Transverse Momentum Dependent distributions and the Color Glass Condensate

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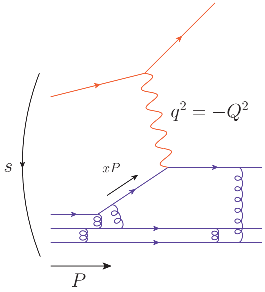
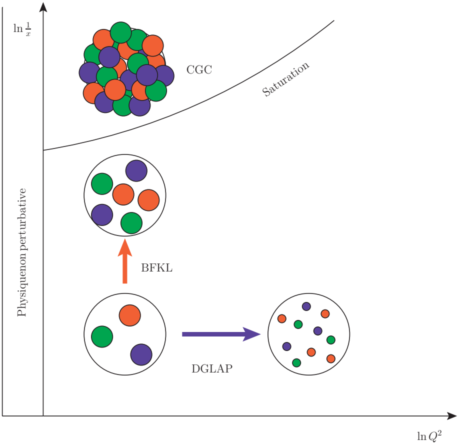
APS GHP Meeting 2021

Accessing the partonic content of hadrons with an electromagnetic probe

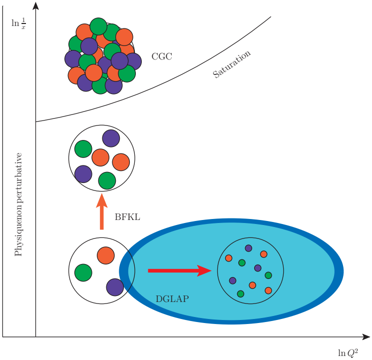


Electron-proton collision (parton model)

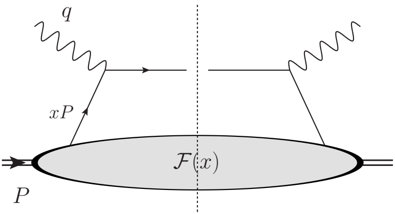


QCD at moderate $x_B = Q^2/s$

$$Q^2 \sim s$$



Collinear factorization:
inclusive processes with a single scale $Q \sim \sqrt{s} \gg \Lambda_{QCD}$



$$\sigma = \mathcal{F}(x, \mu) \otimes \mathcal{H}(\mu)$$

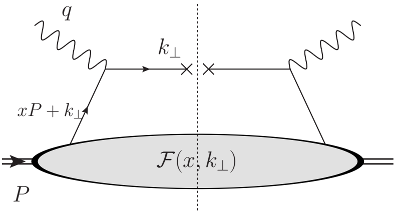
At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A Parton Distribution Function (PDF) $\mathcal{F}(x, \mu)$

μ independence: DGLAP renormalization equation for $\mathcal{F}$

Transverse Momentum Dependent (TMD) factorization: semi-inclusive processes with one hard and one semihard scale

$$Q \sim \sqrt{s} \gg k_\perp$$



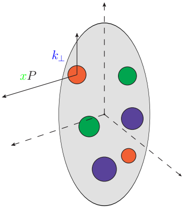
$$\sigma = \mathcal{F}(x, k_\perp, \zeta, \mu) \otimes \mathcal{H}(\mu) \otimes \hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$$

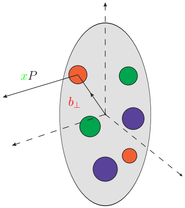
At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A TMD PDF $\mathcal{F}(x, k_\perp, \zeta, \mu)$
- A TMD FF $\hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$

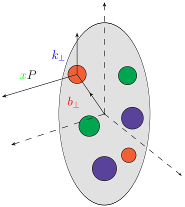
$\mu, \zeta, \hat{\zeta}$ independence: TMD evolution for $\mathcal{F}, \hat{\mathcal{F}}$

