Color charge correlations in the proton on the light front

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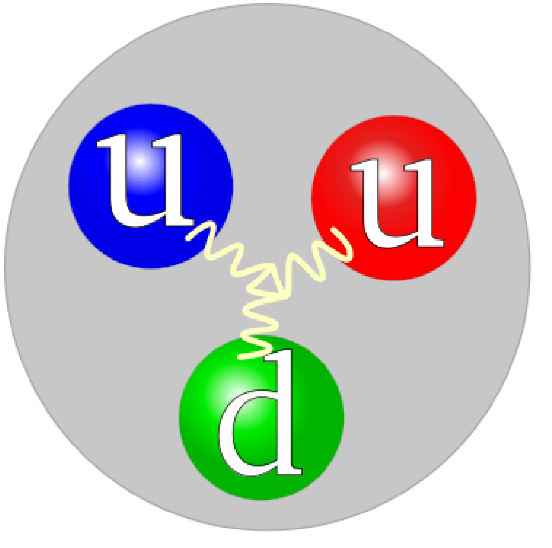
* Color charge correlators (and dipole scattering amplitudes) from the LFwf

wikipedia



What's that ?

talk based on collaborations with
H. Mäntysaari, R. Paatelainen: 2103.11682
R. Paatelainen: 2010.11245
T. Stebel, V. Skokov: 2001.04516
T. Stebel: 1903.07660
G. Miller, R. Venugopalan: 1808.02501

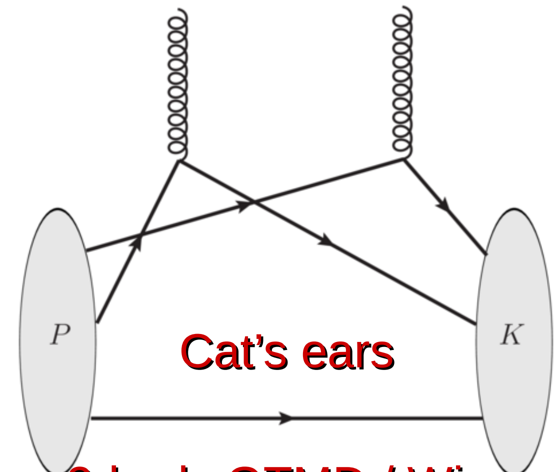
MV-like charge correlators :

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\right\rangle = g^2\mu^2\,\delta^{ab}\,(2\pi)^2\,\delta(\vec{q}_1+\vec{q}_2)\,,\qquad \left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\right\rangle = 0$$

(note: μ^2 a positive constant)

“impact parameter dependent” MV:

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\right\rangle = g^2\mu^2\,\delta^{ab}\int d^2b\,e^{-i(\vec{q}_1+\vec{q}_2)\cdot\vec{b}}\,T_p(b)$$

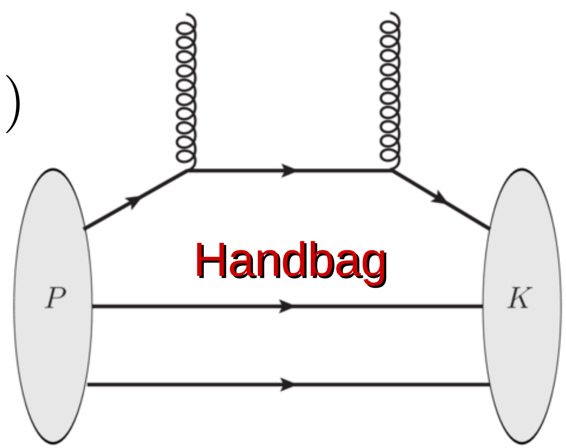


2-body GTMD / Wigner

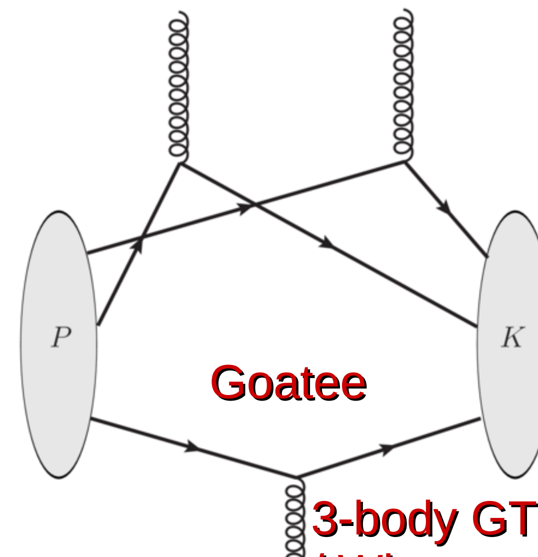
proton :

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\right\rangle \sim g^2\,\delta^{ab}\,G_2(\vec{q}_1,\vec{q}_2)$$

LO diagrams:



1-body GTMD / Wigner



3-body GTMD / Wigner

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\right\rangle_{C=-} \sim g^3\,d^{abc}\,G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\right\rangle_{C=+} \sim i g^3\,f^{abc}\,G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

Motivation :

1) Two- (and three-) body correlations in the proton on the light cone at x ~ 0.1 – 0.01
   b-dependence, q_T dependence, angular dependence (!)

2) Initial conditions for small-x evolution:

nice fits to HERA $F_2(x,Q^2)$ from r.c. & NLO BK / JIMWLK evolution:

– r.c. BK: Albacete et al, PRD 80 (2009), EPJ-C 71 (2011),
  Mäntysaari & Schenke, 1806.06783

– coll. improved BK: Iancu et al, PLB 750 (2015), Ducloue et al, 1912.09196,
  Beuf, Hänninen, Lappi, Mäntysaari, 2007, 2008

However,

– ad hoc initial conditions at x_0 = 0.01, parameters adjusted so that BK fit is optimal

– how do they depend on x_0 ? (important for NLO BK; Ducloue et al, 1902.06637)

– no b, r*b dependence (sometimes modelled via MV)

Our goal is to relate (*x-dependent*) initial condition to light-front w.f. of proton, take advantage of "proton imaging" at EIC $\longrightarrow$ $N_0(\vec{r},\vec{b};x)$

The proton on the light front

The proton on the light front (valence quark Fock state; L.C. time $x^+ = 0$)

$$|P\rangle \;=\; \frac{1}{\sqrt{6}}\int\frac{dx_1dx_2dx_3}{\sqrt{x_1x_2x_3}}\delta(1-x_1-x_2-x_3)\int\frac{d^2k_1d^2k_2d^2k_3}{(16\pi^3)^3}16\pi^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)$$
$$\times\;\psi(k_1,k_2,k_3)\sum_{i_1,i_2,i_3}\epsilon_{i_1i_2i_3}|p_1,i_1;\,p_2,i_2;\,p_3,i_3\rangle$$

+ higher Fock states

P. Lepage & Brodsky, 1979 - ....
Brodsky, Pauli, Pinsky, PR (1998)

* Fock space amplitude ψ is gauge invariant, universal, and process independent

* encodes the non-perturbative structure of hadrons (QCD eigenstates)

$\longrightarrow$ Evaluate color charge $\overline{q}\gamma^+t^aq$ correlators explicitly !

