

Gluon TMD studies using heavy quark production processes

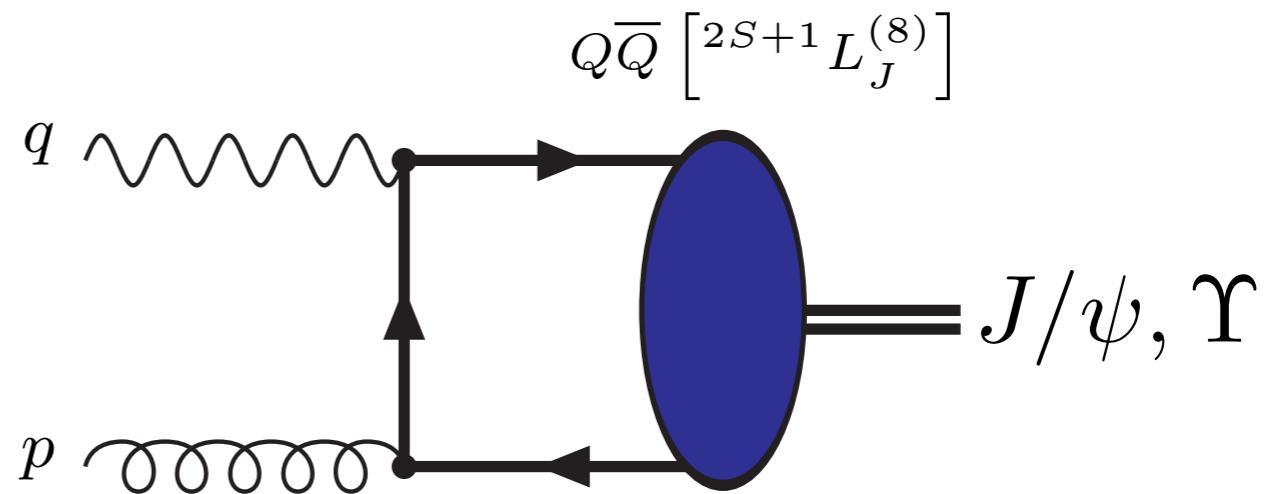
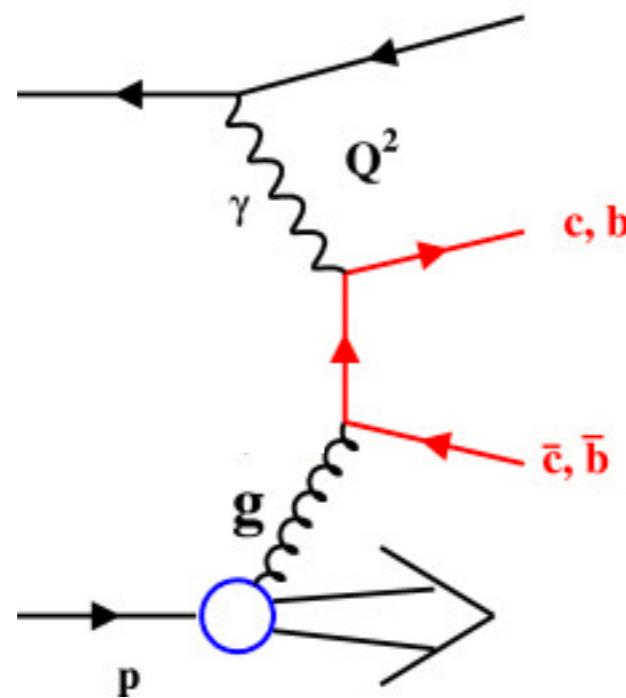
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9th Biennial Workshop of the APS Topical Group on Hadronic Physics
(GHP₂₀₂₁) - April 16, 2021



/ university of
groningen

Probing gluon TMDs using heavy quarks



$$ep \rightarrow e' Q\bar{Q}X$$

$$ep \rightarrow e' Q X$$

Open heavy quark pair production and quarkonium production are processes that are sensitive to the transverse momentum of gluons

Studied at HERA to a limited extent, e.g. J/ψ photoproduction
At EIC much more can be done, also including polarized beams

Gluons TMDs

Gluon TMD correlator: $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P | F^{+\nu}(0) F^{+\mu}(\xi) | P \rangle$

↑
transverse momentum dependent (TMD)

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, p_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, p_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, p_T^2) \right\}$$

↑
unpolarized gluon TMD ↑
linearly polarized
gluon TMD

Gluons inside *unpolarized* protons can be polarized!

[Mulders, Rodrigues '01]

For transversely polarized protons:

$$\Gamma_T^{\mu\nu}(x, p_T) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, p_T^2) + \dots \right\}$$

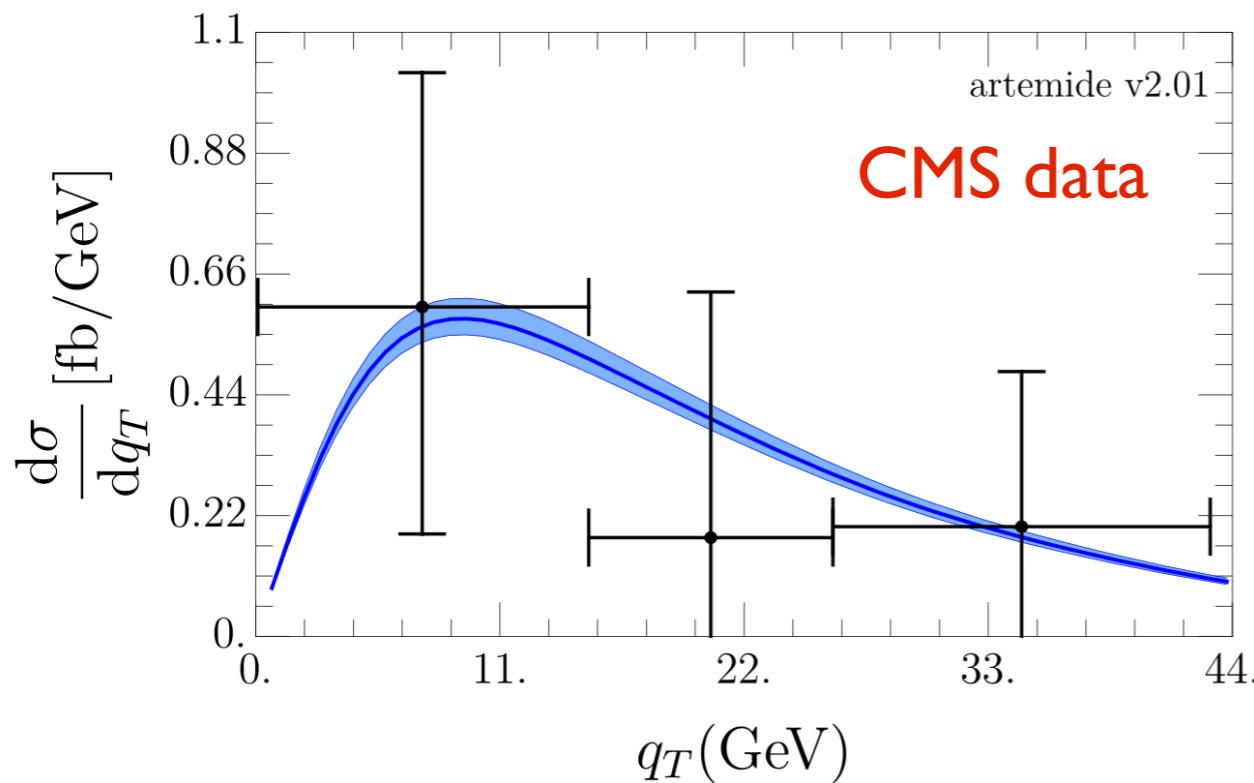
gluon Sivers TMD

Perhaps surprisingly, no gluon TMD has been extracted from experiments yet

Entering the gluon TMD era

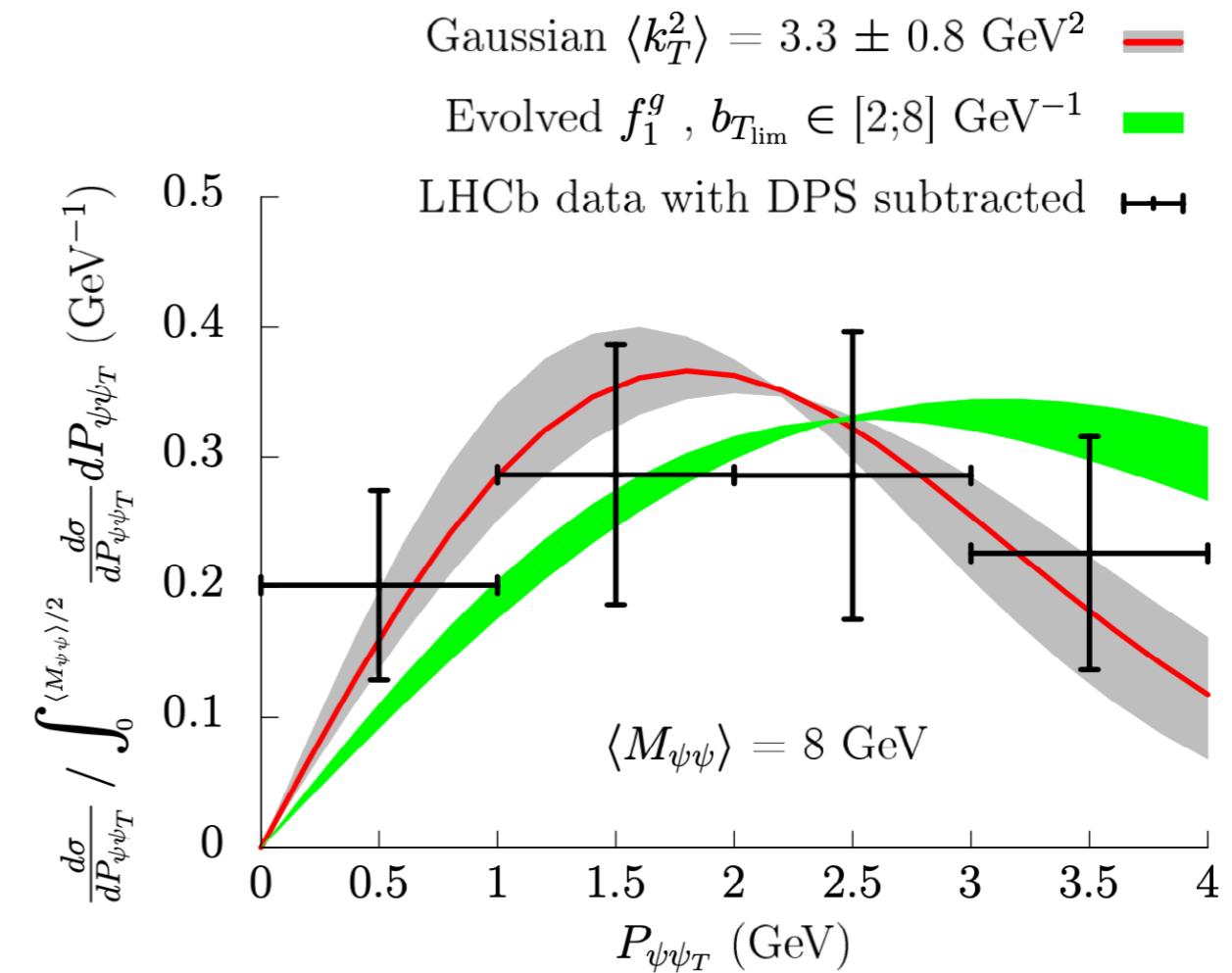
Higgs p_T distribution

$$pp \rightarrow H(\rightarrow \gamma\gamma) + X$$



[Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, 2019]

J/ψ pair production



[Scarpa et al., 2020]

Sivers asymmetry in high-p_T hadron pair production

$$A_{Siv} = -0.23 \pm 0.08 \text{ (stat)} \pm 0.05 \text{ (syst)} \text{ at } \langle x_g \rangle = 0.15$$

[COMPASS Collab., 2017]

Studies of $A_N^{\pi,D}$

[D'Alesio, Murgia, Pisano, 2015;
& Taels, 2017]

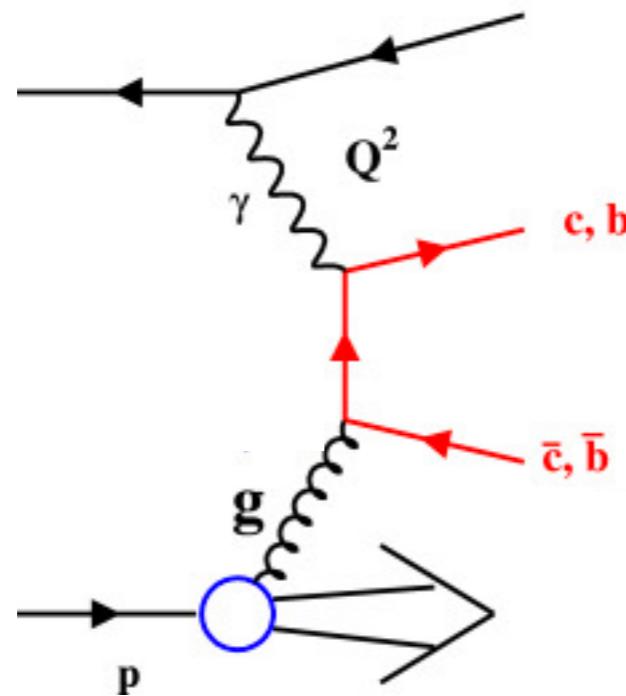
Main future opportunities

f_1^g	$pp \rightarrow \gamma J/\psi X$ $pp \rightarrow \gamma \Upsilon X$ $pp \rightarrow \gamma \text{jet} X$	LHC LHC LHC & RHIC
$h_1^{\perp g}$	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' \text{jet jet} X$ $pp \rightarrow \eta_{c,b} X$ $pp \rightarrow H X$ $pp \rightarrow \gamma^* \text{jet} X$	EIC EIC LHC & NICA LHC LHC & RHIC
$f_{1T}^{\perp g}$	$ep^\uparrow \rightarrow e' Q \bar{Q} X$ $ep^\uparrow \rightarrow e' \text{jet jet} X$ $p^\uparrow p \rightarrow \gamma \gamma X$ $p^\uparrow A \rightarrow \gamma^{(*)} \text{jet} X$ $p^\uparrow A \rightarrow h X (x_F < 0)$	EIC EIC RHIC RHIC RHIC & NICA

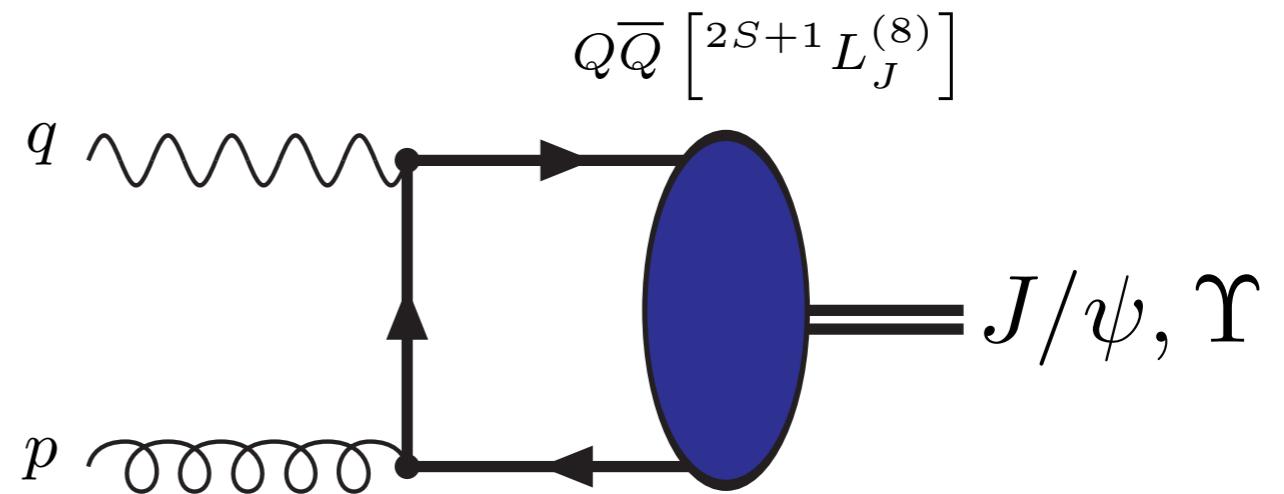
many options involve heavy quark production

Passing over the process dependence of the TMDs → talk by Renaud Boussarie

Probing gluon TMDs using heavy quarks at EIC



$$ep \rightarrow e' Q\bar{Q}X$$

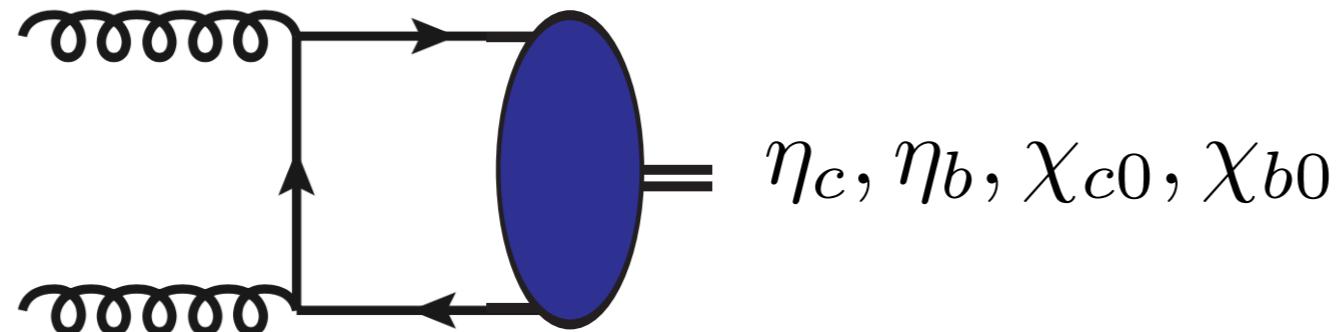


$$ep \rightarrow e' Q X$$

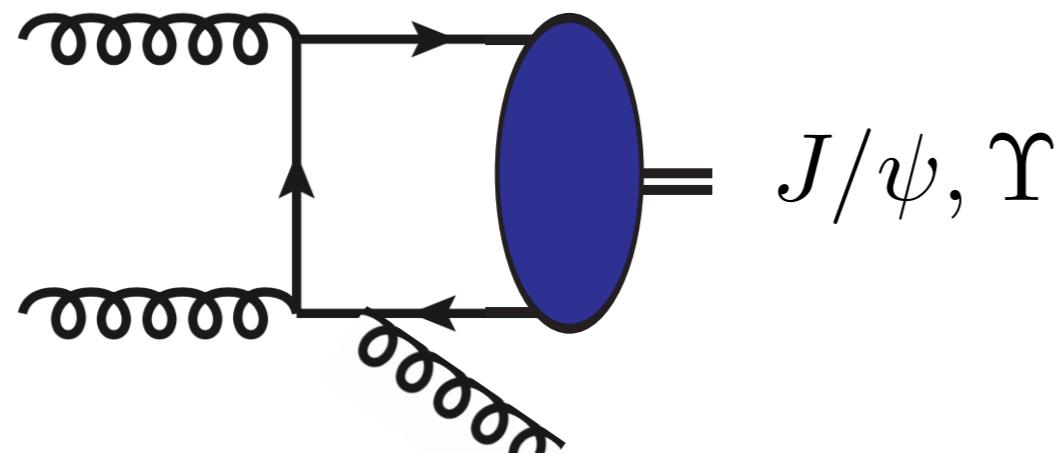
The theoretically simplest and cleanest options are:
open heavy quark pair production and single quarkonium production

Possible at EIC

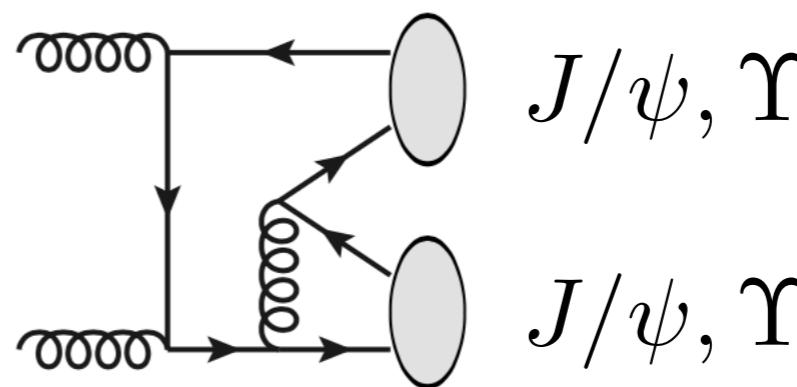
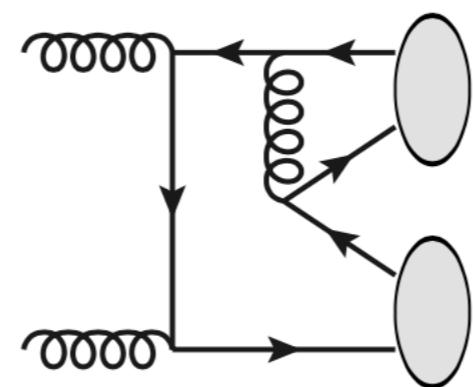
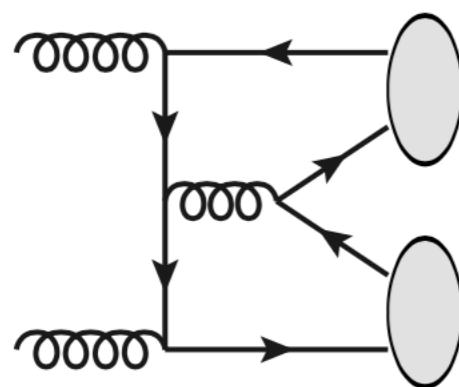
Probing gluon TMDs using heavy quarks in pp



At LHC, NICA & RHIC one can study convolutions of gluon TMDs in several processes



Color Singlet (CS) quarkonium production is possible, but Color Octet (CO) may contribute, TMD factorization may not hold



CS-CS > CO-CO

[Scarpa et al., 2020]

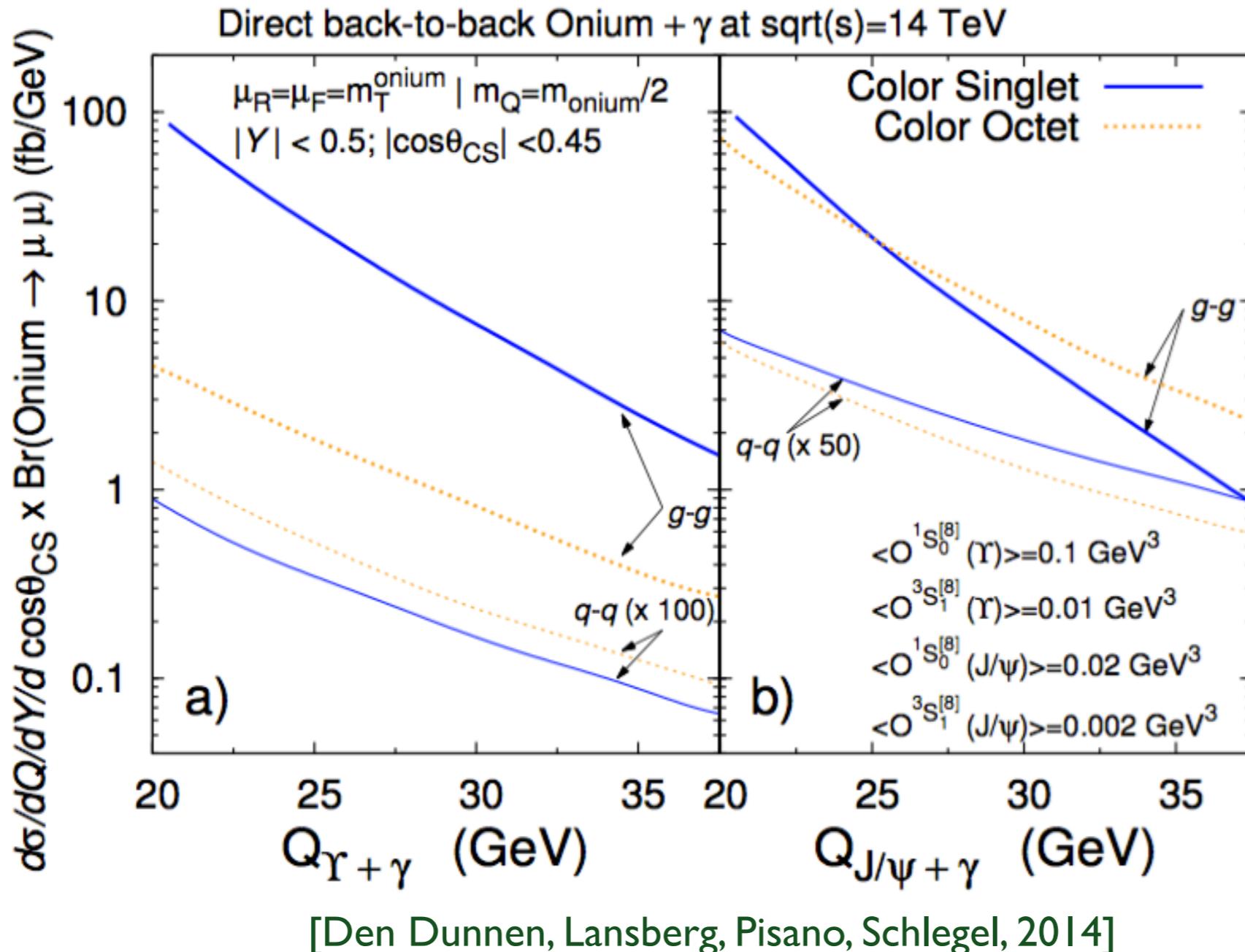
$p\bar{p} \rightarrow Q\bar{Q} X$

Never CS

TMD factorization is a concern here [Rogers, Mulders, 2010; Catani, Grazzini, Torre, 2015]