TMD



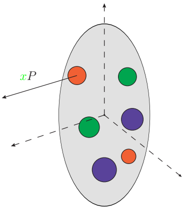


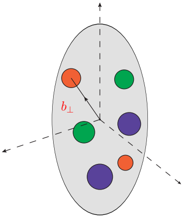
GPD



Wigner

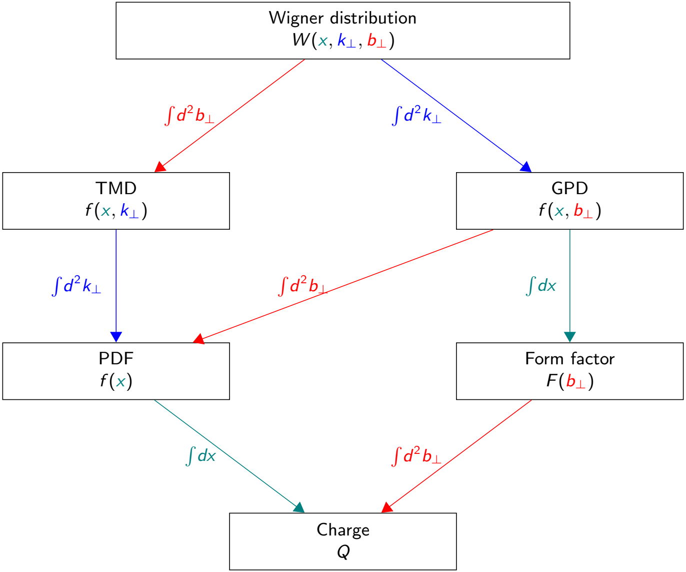
PDF





Form Factor

The family tree of parton distributions



Leading twist quark TMD distributions

Hadron pol. \ Parton	Unpolarized	Chiral	Transverse
Unpolarized	f_1	$\emptyset$	$h_1^\perp$
Longitudinal	$\emptyset$	g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

PDF-spanning

- Unpolarized f_1
- Helicity g_{1L}
- Transversity h_1

Naive T-even pure TMDs

- Worm-gear $h_{1L}^\perp, g_{1T}$
- Pretzelosity $h_{1T}^\perp$
- [Mulders, Tangerman]

Naive T-odd pure TMDs

- Boer-Mulders $h_1^\perp$
- Sivers $f_{1T}^\perp$

Leading twist gluon TMD distributions

Hadron pol. \ Parton	Unpolarized	Circular	Linear
Unpolarized	f_1^g	$\emptyset$	$h_1^{\perp g}$
Longitudinal	$\emptyset$	g_{1L}^g	$h_{1L}^{\perp g}$
Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

PDF-spanning

- Unpolarized f_1^g
- Helicity g_{1L}^g

Naive T-even pure TMDs

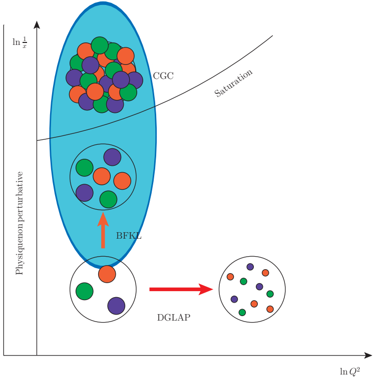
- Worm-gear $h_{1L}^{\perp g}, g_{1T}^g$
- Pretzelosity $h_{1T}^{\perp g}$
- Transversity h_1^g

Naive T-odd pure TMDs

- Boer-Mulders $h_1^{\perp g}$
- Sivers $f_{1T}^{\perp g}$

QCD at small $x_B = Q^2/s$

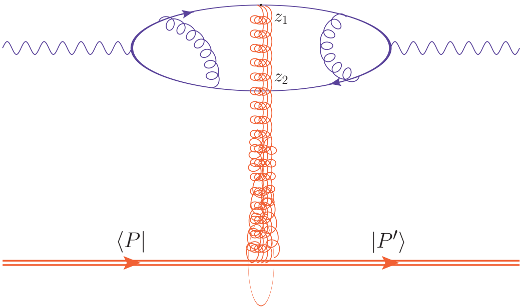
$$Q^2 \ll s$$



Factorized picture

Semi-classical approach to small x physics

[McLerran, Venugopalan], [Balitsky]



$$\mathcal{S} = \int d^2 z_{1\perp} d^2 z_{2\perp}\, \Phi^{Y_c}(z_{1\perp}, z_{2\perp})\, \langle P'|[\mathrm{Tr}(U_{z_{1\perp}}^{Y_c} U_{z_{2\perp}}^{Y_c \dagger}) - N_c]|P\rangle$$

Written similarly for any number of Wilson lines in any color representation!

