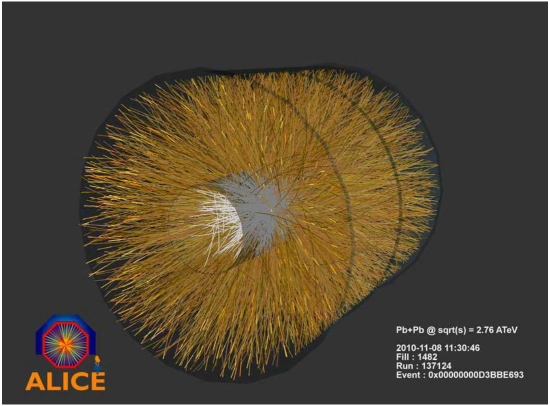
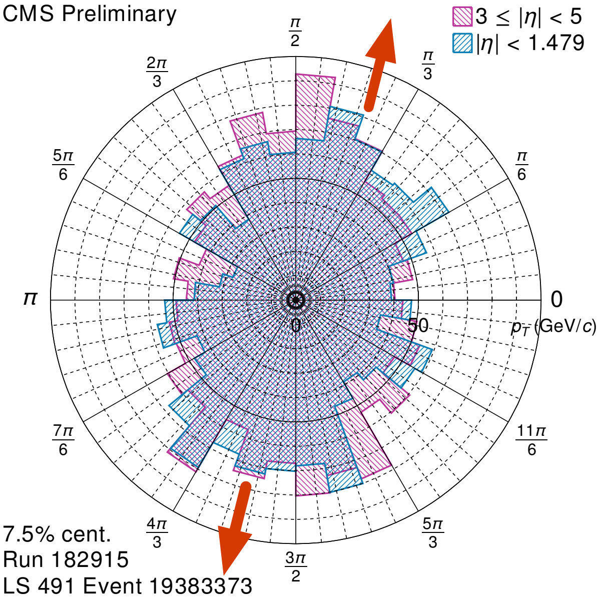
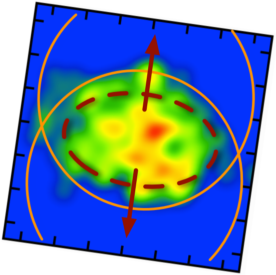
Hydrodynamics, Kinetics, and Thermalization of QGP

Derek Teaney
Stony Brook University

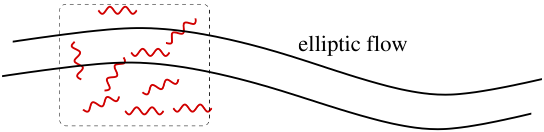


Hydrodynamics in action in high energy nuclear collisions





Viscous Hydrodynamics Equations:



$$T^{\mu\nu} = \underbrace{(e+p)\,u^\mu u^\nu + p\,g^{\mu\nu}}_{\text{equilibrium}} + \underbrace{\pi^{\mu\nu}}_{\text{non-equilibrium}}$$

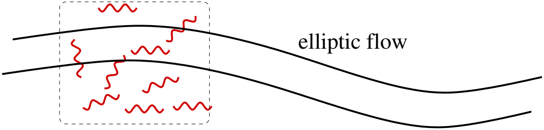
- Use a gradient expansion to characterize corrections order by order

$$\pi^{\mu\nu} = \underbrace{-\eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla\cdot u\right)}_{\text{shear strain } O(\partial)} + \underbrace{\cdots}_{\text{2nd order } O(\partial^2)}$$

- Want to calculate the parameters $p(T)$ and $\eta(T)$ from theory

To compute the kinetic coefficients $\eta(T)$ need QCD kinetic theory!