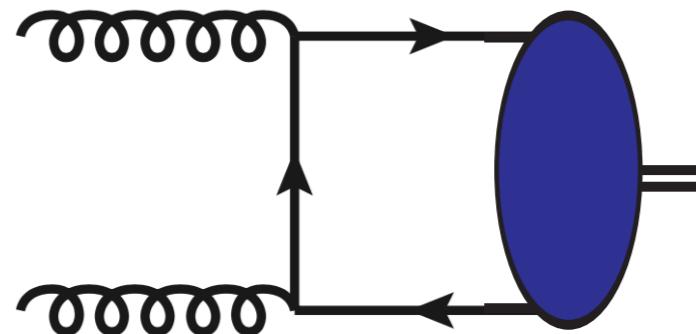
Associated production

$p p \rightarrow Q \gamma X$ could be a good process to extract $f_1^g(x, p_T^2)$ at LHC



The CS contribution dominates in $\Upsilon + \gamma$ production and for lower invariant mass of the pair also in $J/\psi + \gamma$ production

Processes for unpolarized gluon TMDs



$\eta_c, \eta_b, \chi_{c0}, \chi_{b0}$

$p p \rightarrow \eta_c X$ or $p p \rightarrow \chi_c X$
possible at NICA or LHC

For C-even (pseudo-)scalar quarkonium production $g g \rightarrow CS$ is leading contribution

In LO NRQCD the differential cross sections in $p p$ and $p A$ are:

$$\frac{d\sigma(\eta_Q)}{dy d^2 q_T} = \frac{2}{9} \frac{\pi^3 \alpha_s^2}{M^3 s} \langle 0 | \mathcal{O}_1^{\eta_Q}(^1 S_0) | 0 \rangle \mathcal{C} [f_1^g f_1^g] [1 - R(q_T^2)]$$

$$\frac{d\sigma(\chi_{Q0})}{dy d^2 q_T} = \frac{8}{3} \frac{\pi^3 \alpha_s^2}{M^5 s} \langle 0 | \mathcal{O}_1^{\chi_{Q0}}(^3 P_0) | 0 \rangle \mathcal{C} [f_1^g f_1^g] [1 + R(q_T^2)]$$

$$\frac{d\sigma(\chi_{Q2})}{dy d^2 q_T} = \frac{32}{9} \frac{\pi^3 \alpha_s^2}{M^5 s} \langle 0 | \mathcal{O}_1^{\chi_{Q2}}(^3 P_2) | 0 \rangle \mathcal{C} [f_1^g f_1^g]$$

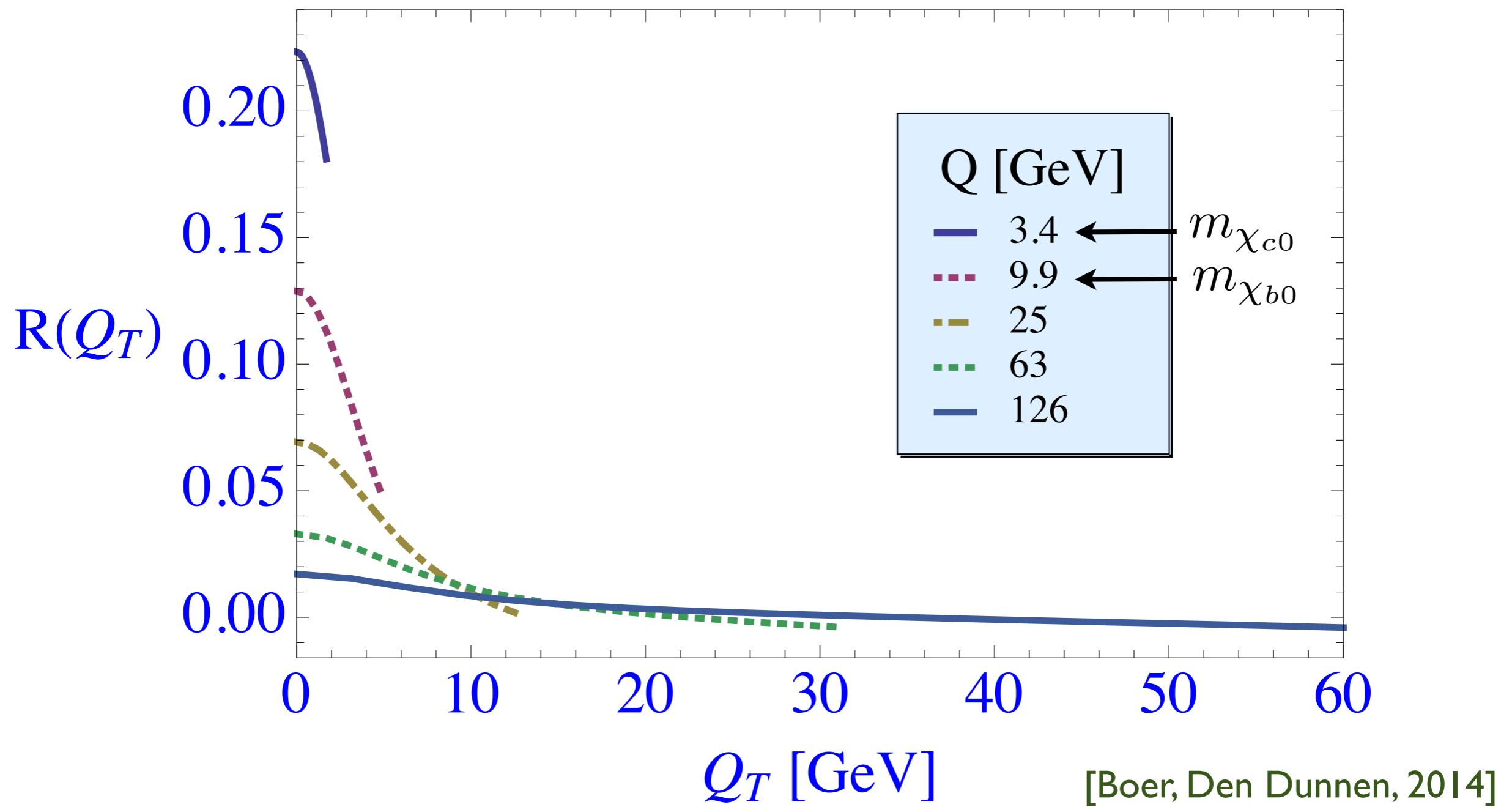
[Boer, Pisano, 2012]

$R(q_T^2)$ is the contribution from the linearly polarized gluon TMD $h_{I^\perp g}$

Quarkonium production in pp

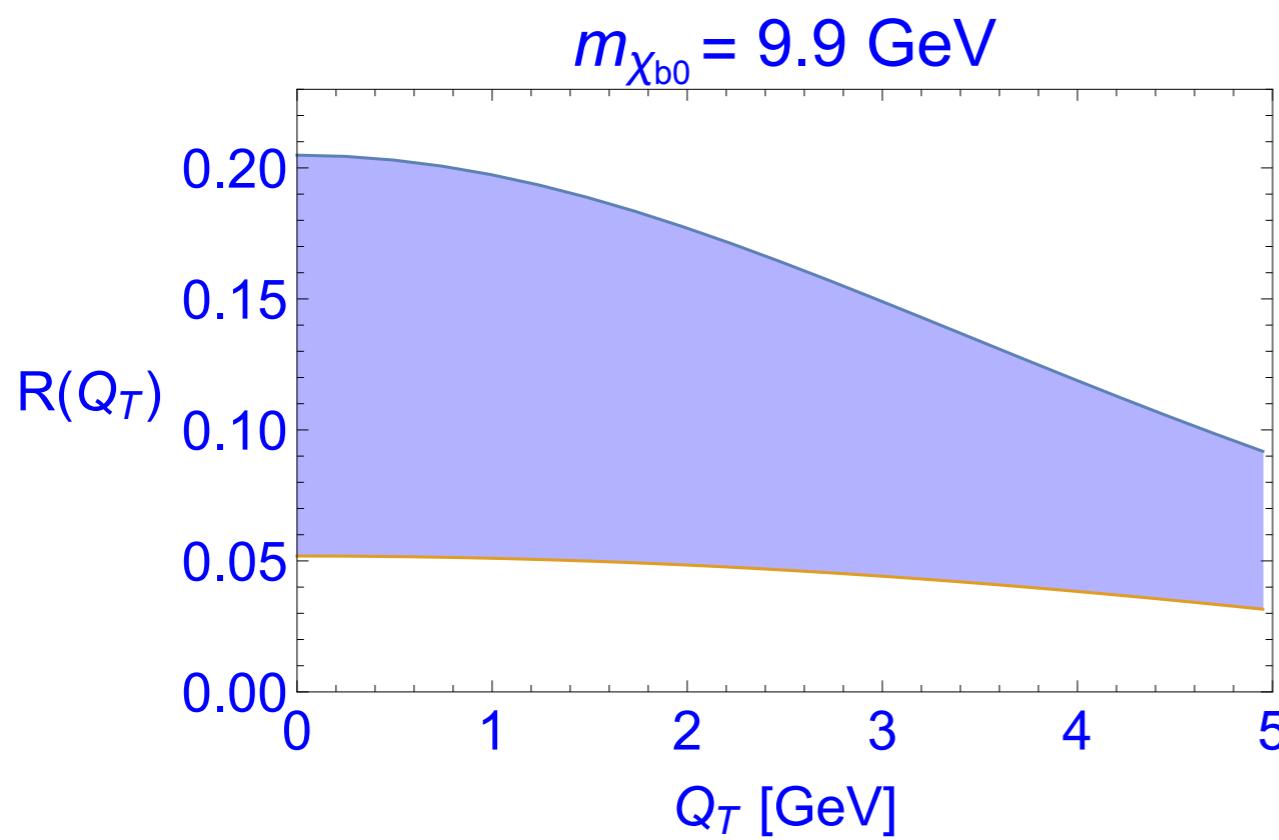
$\text{pp} \rightarrow \eta_c X$ or $\text{pp} \rightarrow \chi_c X$ allow to probe the linearly polarized gluon TMD

The relative contribution w.r.t. the unpolarized gluons decreases with increasing mass of the produced state (effect of TMD evolution):

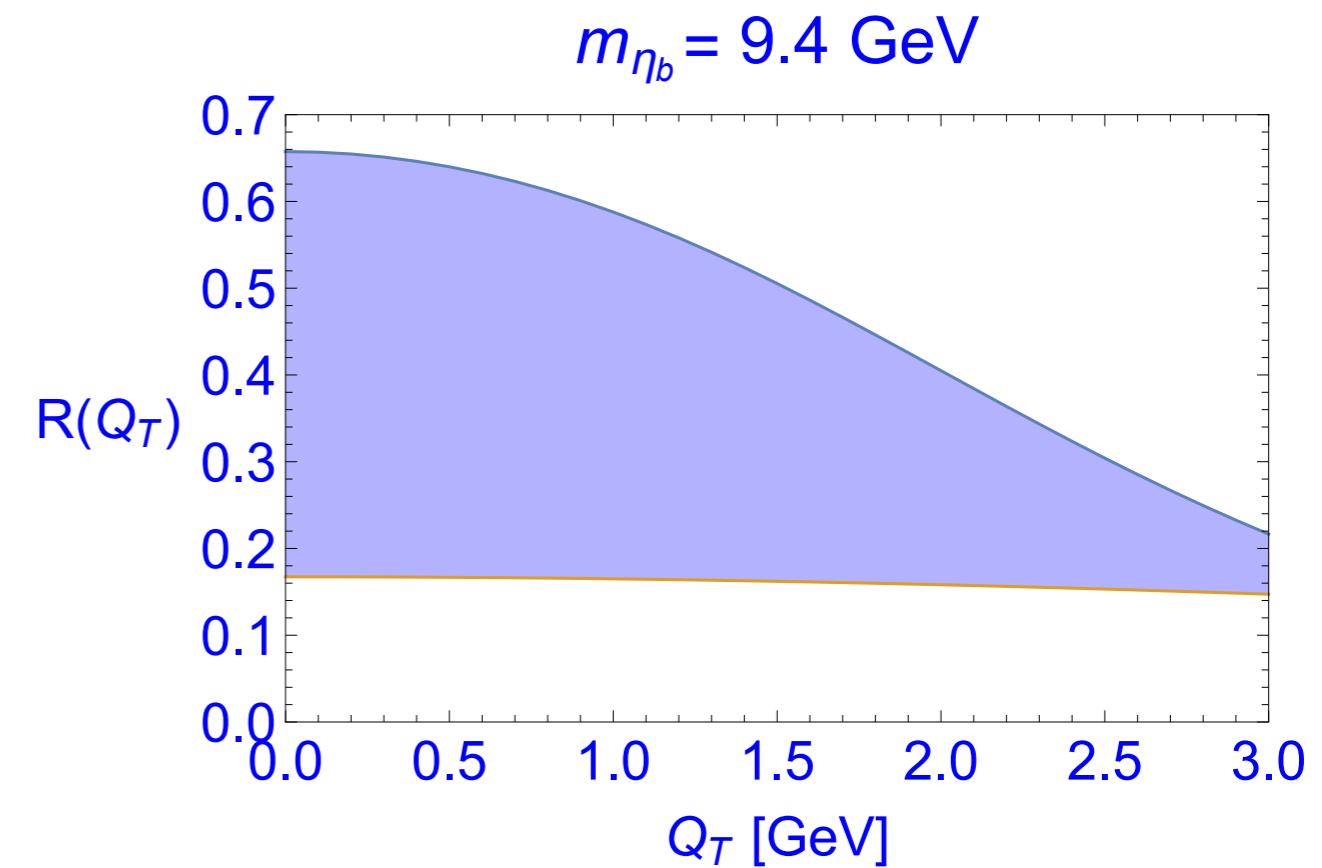


Bottomonium production in pp

The range of predictions for C-even (pseudo-)scalar bottomonium production:



[Boer & Den Dunnen, 2014]

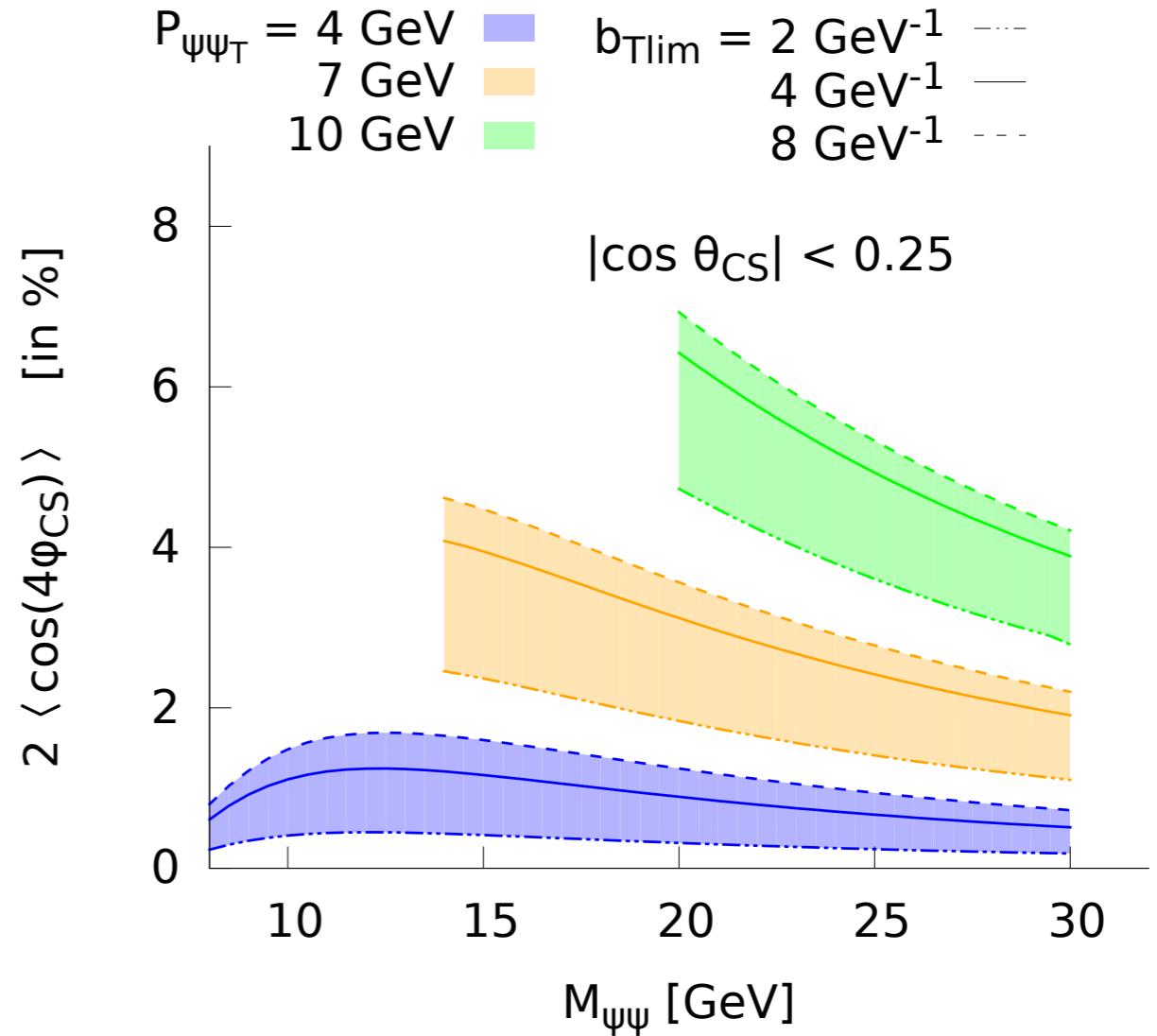
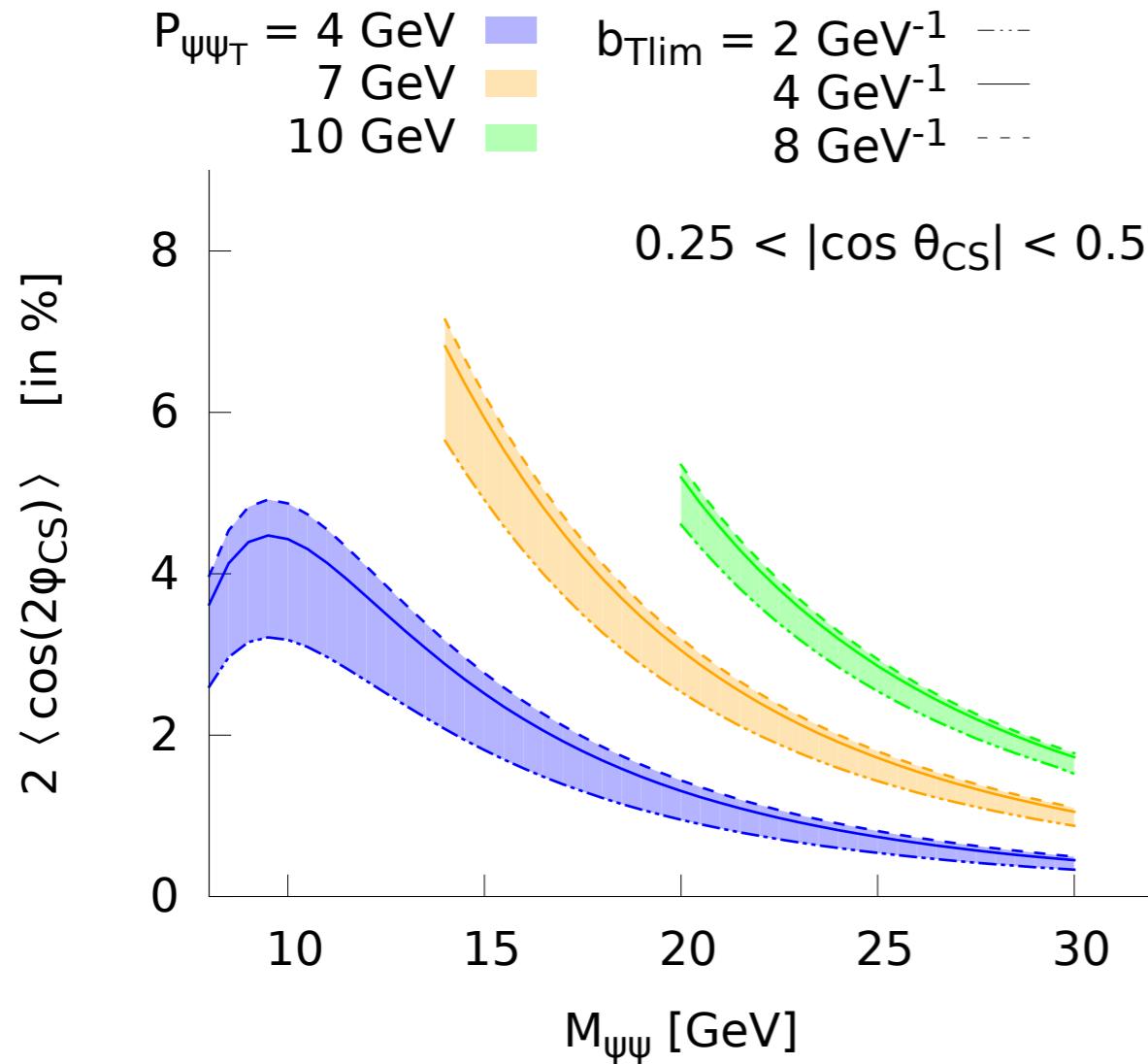


[Echevarria, Kasemets, Mulders, Pisano, 2015]

Very large theoretical uncertainties (from the nonperturbative part of the TMDs), even more for charmonium production, but contribution of 20% or more expected

Linear gluon polarization in di- J/Ψ production

$h_{I^\perp g}$ can be probed through angular modulations in $p p \rightarrow J/\psi J/\psi X$



Estimated to lead to 1-5% level azimuthal modulations at LHC (incl. TMD evolution)

[Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, 2019]

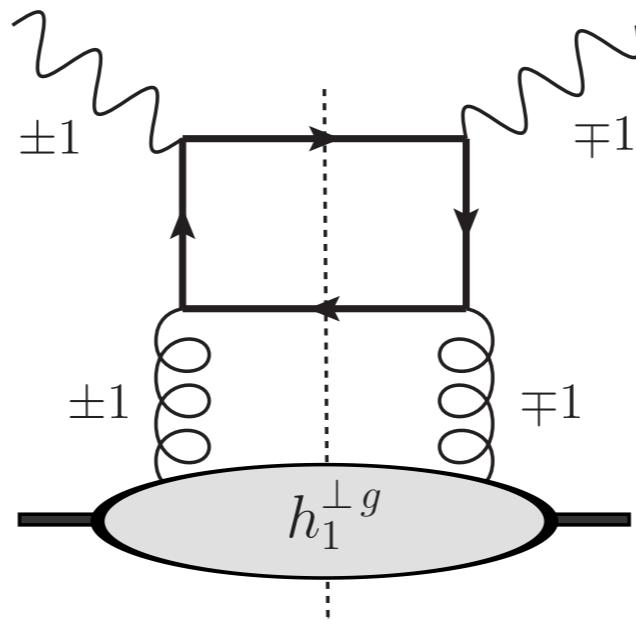
CO contributions estimated to be below the percent level, except at large Δy

Open heavy quark electro-production

Unpolarized open heavy quark production at EIC allows to probe $h_1^{\perp g}(x, p_T^2)$

In pp and pA collisions always convolution of 2 TMDs → advantage of EIC

$$ep \rightarrow e' Q \bar{Q} X$$



no convolution!

