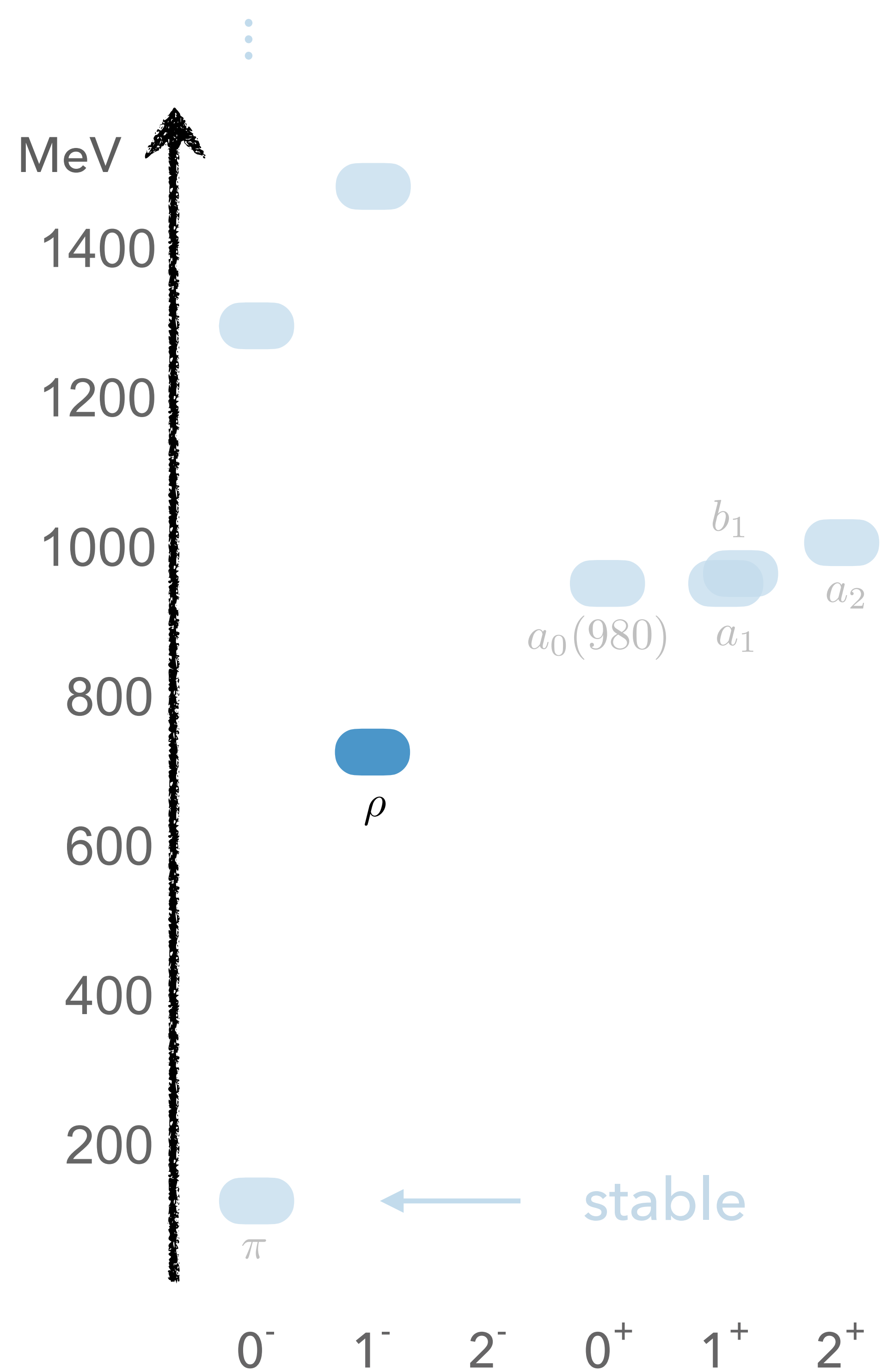
goal: what does QCD say about the excited hadron spectrum?

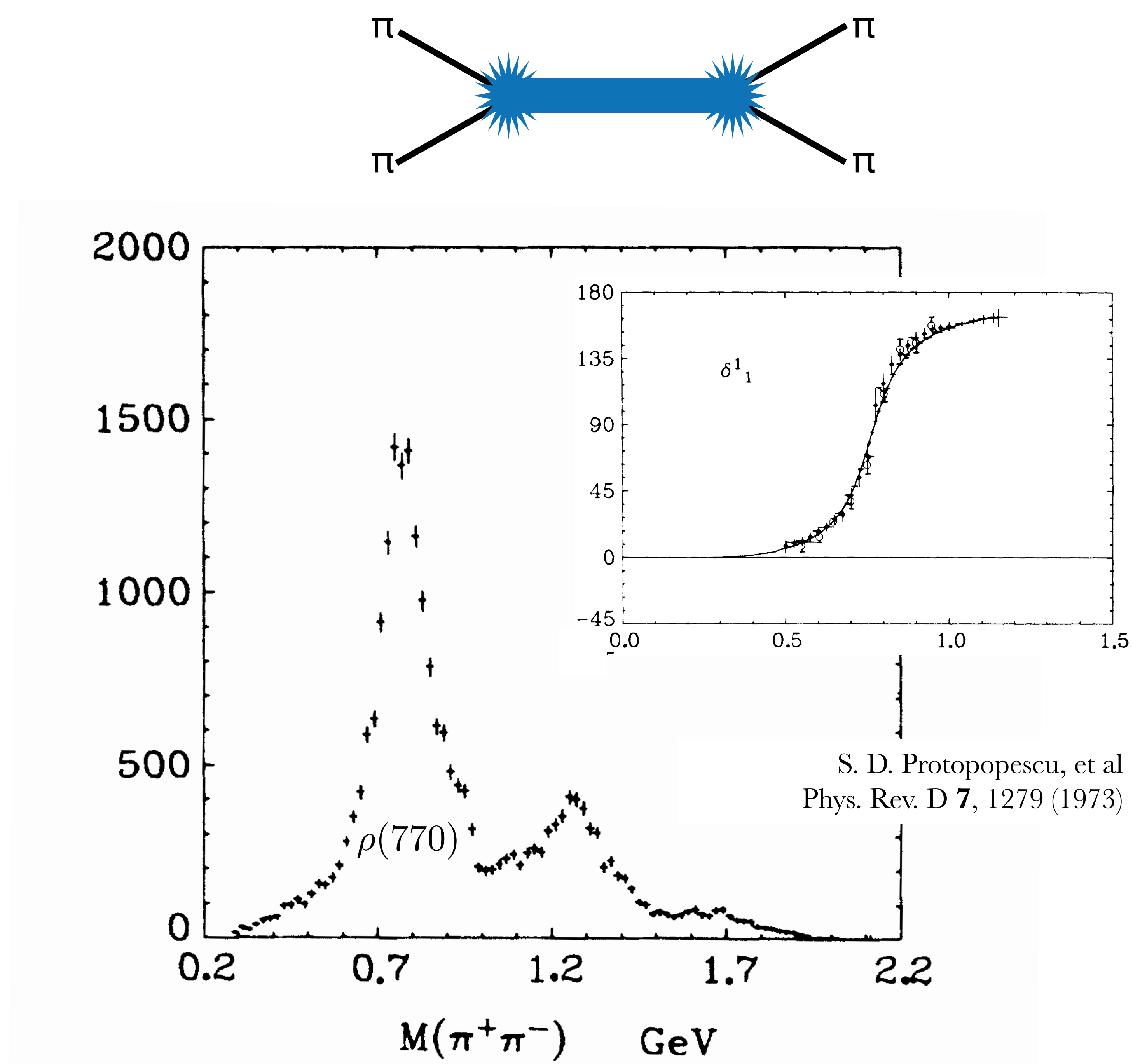


Light mesons with Isospin=1

Light mesons with Isospin=1







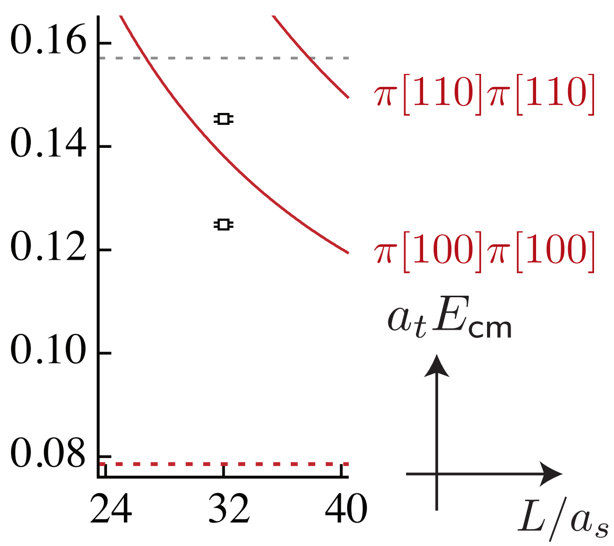
S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)

"most rigorous" quantity for unstable states

$$t \sim \frac{c^2}{s_{\text{pole}} - s} \qquad \sqrt{s_{\text{pole}}} = m \pm \tfrac{i}{2}\Gamma$$