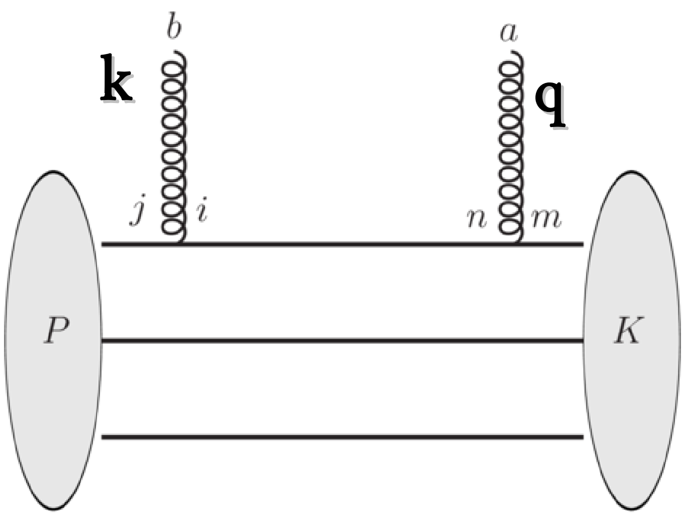
<ρ^aρ^b> correlator

$$\rho^a(\vec{x}) = g(t^a)_{ij} \int \mathrm{d}x^- \, \overline{\psi}_i(x)\gamma^+\psi_j(x)$$

$$\begin{aligned}
\langle\, \rho^a(\vec{q})\, \rho^b(\vec{k})\, \rangle_{K_\perp} \quad &= \quad g^2 \,\mathrm{tr}\; t^a t^b \int [dx_i][d^2p_i] \\
&\quad \Big\{ \psi^* \left(\vec{p}_1 + (1-x_1)\vec{K}_T, \vec{p}_2 - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T\right) \\
&\quad - \psi^* \left(\vec{p}_1 - \vec{q} - x_1\vec{K}_T, \vec{p}_2 - \vec{k} - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T\right)\Big\} \\
&\quad \psi\left(\vec{p}_1, \vec{p}_2, \vec{p}_3\right) \\
&\sim \quad g^2\, \delta^{ab}\, G_2(\vec{q}, \vec{k})
\end{aligned}$$

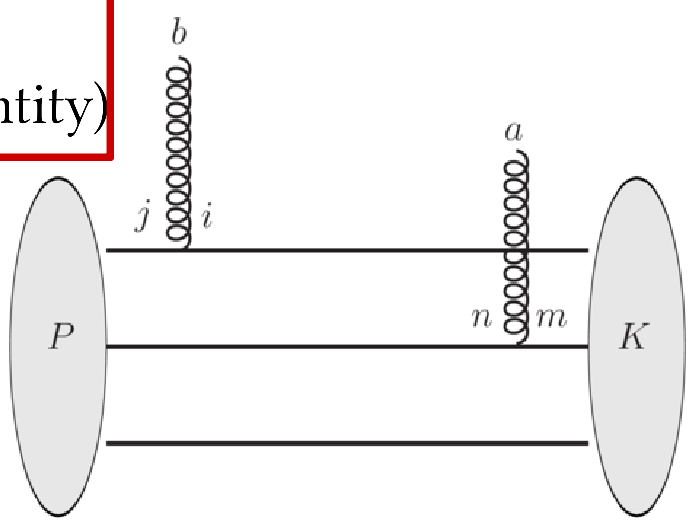
$$(\vec{q} + \vec{k} + \vec{K}_T = 0)$$

involve 1- and 2-particle GTMDs,
sum vanishes in IR
(color neutrality / Ward identity)



← dominates when $q^2, k^2 \gg K_T^2$

→ dominates when $\vec{q} \sim \vec{k} \sim -\vec{K}_T/2$

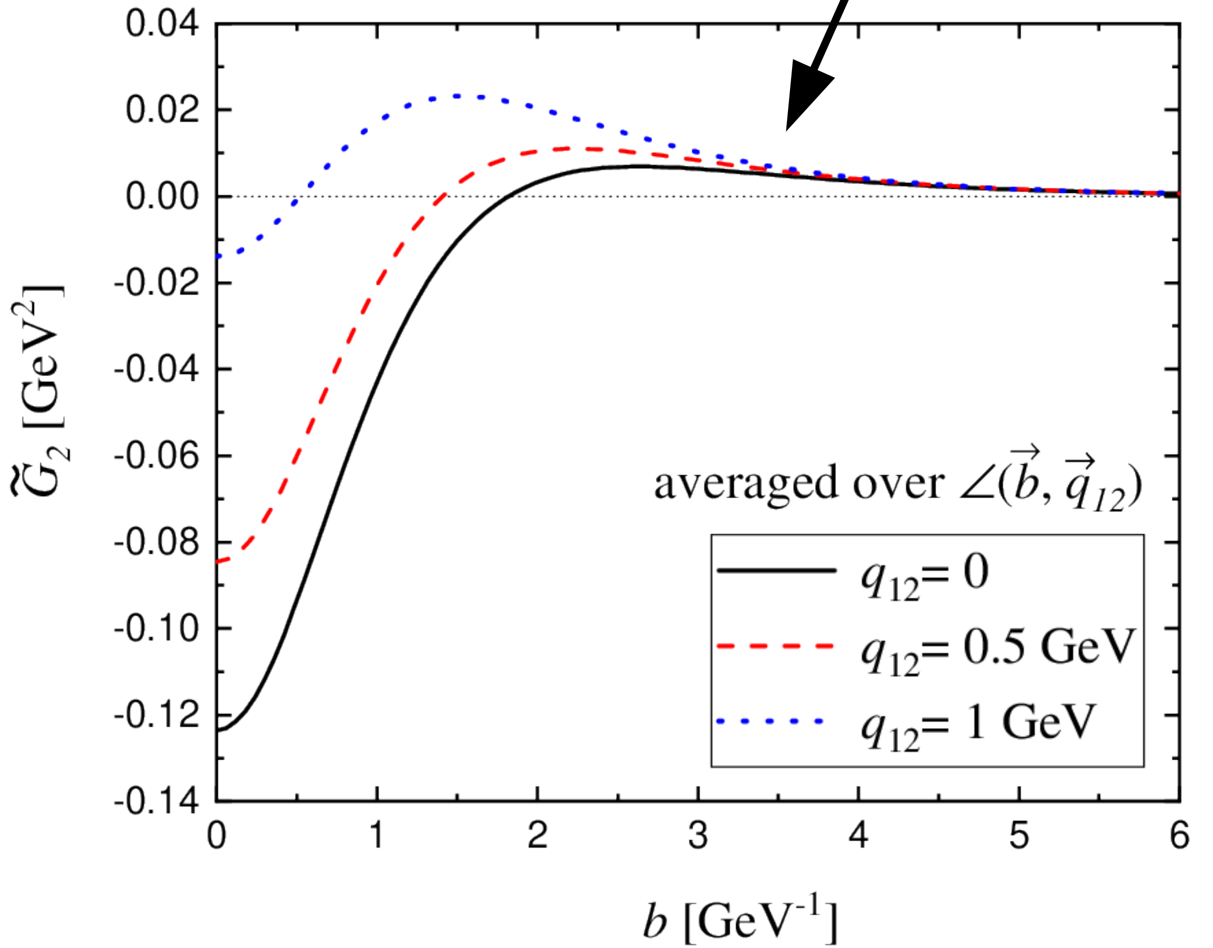


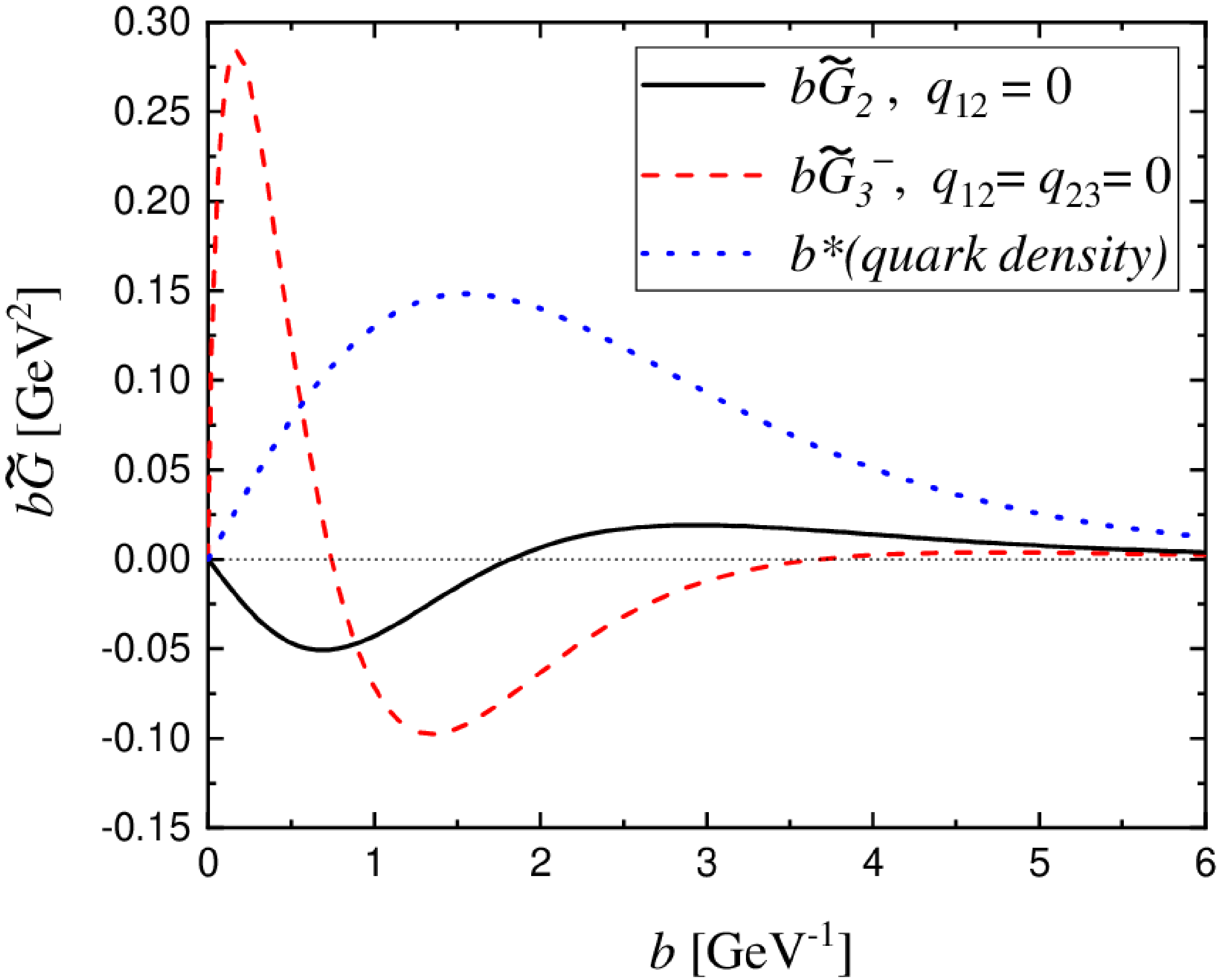
Numerical results for G_2 correlator at LO