Y_c independence: BK-JIMWLK hierarchy of equations
[Balitsky, Kovchegov, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

(So-called) non-universality of TMD distributions:
The importance of gauge links

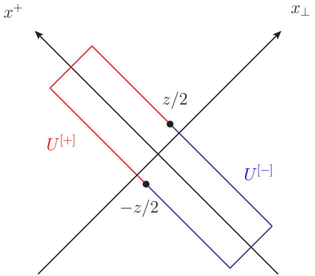
[Collins, Soper, Sterman], [Brodsky, Hwang, Schmidt], [Belitsky, Ji, Yuan], [Bomhof, Mulders, Pijlman], [Boer, Mulders, Pijlman]

[Kharzeev, Kovchegov, Tuchin]

TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



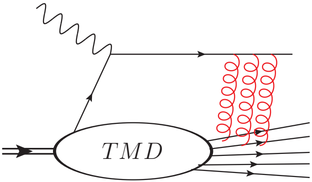
$$q^{[+]}(x,k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

$$q^{[-]}(x,k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

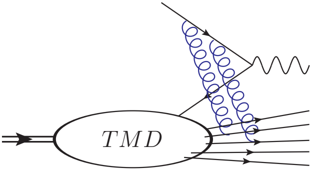
For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers sign change

The Sivers effect

SIDIS



Drell-Yan



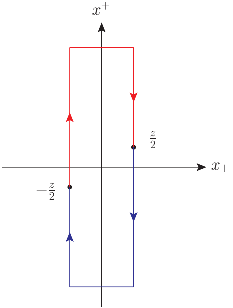
Final state interactions: $q^{[+]}$

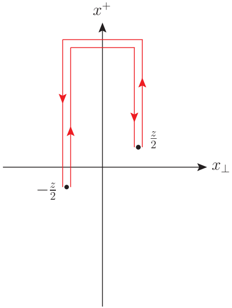
Initial state interactions: $q^{[-]}$

The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions



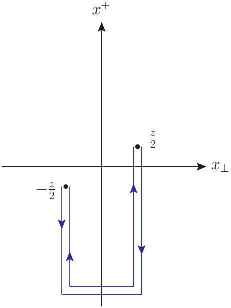


$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

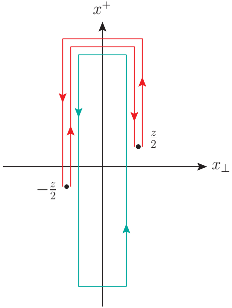
$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

TMD gauge links

"Non-universality" of gluon TMD distributions



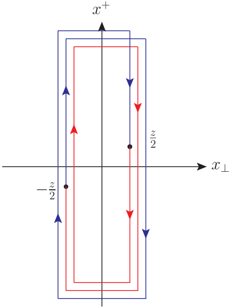
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[-]\dagger}F^{i-}\mathcal{U}^{[-]}\right]$$



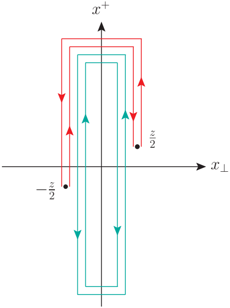
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]$$

TMD gauge links

"Non-universality" of gluon TMD distributions



$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]$$

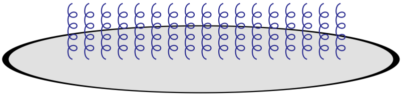


$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]$$

TMD distributions from semiclassical small x physics

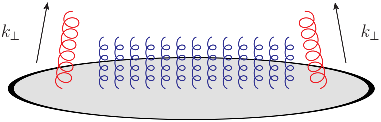
From the CGC to a TMD

From Wilson lines...



$$\left\langle P \left| \mathrm{Tr}\left(U_{\frac{r}{2}} U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

To a parton distribution



$$\left\langle P \left| \mathrm{Tr}\left(\partial^i U_{\frac{r}{2}} \partial^i U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

From the CGC to a TMD

Staple gauge links from a Wilson line operator

[Dominguez, Marquet, Xiao, Yuan]

Consider the derivative of a path-ordered Wilson line, denoting

$$[x_1^+, x_2^+]_{x_\perp} \equiv \mathcal{P} \exp \left[ig \int_{x_1^+}^{x_2^+} dx^+ b^-(x^+, x_\perp) \right]$$

For a given shockwave operator $U_{x_\perp} = [-\infty, +\infty]_{x_\perp}$

$$\partial^i U_{x_\perp} = ig \int dx^+ [-\infty, x^+]_{x_\perp} F^{-i}(x^+, x_\perp) [x^+, +\infty]_{x_\perp}$$

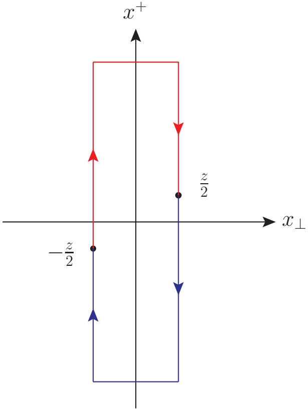
$$\partial^j U^\dagger_{x_\perp} = -ig \int dx^+ [+\infty, x^+]_{x_\perp} F^{-j}(x^+, x_\perp) [x^+, -\infty]_{x_\perp}$$

$$(\partial^i U^\dagger_{x_\perp}) U_{x_\perp} = -ig \int dx^+ [+\infty, x^+]_{x_\perp} F^{-i}(x^+, x_\perp) [x^+, +\infty]_{x_\perp}$$

