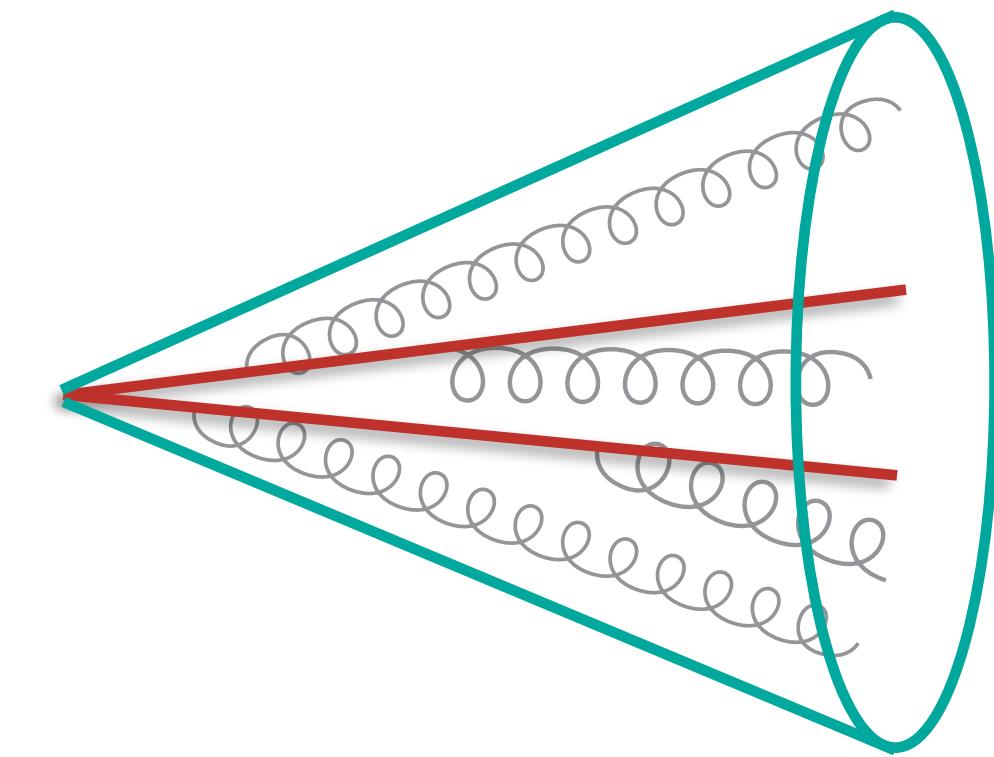


# PINNING DOWN THE SPACE-TIME EVOLUTION OF JETS WITH SUBSTRUCTURE



Alba Soto-Ontoso



9<sup>th</sup> APS Hadronic physics Workshop

Remote, 15th April, 2021



# Outline and disclaimer

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## 1 Jet substructure in proton-proton collisions at LHC energies

- Theoretical calculations with pQCD: logarithmic accuracy and resummation
- Observables: Lund plane density,  $z_g$ -distribution, dynamical grooming

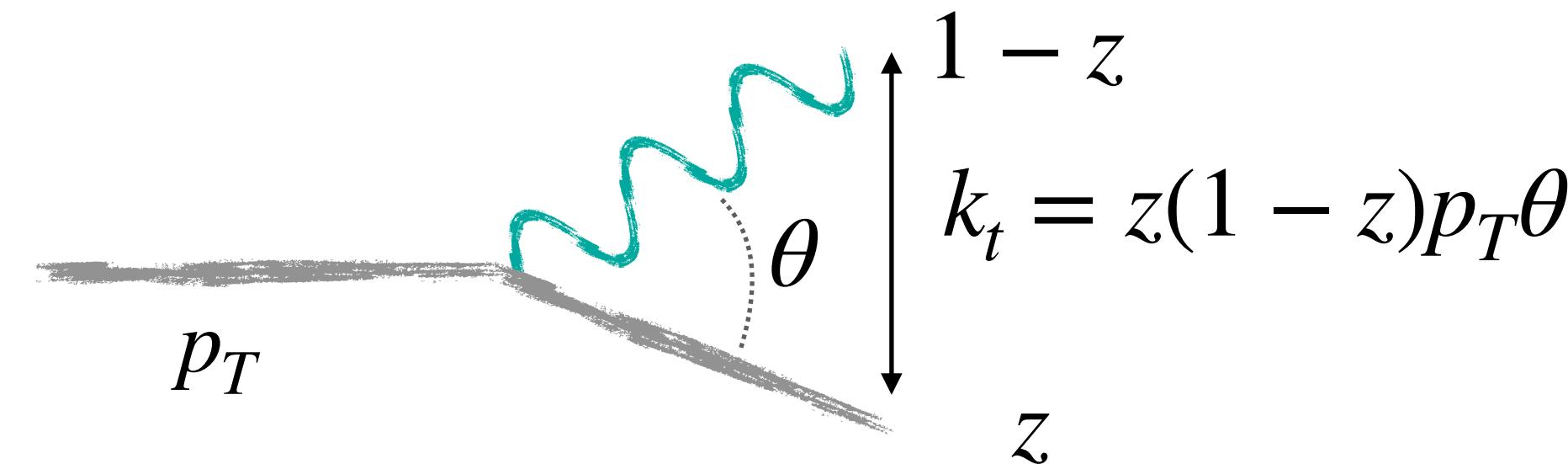
## 2 Jet substructure in Pb+Pb collisions at LHC energies

- Analytic approach to in-medium jet evolution: BDMPS-Z, GLV and IOE
- Observables:  $z_g$ -distribution,  $\theta_g$  with dynamical grooming

## Topics not covered (but very relevant)

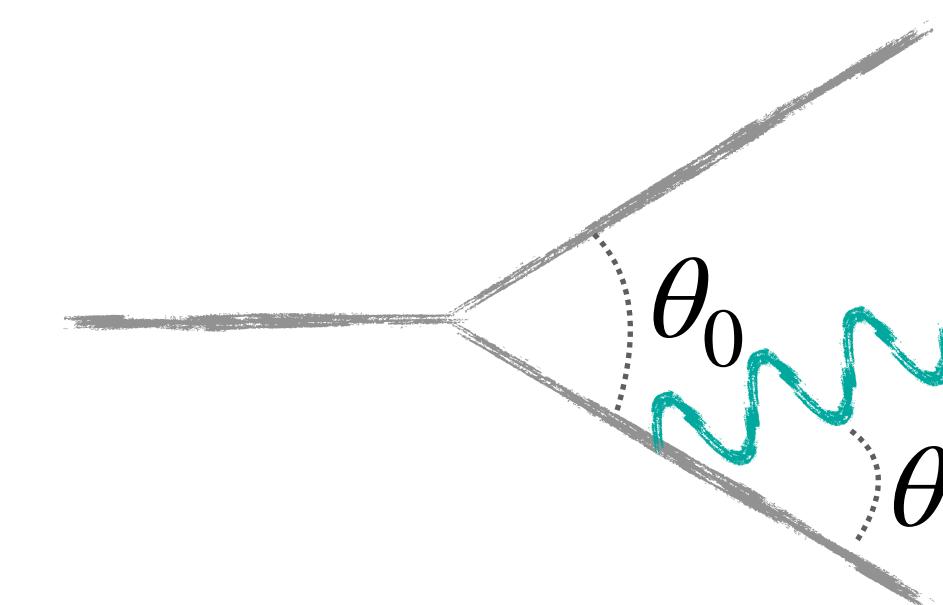
- Machine learning: [Dreyer et al. JHEP 03 (2021) 052, Du et al JHEP 03 (2021) 206, Lai et al 2012.06582]
- Jet physics at the EIC and RHIC: [EIC Yellow Report 2103.05419, STAR Phys.Lett.B 811 (2020), STAR NPA 967 (2017)]
- New clustering algorithms: [Arratia et al 2006.10751, Apolinario et al 2012.02199, Wei et al PRD 101 (2020) 9, 094015]

# Parton branching in vacuum



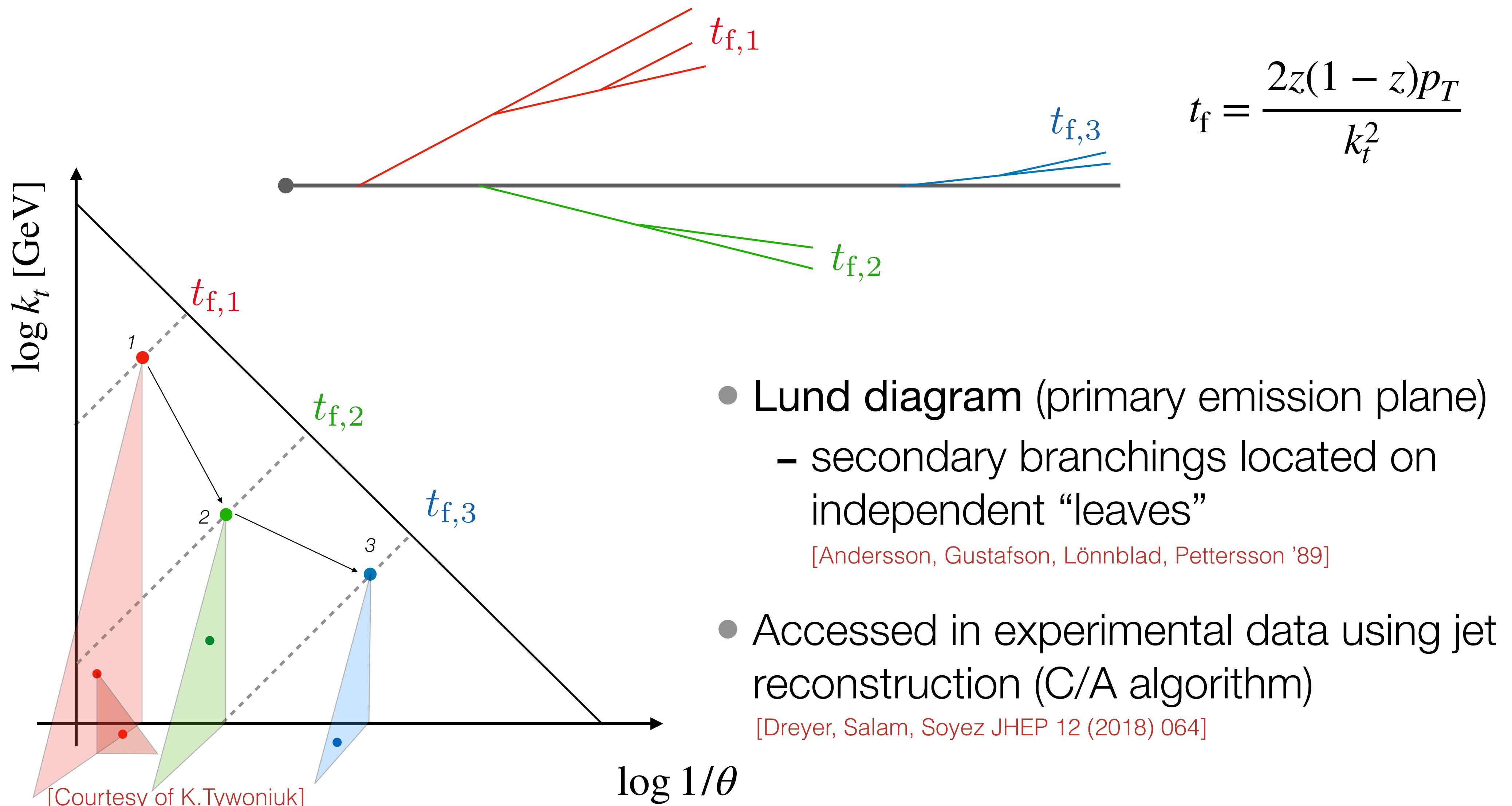
$$d\Pi_{a \rightarrow bc} = \frac{C_R \alpha_s(k_t)}{\pi} \frac{d\theta}{\theta} P_{a \rightarrow bc}(z) dz$$

- Soft and collinear divergences:  $z \ll 1, \theta \ll 1 \longrightarrow d\Pi_{LL} = \frac{2C_R \alpha_s(k_t)}{\pi} \frac{d\theta}{\theta} \frac{dz}{z}$
- Angular ordering:



$$dN^{z \ll 1} \propto C_R \alpha_s \frac{dz}{z} \frac{\sin \theta d\theta}{1 - \cos \theta} [\Theta(\cos \theta_0 - \cos \theta)]$$

# Space-time picture of a jet: Lund plane representation



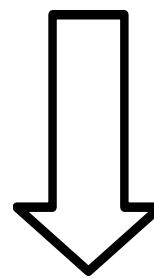
# Engineering jet substructure observables

Minimal checklist to keep in mind for potential jet substructure observables

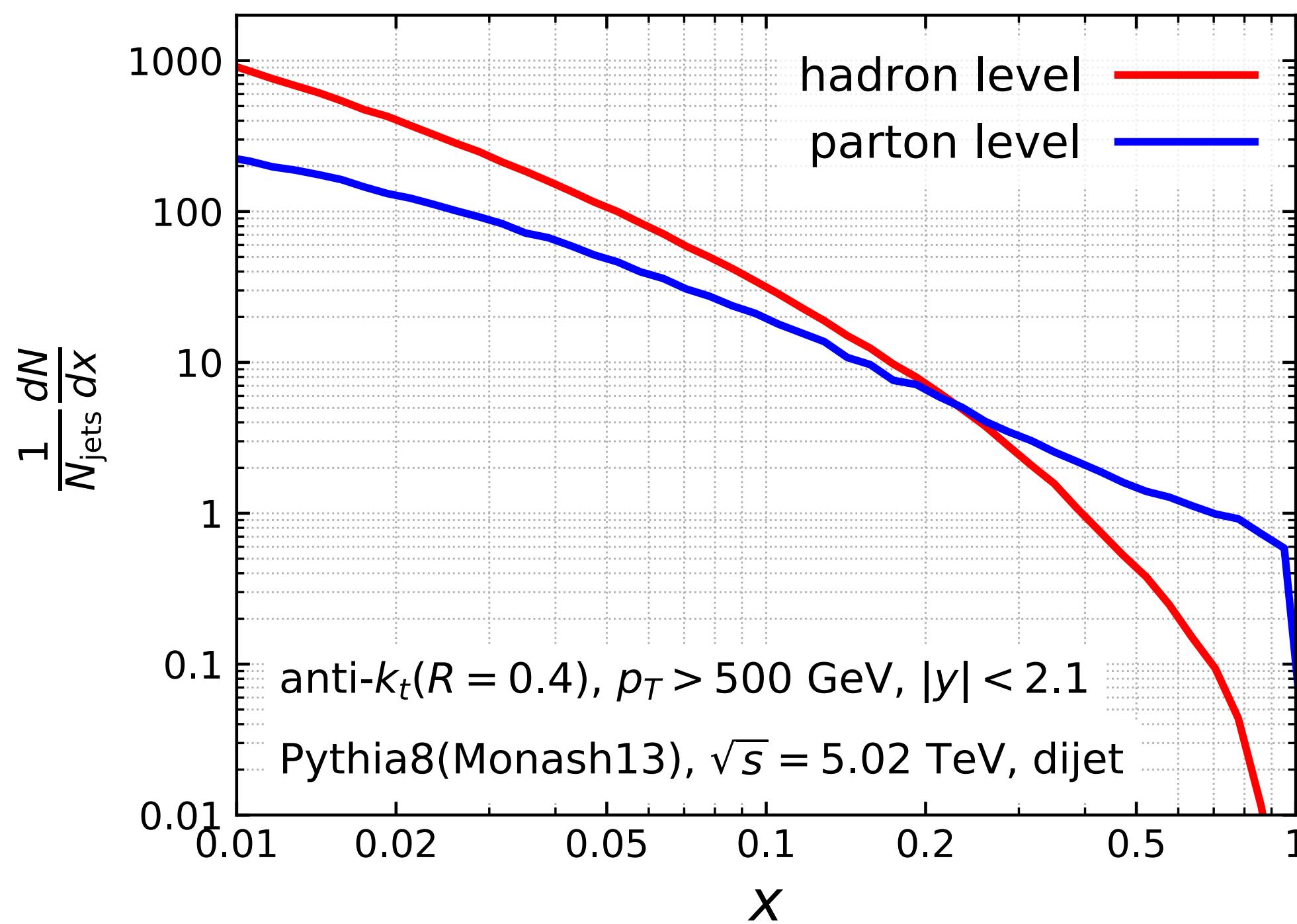
- Theoretically calculable: IRC (or at least Sudakov) safety

e.g. fragmentation function  $D(x) = \frac{1}{N_{\text{jets}}} \frac{dN}{dx}$  with  $x = \frac{p_t \cos \theta}{p_{t,\text{jet}}}$

$$x D_{\text{LL}}(x) = 2 \frac{\alpha_s C_R}{\pi} \int_0^R \frac{d\theta}{\theta} f(1/x, 1/\theta)$$