(using Brodsky & Schlumpf LFwf)

- depends on b as well as on q_{12} = q1-q2 (resp. r)
- and on their relative angle
- note: small b is large K_T, “cat’s ears” dominates !

here it resembles a “shape function”

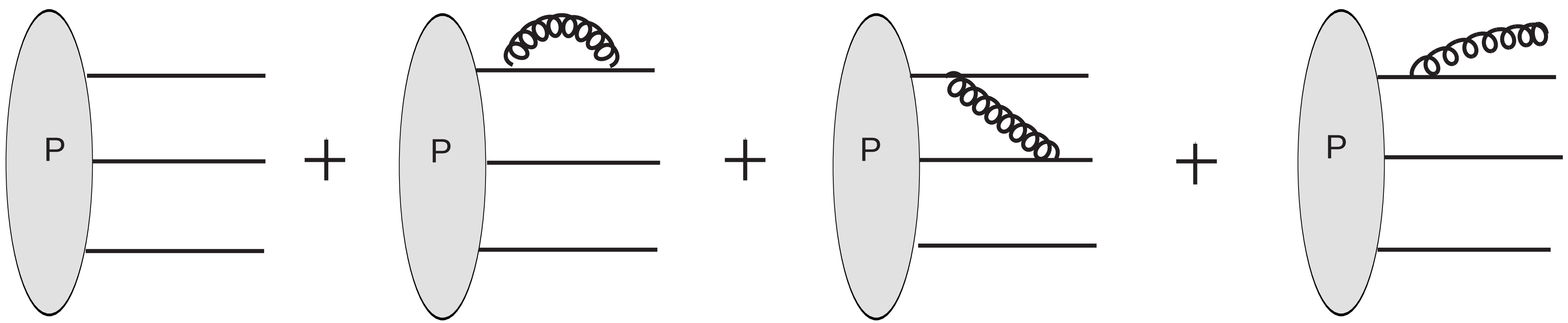




* G_2, G_3 are n-body GTMDs, compare to 1-body quark density (thickness func.)

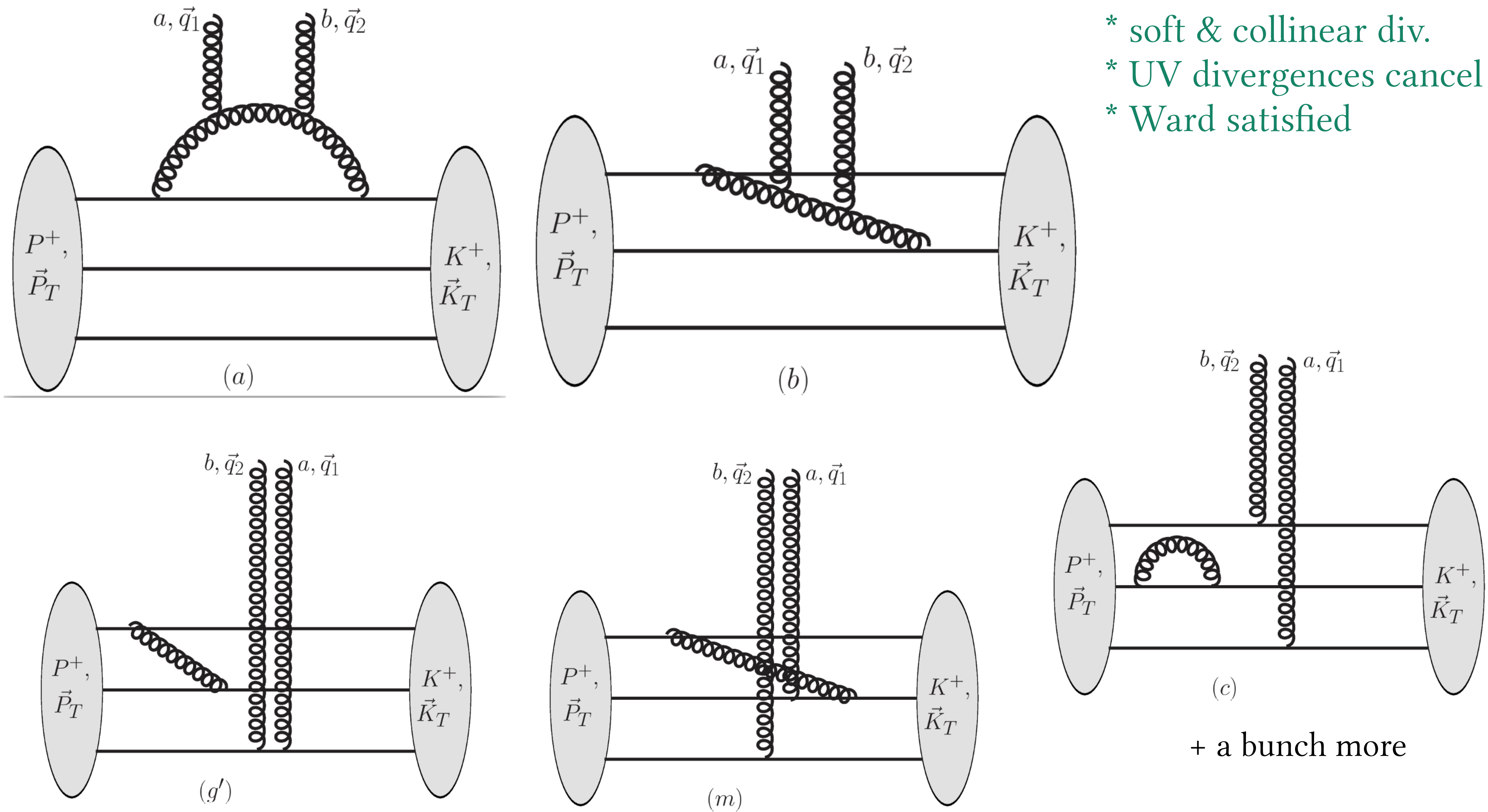
Now to $|P\rangle \sim \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle$

computed in perturbation theory, 1-gluon emission / exchange,
w/o employing small-x approximation