$m_\pi = 239$ MeV

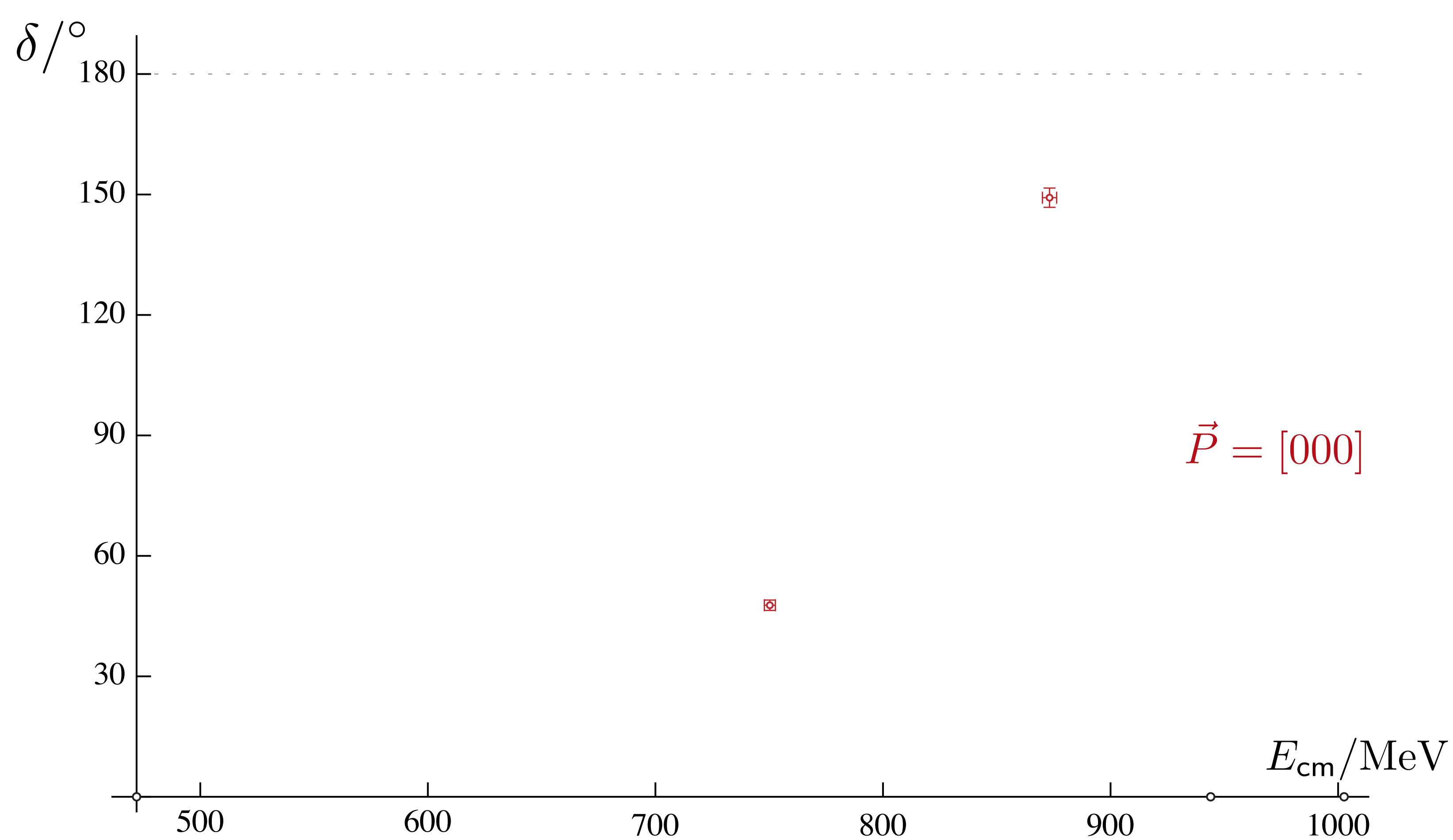
$[000]\ T_1^-$



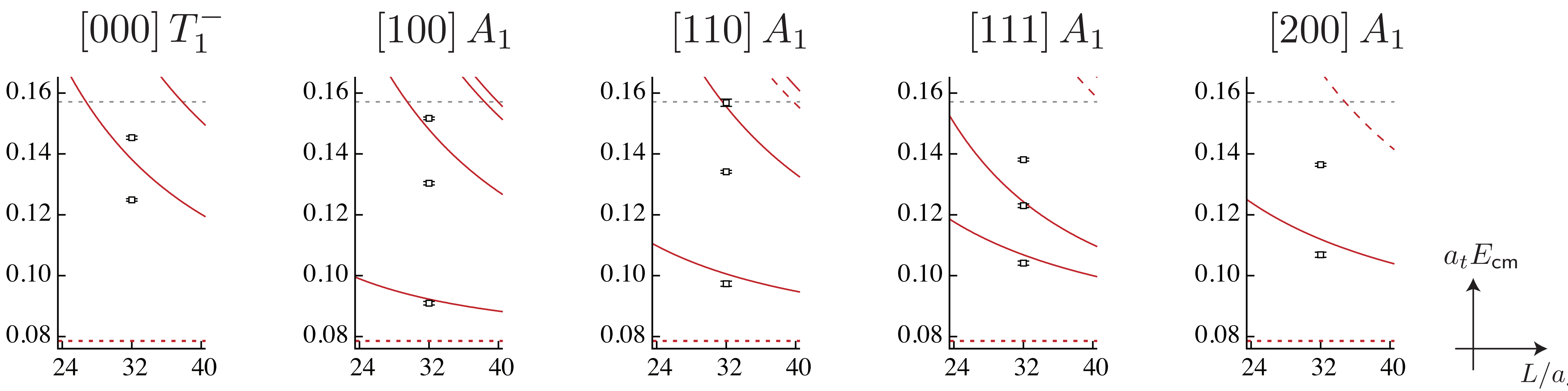
L/a_s=32

momentum is quantized in a finite volume $\vec{p} = \frac{2\pi\vec{n}}{L}$

at-rest

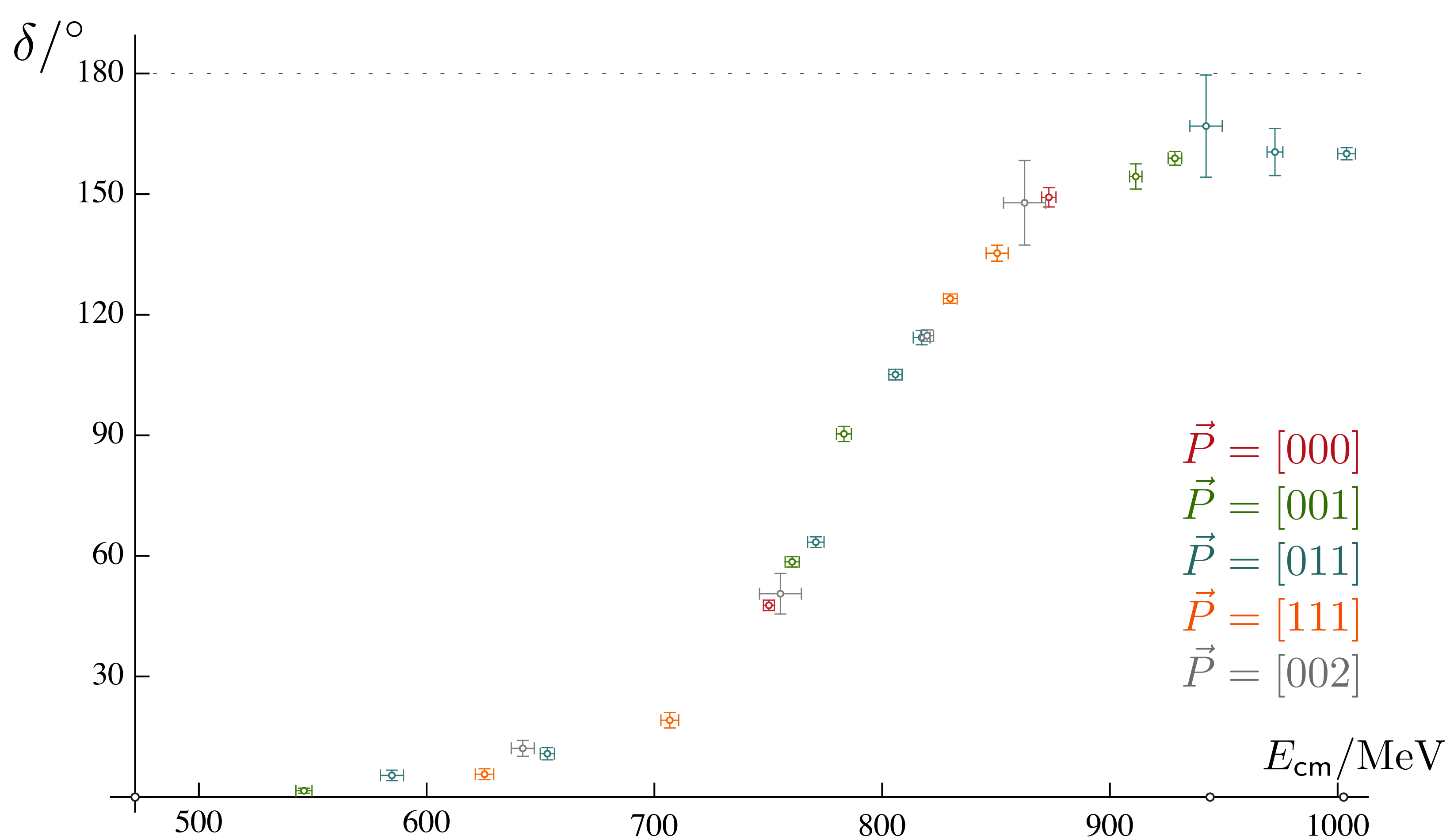


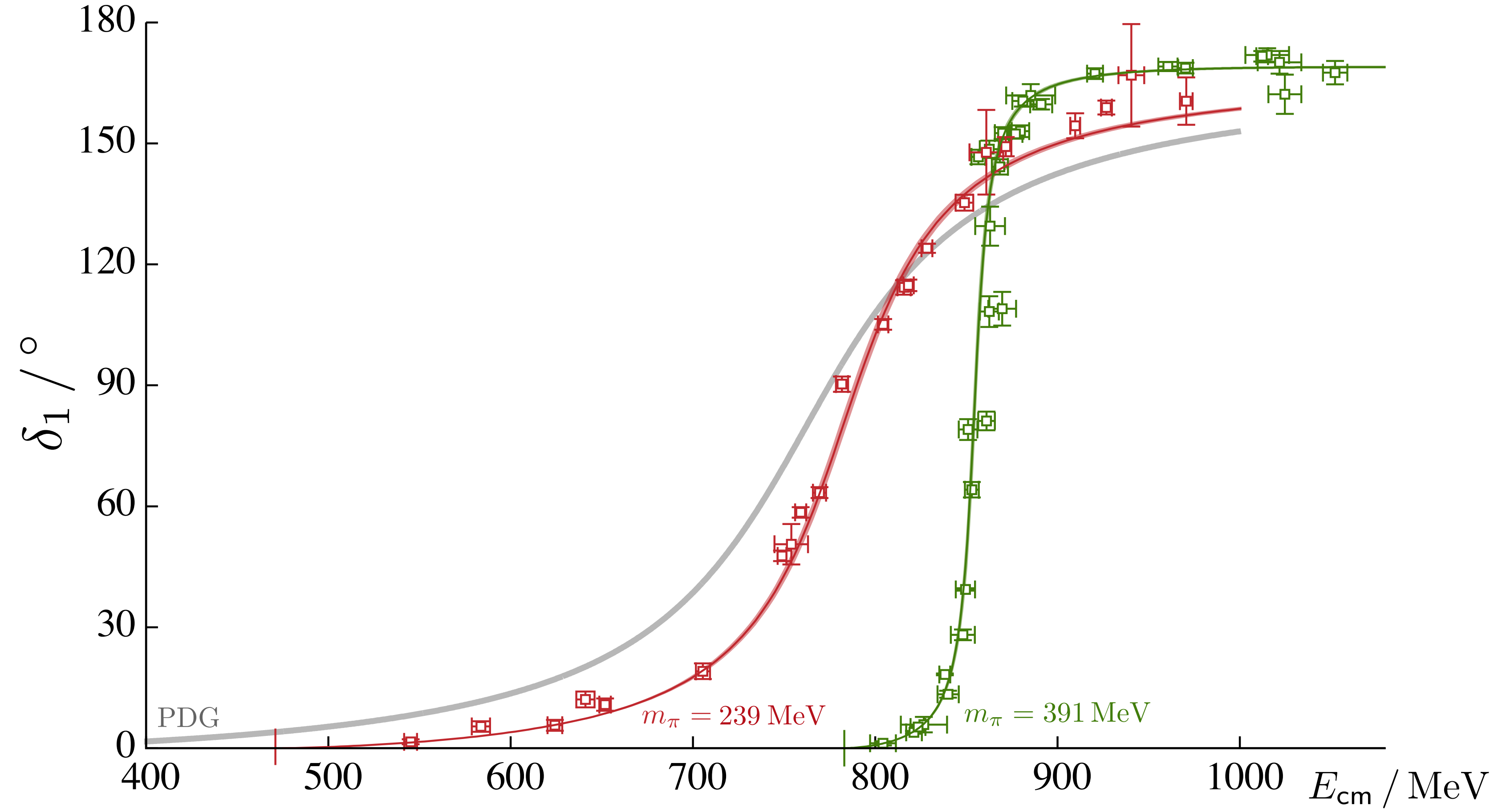
$m_\pi = 239$ MeV



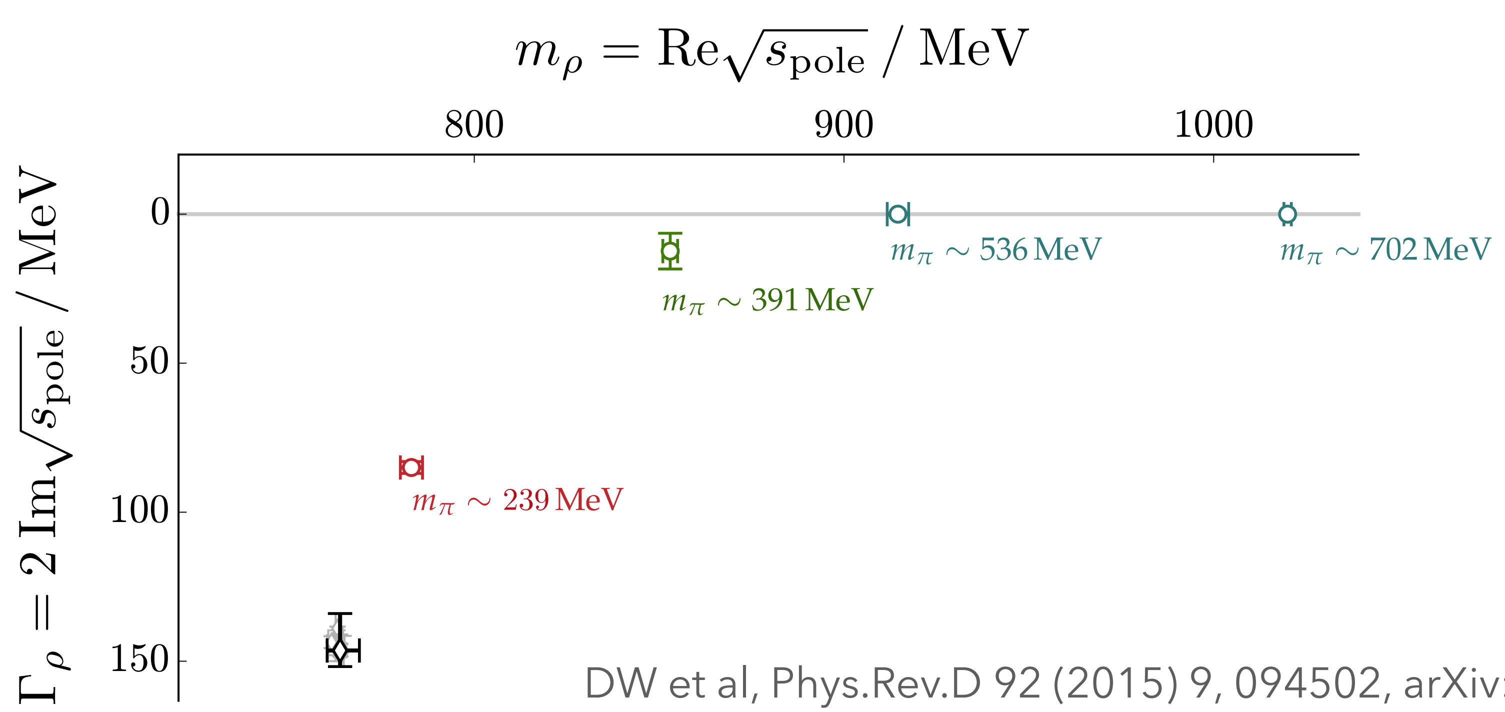
at-rest

non-zero overall momentum

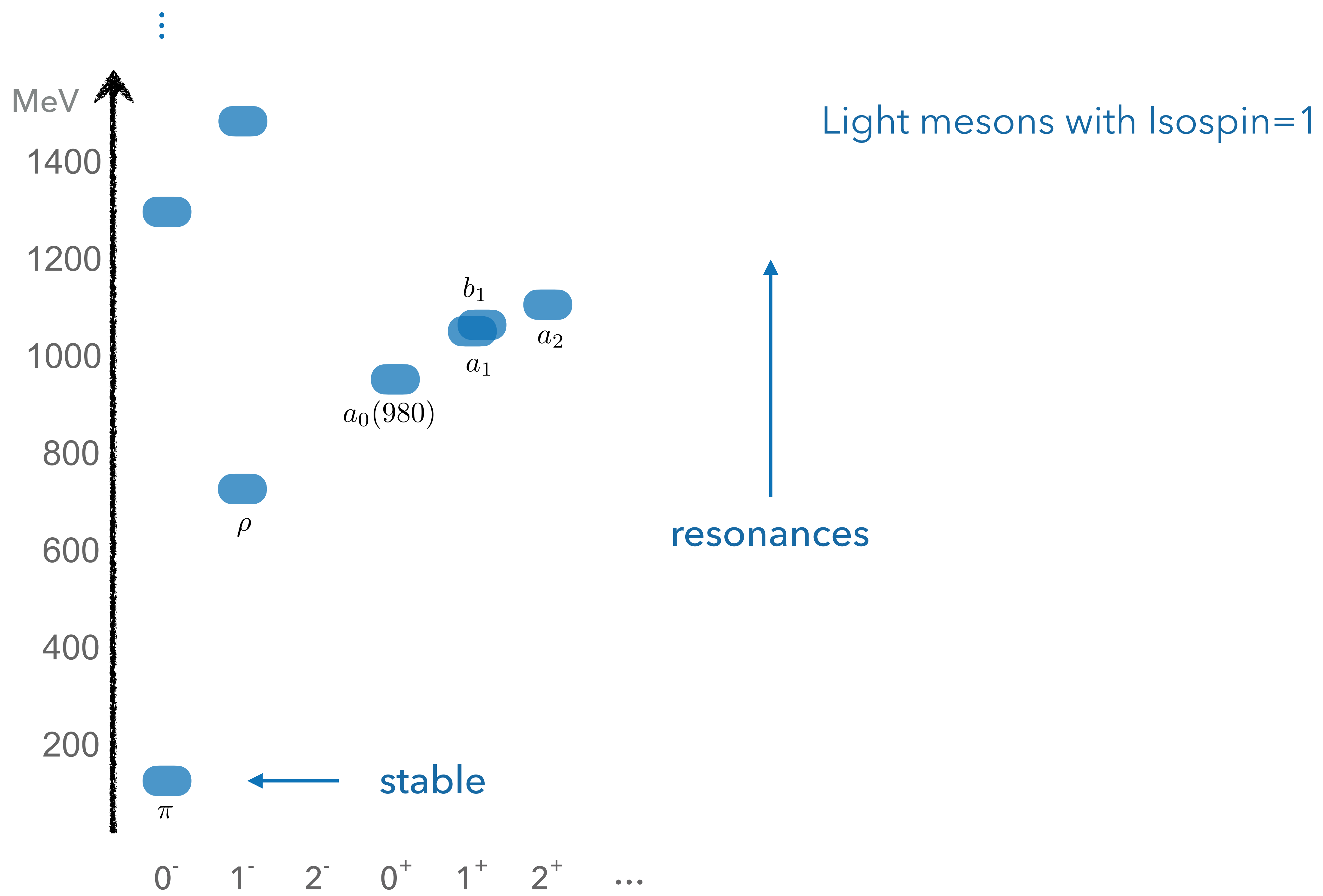


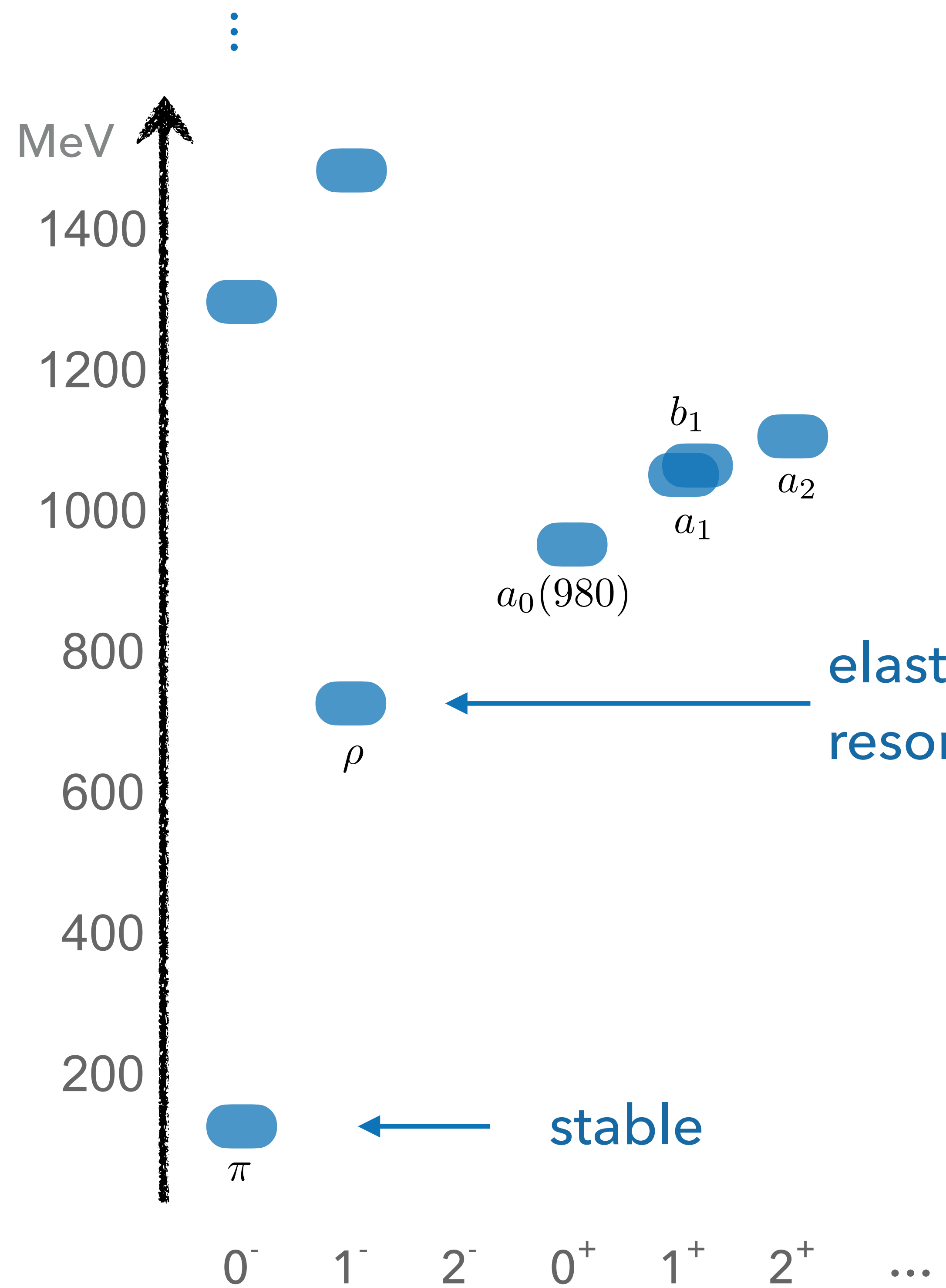


$$t \sim \frac{c^2}{s_{\rm pole} - s}$$



DW et al, Phys.Rev.D 92 (2015) 9, 094502, arXiv: 1507.02599
J. Dudek et al, Phys.Rev.D 87 (2013) 3, 034505, arXiv:1212.0830


MeV
1400
1200
1000
800
600
400
200
b_1
a_2
a_1
$a_0(980)$
ρ
π
stable
resonances
0^- 1^- 2^- 0^+ 1^+ 2^+ ...
Light mesons with Isospin=1



stable

elastic resonances

resonances in multiple channels

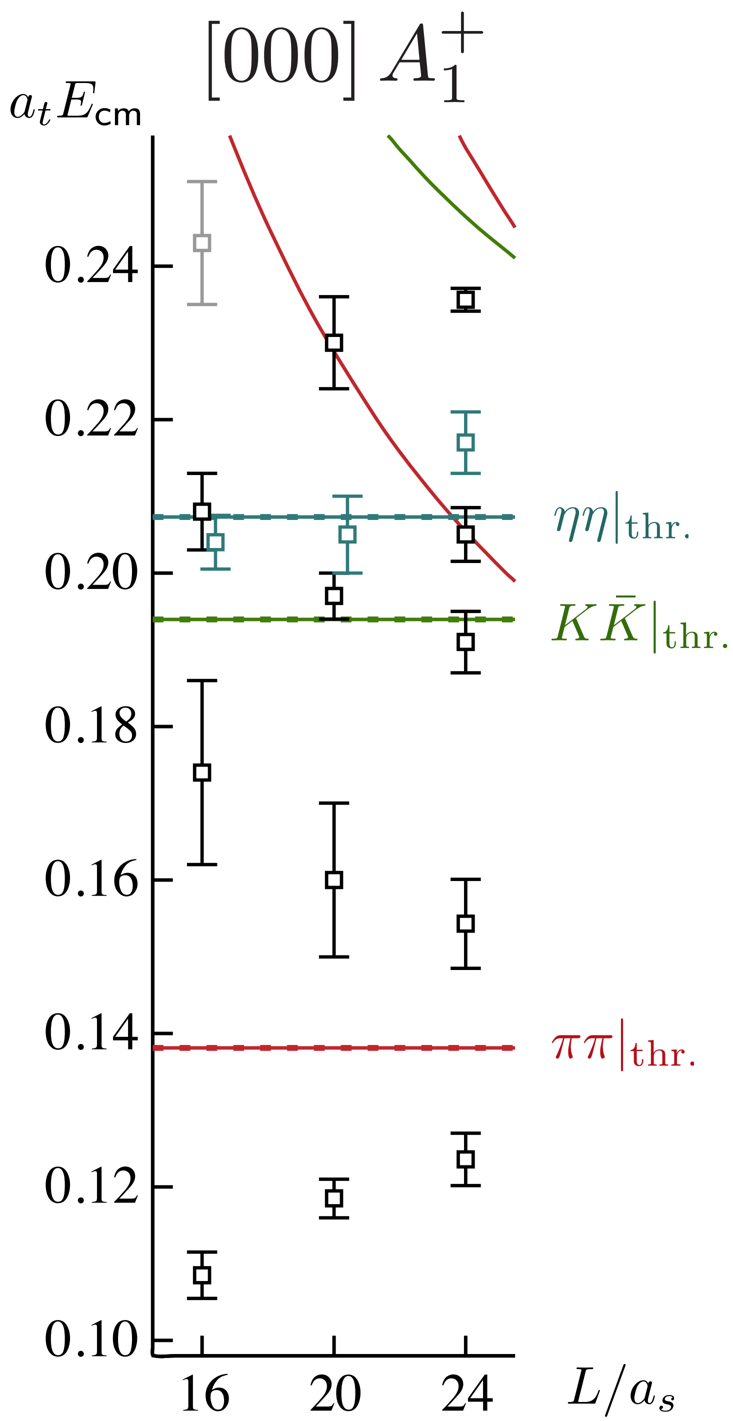
e.g. $\pi\eta \to \pi\eta$

$\to K\bar{K}$

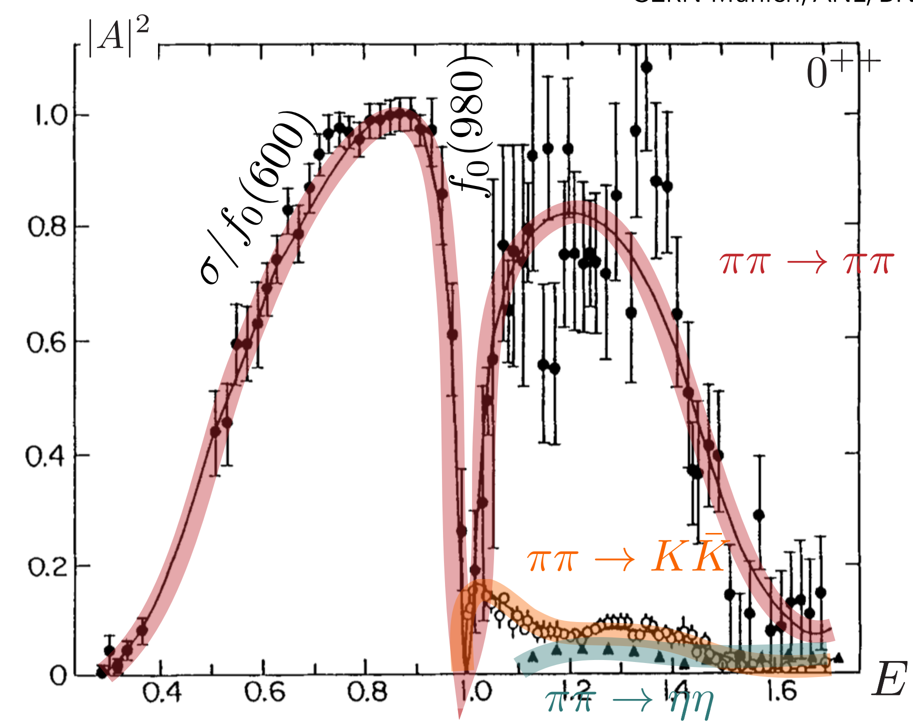
$\pi\pi \to \pi\pi$

$\to K\bar{K}$

$m_\pi = 391$ MeV

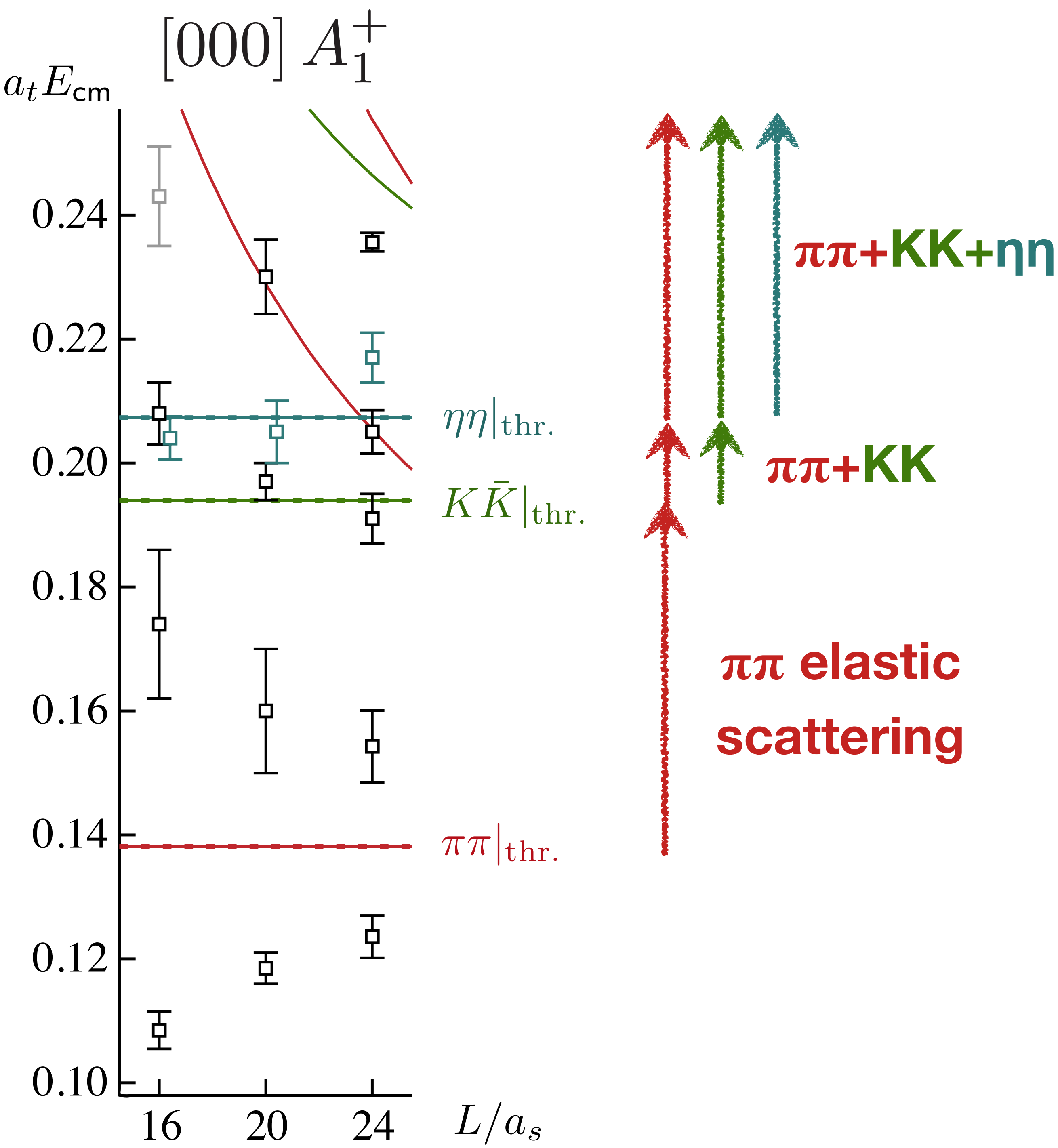


CERN-Munich, ANL, BNL



R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$m_\pi = 391$ MeV



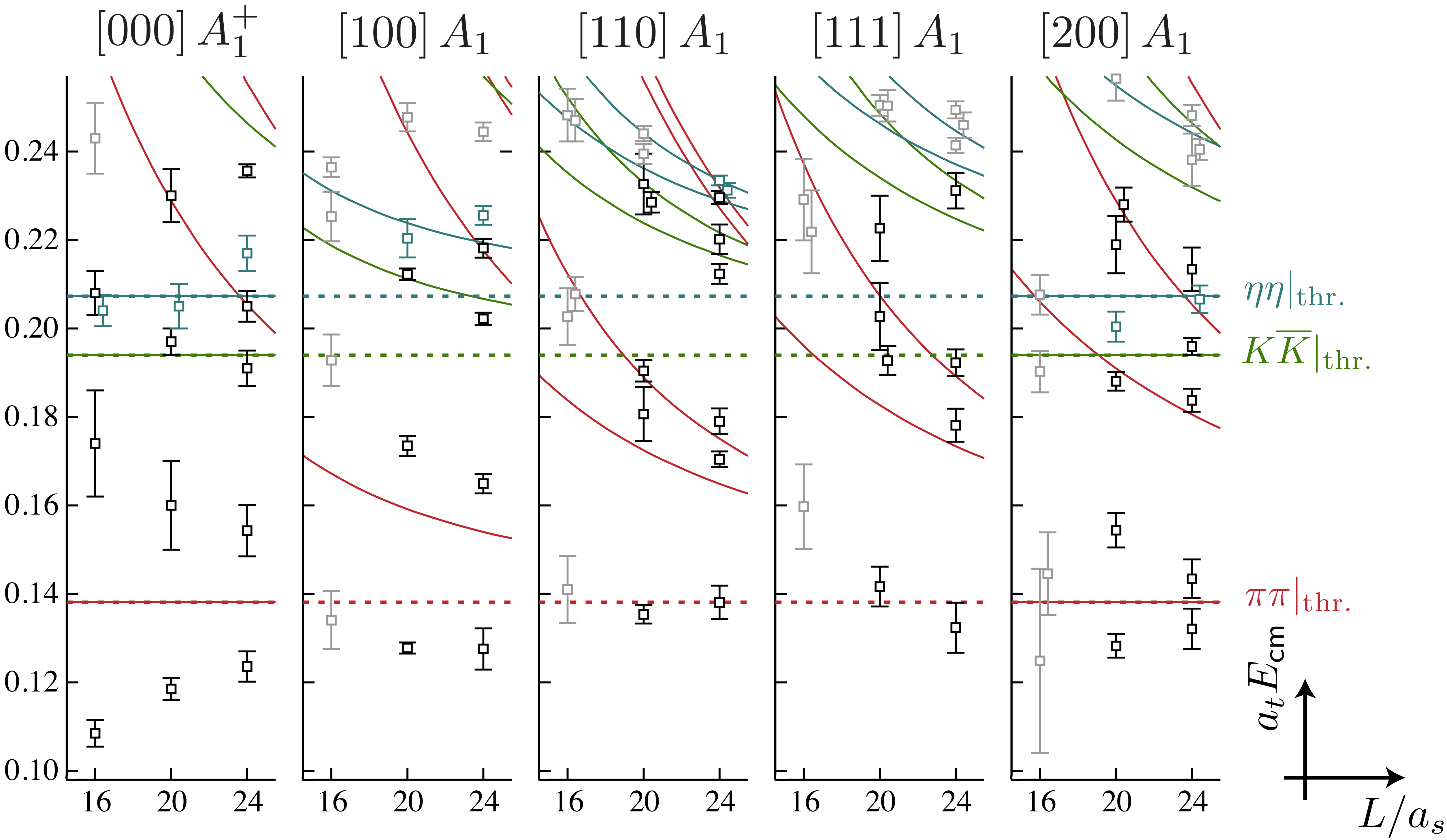
$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ & K\bar{K} \to K\bar{K} \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi \end{pmatrix}$$

R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

local $q\bar{q}$ & 2-hadron operators
conservatively 57 energy levels
dominated by S-wave interactions



R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

Lüscher et al

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

determinant condition:

- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations

- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

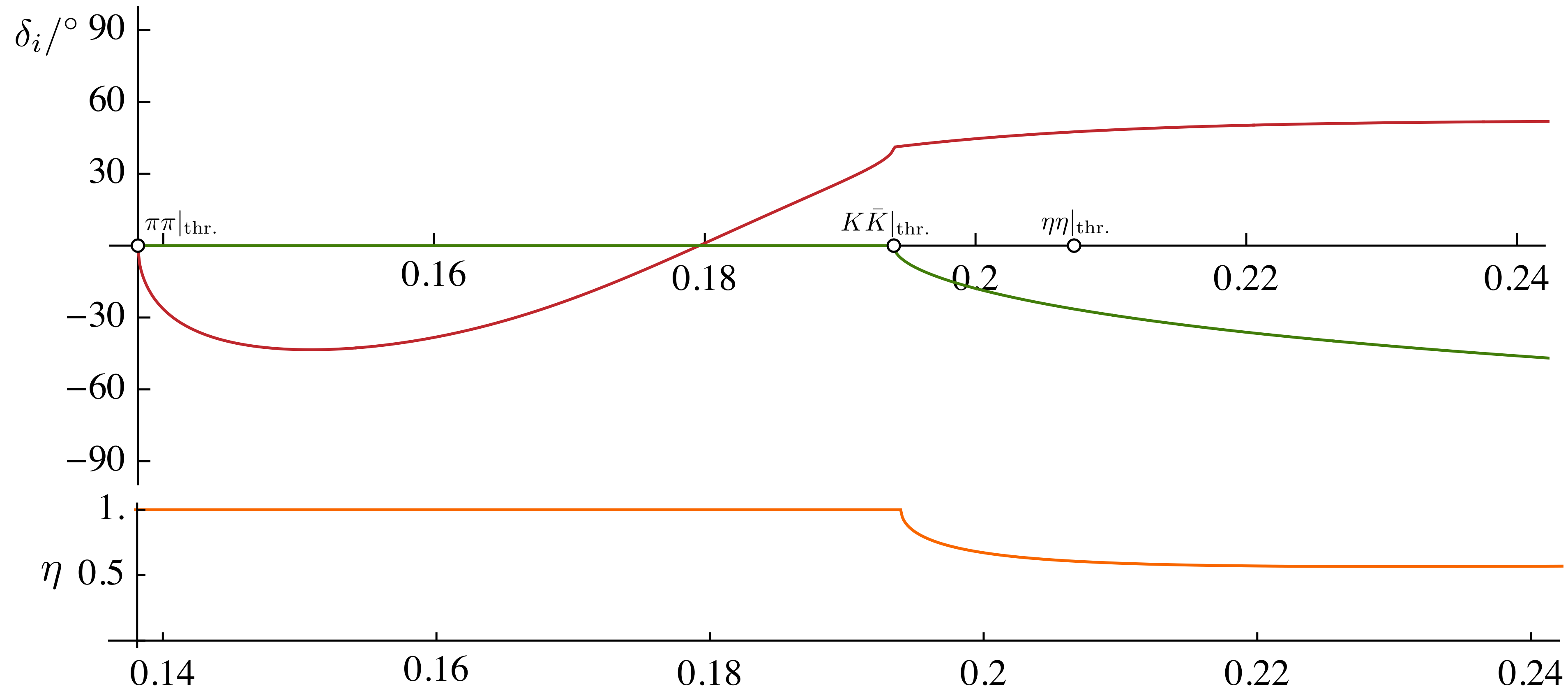
use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