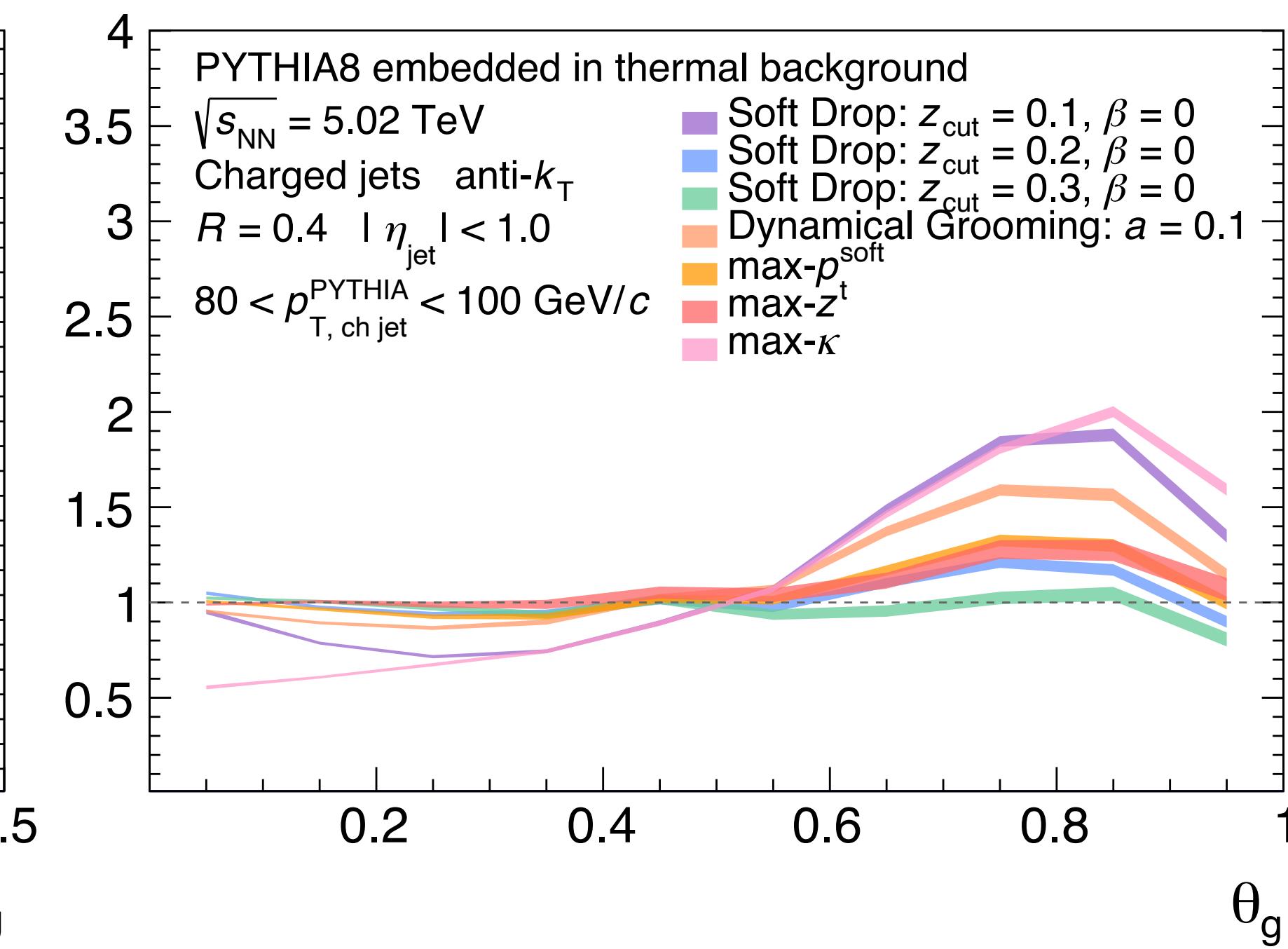
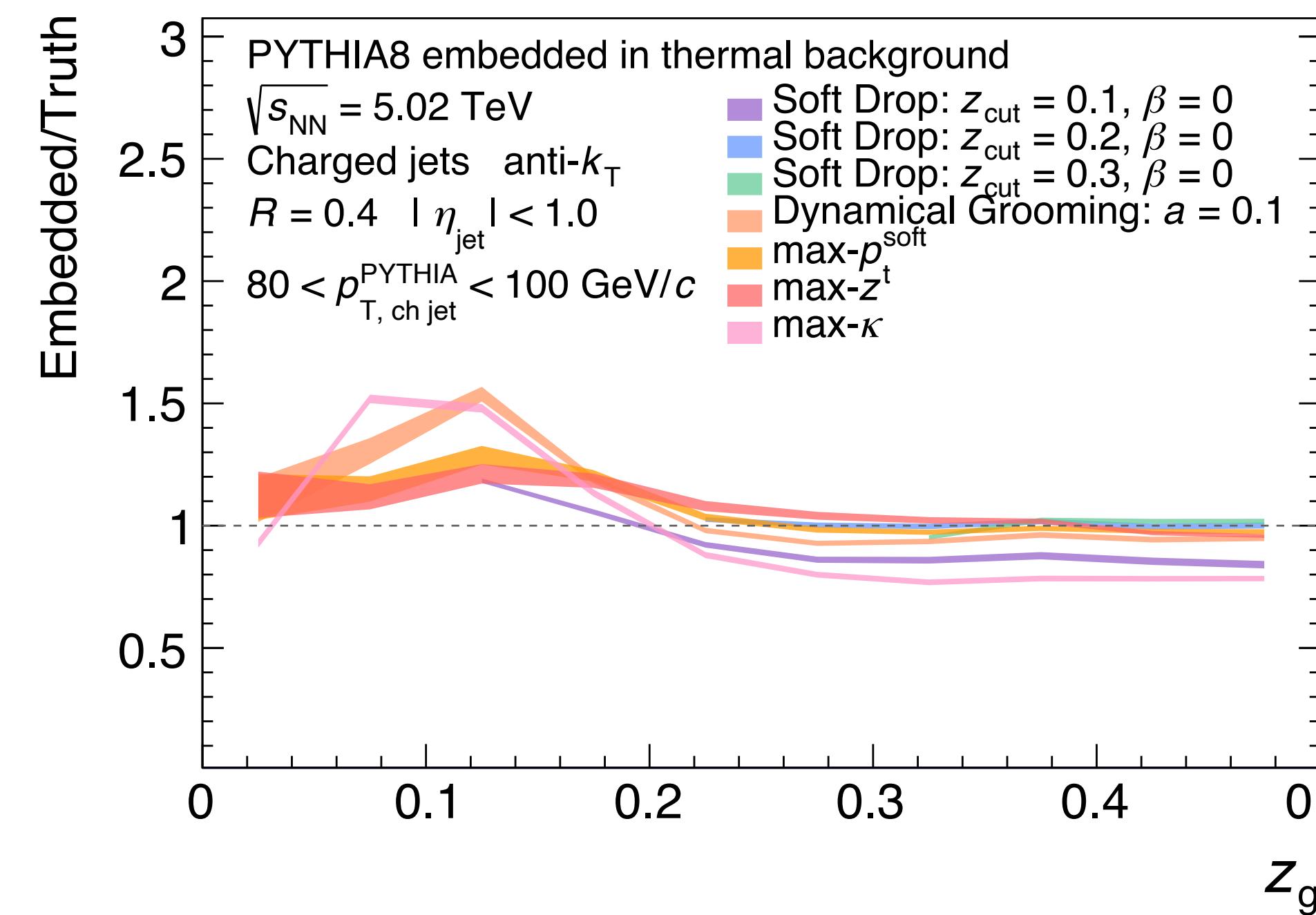
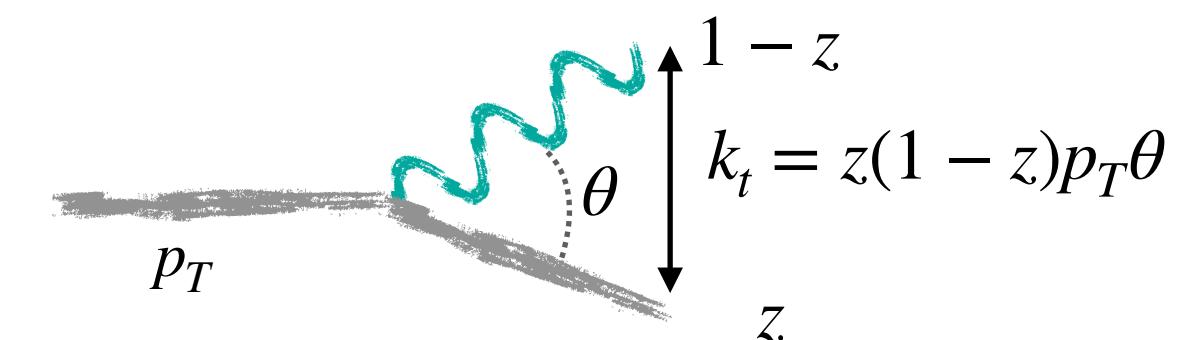
$$x D_{\text{LL}}(x) = 2 \frac{\alpha_s C_R}{\pi} \int_{k_{\perp,\min}/(xp_t)}^R \frac{d\theta}{\theta} f(1/x, 1/\theta)$$



# Engineering jet substructure observables

Minimal checklist to keep in mind for potential jet substructure observables

- Theoretically calculable: IRC (or at least Sudakov) safety
- Experimentally measurable, e.g. [thermal background](#) impact on