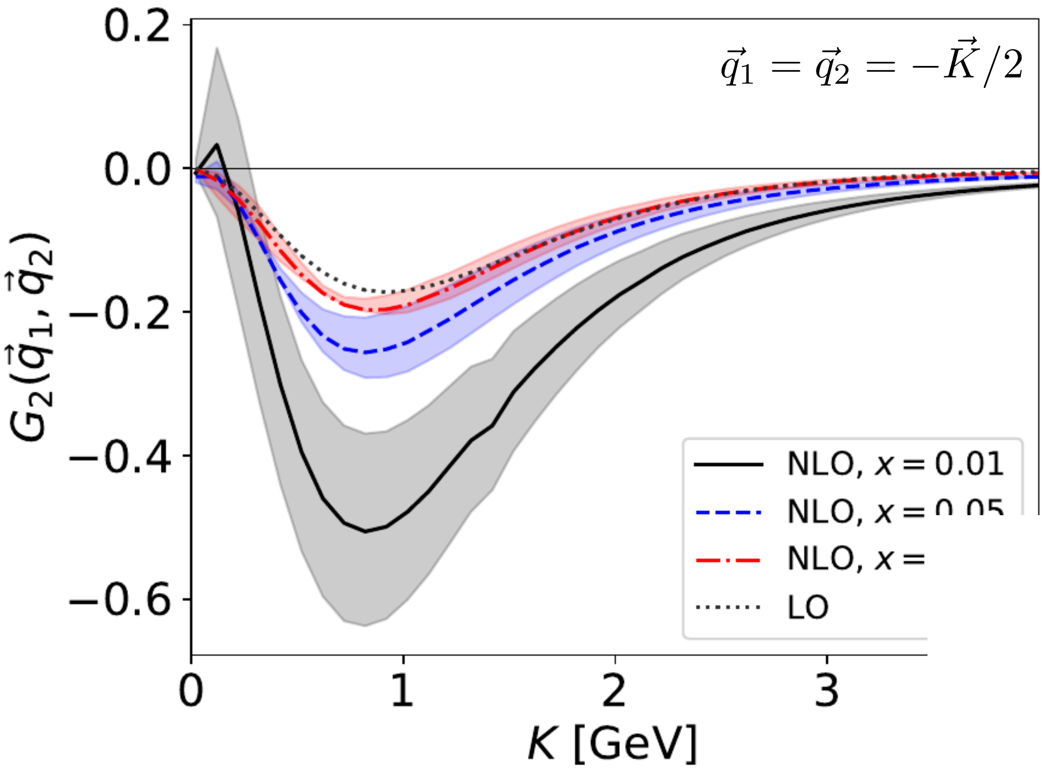
NLO color charge correlators

notes:
* soft & collinear div.
* UV divergences cancel
* Ward satisfied

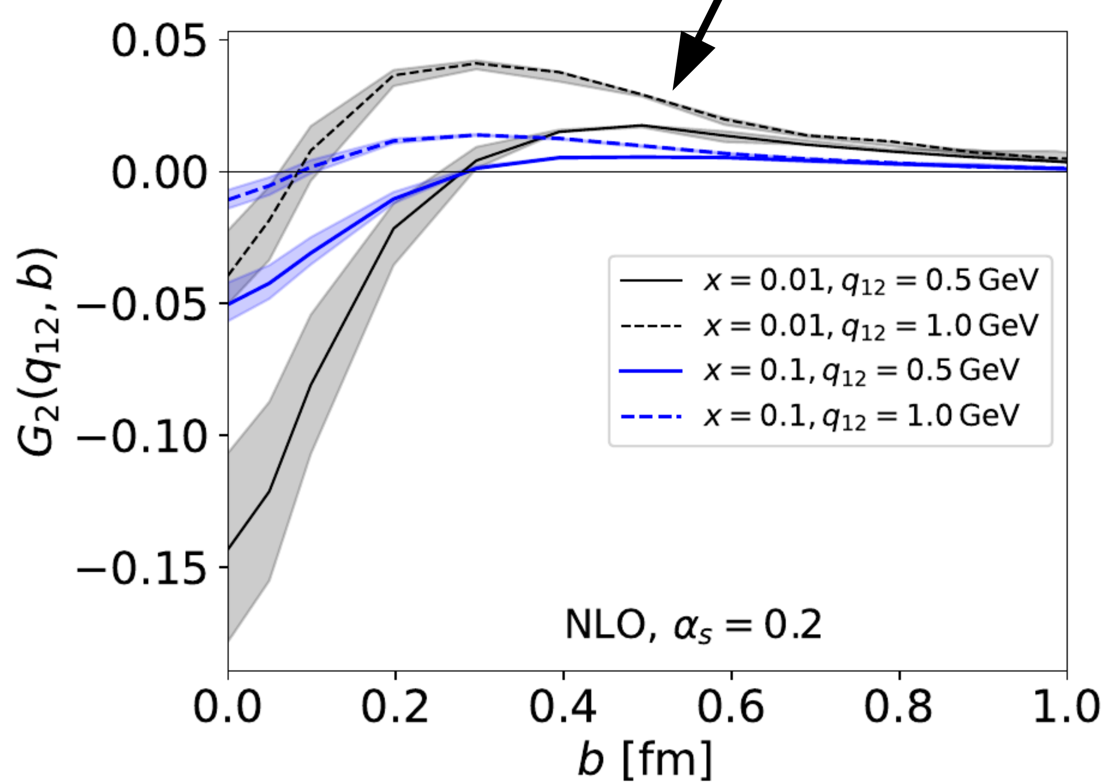


+ a bunch more

The effect of adding the gluon:



~ T_p(b) profile fct.

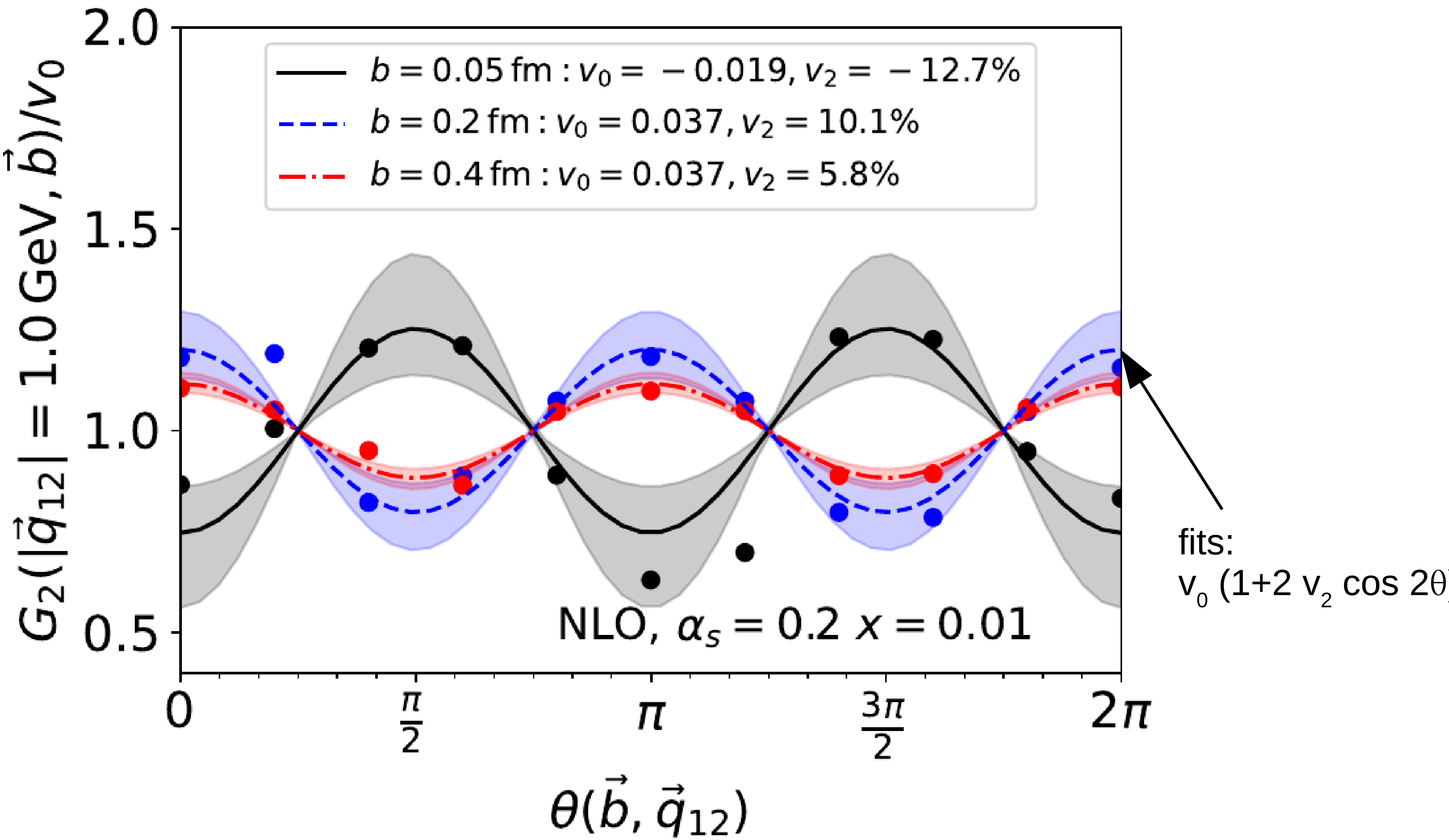


* modest effect at x=0.1

* big effect at x = 0.01 !