Computing the shear viscosity with weak coupling kinetics:



- The shear to entropy ratio determines a relaxation time:

$$\tau_R \equiv \underbrace{\frac{\eta}{sT}}_{\text{momentum relaxation time}}$$

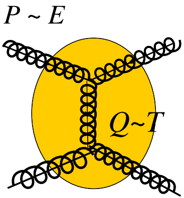
- *The perturbation theory is in g not $\alpha_s = g^2/4\pi$:*

$$\frac{\eta}{s} = \frac{1}{g^4}\Big(\underbrace{C + C\log(g)}_{\text{LO Boltzmann}} + \underbrace{Cg + C\,g\log(g)}_{\text{“NLO” from soft gluons}} + \ldots \Big)$$

Three rates for QCD Kinetic Theory at LO and "NLO" in QGP:

$$(\partial_t + \boldsymbol{v_p} \cdot \partial_{\mathbf{x}})\, f(t, \boldsymbol{x}, \boldsymbol{p}) = -C[f]$$

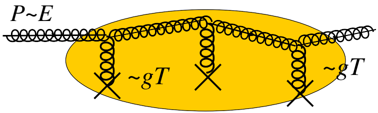
1. Hard Collisions: $2 \leftrightarrow 2$ (trivial)



$$\underbrace{C^{2\leftrightarrow 2}[\mu_\perp]}_{\text{vacuum matrix elements}}$$

2. Collisions with soft random classical field (Braaten,Pisarski 1995)

soft fields have $p \sim gT$ and *large occupation numbers* $n_B \sim \frac{T}{p} \sim \frac{1}{g}$



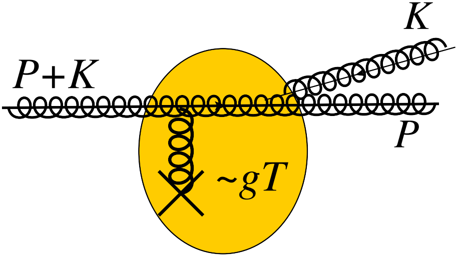
$$\underbrace{\hat{q}(\mu_\perp) = \frac{\langle p_T^2 \rangle}{L}}_{\text{momentum broadening}}$$

The most important process: collinear radiation

3. Collinear Bremm: $1 \leftrightarrow 2$

Baier,Dokshitzer,Mueller,Peigne, Schiff; Arnold, Moore, Yaffe 2001

- Random walk induces collinear bremsstrhalung at LO

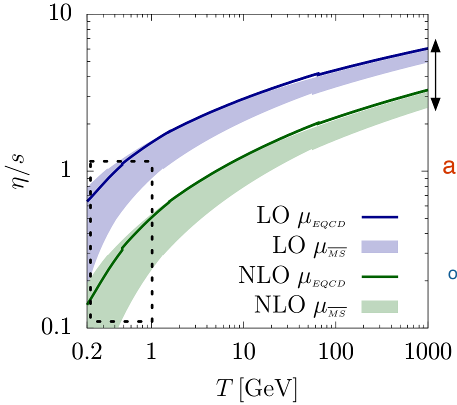


- Includes multiple scattering in the bremm rate (the LPM effect)
- The same collinear radiation cause the energy loss of jets in QGP

At "NLO" need to understand the overlap between these processes

The shear viscosity versus temperature

LO: Arnold, Moore, Yaffe; NLO: S. Caron-Huot; Ghiglieri, Moore, Teaney



Poor convergence!

The NLO correction is large and makes the viscosity smaller

Can the error be reduced with classical statistical simulations of quasi-particles? (see Boguslavski et al; Moore and Schlusser)

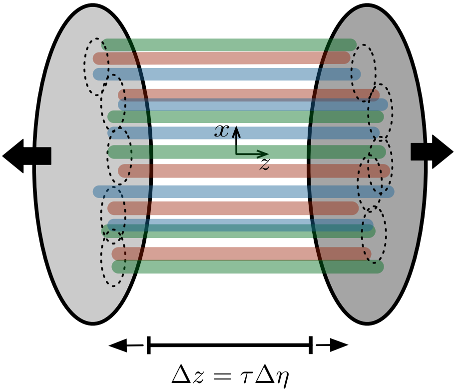
In the temperature range relevant to heavy ion collisions:

$$\tau_R = \frac{\eta}{sT} \simeq \frac{(2 \leftrightarrow 8)\hbar}{4\pi T}$$

Larger than hydro fits where $4\pi\eta/sT \simeq 1.0$

see Bernhard,Morland,Bass,Liu,Heinz

Using QCD kinetics for Heavy Ion Collisions

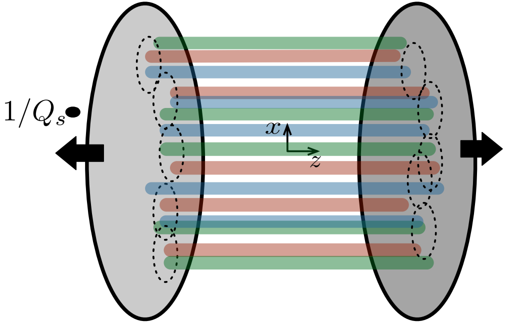


Kinetic theory in the expanding geometry:

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + \underbrace{\tau^2 d\eta^2}_{dz^2}$$

Kurkela, Zhu; Keegan, Kurkela Mazeliauskas, Teaney; Kurkela, Mazeliauskas, Paquet, Schlichting

The initial production and the Color Glass Condensate (CGC)



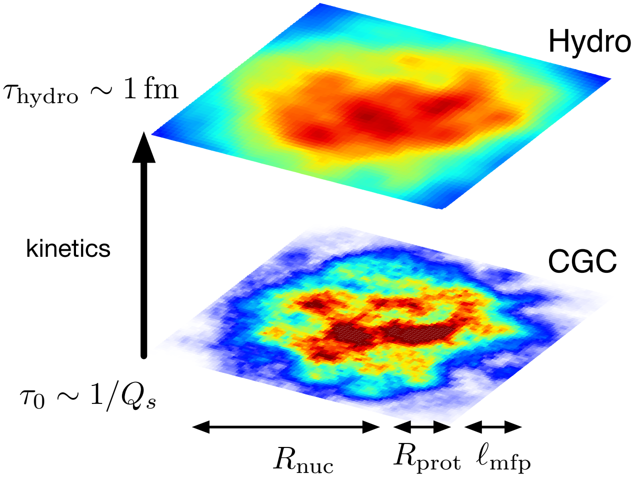
At high energies and large nuclei, the gluon density gets very large

$$\frac{1}{\pi R_A^2}\frac{dN}{dy} \sim \frac{Q_s^2}{\alpha_s} \qquad Q_s \gg \Lambda_{QCD}$$

Then, the initial passage can be treated with classical QCD (the CGC)

McLerran, Venugopalan

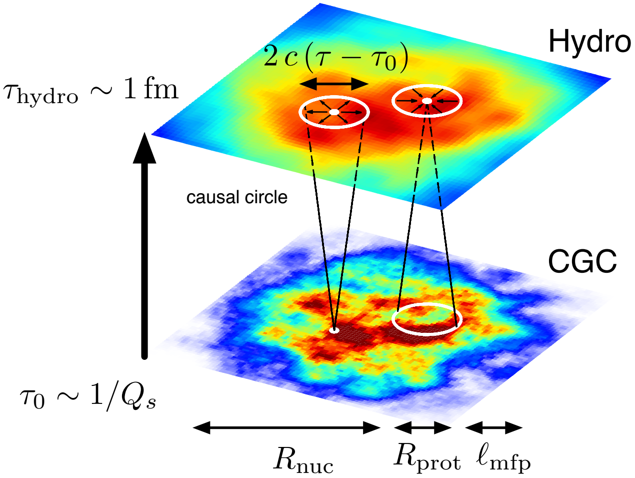
Mapping the fluctuating CGC initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics

$$R_{\mathrm{nuc}} \gg R_{\mathrm{prot}} \sim \ell_{\mathrm{mfp}} \gg 1/Q_s$$

Mapping the fluctuating CGC initial conditions to hydro

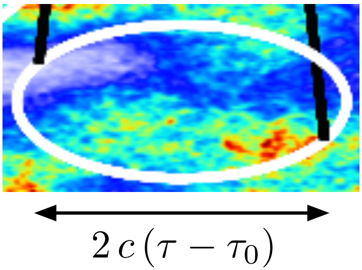


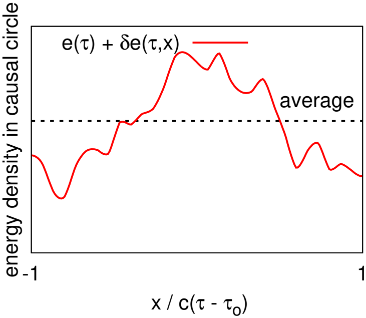
Causality limits the equilibration dynamics within a causal circle