Taking the derivative of a shockwave operator allows to extract a physical gluon

From the CGC to a TMD

The dipole TMD

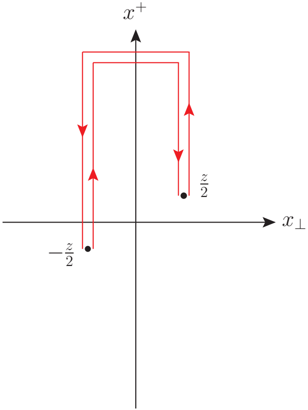


$$\mathcal{F}_{qg}^{(1)}(x, k_\perp) \propto \int d^4z\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]\right| P\right\rangle$$

$$\rightarrow \int d^2z_\perp e^{i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U^\dagger_{\frac{z}{2}}\right)\left(\partial^i U_{-\frac{z}{2}}\right)\right]\right| P\right\rangle$$

From the CGC to a TMD

The Weizsäcker-Williams TMD



$$\mathcal{F}_{gg}^{(3)}(x, k_\perp) \propto \int d^4z\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right) \mathcal{U}^{[+]\dagger} F^{i-}\left(-\frac{z}{2}\right) \mathcal{U}^{[+]} \right] \right| P \right\rangle$$

$$\rightarrow \int dz_\perp e^{i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U_{\frac{z}{2}}\right) U^\dagger_{-\frac{z}{2}} \left(\partial^i U_{-\frac{z}{2}}\right) U^\dagger_{\frac{z}{2}} \right] \right| P \right\rangle$$

CGC amplitudes and TMD amplitudes

Small dipole "correlation" expansion

[Dominguez, Marquet, Xiao, Yuan]

Taylor expansion of Wilson line operators

$$U_{\mathrm{b}+\frac{\mathrm{r}}{2}} U^{\dagger}_{\mathrm{b}-\frac{\mathrm{r}}{2}} - 1 = \frac{\mathrm{r}^i}{2}\left[\left(\partial^i U_{\mathrm{b}}\right) U^{\dagger}_{\mathrm{b}} - U_{\mathrm{b}}\left(\partial^i U^{\dagger}_{\mathrm{b}}\right)\right] + O\left(\mathrm{r}^2\right)$$

leading twist correspondence:

CGC in the "correlation" limit = TMD in the small x limit

Beyond the correlation limit

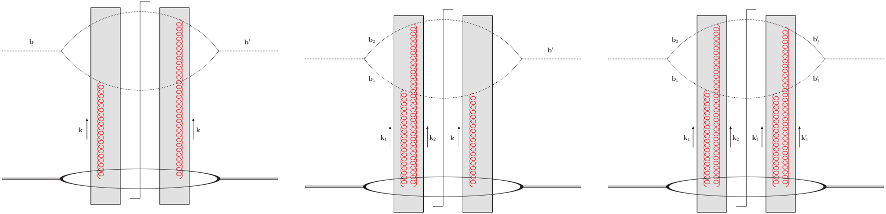
[Altinoluk, RB, Kotko], [Altinoluk, RB]

CGC = infinite twist TMD in the small x limit

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]

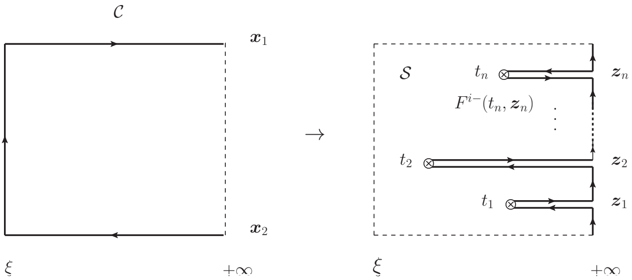


$$
\begin{aligned}
\sigma =\; & \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \middle| F^{-i} W F^{-j} W \middle| P \right\rangle \\
& + \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W F^{-k} W \middle| P \right\rangle \\
& + \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \middle| P \right\rangle
\end{aligned}
$$

The Wilson line ↔ parton distribution equivalence

Most general equivalence: use the Non-Abelian Stokes theorem

[RB, Mehtar-Tani]