G_2 at $b \longrightarrow 0$ even more negative than LO cat's ears diagram

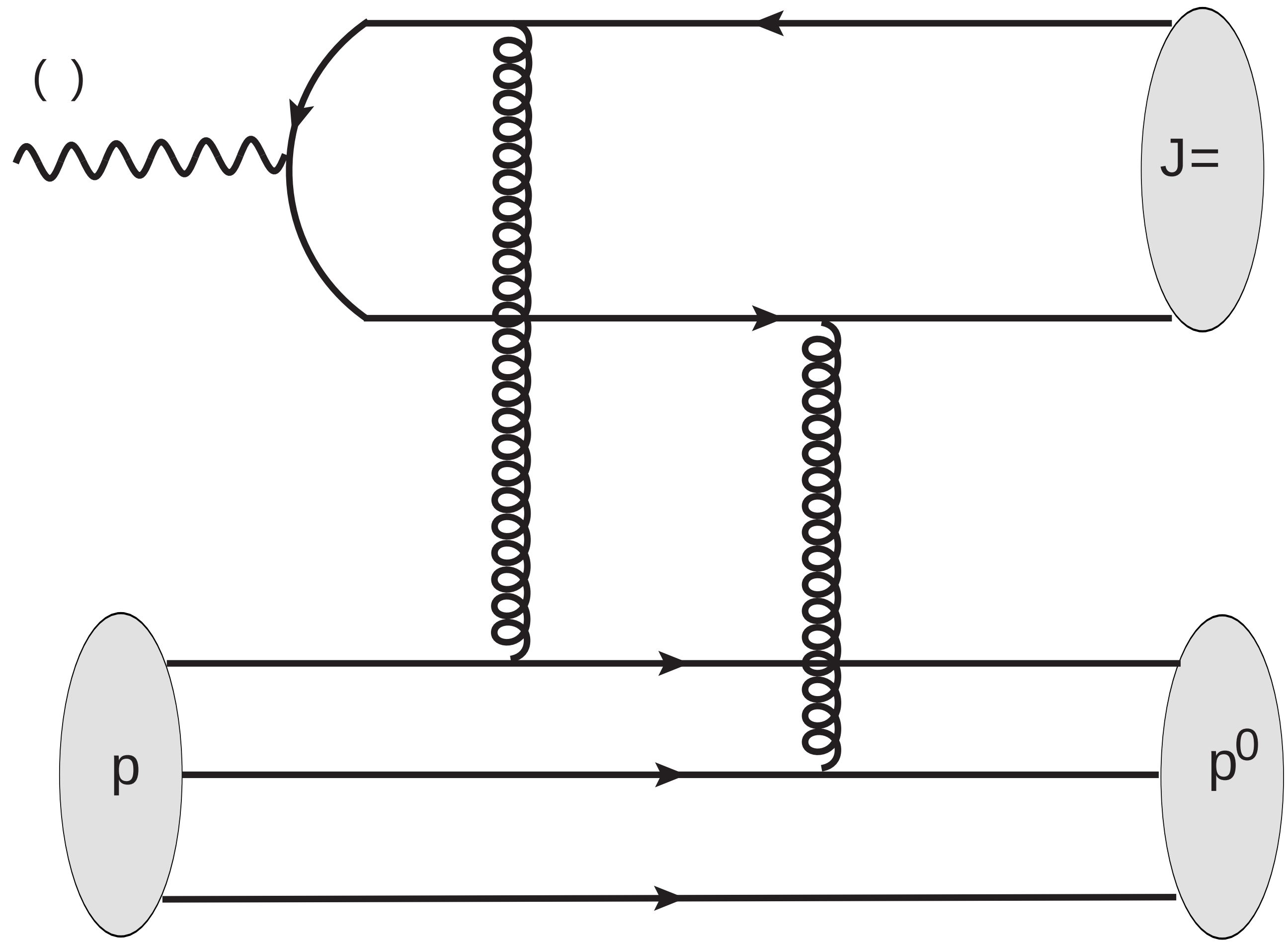
Color charge correlator exhibits angular dependence:



* band = variation of coll. regulator 0.1 – 0.4 GeV

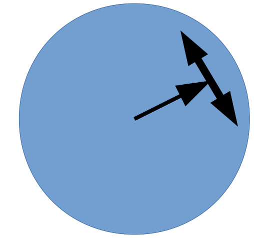
* sign and magnitude of <cos 2θ> = v_2 changes drastically with b, q_{12}

From color charge correlator to dipole scattering amplitude :



$$N(\vec{r},\vec{b}) \;=\; -g^4 C_F \int_{\vec{q},\vec{K}} \frac{\cos\vec{b}\cdot\vec{K}}{q^2\,(\vec{q}+\vec{K})^2}$$
$$\left[\cos\left(\vec{r}\cdot\left(\vec{q}+\frac{1}{2}\vec{K}\right)\right)-\cos\left(\frac{\vec{r}\cdot\vec{K}}{2}\right)\right]G_2(\vec{q},-\vec{q}-\vec{K})$$

Azimuthal anisotropy of dipole scatt. amplitude

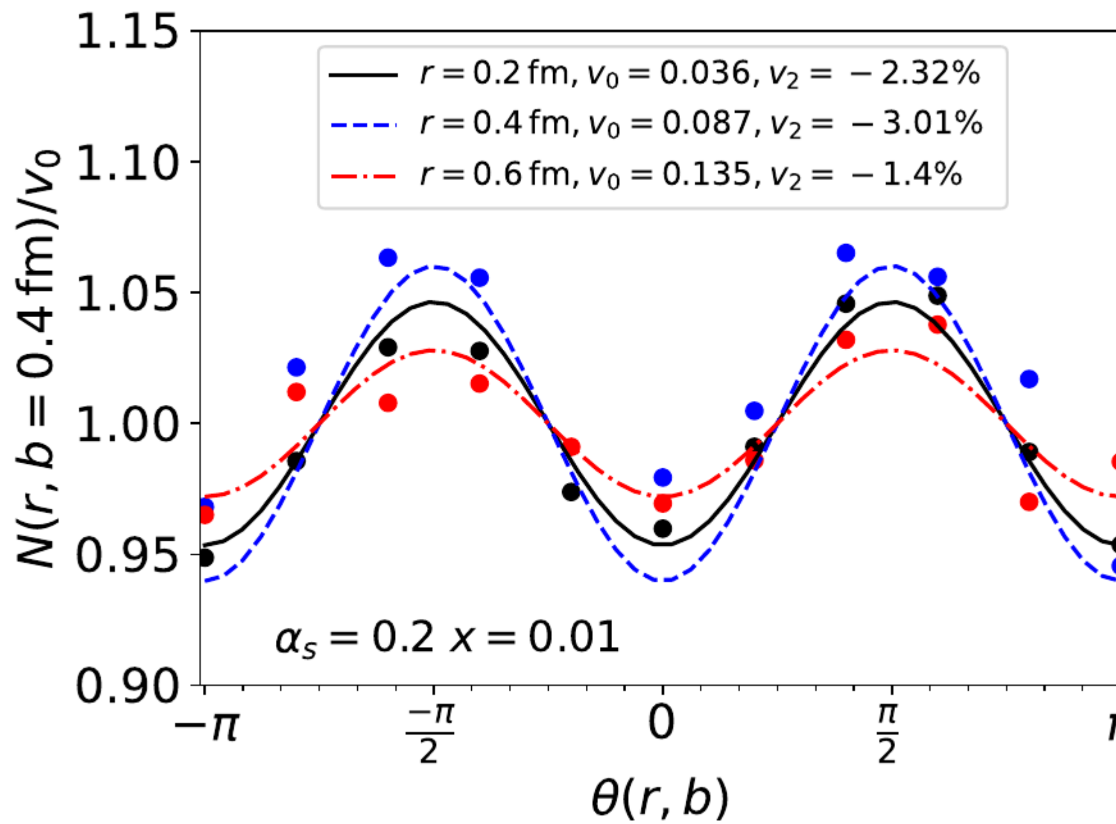


If the color charge correlator is simply proportional to the "proton shape function" $\langle \rho^a(\vec{x})\, \rho^b(\vec{y}) \rangle \sim \delta^{ab}\, \mu^2(\vec{b})\, \delta(\vec{x}-\vec{y})$

then $N(\vec{r},\vec{b}) = f(r,b)\ \left(1 + c\, r^2\, b^2\ \cos 2\theta\right)$
at small r, b; with $c > 0$!