$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \sim c\tau_{\rm hydro} \gg 1/Q_s$$

An approximation scheme for the equilibration dynamics:

look in causal circle





1. Determine the evolution of the average (homogeneous) background

   Bottom-Up Thermalization

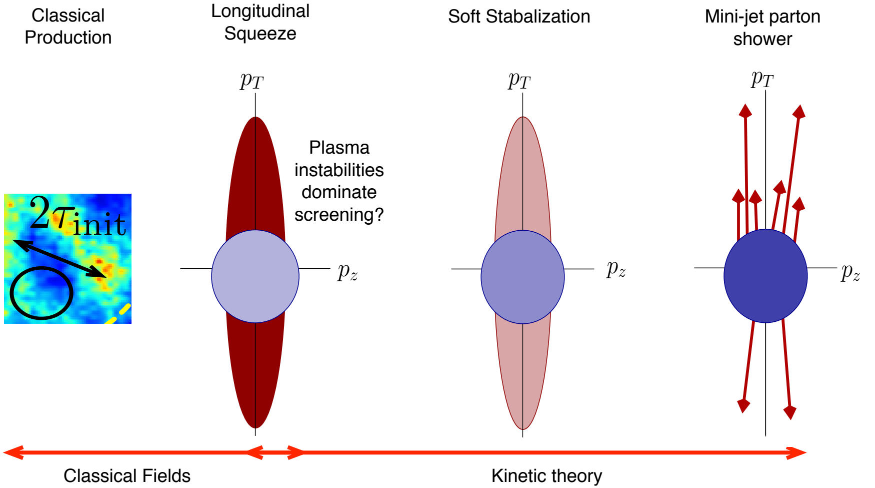
2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}'\, G(\boldsymbol{x} - \boldsymbol{x}'; \tau, \tau_o) \underbrace{\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

The background and "bottom-up" thermalization

Baier, Mueller, Schiff, Son (2001);

Berges, Boguslavski, Schlichting, Venugopalan (2014); Berges, Mace, Schlichting; Boguslaski,Kurkela, Lappi, Peuron (2018)

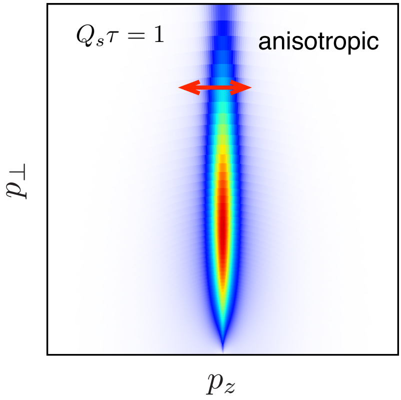


Reach a thermal state in $\tau_{\text{hydro}} \sim 1/(\alpha_s^{13/5} Q_s)$

A numerical realization of bottom-up

- Builds upon the first numerical realization — Kurkela, Zhu PRL (2015)

$p^2 f(p_\perp, p_z)$



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling of $\alpha_s = 0.3$ corresponding to

$$\frac{\eta}{s} = 0.6 = \frac{7.5}{4\pi}$$

also use $\lambda \equiv g^2 N$.

Can see "Bottom-Up" in the computer code.

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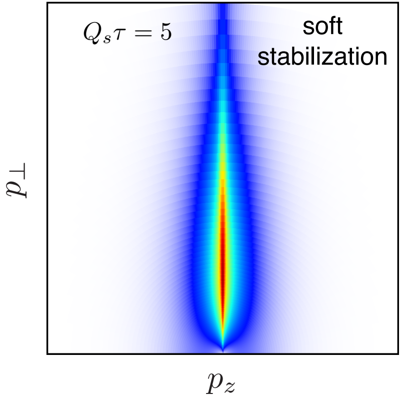

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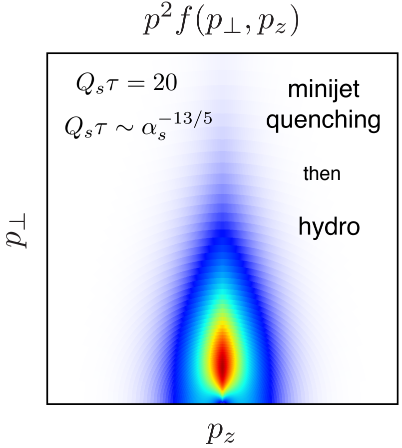