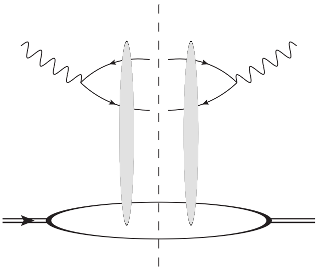


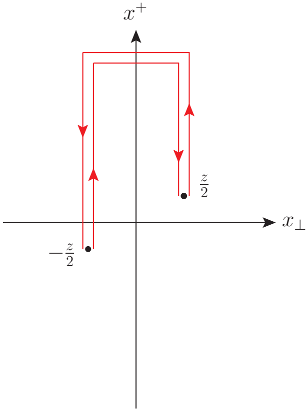
$$\mathcal{P}\exp\left[\oint_{\mathcal{C}} dx_\mu A^\mu(x)\right] = \mathcal{P}\exp\left[\int_{\mathcal{S}} d\sigma_{\mu\nu}\, U F^{\mu\nu} U^\dagger\right]$$

$$U_{x_{1\perp}} U^\dagger_{x_{2\perp}} = [\hat{x}_{1\perp}, \hat{x}_{2\perp}]$$

Dijet electro- or photoproduction

Weizsäcker-Williams TMD

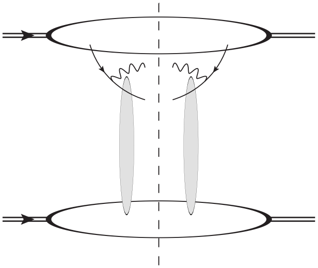


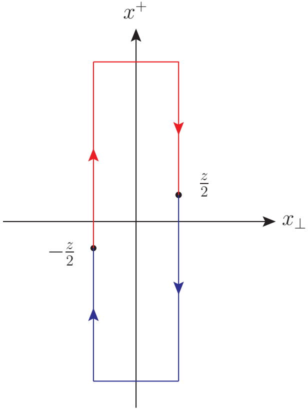


$$\mathcal{F}_{gg}^{(3)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{Tr}(\partial^i U_{\frac{z}{2}}^\dagger) U_{\frac{z}{2}} (\partial^i U_{-\frac{z}{2}}^\dagger) U_{-\frac{z}{2}} | P \rangle$$

Jet+photon production in pA collisions

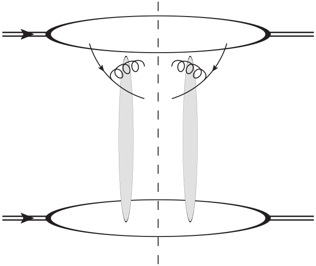
Dipole TMD

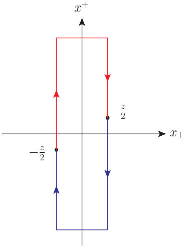




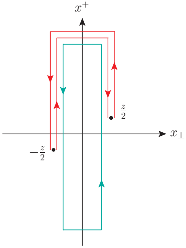
$$\mathcal{F}_{gg}^{(1)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{tr}(\partial^i U_{\frac{z}{2}})(\partial^i U^\dagger_{-\frac{z}{2}}) | P \rangle$$