e.g. A. Rezaeian & E. Iancu, 1702.03943
Kovner & Lublinsky, 1211.1928
E. Levin & A. Rezaeian, 1105.3275

- Recall that "cat's ears" diagram involves 2-body correlations and breaks <ρρ> ~ T_p(b)
- v2 from $N(\vec{r},\vec{b})$ is *negative*
- *magnitude of v2 is not proportional to r^2*



Summary

* color charge correlators $<\rho^2>$, $<\rho^3>$ etc provide insight into n-body correlations in the proton.
  Many-body diagrams dominant at high |t| or small b

* $<\rho^2>$ very different from MV model at small b (cat's ears !)

* Explicit relations to LFwf of the proton

* initial conditions for (NLO) BK incl. $b, \vec{r}\cdot\vec{b}$, x dependence
  (and C-odd contribution from 3g exchange)

* Angular dependence of $N(\vec{r},\vec{b})$ appears to be quite different from models assuming $<\rho^2> \sim T_p(b)$;

  $v_2 = \langle \cos 2\theta_{\vec{r},\vec{b}} \rangle < 0$!

Backup Slides

NLO BK: evolution in terms of target rapidity (i.e. in x) :

B. Ducloué et al: 1902.06637

$$\begin{aligned}
\frac{\partial \bar{S}_{\boldsymbol{xy}}(\eta)}{\partial \eta} &= \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\Theta(\eta-\delta_{\boldsymbol{xyz}})\left[\bar{S}_{\boldsymbol{xz}}(\eta-\delta_{\boldsymbol{xz};r})\bar{S}_{\boldsymbol{zy}}(\eta-\delta_{\boldsymbol{zy};r})-\bar{S}_{\boldsymbol{xy}}(\eta)\right] \\
&- \frac{\bar{\alpha}_s^2}{4\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\ln\frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\left[\bar{S}_{\boldsymbol{xz}}(\eta)\bar{S}_{\boldsymbol{zy}}(\eta)-\bar{S}_{\boldsymbol{xy}}(\eta)\right] \\
&+ \frac{\bar{\alpha}_s^2}{2\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{z}\,\mathrm{d}^2\boldsymbol{u}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{u})^2(\boldsymbol{u}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[\ln\frac{(\boldsymbol{u}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}+\delta_{\boldsymbol{uy};r}\right]\bar{S}_{\boldsymbol{xu}}(\eta)\left[\bar{S}_{\boldsymbol{uz}}(\eta)\bar{S}_{\boldsymbol{zy}}(\eta)-\bar{S}_{\boldsymbol{uy}}(\eta)\right] \\
&+ \bar{\alpha}_s^2 \times \text{“regular”}, \qquad (6.4)
\end{aligned}$$

non-local in rapidity, involves S at rapidities $\eta < \eta_0 = \log 1/x_0$!

“Impact parameter dependent MV”, version by Kovner & Skokov

arXiv:1805.09297

A. Projectile averaging

We will perform the averaging over the projectile charge density ρ using the MV model, which is equivalent to pairwise Wick contraction of ρ with the basic “propagator”

ok

$$\left\langle \rho^a(\underline{p})\rho^b(\underline{k})\right\rangle_\rho = \mu^2(\underline{p},\underline{k})\delta^{ab}\,. \tag{5}$$

In the original MV model the function μ^2 is taken to be proportional to $\delta^2(\underline{p}+\underline{q})$. This form assumes translational invariance in the transverse plane. Since we wish to explore the dependence on the finite size and shape of the projectile, we generalize it in the following way

$G_2(p,k)$ does not factorize

$$\mu^2(\underline{p},\underline{k}) = \mu^2(\underline{p}+\underline{k})F\left(\frac{(\underline{p}-\underline{k})^2}{\Lambda^2}\right).$$

This factorized form albeit not generic, but is intuitive and we believe captures the main features of the projectile charge distribution. The function $\mu^2(\underline{p}+\underline{k})$ arises as a Fourier transform of the charge density in the transverse plane

$$\mu^2(\underline{p}) = \int d^2b e^{i\underline{p}\underline{b}}\mu^2(\underline{b}). \tag{7}$$

Thus the coordinate space density profile is directly reflected in $\mu^2(\underline{p})$. Naturally, we expect this function to vanish for momenta much greater than the inverse of the linear dimension of the projectile R. The spatial eccentricity will also be directly encoded in μ^2. The numerical calculations are performed for a Gaussian profile

$$\mu^2(\underline{b}) = Ce^{-\frac{b_1^2}{a^2R^2}}e^{-\frac{a^2b_2^2}{R^2}}\,. \tag{8}$$

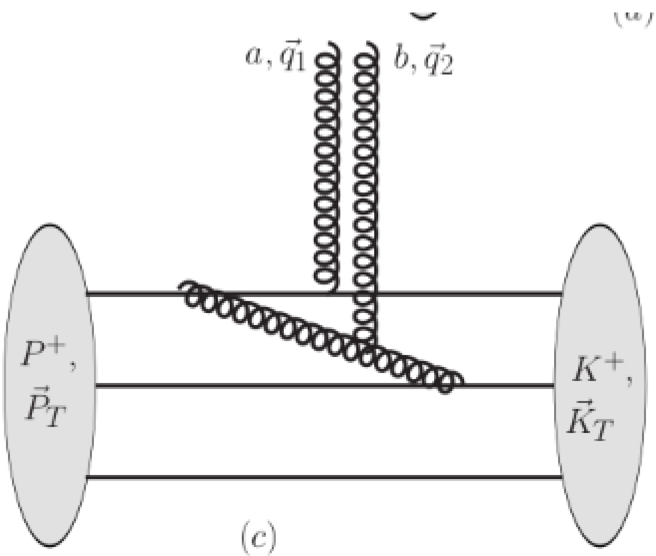
The normalization constant C is fixed by

should be =0 (for p=k) due to Ward id.

$$\int d^2b\mu^2(\underline{b}) = S_\perp\mu_0^2\,. \tag{9}$$

Note that this way of introducing eccentricity *preserves* the area of the projectile, and therefore the single inclusive gluon production cross section.