Allows to probe
the sign of $h_{1\perp}$
(a T-even TMD)

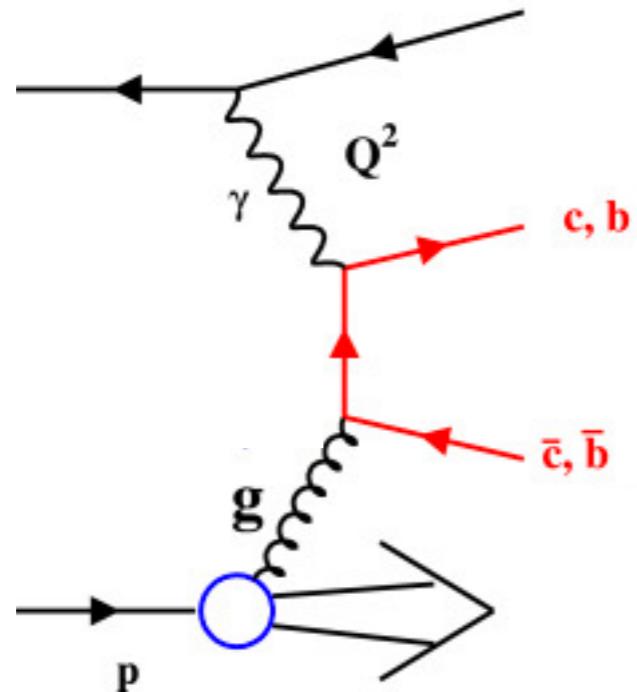
[Boer, Brodsky, Mulders & Pisano, 2010]

The individual transverse momenta have to be large but their sum has to be small

The sum q_T is then related to the transverse momentum of the initial gluon

The linear polarization of gluons will show up as an angular modulation

Open heavy quark electro-production



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_{\perp} = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_{\perp}|$$

ϕ_T, ϕ_{\perp} are the angles of q_T, K_{\perp}

Linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_{\perp})$ modulation

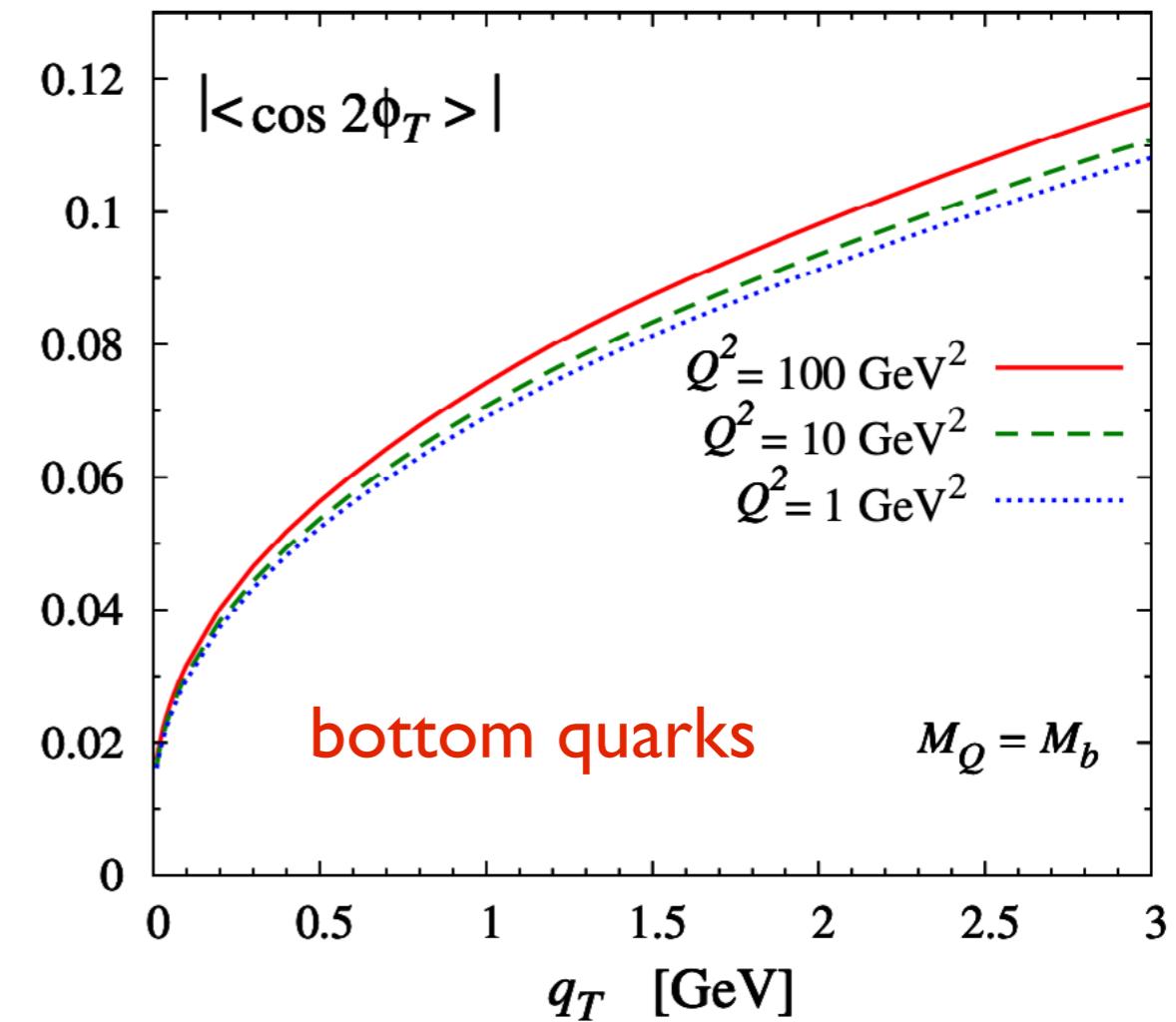
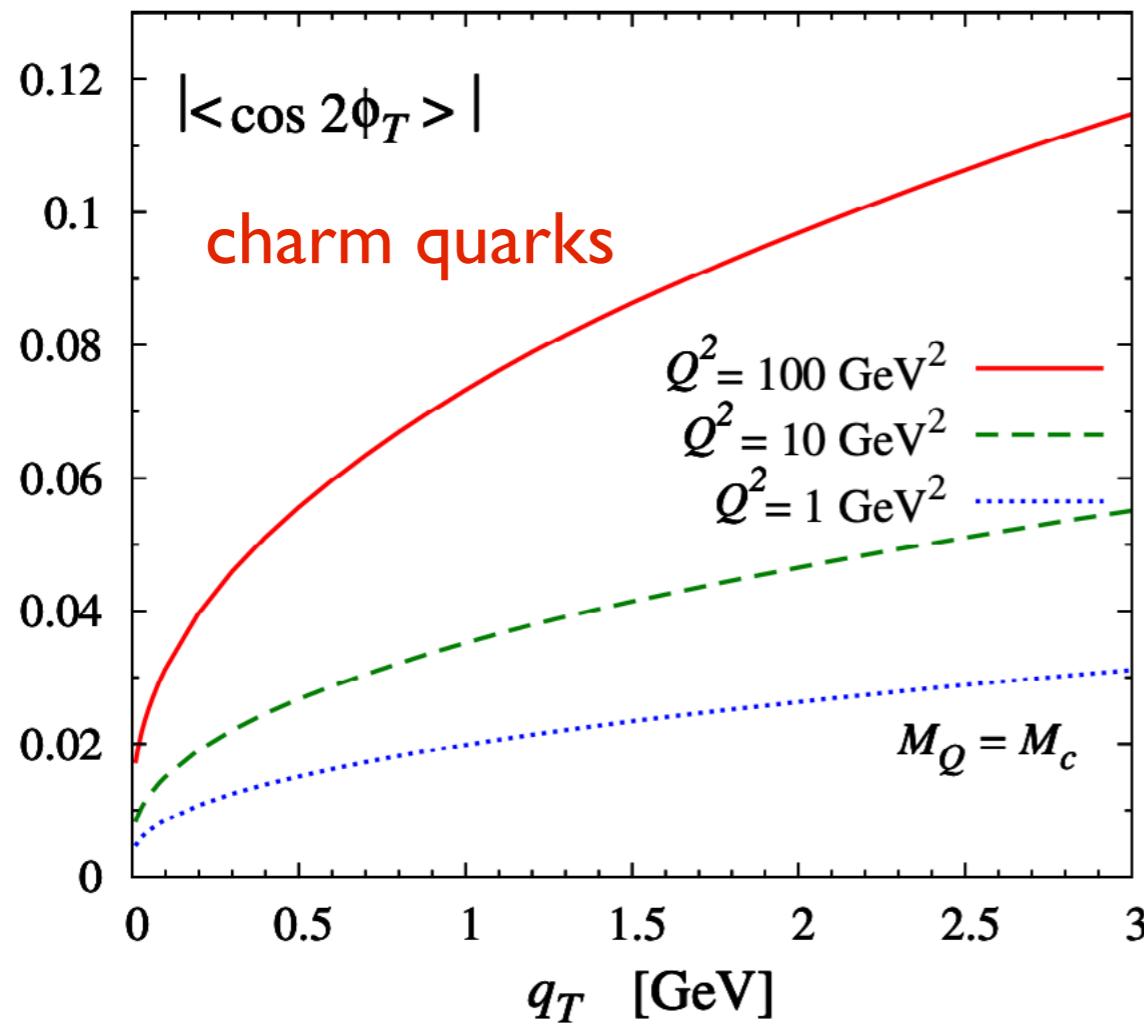
$$|\langle \cos 2\phi_T \rangle| = \left| \frac{\int d\phi_{\perp} d\phi_T \cos 2\phi_T d\sigma}{\int d\phi_{\perp} d\phi_T d\sigma} \right| = \frac{q_T^2}{2M^2} \frac{\left| h_1^{\perp g}(x, \mathbf{p}_T^2) \right|}{f_1^g(x, \mathbf{p}_T^2)} \frac{\left| \mathcal{B}_0^{eg \rightarrow eQ\bar{Q}} \right|}{\mathcal{A}_0^{eg \rightarrow eQ\bar{Q}}}$$

Asymmetries in heavy quark pair production

$$|\langle \cos 2\phi_T \rangle| = \left| \frac{\int d\phi_\perp d\phi_T \cos 2\phi_T d\sigma}{\int d\phi_\perp d\phi_T d\sigma} \right| = \frac{q_T^2}{2M^2} \frac{|h_1^{\perp g}(x, p_T^2)|}{f_1^g(x, p_T^2)} \frac{|\mathcal{B}_0^{eg \rightarrow eQ\bar{Q}}|}{\mathcal{A}_0^{eg \rightarrow eQ\bar{Q}}}$$

Especially interesting in the small x region

$h_1^{\perp g}$ expected to keep up with growth of the unpolarized gluons TMD as $x \rightarrow 0$



MV model prediction for $|K_\perp|=6 \text{ GeV}$, $z=0.5$, $y=0.1$

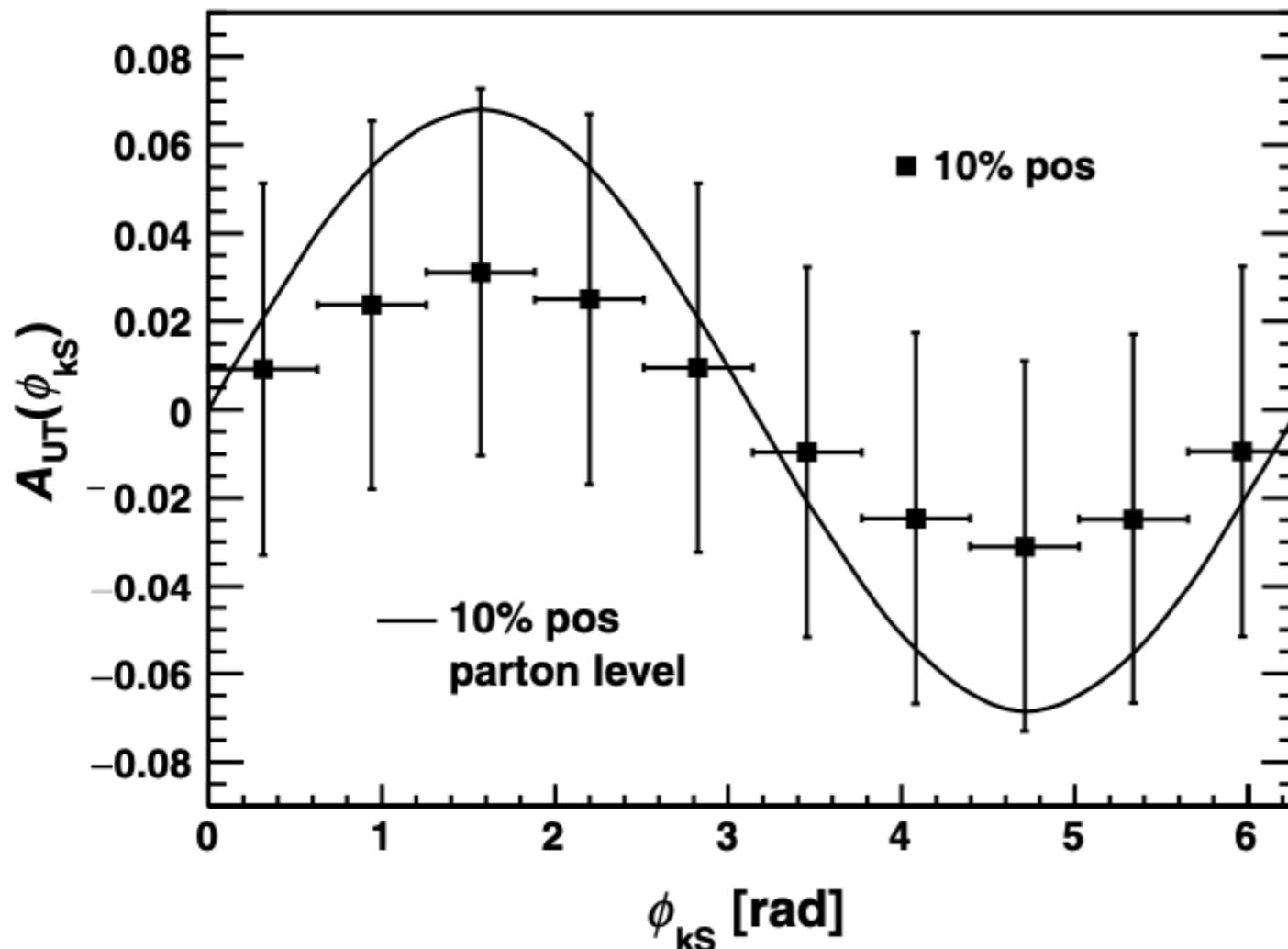
[Boer, Pisano, Mulders, Zhou, 2016]

Asymmetries in heavy quark pair production

For polarized protons one can study azimuthal spin asymmetries, like

$$A_N^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

Sivers asymmetry in D-meson pair production at EIC:



Assumes gluon Sivers TMD to be 10% of its bound

Luminosity of 10 fb^{-1}

Zheng, Aschenauer, Lee, Xiao, Jin, 2018

$\langle x_B \rangle = 0.0012$

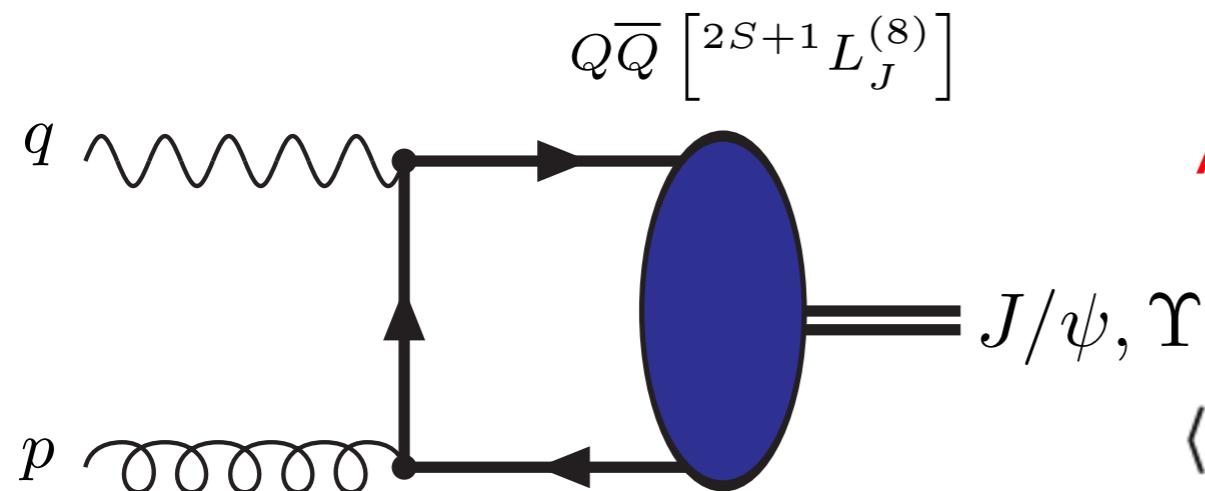
The (f-type) Sivers TMD lacks the $1/x$ growth of the unpolarized gluon TMD

[Boer, Echevarria, Mulders, J. Zhou, 2016]

Quarkonium production in ep

$e p \rightarrow e' Q X$ with Q either a J/ψ or a Υ meson

[Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, Boer, Pisano, Taels, 2018; Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; ...]



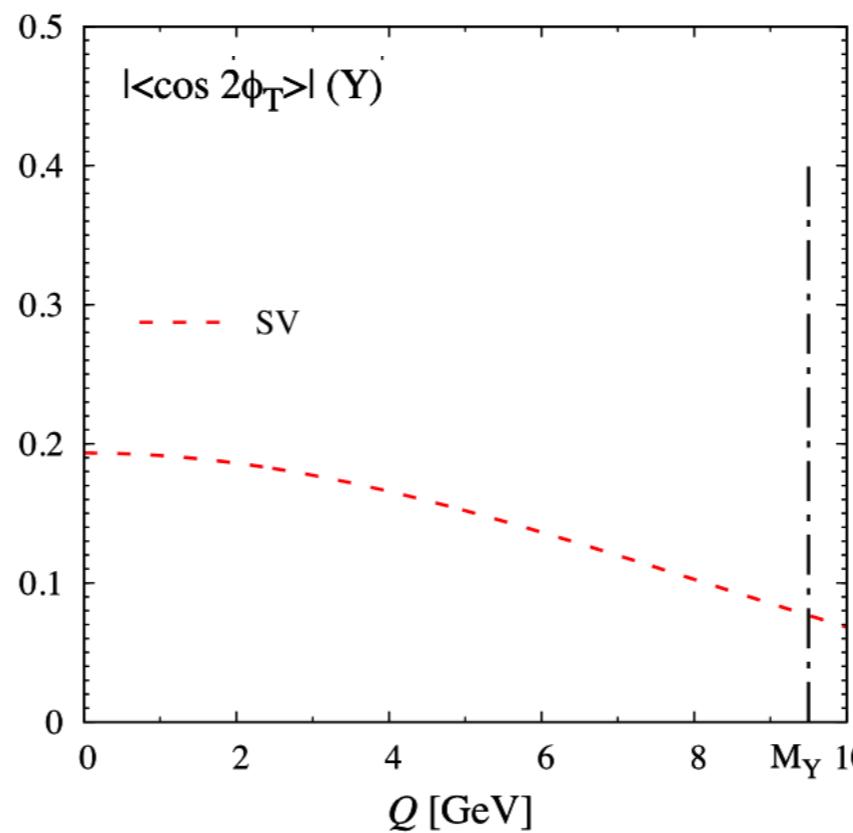
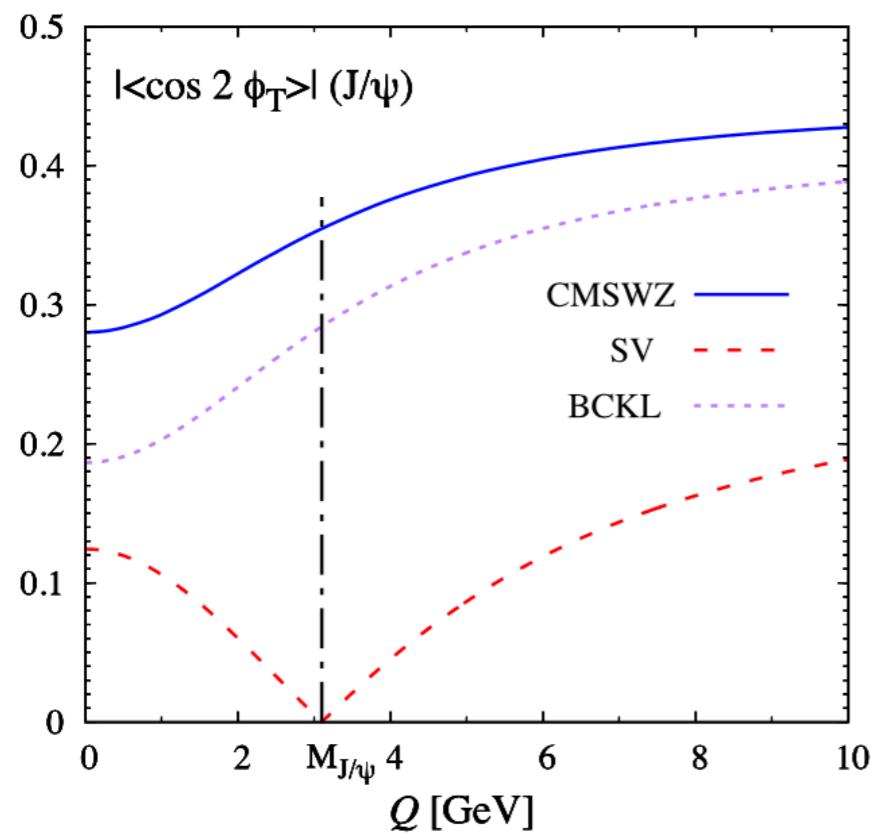
A $\cos(2\phi_T)$ asymmetry probes $h_1^{\perp g}$

$$\langle \cos 2\phi_T \rangle = \frac{(1-y) \mathcal{B}_T^{\gamma^* g \rightarrow Q}}{[1 + (1-y)^2] \mathcal{A}_{U+L}^{\gamma^* g \rightarrow Q} - y^2 \mathcal{A}_L^{\gamma^* g \rightarrow Q}} \times \frac{\mathbf{q}_T^2}{2M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}.$$

In LO NRQCD the prefactor of the asymmetry depends on y, Q, M_Q and on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

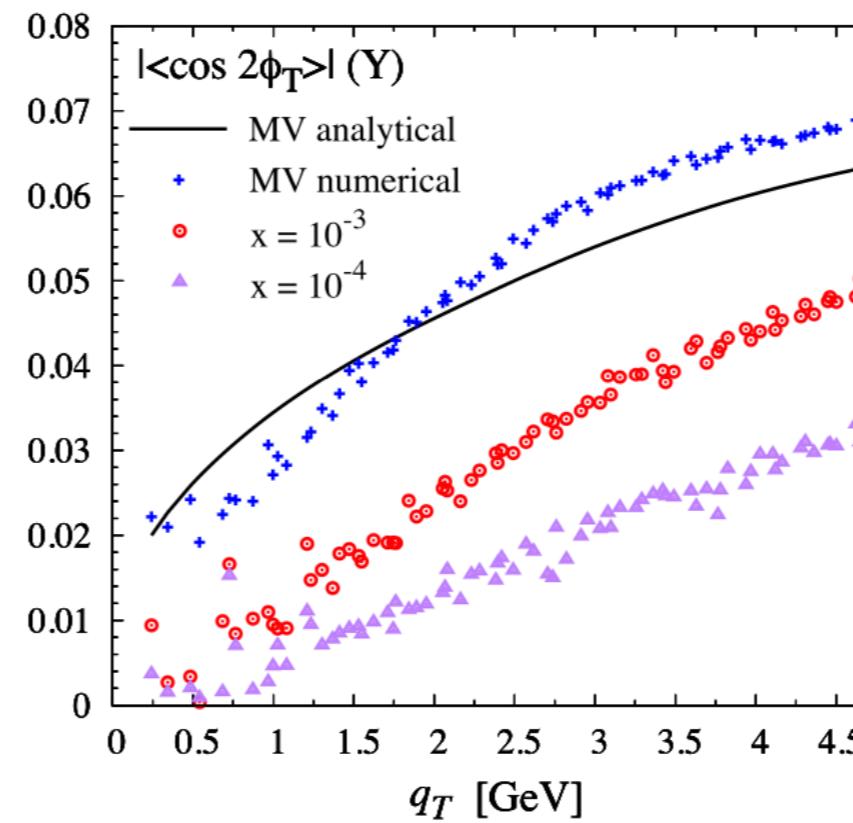
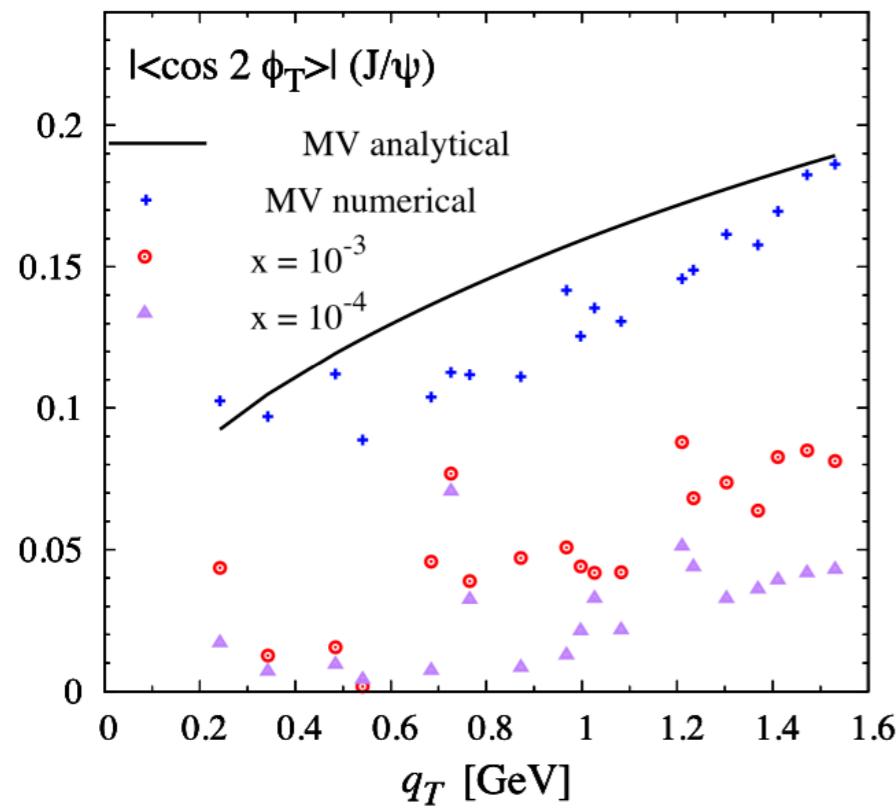
We have studied the maximal asymmetry and a small- x model

[Bacchetta, Boer, Pisano, Taels, 2018]



Maximal asymmetries depend on LDME fit

$y=0.1$



Asymmetries for $Q=M_Q$ & $y=0.1$ in the small- x MV model and numerical implementation on a 2D lattice including nonlinear evolution