Forward dijet production in pA collisions



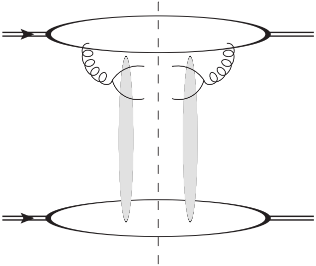


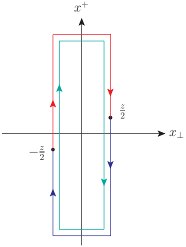
$\mathcal{F}_{qg}^{(1)}$



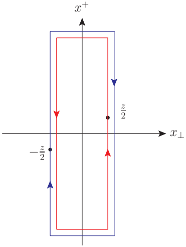
$\mathcal{F}_{qg}^{(2)}$

Forward dijet production in pA collisions

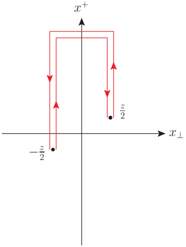




$\mathcal{F}_{gg}^{(1)}$

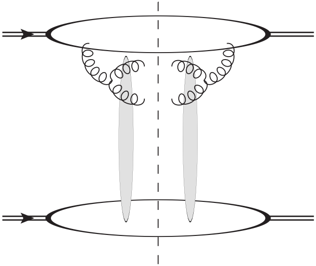


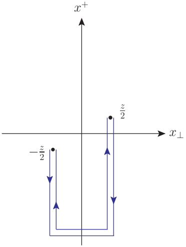
$\mathcal{F}_{gg}^{(2)}$



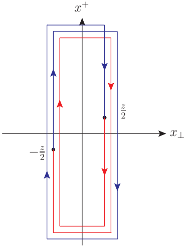
$\mathcal{F}_{gg}^{(3)}$

Forward dijet production in pA collisions

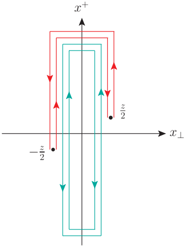




$\mathcal{F}_{gg}^{(4)}$



$\mathcal{F}_{gg}^{(5)}$



$\mathcal{F}_{gg}^{(6)}$

Common tools

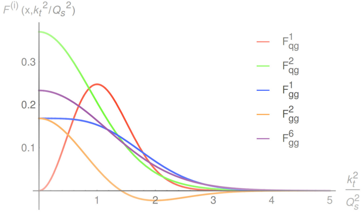
The CGC/TMD equivalence allows to use some TMD tools for the CGC:

- Target Sudakov log resummation for small x processes [Mueller, Xiao, Yuan], [Xiao, Yuan, Zhou]
- Phenomenological Sudakov log simulation [Kotko, Kutak, Sapeta, Stasto, Strikman], [Van Hameren, Kotko, Kutak, Sapeta]

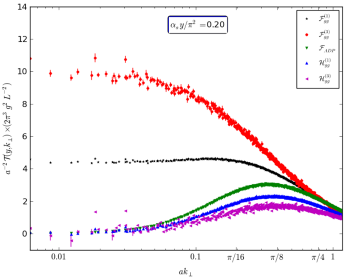
and some CGC tools for small x TMD distributions:

- Golec-Biernat Wüsthoff model for a TMD [Van Hameren, Kotko, Kutak, Marquet, Petreska, Sapeta]
- McLerran-Venugopalan model for a TMD, with JIMWLK evolution [Marquet, Petreska, Roiesnel], [Marquet, Petreska, Taels]

TMD from the CGC



TMD in the GBW model
[Van Hameren, Kotko, Kutak, Marquet, Petreska, Sapeta]



TMD in the MV model with JIMWLK evolution
[Marquet, Petreska, Roiesnel], [Marquet, Roiesnel, Taels]

Linearly polarized gluons in the CGC

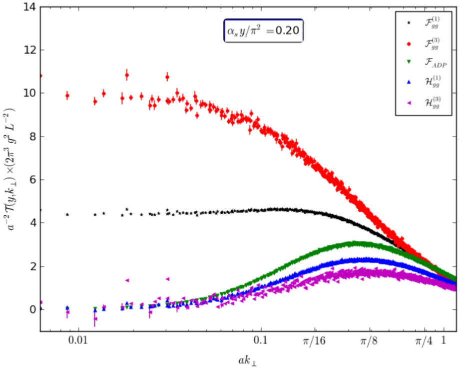
Polarized TMD in the CGC

Wilson line operators also contain linearly polarized gluon TMDs

$$\left\langle P \left| F^{i-} W F^{j-} W \right| P \right\rangle \to \frac{\delta^{ij}}{2} \mathcal{F}(k_\perp) + \left(\frac{k_\perp^i k_\perp^j}{k_\perp^2} - \frac{\delta^{ij}}{2} \right) \mathcal{H}(k_\perp)$$

- They can be computed in the MV model [Metz, Zhou]
- They can be observed in processes with massive quarks [Marquet, Roiesnel, Taels]
- Or in processes with 3 body final states (requires an extension of the notion of the correlation limit) [Altinoluk, RB, Marquet, Taels]
- Can also be seen from loop corrections to 2-body observables, for example prompt photon+jet production in *pA* collisions [Benić, Dumitru], based on a computation by [Benić, Fukushima, Garcia-Montero, Venugopalan]

Polarized TMD in the CGC



In the large $k_\perp \sim Q$ limit (BFKL limit), all TMDs are equal:

$$\mathcal{F}(k_\perp) = \mathcal{H}(k_\perp), \text{ then } \left\langle P \left| \partial^i U \partial^j U \right| P \right\rangle \to \frac{k_\perp^i k_\perp^j}{k_\perp^2} \mathcal{F}(k_\perp)$$

We can recognize the so-called *non-sense polarization* in lightcone gauge: $\frac{k_\perp^i}{|k_\perp|}$.