Message: we should not identify charge correlation with 1-body density !



$$
\text{fig. 3c} = \frac{g^4}{12 \cdot 16\pi^3} \operatorname{tr} T^a T^b \int [\mathrm{d}x_i] \int [\mathrm{d}^2 k_i] \; \Psi_{qqq}(x_1, \vec{k}_1; x_2, \vec{k}_2; x_3, \vec{k}_3)
$$

$$
\int_x^{\min(x_1, 1-x_2)} \frac{\mathrm{d}x_g}{x_g} \left(1 - \frac{z_1 + z_2}{2} + \frac{z_1 z_2}{6}\right) \sqrt{\frac{x_1}{x_1 - x_g}} \sqrt{\frac{x_2}{x_2 + x_g}}
$$

$$
\int \mathrm{d}^2 k_g \frac{z_1 \vec{p}_1 - \vec{k}_g}{\left(z_1 \vec{p}_1 - \vec{k}_g\right)^2} \cdot \frac{z_2 \vec{p}_2 - (1 - z_2)(\vec{k}_g - \vec{q}_2)}{\left(z_2 \vec{p}_2 - (1 - z_2)(\vec{k}_g - \vec{q}_2)\right)^2}
$$

$$
\Psi^*_{qqq}(x_1 - x_g, \vec{k}_1 + x_1 \vec{q} - \vec{q}_1 - \vec{k}_g + x_g \vec{K}; x_2 + x_g, \vec{k}_2 + x_2 \vec{q} - \vec{q}_2 + \vec{k}_g - x_g \vec{K}; x_3, \vec{k}_3 + x_3 \vec{q})
$$

(see arXiv:2010.11245 for details and other diagrams)

Aside: <$\rho^a\rho^b\rho^c$> correlator (C odd part, LO)

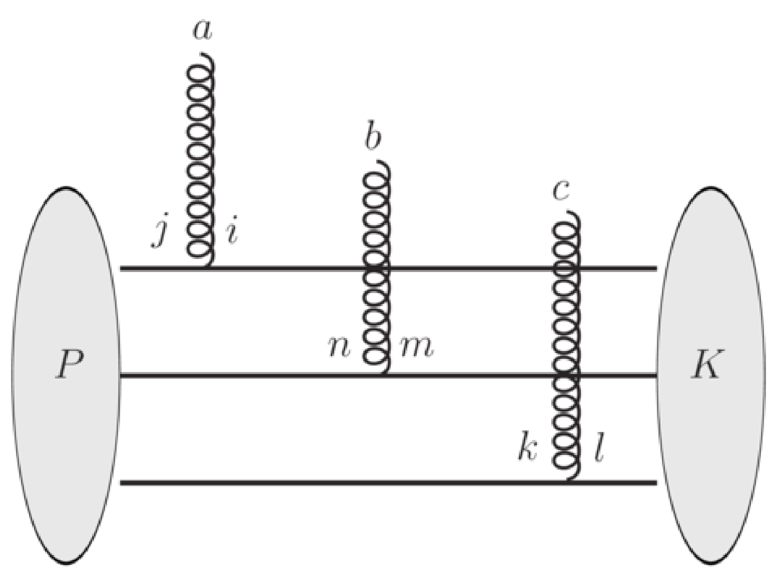
does not vanish (color charge fluct. <u>not Gaussian</u>) :

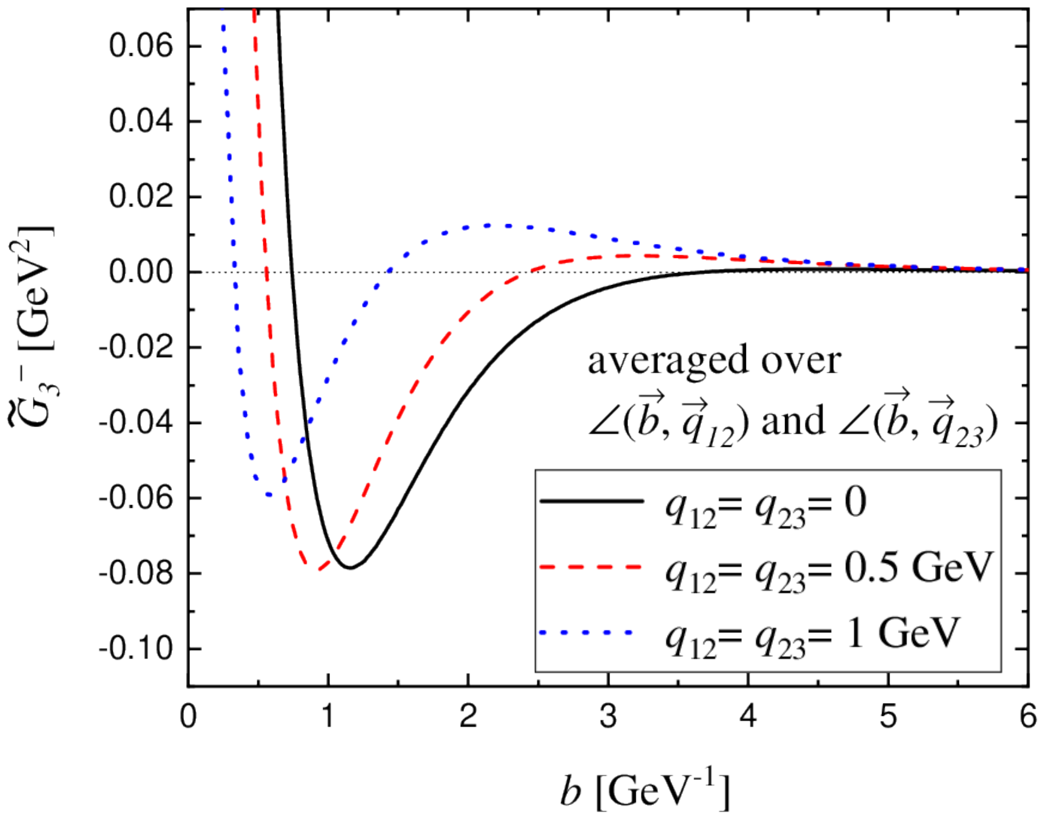
$$\left\langle \rho^a(\vec{q}_1)\, \rho^b(\vec{q}_2)\, \rho^c(\vec{q}_3)\, \right\rangle_{K_\perp} \Big|_{\mathcal{C}=-} \equiv \frac{g^3}{4}\, d^{abc}\, G_3^-(\vec{q}_1, \vec{q}_2, \vec{q}_3)$$

$$\begin{aligned} G_3^-(\vec{q}_1, \vec{q}_2, \vec{q}_3) = & \int [dx_i][dp_i] \\ & \Big[\psi^*(\vec{p}_1 + (1-x_1)\vec{K}_\perp, \vec{p}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1 - \vec{q}_1 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + (1-x_2)\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1 + \vec{q}_2 + (1-x_1)\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1 - \vec{q}_1 - \vec{q}_2 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + \vec{q}_2 + (1-x_2)\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\ & +2\,\psi^*(\vec{p}_1 - \vec{q}_1 - x_1\vec{K}_\perp, \vec{p}_2 + \vec{q}_1 + \vec{q}_2 + (1-x_2)\vec{K}_\perp, \vec{p}_3 - \vec{q}_2 - x_3\vec{K}_\perp) \\ & \Big]\, \psi(\vec{p}_1, \vec{p}_2, \vec{p}_3) \end{aligned}$$

* 1-, 2- and 3-particle GTMDs,
  sum vanishes when either $q_i \longrightarrow 0$
  (Ward identities)

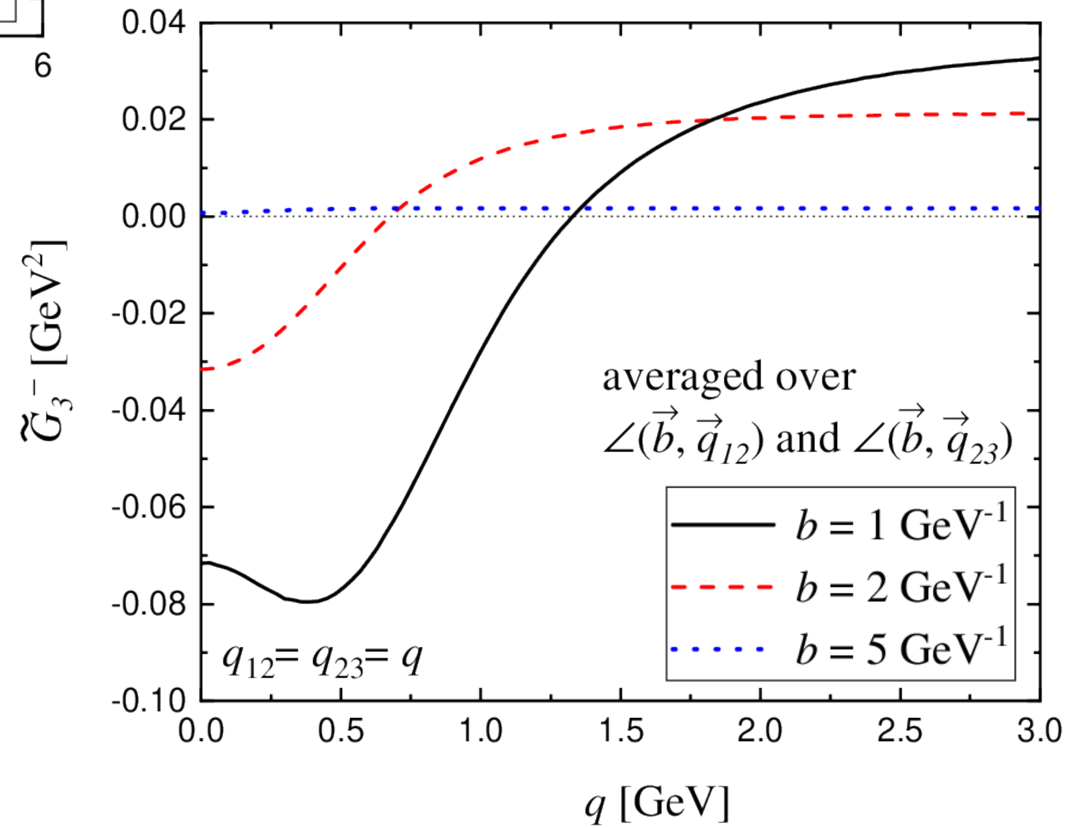
* "3-body" diagrams not (power-) suppressed
  when $\vec{q}_1 \sim \vec{q}_2 \sim \vec{q}_3 \sim -\vec{K}_T/3 \gg \Lambda_{\rm QCD}$
  but actually dominant !