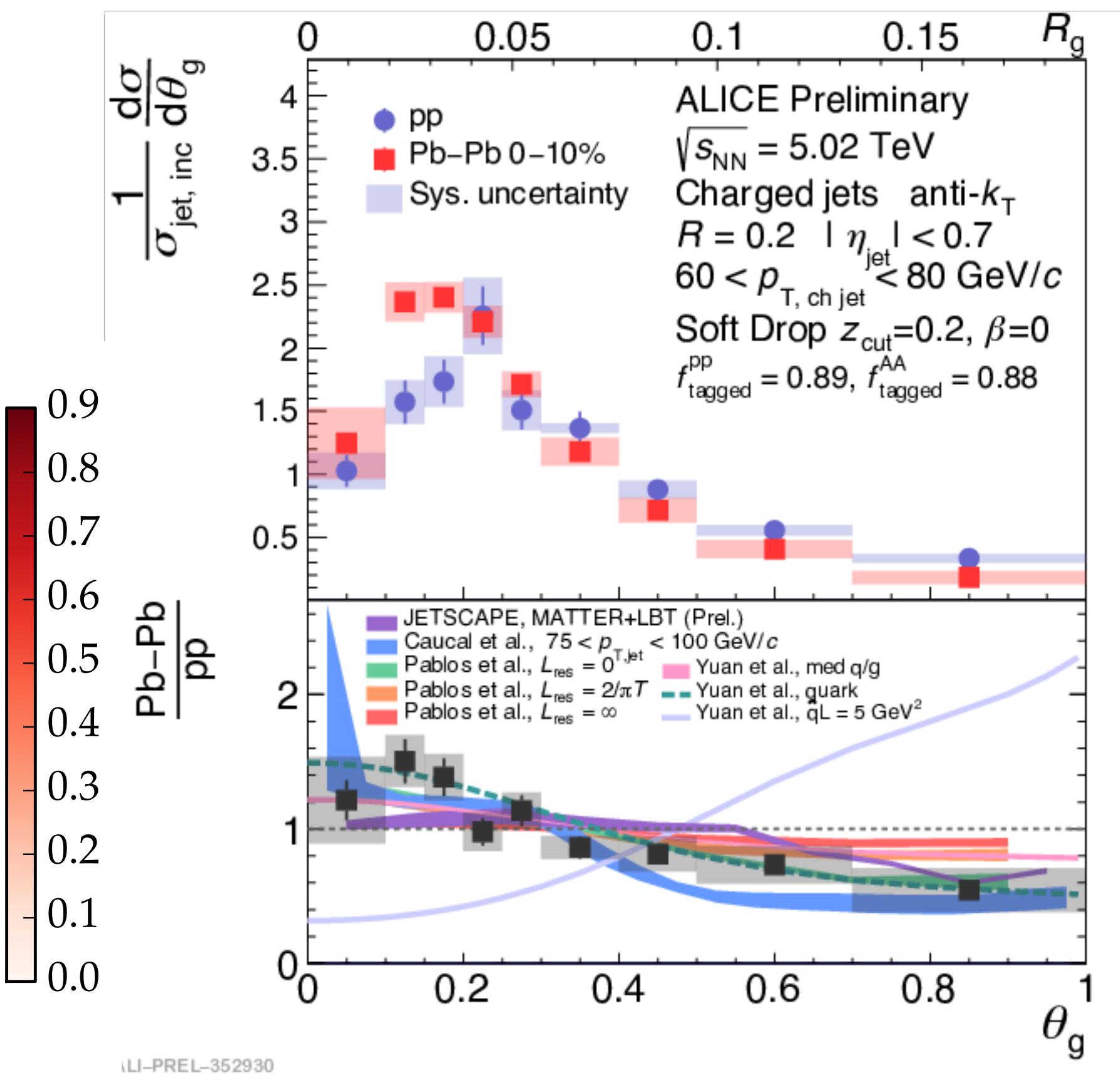
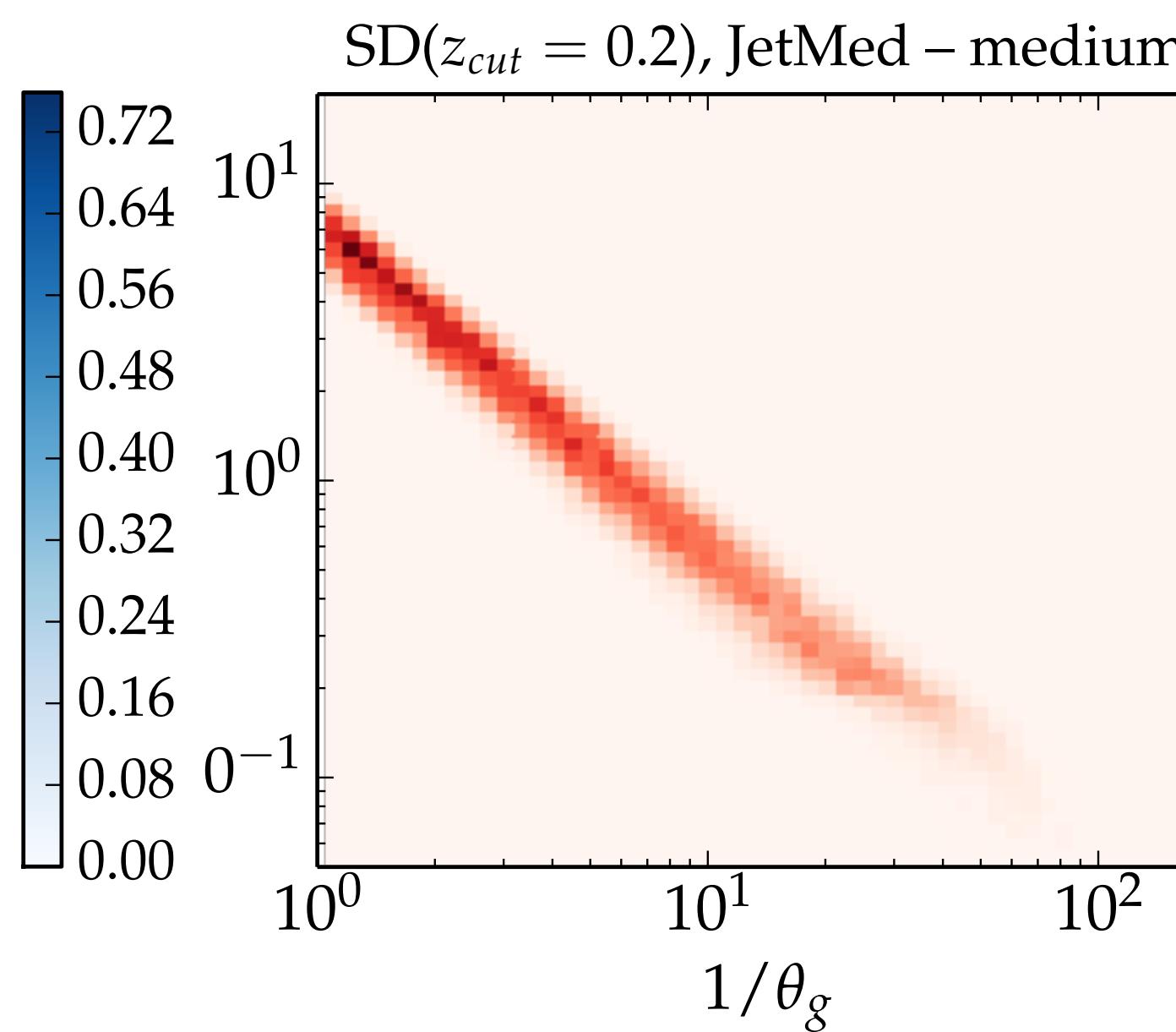
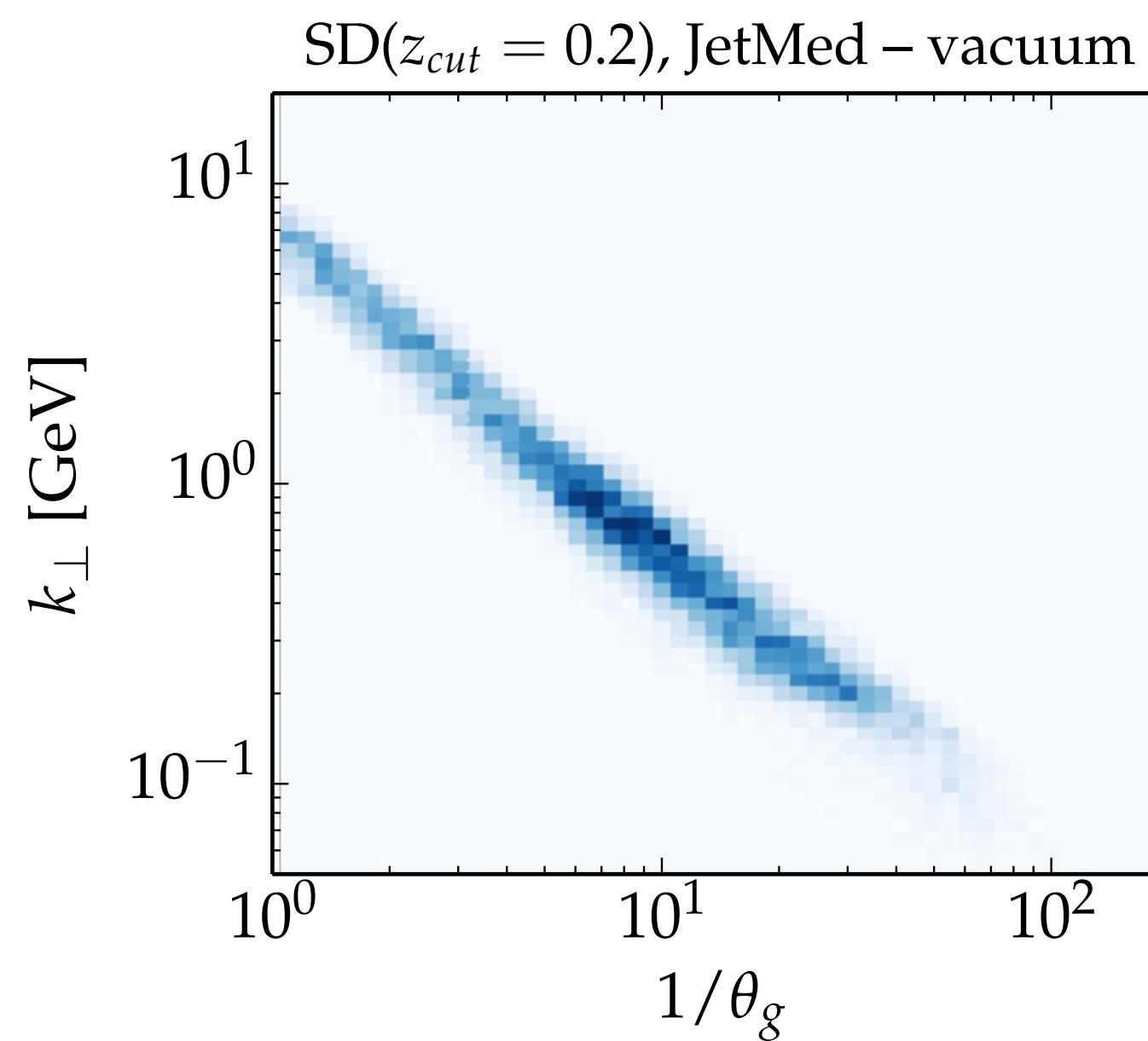


[Mulligan, Ploskon PRC 102(2020) 4]

# Engineering jet substructure observables

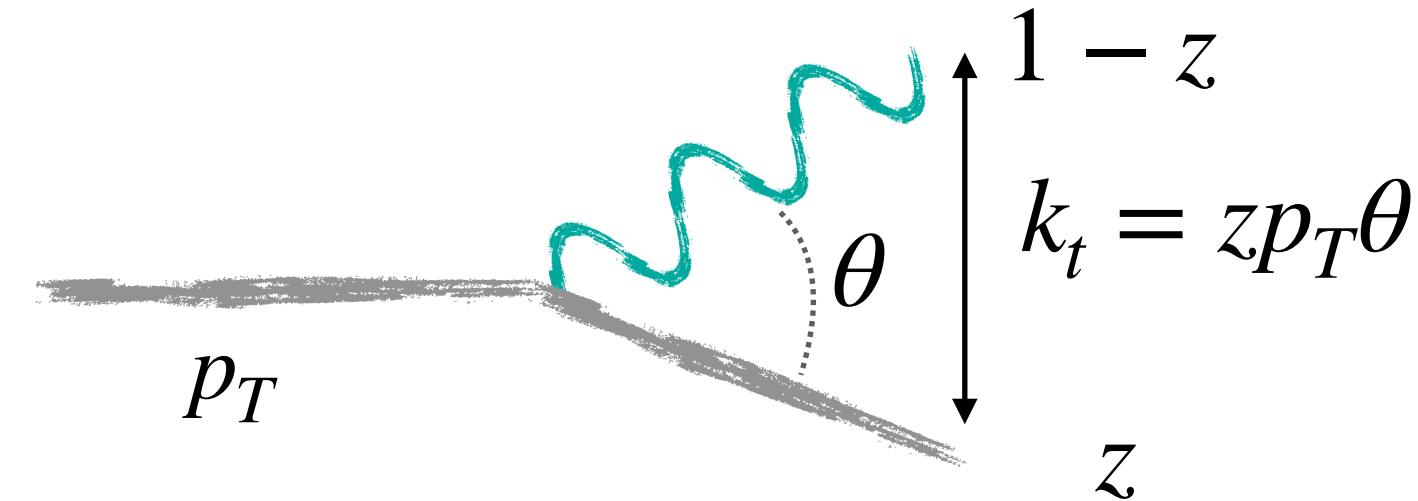
Minimal checklist to keep in mind for potential jet substructure observables

- Theoretically calculable: IRC (or at least Sudakov) safety
- Experimentally measurable
- Designed to probe a particular physics mechanism  
e.g. opening angle of the splitting with  $z > z_{\text{cut}} \theta_g^\beta$



# Analytic insight: $k_t$ -distribution of the hardest splitting

Fixed order

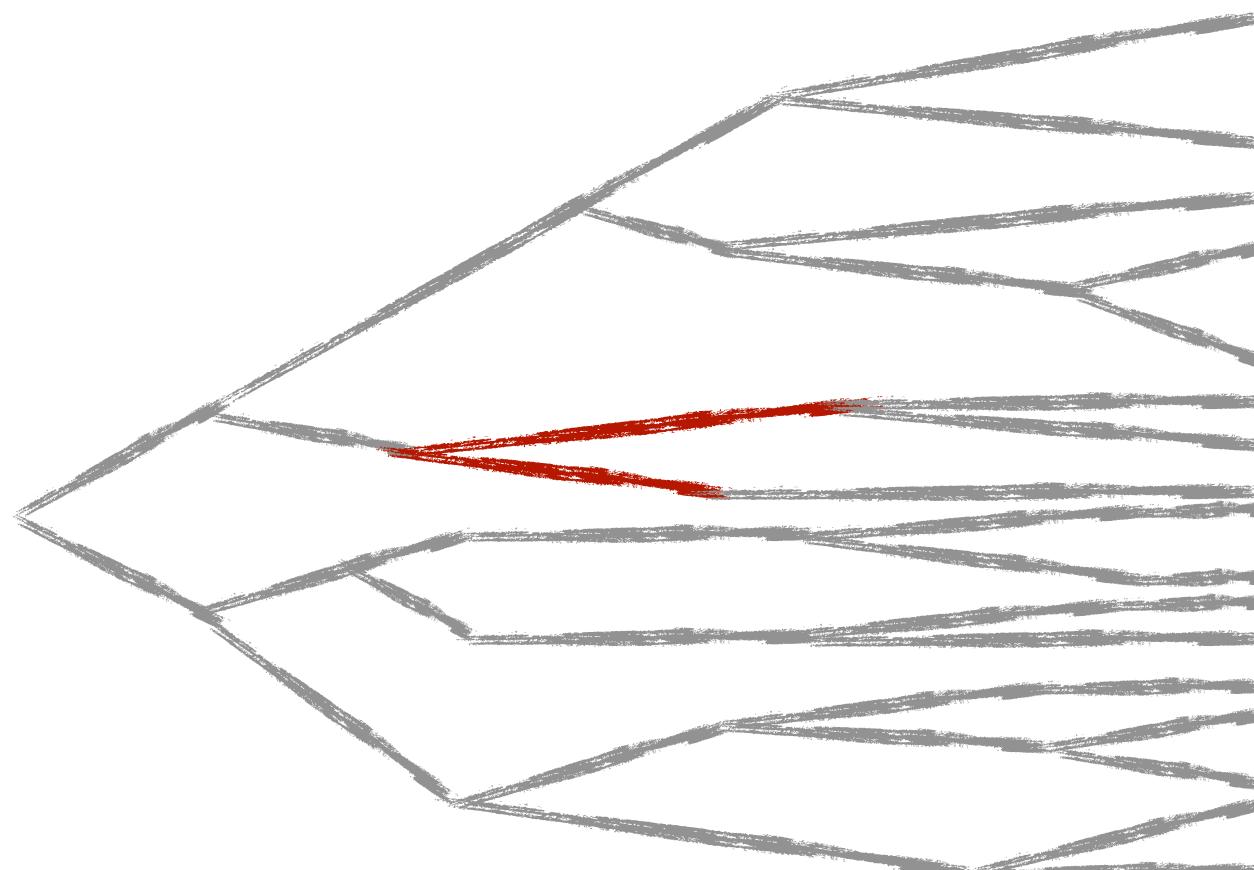


In the  $\alpha_s \ll 1, k_{t,g} \ll 1$  regime,

$$\mathcal{O}(\alpha_s) : \frac{1}{\sigma} \frac{d\sigma}{dk_{t,g}} \sim \alpha_s \frac{\ln(k_{t,g})}{k_{t,g}} \quad \longrightarrow \quad \mathcal{O}(\alpha_s \ln(k_{t,g})) = 1$$

Large logarithms enter order-by-order and must be resummed

Resummation



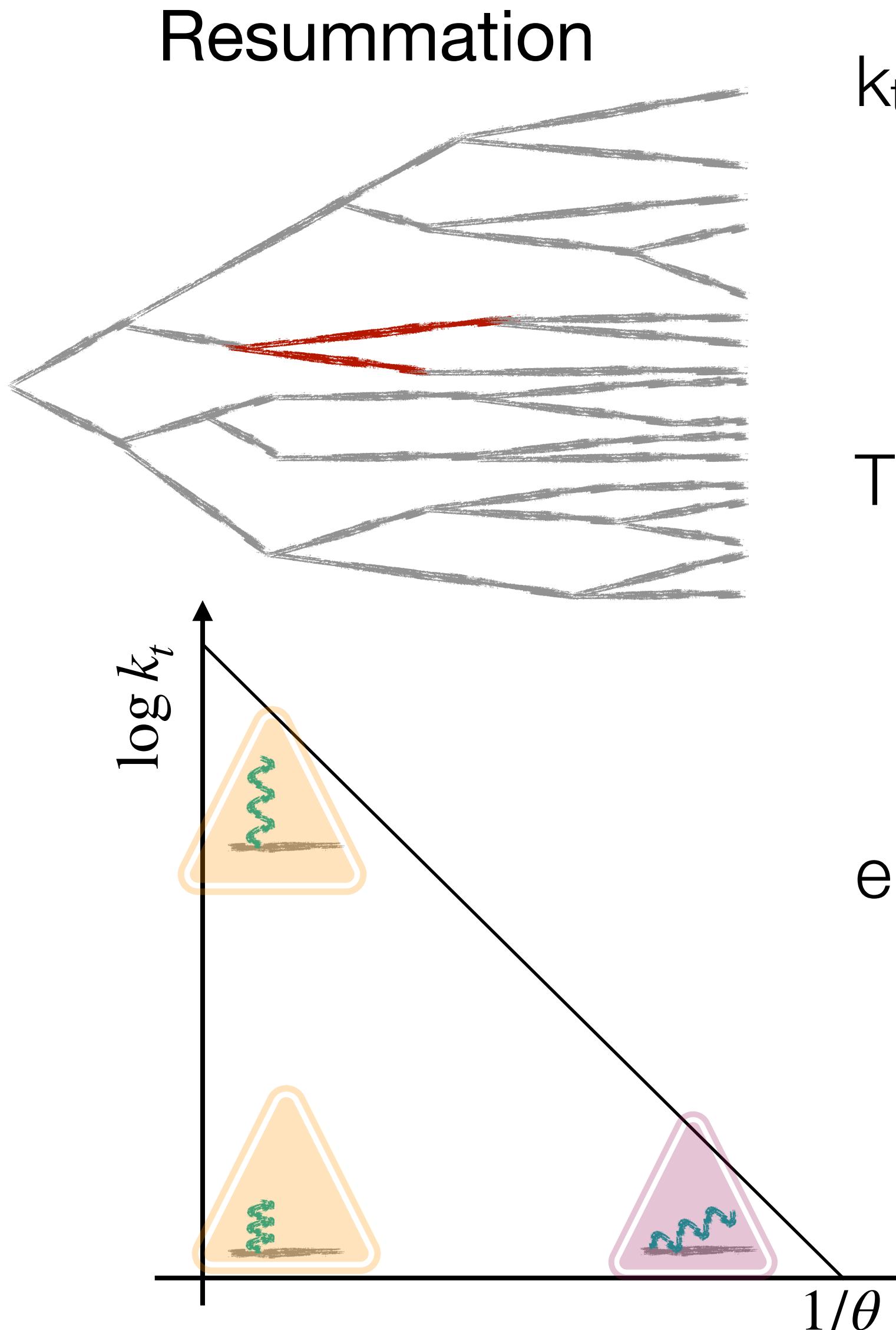
2D probability for a splitting to the hardest

$$\frac{d^2 \mathcal{P}_i(z, \theta)}{d\theta dz} = \widetilde{P}_i(z, \theta) \Delta_i(k_t)$$