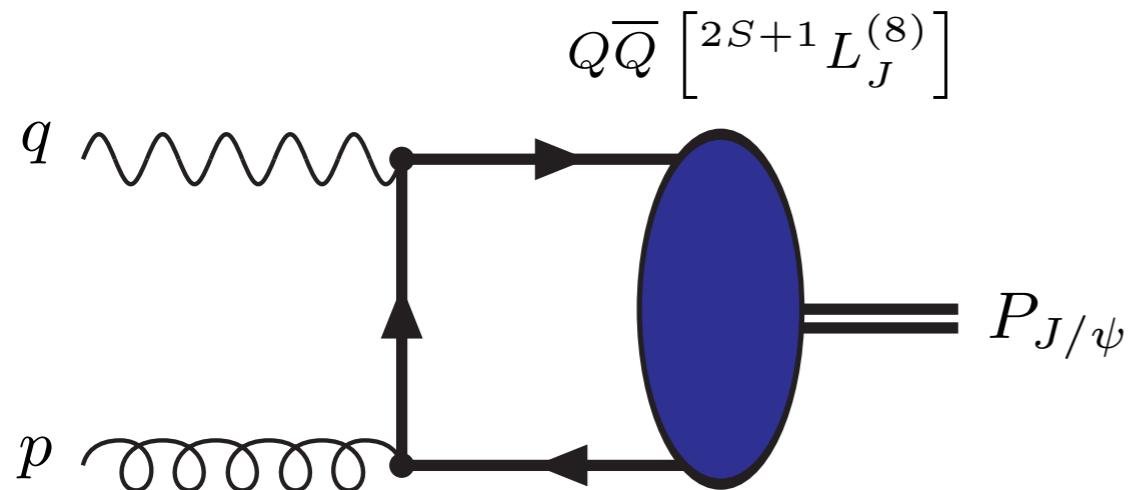
Despite the uncertainties sizable $\cos 2\phi_T$ asymmetries are possible

[Bacchetta, Boer, Pisano, Taels, 2018]

Quarkonium production in $e p^\uparrow$

$e p^\uparrow \rightarrow e' Q X$ with Q either a J/ψ or a Υ meson

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2016; Rajesh, Kishore, Mukherjee, 2018]



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

Other asymmetries depend on the quite uncertain CO NRQCD LDMEs, but one can consider ratios of asymmetries to cancel them out

[Bacchetta, Boer, Pisano, Taelis, 2018]

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S + \phi_T)}} = \frac{\mathbf{q}_T^2}{M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{h_1^g(x, \mathbf{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S - 3\phi_T)}} = -\frac{1}{2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S - 3\phi_T)}}{A^{\sin(\phi_S + \phi_T)}} = -\frac{\mathbf{q}_T^2}{2M_p^2} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{h_1^g(x, \mathbf{q}_T^2)}$$

Color Octet LDMEs from EIC

One can also consider ratios where the TMDs cancel out

Allows to obtain new experimental information on the poorly known CO LDMEs and to learn more about the quarkonium production mechanism

One way is to exploit the polarization ($P=L$ or T) of the quarkonium state

$$D^{\mathcal{Q}_P} \equiv \int d\phi_T \frac{d\sigma^{UP}}{dy dx_B d^2\mathbf{q}_T} = 2\pi \mathcal{N} A^{UP} f_1^g(x, \mathbf{q}_T^2),$$

$$N^{\mathcal{Q}_P} \equiv \int d\phi_T \cos 2\phi_T \frac{d\sigma^{UP}}{dy dx_B d^2\mathbf{q}_T} = \pi \mathcal{N} B^{UP} \frac{\mathbf{q}_T^2}{M_p^2} h_1^{\perp g}(x, \mathbf{q}_T^2).$$

At LO the ratios A^{UL}/A^{UU} , A^{UT}/A^{UU} , B^{UL}/B^{UU} , B^{UT}/B^{UU} only depend on y, Q and the two CO LDMEs, for example for $Q^2=4M_Q^2$:

$$\frac{D^{\mathcal{Q}_L}}{D^{\mathcal{Q}_U}} = \frac{(1 + (1 - y)^2) \mathcal{O}_8^S / 3 + (6 - 6y + y^2) \mathcal{O}_8^P / M_Q^2}{(1 + (1 - y)^2) \mathcal{O}_8^S + (10 - 10y + 3y^2) \mathcal{O}_8^P / M_Q^2}, \quad \mathcal{O}_8^S \equiv \langle 0 | \mathcal{O}_8^Q ({}^1 S_0) | 0 \rangle$$

$$\frac{N^{\mathcal{Q}_L}}{N^{\mathcal{Q}_U}} = \frac{\mathcal{O}_8^S / 3 - \mathcal{O}_8^P / M_Q^2}{\mathcal{O}_8^S - \mathcal{O}_8^P / M_Q^2}. \quad \mathcal{O}_8^P \equiv \langle 0 | \mathcal{O}_8^Q ({}^3 P_0) | 0 \rangle$$

CO LDMEs

We considered 5 different extractions that differ considerably,
most have $O_8^P \ll O_8^S$ but one has $O_8^P = O_8^S$

Extractions of the Color Octet Long Distance Matrix Elements

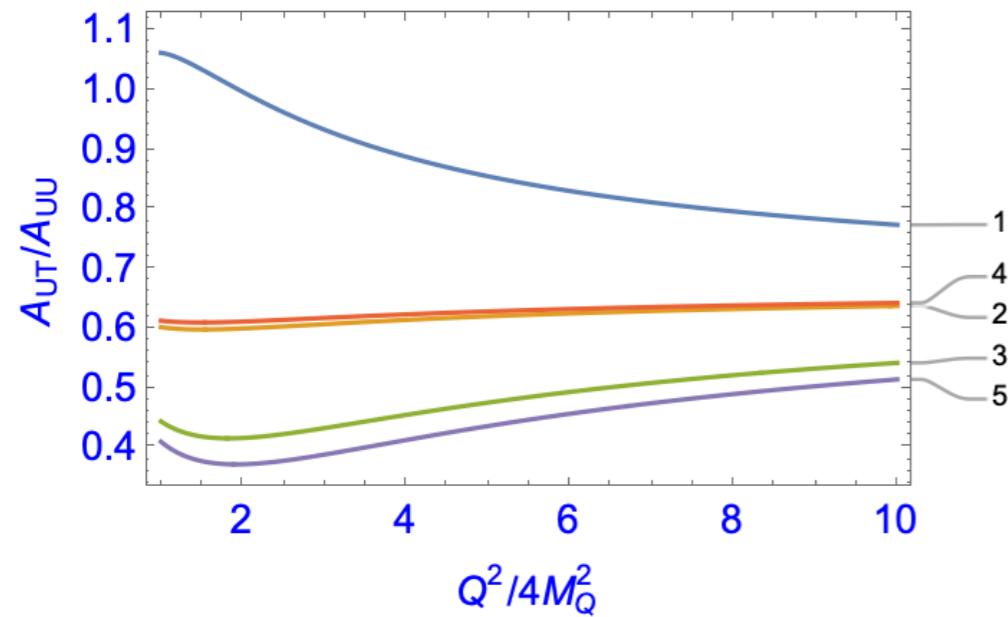
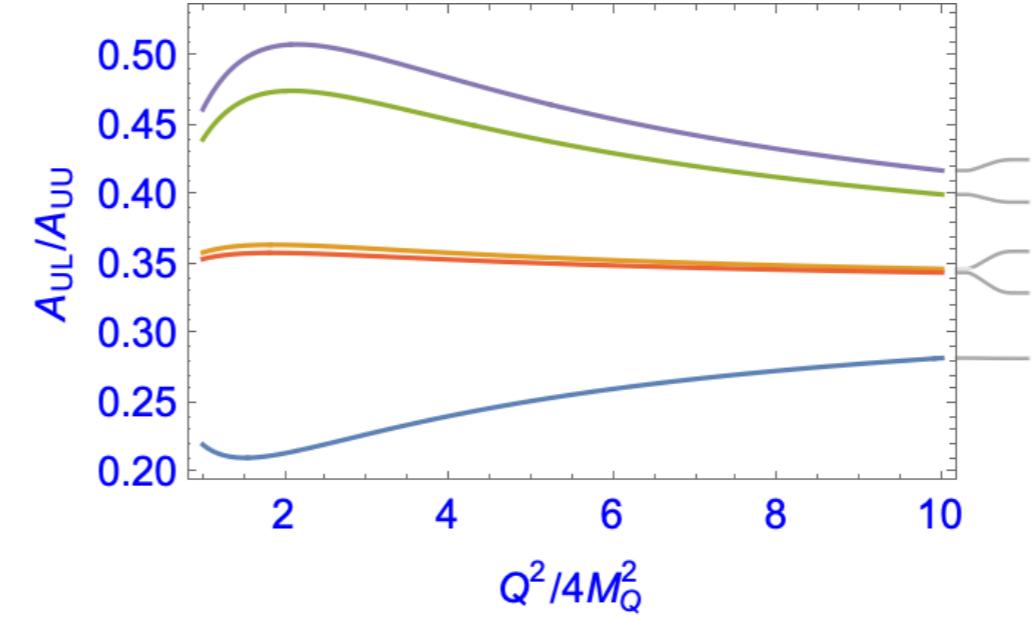
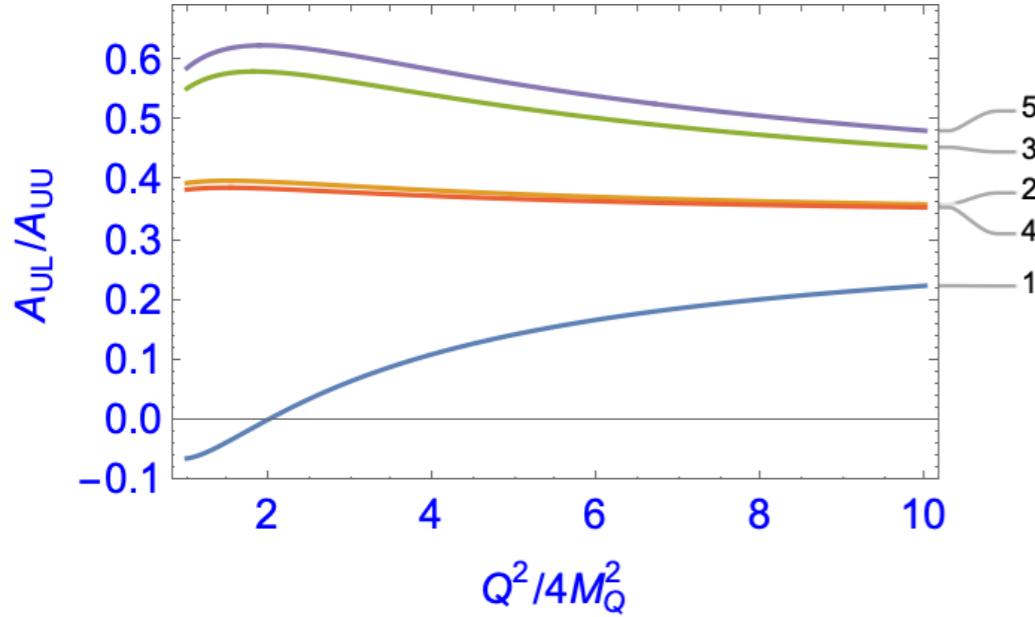
Fit No.	Reference	$\langle 0 \mathcal{O}_8^{J/\psi} ({}^1 S_0) 0 \rangle$	$\langle 0 \mathcal{O}_8^{J/\psi} ({}^3 P_0) 0 \rangle / M_c^2$	Units	M_c
1	Butenschön & Kniehl [14]	4.50 ± 0.72	-0.54 ± 0.16	$\times 10^{-2} \text{ GeV}^3$	1.5 GeV
2	Chao <i>et al.</i> [15]	8.9 ± 0.98	0.56 ± 0.21	$\times 10^{-2} \text{ GeV}^3$	not specified
3	Sharma & Vitev [16]	1.8 ± 0.87	1.8 ± 0.87	$\times 10^{-2} \text{ GeV}^3$	1.4 GeV
4	Bodwin <i>et al.</i> [17]	9.9 ± 2.2	0.49 ± 0.44	$\times 10^{-2} \text{ GeV}^3$	1.5 GeV

Fit No.	Reference	$\langle 0 \mathcal{O}_8^{\Upsilon(1S)} ({}^1 S_0) 0 \rangle$	$\langle 0 \mathcal{O}_8^{\Upsilon(1S)} ({}^3 P_0) 0 \rangle / (5M_b^2)$	Units	M_b
5	Sharma & Vitev [16]	1.21 ± 4.0	1.21 ± 4.0	$\times 10^{-2} \text{ GeV}^3$	4.88 GeV

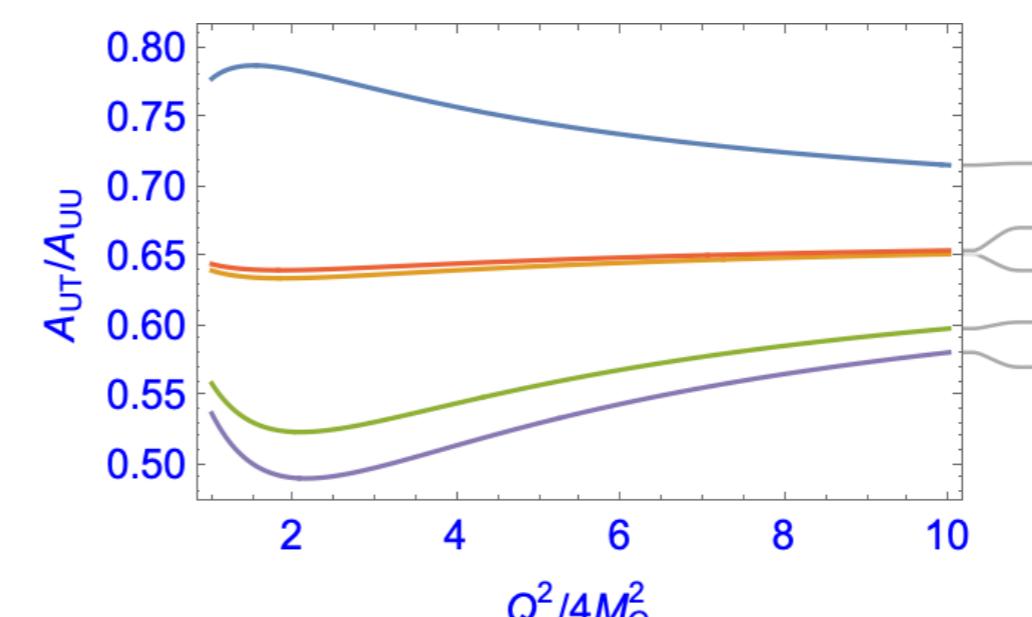
- Fit 1 - Butenschoen & Kniehl, PRL 106, 022003 (2011)
- Fit 2 - CSMWZ = Chao *et al.*, PRL 108, 242004 (2012)
- Fit 3 - SV= Sharma & Vitev, PRC 87, 044905 (2013)
- Fit 4 - BCKL = Bodwin *et al.*, PRL 113, 022001 (2014)
- Fit 5 - Sharma & Vitev, PRC 87, 044905 (2013)

CO LDMEs from polarized quarkonia at EIC

Ratios A^{UL}/A^{UU} and A^{UT}/A^{UU} as a function of Q^2 for the 5 different fits



$y=0.1$



$y=0.8$

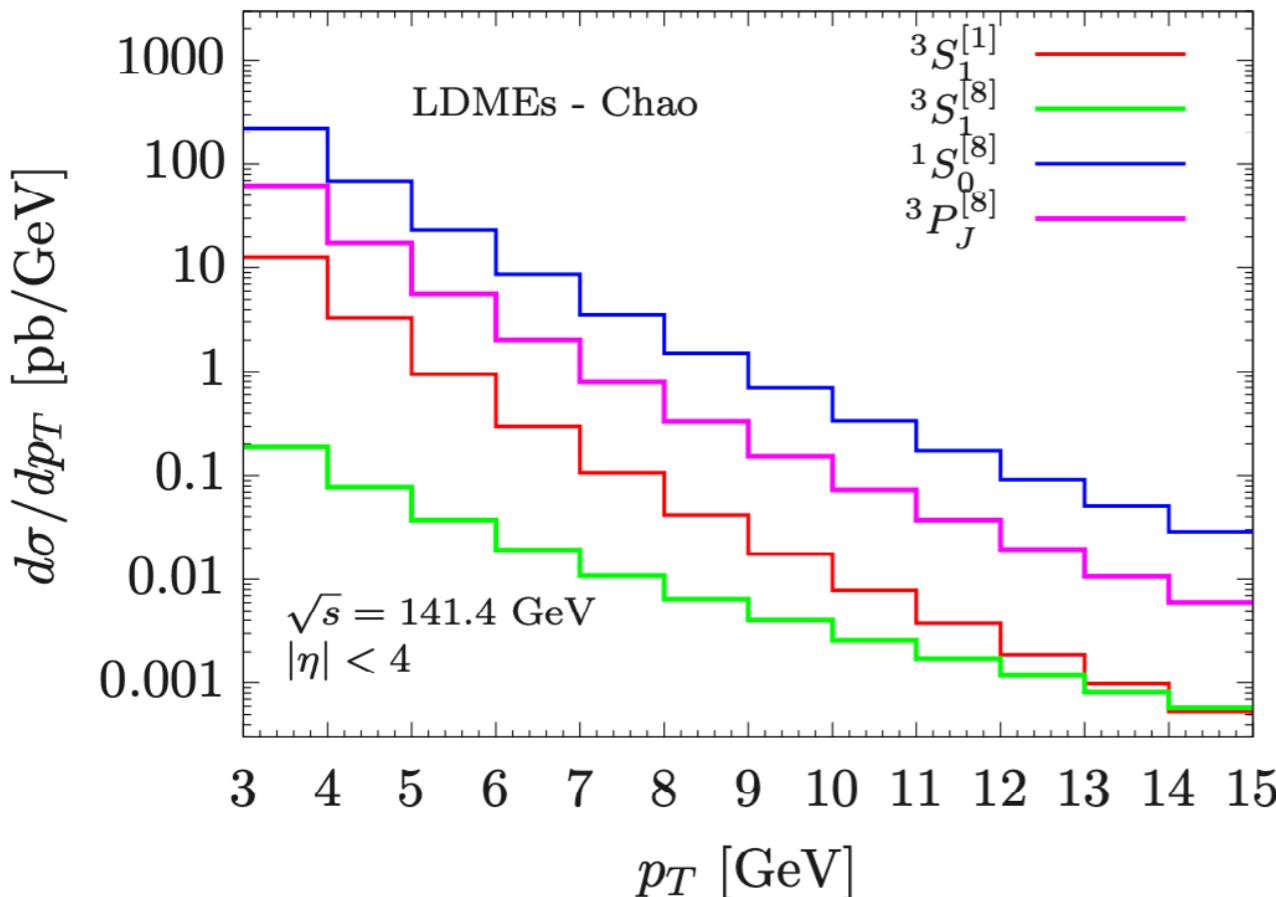
The quarkonium polarization measurement at EIC could help to reduce the uncertainty on the CO LDMEs

[Boer, Pisano, Taels, 2021]

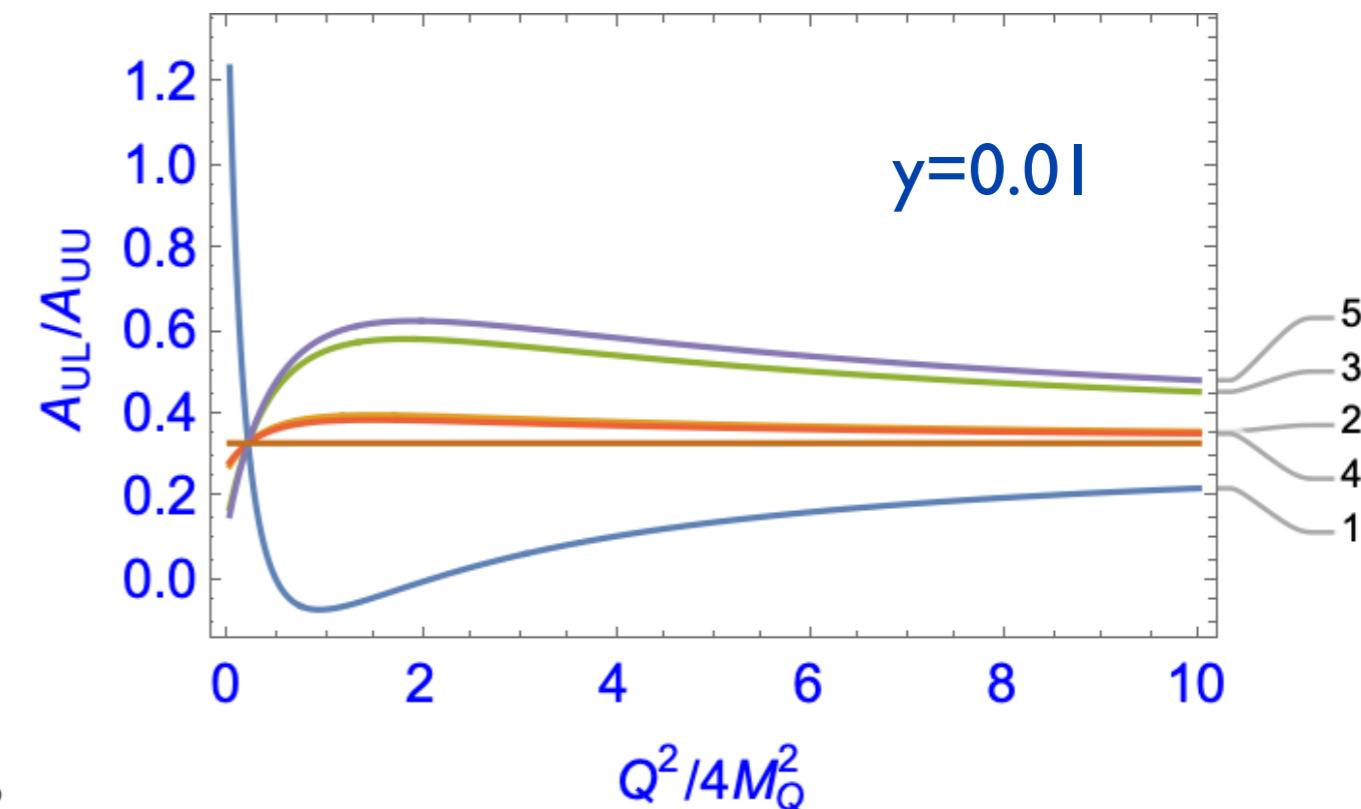
Polarized quarkonia at EIC

Will the quarkonium state be produced polarized at EIC?