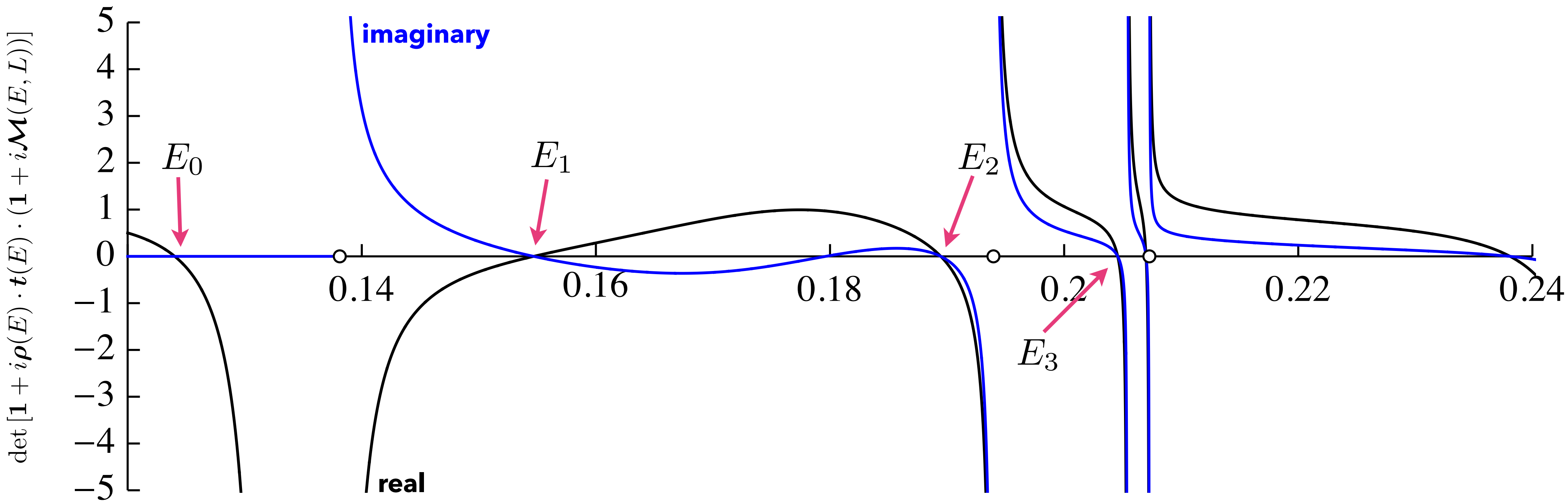
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

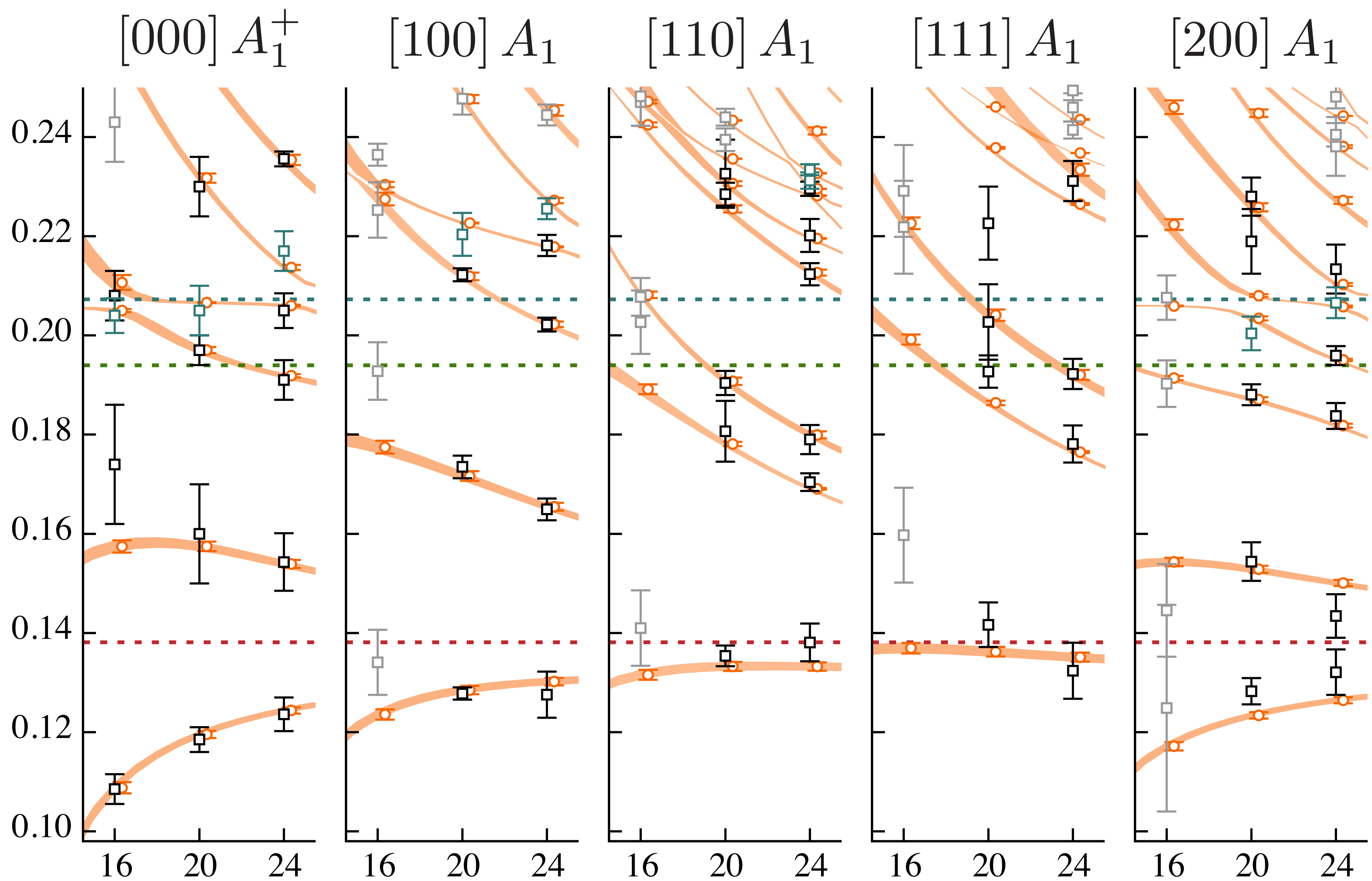
$$S_{ii} = \eta e^{2i\delta_i}$$



we can identify the solutions



$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$



$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

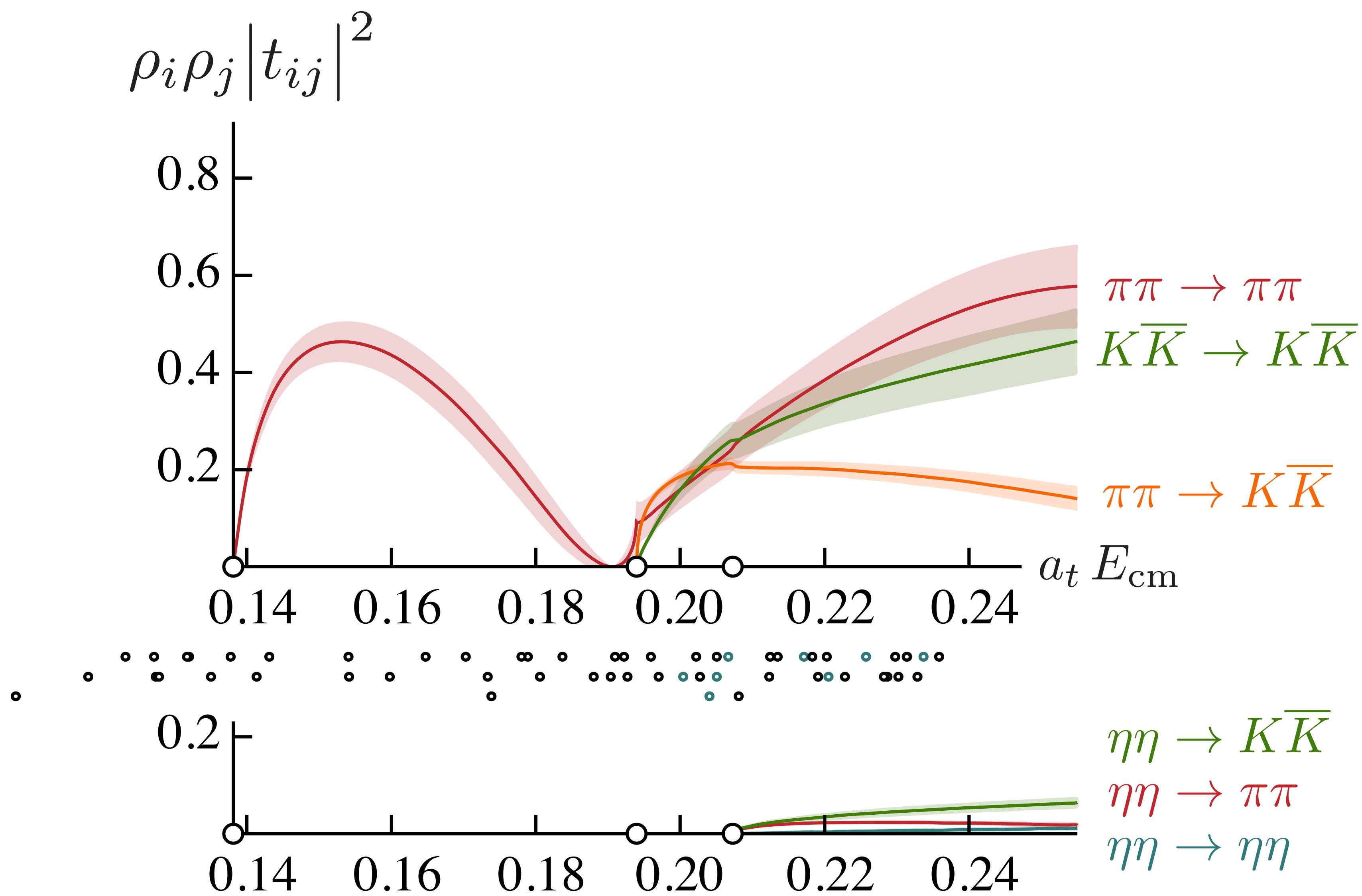
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

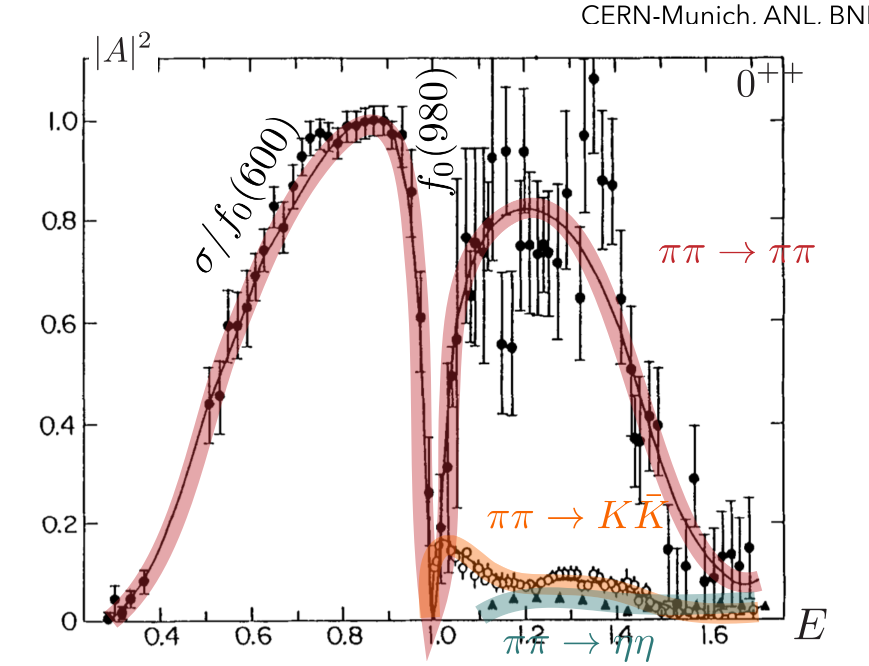
$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\rm dof} = \frac{44.0}{57-8} = 0.90$$

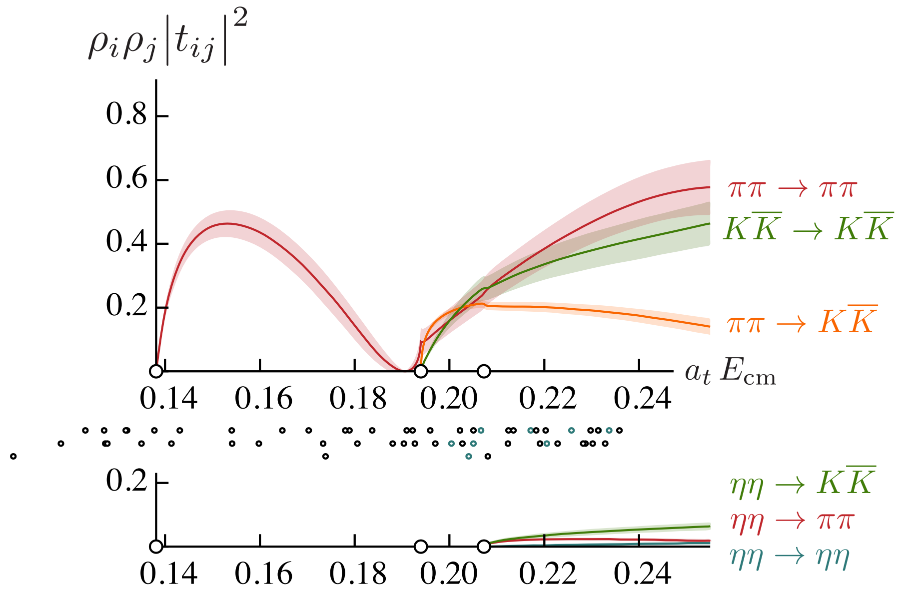
57 energy levels

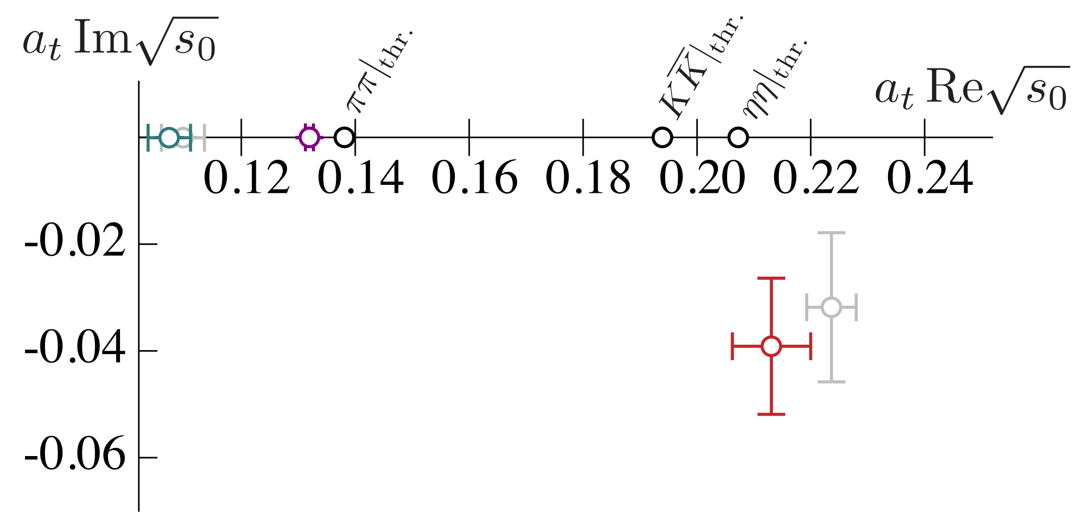


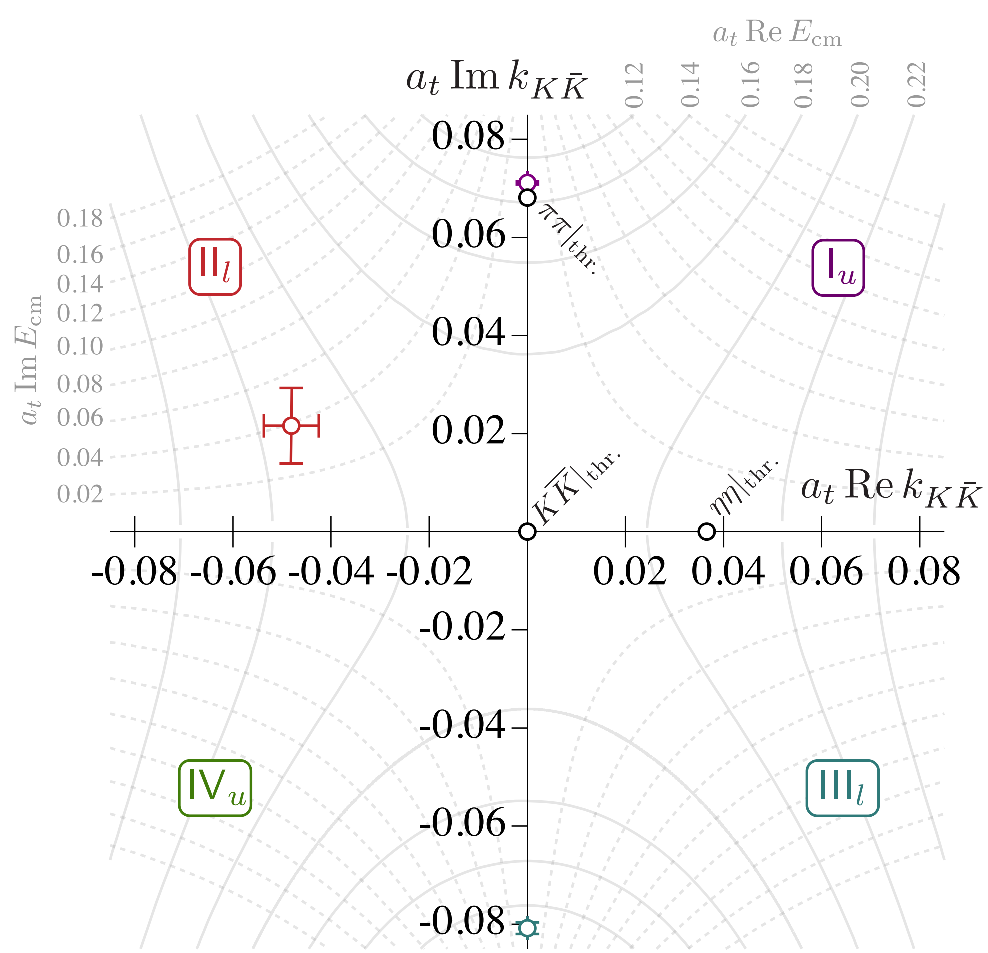
CERN-Munich. ANL. BNL

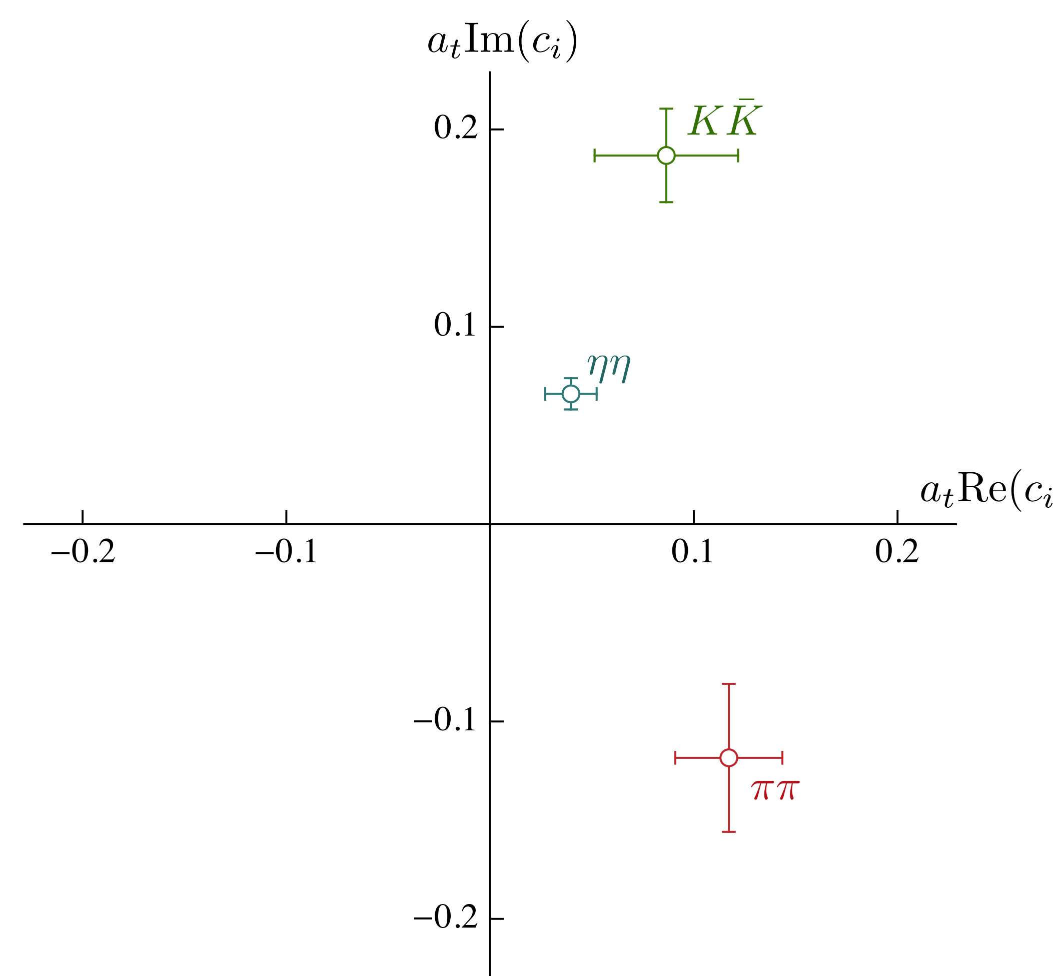


a pole in multiple channels: $t_{ij} \sim \frac{c_i c_j}{s_{\text{pole}} - s}$ $\sqrt{s_{\text{pole}}} = m \pm \frac{i}{2}\Gamma$