BFKL contains as many linearly polarized gluon pairs as unpolarized ones. At large $k_\perp$, the CGC is maximally polarized [Boer, Mulders, Zhou, Zhou]

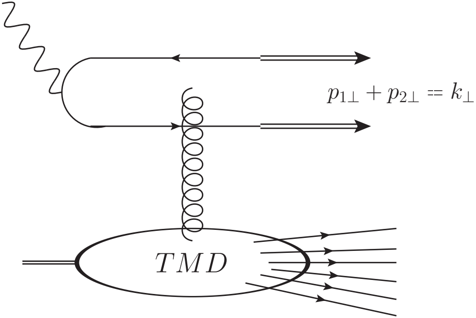
Azimuthal harmonics from TMDs

Azimuthal harmonics in inclusive processes can arise from polarized TMDs

[Boer, Mulders, Pisano], [Metz, Zhou], [Dominguez, Qiu, Xiao, Yuan] , [Dumitru, Skokov]
[RB, Mäntysaari, Salazar, Schenke, *in progress*]

$$\left\langle P \left| F^{-i}(z) W^{[\pm]}_{z,0} F^{-j}(0) W^{[\pm]}_{z,0} \right| P \right\rangle_{z^-=0} \to \frac{\delta^{ij}}{2}\mathcal{F}(k_\perp) + \left(\frac{k^i k^j}{k^2} - \frac{\delta^{ij}}{2}\right)\mathcal{H}(k_\perp)$$

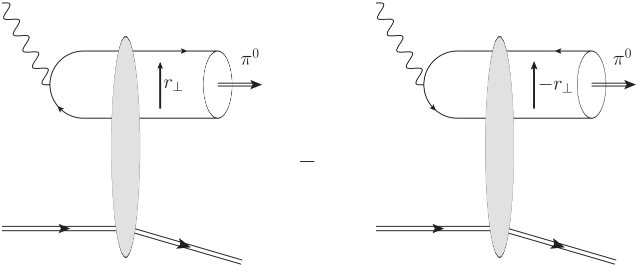
$$\langle P | FWFW | P \rangle \times \mathcal{H} \Rightarrow v_0 \mathcal{F}(k_\perp) + v_2 \cos(2\phi) \mathcal{H}(k_\perp)$$



Proton spin physics with unpolarized proton beams

DVMP and the Odderon(s)

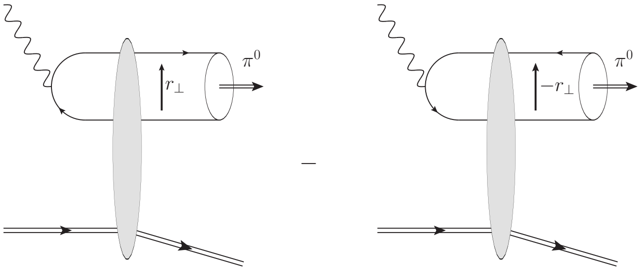
Odderon exchange: C even meson production



$$\frac{1}{2}\left[\mathrm{Tr}\left(U_{b+\frac{r}{2}}U^\dagger_{b-\frac{r}{2}}\right) - \mathrm{Tr}\left(U_{b-\frac{r}{2}}U^\dagger_{b+\frac{r}{2}}\right)\right]$$

DVMP and the Odderon(s)

Odderon exchange: C even meson production



Forward limit of the dipole-type Odderon GTMD = Sivers TMD: $xf_{1T}^{\perp}(x, \boldsymbol{k}^2)$

$$\int d^2\boldsymbol{v}\, e^{-i(\boldsymbol{k}\cdot\boldsymbol{v})}\boldsymbol{k}^2\langle P', S'|\mathcal{O}(\boldsymbol{v})|P, S\rangle$$
$$= -\frac{g_s^2}{4}N_c(2\pi)^2\delta(P'^{+} - P^{+})\frac{\boldsymbol{k}^j}{M}\left(\bar{u}_{P',S'}\sigma^{+j}u_{P,S}\right) xf_{1T}^{\perp}(x, \boldsymbol{k}^2).$$

Probing the Sivers function

Thanks to the Odderon/GTMD equivalence, the cross section for exclusive π^0 electroproduction at small x and small t with unpolarized lepton and proton beams is a direct probe for the gluon Sivers function