In $e p \rightarrow Q X$ (untagged e', dominated by $Q^2 \approx 0$) in collinear factorization at large p_T the 1S_0 state dominates in Fit No 2 (see left figure) \approx unpolarized



[Qiu, Wang, Qing, 2020]



[Boer, Pisano, Taels, 2021]

In our LO NRQCD study in the TMD regime, for some fits: A_{UL}/A_{UU} far from 1/3, therefore, polarized production is very well possible, showing relevance of 3P_0

Conclusions

Conclusions

- Gluon TMDs can be studied using heavy quark production processes at EIC, LHC, NICA and RHIC
- The linear polarization of gluons inside unpolarized hadrons is expected to lead to sizable $\cos 2\phi$ (& $\cos 4\phi$) asymmetries, especially at smaller x and higher Q^2
- J/Ψ or Υ production in $e p/e A$ collisions allow to probe gluon TMDs, but involve (in LO NRQCD) 2 Color Octet LDMEs (1S_0 & 3P_0) that are still poorly known
- In the TMD regime these CO LDMEs can be extracted by exploiting the quarkonium polarization or a comparison to open heavy quark pair production which allows to learn more about the quarkonium production mechanism
- In TMD regime unpolarized quarkonium production not a prediction of NRQCD
- Heavy quark probes of gluon TMDs look very promising, but experimental projections and higher order calculations are still lacking to a large extent

Back-up slides

Gluon polarization inside unpolarized protons

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

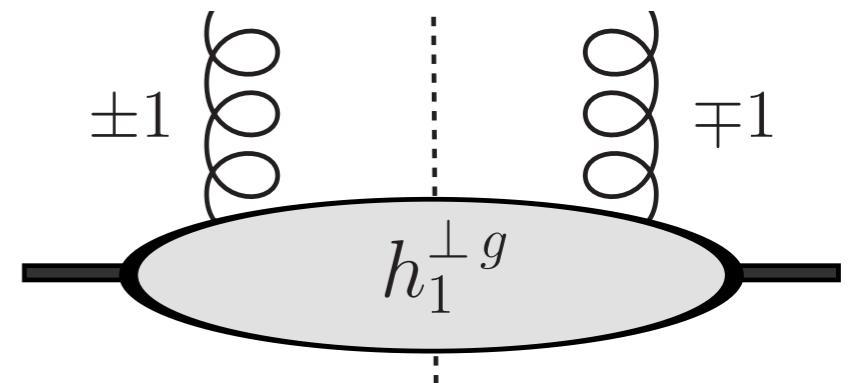
Linearly polarized gluons can exist in
unpolarized hadrons

[Mulders, Rodrigues, 2001]

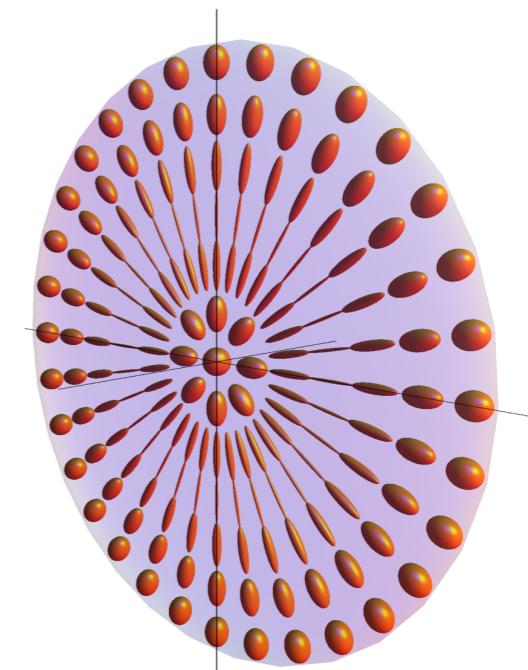
It requires nonzero transverse momentum

For $h_1^{\perp g} > 0$ gluons are linearly polarized along \mathbf{k}_T ,
with a $\cos 2\phi$ distribution around it, where $\phi = \angle(\mathbf{k}_T, \varepsilon_T)$

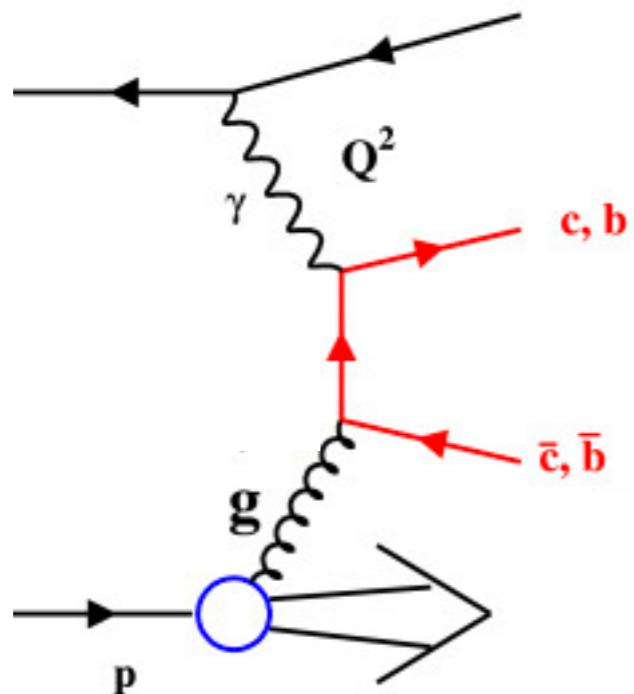
The linearly polarized gluons can affect the
transverse momentum spectrum



an interference between
 ± 1 helicity gluon states



Parallels between SIDIS and HQ pair production



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_{\perp} = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_{\perp}|$$

ϕ_T, ϕ_{\perp} are the angles of q_T, K_{\perp}

Linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_{\perp})$ distribution

Despite the differences in properties of some of the quark and gluon TMDs, the asymmetries they lead to are analogous for SIDIS and HQ pair production

There is a “Collins” asymmetry without a Collins function, but it does probe h_{1g} which is not transversity however

Parallels between quarks and gluons

$$\Phi_U(x, \mathbf{k}) = \frac{1}{2} \left[\not{n} f_1(x, \mathbf{k}^2) + \frac{\sigma_{\mu\nu} k_T^\mu \bar{n}^\nu}{M} h_1^\perp(x, \mathbf{k}^2) \right],$$

$$\Phi_L(x, \mathbf{k}) = \frac{1}{2} \left[\gamma^5 \not{n} S_L g_1(x, \mathbf{k}^2) + \frac{i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^\nu S_L}{M} h_{1L}^\perp(x, \mathbf{k}^2) \right],$$

$$\begin{aligned} \Phi_T(x, \mathbf{k}) = & \frac{1}{2} \left[\frac{\not{n} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{\gamma^5 \not{n} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ & \left. + i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu S_T^\nu h_1(x, \mathbf{k}^2) - \frac{i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^\nu S_{T\rho}}{M^2} h_{1T}^\perp(x, \mathbf{k}^2) \right] \end{aligned}$$

$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[\delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right],$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} \alpha k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right],$$

$$\begin{aligned} \Gamma_T^{ij}(x, \mathbf{k}) = & x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ & \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} \alpha k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right] \end{aligned}$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

For gluons $h_{1\perp}$ is T-even and h_1 is k_T -odd, T-odd and unrelated to transversity

Asymmetries in HQ pair production at EIC

LO asymmetries in HQ pair production:

$$|\langle \cos 2\phi_T \rangle| = \left| \frac{\int d\phi_\perp d\phi_T \cos 2\phi_T d\sigma}{\int d\phi_\perp d\phi_T d\sigma} \right| = \frac{\mathbf{q}_T^2 |B_0^U|}{2 A_0^U} = \frac{\mathbf{q}_T^2}{2M^2} \frac{|h_1^{\perp g}(x, \mathbf{p}_T^2)|}{f_1^g(x, \mathbf{p}_T^2)} \frac{|\mathcal{B}_0^{eg \rightarrow eQ\bar{Q}}|}{\mathcal{A}_0^{eg \rightarrow eQ\bar{Q}}}$$

$$A_N^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{A_0^T}{A_0^U} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A_N^{\sin(\phi_S + \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{{B'_0}^T}{A_0^U} = \frac{2(1-y) \mathcal{B}_{0T}^{\gamma^* g \rightarrow Q\bar{Q}}}{[1 + (1-y)^2] \mathcal{A}_{U+L}^{\gamma^* g \rightarrow Q\bar{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \rightarrow Q\bar{Q}}} \frac{|\mathbf{q}_T|}{M_p} \frac{h_1^g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

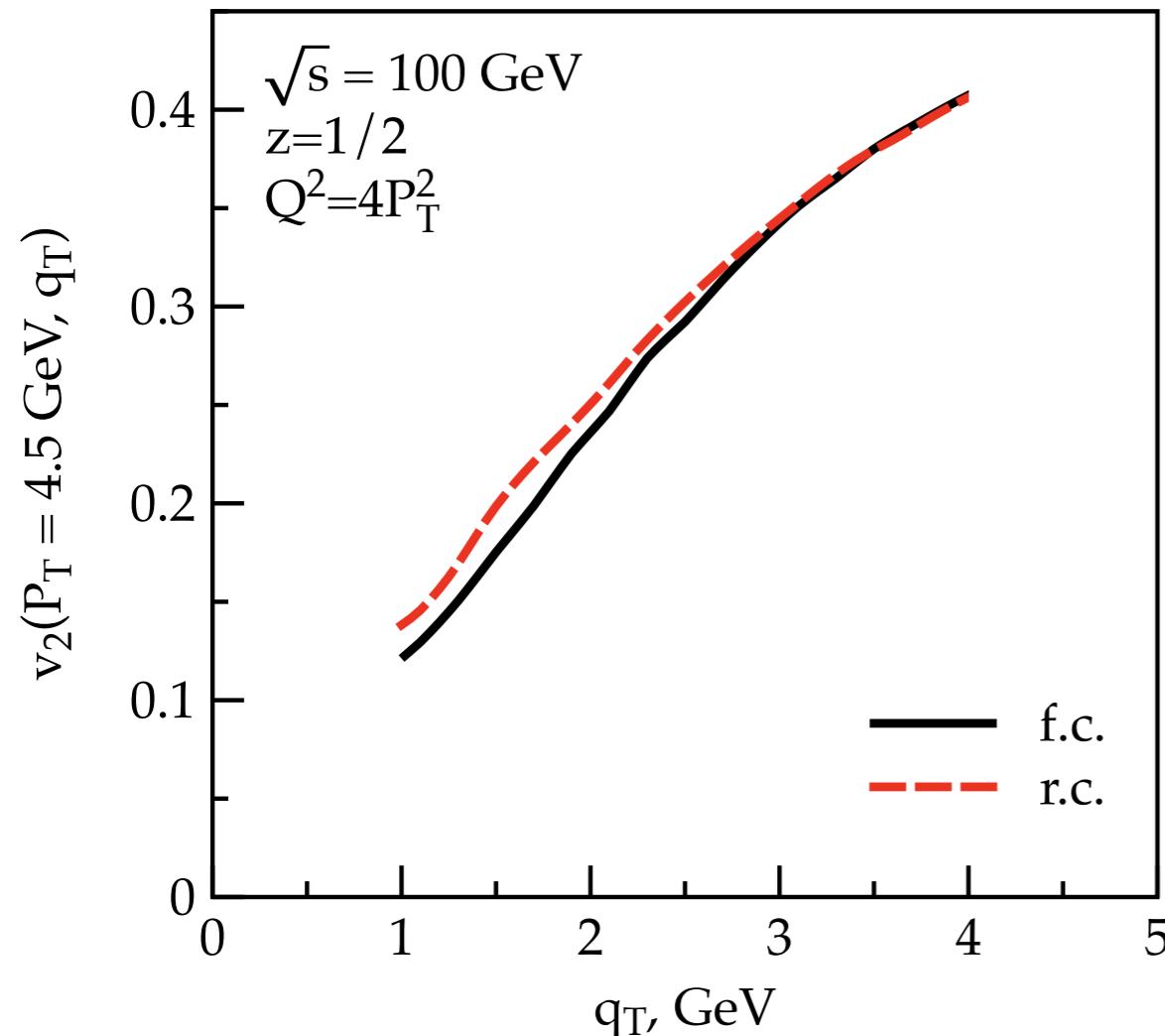
$$A_N^{\sin(\phi_S - 3\phi_T)} = -\frac{|\mathbf{q}_T|^3}{M_p^3} \frac{B_0^T}{2A_0^U} = -\frac{2(1-y) \mathcal{B}_{0T}^{\gamma^* g \rightarrow Q\bar{Q}}}{[1 + (1-y)^2] \mathcal{A}_{U+L}^{\gamma^* g \rightarrow Q\bar{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \rightarrow Q\bar{Q}}} \frac{|\mathbf{q}_T|^3}{2M_p^3} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

Dijet production at EIC

$h_{1\perp g}$ (WW) is also accessible in dijet production at EIC

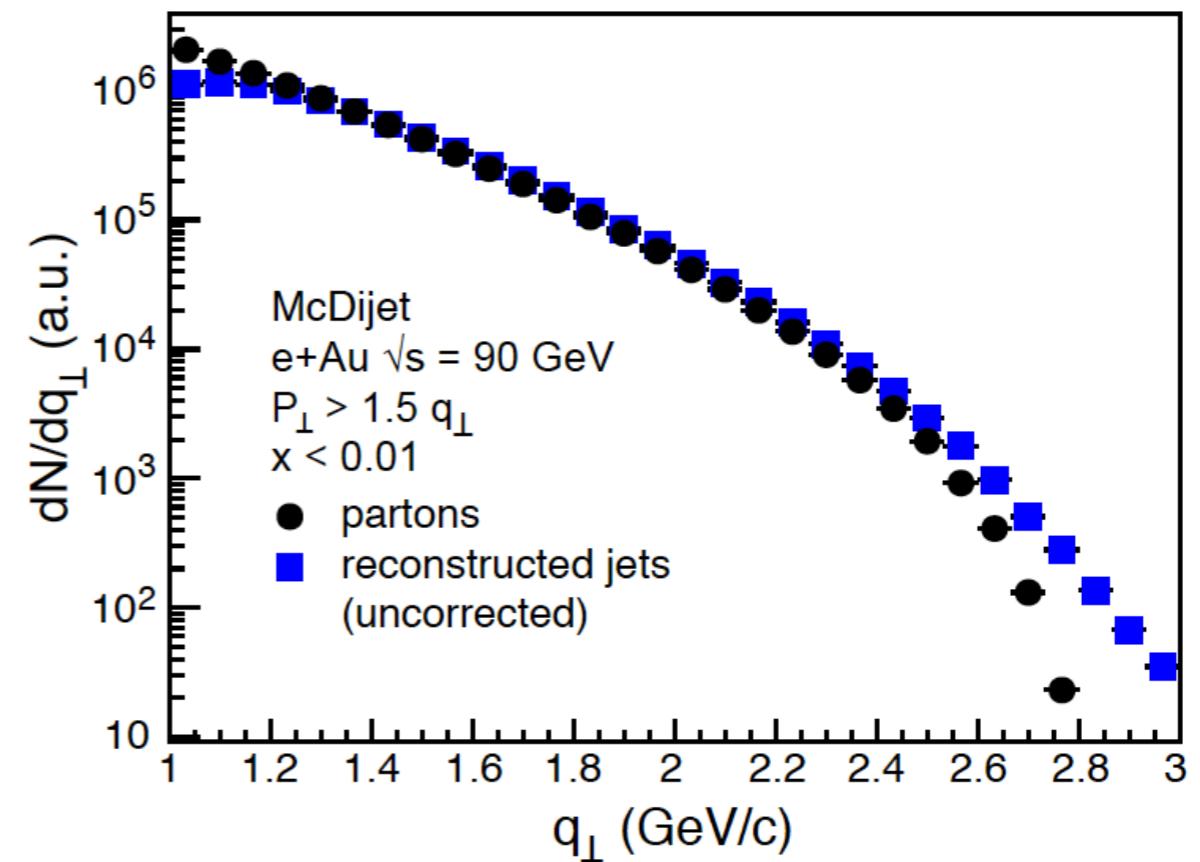
[Metz, Zhou 2011; Pisano, Boer, Brodsky, Buffing, Mulders, 2013; Boer, Pisano, Mulders, Zhou, 2016]

Linear gluon polarization shows itself through a $\cos 2\phi$ distribution (“ v_2 ”)



Large effects are found

Dumitru, Lappi, Skokov, 2015



Jets are reasonable proxies for outgoing quarks concerning the q_T distribution

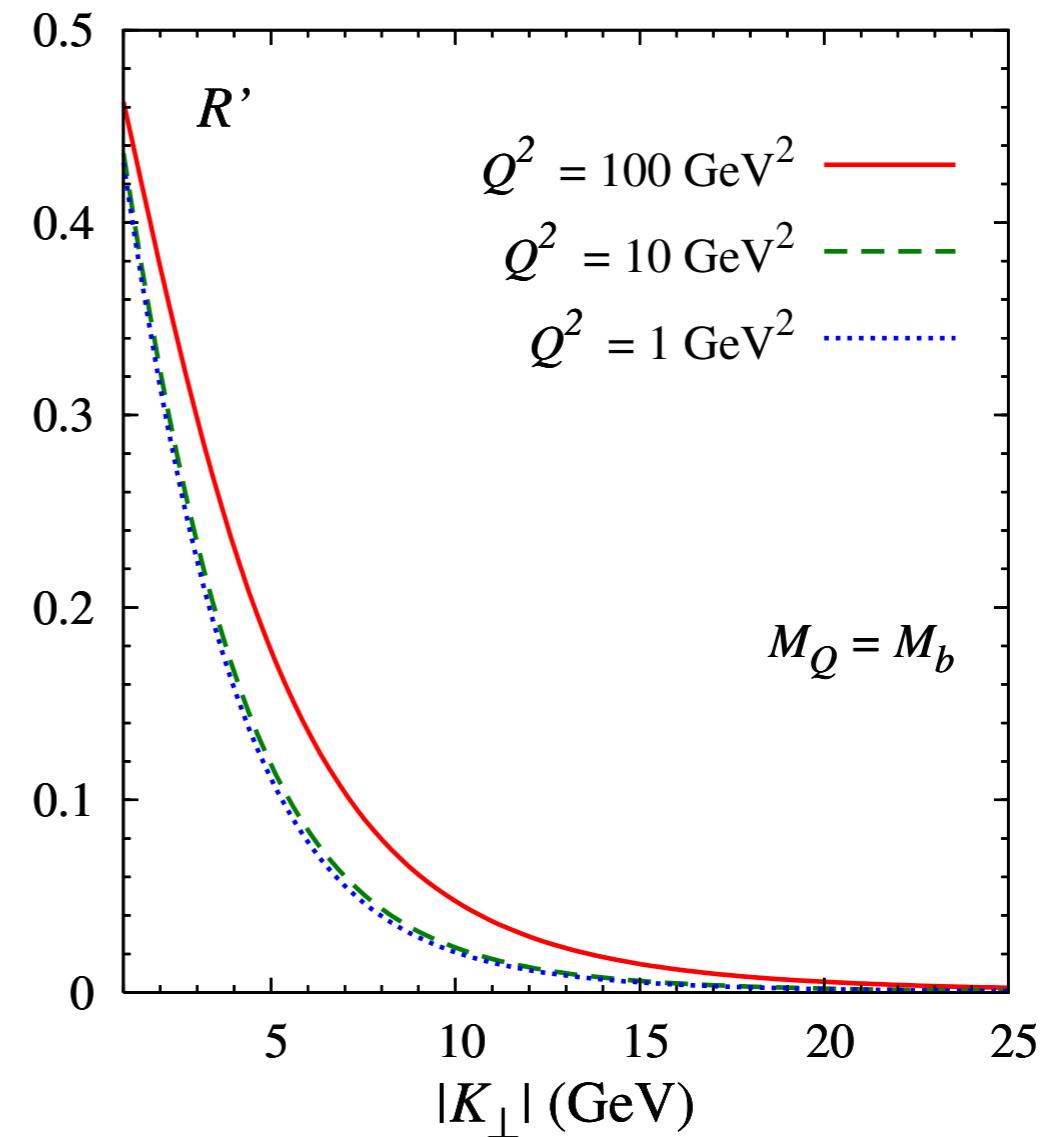
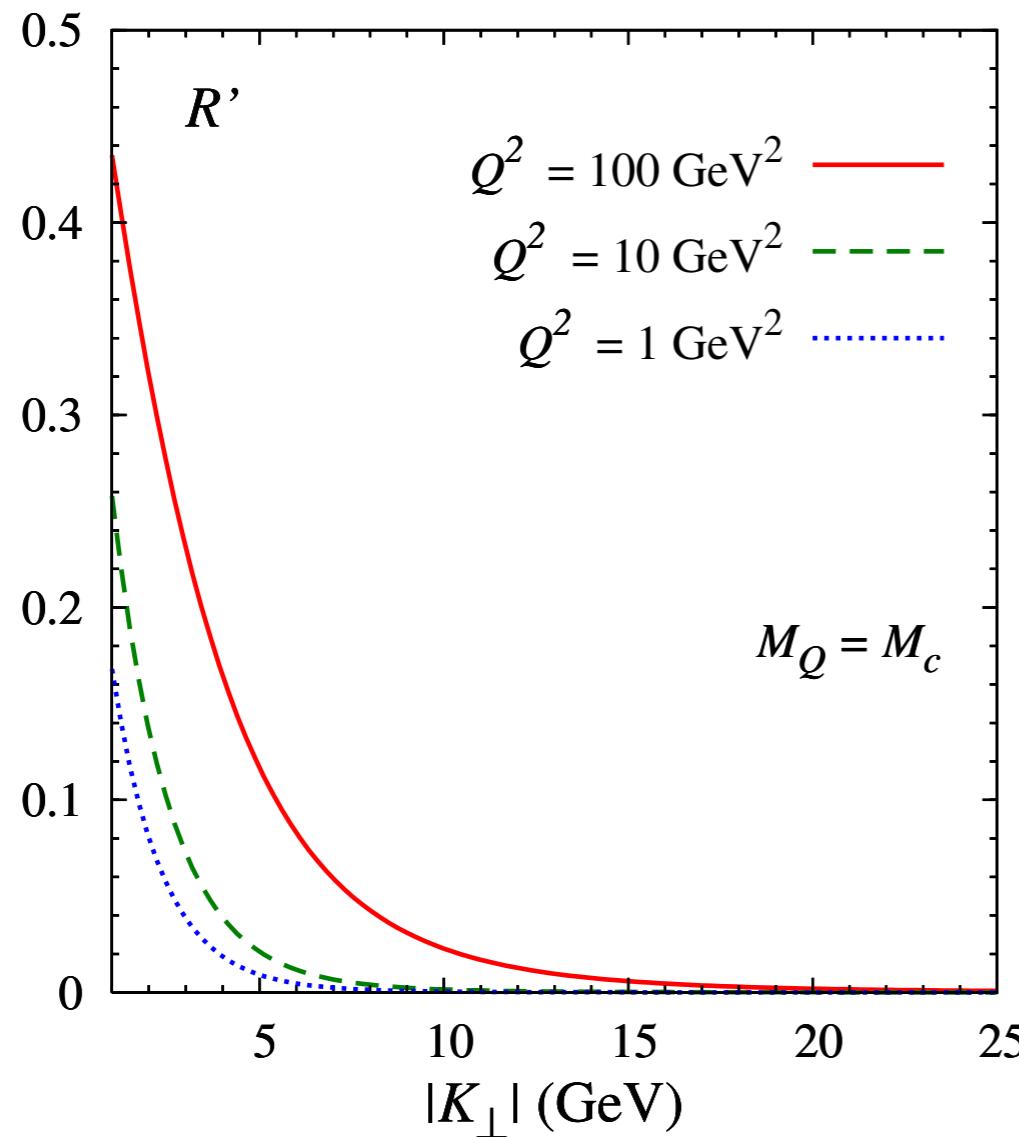
Dumitru, Skokov, Ullrich, 2018

Maximum asymmetries in heavy quark production

There are also angular asymmetries w.r.t. the lepton scattering plane, whose maxima are large at smaller $|K_\perp|$

$$ep \rightarrow e' Q \bar{Q} X$$

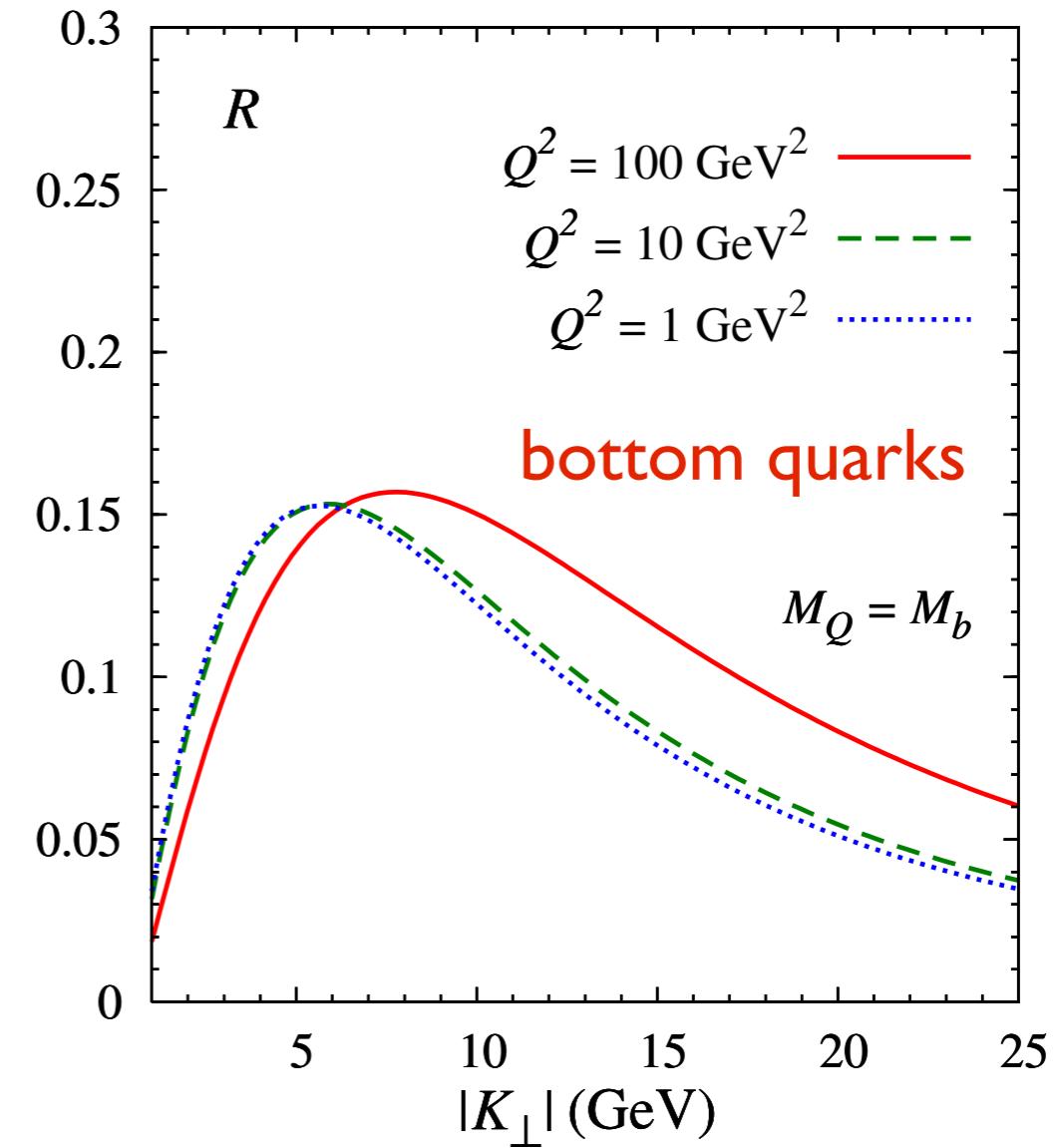
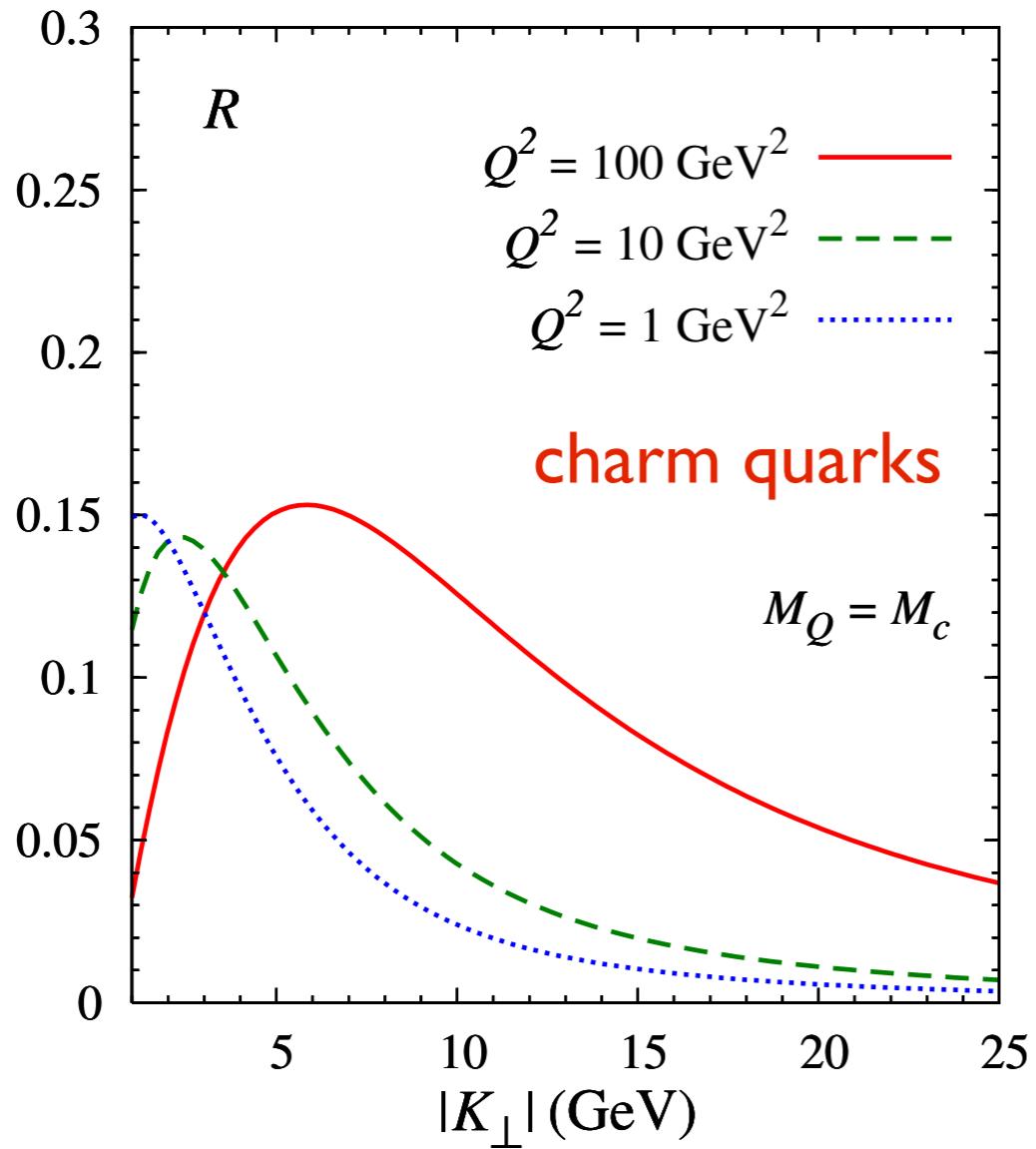
$$R' = \text{bound on } |\langle \cos 2(\phi_\ell - \phi_T) \rangle|$$