== branching kernel  $\times$  Sudakov form factor

$$\ln \Delta_i(k_{t,g}) = - \int_0^1 dz' \int_0^1 d\theta' \widetilde{P}_i(z', \theta') \Theta(z'\theta' - k_{t,g})$$

# Analytic insight: $k_t$ -distribution of the hardest splitting



$k_t$  distribution of the hardest splitting

$$\frac{1}{\sigma} \frac{d\sigma}{dk_{t,g}} = \int_0^1 d\theta \int_0^1 dz \mathcal{P}_i(z, \theta) \delta(z\theta - k_{t,g})$$

This expression admits a perturbative expansion

$$\frac{1}{\sigma} \frac{d\sigma}{dk_{t,g}} \propto e^{\ln(k_{t,g})g_1(x) + g_2(x) + \alpha_s g_3(x) + \mathcal{O}(\alpha_s^{n+2} \ln^n k_{t,g})}$$

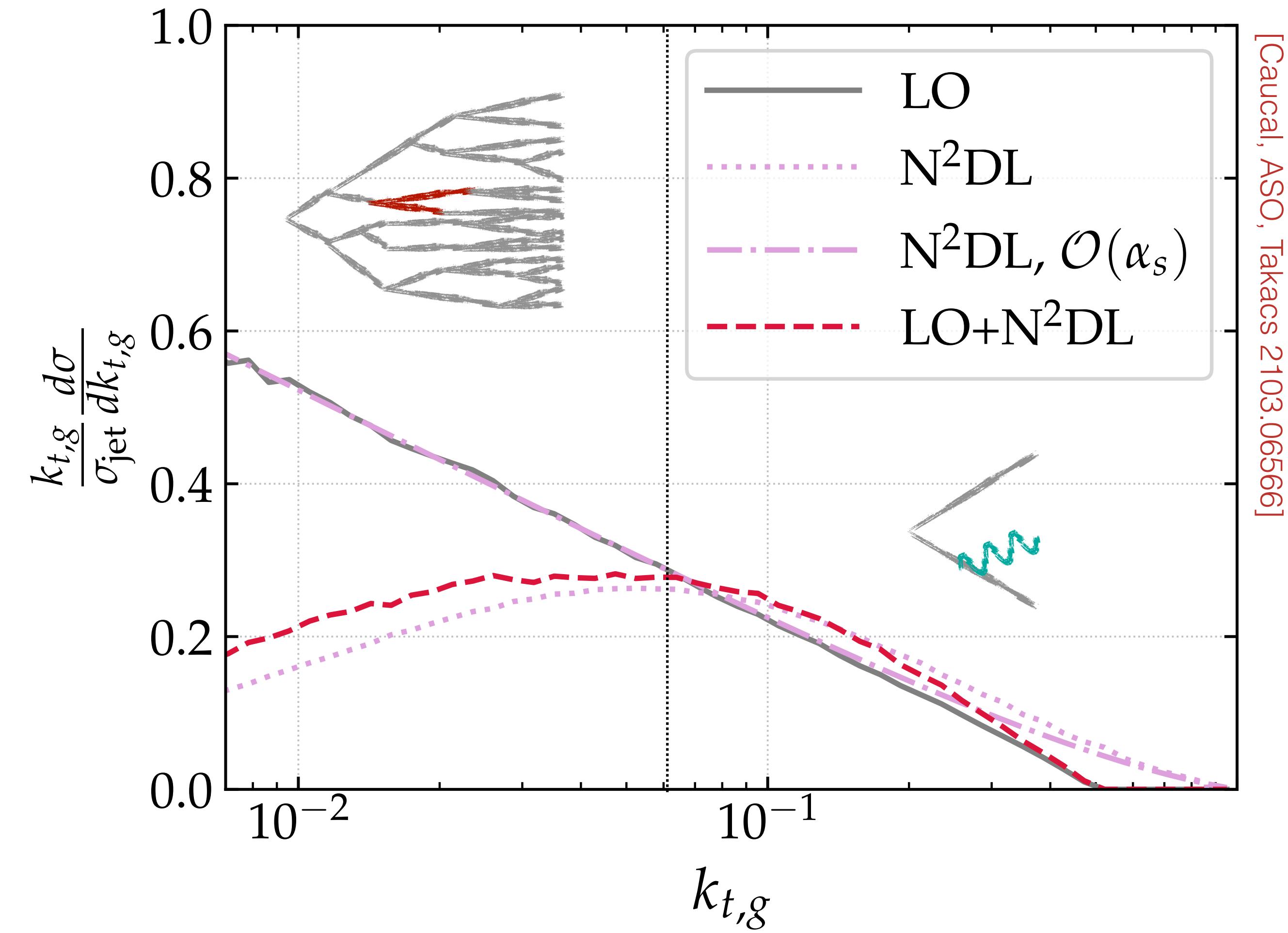
LL	NLL	NNLL
$\mathcal{O}(1/\alpha_s)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

e.g leading-log is achieved by taking soft and collinear limit

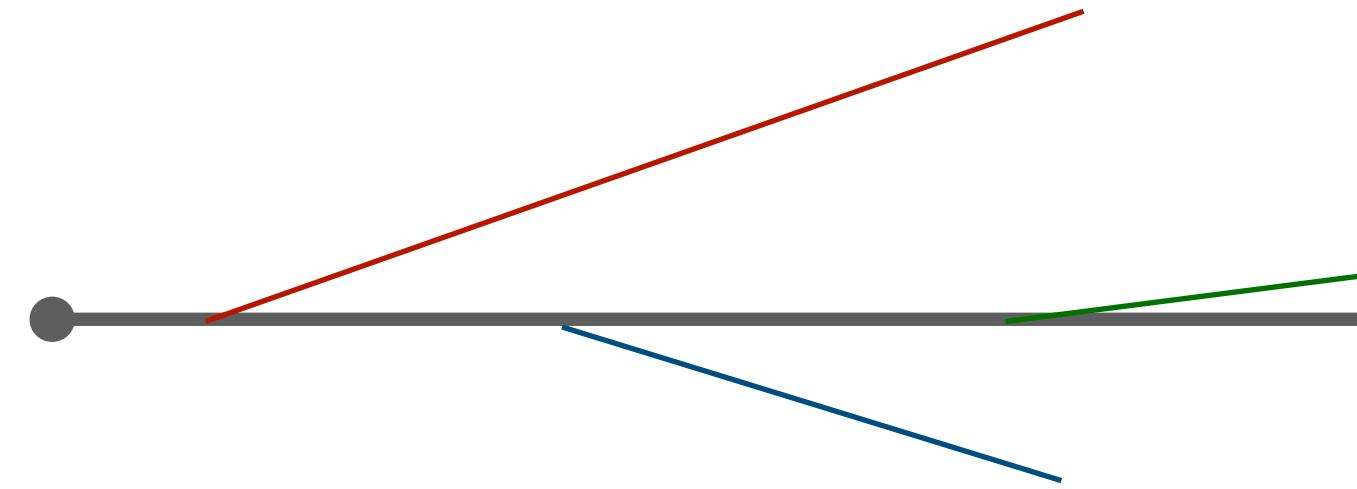
$$\widetilde{P}_i(z, \theta) = \frac{\alpha_s^{1l}(k_t)}{\theta\pi} P_i(z) \quad \text{with} \quad P_i(z) = \frac{2C_R}{z}$$

# Analytic insight: $k_t$ -distribution of the hardest splitting

Analytical full coverage of  $k_{t,g}$  is obtained by matching  $d^2\sigma_i^{\text{LO}+\text{N}^2\text{DL}} = \frac{d^2\sigma_i^{\text{LO}} \times d^2\sigma_i^{\text{N}^2\text{DL}}}{d^2\sigma_{i,1}^{\text{N}^2\text{DL}}}$



# Observable #1: Primary Lund jet plane density at NLL

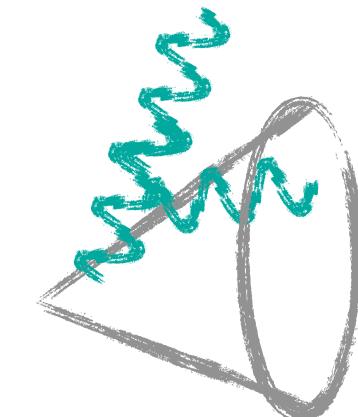


**Collinear ingredients:**

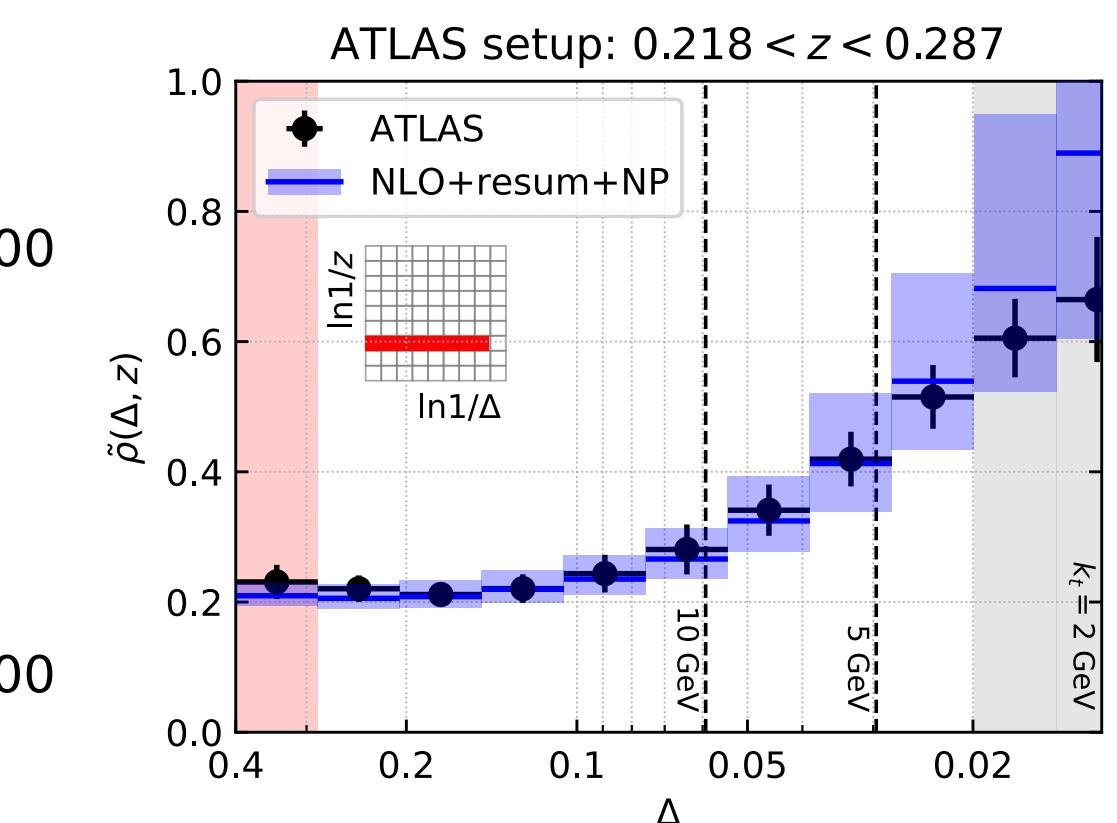
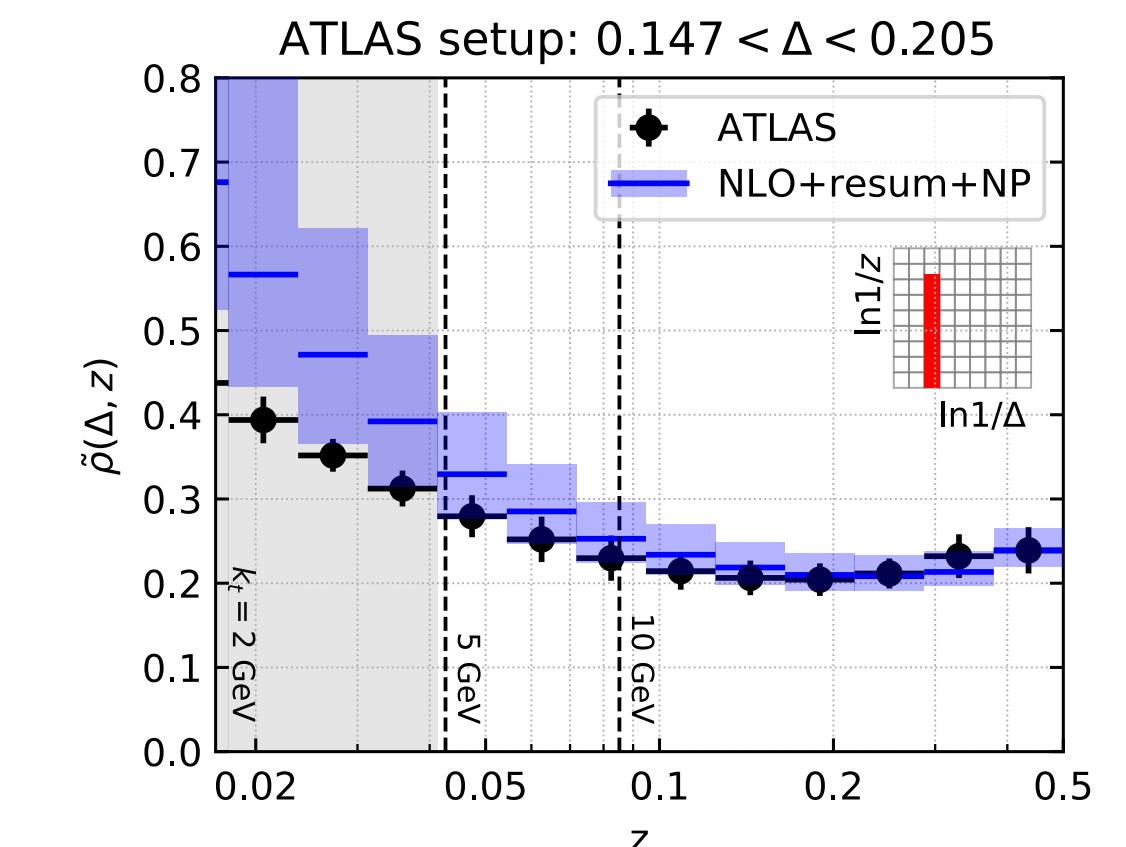
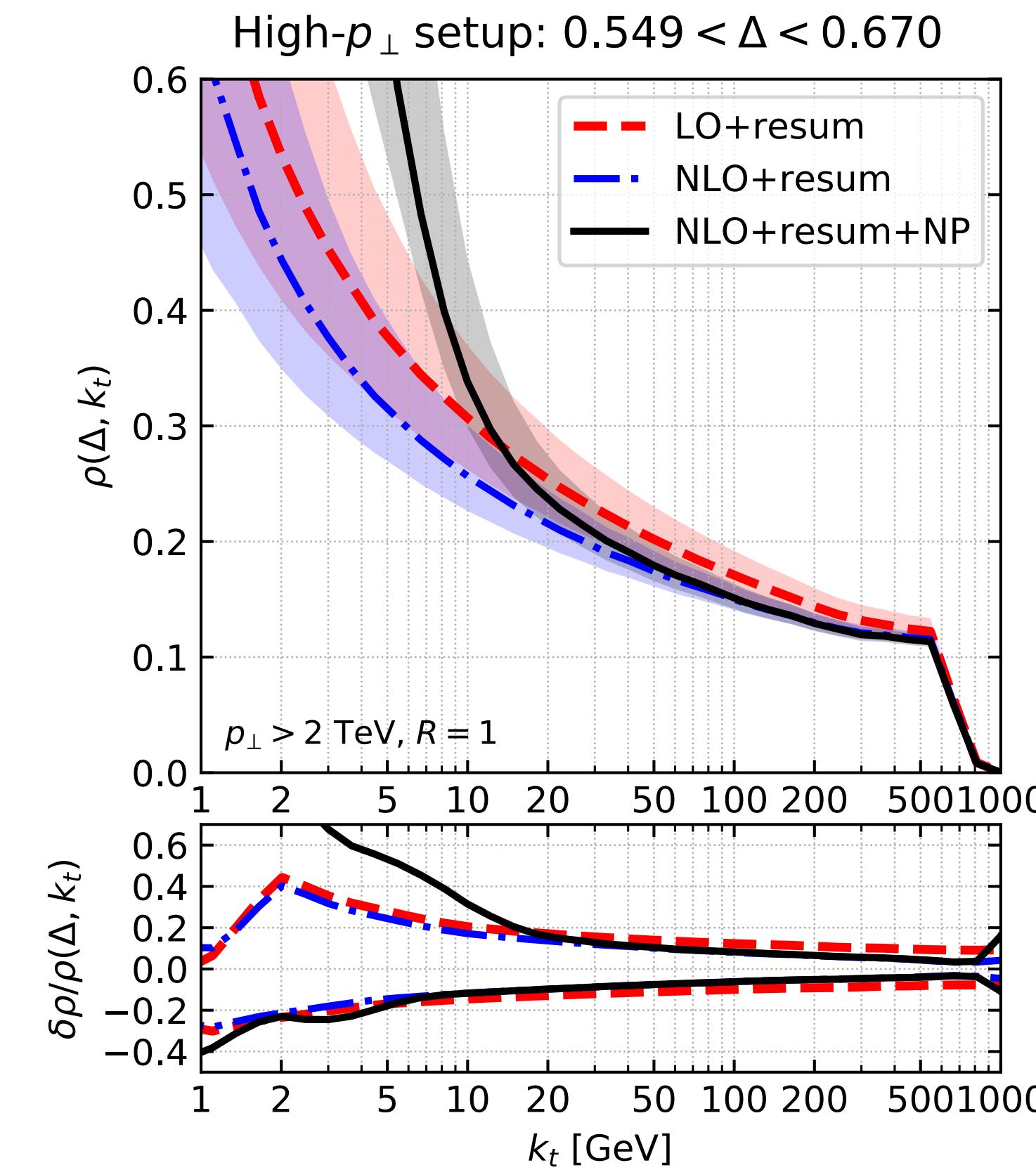
- Running coupling  $\alpha_s^{2l}(k_t)$
- $p_t$ -degradation  $k_t = xp_t\theta z$
- Flavor changing  $p(x, i | j)$