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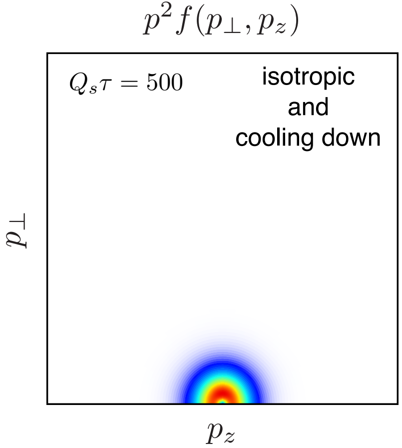

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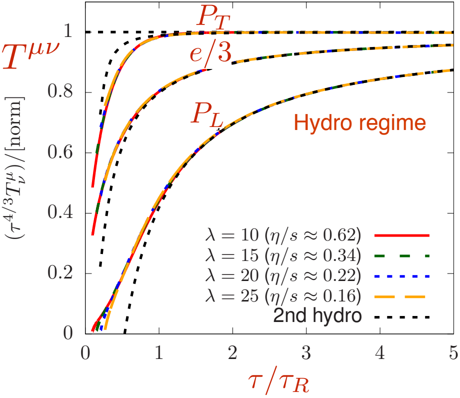
When does the stress tensor approach 2nd order hydrodynamics?



Different values of coupling give different η/s

However, all couplings thermalize at same scaled time τ/τ_R

Measure time in a physical relaxation time given by τ_R:

$$\frac{\tau}{\tau_R} \equiv \frac{\tau T_{\rm eff}(\tau)}{4\pi\eta/s} \qquad \text{with} \qquad e(\tau) \equiv \frac{\pi^2}{30}\nu_{\rm eff} T^4_{\rm eff}(\tau)$$

Can start hydro when $\tau T_{\rm eff}(\tau)/4\pi\eta/s \sim 1$

Translating "thermalization time" into physical units:

- The calculation used units which fixed the final entropy τs:
  - The final entropy is highly constrained by hydrodynamic fits, yielding:

$$\tau_{\mathsf{hydro}} \approx 1.1\,\mathrm{fm}\,\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{1/2}\underbrace{\left(\frac{4.1\,\mathrm{GeV}}{\langle\tau s\rangle}\right)^{-1/2}}_{\text{entropy per rapidity}}$$

- Require that this time be smaller than the radius of the system:
  - This yields an estimate for the smallest entropy where hydro applies

$$\left(\frac{dN_{\mathrm{ch}}}{d\eta}\right)_{\mathrm{hydro}} = 63\left(\frac{4\pi(\eta/s)}{2}\right)^{3}\left(\frac{\nu_{\mathrm{eff}}}{40}\right)\left(\frac{S/N_{\mathrm{ch}}}{7}\right)^{-1}$$

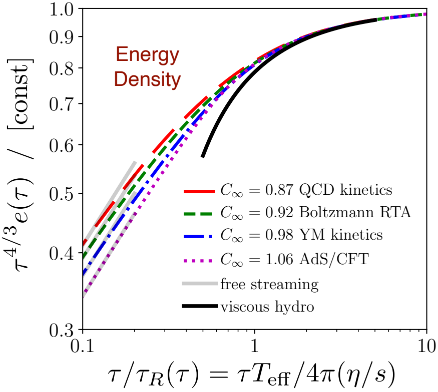
Gives constraints on smallest systems where hydrodynamics applies.
These systems are being studied in very peripheral collisions.