Maximum asymmetries in heavy quark pair production

$$ep \rightarrow e' Q \bar{Q} X$$

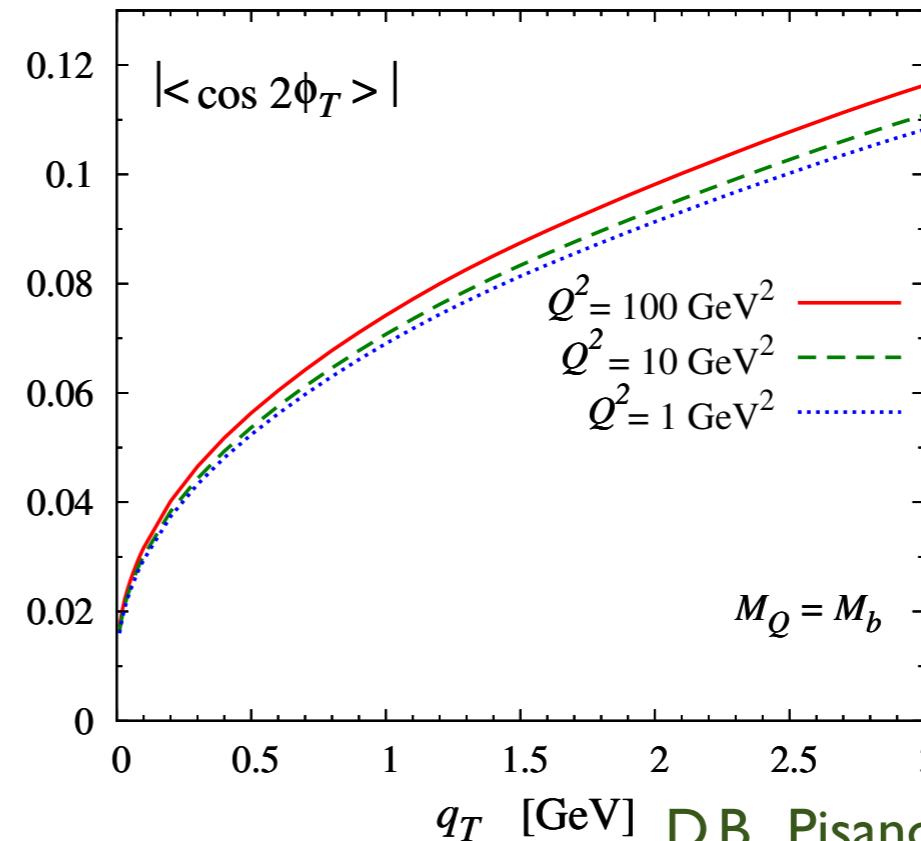
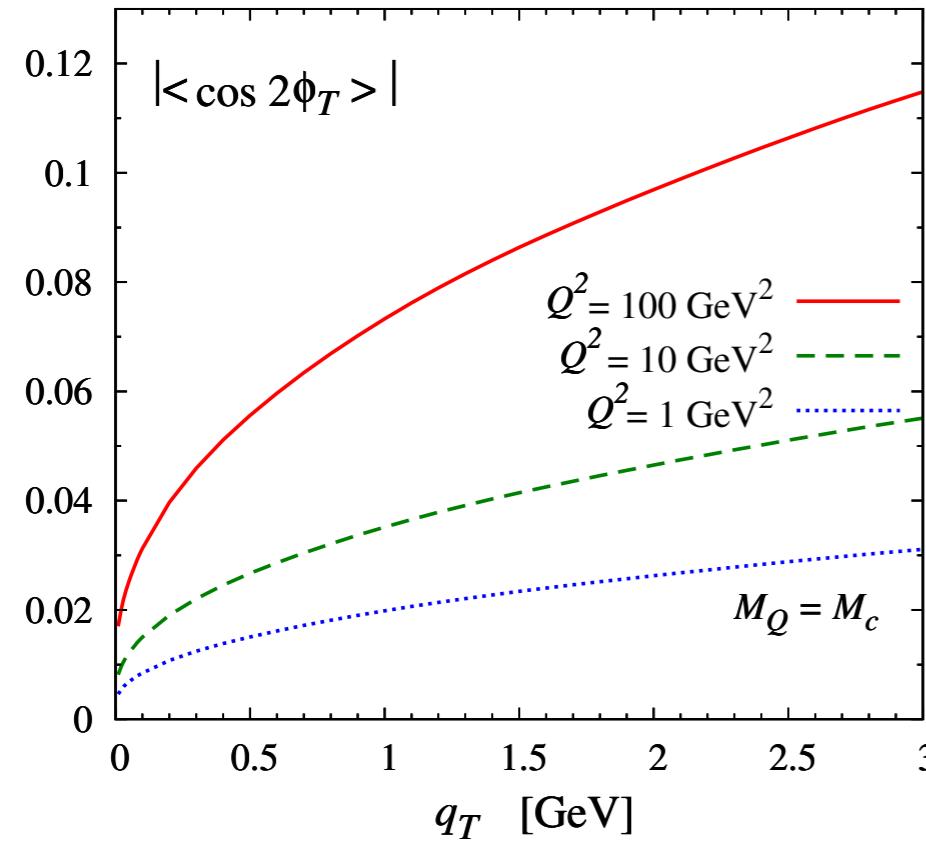
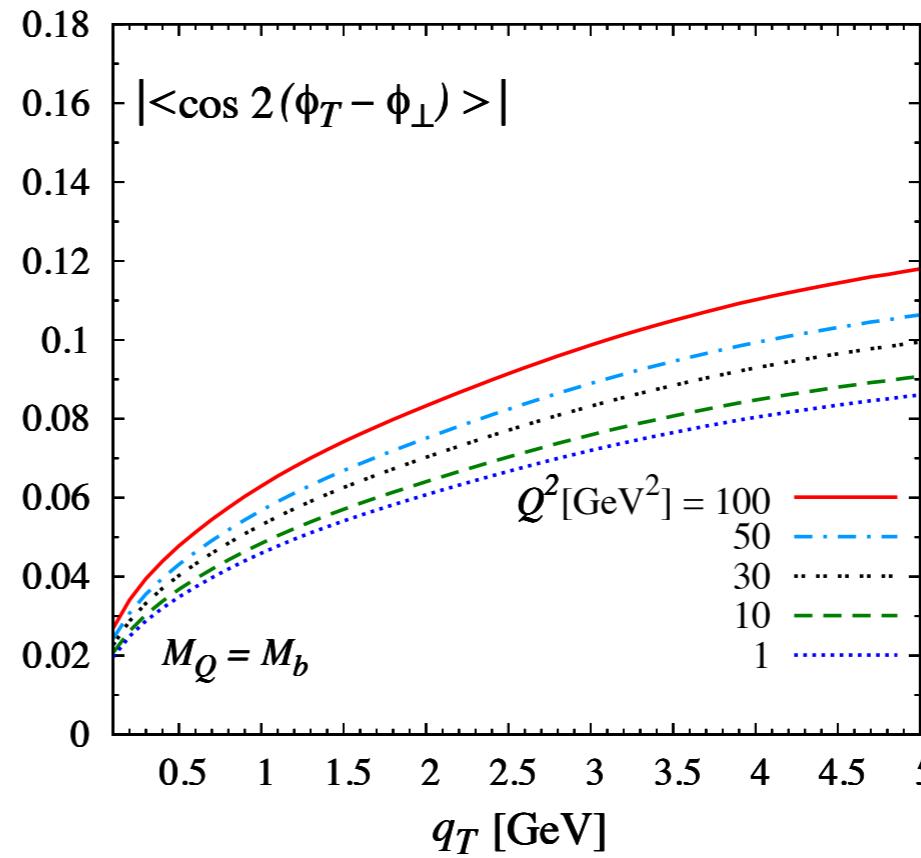
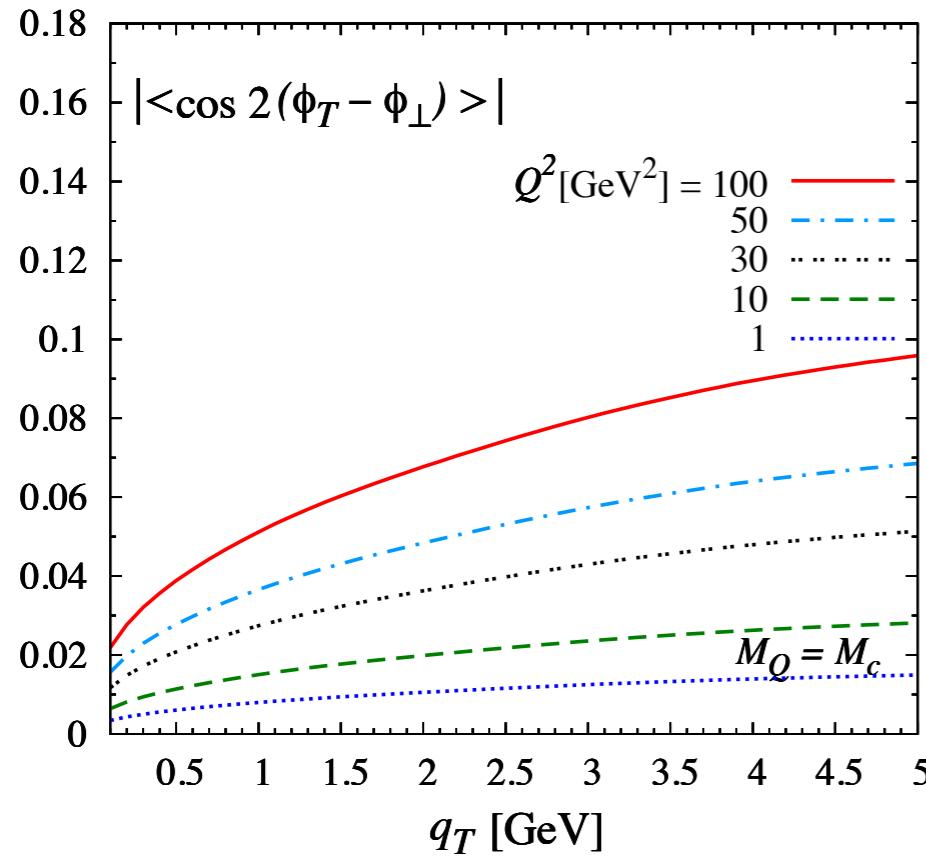
$$R = \text{bound on } |\langle \cos 2(\phi_T - \phi_\perp) \rangle|$$



[Pisano, Boer, Brodsky, Buffing & Mulders, 2013]

Maximal asymmetries can be substantial (for any Q^2 and for both charm & bottom)

Heavy quark pair production at EIC



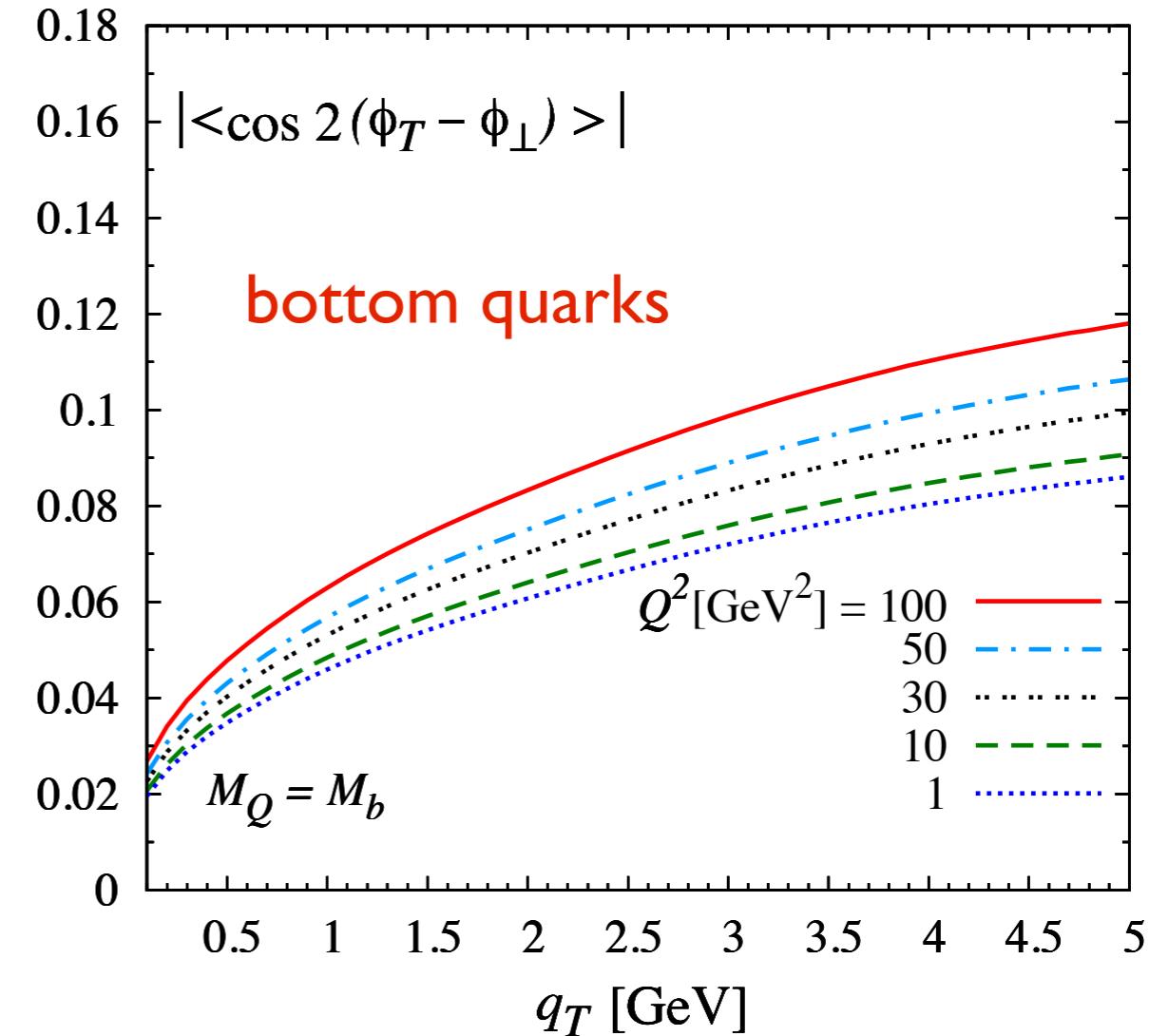
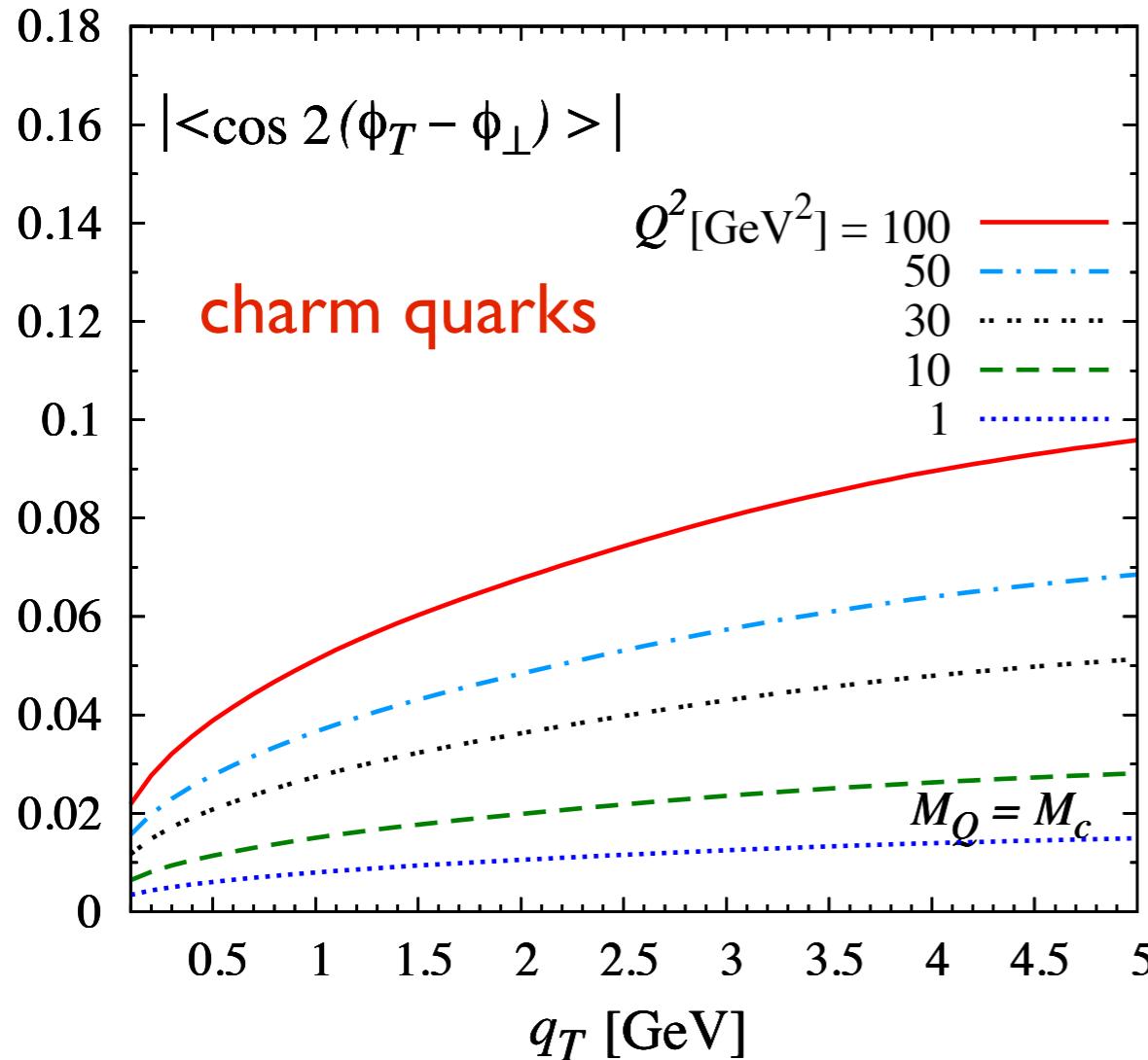
small \times
MV model

$|K_{\perp}| = 10$ GeV
 $z = 0.5$
 $y = 0.3$

$|K_{\perp}| = 6$ GeV
 $z = 0.5$
 $y = 0.1$

Asymmetries in heavy quark pair production

$h_1^{\perp g}$ expected to keep up with growth of the unpolarized gluons TMD as $x \rightarrow 0$



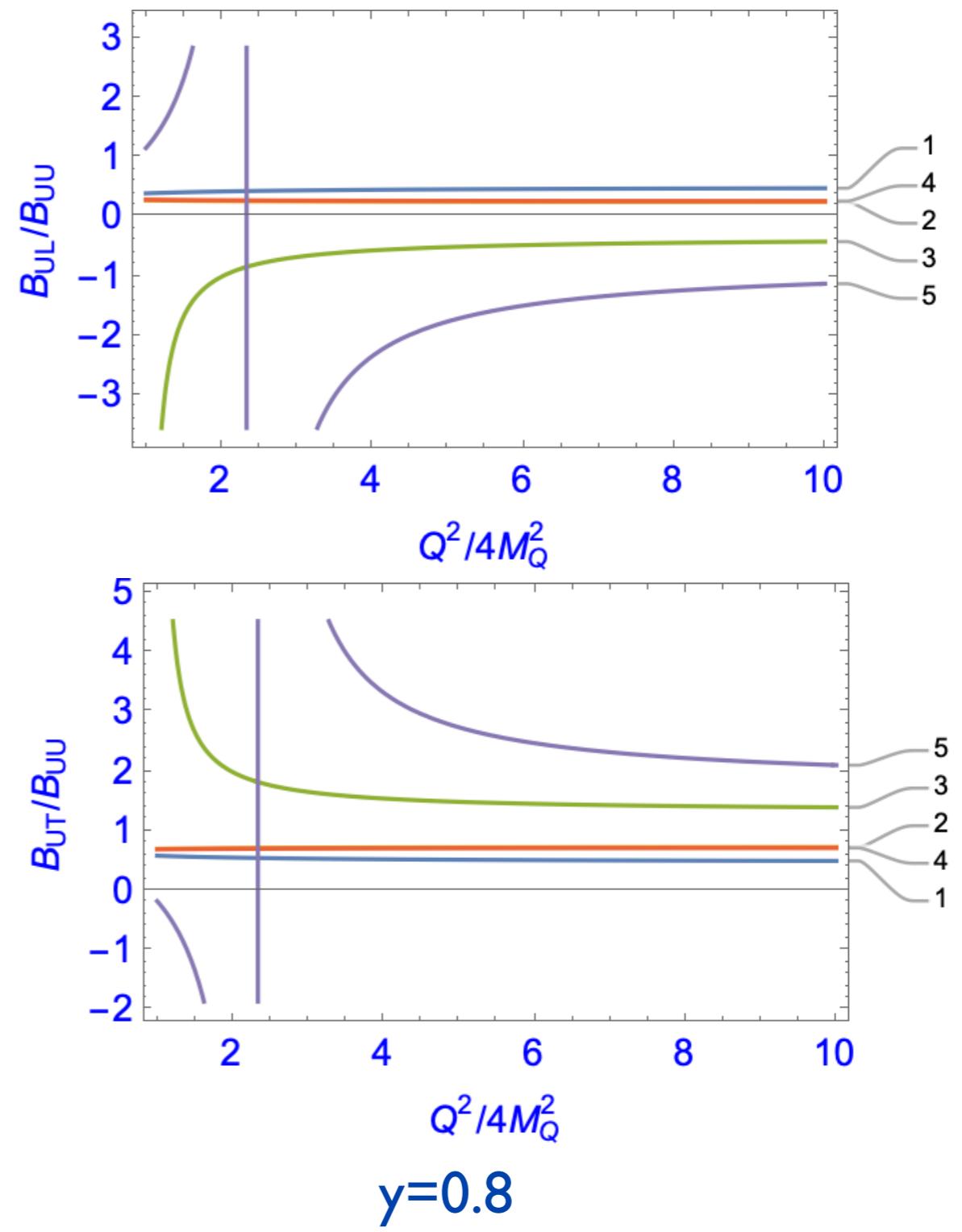
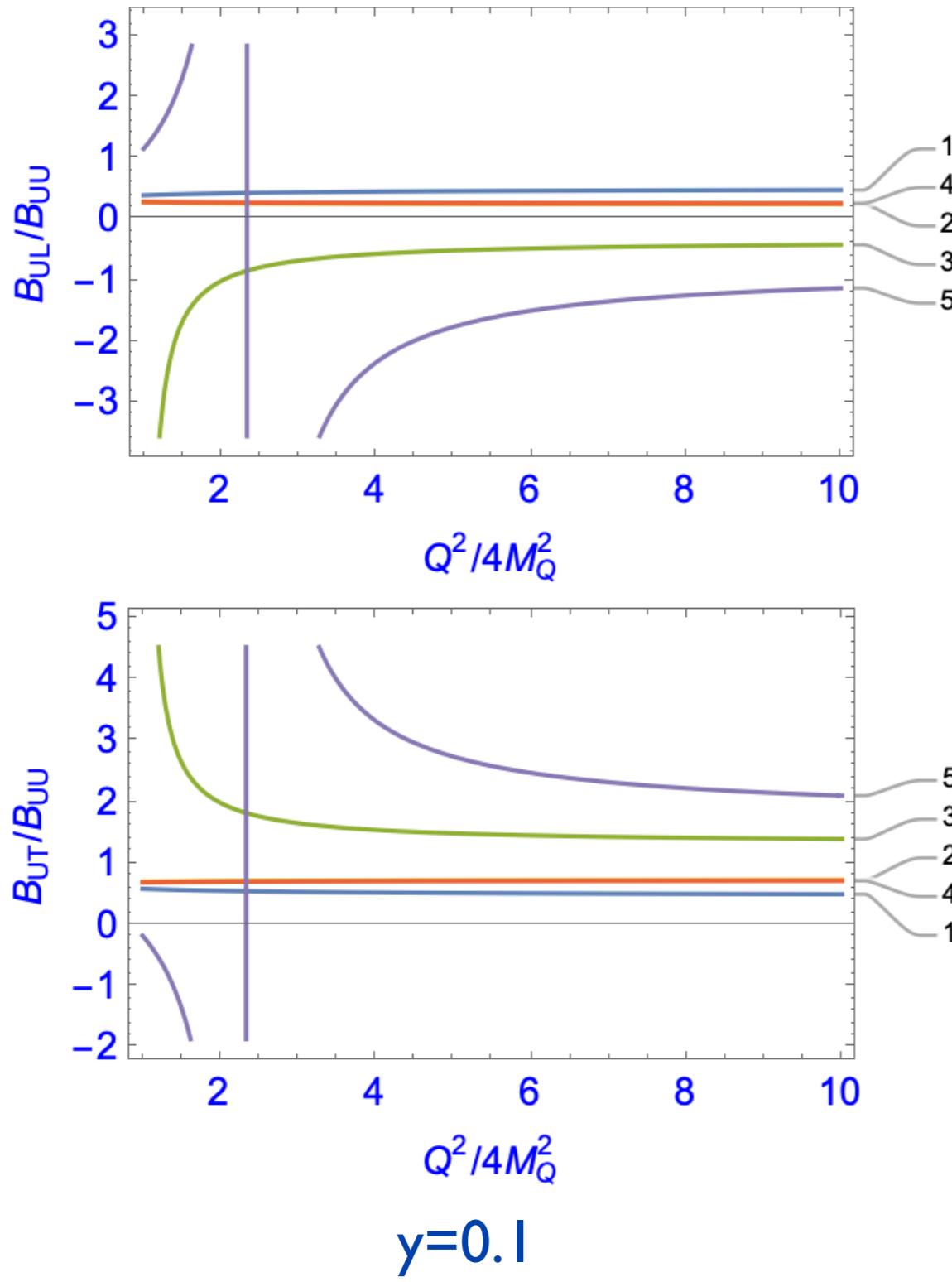
small x
MV model