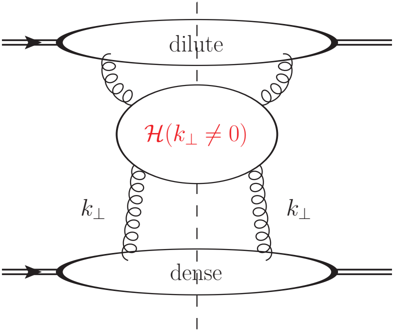
$$\frac{d\sigma}{d\xi dQ^2 d\left|t\right|} \simeq (2\pi)^3 \frac{\alpha_{\rm em}^2 \alpha_s^2 f_\pi^2}{8\xi N_c M^2 Q^2}(1-y+\frac{y^2}{2}) \times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}Q^2} \int d\boldsymbol{k}^2 \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + z\bar{z}Q^2} x f_{1T}^{\perp}(x, \boldsymbol{k}^2)\right]^2 .$$

We can thus understand the gluonic content of the transversely polarized protons without polarizing the proton beam.

Saturation in terms of TMD distributions

Small x improved TMD framework (iTMD)

A hybrid framework with off-shell gluons from the target
[Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren]

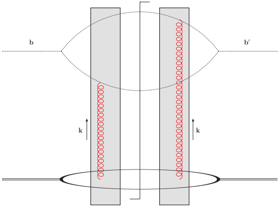


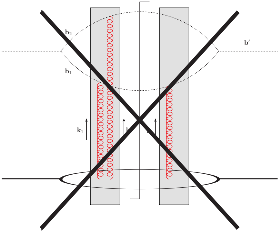
- QCD gauge invariance for multileg amplitudes with an off-shell leg restored with target counterterms [Kotko]
- TMD gauge links are built from the [Bomhof, Mulders, Pijlman] techniques
- Eventually, looks like BFKL, but with distinct TMD distributions for different color flow structures. Interpolates between the TMD regime $|k_\perp| \ll Q$ and the BFKL regime $|k_\perp| \sim Q$

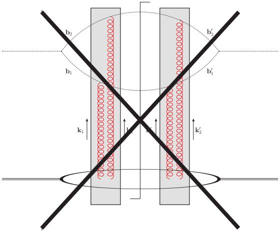
Inclusive low x cross section

Inclusive low x cross section + WW = iTMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]







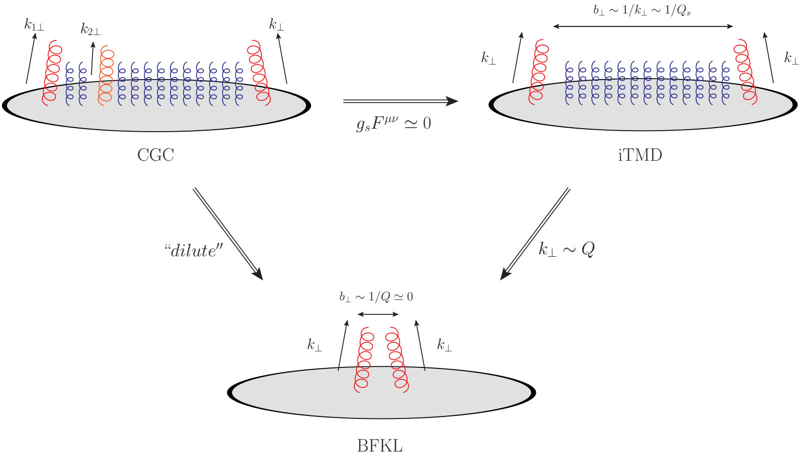
$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \middle| F^{-i} W F^{-j} W \middle| P \right\rangle$$

$$+\mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W F^{-k} W \middle| P \right\rangle$$

$$+\mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \middle| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \middle| P \right\rangle$$

The dilute limit

The dilute limit in terms of TMD distributions



higher kinematic twists $\propto |k_\perp|/Q$
vs genuine higher twists $\propto Q_s/Q$

Kinematic saturation vs genuine saturation

Correlations of semi-inclusive processes with 2 fully reconstructed outgoing final states in the forward region will give insight on which effects are kinematic ($|k_\perp|/Q$) and which effects are density effects (Q_s/Q)

- Forward quark dijet production in *pp* and *pA*
  [Fujii, Marquet, Watanabe]
- Lepto- and hadro- production of a heavy quark pair
  [Altinoluk, Marquet, Taels]
- Electroproduction of a dijet
  [RB, Mäntysaari, Salazar, Schenke, *work in progress*]

Conclusions

- TMD distributions are what allows to match standard parton distributions and semi-classical descriptions of small x physics
- Color Glass Condensate models can give insights on TMDs at small x
- The reformulation of the CGC in terms of TMD distributions allows to access transverse spin physics in the CGC
- Two distinct kinds of multiple scattering effects must be distinguished and measured separately to understand gluonic saturation