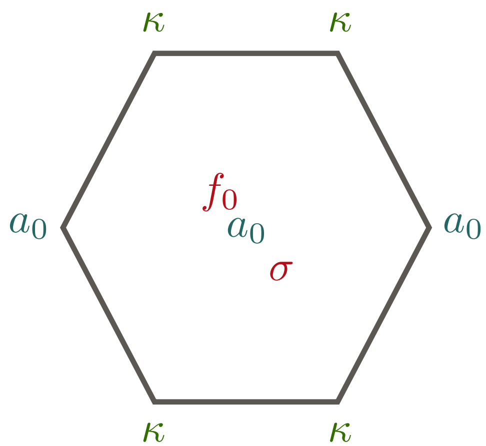


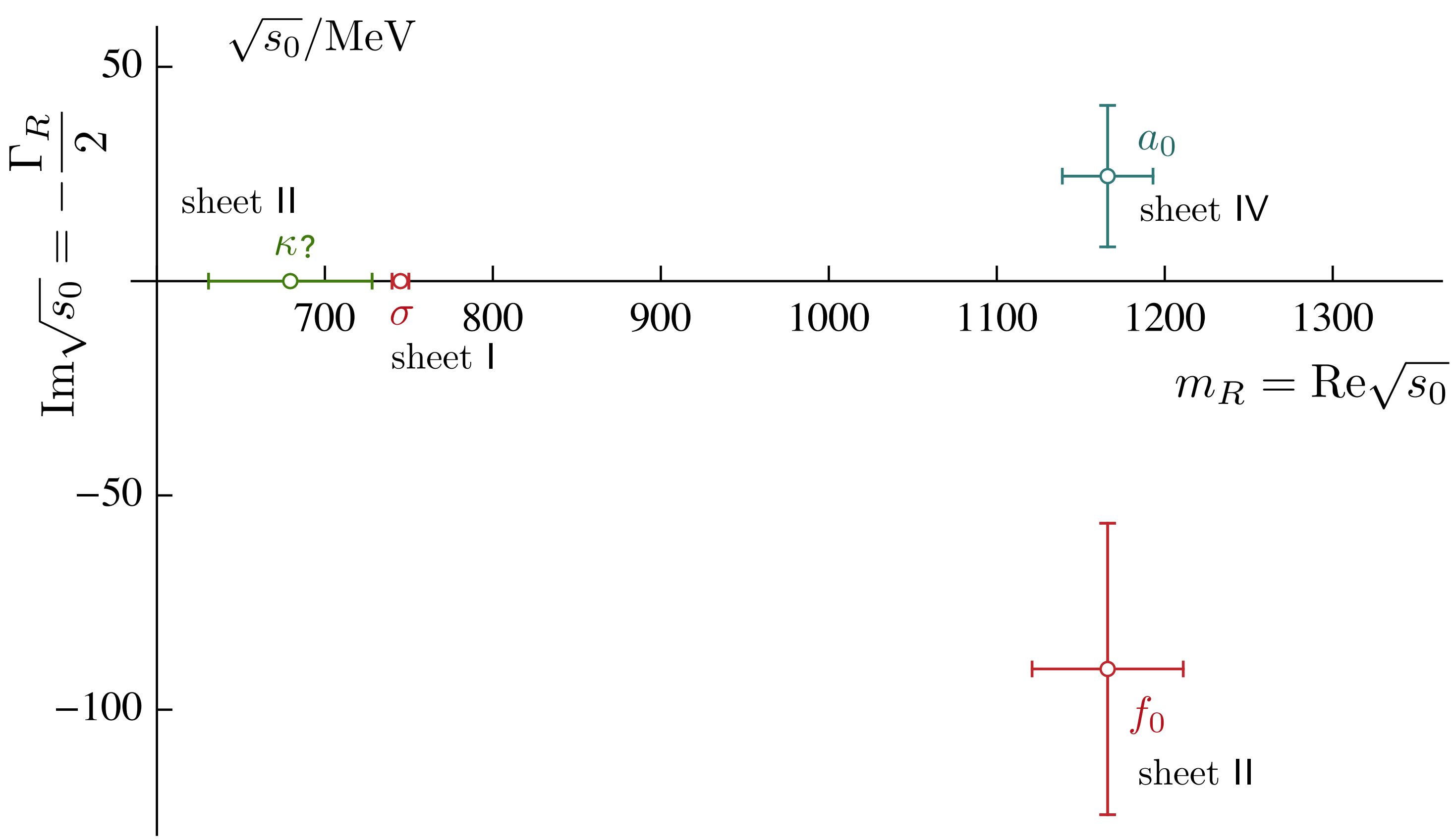


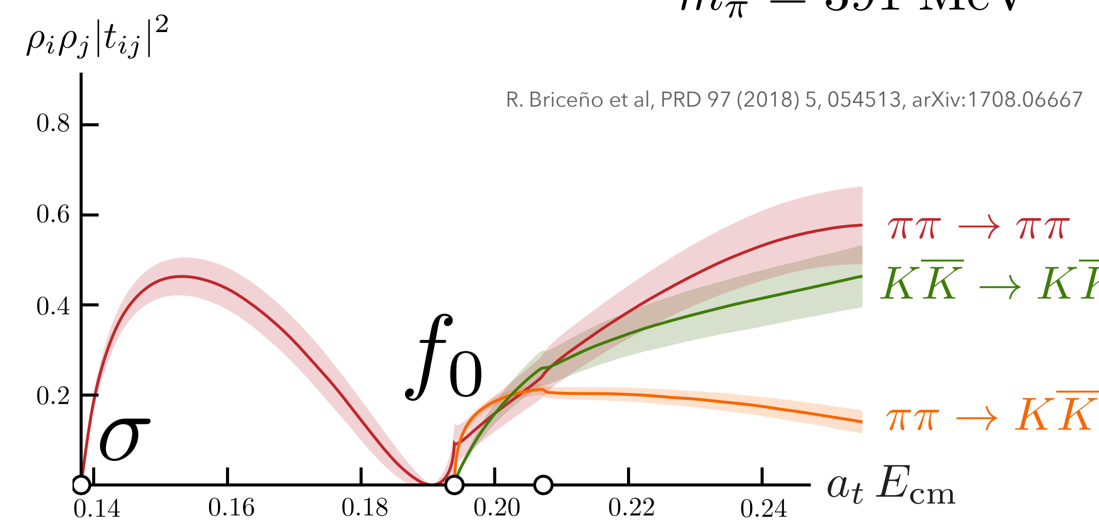


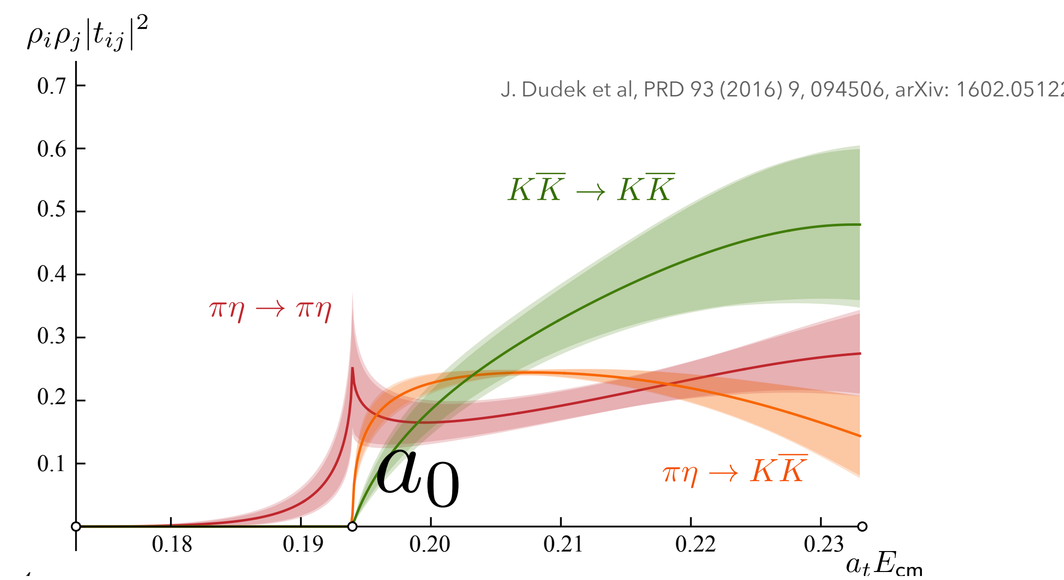


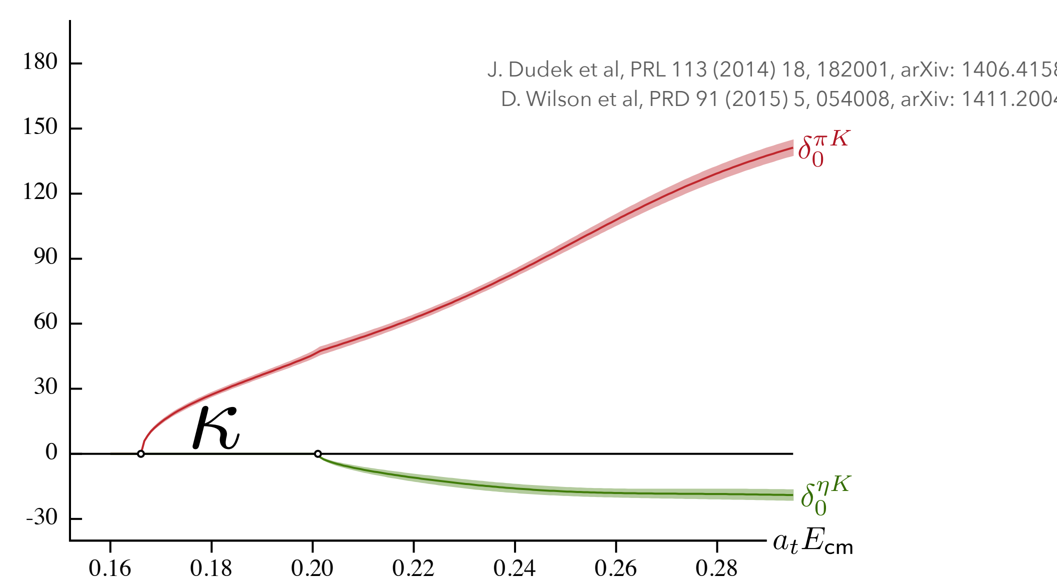
$m_\pi = 391$ MeV


κ
κ
f_0
a_0
a_0
a_0
σ
κ
κ


$\sqrt{s_0}$/MeV
50
−50
−100
$\mathrm{Im}\sqrt{s_0} = -\frac{\Gamma_R}{2}$
sheet II
κ?
700
σ
800
900
1000
1100
1200
1300
sheet I
a_0
sheet IV
$m_R = \mathrm{Re}\sqrt{s_0}$
f_0
sheet II


$\rho_i \rho_j |t_{ij}|^2$
R. Briceño et al, PRD 97 (2018) 5, 054513, arXiv:1708.06667
0.8
0.6
0.4
0.2
$\pi\pi \to \pi\pi$
$K\overline{K} \to K\overline{K}$
f_0
$\pi\pi \to K\overline{K}$
σ
0.14
0.16
0.18
0.20
0.22
0.24
$a_t E_{cm}$

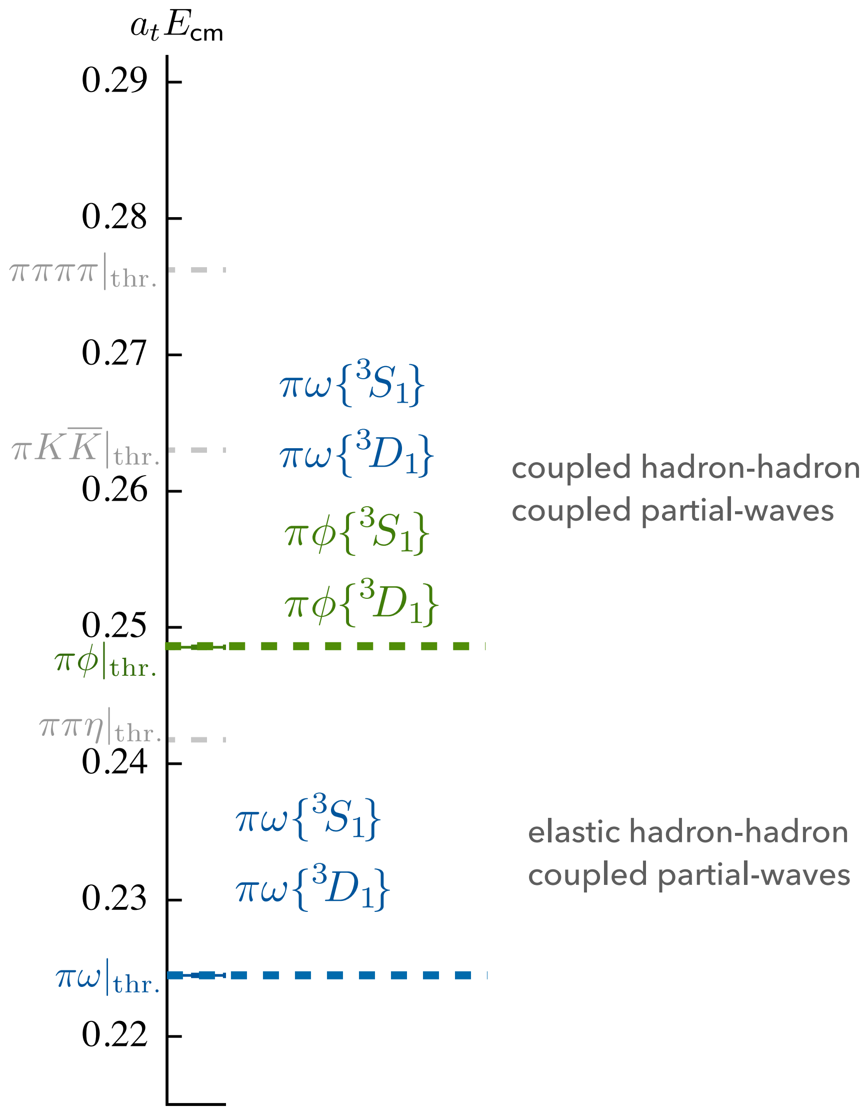

$\rho_i \rho_j |t_{ij}|^2$
J. Dudek et al, PRD 93 (2016) 9, 094506, arXiv: 1602.05122
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$K\overline{K} \to K\overline{K}$
$\pi\eta \to \pi\eta$
a_0
$\pi\eta \to K\overline{K}$
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{cm}$

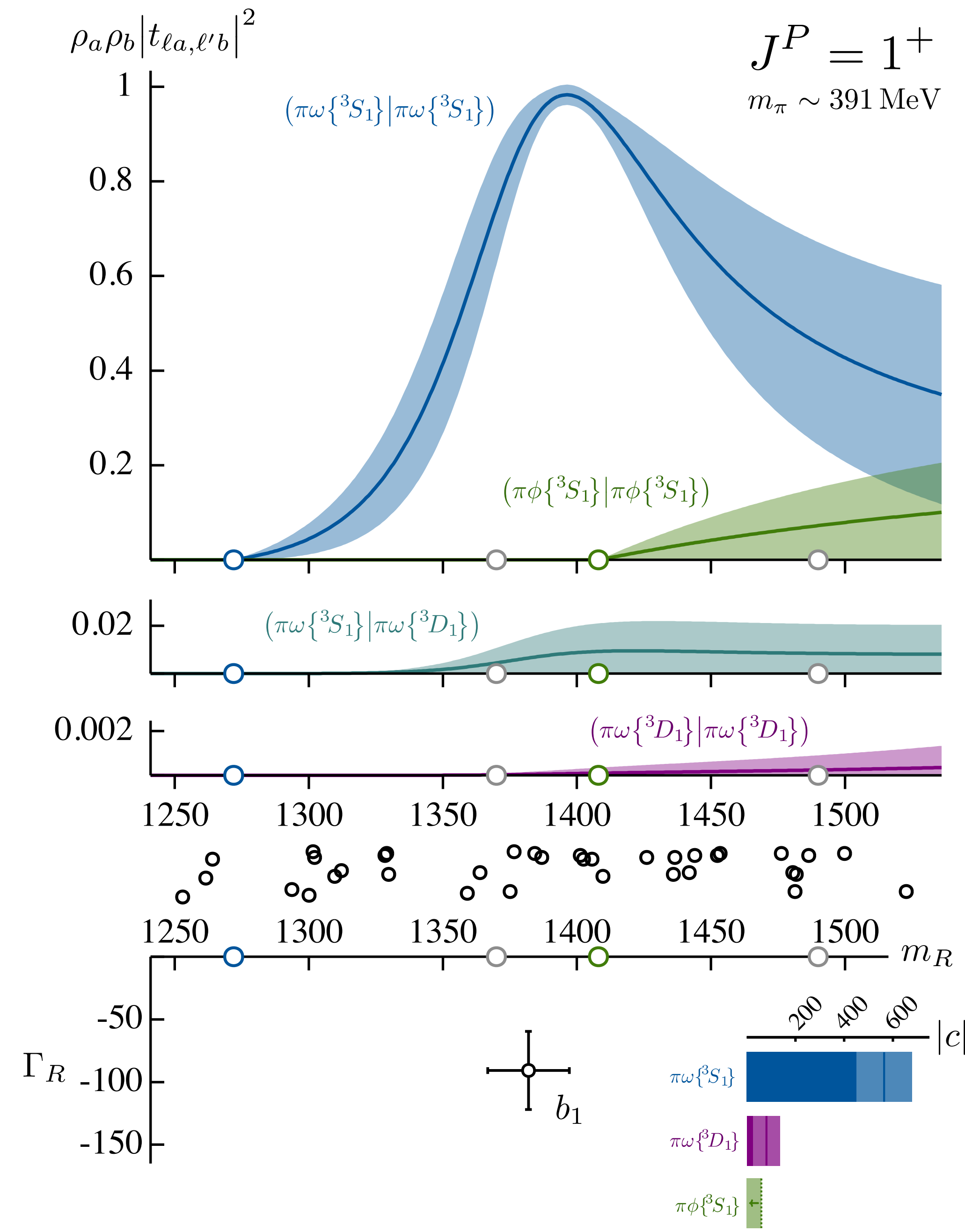

J. Dudek et al, PRL 113 (2014) 18, 182001, arXiv: 1406.4158
D. Wilson et al, PRD 91 (2015) 5, 054008, arXiv: 1411.2004
180
150
120
90
60
30
0
-30
$\delta_0^{\pi K}$
κ
$\delta_0^{\eta K}$
0.16
0.18
0.20
0.22
0.24
0.26
0.28
$a_t E_{cm}$

scattering with spinning particles

A. J. Woss, et al, PRD100 (2019) 5, 054506, arXiv: 1904.04136

stable vector ω scattering

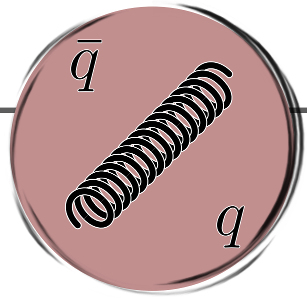




$b_1(1235)$ $I^G(J^{PC}) = 1^+(1^{+-})$

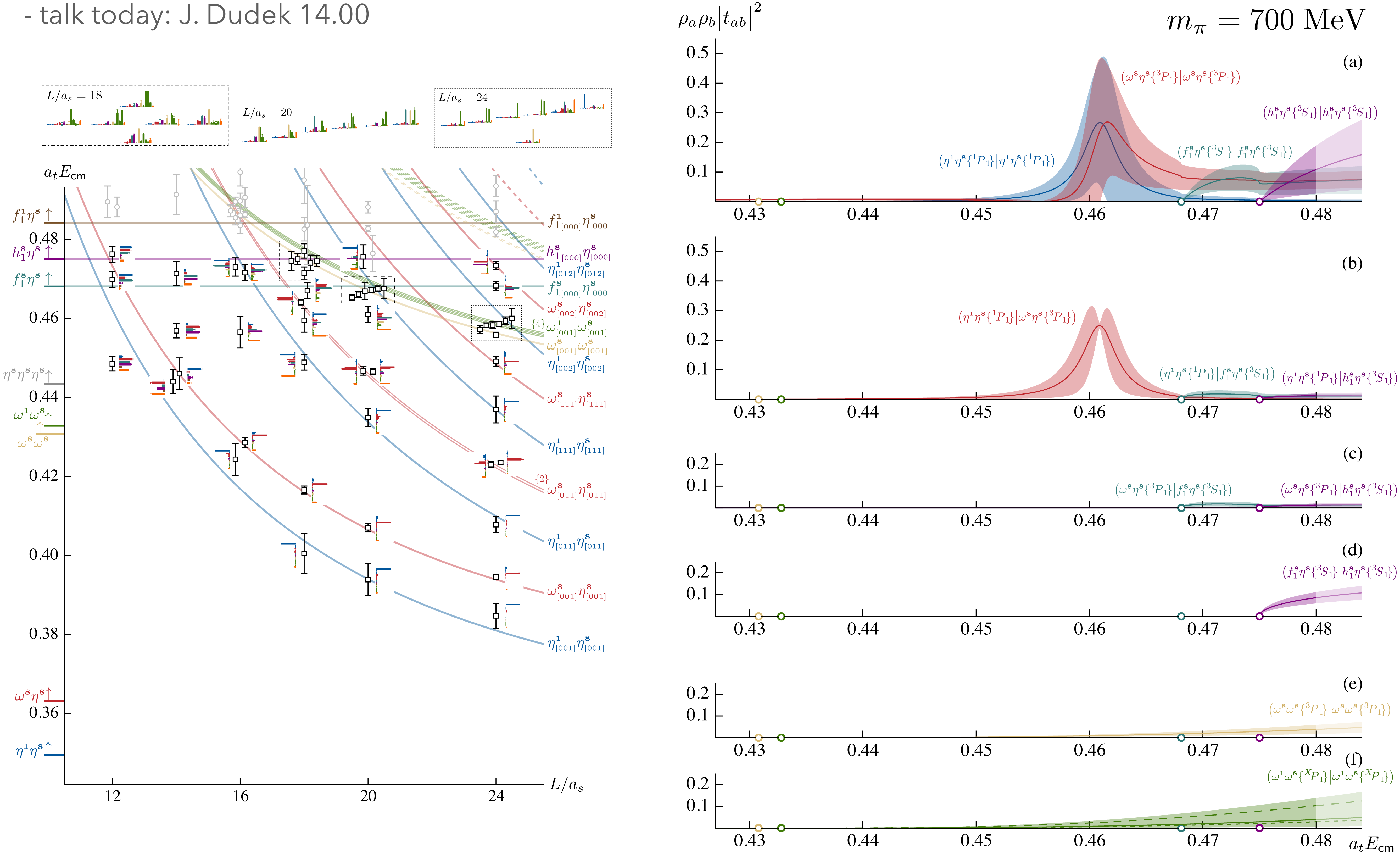
Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	dominant		348
$[D/S$ amplitude ratio $= 0.277 \pm 0.027]$			