**Soft, wide angle contributions:**

- Boundary effects: anti- $k_t$ +C/A
- Non-global logarithms

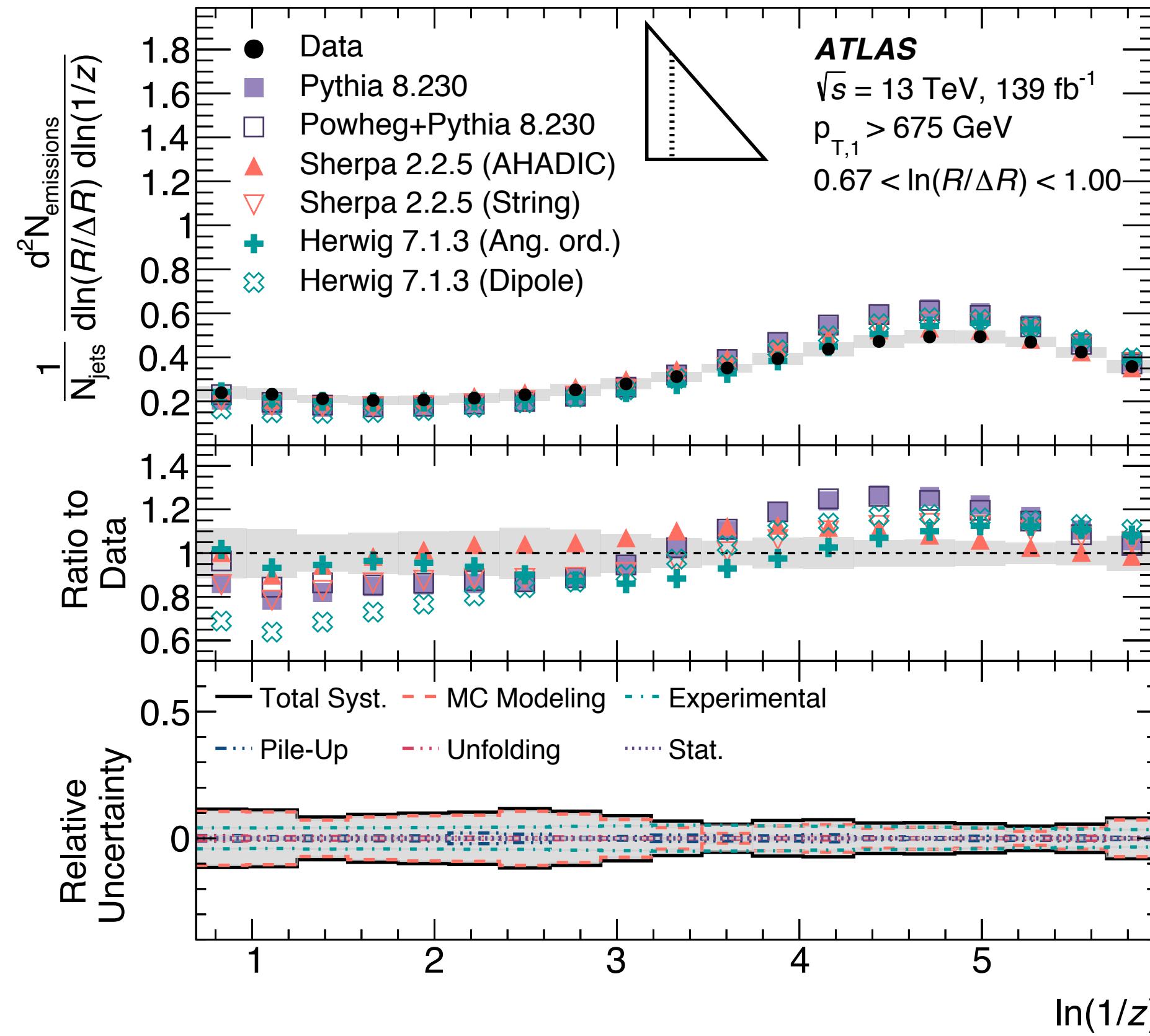


$$\rho(\Delta \equiv \theta, k_t) = \frac{1}{N_{\text{jets}}} \frac{dN}{d \ln 1/\Delta d \ln k_t} \xrightarrow[\text{LO}]{\text{2}\alpha_s C_R} \frac{2\alpha_s C_R}{\pi}$$

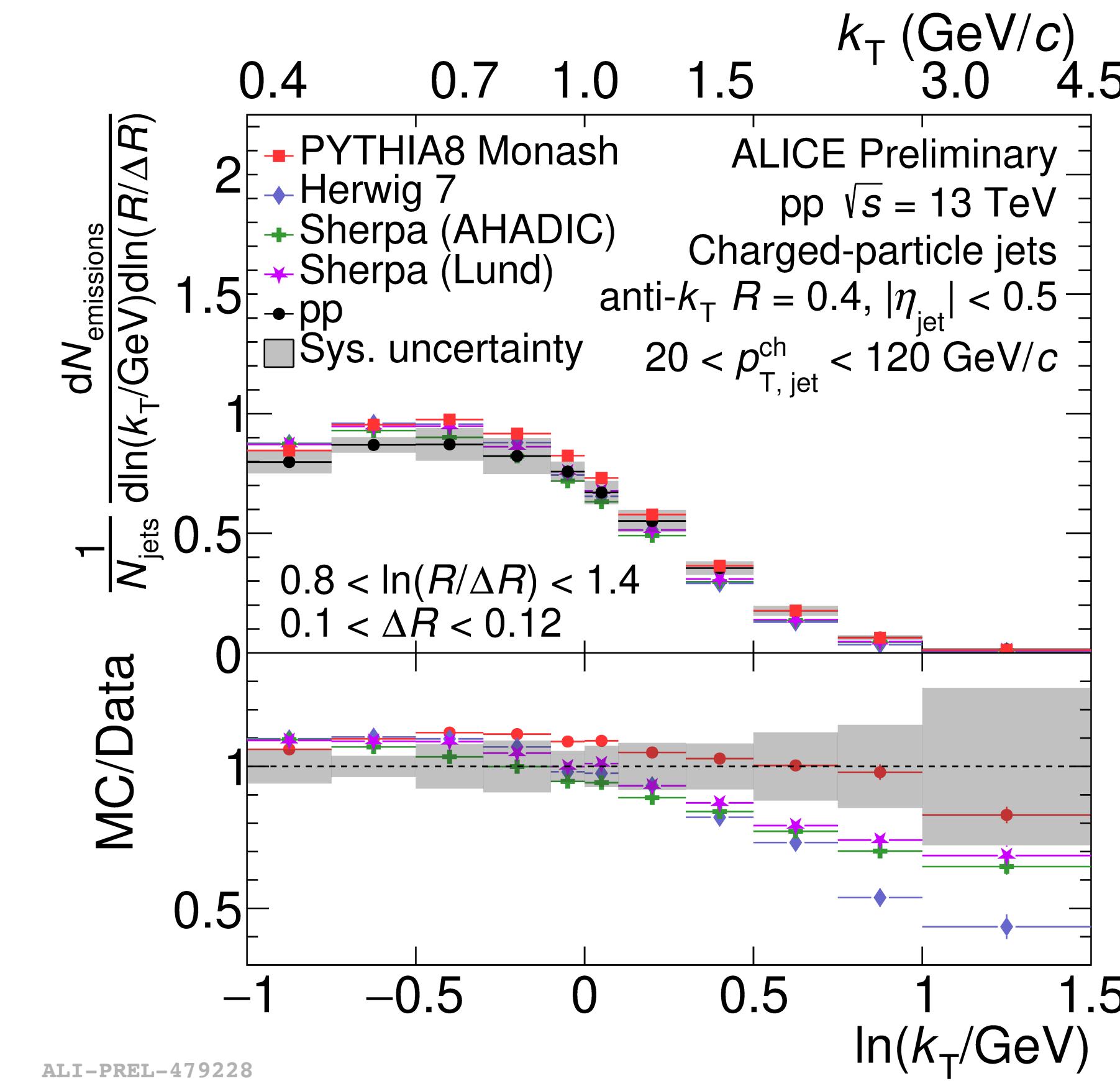


# Observable #1: Primary Lund jet plane density

$p_{t,\text{jet}} > 675 \text{ GeV}$



$20 < p_{t,\text{jet}} < 120 \text{ GeV}$

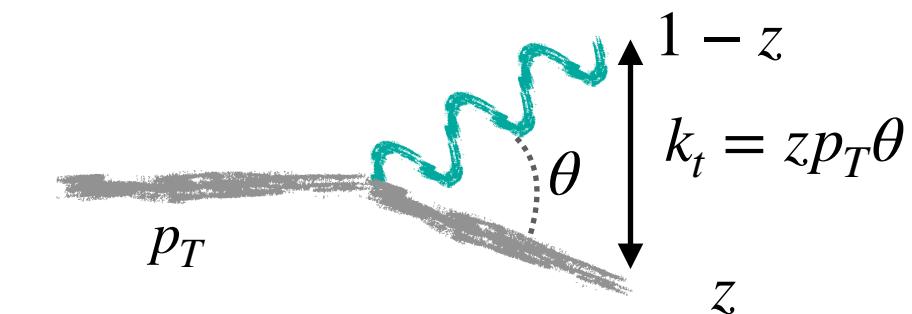


Complementary measurements. Challenge for pQCD calculations.