$|K_\perp| = 10$ GeV
 $z = 0.5$
 $y = 0.3$

Sizable asymmetries at EIC
[Boer, Pisano, Mulders, Zhou, 2016]

CO LDMEs from polarized quarkonia at EIC

Ratios B_{UL}/B_{UU} and B_{UT}/B_{UU} as a function of Q^2 for the 5 different fits



Color Octet LDMEs from EIC

If the quarkonium polarization cannot be determined one can consider other ratios to cancel out the TMDs

This requires a comparison of $e p \rightarrow e' Q X$ and $e p \rightarrow e' Q \bar{Q} X$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int d\phi_T \cos 2\phi_T d\sigma^Q(\phi_S, \phi_T)}{\int d\phi_T d\phi_\perp \cos 2\phi_T d\sigma^{Q\bar{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R} = \frac{\int d\phi_T d\sigma^Q(\phi_S, \phi_T)}{\int d\phi_T d\phi_\perp d\sigma^{Q\bar{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0 | \mathcal{O}_8^Q ({}^1 S_0) | 0 \rangle$$

$$\mathcal{O}_8^P \equiv \langle 0 | \mathcal{O}_8^Q ({}^3 P_0) | 0 \rangle$$

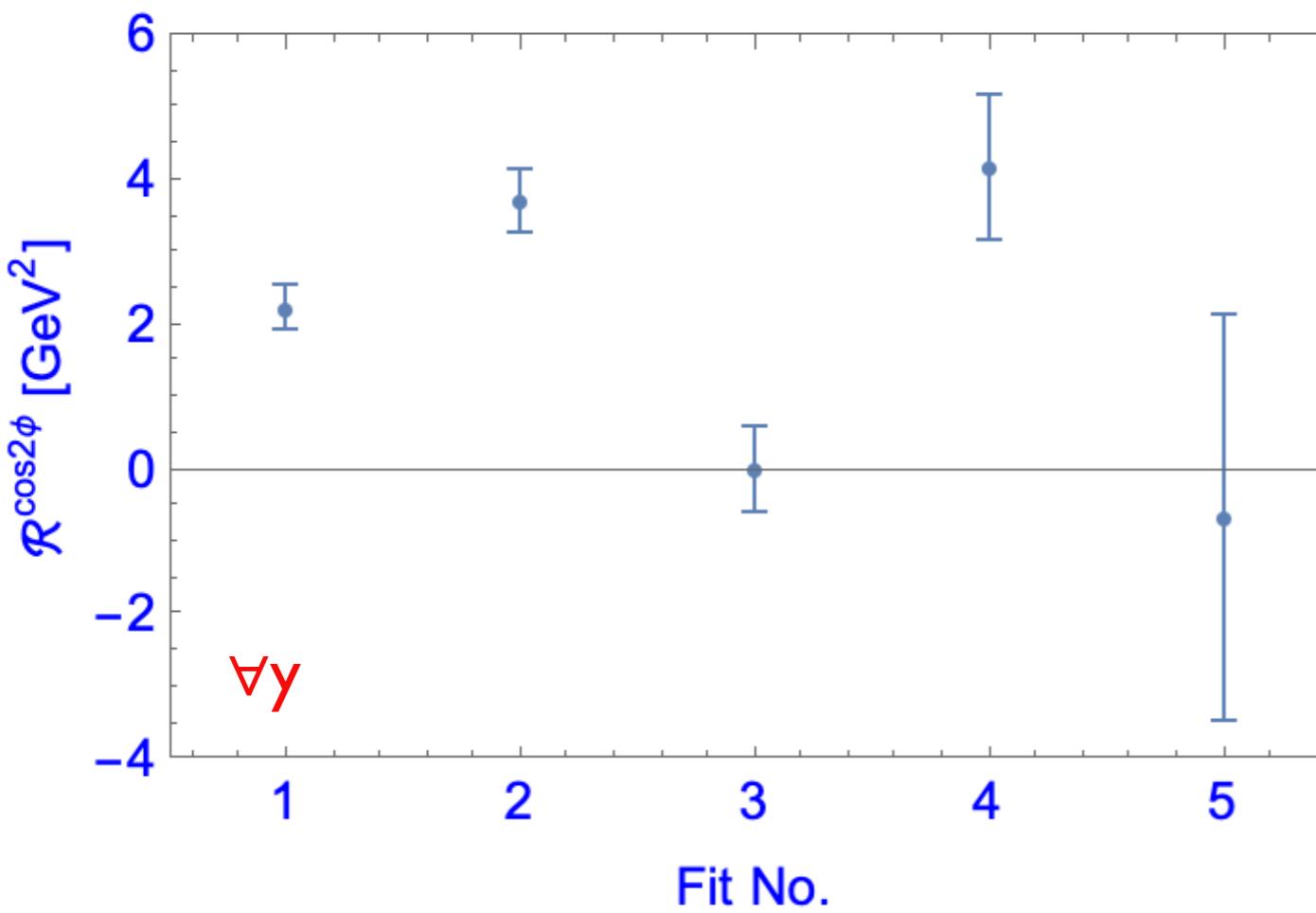
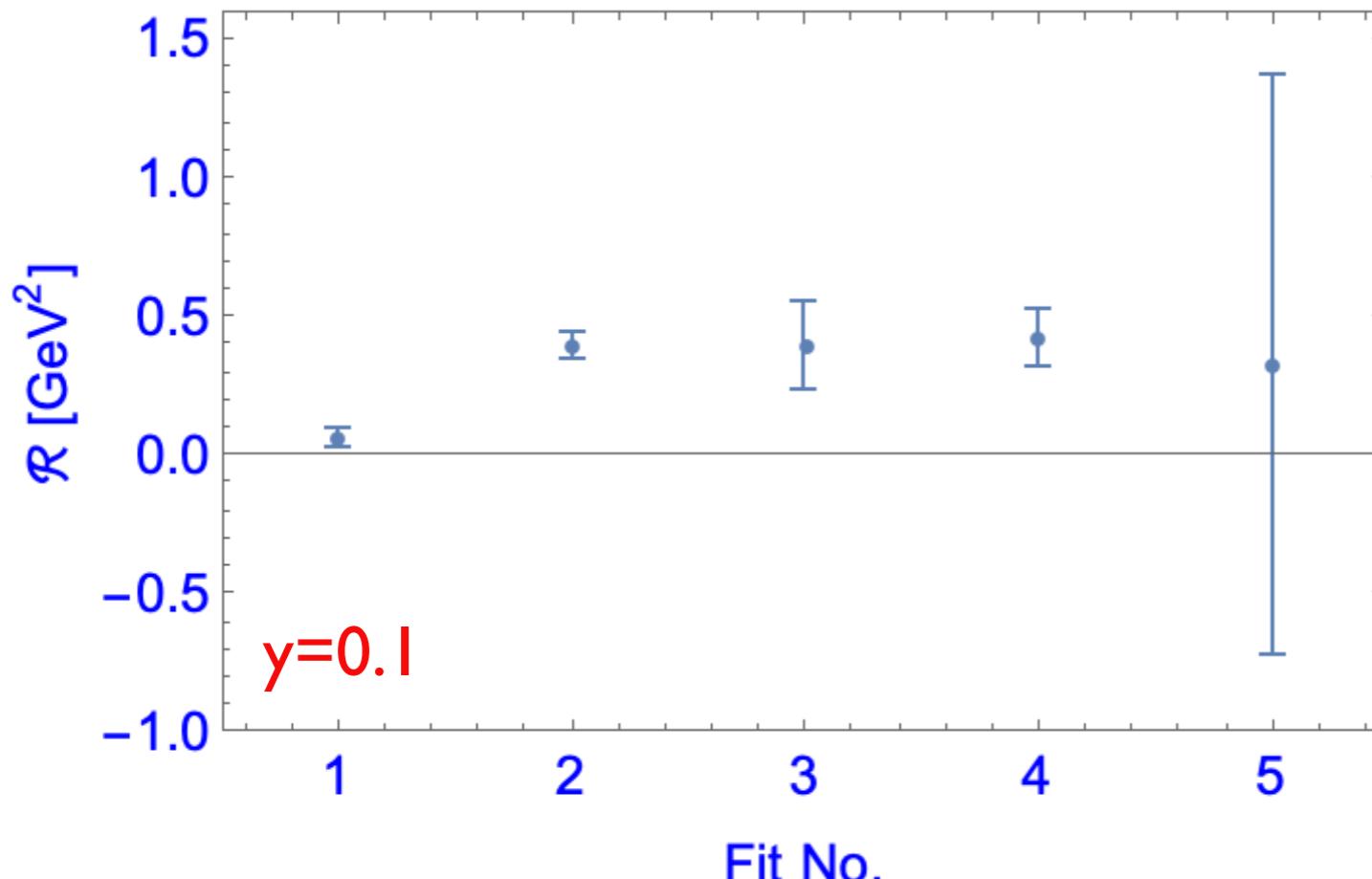
$$\mathcal{R}^{\cos 2\phi_T} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[\mathcal{O}_8^S - \frac{1}{M_Q^2} \mathcal{O}_8^P \right]$$

$$\mathcal{R} = \frac{27\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1-y)^2] \mathcal{O}_8^S + (10 - 10y + 3y^2) \mathcal{O}_8^P / M_Q^2}{26 - 26y + 9y^2}$$

$z = 1/2$

To avoid evolution we chose $K_\perp = Q = 2M_Q$

[Bacchetta, Boer, Pisano, Taels, 2018]

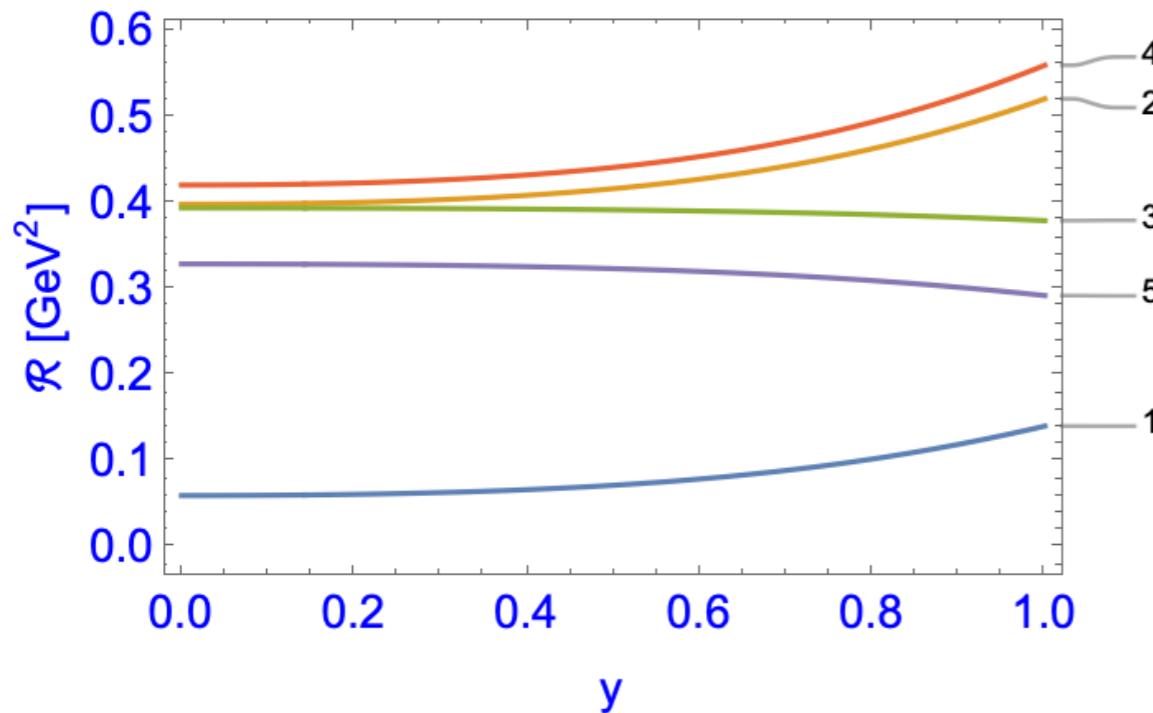


Ratios not normalized to
[0,1] for \mathcal{R} or [-1,1] for $\mathcal{R}^{\cos(2\phi)}$

$$\frac{\mathcal{R}^{\cos 2\phi_T}}{\mathcal{R}} = \frac{\langle \cos 2\phi_T \rangle_Q}{\langle \cos 2\phi_T \rangle_{Q\bar{Q}}}$$

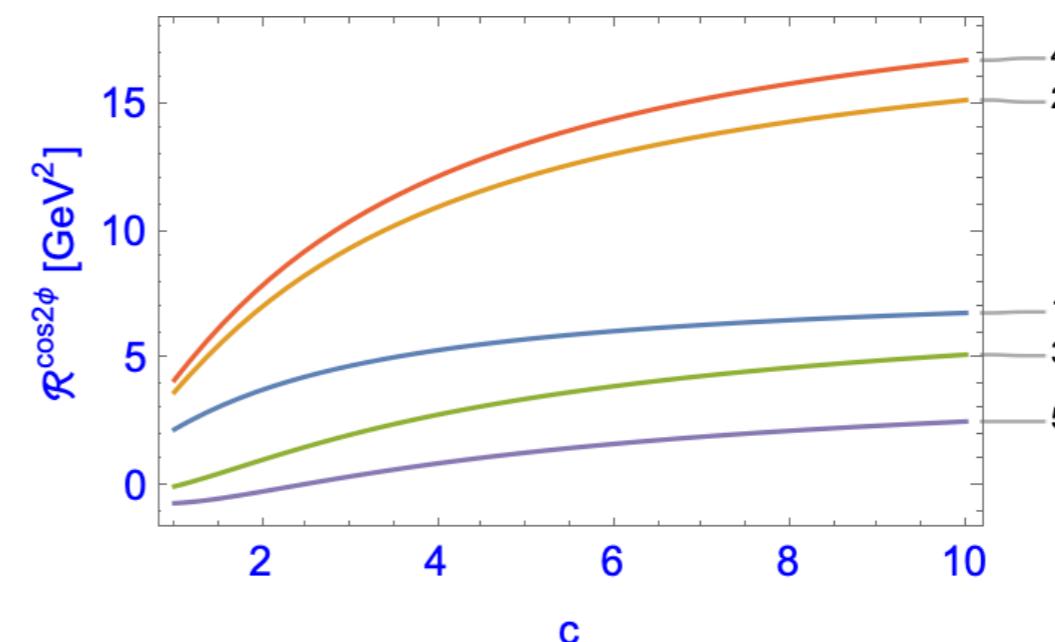
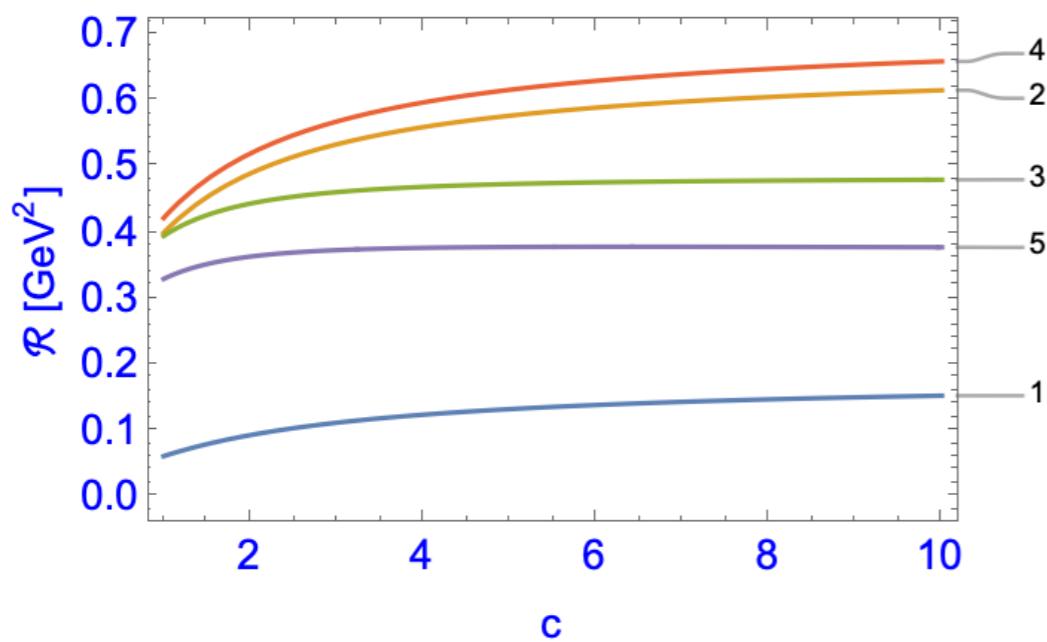
Based on fits the ratio of asymmetries could be anywhere between 0 and ∞

But rough average of the fits would indicate that the $\cos(2\phi_T)$ asymmetry could be of $\mathcal{O}(10)$ times larger in J/Ψ production



Central values shown only
 Possible correlations of errors not included
 Moderate y dependence
 Numerator and denominator of $\mathcal{R}^{\cos(2\phi)}$
 have prefactor $(1-y)$ so vanish at $y=1$

No projections for EIC available, i.e. there is no information on how well a $\cos(2\phi_T)$ asymmetry in J/ Ψ could be determined



Moderate $c=Q^2/(4M_Q^2)$ dependence, but both numerator and denominator fall off as $1/c^2$, therefore, larger statistical error at large c expected

Exploiting polarization

There are different equations for polarized quarkonium production that involve the same two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0 | \mathcal{O}_8^Q ({}^1 S_0) | 0 \rangle \quad \mathcal{O}_8^P \equiv \langle 0 | \mathcal{O}_8^Q ({}^3 P_0) | 0 \rangle$$

$$\mathcal{R}_L = \frac{9\pi^2}{4} \frac{1}{M_Q} \frac{[1+(1-y)^2]\mathcal{O}_8^S + 3(6-6y+y^2)\mathcal{O}_8^P/M_Q^2}{26 - 26y + 9y^2}$$

$$Q^2=4M_Q^2$$

$$\mathcal{R}_T = \frac{9\pi^2}{2} \frac{1}{M_Q} \frac{[1+(1-y)^2]\mathcal{O}_8^S + 3(2-2y+y^2)\mathcal{O}_8^P/M_Q^2}{26 - 26y + 9y^2}$$