Attractors and a universal description:

Giacalone, Mazeliauskas, Schlichting

RTA: Almaalol, Strickland, Noronha, Denicol; Blaizot, Yan ; Behtash, Martinez, Kamata; AdS/CFT: Heller, Spalinski, Janik; QCD kinetcs: Mazeliauskas, Kurkela

$$\underbrace{\frac{d(\tau e)}{d\tau}}_{\text{energy density}} = - \underbrace{T^{zz}}_{\text{longitudinal pressure}}$$



In viscous hydro $T^{zz} = 1/3e$:

$$\tau^{4/3} e(\tau) \propto 1 - \frac{2}{3\pi}\left(\frac{\tau_R}{\tau}\right)$$

In free streaming $T^{zz} = 0$

$$e(\tau) \propto \frac{1}{\tau}$$

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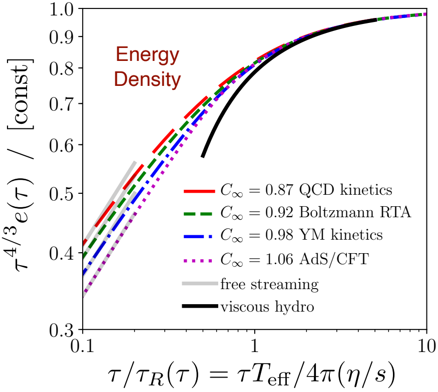

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$$\tau^{4/3}e(\tau) \propto C_\infty \left(\frac{\tau}{\tau_R(\tau)}\right)^{4/9}$$

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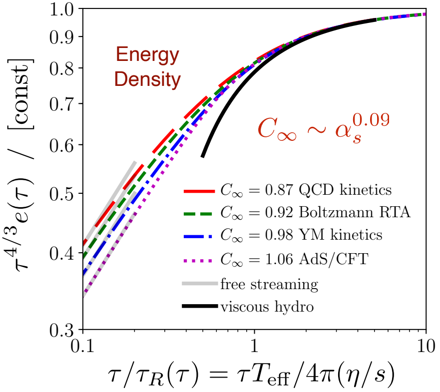

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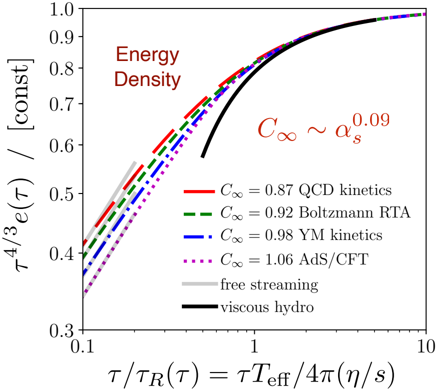

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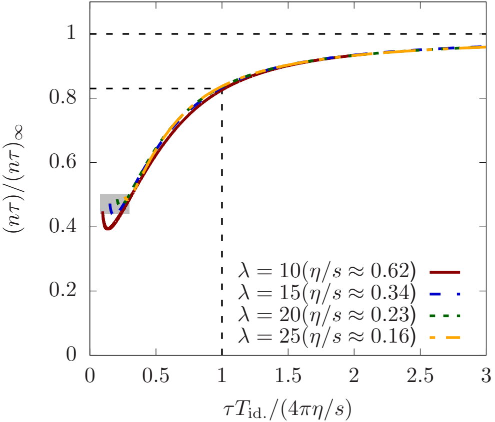
The initial energy (read Q_s) is tightly constrained by hydro+non. eq. attractor