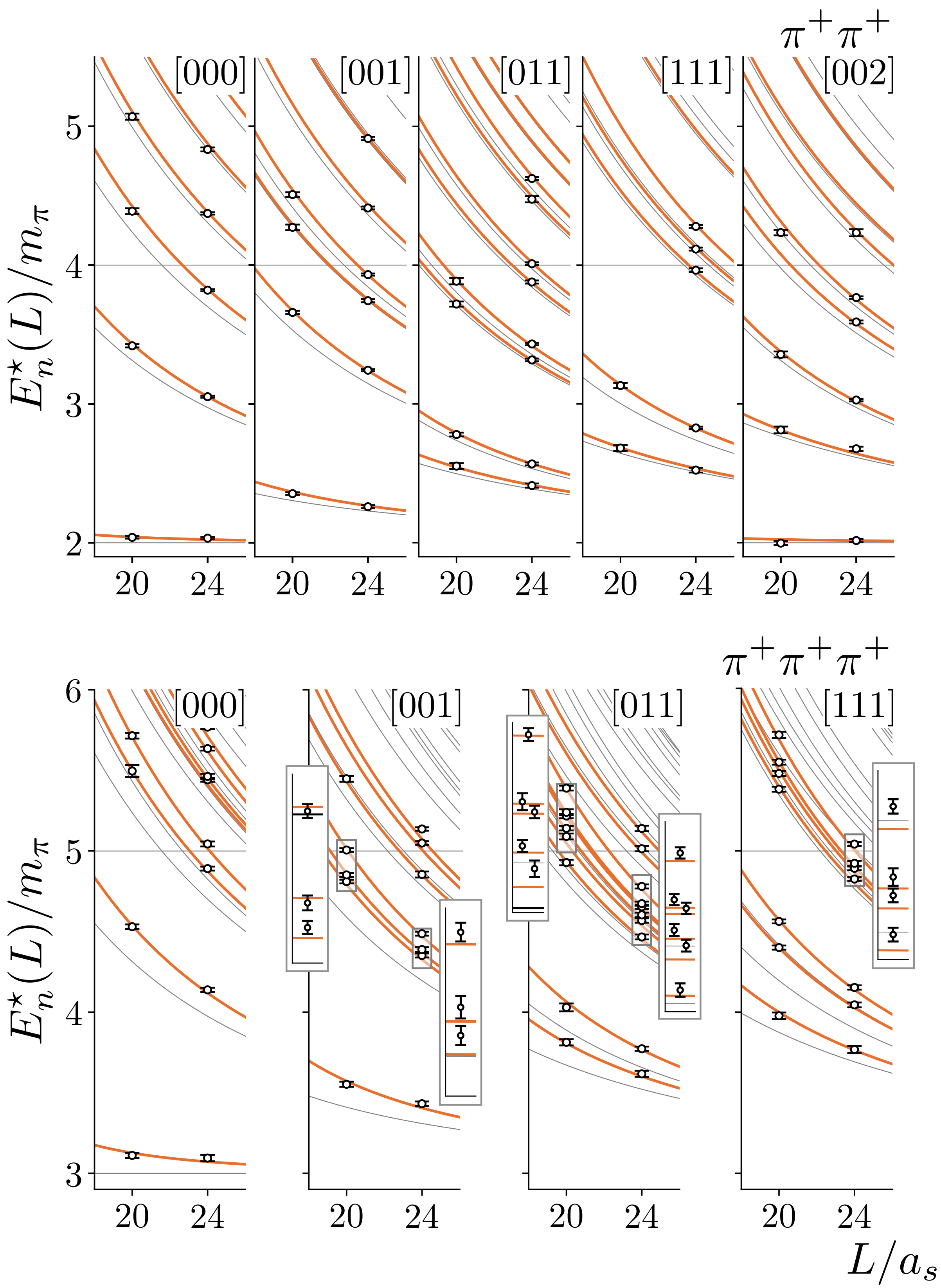


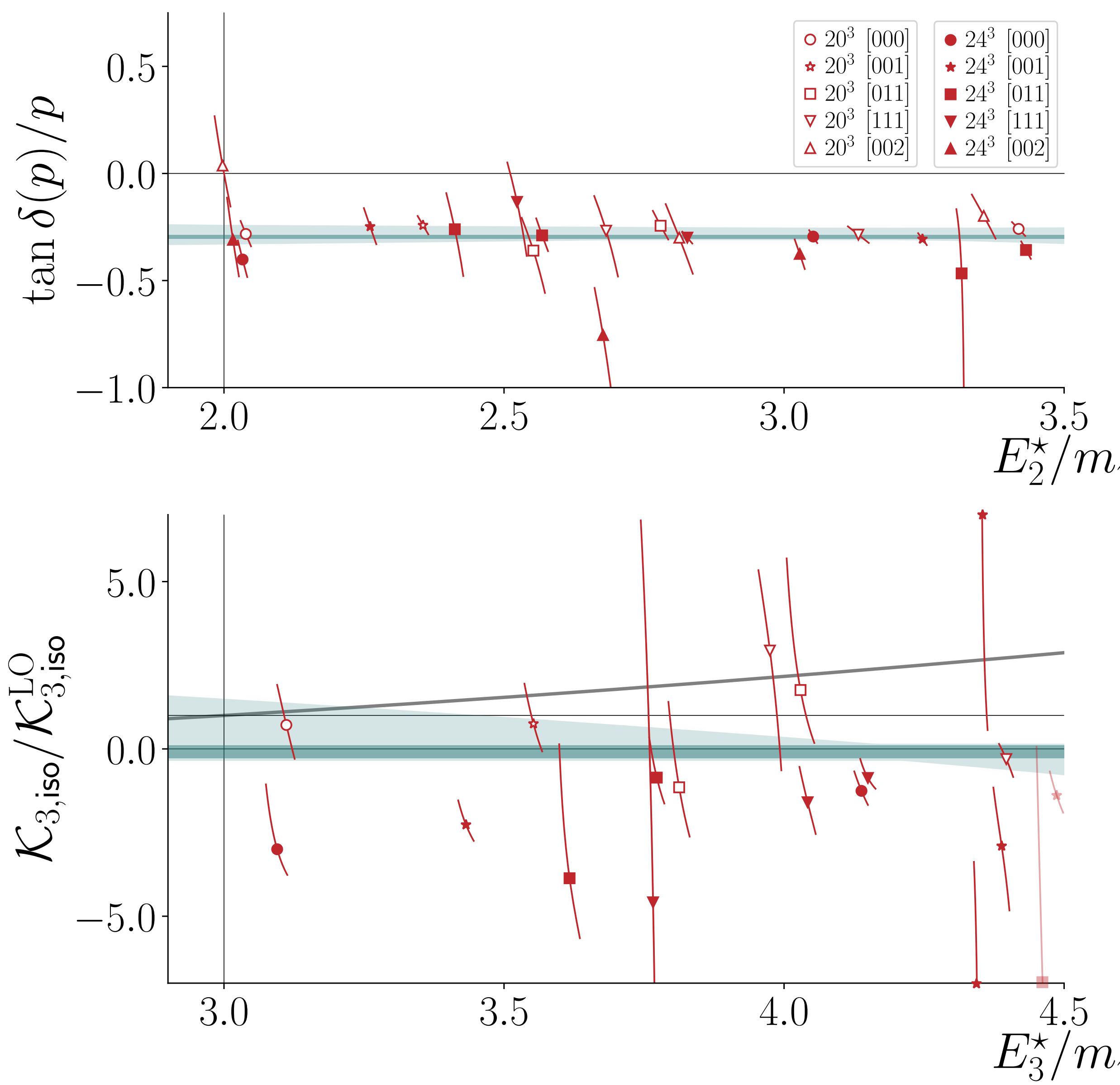
A. J. Woss et al, PRD103 (2021) 5, 054502 arXiv:2009.10034

- first computation of the decays of an exotic hybrid
- talk today: J. Dudek 14.00

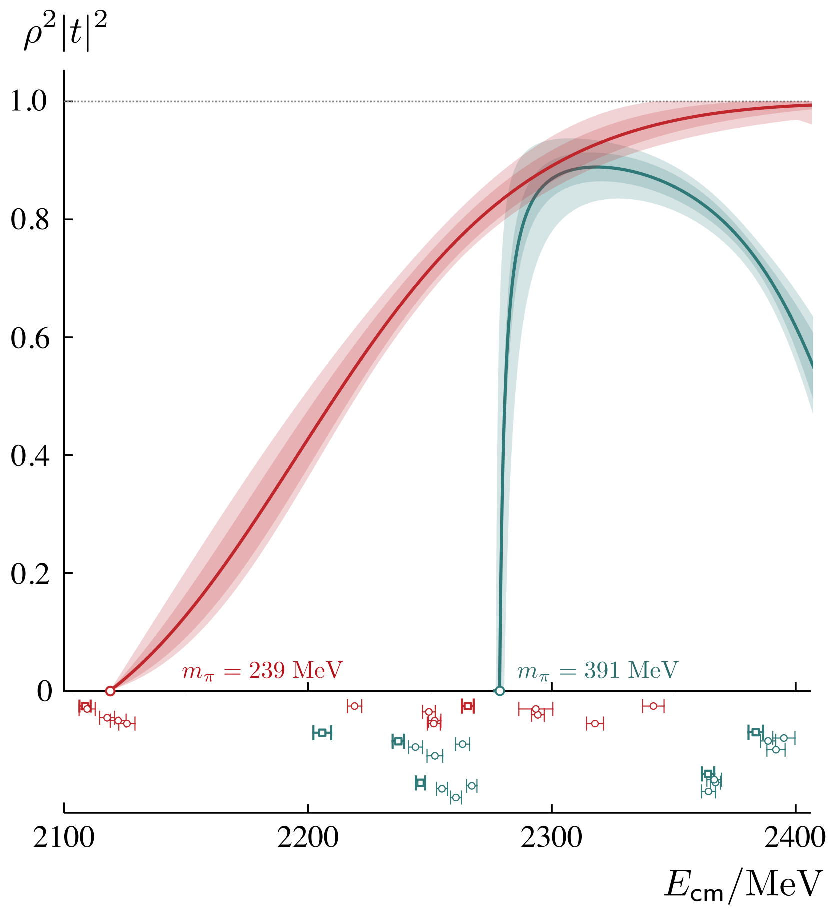


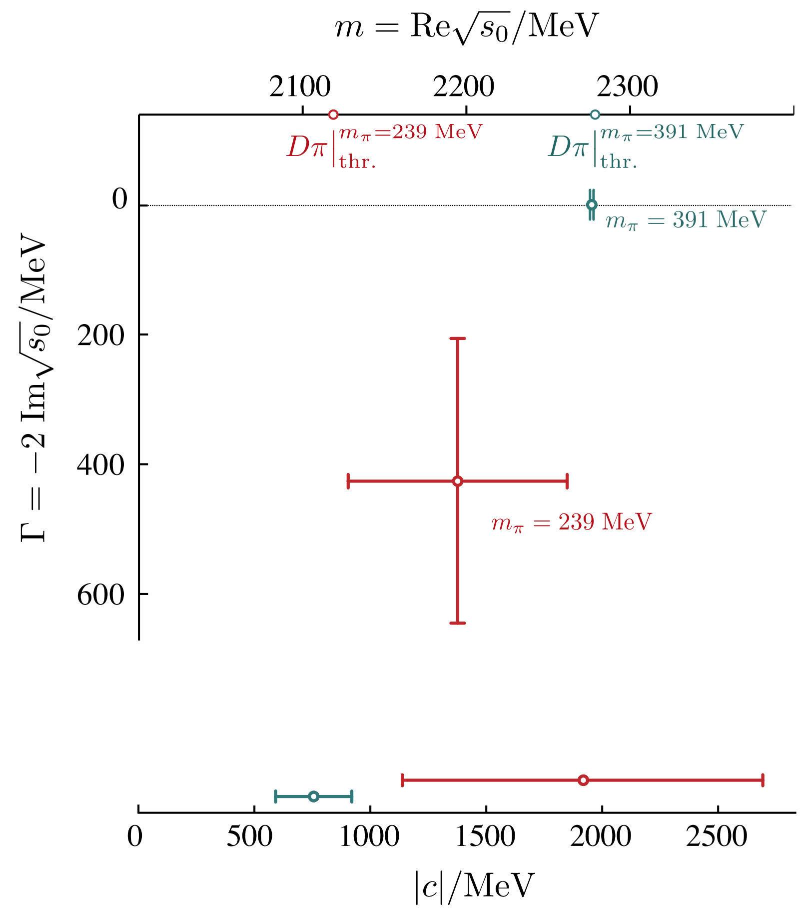
M. T. Hansen et al, PRL 126 (2021) 012001, arXiv:2009.04931
- talk Friday 13:50



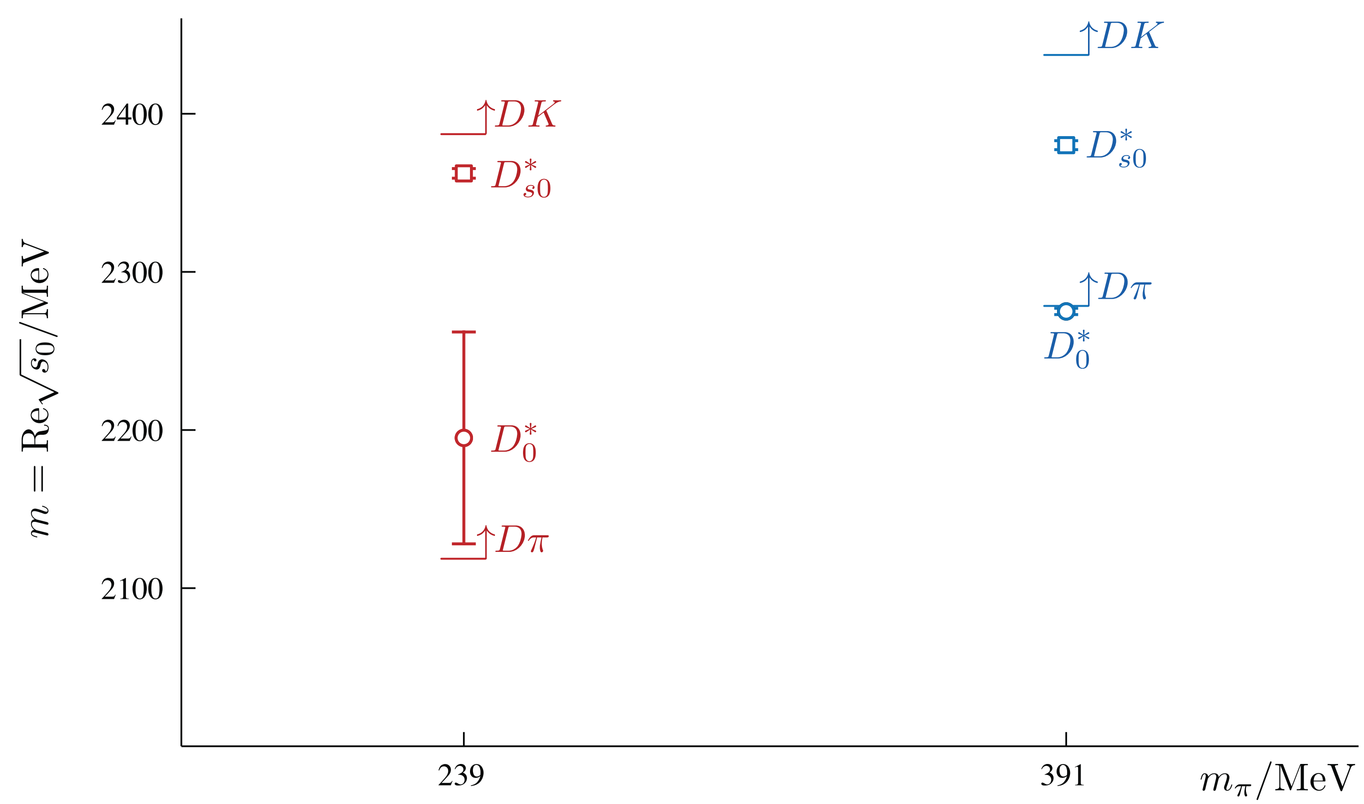


G. Cheung et al, JHEP 02 (2021) 100 arXiv: 2008.06432 - talk C. Thomas today 15.50
L. Gayer, N. Lang et al, arXiv:2102.04973 - talk N. Lang today 16.10





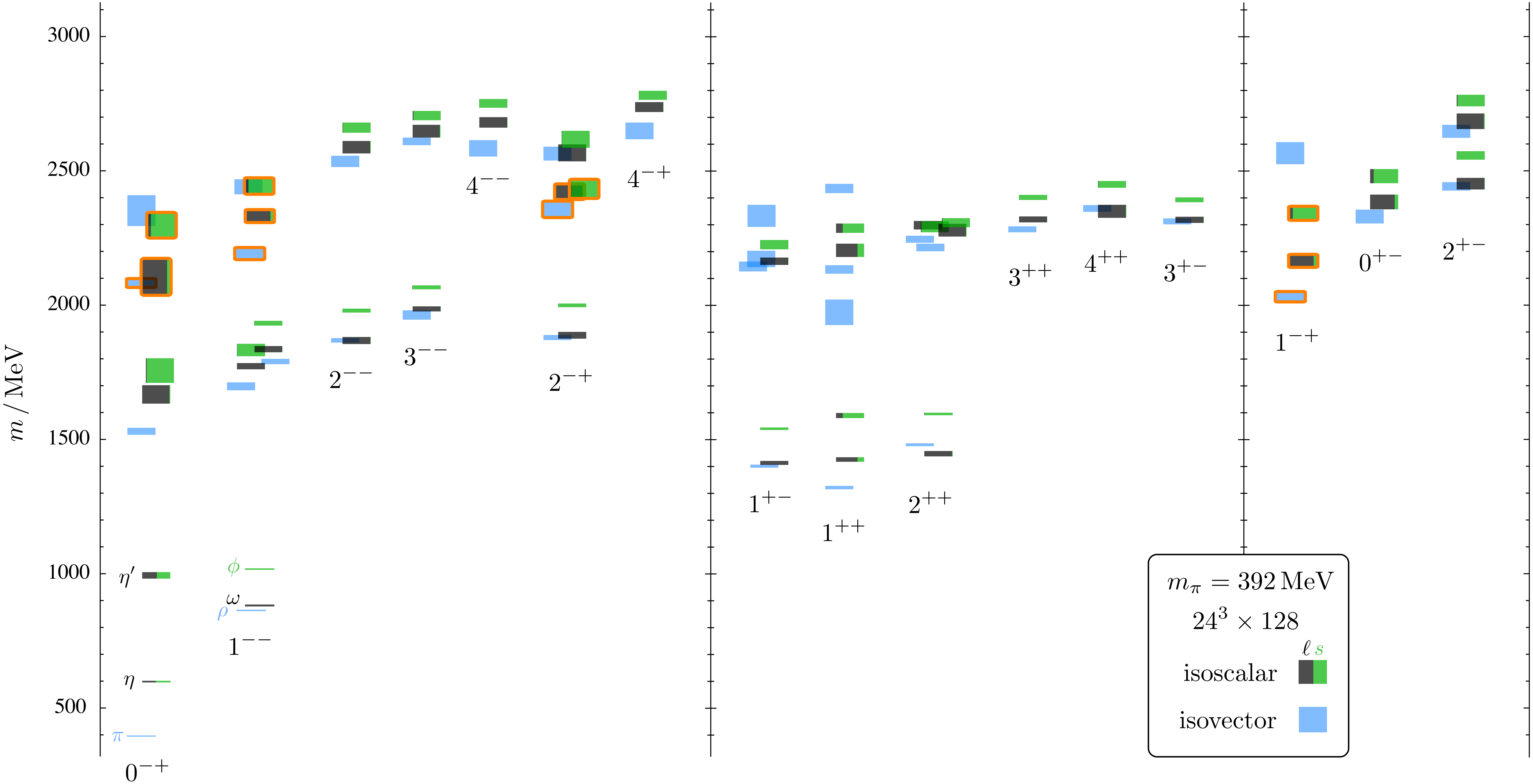
suggestive of a much lighter D_0^* compared with the D_{s0}^*



Lattice QCD provides a first-principles tool
to do hadron spectroscopy

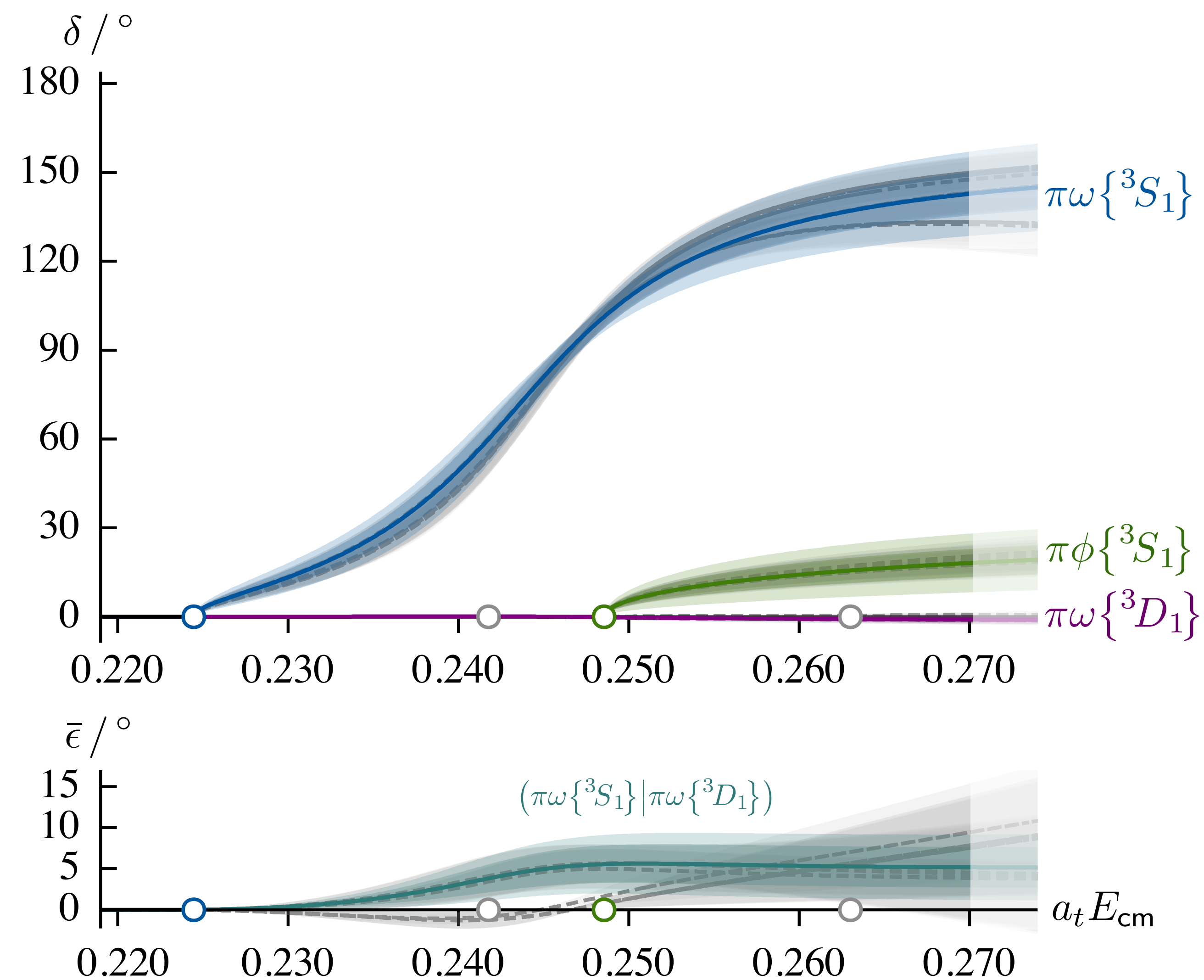
These methods are widely applicable
- coupled-channel scattering
- baryons
- charm quarks, beauty quarks
- form factors, radiative transitions (incl. resonances)
…

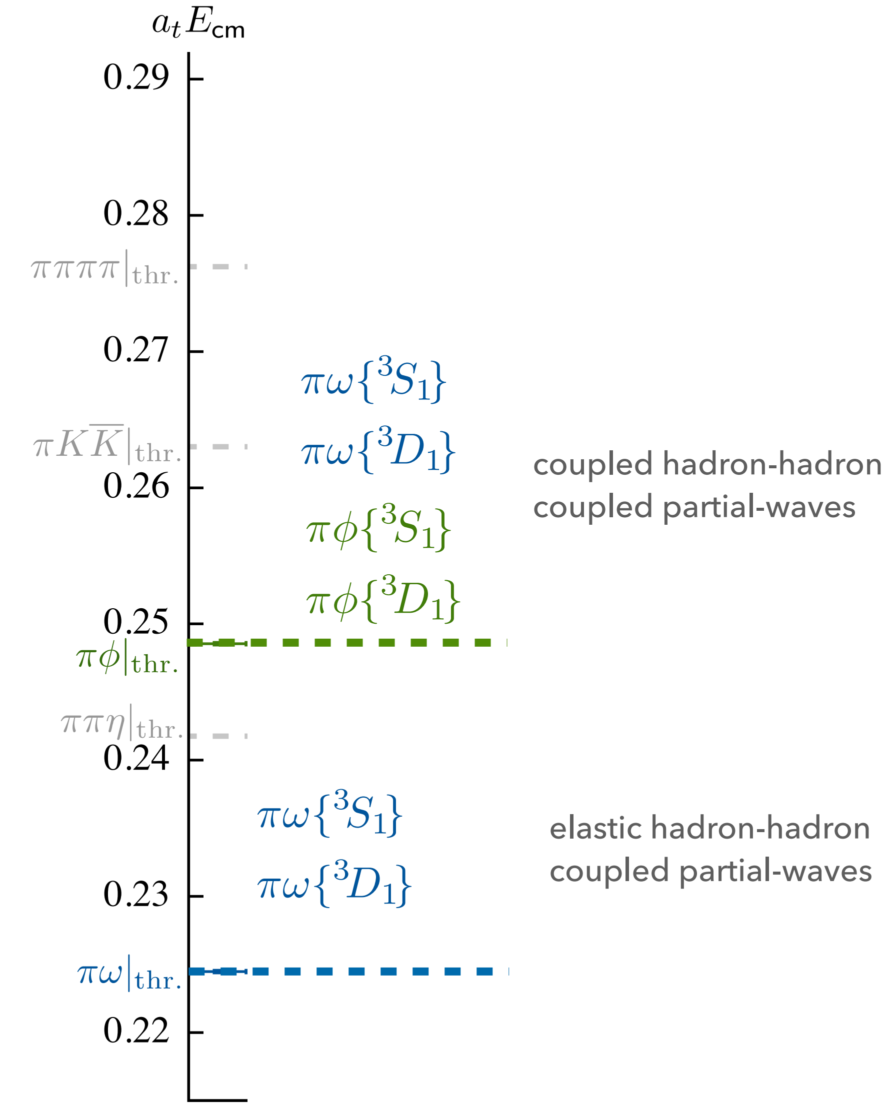
Control of 3+ body effects needed for
- lighter pion masses
- higher resonances