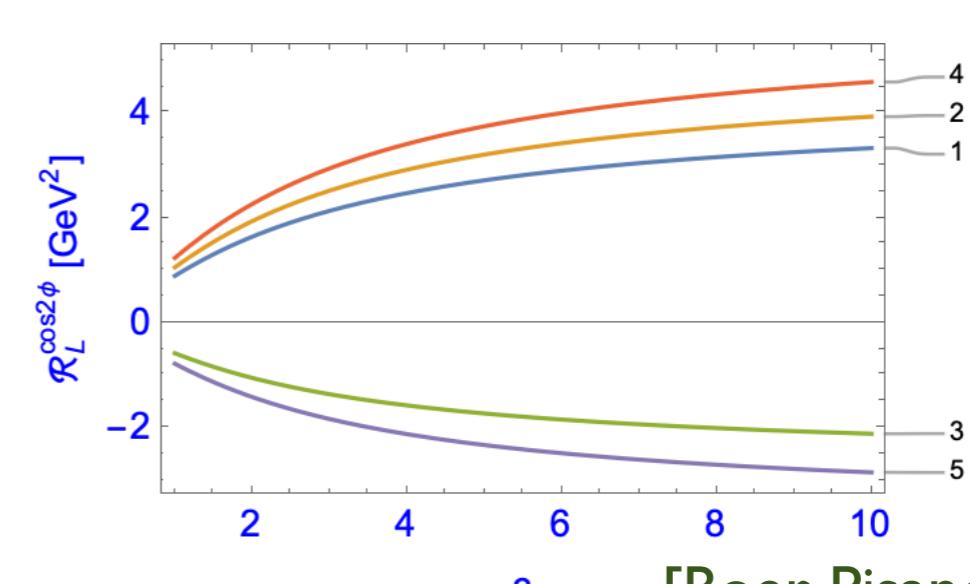
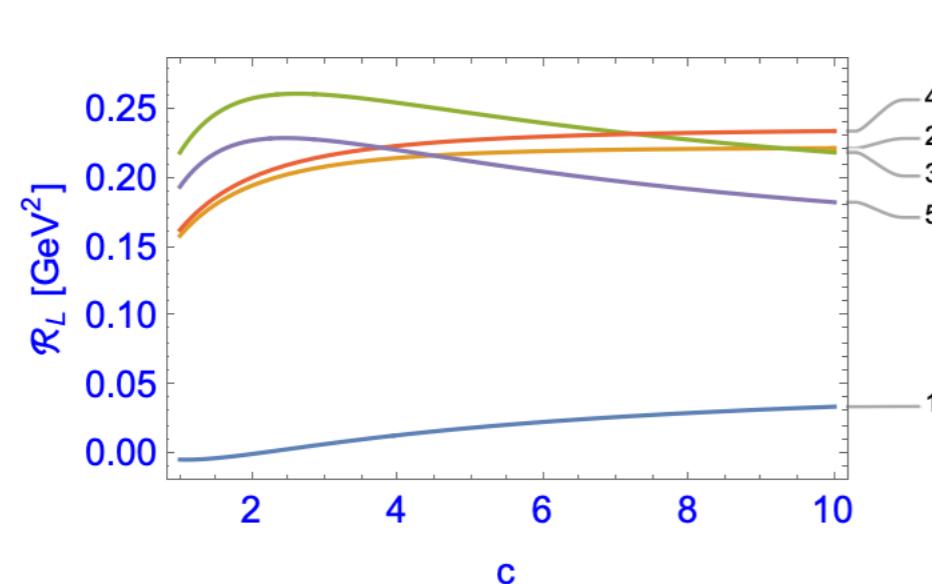
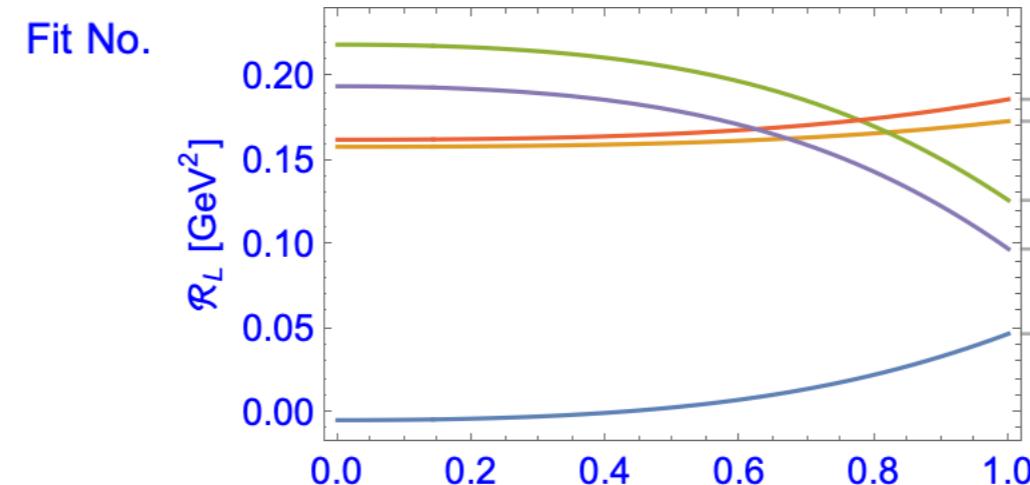
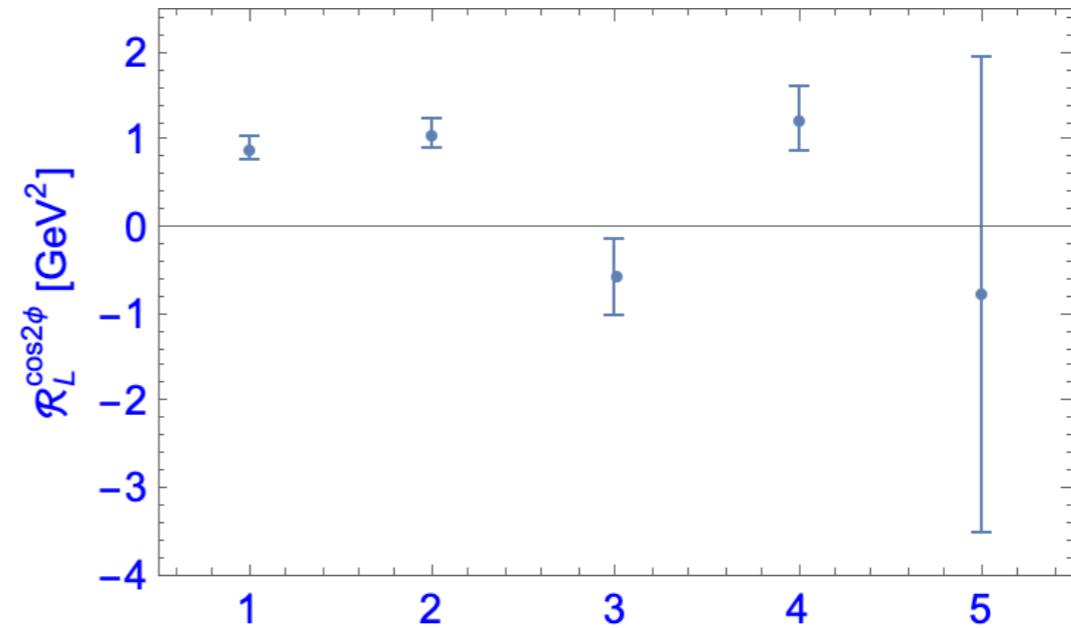
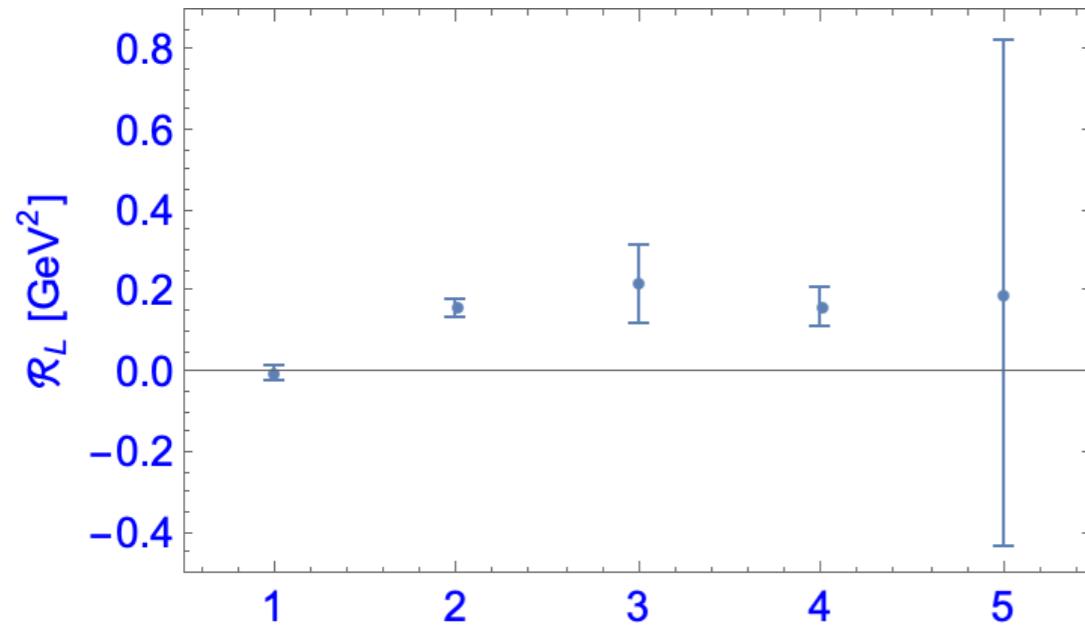
$$\mathcal{R}_L^{\cos 2\phi_T} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[\frac{1}{3} \mathcal{O}_8^S - \frac{1}{M_Q^2} \mathcal{O}_8^P \right] \quad \mathcal{R}_T^{\cos 2\phi_T} = \frac{9\pi^2}{2} \frac{1}{M_Q} \mathcal{O}_8^S$$

$$\mathcal{R}_L^{\cos 2\phi_T} + \mathcal{R}_T^{\cos 2\phi_T} = \mathcal{R}^{\cos 2\phi_T}$$
$$\mathcal{R}_L + \mathcal{R}_T = \mathcal{R}.$$

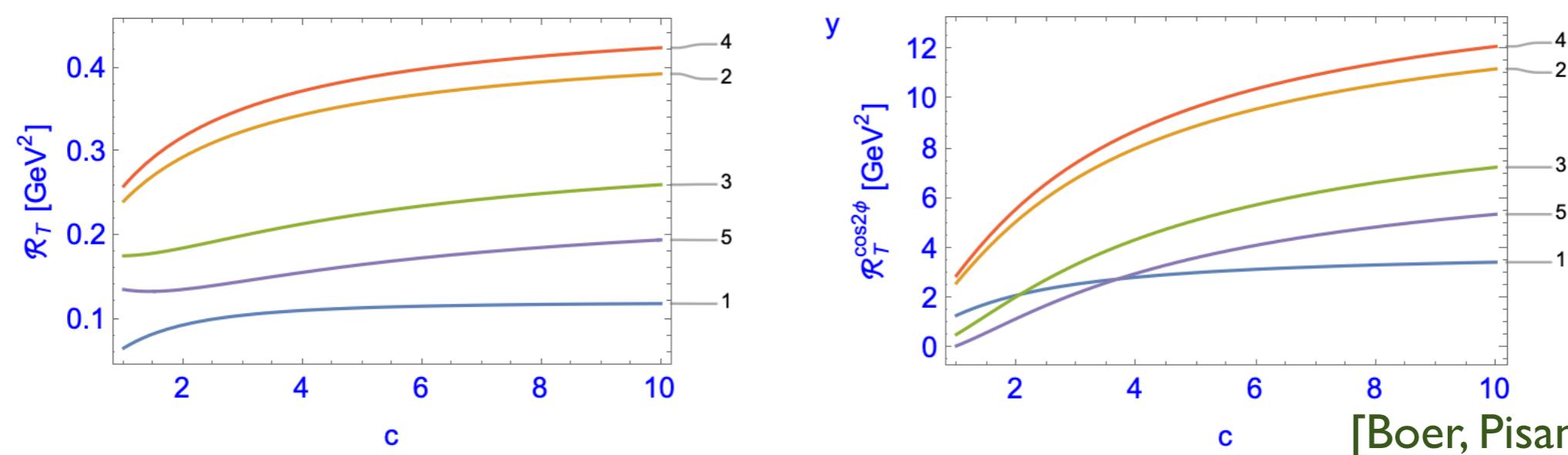
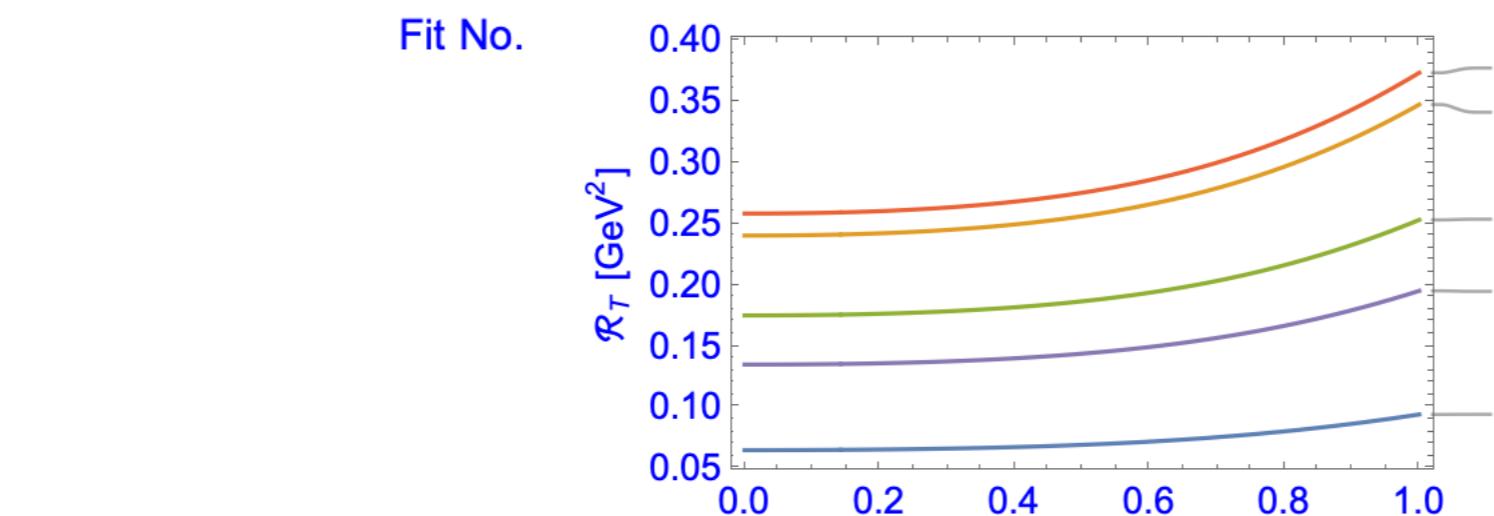
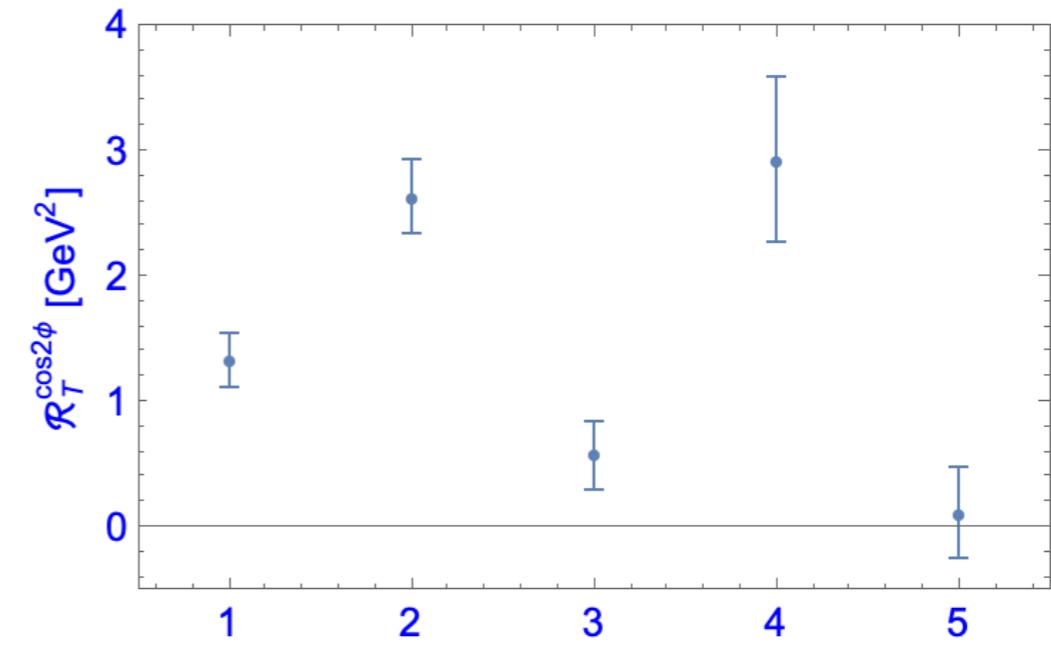
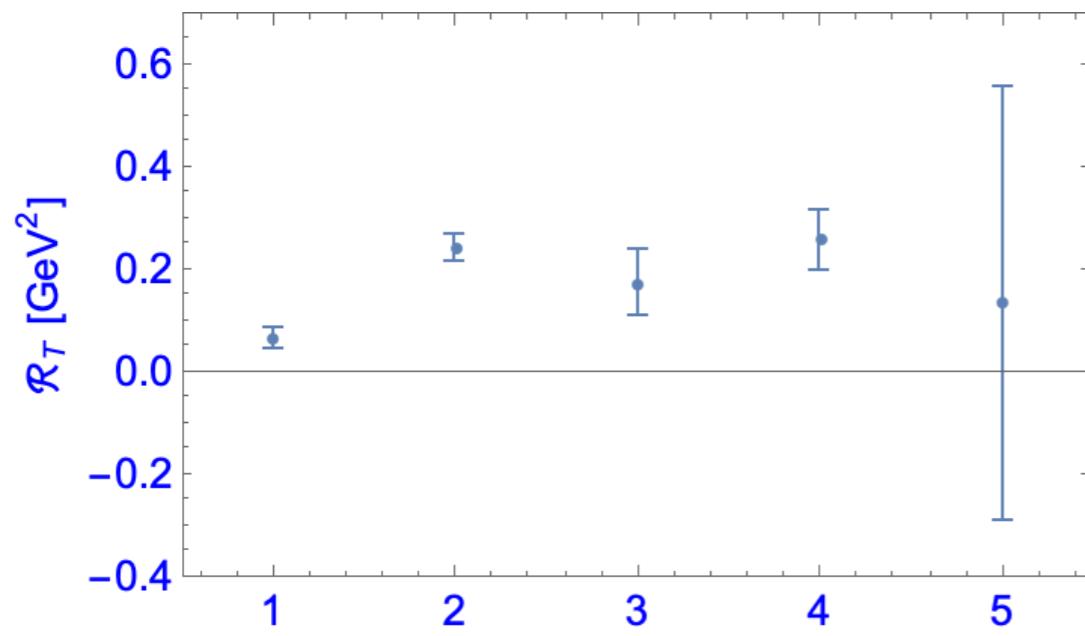
Overconstrained system allows to cross check the extraction and to estimate the uncertainty

[Bacchetta, Boer, Pisano, Taelis, 2018]

Longitudinal polarization



Transverse polarization



Effect of smearing

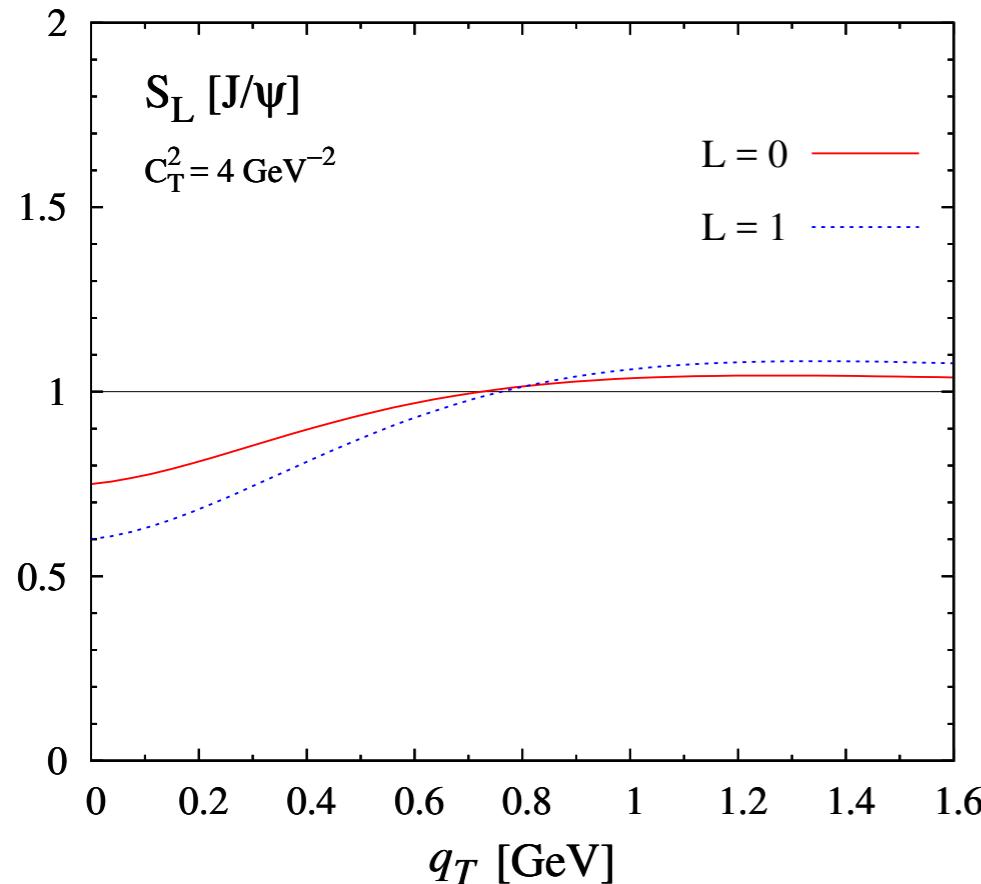
In reality the process of $Q\bar{Q} \rightarrow J/\Psi$ involves some k_T -smearing

The factorization involves still unknown “shape functions”

[Echevarria, 2019; Fleming, Makris & Mehen, 2019]

If L dependent this smearing would affect the extraction of CO LDMEs:

$$\mathcal{R} = \frac{27\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1 - y)^2] \mathcal{O}_8^S S_0(x, \mathbf{q}_T^2) + (10 - 10y + 3y^2) \mathcal{O}_8^P / M_Q^2 S_1(x, \mathbf{q}_T^2)}{26 - 26y + 9y^2}$$

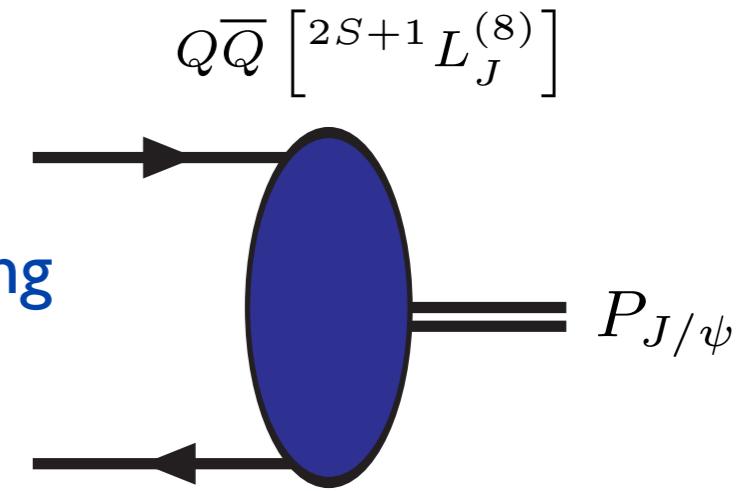


$$S_L(x, \mathbf{q}_T^2) = \frac{\mathcal{C}[f_1^g \Delta_L](x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

← Example study using positronium-like functions
[Bacchetta, Boer, Pisano, Taels, 2018]

Note that these are not the wave functions
of the quarkonia however ($\Delta \neq \Psi$)

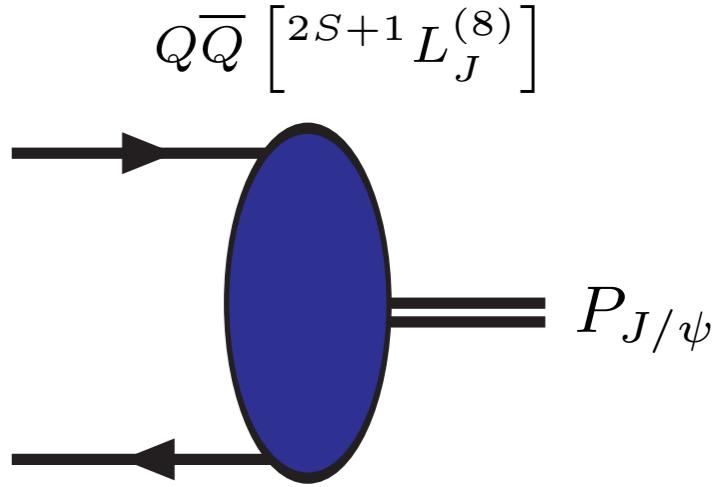
Perturbative tail is L independent
[Boer, D'Alesio, Murgia, Pisano, Taels, 2020]



Effect of smearing

$$\mathcal{R} = \frac{27\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1 - y)^2] \mathcal{O}_8^S S_0(x, \mathbf{q}_T^2) + (10 - 10y + 3y^2) \mathcal{O}_8^P / M_Q^2 S_1(x, \mathbf{q}_T^2)}{26 - 26y + 9y^2}$$

$$S_L(x, \mathbf{q}_T^2) = \frac{\mathcal{C}[f_1^g \Delta_L](x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$



To the best of our knowledge, no parametrization is so far available for the smearing functions Δ_L . Therefore we propose a model based on the properties of the radial wave function of the hydrogen atom in momentum space, namely:

- For large \mathbf{p}_T , Δ_L vary as $(\mathbf{p}_T^2)^{-(L+4)}$, with $L = 0, 1$, independently of the heavy quark mass.
- For small \mathbf{p}_T , Δ_L vary as $(\mathbf{p}_T^2)^L$, hence Δ_1 vanishes at $\mathbf{p}_T = 0$, while Δ_0 does not.

Furthermore, the normalization is fixed by imposing

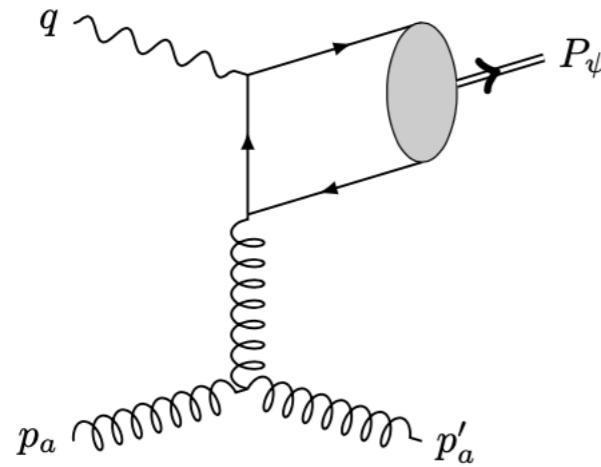
$$\int d^2 \mathbf{k}_T \Delta_L(\mathbf{k}_T^2) = 1. \quad (75)$$

Explicitly we have

$$\Delta_0(\mathbf{k}_T^2) = \frac{3C_T^2}{\pi} \frac{1}{(1 + \mathbf{k}_T^2 C_T^2)^4}, \quad \Delta_1(\mathbf{k}_T^2) = \frac{12C_T^4}{\pi} \frac{\mathbf{k}_T^2}{(1 + \mathbf{k}_T^2 C_T^2)^5}, \quad (76)$$

where C_T is taken to be independent of L and equal to the width of the TMD distribution in Eq. (74). This guarantees that the transverse momentum distribution for a heavier quarkonium state falls off less fast, reflecting its smaller spatial extent.

Matching high and low transverse momentum



Calculating such diagrams at high P_T and considering the low P_T limit leads to a $\text{Log}(P_T)$ behavior:

$$F_{UU,T} = \sigma_{UU,T} \left[L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) f_1^g(x, \mu^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, \mu^2) \right]$$

$$L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) \equiv 2C_A \ln \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) - \frac{11C_A - 4n_f T_R}{6}$$

This should match with the high P_T limit of the TMD contribution, but that can only work out after including a shape function Δ :

$$\mathcal{F}_{UU,T} = \frac{2\pi^2 \alpha_s e_c^2}{M_\psi(M_\psi^2 + Q^2)} \left[\langle 0 | \mathcal{O}(^1S_0^{[8]}) | 0 \rangle + 4 \frac{(7M_\psi^4 + 2M_\psi^2 Q^2 + 3Q^4)}{M_\psi^2 (M_\psi^2 + Q^2)^2} \langle 0 | \mathcal{O}(^3P_0^{[8]}) | 0 \rangle \right]$$

$$\times f_1^g(x, p_T^2) \Big|_{\mathbf{p}_T = \mathbf{q}_T}$$

$$\int d^2 p_T \int d^2 k_T \delta^2(\mathbf{q}_T - \mathbf{p}_T - \mathbf{k}_T) f_1^g(x, p_T^2; \mu^2) \Delta^{[n]}(\mathbf{k}_T^2, \mu^2)$$

Perturbative tail is L independent

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \langle 0 | \mathcal{O}(n) | 0 \rangle \delta^2(\mathbf{k}_T)$$

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \langle 0 | \mathcal{O}(n) | 0 \rangle \ln \frac{\mu^2}{\mathbf{k}_T^2}$$

[Boer, D'Alesio, Murgia, Pisano, Taels, 2020]