# Observable #2: $z_g$ -distribution

[Larkowski et al. JHEP'14]  
 [Butterworth et al. PRL'08]

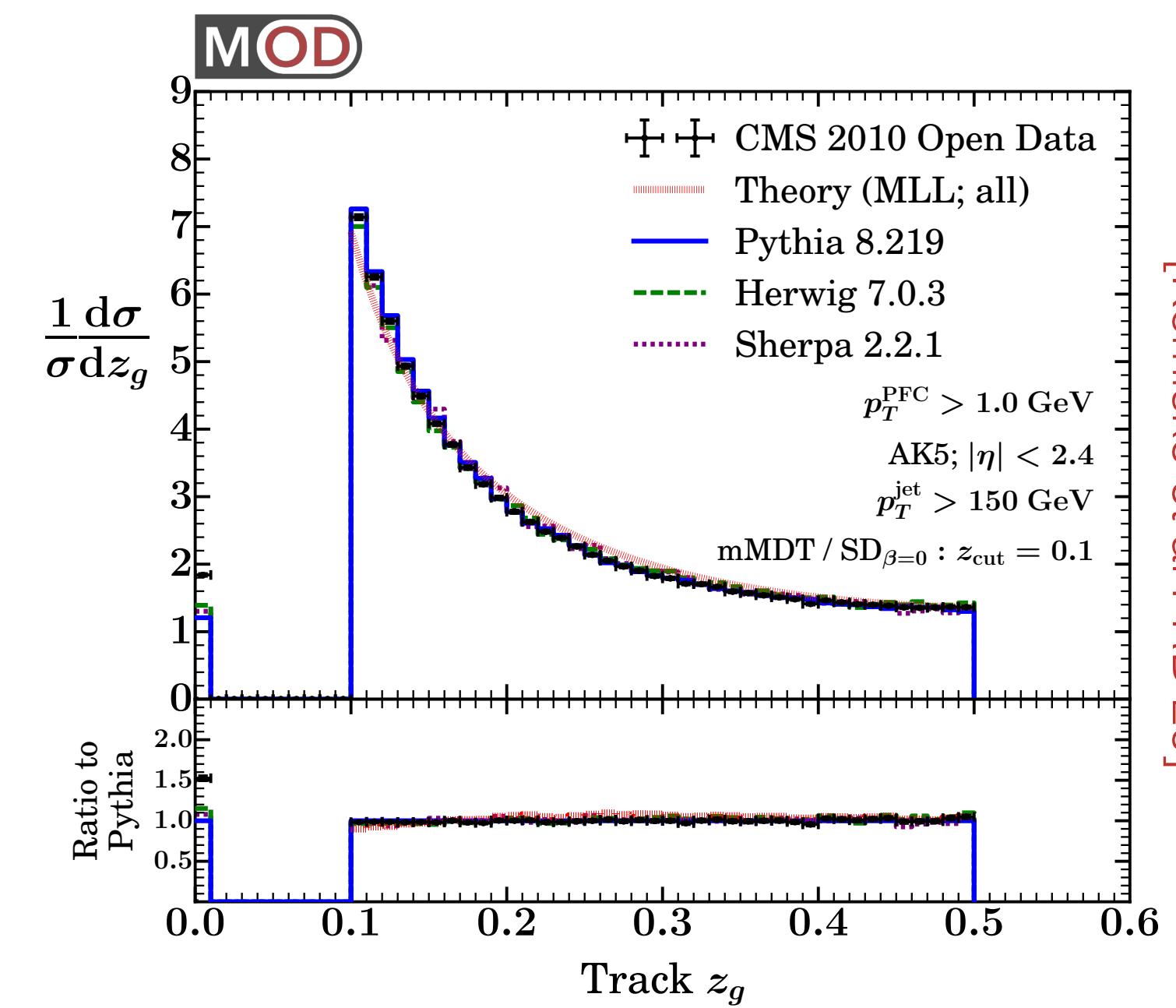
SoftDrop selects the first splitting that satisfies  $z > z_{\text{cut}}\theta^\beta$



Resummation:  $\frac{1}{\sigma} \frac{d\sigma}{dz_g} = \int_0^R \mathcal{P}(z, \theta) \Theta(z - z_{\text{cut}}\theta^\beta) d\theta$  with  $\frac{d^2 \mathcal{P}_i(z, \theta)}{d\theta dz} = \tilde{P}_i(z, \theta) \Delta_i(z > z_{\text{cut}}\theta^\beta)$

$$\mathcal{O}(\alpha_s) : \frac{1}{\sigma} \frac{d\sigma}{dz_g} = \sqrt{\frac{\alpha_s C_R}{\beta}} P_{\text{AP}}(z_g) \quad \text{Non-analytic Taylor series in } \alpha_s \text{ when } \beta > 0 \quad [\text{Marzani et al PRD'15}]$$

**Sudakov safety hampers achieving higher precision, matching...  
 Not ideal baseline for heavy-ion collisions**



[Komiske et al PRD'20]

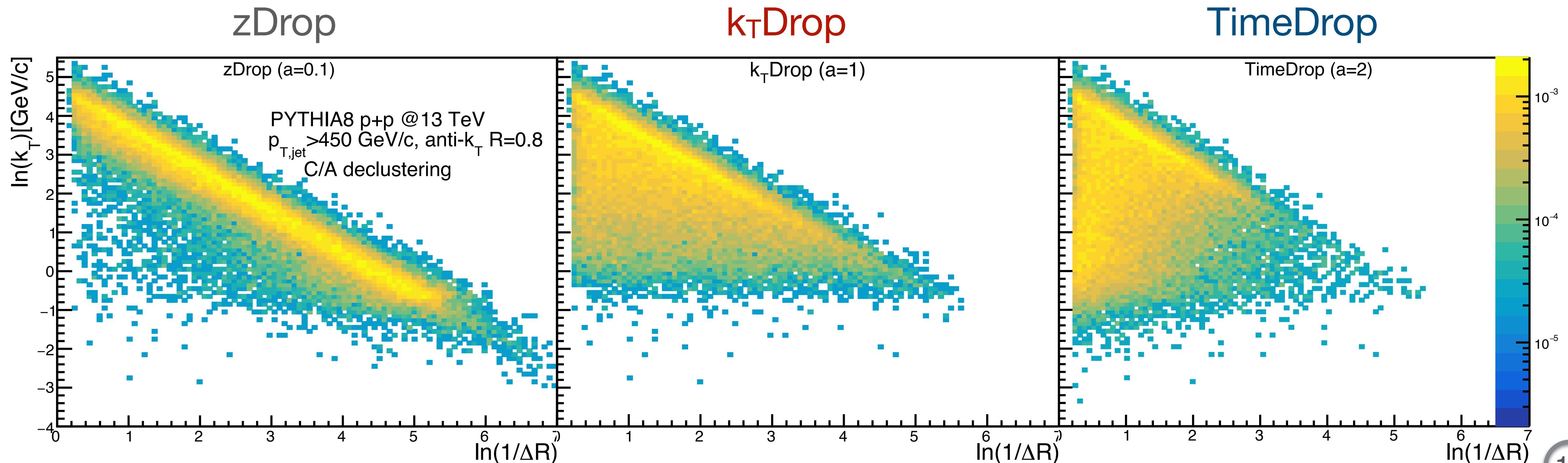
# Observable #3: dynamical grooming

[Mehtar-Tani, ASO, Tywoniuk PRD 101 (2020) 3, 034004]

Find hardest branch in the C/A sequence, i.e.

$$\kappa^{(a)} = \frac{1}{p_T} \max_{i \in \text{C/A}} z_i(1 - z_i)p_{T,i}(\theta_i/R)^a$$

Physical interpretation: •  $a=2$ : TimeDrop    •  $a=1$ :  $k_T$ Drop    •  $a \sim 0$ : zDrop



# Observables #3: $k_{t,g}$ with dynamical grooming

[Caucal, ASO, Takacs 2103.06566]

Resummation: the cumulative distribution  $\Sigma(\nu) = \int_0^\nu d\nu' \frac{1}{\sigma} \frac{d\sigma}{d\nu'}$  reads

$$\Sigma(k_{t,g} | a) = \int_0^1 dz \int_0^1 d\theta \widetilde{P}(z, \theta) \Delta(\kappa | a) \Theta(k_{t,g} - z\theta) \longrightarrow \Sigma(k_{t,g}) = \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=0}^{2n} c_{nm} \ln^m(k_{t,g})$$

Non-exponentiation leads to  $N^p$ DL type of counting. We reach  $N^2$ DL in the narrow jet limit:

**Collinear** ingredients:

- Running coupling  $\alpha_s^{2l}(k_t)$
- Hard-collinear correction to splitting function

**Soft, wide angle** contributions:

- Non-global logarithms  $\propto \ln(z_g)$

Flavor switching leads to a power correction,  $p_t$ -degradation of the primary branch is  $N^3$ DL

Multiple emissions do not enter by definition and free of clustering logarithms

# Observables #3: $k_{t,g}$ with dynamical grooming

[Caucal, ASO, Takacs 2103.06566]

Resummation: we reach  $N^2DL$  in the narrow jet limit by including

Collinear ingredients:

- Running coupling
- Hard-collinear correction to splitting function

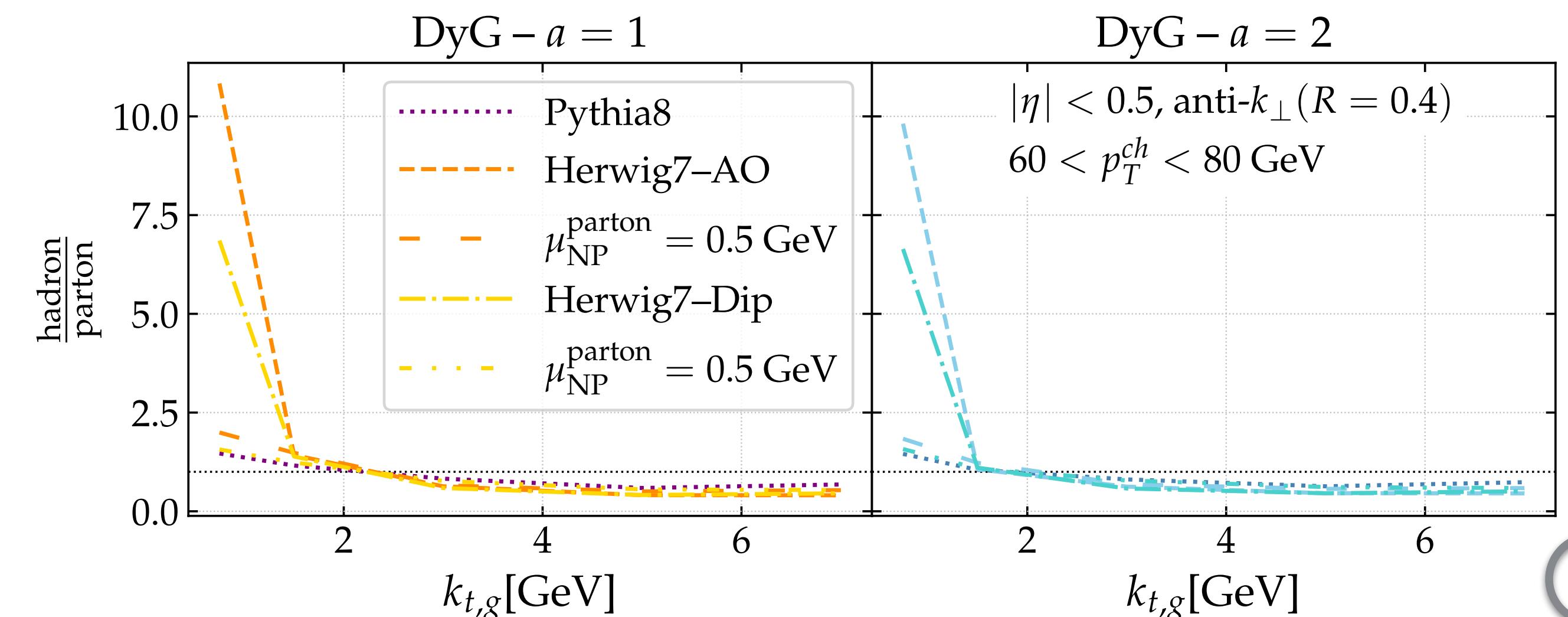
Soft, wide angle contributions:

- Non-global logarithms

Fixed order: exact matrix element at  $\mathcal{O}(\alpha_s)$  with MadGraph.  $C_1$  term through matching

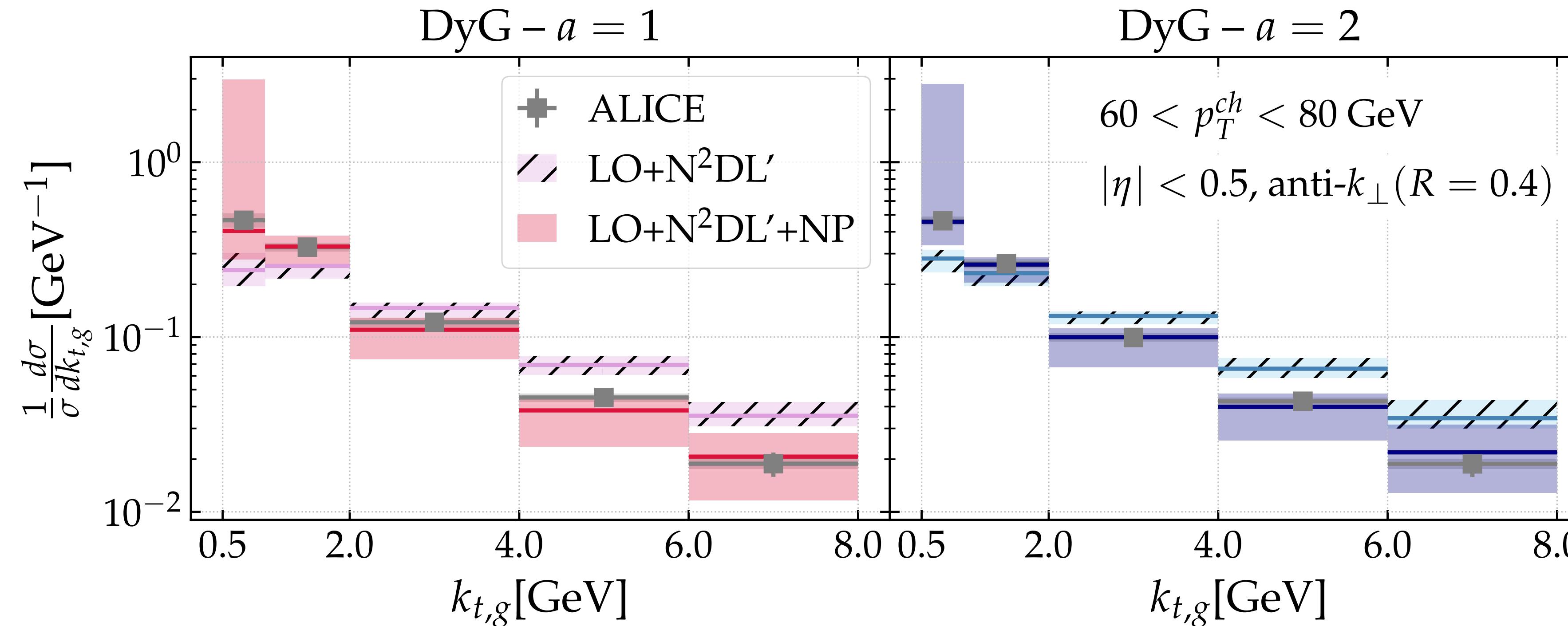
Non-perturbative corrections:

strong sensitivity to the infra-red cutoff,  
i.e.  $\alpha_s(\mu_{NP})=0$ , of the parton shower



# Observables #3: $k_{t,g}$ with dynamical grooming

[Caucal, ASO, Takacs 2103.06566]



Very good description of ALICE data (also for  $z_g, \theta_g$ ) after including NP