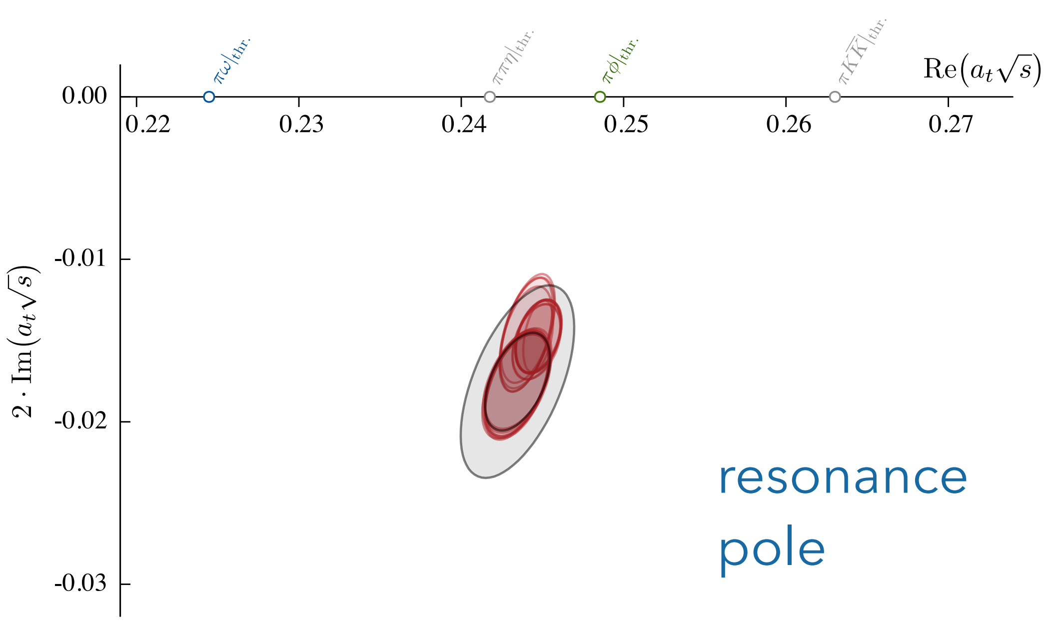


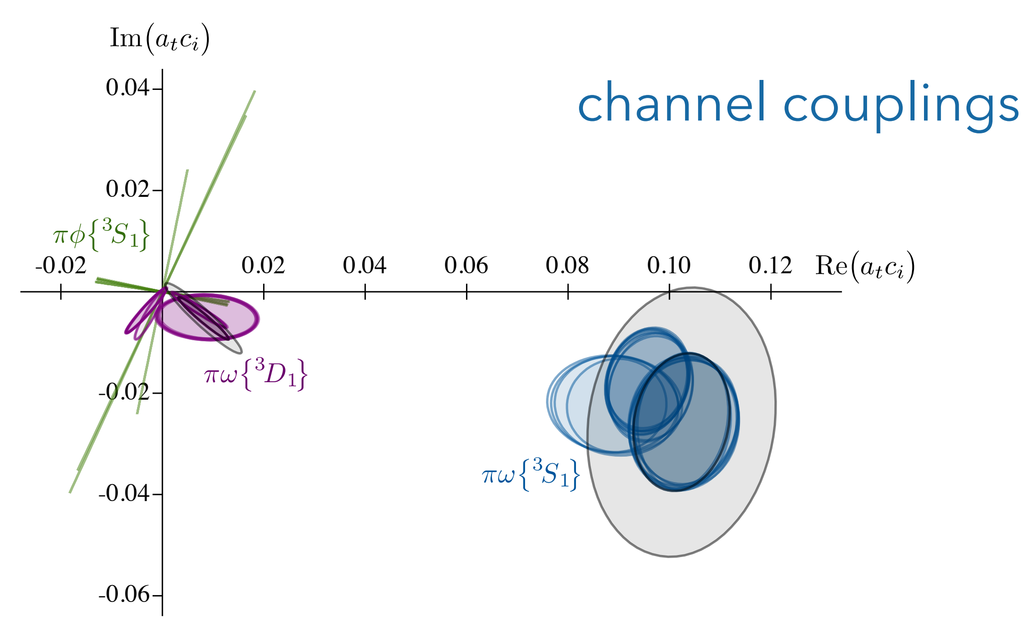
J. Dudek et al, Phys.Rev.D 88 (2013) 9, 094505, arXiv:1309.2608
for a recent example with b-quarks see also S. Ryan and DW, JHEP 02 (2021) 214, arXiv: 2008.02656
- talk on Weds 14th @ 13.30

A. J. Woss, et al, PRD100 (2019) 5, 054506, arXiv: 1904.04136







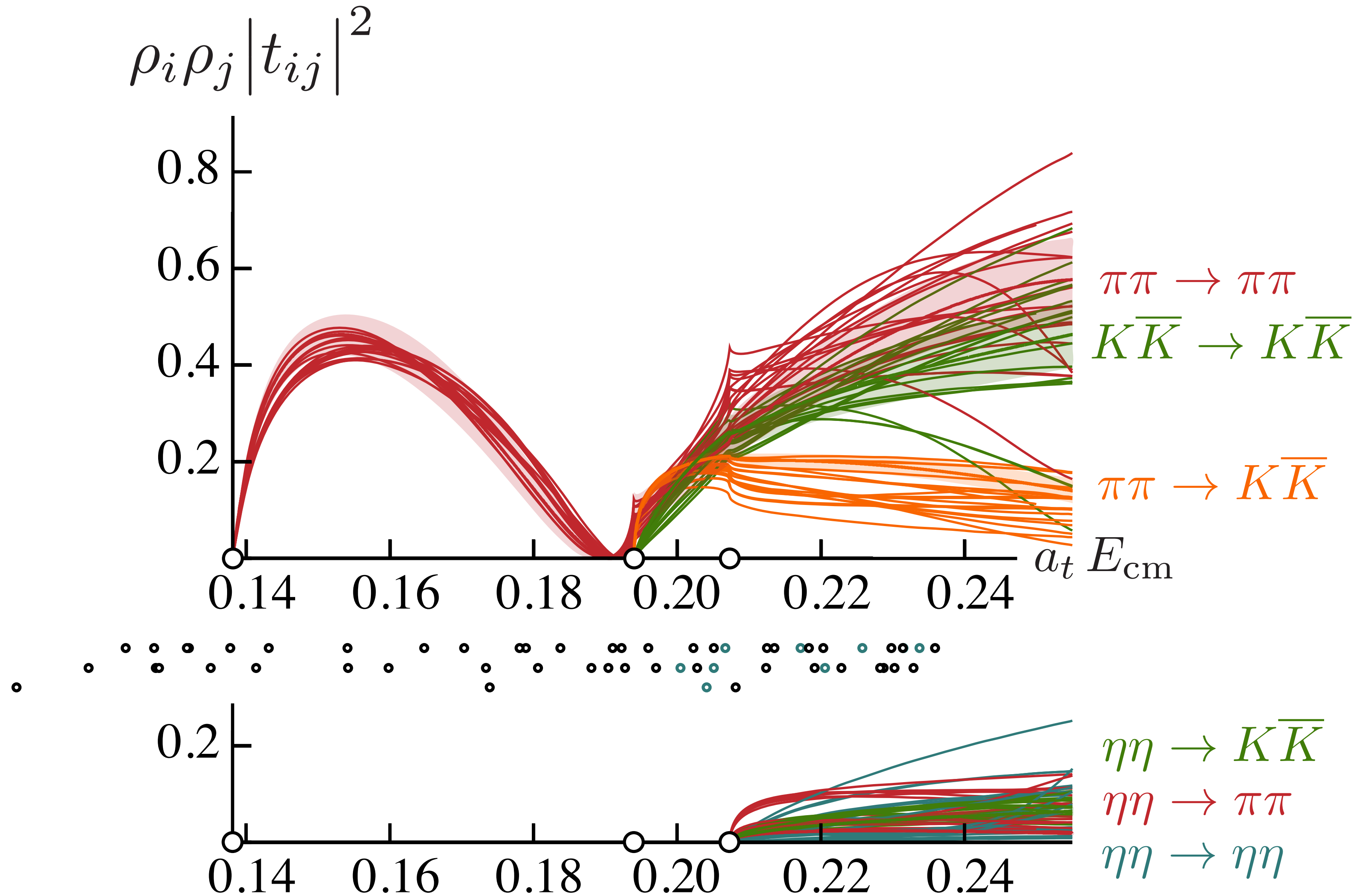


S-wave amplitude variation

20 amplitudes

$\chi^2/N_{\rm dof} < 1.05$

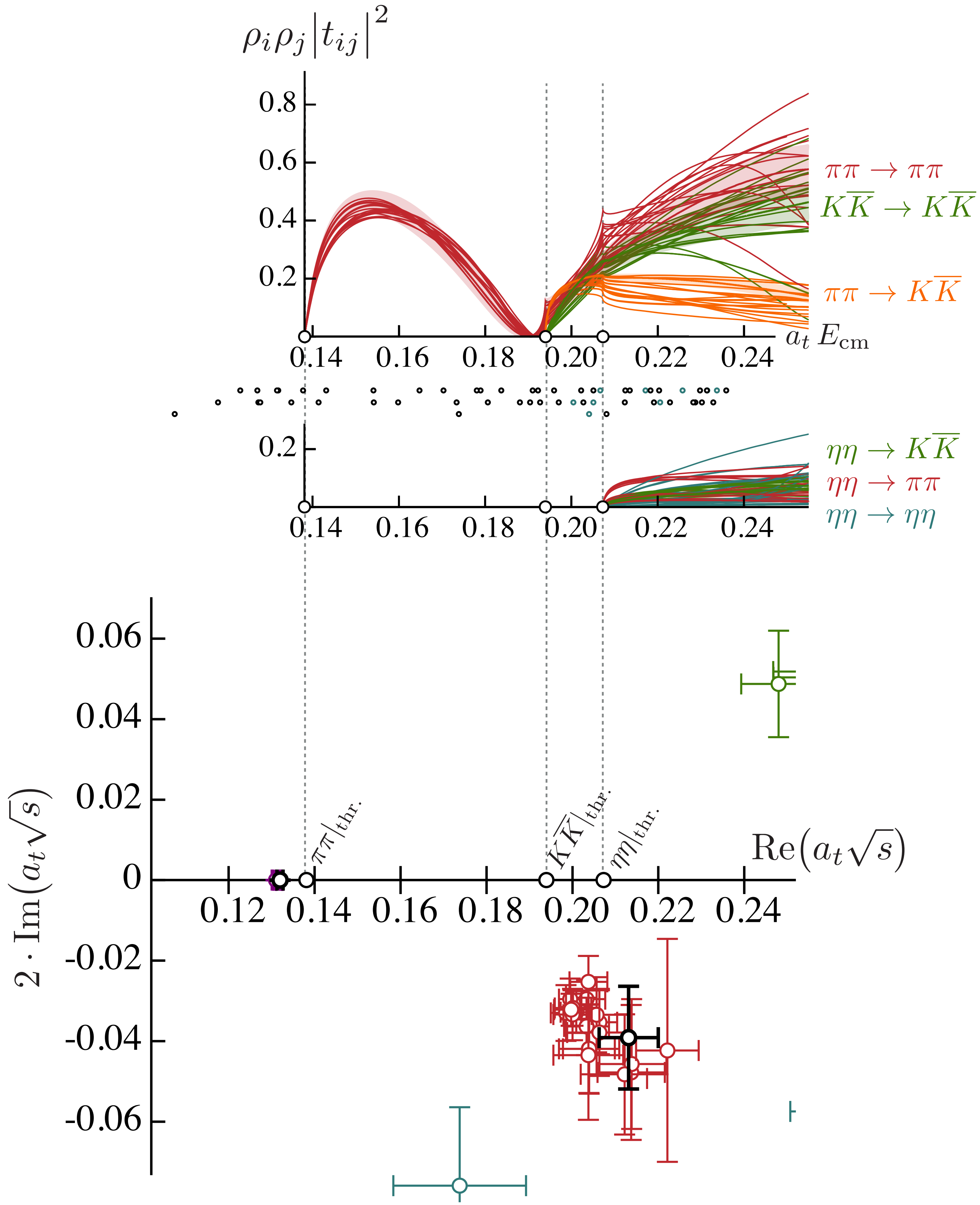
57 energy levels



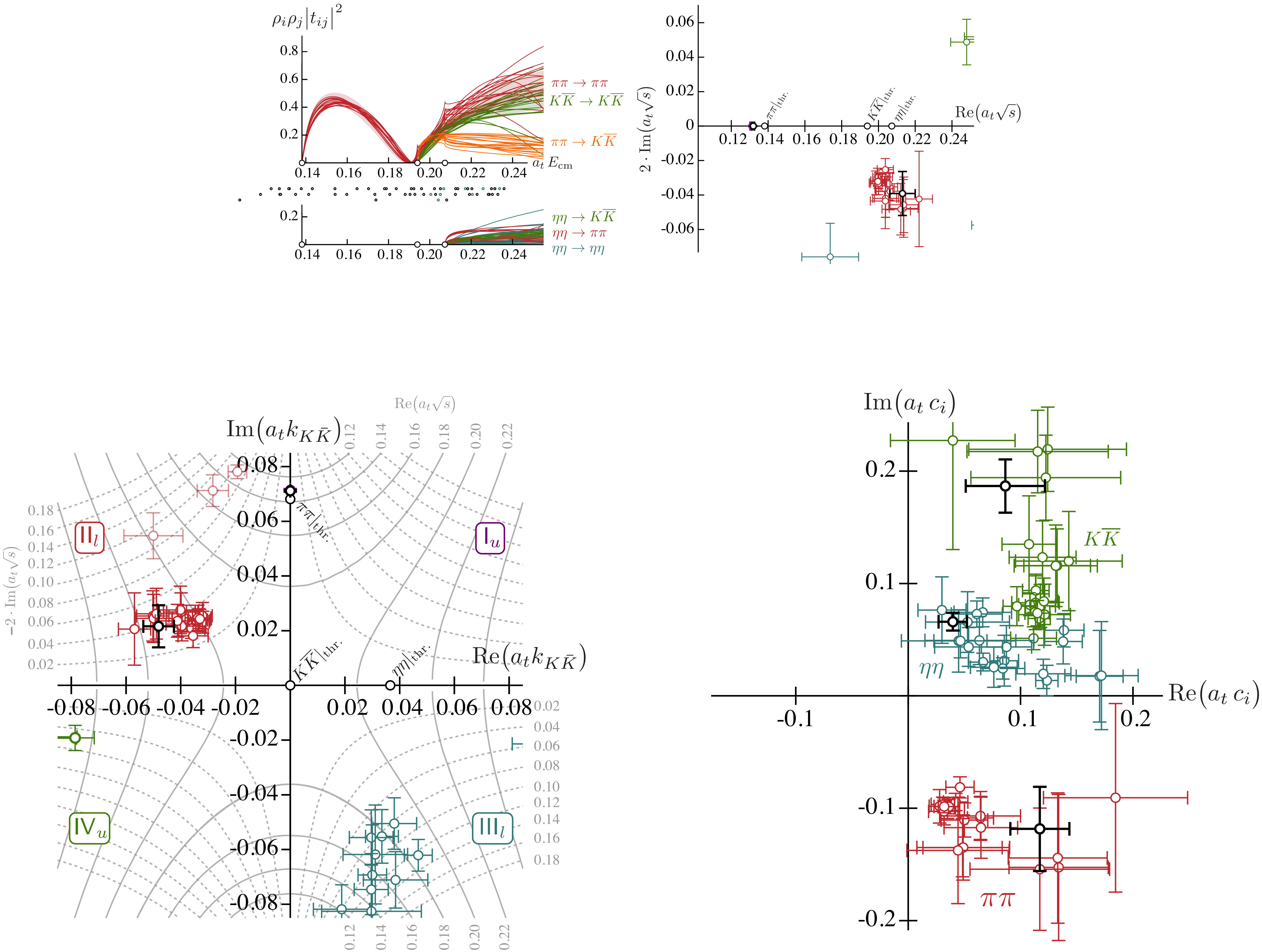
Scalar f0 poles

Near a t-matrix pole

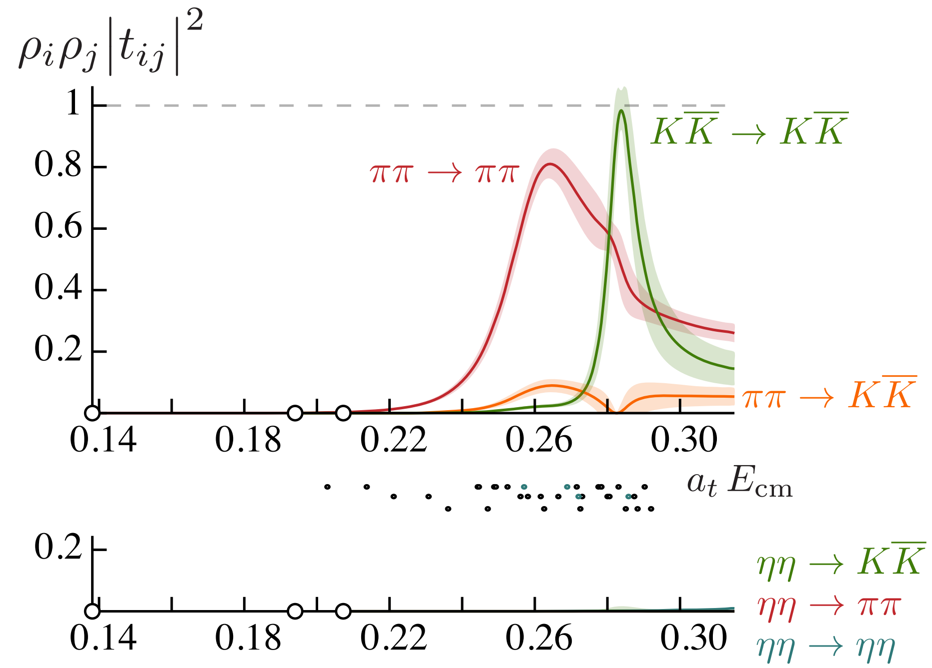
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$



Scalar f0 resonance poles

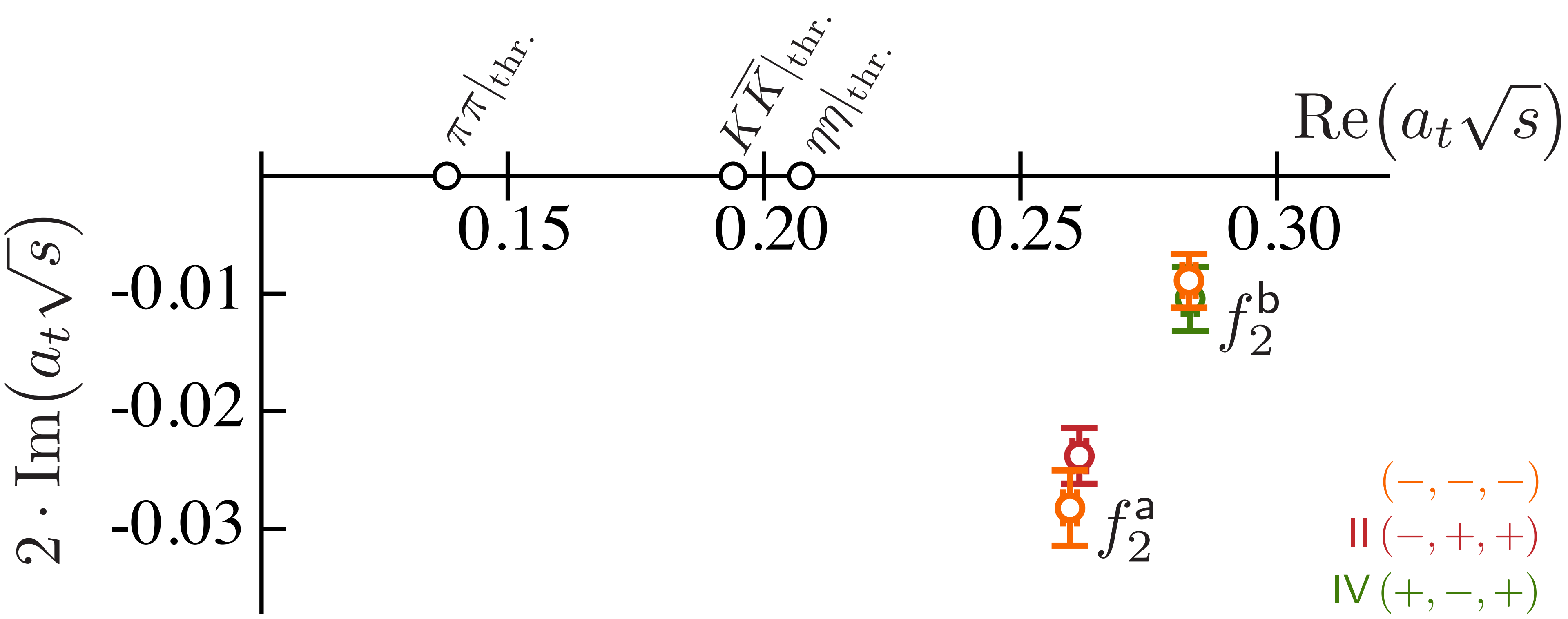


Tensor resonance poles

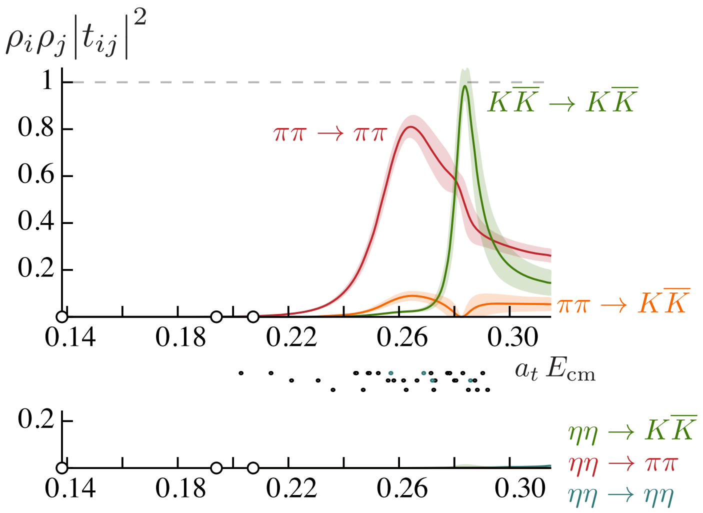


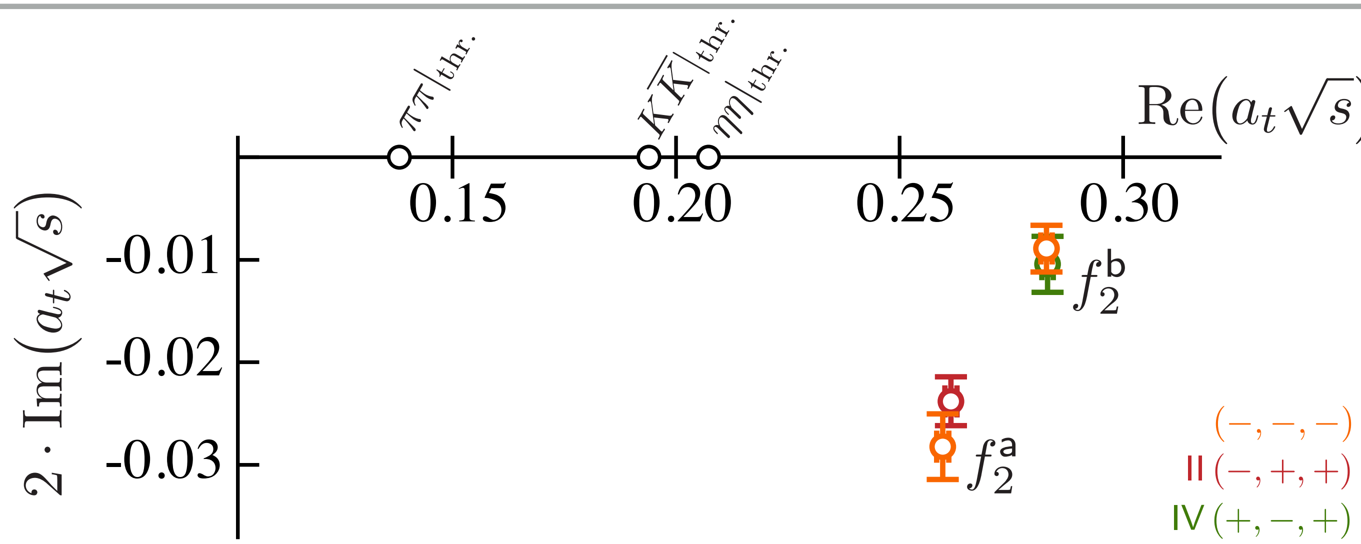
Near a t-matrix pole

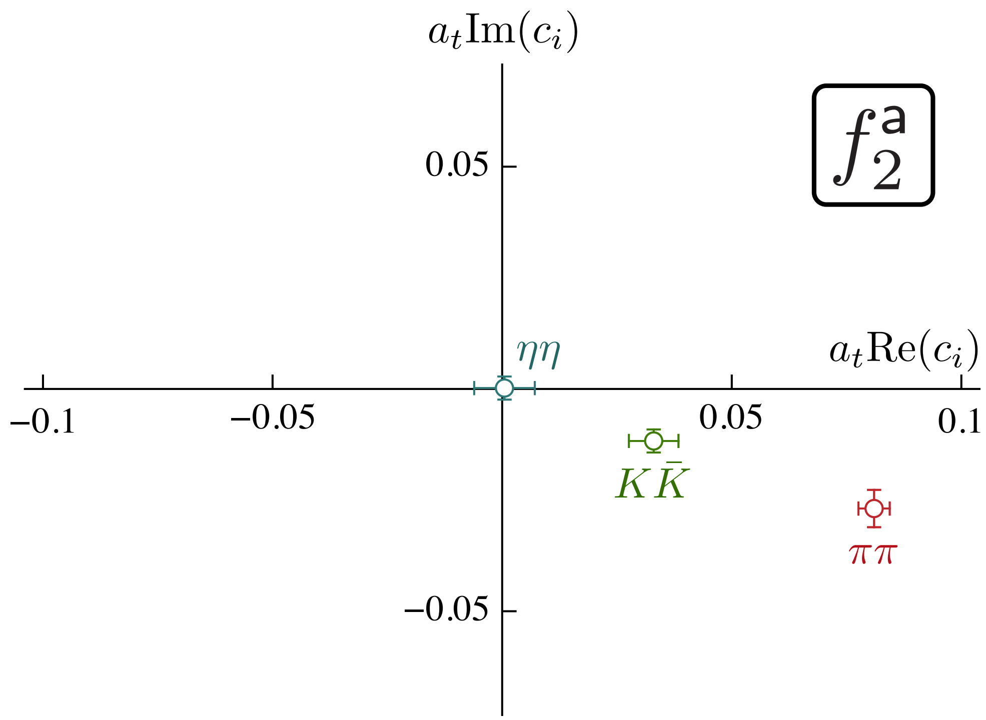
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