Changes during the equilibration process

1. Increase in multiplicity.
2. Change in chemical composition.

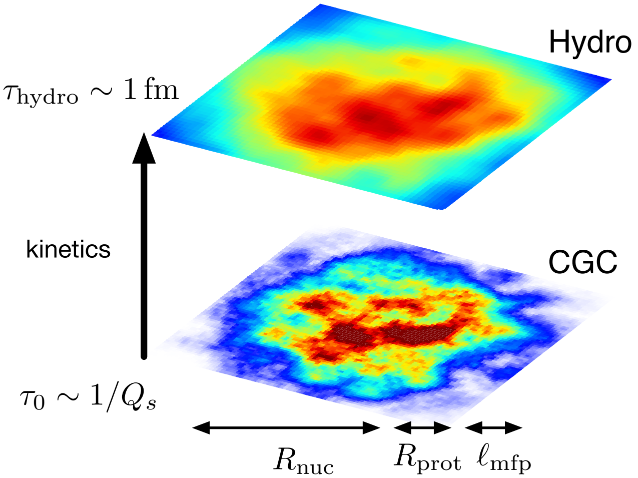
Kurkela and Mazeliauskas



$$n\tau = \frac{1}{A}\frac{dN}{dy}$$

The final gluon multiplicity is 2.5 times the initial gluon multiplicity

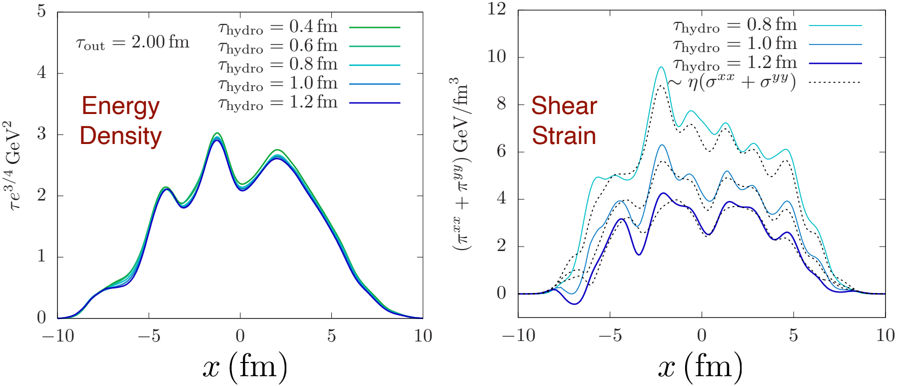
Mapping the fluctuating initial conditions to hydro



Final results should be insensitive to the switching time $\tau_{\rm hydro}$

Do hydro results depend on $\tau_{\rm hydro}$?

Kinetics runs from $\tau_0 = 0.1$ up to $\tau_{\rm hydro}$, then hydro runs up to $\tau_{\rm out}$.

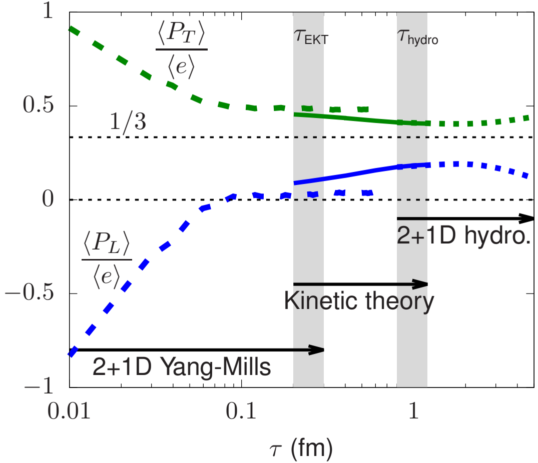


Energy density insensitive to $\tau_{\rm hydro}$ as we want.

The shear strain agrees with the hydro constitutive relations

Code starting to be used for real stuff: Gale, Paquet, Schenke, Shen (thermal photons); da Silva et al arXiv:2006.023424 (flow and $\langle p_T \rangle$; Shen, Schenke, Teaney $\langle p_T \rangle$; Coquet, Xiaojian Du, et at all 2104.07622 (dileptons)

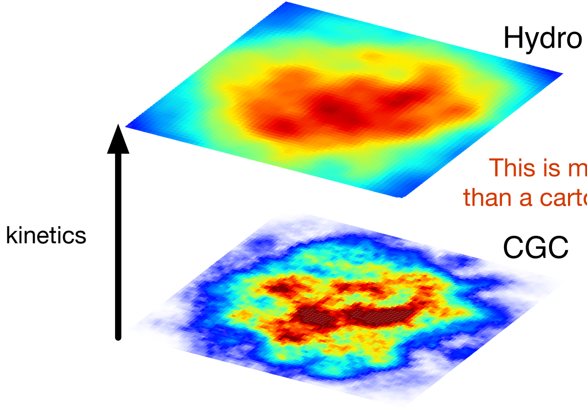
Overlapping effective theories for heavy ion collisions



Gives a 0th order description of the whole heavy ion event:
from wave function (Classical Yang-Mills), to kinetics, to hydro

Summary of QCD kinetics

1. Computed transport coefficients with QCD kinetics to "NLO"
   - Convergence is poor, but the picture is robust
2. Described a first principles (but asymptotic) picture of thermalization
   - Have a useful computer code to take to the initial state to hydro



This is much more than a cartoon by now!