- magnitude for generic b, q comparable to G_2
- diverges for $b \longrightarrow 0$ due to contribution from high K_T

$$\left(\int \frac{\mathrm{d}^2 K_T}{K_T^2} \; e^{-i\vec{b}\cdot\vec{K}_T} \right)$$



<ρ^{a}ρ^{b}ρ^c> correlator (C even part)

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\,\right\rangle_{K_\perp}\Big|_{\mathcal{C}=+} \quad\equiv\quad \frac{g^3}{4}\,if^{abc}\,G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$\begin{aligned}
G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3) \quad &= \quad \int [dx_i] \int [d^2p_i] \\
&\Big[\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&-\psi^*(\vec{p}_1-\vec{q}_2-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_1-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&+\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&-\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_3-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&\Big]\,\psi(\vec{p}_1,\vec{p}_2,\vec{p}_3) \\
= \quad & G_2(\vec{q}_1+\vec{q}_2,\vec{q}_3) - G_2(\vec{q}_1+\vec{q}_3,\vec{q}_2) + G_2(\vec{q}_1,\vec{q}_2+\vec{q}_3)
\end{aligned}$$

(like Reggeized 2-gluon exchange)

e.g. C. Ewerz, hep-ph/0103260, hep-ph/0306137

G_3^+ vanishes when $q_1 \longrightarrow 0$ or $q_3 \longrightarrow 0$ but not for $q_2 \longrightarrow 0$

<ρ^{a}ρ^{b}ρ^{c}ρ^d> correlator

$$
\begin{aligned}
&\left\langle \rho^a(\vec q_1)\,\rho^b(\vec q_2)\,\rho^c(\vec q_3)\,\rho^d(\vec q_4)\,\right\rangle = g^4 \int [dx_i] \int [d^2p_i]\,\psi(\vec p_1,\vec p_2,\vec p_3)\\
&\Big\{ \mathrm{tr}\, t^a t^b t^c t^d \;\; \psi^*(\vec p_1 + (1-x_1)\vec K_T, \vec p_2 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^b\, \mathrm{tr}\, t^c t^d - \mathrm{tr}\, t^a t^b t^c t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_2 - x_1 \vec K_T, \vec p_2 - \vec q_3 - \vec q_4 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^c\, \mathrm{tr}\, t^b t^d - \mathrm{tr}\, t^a t^c t^b t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_3 - x_1 \vec K_T, \vec p_2 - \vec q_2 - \vec q_4 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^d\, \mathrm{tr}\, t^b t^c - \mathrm{tr}\, t^a t^d t^b t^c\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_4 - x_1 \vec K_T, \vec p_2 - \vec q_2 - \vec q_3 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&- \mathrm{tr}\, t^a t^b t^c t^d \;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_2 - \vec q_3 - x_1 \vec K_T, \vec p_2 - \vec q_4 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&- \mathrm{tr}\, t^a t^b t^c t^d \;\; \psi^*(\vec p_1 - \vec q_1 - x_1 \vec K_T, \vec p_2 - \vec q_2 - \vec q_3 - \vec q_4 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&- \mathrm{tr}\, t^a t^b t^d t^c \;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_2 - \vec q_4 - x_1 \vec K_T, \vec p_2 - \vec q_3 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&- \mathrm{tr}\, t^a t^c t^d t^b \;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_3 - \vec q_4 - x_1 \vec K_T, \vec p_2 - \vec q_2 - x_2 \vec K_T, \vec p_3 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^b t^c t^d + \mathrm{tr}\, t^a t^b t^d t^c - \mathrm{tr}\, t^a t^b\, \mathrm{tr}\, t^c t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_2 - x_1 \vec K_T, \vec p_2 - \vec q_3 - x_2 \vec K_T, \vec p_3 - \vec q_4 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^c t^b t^d + \mathrm{tr}\, t^a t^c t^d t^b - \mathrm{tr}\, t^a t^c\, \mathrm{tr}\, t^b t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_3 - x_1 \vec K_T, \vec p_2 - \vec q_2 - x_2 \vec K_T, \vec p_3 - \vec q_4 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^d t^b t^c + \mathrm{tr}\, t^a t^d t^c t^b - \mathrm{tr}\, t^a t^d\, \mathrm{tr}\, t^b t^c\right)\;\; \psi^*(\vec p_1 - \vec q_1 - \vec q_4 - x_1 \vec K_T, \vec p_2 - \vec q_2 - x_2 \vec K_T, \vec p_3 - \vec q_3 - x_3 \vec K_T)\\
&+ \left(\mathrm{tr}\, t^a t^b t^c t^d + \mathrm{tr}\, t^a t^d t^b t^c - \mathrm{tr}\, t^a t^d\, \mathrm{tr}\, t^b t^c\right)\;\; \psi^*(\vec p_1 - \vec q_1 - x_1 \vec K_T, \vec p_2 - \vec q_2 - \vec q_3 - x_2 \vec K_T, \vec p_3 - \vec q_4 - x_3 \vec K_T)\\
&+ \left(t^a t^b t^c t^d + t^a t^c t^d t^b - t^a t^b\, t^c t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - x_1 \vec K_T, \vec p_2 - \vec q_2 - x_2 \vec K_T, \vec p_3 - \vec q_3 - \vec q_4 - x_3 \vec K_T)\\
&+ \left(t^a t^b t^d t^c + t^a t^c t^b t^d - t^a t^c\, t^b t^d\right)\;\; \psi^*(\vec p_1 - \vec q_1 - x_1 \vec K_T, \vec p_2 - \vec q_2 - \vec q_4 - x_2 \vec K_T, \vec p_3 - \vec q_3 - x_3 \vec K_T)\Big\}\,,
\end{aligned}
$$

where $\vec K_T = -(\vec q_1 + \vec q_2 + \vec q_3 + \vec q_4)$

Note: $\neq \langle \rho^a(\vec q_1)\,\rho^b(\vec q_2)\rangle\,\langle \rho^c(\vec q_3)\,\rho^d(\vec q_4)\rangle + \mathrm{perm.}$

Model LFwf for the proton (Brodsky & Schlumpf, PLB 329, 1994)

$$\begin{aligned}\psi_{\mathrm{H.O.}}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\mathrm{H.O.}}\exp(-\mathcal{M}^2/2\beta^2)\ ,\\ \psi_{\mathrm{Power}}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\mathrm{Power}}(1+\mathcal{M}^2/\beta^2)^{-p}\ .\end{aligned}$$

$$\mathcal{M}^2 = \sum_{i=1}^{3}\frac{\vec{k}^2_{\perp i}+m^2}{x_i}$$

m = 0.26 GeV, β = 0.55 for H.O. wf

m = 0.263, β = 0.607, p = 3.5 for PWR wf

With these parameters they fit:

- proton radius $R^2 = -6\ \frac{dF_1(Q^2)}{dQ^2}\Big|_{Q^2=0} = (0.76\ \mathrm{fm})^2$
- proton / neutron magnetic moments $1+F_2(Q^2\to 0) = 2.81\ /\ -1.66$
- axial vector coupling g_A = 1.25