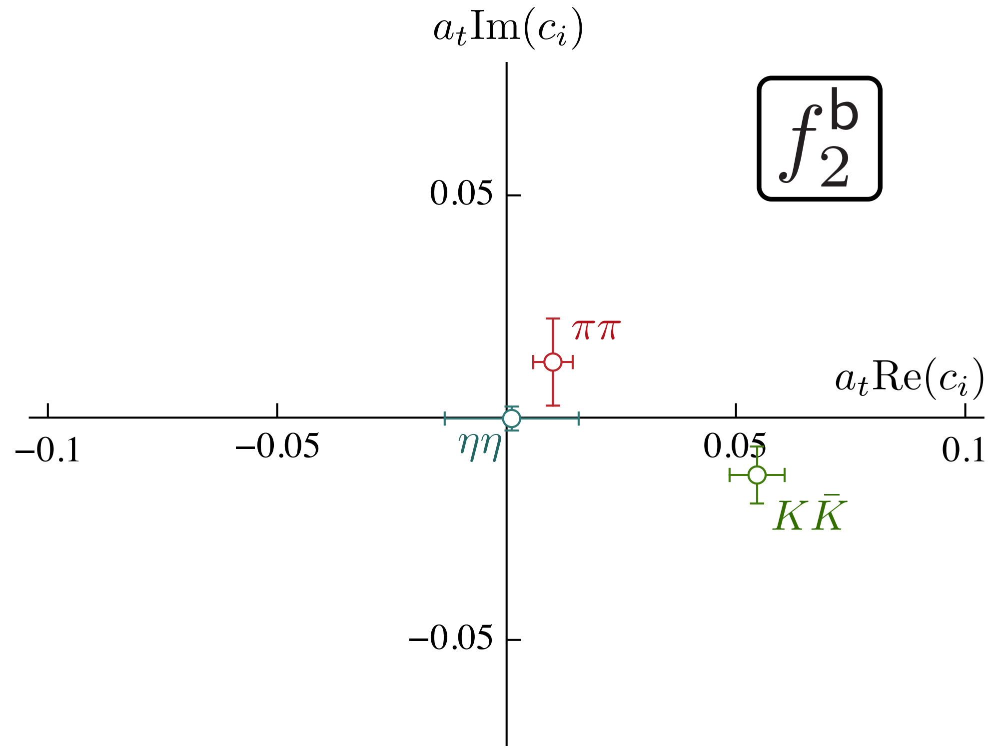


Tensor resonance poles









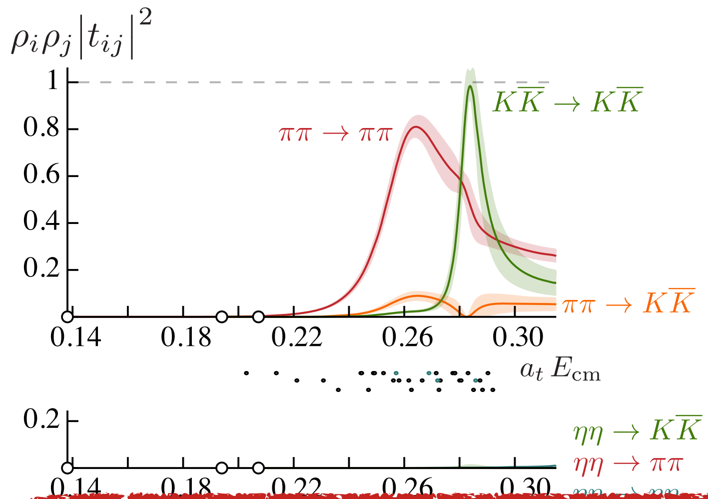
$f_2^a: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

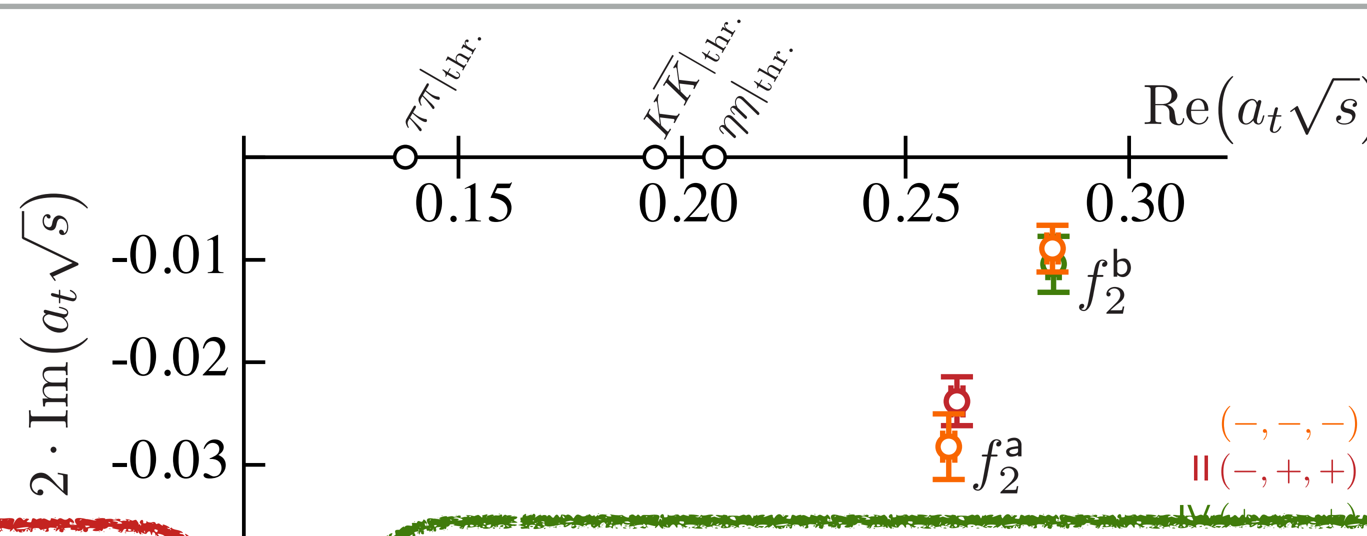
$\mathrm{Br}(f_2^a \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^a \to K\overline{K}) \sim 12\%$

$f_2^b: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^b \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^b \to K\overline{K}) \sim 92\%$

Just for fun…





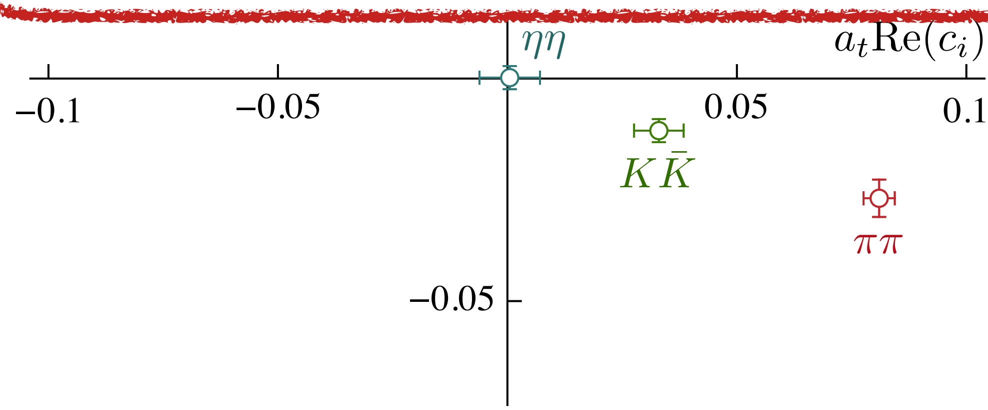
$f_2(1270)$ DECAY MODES

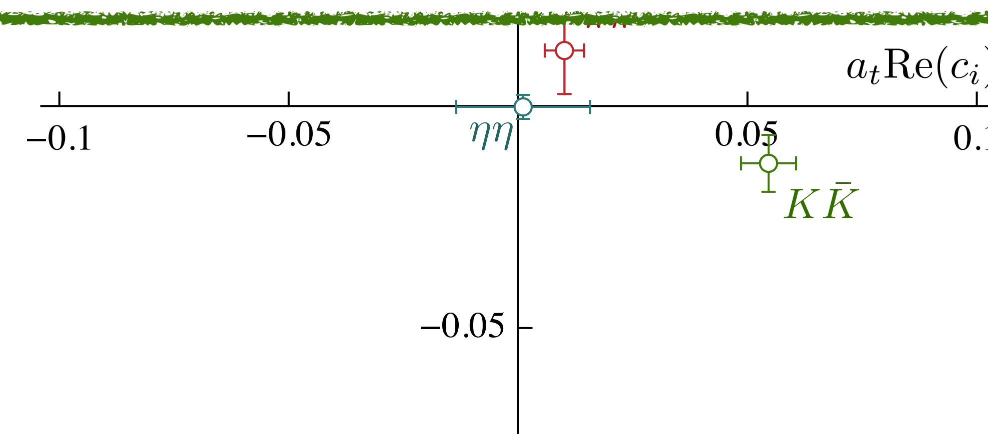
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ {}^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(\ 7.7\ {}^{+1.1}_{-3.2})$ %
Γ_3	$K\bar{K}$	$(\ 4.6\ {}^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	$(\ 2.8\ \pm0.4)$ %

PDG2017

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\bar{K}$	$(88.7\ \pm2.2)$ %
Γ_2	$\eta\eta$	$(10.4\ \pm2.2)$ %
Γ_3	$\pi\pi$	$(\ 8.2\ \pm1.5) \times 10^{-3}$
Γ_4	$K\bar{K}^*(892)+$ c.c.	





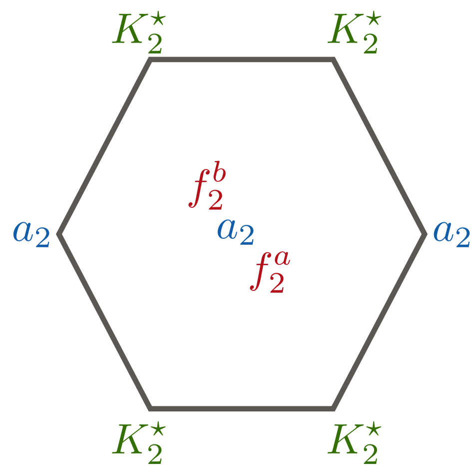
$$f_2^a:\quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$$

$$\mathrm{Br}(f_2^a\to\pi\pi)\sim 85\%,\quad \mathrm{Br}(f_2^a\to K\bar{K})\sim 12\%$$

$$f_2^b:\quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)\ \mathrm{MeV}$$

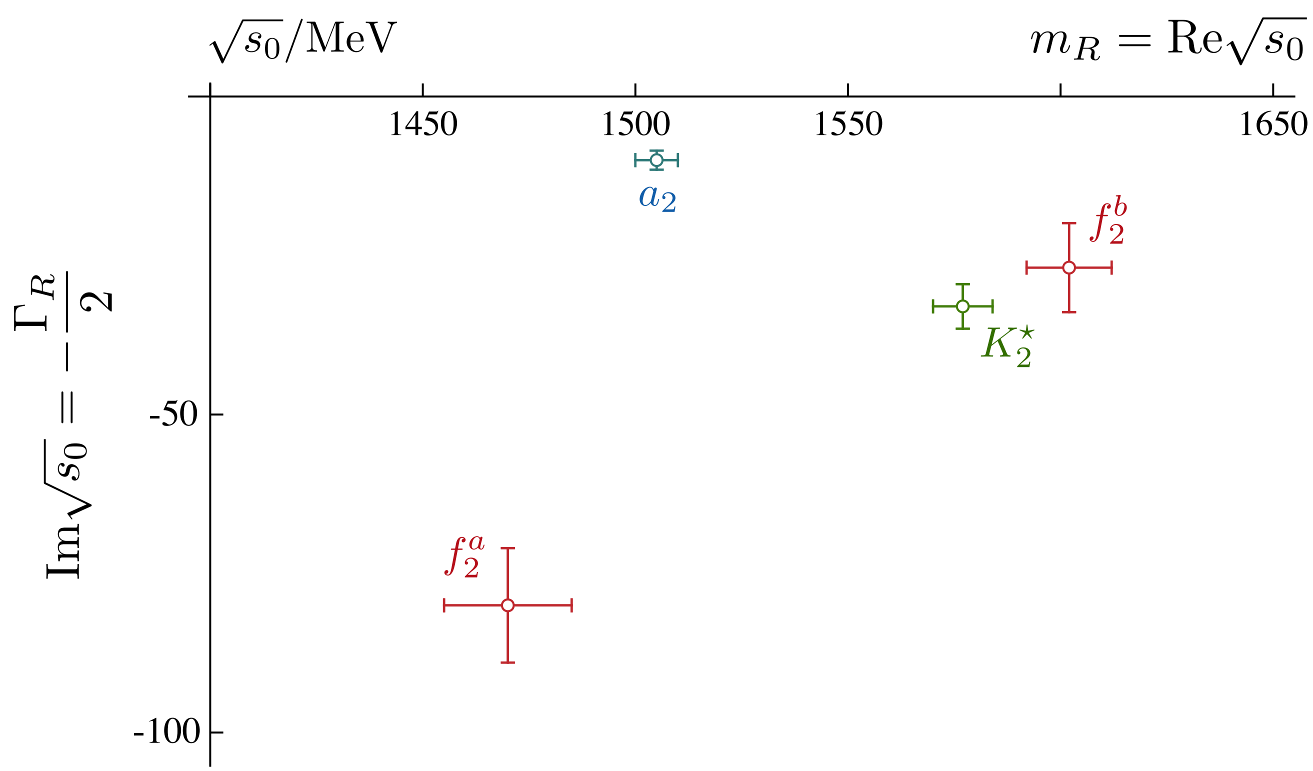
$$\mathrm{Br}(f_2^b\to\pi\pi)\sim 8\%,\quad \mathrm{Br}(f_2^b\to K\bar{K})\sim 92\%$$

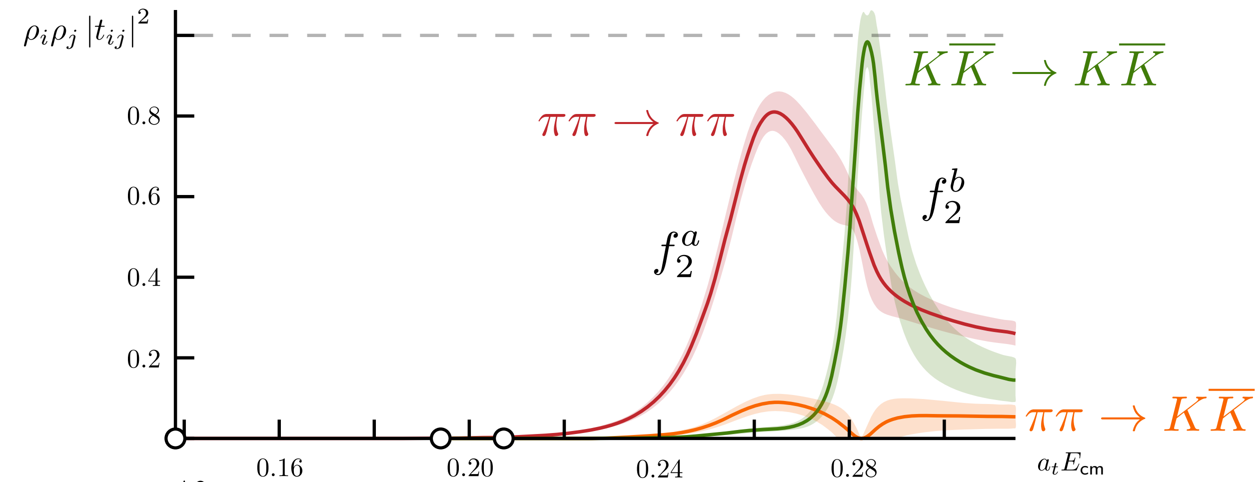
The lightest tensors of QCD at m_π=391 MeV

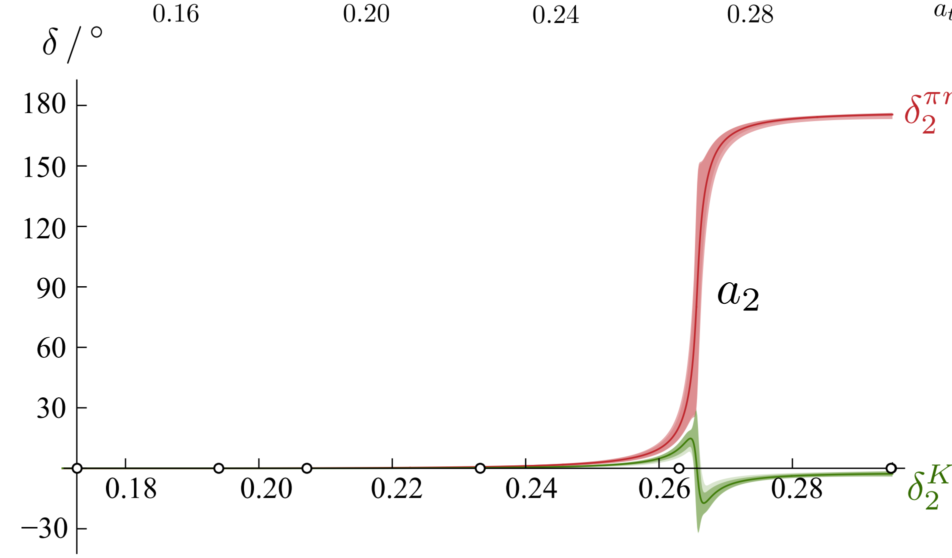


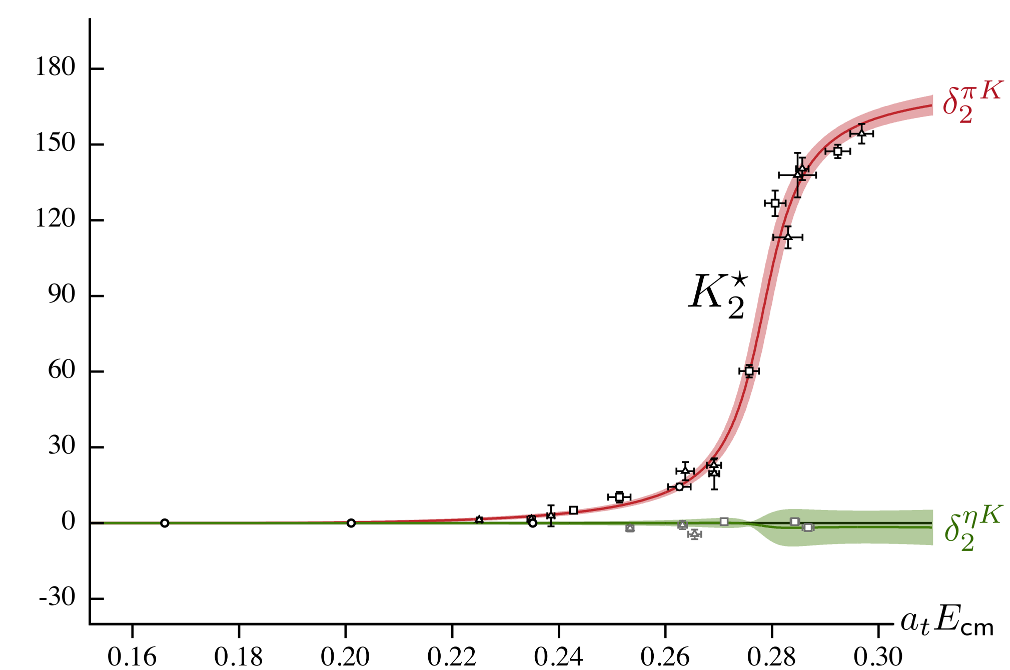
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$