# Space-time picture of an in-medium jet

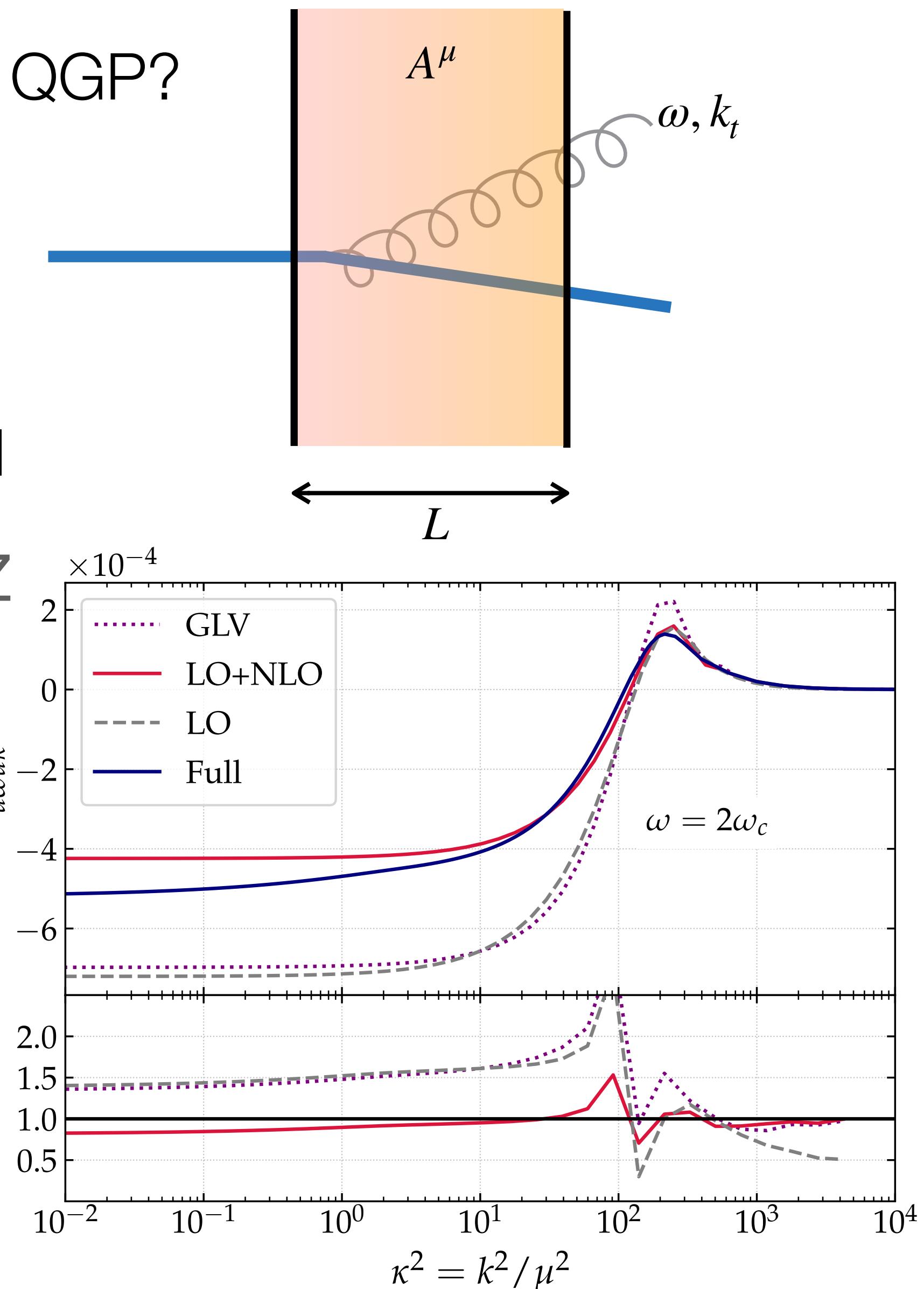
How is the branching kernel,  $\tilde{P}_i(z, \theta)$ , modified by the QGP?

$$\tilde{P}^{\text{vac}}(z, \theta) \propto \frac{d\theta}{\theta} P_{\text{AP}}(z) \rightarrow \frac{dI}{d\omega d^2 k_t}$$

Analytic approaches to compute the branching kernel

- Multiple soft scattering approximation/BDMPS-Z  
[Baier, Dokshitzer, Mueller, Peigné, Schiff'96]  
[Zakharov'96]
- Opacity expansion/GLV LO (N=0): vacuum radiation. NLO (N=1): single, hard scattering  
[Gyulassy, Levai, Vitev'01] [Sievert, Vitev'19]  
[Wiedemann'00]
- Improved Opacity expansion LO: BDMPS-Z,  
NLO includes N=1 GLV [Mehtar-Tani, Barata, Tywoniuk, ASO'19-21]

All-orders numerical solution [Andres, Apolinario, Dominguez'20]



[Barata, Mehtar-Tani, ASO,  
Tywoniuk to appear]

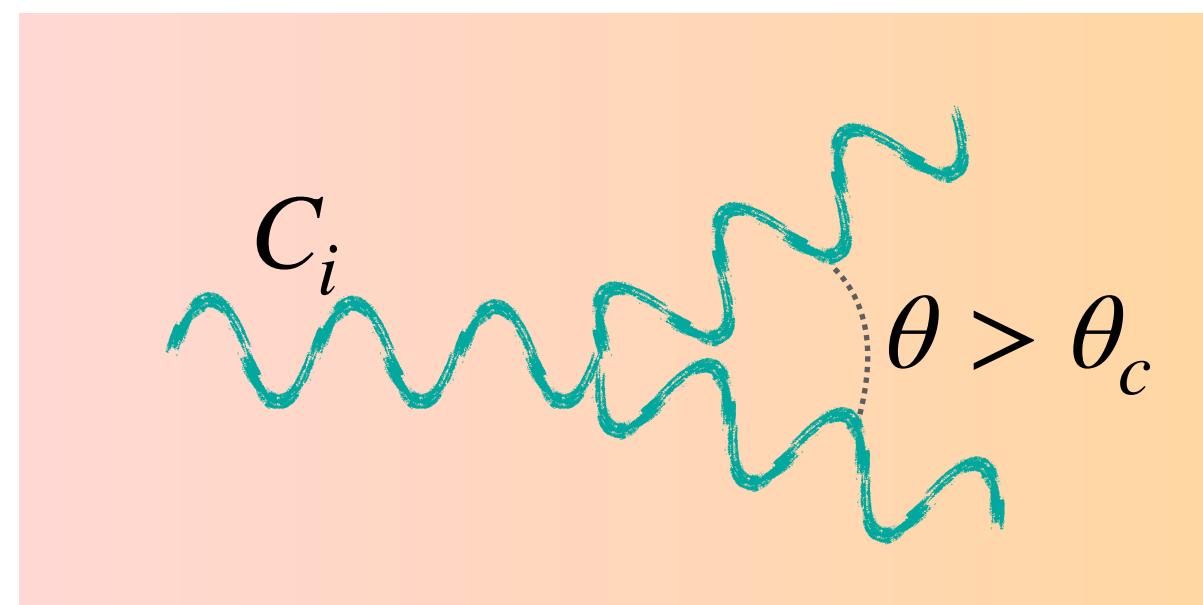
# Space-time picture of an in-medium jet

Not only the branching kernel gets modified but also the prongs lose energy

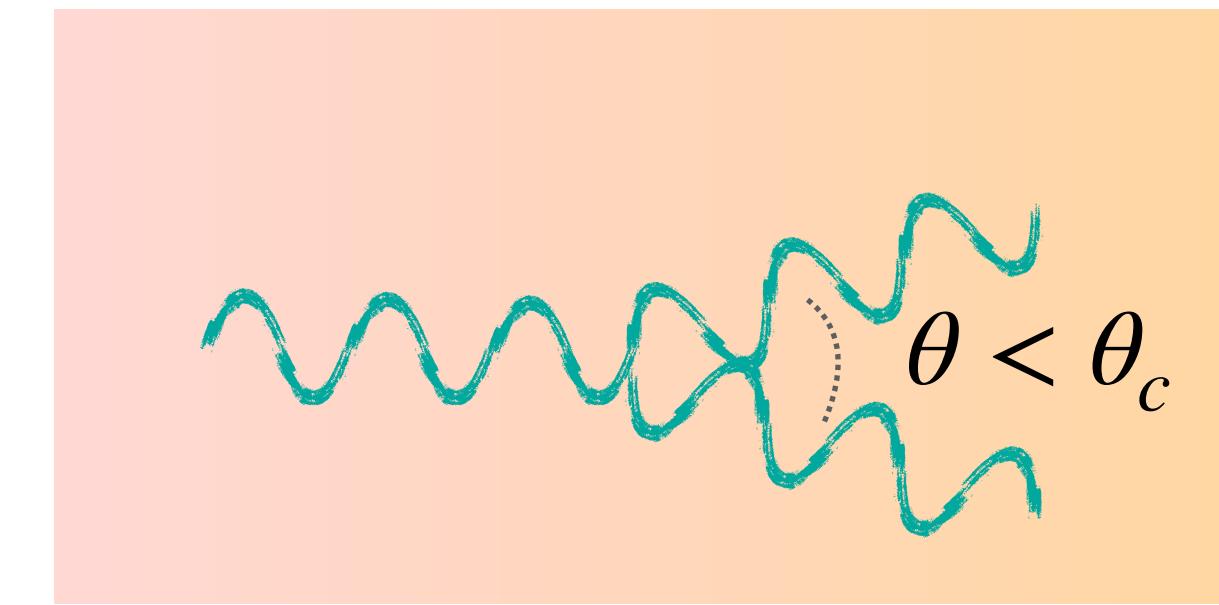
e.g. opening angle of the hardest splitting as tagged by DyG

$$\frac{1}{\sigma} \frac{d\sigma}{d\theta_g|_{p_T}} = \mathcal{N}_{\text{med}}^{-1} \sum_{i \in \{q,g\}} \int d\varepsilon \frac{d\sigma_i^h}{dp_{T0}}(p_T + \varepsilon) \mathcal{E}_{i,p_T,R}(\varepsilon | \theta_g) \times \int dz'_g \mathcal{P}_i(z'_g, \theta_g)$$

with  $\mathcal{E}_{i,p_T,R}(\varepsilon | \theta_g)$  the energy loss probability distribution that depends on  $\theta_g$ : color coherence



$$\langle \varepsilon \rangle \propto C_A + C_i$$



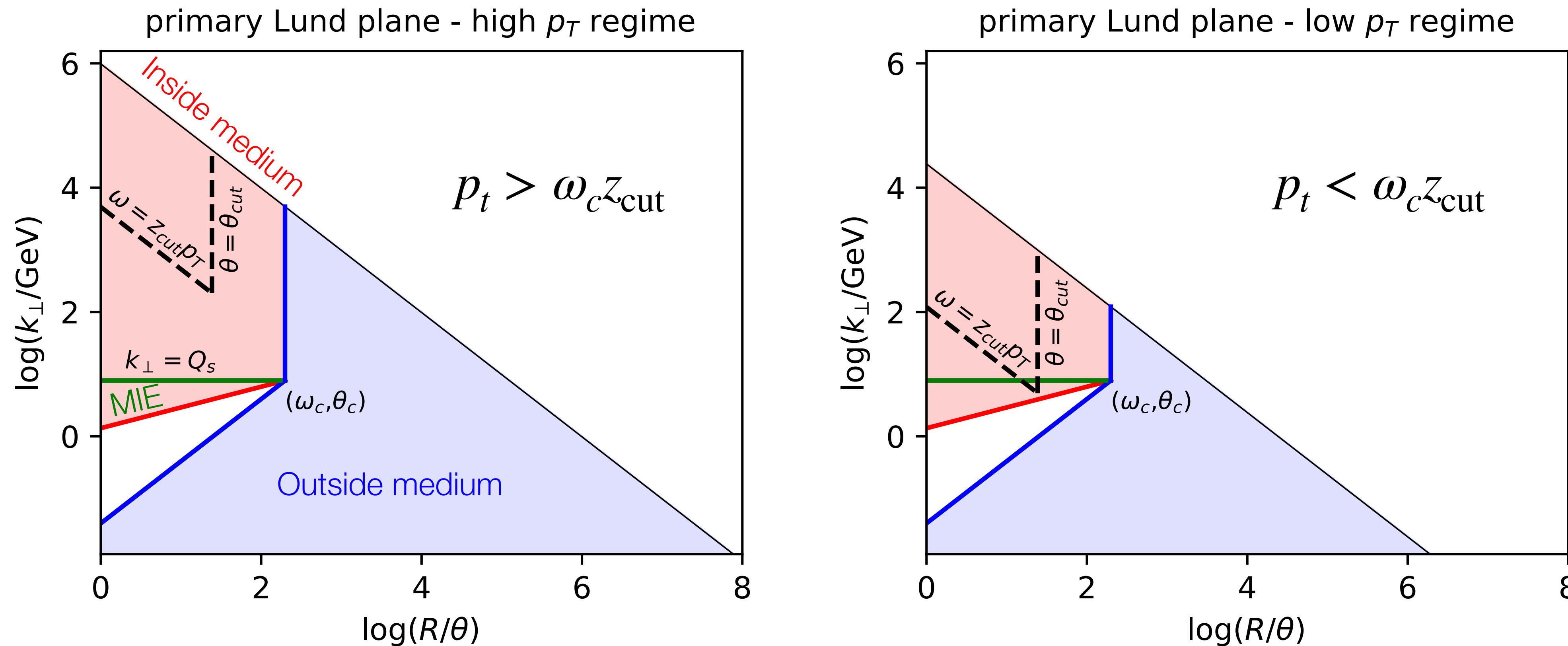
$$\langle \varepsilon \rangle \propto C_i$$

$$\theta_c = (\hat{q}L^3)^{-1/2}$$

# Observable #1: $z_g$ -distribution

[Caucal, Iancu, Soyez JHEP10 (2019) 273]