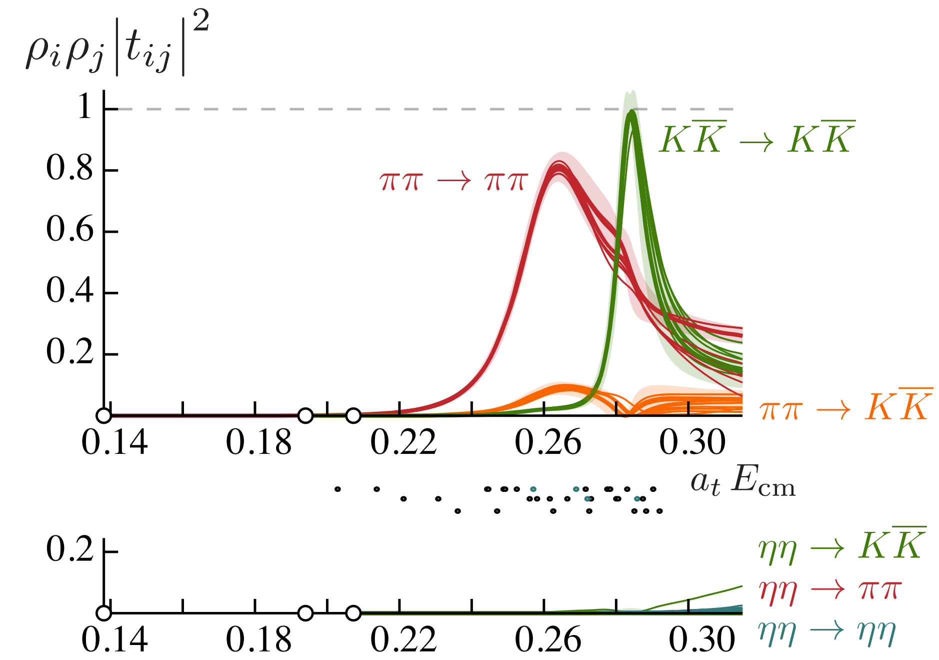






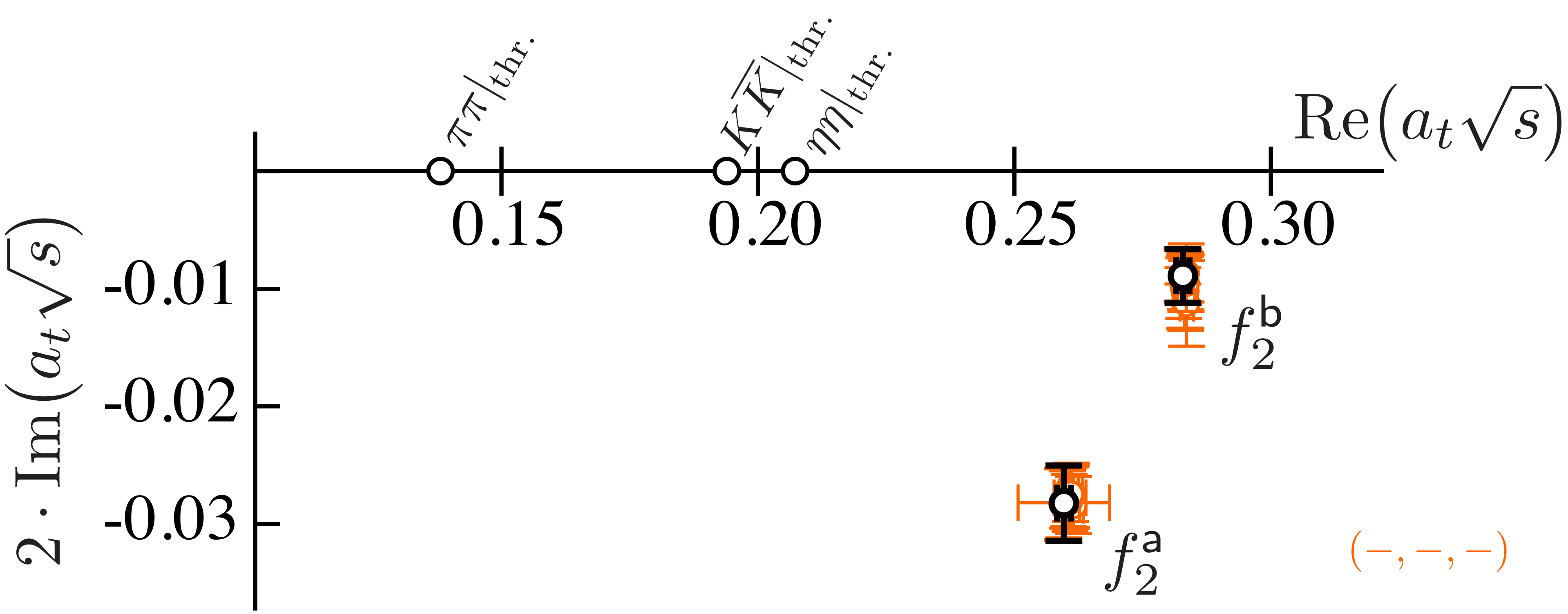


Tensor f2 resonance poles

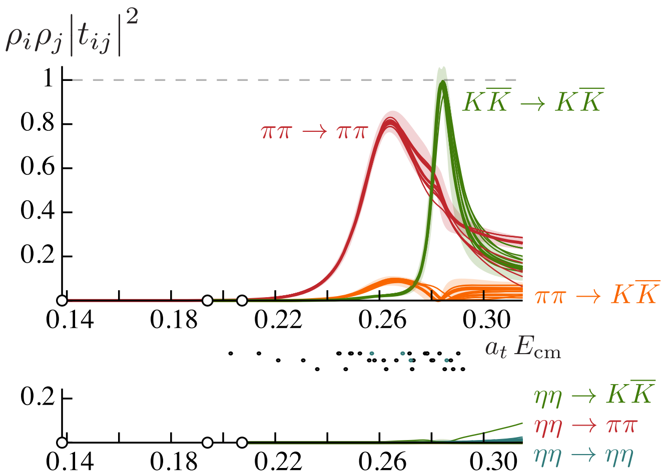


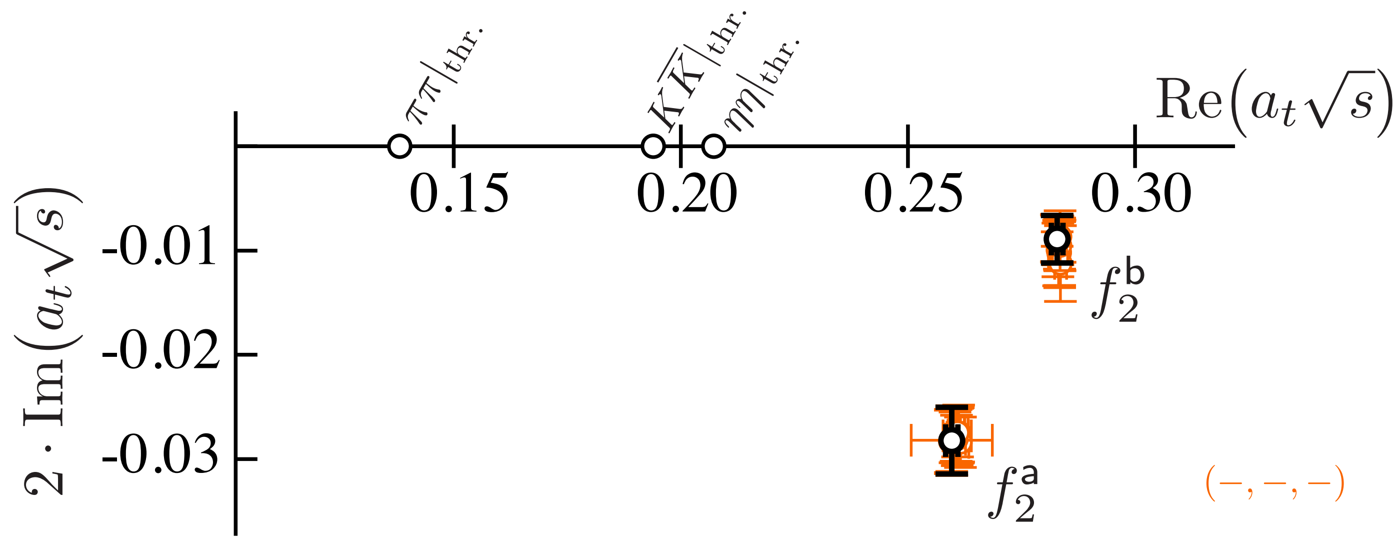
Near a t-matrix pole

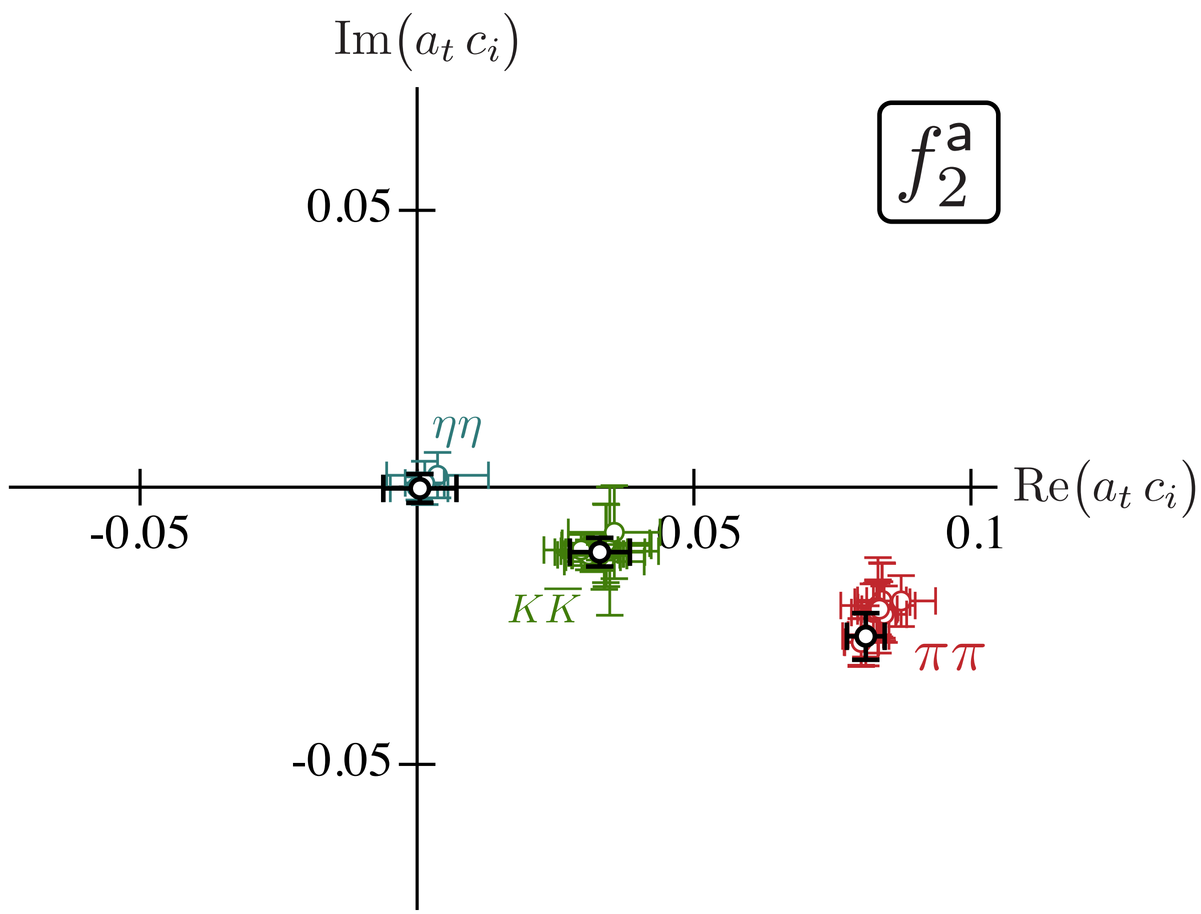
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

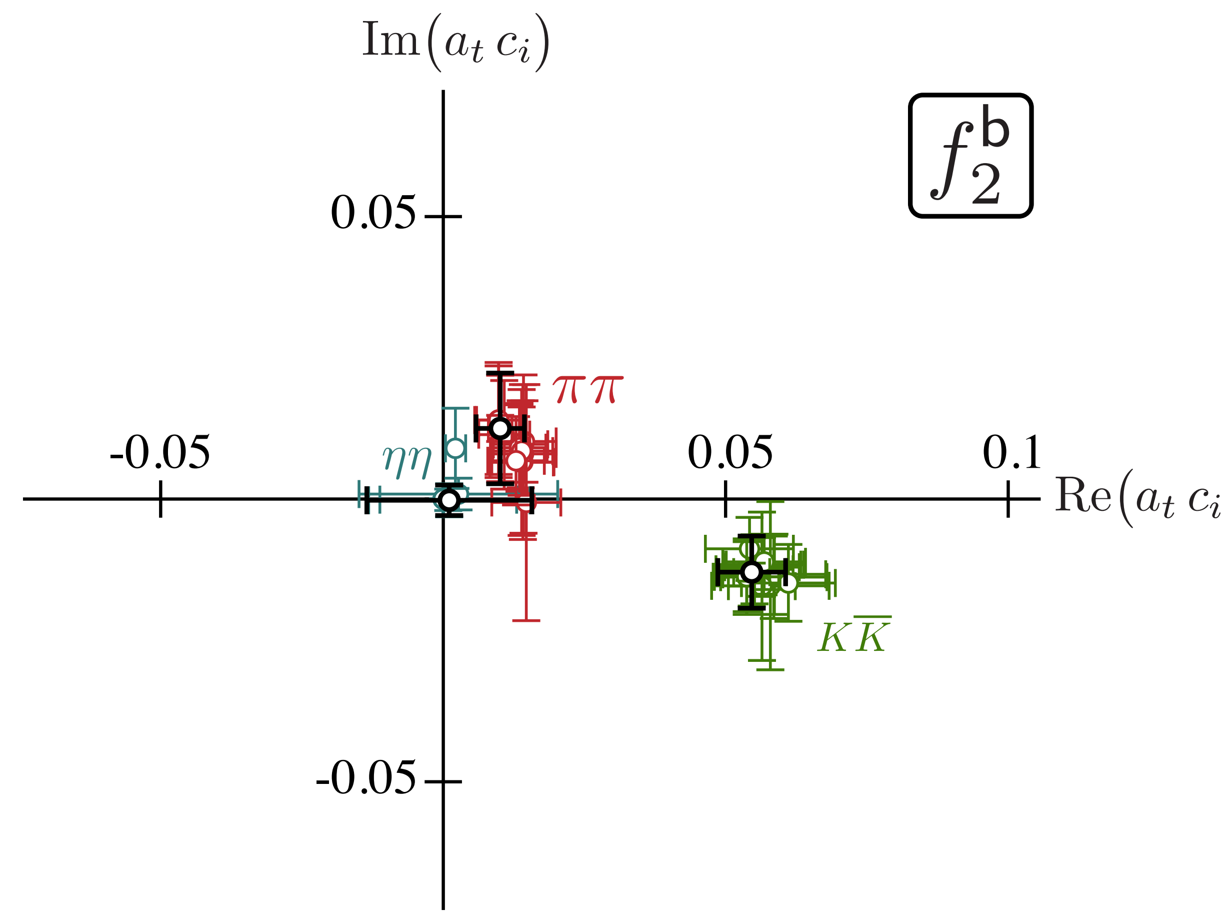


Tensor f2 resonance poles









$f_2^{\mathsf{a}}:\quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$

$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

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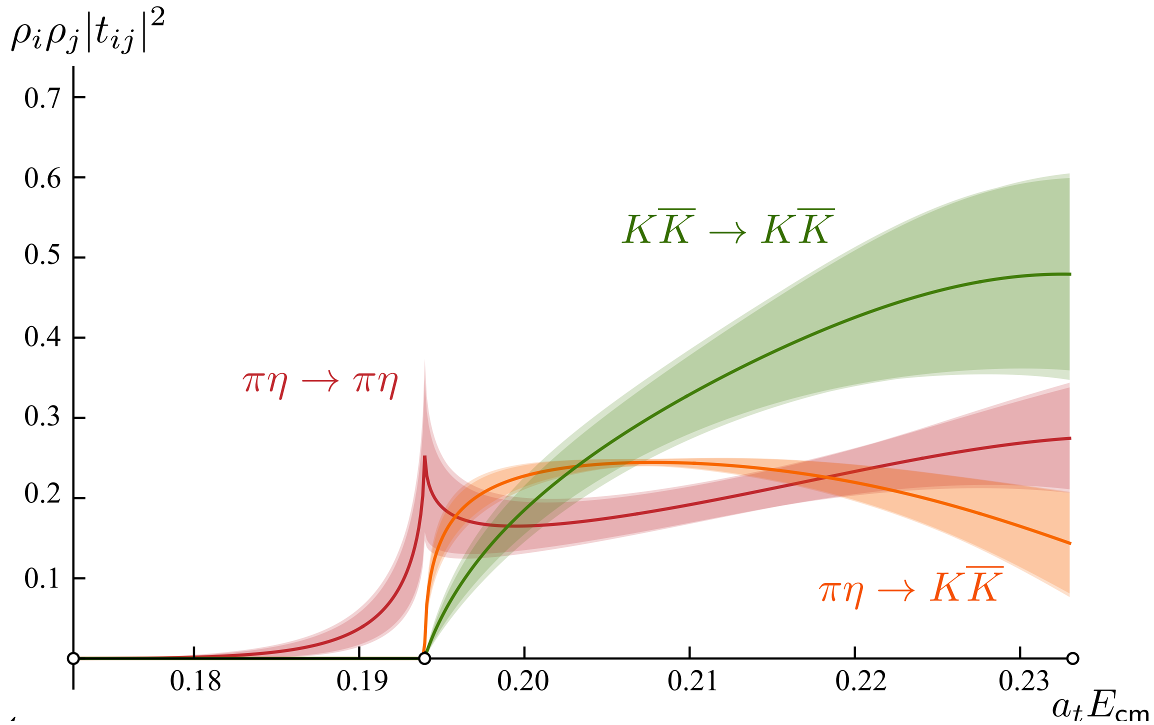
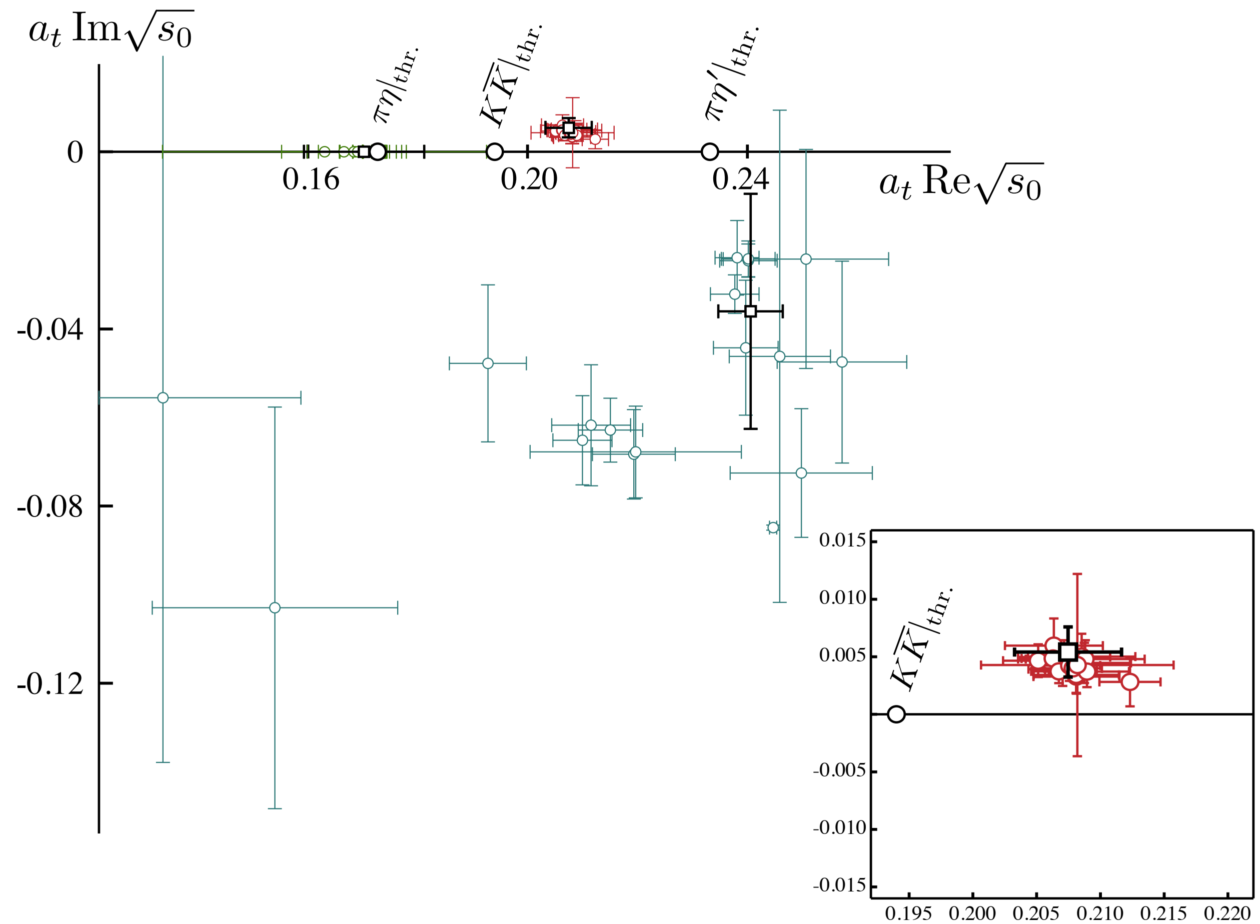
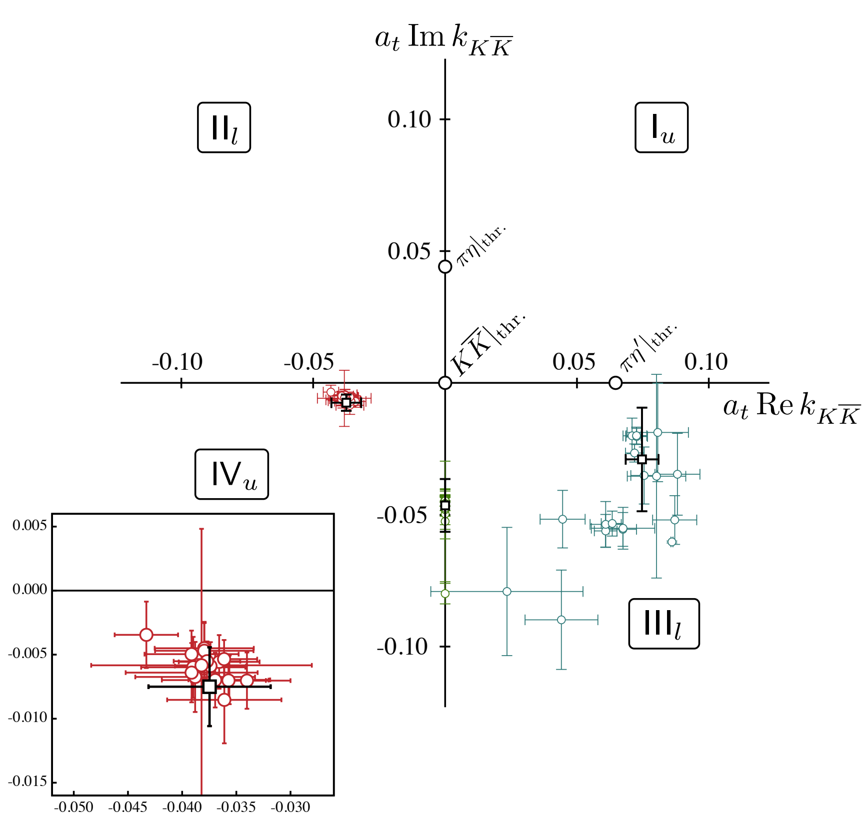
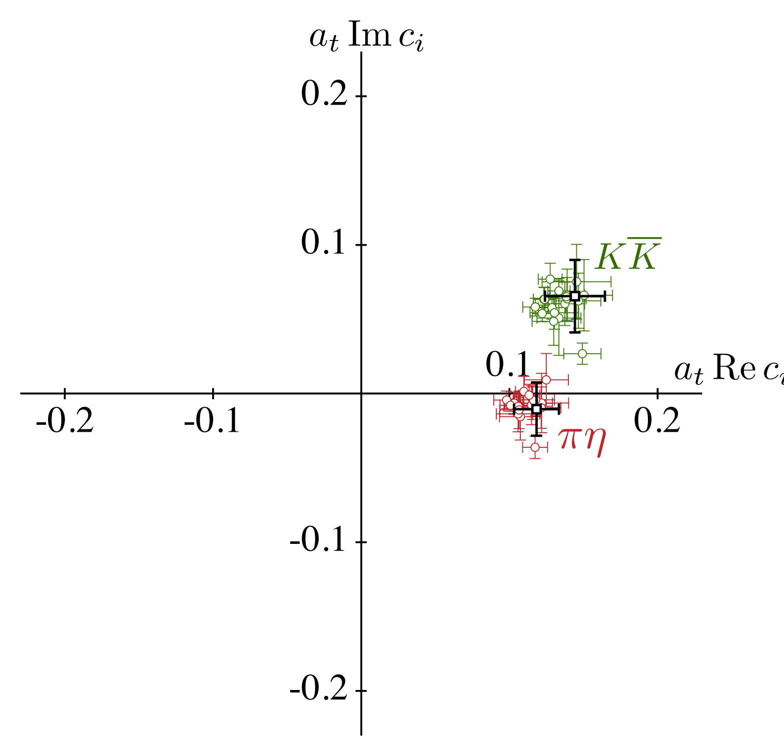
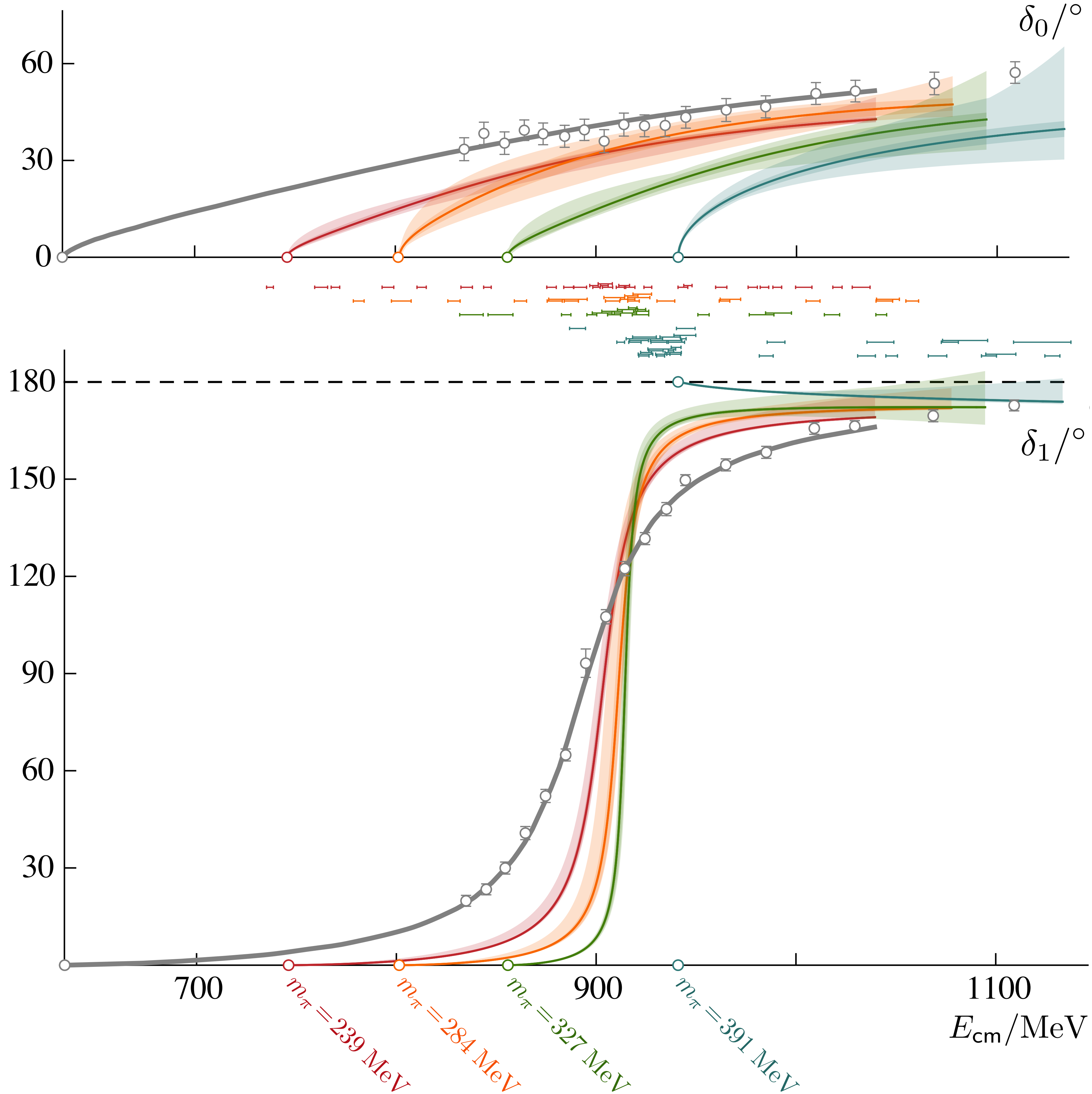
a0 resonance poles











δ₀/°
60
30
0
180
150
120
90
60
30
δ₁/°
700
900
1100
E_cm/MeV
m_π = 239 MeV
m_π = 284 MeV
m_π = 327 MeV
m_π = 391 MeV

resonance poles

scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$