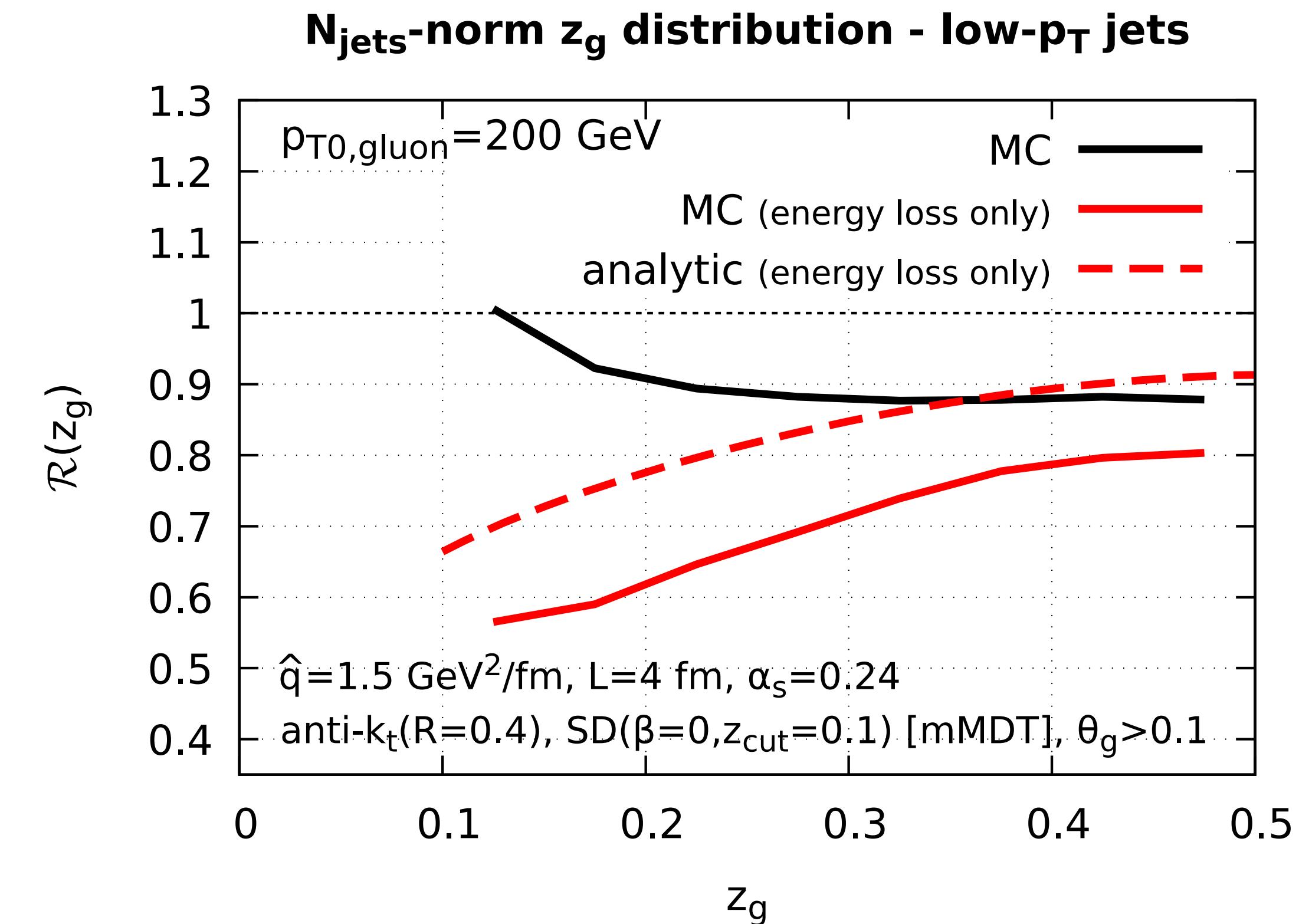
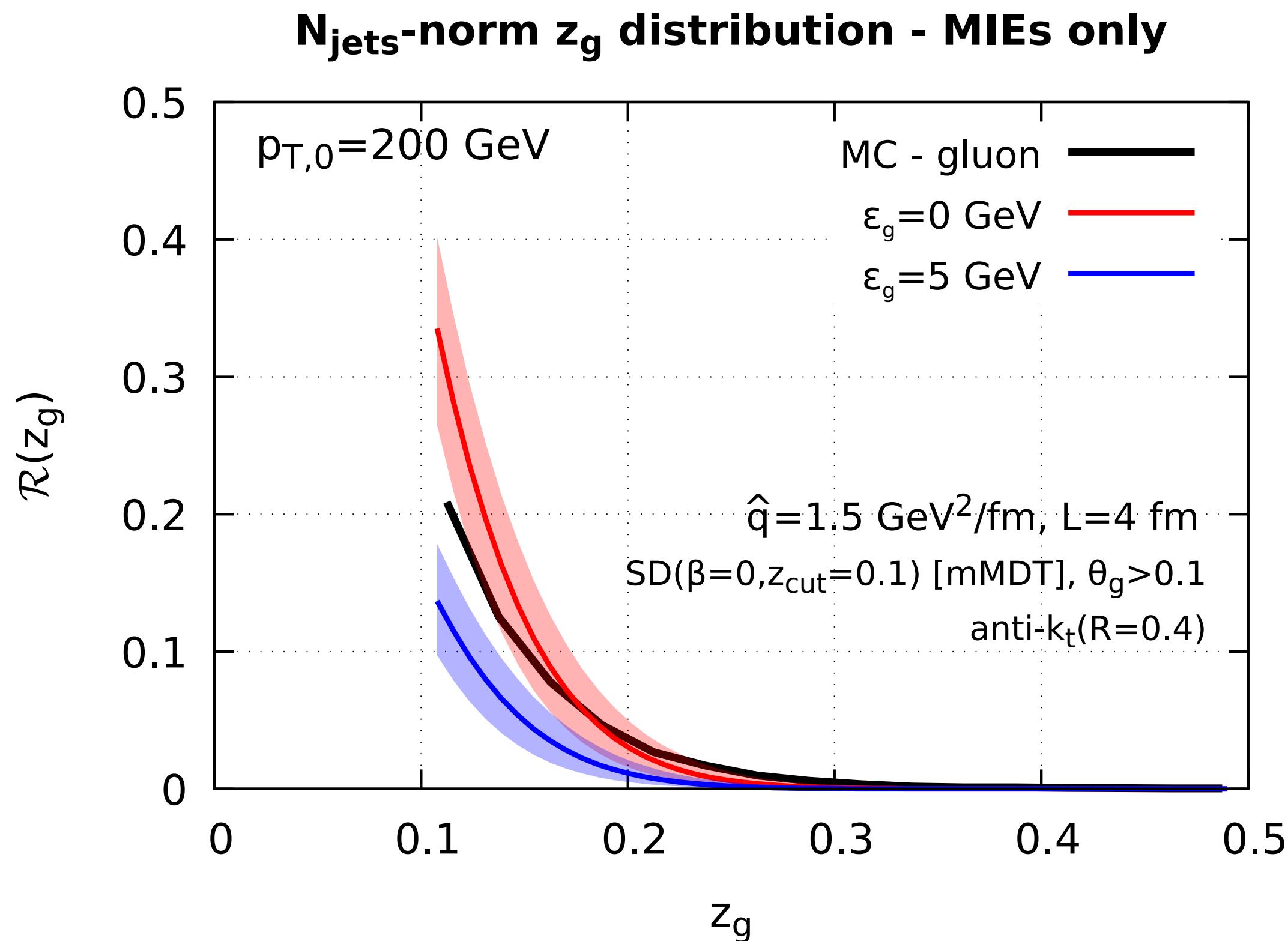
$$z > z_{\text{cut}} + \theta > \theta_{\text{cut}}$$



Tune the grooming condition to explore different in-medium physics

# Observable #1: $z_g$ -distribution

[Caucal, Iancu, Soyez JHEP10 (2019) 273]

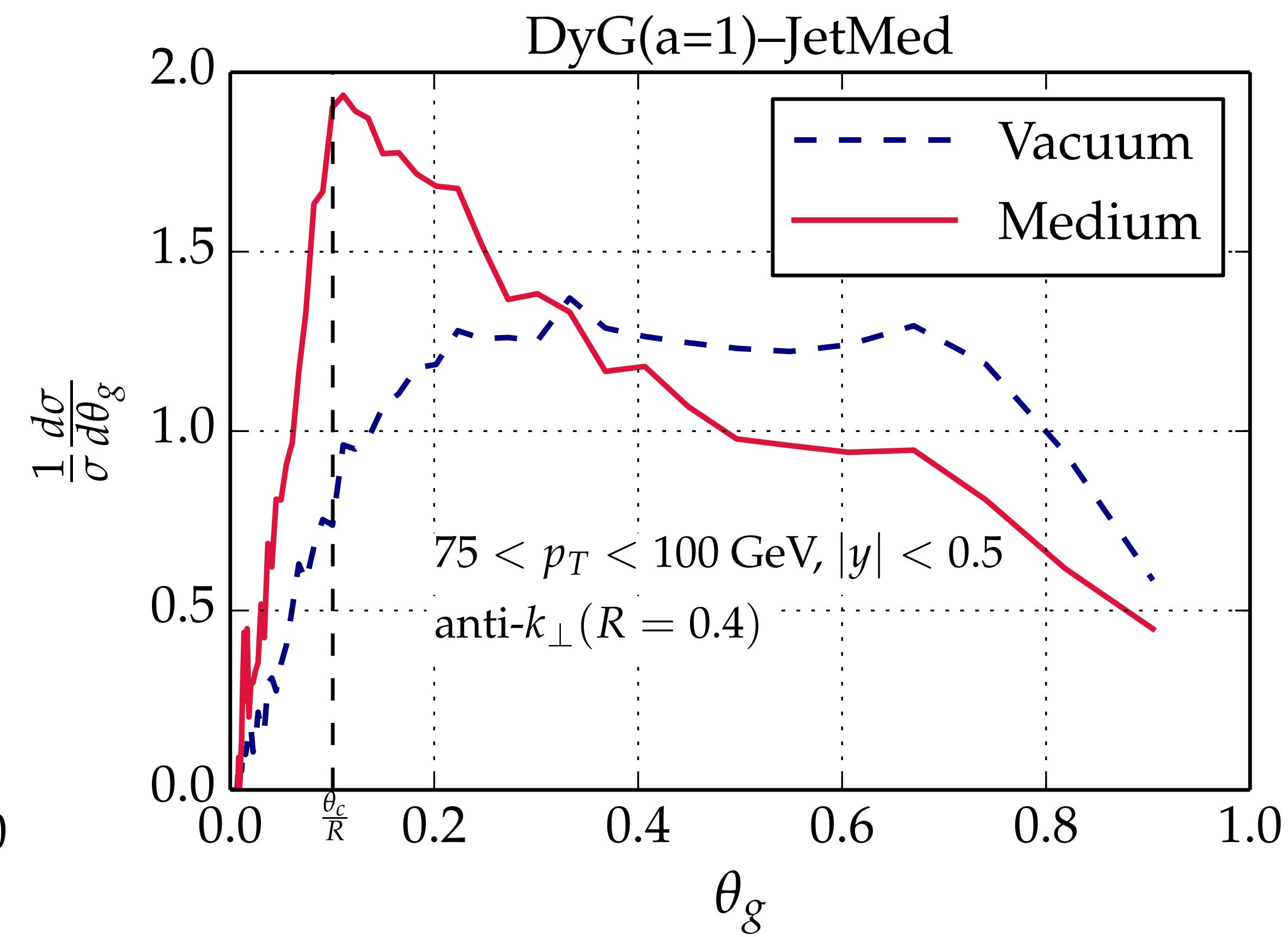
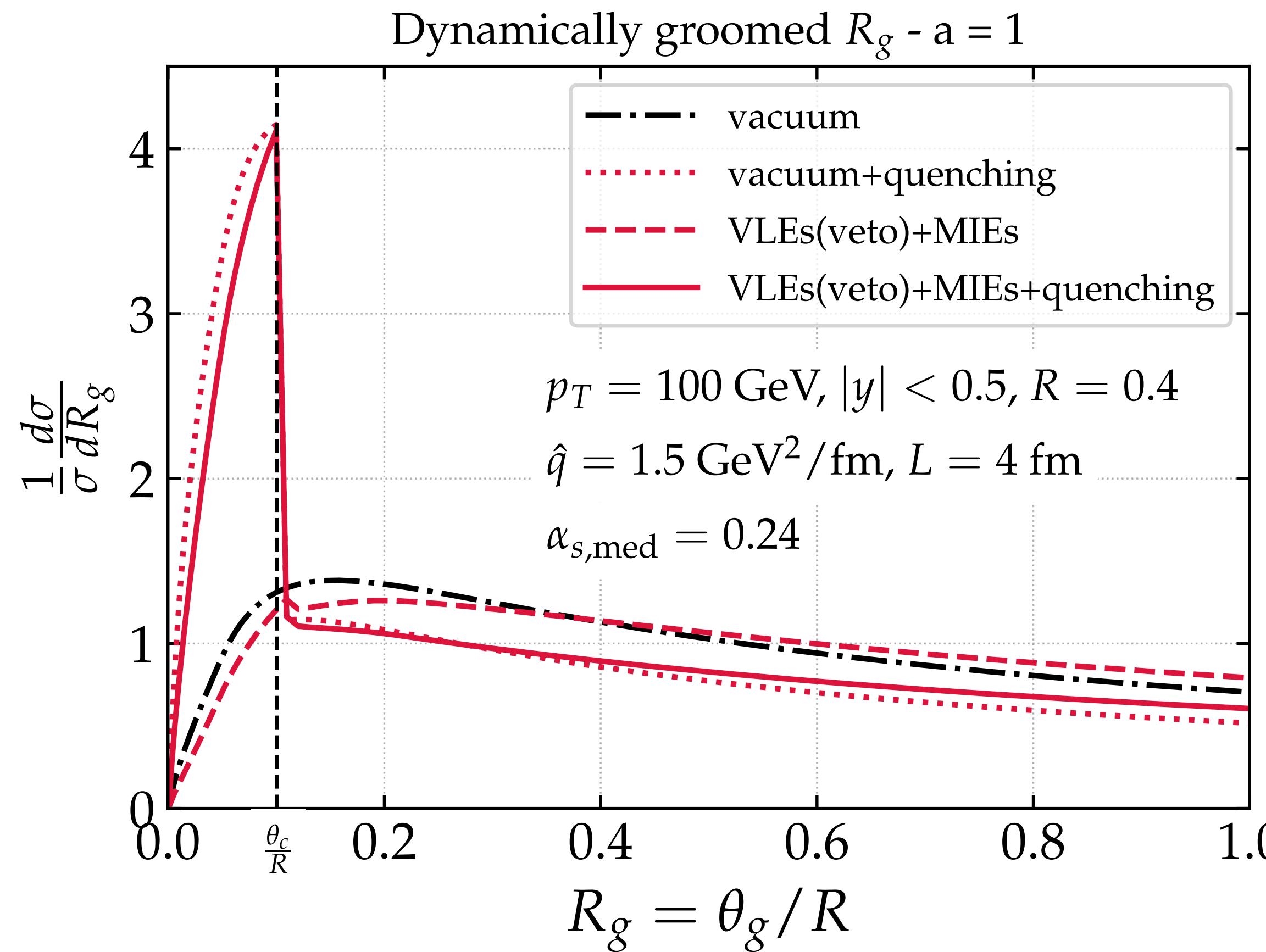


Small  $z_g$ : hard medium-induced emissions  $P^{\text{BDMPS}}(z)/P_{\text{AP}}(z) \sim z^{-1/2}$

Large  $z_g$ : incoherent energy loss

# Observable #2: $\theta_g$ -distribution

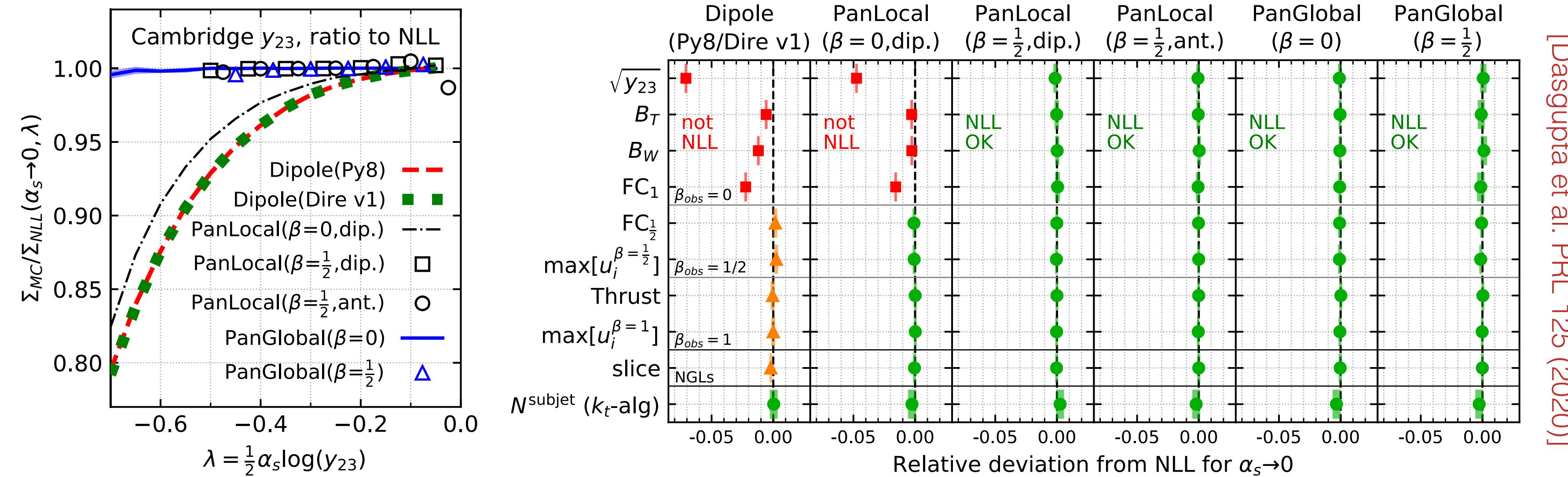
[Caucal, ASO, Takacs in preparation]



New opportunity to measure the QGP resolution angle  $\theta_c$

# Wrap-up

- Analytic understanding of jet substructure observables to probe the different regimes of radiation phase space both in vacuum and in the medium
- In vacuum, jet substructure as tool to gauge the accuracy of parton showers



- Recent developments in jet quenching theory are paving the way to quantitative theory-to-data comparisons